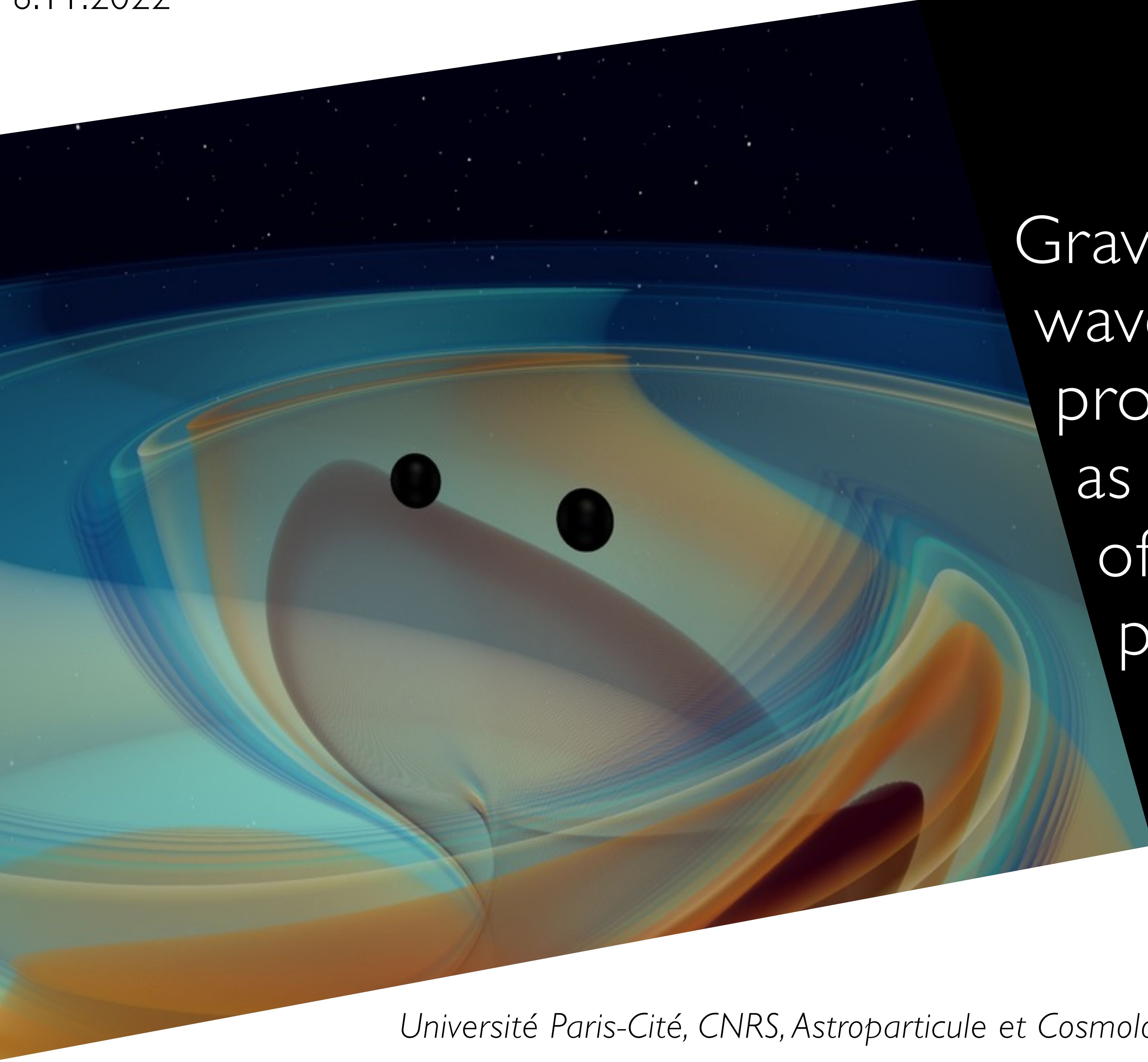
4th Summit on Exploring the Dark Side of the Universe (EDSU2022)
8.11.2022



Gravitational waves propagation as a probe of fundamental physics

Leïla Haegel

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Gravitational waves detection: a summary

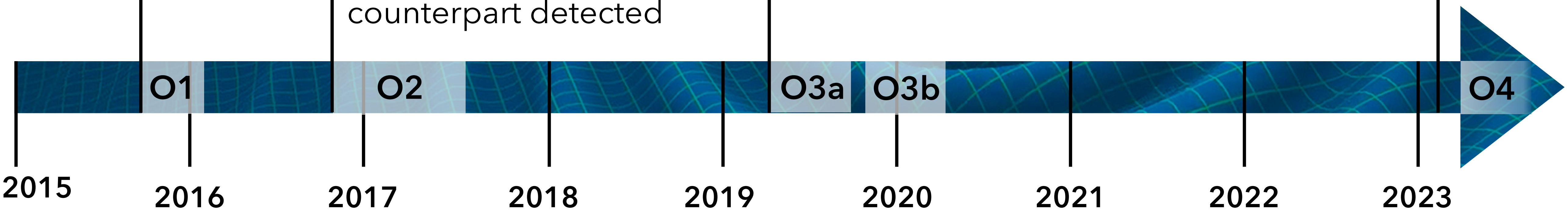
arXiv
1811.12907
2108.01045
2111.03606

3 GW detection during O1
- First direct detection of GW
- From coalescing binary systems of black holes

8 GW detection during O2
- 1 coalescing binary system of neutron stars: electromagnetic counterpart detected

79 GW detection during O3
- 44 during O3a, including 1 confirmed binary system of neutron stars
- 35 during O3b, including 2 confirmed systems of neutron stars - black holes
- No electromagnetic counterpart

O4 to start in 2023
- Duration: 1 year



90 GW detections reported

Coalescence of black holes and neutron stars

1 multimessenger event (GW + EM observation)

Mass range 1.2 → 107 $M_\odot$ (stellar)

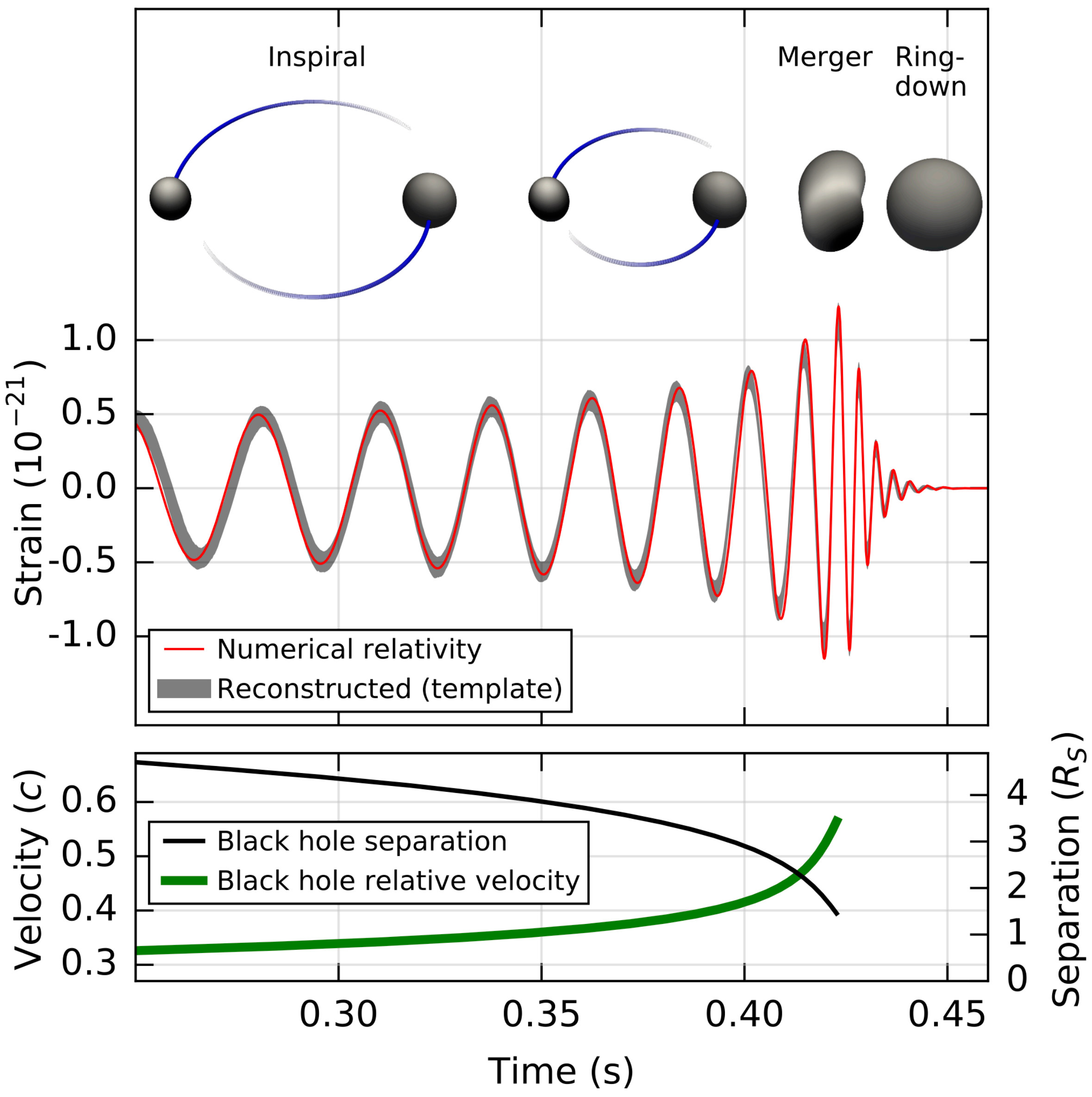
Distance range 40 Mpc → 8 Gpc (z → 1.14)

Introduction

- **Gravitational waves (GW) enable to test fundamental physics in the gravitational sector**
  ↳ complementary to tests with solar system, pulsars, gravitational lensing…

- **Several approaches to test for deviation from General Relativity**
  ↳ consistency tests
  ↳ search for phenomena impacting GW generation
  ↳ search for exotic compact objects…

- **New physics may affect the propagation of GW**
  ↳ gravitational coupling
  ↳ overall effect on the signal (independent of the source)
  ↳ cumulative effect
  ↳ dynamical regime at large distance due to Universe expansion



Gravitational waves propagation

General relativity (GR) case:

$$h''_{ij} + 2\,H\,h_{ij} + c^2\,k^2\,h_{ij} = 0$$

Modified GR

Nishizawa, Phys. Rev. D 97, 104037 (2018)

$$h''_{ij} + (2+\nu)\,H\,h_{ij} + (c_T^2 k^2 + a^2\mu^2)\,h_{ij} = a^2\,\Gamma\,\gamma_{ij}$$

GW friction

Amplitude does not scale as 1/distance

Speed of gravity

Speed of GW $\neq c$

Mass of graviton

Non-0 graviton mass

Polarisation mixing

h_+ and $h_\times$

Can be probed with multimessenger events

Can be probed from GW signal (pattern & polarisation)

Gravitational waves propagation

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Mass of graviton

Non-0 graviton mass

Lorentz invariance violation induced GW dispersion

- **GW from the coalescence of compact binaries have a characteristic signal**

$$h(t) = |h(t)|\; e^{-i\,(\omega(t)\,+\phi_c)}$$

with $|h(t)|, \omega(t)$ increasing until merger

- **Breaking of Lorentz symmetry & massive graviton modify the energy relation:**

$$E^2 = p^2c^2 + m_g^2c^4 + \boxed{\mathbb{A}p^\alpha c^\alpha}$$

- The extra term in $\mathbb{A}$ creates a frequency-dependent dispersion of the GW

$$\tilde{h}(f) = |\tilde{h}(f)|\; e^{-i\,(\tilde{\phi}_{GR}(f)\,+\,\boxed{\delta\tilde{\phi}(f)})}$$

- The dispersion is:
  - isotropic
  - polarisation independent
  - possibly mapped to alternative theories of gravitation

[Mirshekari, Yunes & Will
Phys. Rev. D85: 024041 (2012)]

IN A NUTSHELL

Question:

- Are gravitational waves dispersed due to Lorentz Invariance violation ?

Test:

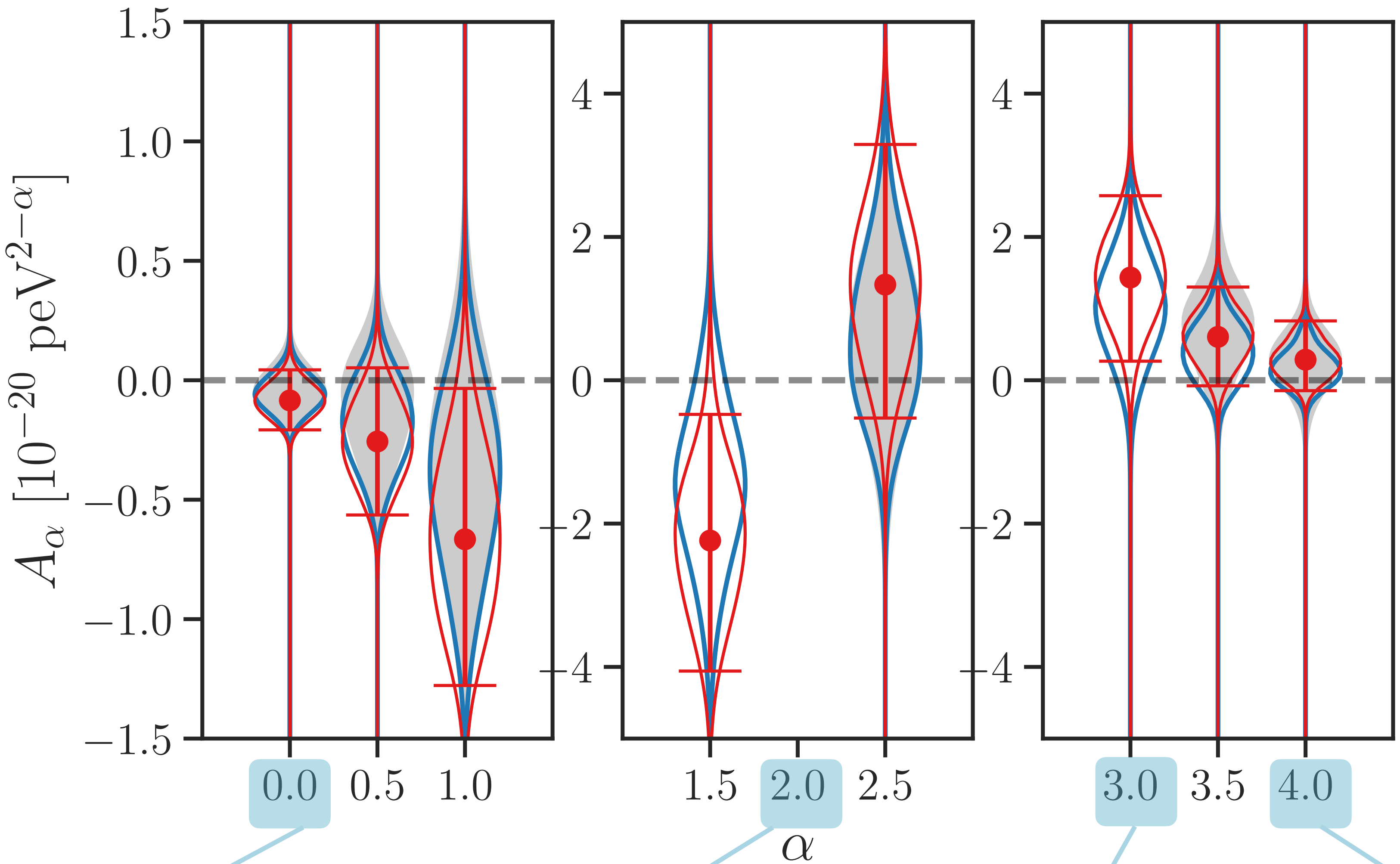
- Compare modified waveform signals with inference of source and $\mathbb{A}$ parameters

Constraints on Lorentz invariance violation

— w/ GW200219_094415 & GW200225_060421
— w/o GW200219_094415 & GW200225_060421

2 events presenting a bias
with lowest p-value in residual tests

$$E^2 = p^2c^2 + m_g^2c^4 + \boxed{\mathbb{A}p^\alpha c^\alpha}$$



LVK, arxiv:2112.06861

massive graviton
$m_g < 1.76 \cdot 10^{-23} eV/c^2$

degeneracy with
coalescence time

Doubly Special
Relativity

extra dimensions /
Hořava-Lifshitz /
non commutative geometry

Gravitational waves propagation

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Polarisation mixing

h_+ and $h_\times$

EFT for spacetime symmetry breaking

- **Breaking of spacetime symmetries (CPT, Lorentz)** can be studied with an effective field theory (EFT) formalism (Standard Model Extension, or SME):

$$\mathscr{L} = \mathscr{L}_{GR} + \boxed{\frac{1}{4} h_{\mu\nu} \left(\hat{s}^{\mu\nu\rho\sigma} + \hat{q}^{\mu\nu\rho\sigma} + \hat{k}^{\mu\nu\rho\sigma}\right) h_{\rho\sigma}}$$

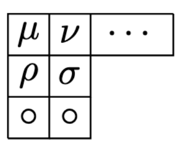
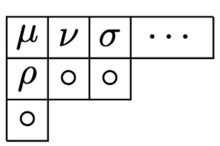
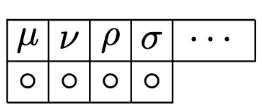
Tableau	Operator $\widehat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$	CPT	d	Number
	$s^{(d)\mu\rho\circ\nu\sigma\circ\circ^{d-4}}$	even	even, ≥ 4	$(d-3)(d-2)(d+1)$
	$q^{(d)\mu\rho\circ\nu\circ\sigma\circ\circ^{d-5}}$	odd	odd, ≥ 5	$\frac{5}{2}(d-4)(d-1)(d+1)$
	$k^{(d)\mu\circ\nu\circ\rho\circ\sigma\circ\circ^{d-6}}$	even	even, ≥ 6	$\frac{5}{2}(d-5)d(d+1)$

TABLE I: Gauge-invariant operators in the quadratic gravitational action.

Impact GW momentum:
dispersion effect,
this study

Kostelecky & Mewes, Phys. Lett. B757:510-514 (2016)

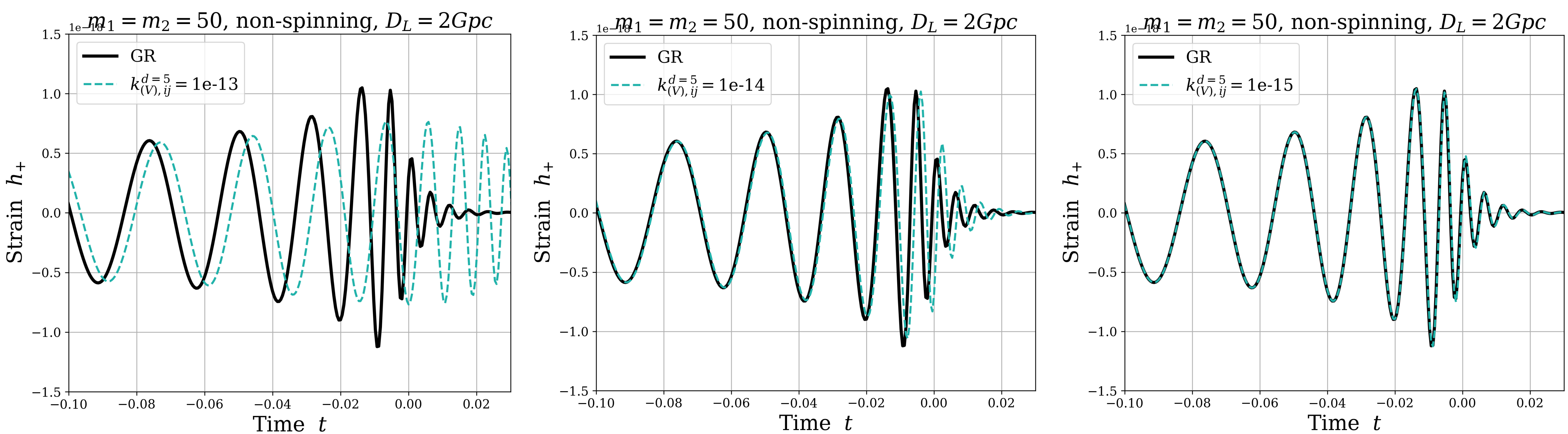
Polarisation-dependent, anisotropic dispersion

- The coupling of new fields with the metric modified the GW propagation in a frequency-dependent, polarisation-dependent, anisotropic way:

$$h^{SME}_{+,\times} = e^{i\delta}(\cos\beta \mp i \sin\vartheta \cos\varphi \sin\beta)\; h^{GR}_{+,\times} - e^{i\delta}(\cos\vartheta \pm i \sin\vartheta \sin\varphi)\;\; \sin\beta\; h^{GR}_{\times,+}$$

$$\delta, \beta, \vartheta, \varphi \propto \zeta^{1,2,3} \simeq \sum_{djm} \omega^{d-4}\; Y_{jm}(\theta, \phi)\; k^{(d)}_{(I,E,B,V)jm}$$

Mewes, Phys. Rev. D 99, 104062 (2019)



Ault-O'Neal, Bailey, Dumerchat, Haegel, Tasson, Universe 2021, 7(10), 380 (2021)

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IN A NUTSHELL

Question:

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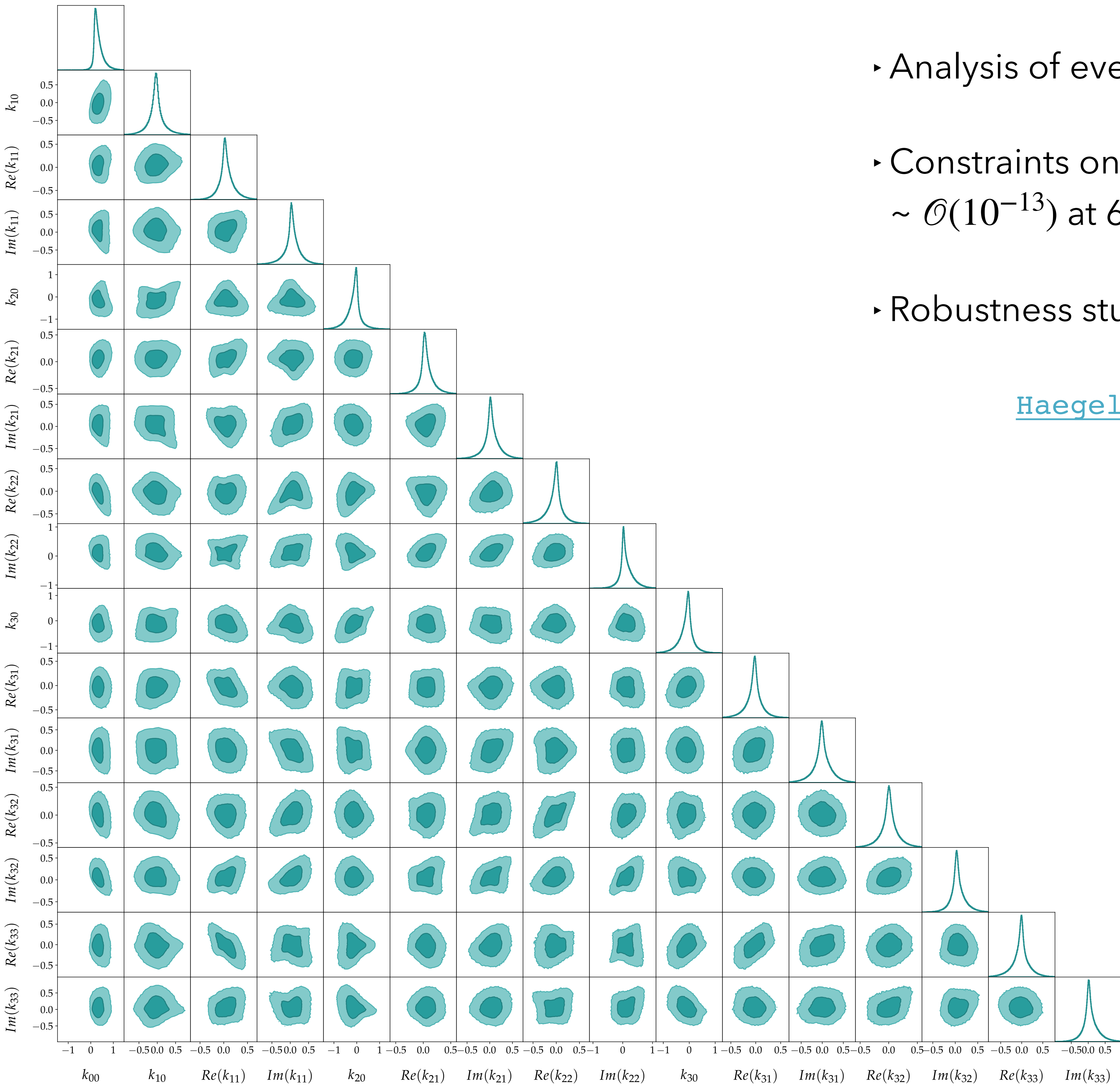
Test:

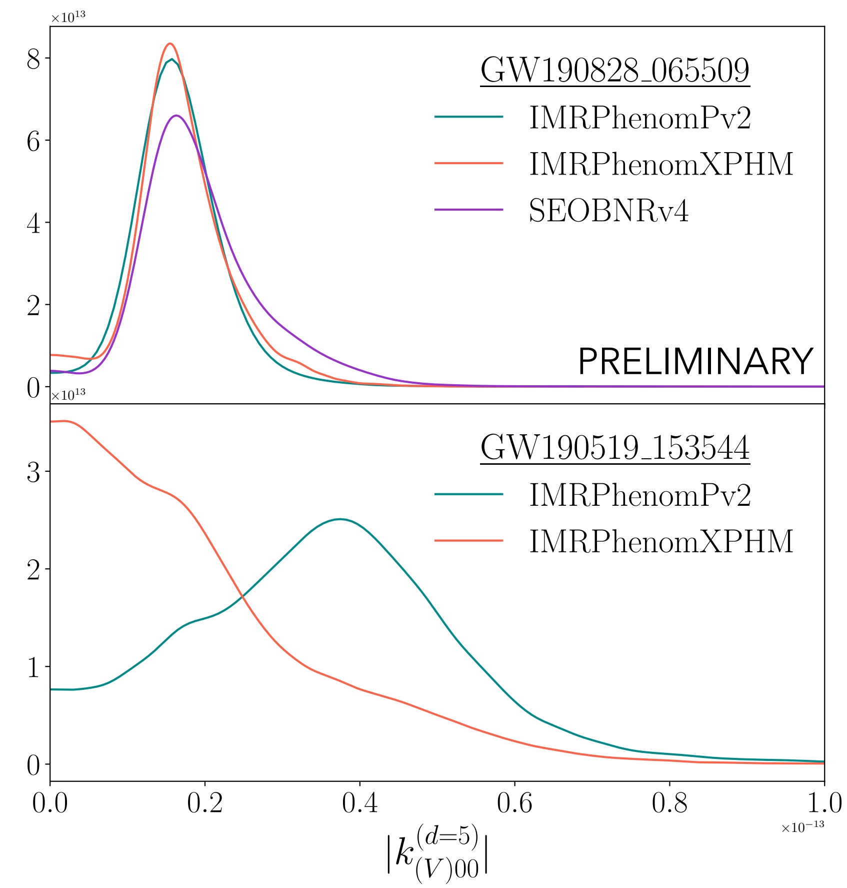
- Compare modified waveform signals with inference of source and $k^{(d=5)}_{(V)jm}$ parameters

Polarisation-dependent, anisotropic dispersion



- Analysis of events from O1 + O2 + O3 events
- Constraints on 16 dispersion parameters at ~ $\mathcal{O}(10^{-13})$ at 68.3% CI
- Robustness study: impact of waveform model

[Haegel et al, arxiv:2210.04481](https://arxiv.org/abs/2210.04481)



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Speed of GW $\neq c$

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Impact GW velocity: constrained with the BNS event GW170817

Kostelecky & Mewes, Phys. Lett. B757:510-514 (2016)

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IN A NUTSHELL

Question:

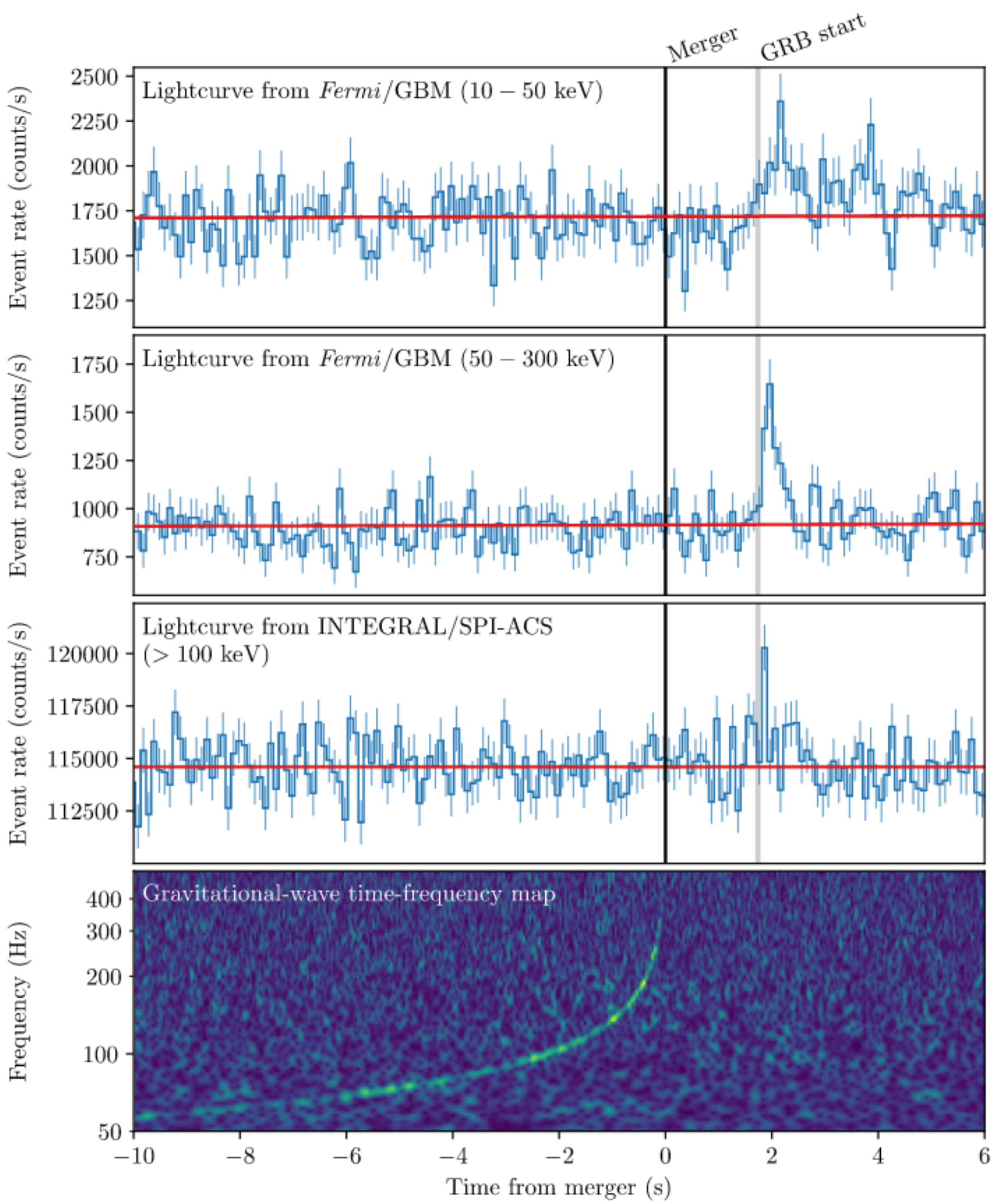
- Is gravitational waves velocity sub or supra luminal ?

Test:

- Compare the arrival time of GW and EM signals for binary neutron star merger

Multimessenger event: GW170817

- **GW170817: binary neutron star merger**



- Difference in time of arrival between electromagnetic and gravitational radiation enables to constrain the speed of gravity:

$$-3 \times 10^{-15} \leqslant \frac{\Delta v}{v_{\mathrm{EM}}} \leqslant +7 \times 10^{-16}$$

- As well as Lorentz-violating parameters in the SME:

$$\Delta v = -\sum_{\substack{\ell m \\ \ell \leqslant 2}} Y_{\ell m}(\hat{n}) \left(\frac{1}{2}(-1)^{1+\ell} \bar{s}^{(4)}_{\ell m} - c^{(4)}_{(I)\ell m} \right).$$

Coefficient	This Work Upper	Previous Upper
$\bar{s}^{(4)}_{00}$	5×10^{-15}	8×10^{-5}
$\bar{s}^{(4)}_{10}$	7×10^{-15}	7×10^{-14}
$-\mathrm{Re}\ \bar{s}^{(4)}_{11}$	2×10^{-15}	8×10^{-14}
$\mathrm{Im}\ \bar{s}^{(4)}_{11}$	7×10^{-15}	9×10^{-14}
$-\bar{s}^{(4)}_{20}$	8×10^{-15}	7×10^{-14}
$-\mathrm{Re}\ \bar{s}^{(4)}_{21}$	2×10^{-15}	7×10^{-14}
$\mathrm{Im}\ \bar{s}^{(4)}_{21}$	8×10^{-15}	8×10^{-14}
$\mathrm{Re}\ \bar{s}^{(4)}_{22}$	3×10^{-15}	8×10^{-14}
$-\mathrm{Im}\ \bar{s}^{(4)}_{22}$	4×10^{-15}	7×10^{-14}

LVC, Fermi, Integral, Astrophys.J.Lett. 848 (2017) 2, L13

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GW friction

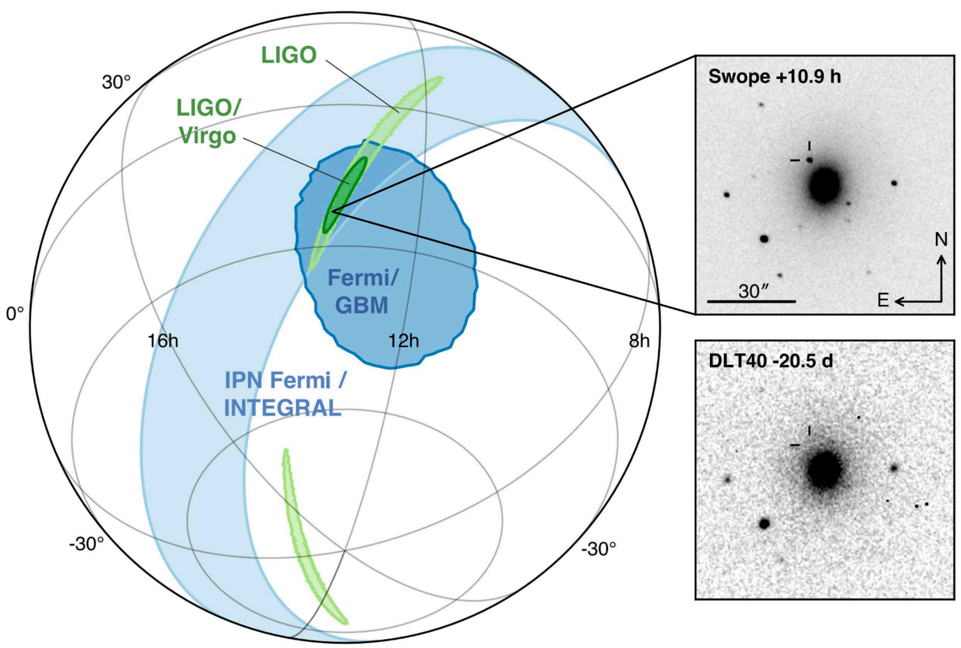
Amplitude does not scale as 1/distance

Standard sirens

- **Standard siren** = simultaneous observation of electromagnetic and gravitational radiation from the same event

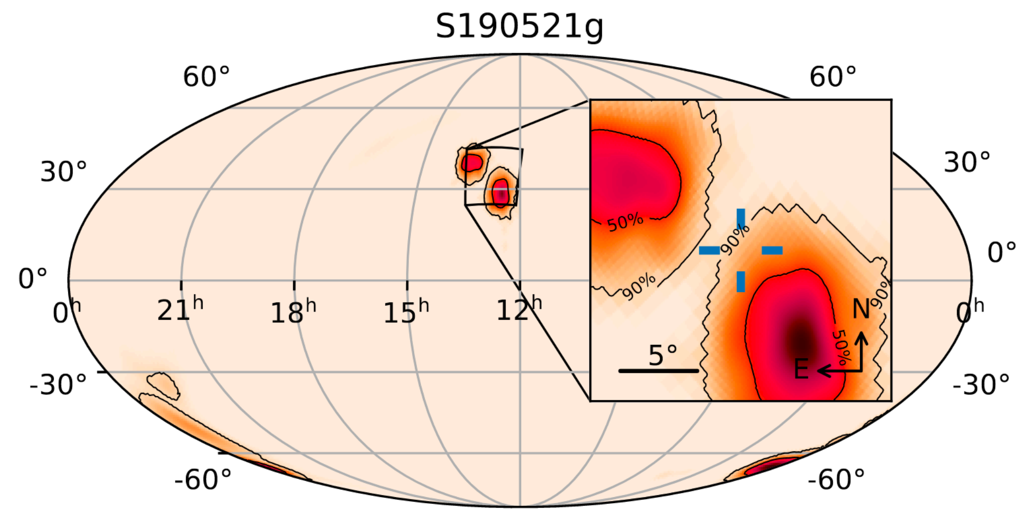
- **The distance (luminosity distance & redshift)** can be separately inferred from the two signals, enabling to measure cosmological and GW friction parameters

- **GW170817 (candidate):**
  - binary neutron star merger
  - $z \simeq 0.01$



LVC, Phys. Rev. Lett. 119, 161101 (2017)

- **GW190521 (assumed):**
  - binary black holes merger
  - potential location in AGN disk creating electromagnetic signal (not confirmed)
  - $z \simeq 0.44$



LVC, Phys. Rev. Lett. 125, 101102 (2020)
Graham et al, Phys. Rev. Lett. 124, 251102 (2020)

Standard sirens

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- **The distance (luminosity distance & redshift)** can be separately inferred from the two signals, enabling to measure cosmological and GW friction parameters

- GW170817:
  - binary neutron star merger
  - $z \simeq 0.01$

- GW190521:
  - binary black holes merger
  - potential location in AGN disk creating electromagnetic signal (not confirmed)
  - $z \simeq 0.44$

IN A NUTSHELL

Question:

- Is gravitational waves amplitude modified due to a friction mechanism ?

Test:

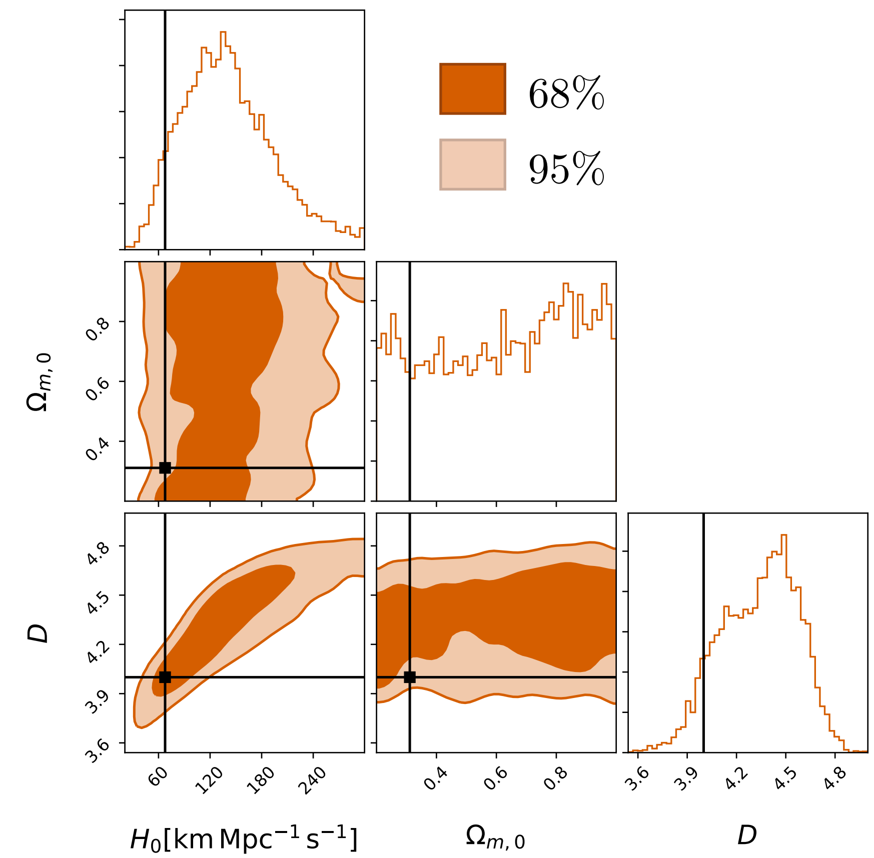
- Compare distance reconstructed with GW and EM signals (taking into account cosmology)

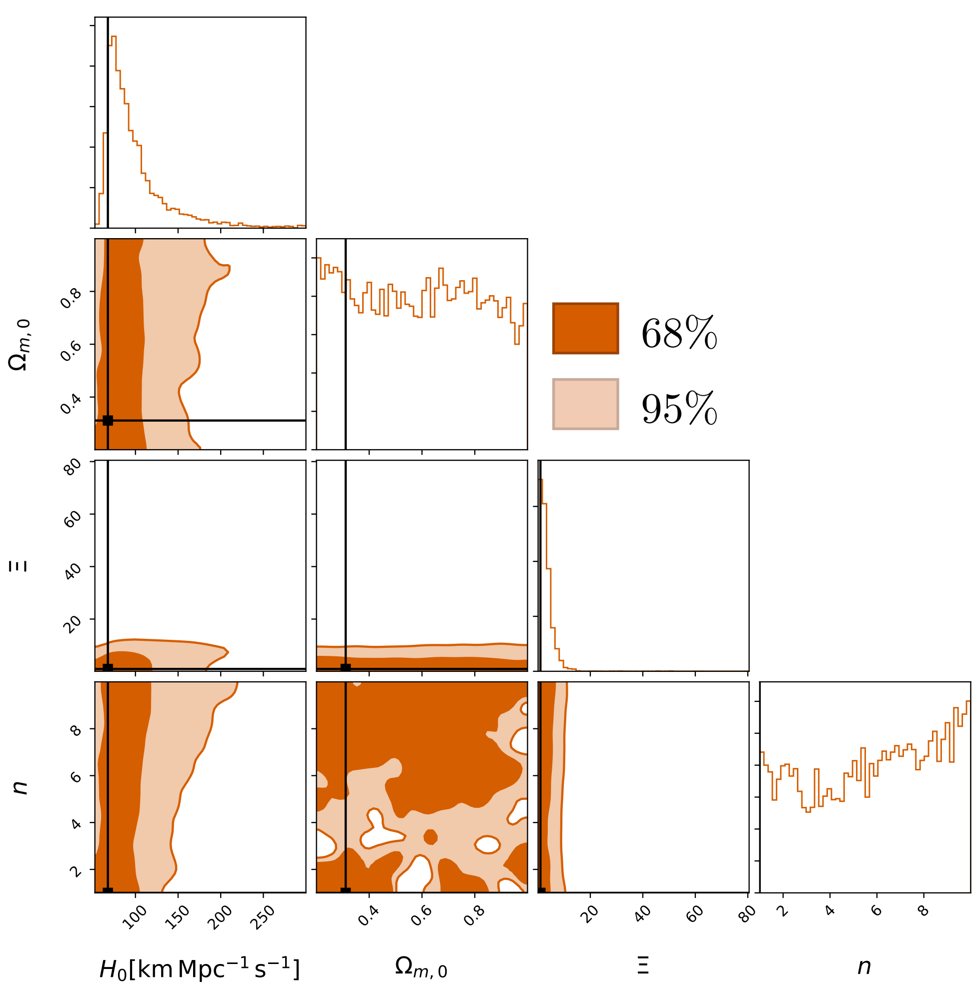
Constraints on GW friction

- **Large extra dimensions** (DGP gravity and other non-compactified extra dimensions quantum gravity theories)

$$d_L^{GW}(z) = \left[1 + \left(\frac{d_L^{EM}(z)}{R_c} \right)^n \right]^{\frac{D-2}{2n}}$$

[Mastrogiovanni, Haegel, Karathanasis, Magana-Hernandez, Steer, JCAP 02 (2021) 043]()





- **Scalar-tensor theories of gravitation parameterisation** (Brans-Dicke, Horndeski, beyond-Horndeski, DHOST)

$$d_L^{GW}(z) = d_L^{EM}(z) \left[\Xi + \frac{1 - \Xi}{(1+z)^n} \right]$$

- Constraints can be improved in case of detection of a lensed event (current study, H. Narola talk at GR23 [C3])

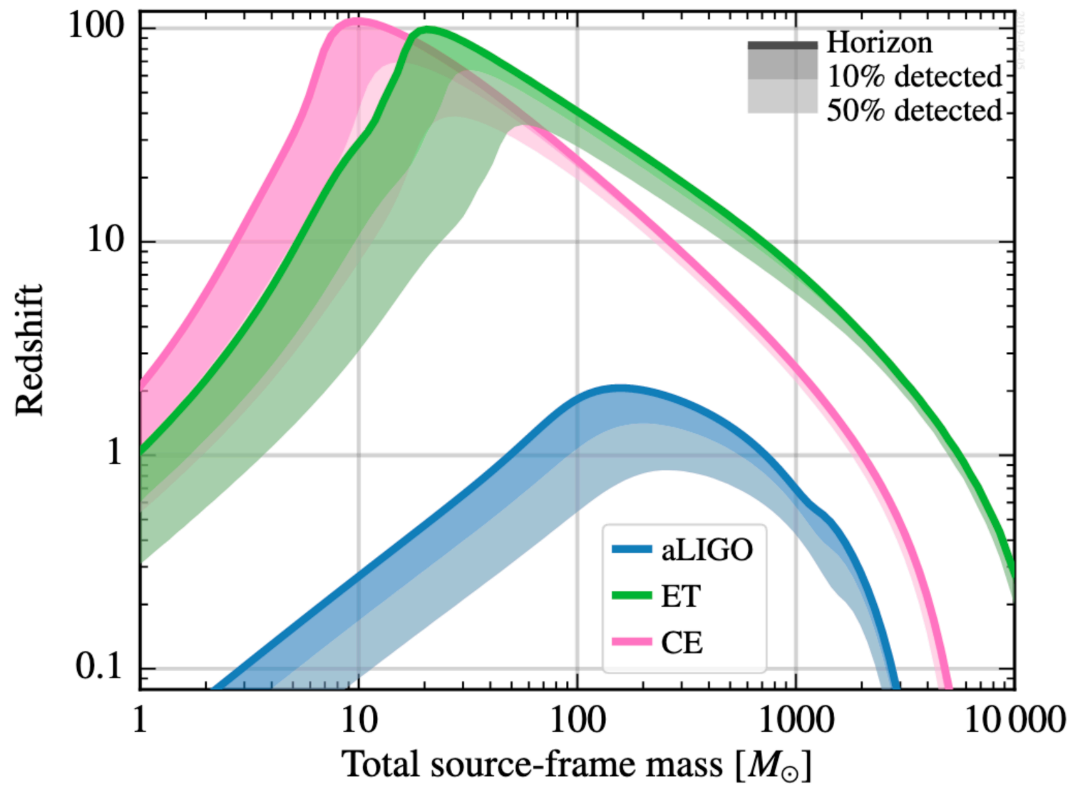
Conclusion

- **GW enable to test several beyond-GR phenomenology:**
  - speed of gravity
  - graviton mass
  - spacetime symmetry breaking
  - scalar-tensor theories of gravitation
  - "agnostic" deviations

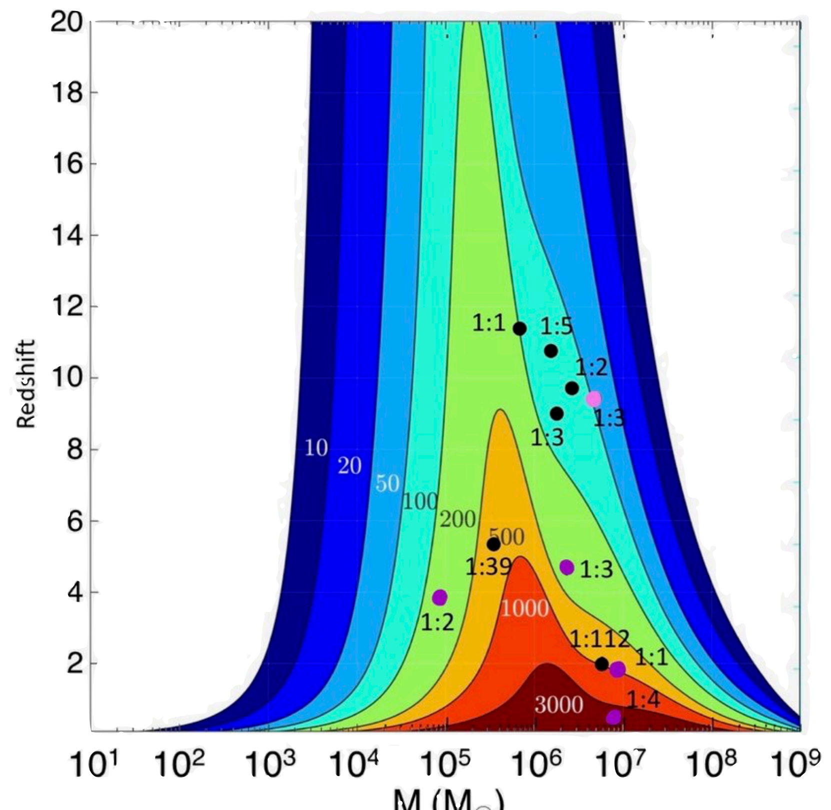
- **Multimessenger studies are complementary to GW-only tests**
  a collaborative effort from the community

- **The best is yet to come**
  towards the advanced Virgo / Einstein Telescope era
  LISA is under construction

Maggiore et al, JCAP 03 (2020) 050



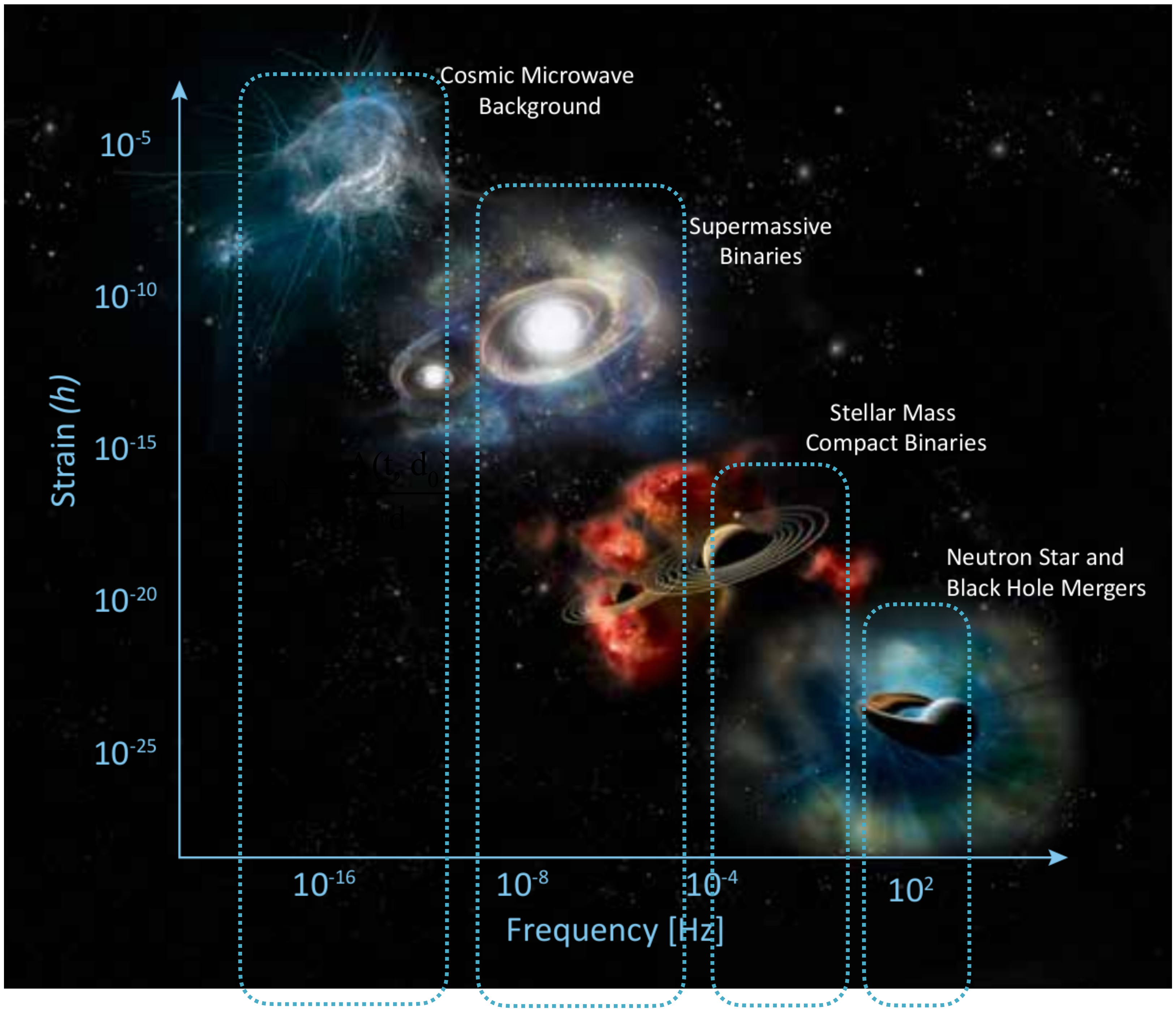
Bellovary et al, MRAS 482 (2019) 3



Bonus

Gravitational waves sources

Credit: SSU EPO/Aurore Simonnet



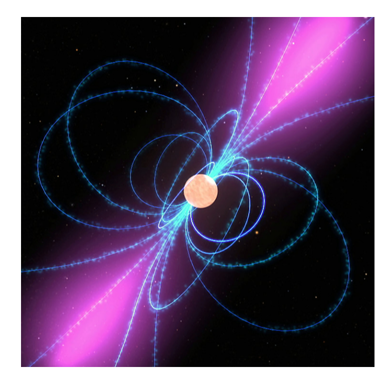
CMB modes

ePTA nanoGRAV

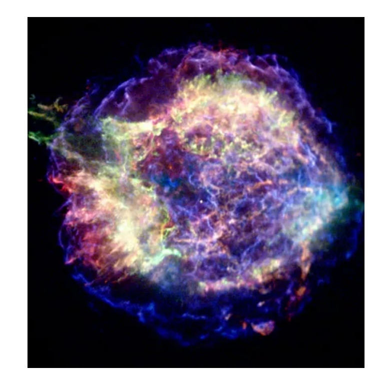
LISA

LIGO-Virgo-KAGRA

+ pulsars



+ supernovae

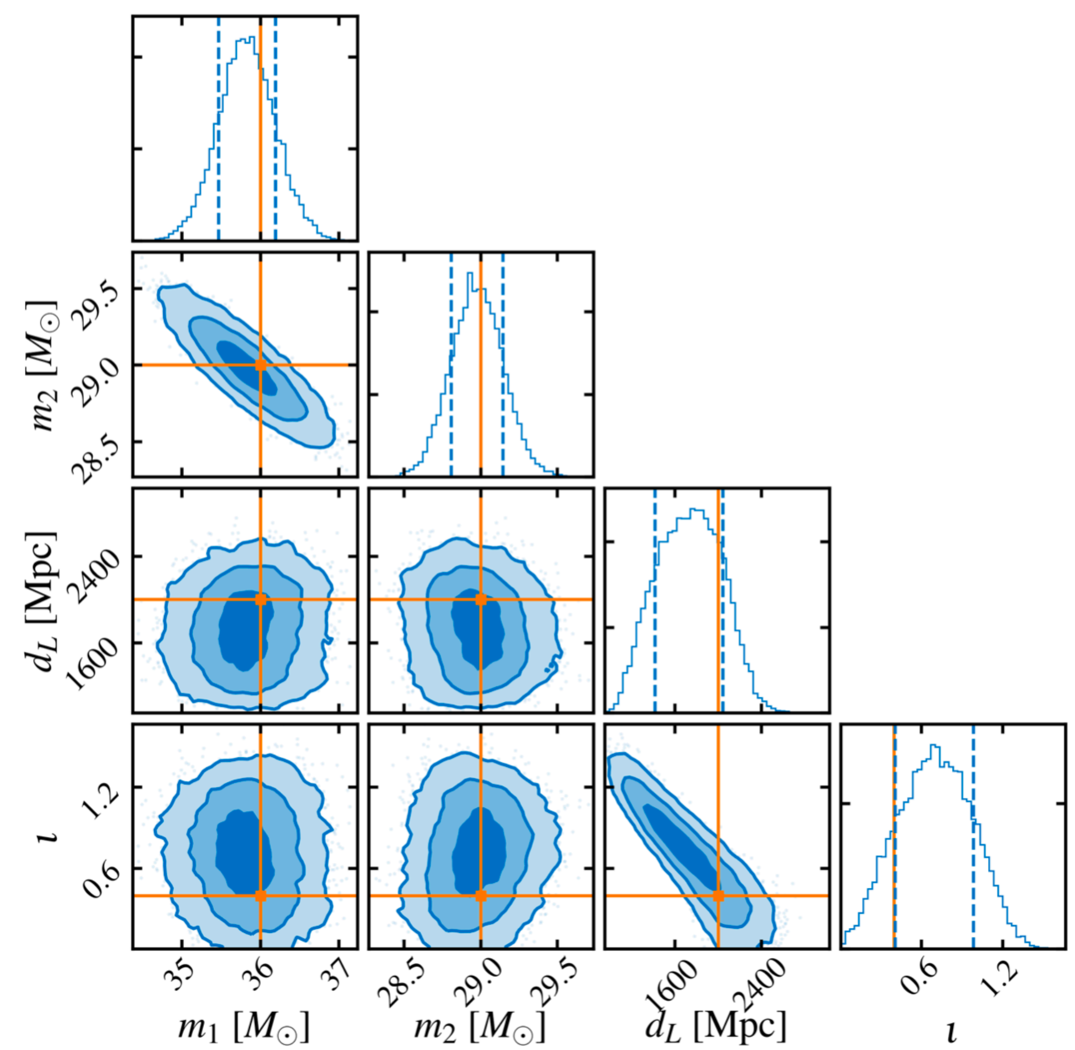


GW parameter estimation for binary coalescence

- **Bayesian analyses**: joint posterior probabilities of source parameters
  Markov chains sampling methods (Nested sampling, MCMC)

- **Binary systems of black holes and/or neutron stars**:
  15 parameters minimum
  - 2 masses
  - 2 spin magnitudes
  - 2 angles for each spin
  - Reference time
  - Orbital phase at reference time
  - Luminosity distance
  - Right ascension & declination
  - Inclination angle
  - Polarisation angle

  \+ tidal parameters in neutron stars



Ashton et al, Astrophys.J.Suppl. 241 (2019) 2, 27

Measuring SME coefficients for d=5

- In practice: look at each mass dimension separately
- Dispersion starts for mass dimension $d = 5$
- It is controlled by 16 coefficients $k^{(d=5)}_{(V),ij}$
- The modified GW signal is:

$$h_+ = \cos\beta\ h_+^{GR} - \sin\beta\ h_\times^{GR}$$

- Constrain $k^{(d=5)}_{(V),eff}$ with individual events, then reinterpret the constraint as a single coefficient / all coefficients constraint

$$\beta = \omega^2\ \tau \Big| \frac{1}{2}\frac{1}{\sqrt{\pi}}\ k_{00} + \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta\ k_{10} - \sqrt{\frac{3}{2\pi}}\sin\theta\ (\cos\phi\ Re(k_{11}) - \sin\phi\ Im(k_{11})) + \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1)\ k_{20} - \sqrt{\frac{15}{2\pi}}\sin\theta\ \cos\theta\ (\cos\phi\ Re(k_{21}) - \sin\phi\ Im(k_{21})) + \frac{1}{2}\sqrt{\frac{15}{2\pi}}\sin^2\theta\ (\cos 2\phi\ Re(k_{22}) - \sin 2\phi\ Im(k_{22})) + \frac{1}{4}\sqrt{\frac{7}{\pi}}(5\cos^3\theta - 3\cos\theta)\ k_{30} - \frac{1}{4}\sqrt{\frac{21}{\pi}}\sin\theta\ (5\cos^2\theta - 1)(\cos\phi\ Re(k_{31}) - \sin\phi\ Im(k_{31})) + \frac{1}{2}\sqrt{\frac{105}{2\pi}}\sin^2\theta\ \cos\theta\ (\cos 2\phi\ Re(k_{32}) - \sin 2\phi\ Im(k_{32})) - \frac{1}{4}\sqrt{\frac{35}{\pi}}\sin^3\theta\ (\cos 3\phi\ Re(k_{33}) - \sin 3\phi\ Im(k_{33}))\Big|$$

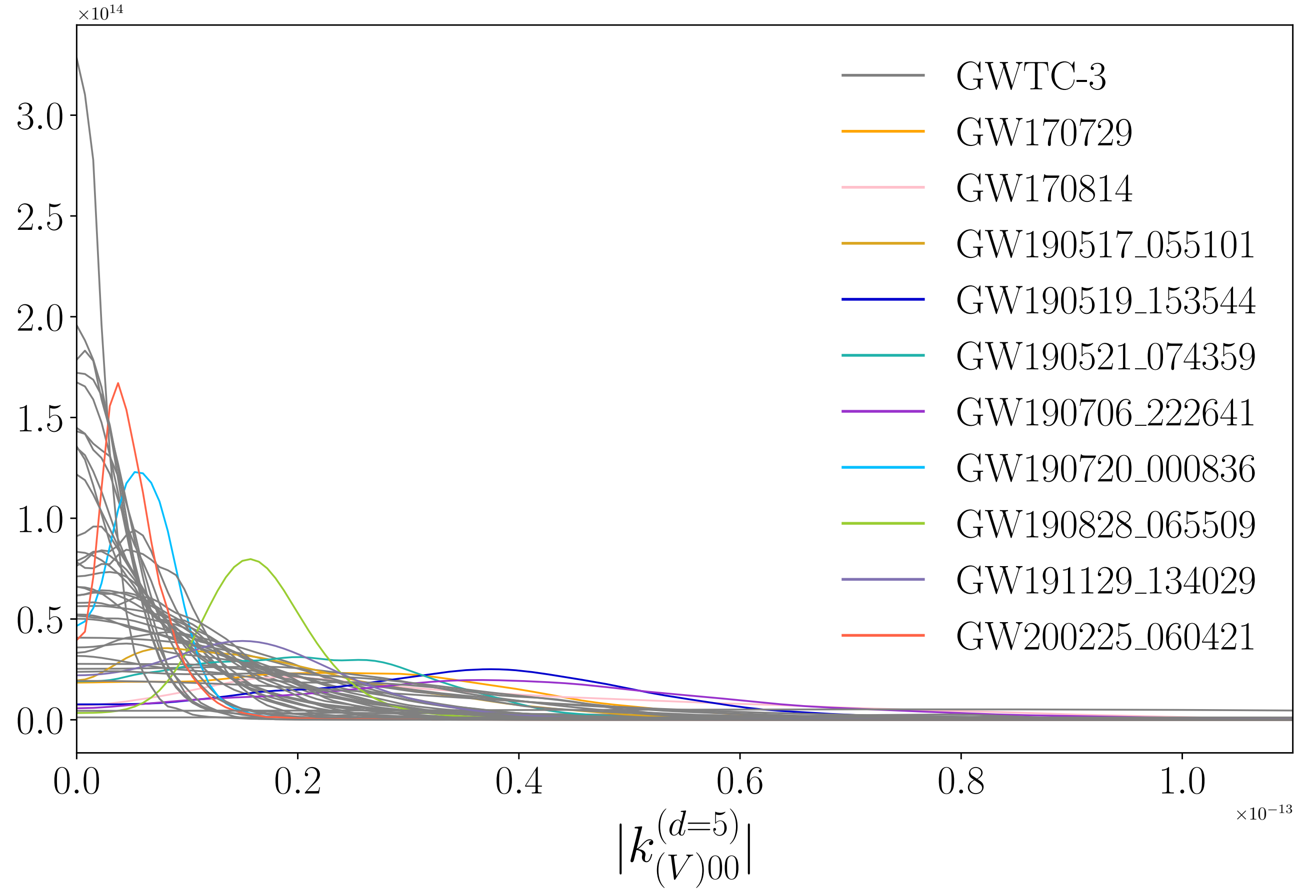
$k^{(d=5)}_{(V),eff}$

[Haegel et al, arxiv:2210.04481](https://arxiv.org/abs/2210.04481)

Measuring isotropic SME coefficients for d=5

- **Analysis**: GWTC-3 (O1 + O2 + O3)
  45 events with higher SNR chosen (FAR > 2 / year)
  joint measurement of source and SME parameters

- **Constraints**: combined events: $k^{(d=5)}_{(V),00} \sim \mathcal{O}(10^{-15})$ (single parameter constraint)
  global analysis: $k^{(d=5)}_{(V),ij} \sim \mathcal{O}(10^{-13})$ (multi parameter constraint)

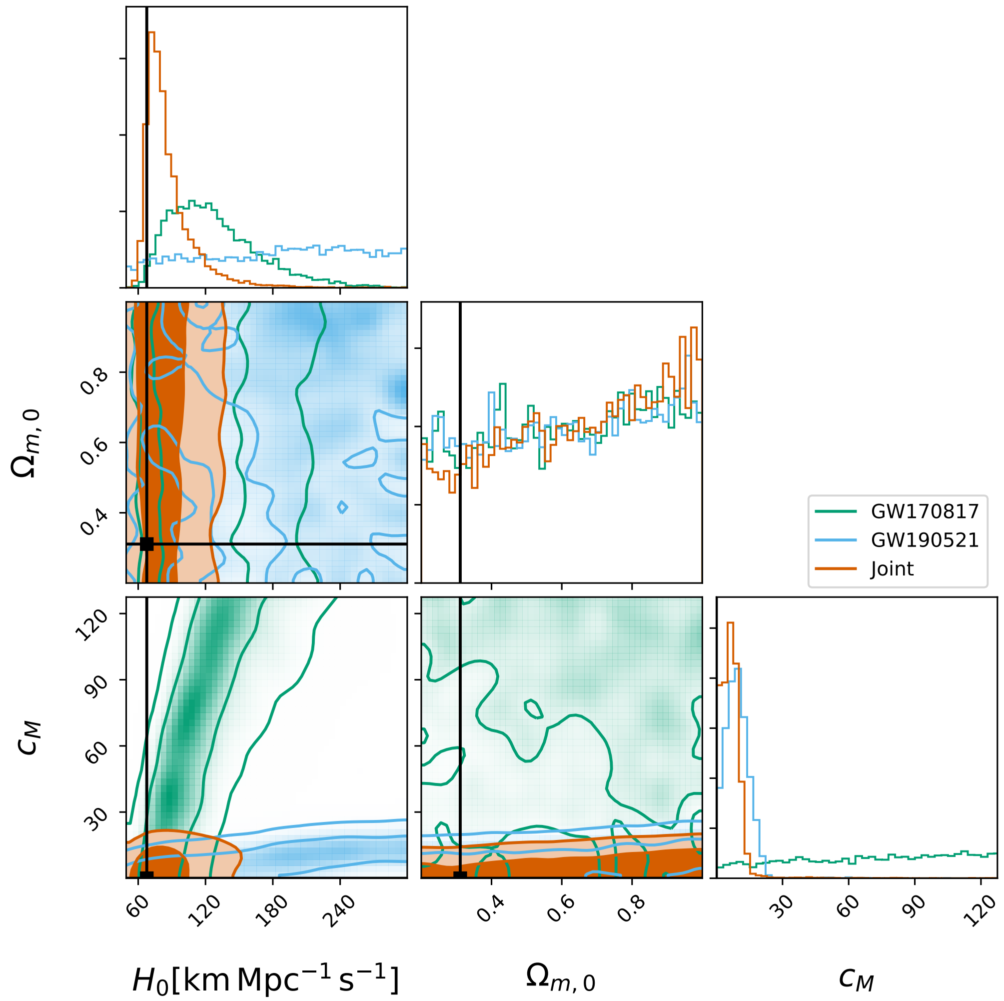


Haegel et al, arxiv:2210.04481

90% lower	68.3% lower	$k^{(5)}_{(V)ij}$ coefficient	68.3% upper	90% upper
0.51	1.21	k_{00}	4.38	7.37
-4.54	-2.13	k_{10}	1.19	3.91
-2.30	-1.00	$Re(k_{11})$	1.73	3.39
-3.64	-1.21	$Im(k_{11})$	2.35	4.45
-7.40	-3.75	k_{20}	1.10	3.78
-1.75	-0.61	$Re(k_{21})$	1.43	3.02
-2.77	-1.16	$Im(k_{21})$	1.71	3.67
-3.58	-1.72	$Re(k_{22})$	1.02	2.55
-2.49	-0.96	$Im(k_{22})$	2.80	5.58
-6.40	-3.31	k_{30}	1.17	3.57
-3.34	-1.65	$Re(k_{31})$	0.98	2.48
-3.90	-1.92	$Im(k_{31})$	1.75	3.87
-2.76	-1.23	$Re(k_{32})$	1.34	2.87
-2.26	-0.90	$Im(k_{32})$	1.82	3.60
-3.95	-1.95	$Re(k_{33})$	1.28	3.18
-3.22	-1.35	$Im(k_{33})$	2.25	4.78

TABLE I. Credible intervals on the $k^{(5)}_{(V)ij}$ coefficients (in 10^{-13} m), determined from the marginalised posterior probability distributions estimated with the joint estimation of the 16 $k^{(5)}_{(V)ij}$ coefficients shown in diagonal in Fig. 2.

GW friction and dark energy motivated friction



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- **Dynamical dark energy models:**
  α_M is linked to the energy content of the Universe

$$\alpha_M = c_M \frac{\Omega_{\Lambda(z)}}{\Omega_{\Lambda(0)}}$$

$$d_L^{GW}(z) = d_L^{EM}(z)\, exp\left[\frac{c_M}{2\Omega_{\Lambda,0}}\, ln \frac{1+z}{\Omega_{m,0}\,(1+z)^3 + \Omega_{\Lambda,0}}\right]$$

$c_M = 0$ is the GR case

GW friction and extra dimensions

- **D=4 is on the edge on the contour** due to the luminosity distance posterior skewed towards large values

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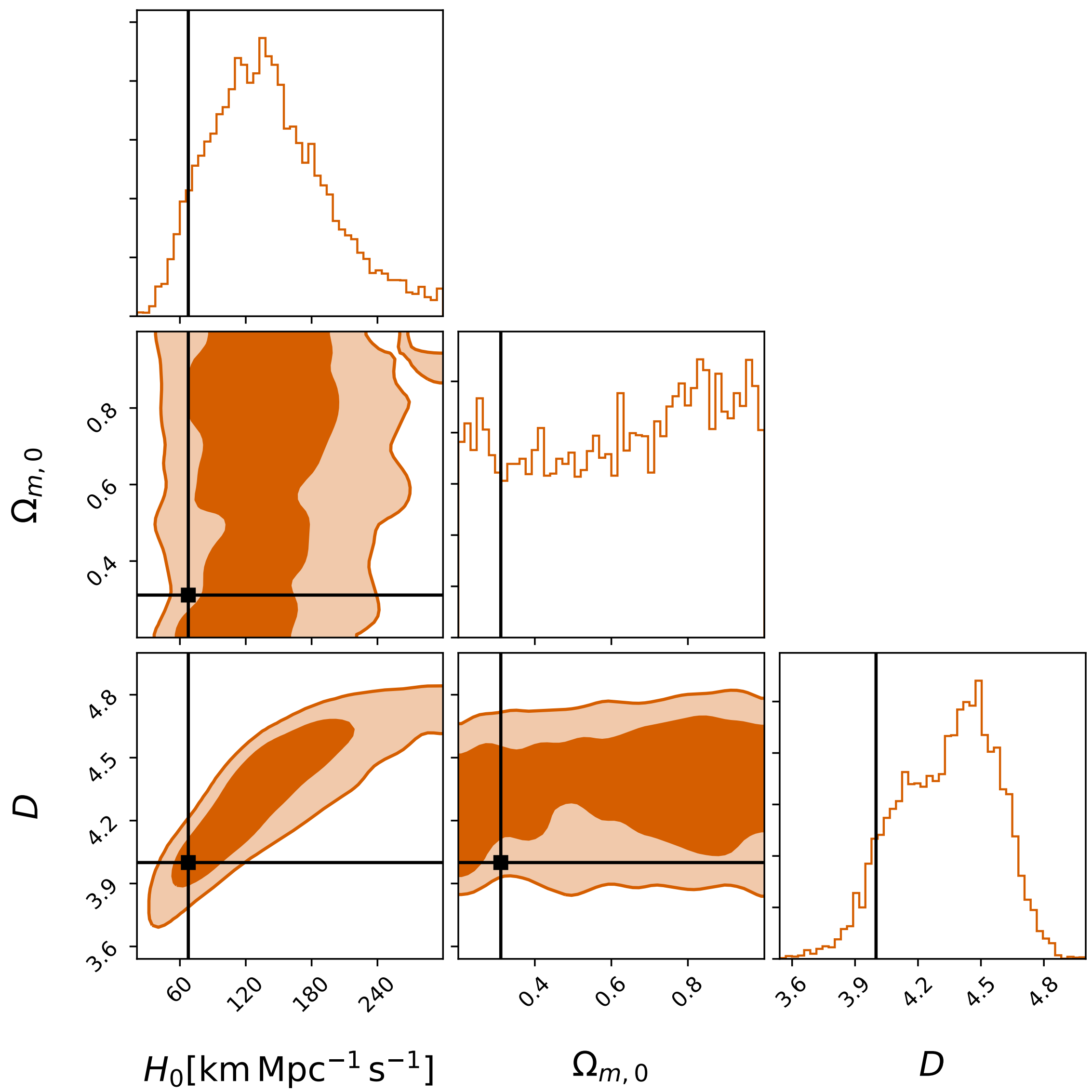
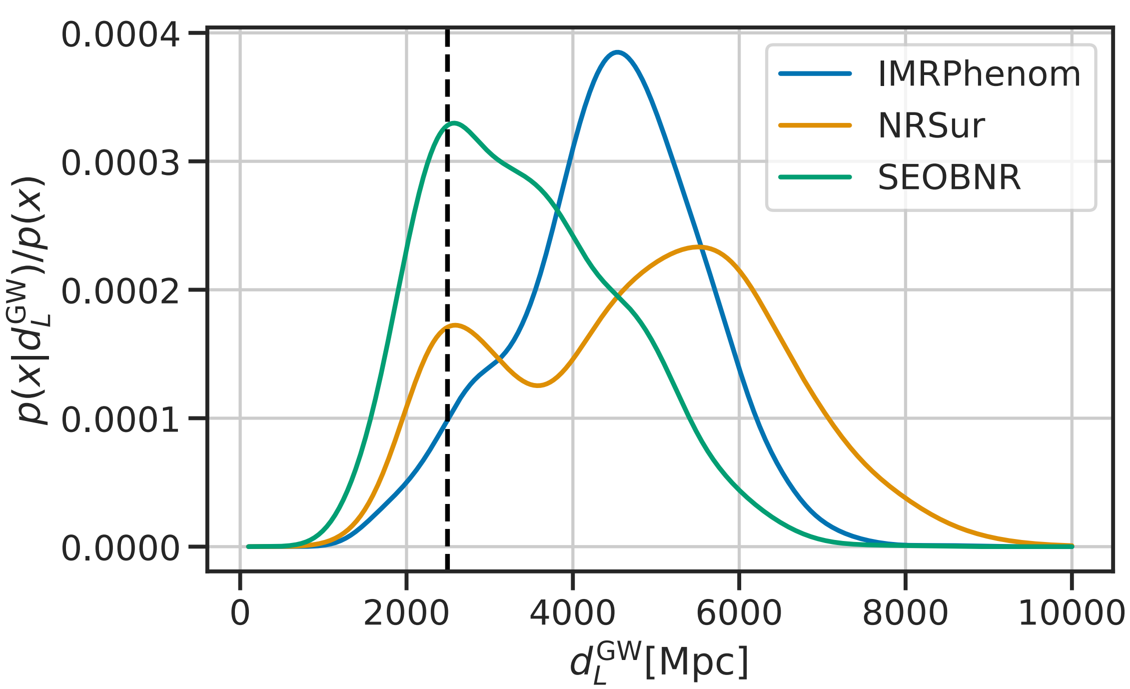
$$d_L^{GW}(z) = \left[1 + \left(\frac{d_L^{EM}(z)}{R_c} \right)^n \right]^{\frac{D-2}{2n}}$$

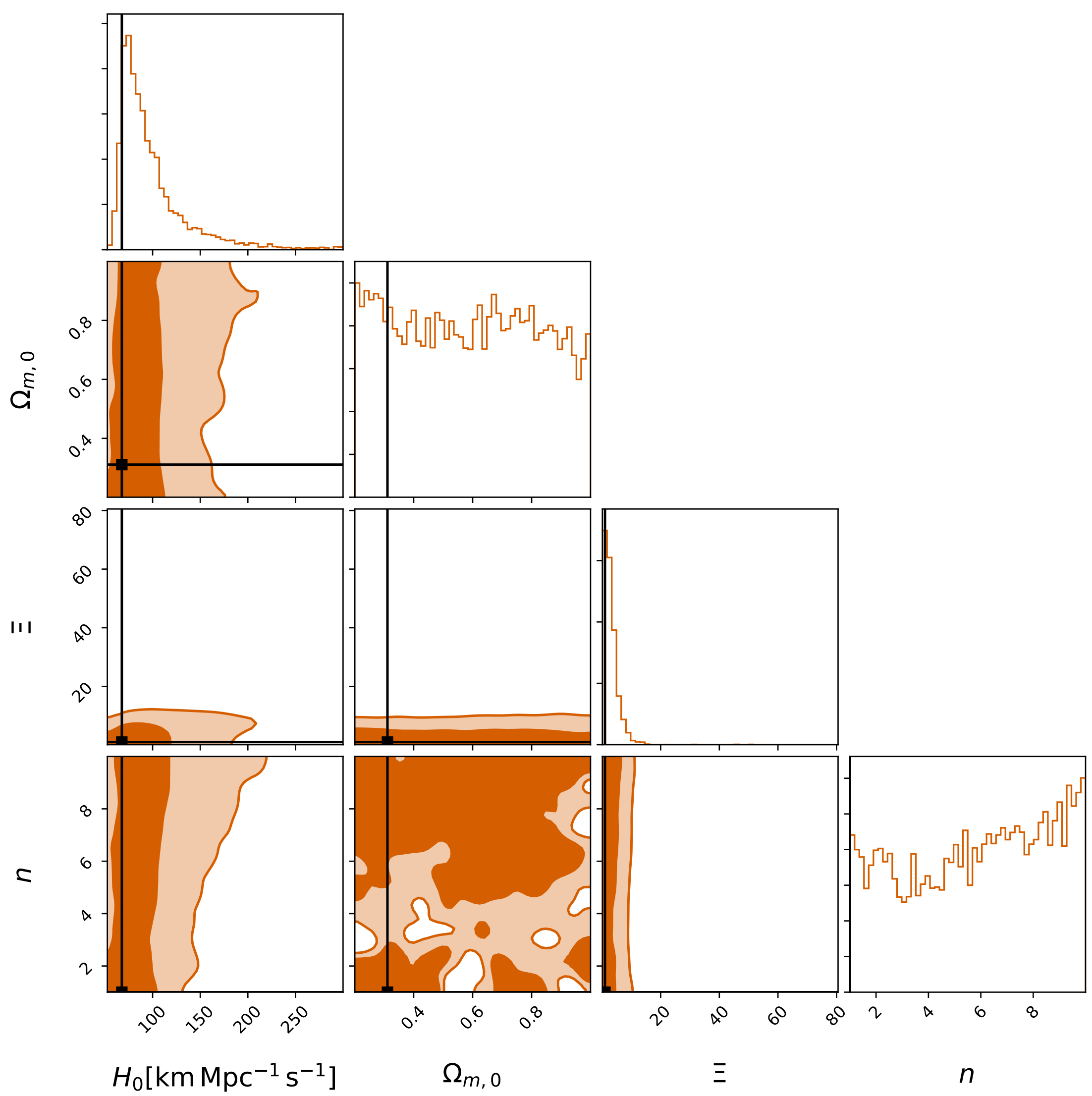




From GW friction to scalar-tensor theories

- **Scalar-tensor theories of gravitation parameterisation** (Brans-Dicke, Horndeski, beyond-Horndeski, DHOST)

$$d_L^{GW}(z) = d_L^{EM}(z)\left[\Xi + \frac{1-\Xi}{(1+z)^n}\right]$$



Model	$\Xi_0 - 1$	n	Refs.
HS $f(R)$ gravity	$\frac{1}{2} f_{R0}$	$\frac{3(\tilde{n}+1)\Omega_m}{4-3\Omega_m}$	[68]
Designer $f(R)$ gravity	$-0.24\Omega_m^{0.76} B_0$	$3.1\Omega_m^{0.24}$	[69]
Jordan–Brans–Dicke	$\frac{1}{2}\delta\phi_0$	$\frac{3(\tilde{n}+1)\Omega_m}{4-3\Omega_m}$	[70]
Galileon cosmology	$\frac{\beta\phi_0}{2M_{\rm Pl}}$	$\frac{\dot{\phi}_0}{H_0\phi}$	[71]
$\alpha_M = \alpha_{M0} a^{\tilde{n}}$	$\frac{\alpha_{M0}}{2\tilde{n}}$	$\tilde{n}$	[67]
$\alpha_M = \alpha_{M0}\frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$	$-\frac{\alpha_{M0}}{6\Omega_\Lambda}\ln\Omega_m$	$-\frac{3\Omega_\Lambda}{\ln\Omega_m}$	[67, 72]
$\Omega = 1 + \Omega_+ a^{\tilde{n}}$	$\frac{1}{2}\Omega_+$	$\tilde{n}$	[6]
Minimal self-acceleration	$\lambda\left(\ln a_{acc} + \frac{C}{2}\chi_{acc}\right)$	$\frac{C/H_0 - 2}{\ln a_{acc}^2 - C\chi_{acc}}$	[66]

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Belgacem et al, JCAP07 (2019) 024

Alternatives theories of gravity

- **Doubly special relativity:**
  Modification of special relativity with the addition of an observer-independent maximum energy ; length scale (Planck length / energy). Motivation: same scale of quantum gravity effects for all observers
  `Amelino-Camelia, Symmetry 2 (2010) 230-271`

- **Hořava-Lifshitz gravity:**
  Quantisation of gravitation with a QFT approach, where ghosts are avoided by introducing anisotropic scaling between space and time at high energies
  `Wang, Int. J. Mod. Phys. D26 (2017) 1730014`

- **DGP gravity:**
  Extension of the Einstein-Hilbert action to a 4+1 Minkowski space. Motivation: acceleration of the Universe expansion without Λ.
  `Dvali, Gabadadze, Porrati. Phys. Lett. B 485:208-214 (2000)`

- **Horndeski (and beyond) gravity:**
  General formulation of scalar-tensor theory of gravitation (includes Brans-Dicke, DHOST, linked to Gauss-Bonet). Particularly used to study inflation, metric perturbation, cosmological effects.
  `Kobayashi. Rept. Prog. Phys. 82 (2019) no.8, 086901`

- **f(R) gravity:**
  Class of beyond-GR theory where the Ricci scalar R follows an arbitrary function. Presence of equivalence with scalar-tensor theories for GW.
  `Sotiriou, Faraoni. Rev. Mod. Phys. 82:451-497 (2010)`