



Experimental Overview on g-2 Experiments

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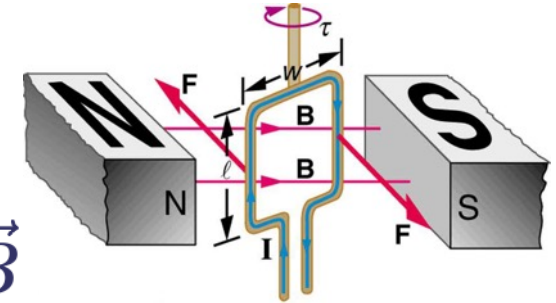
4th World Summit on Exploring the Dark Side of the Universe

Outline

- General introduction to magnetic moments
- Recent electron $g-2$ results
- Muon $g-2$ (bulk of talk)
- Briefly mention tau $g-2$
- Theory talk by Laurent Lellouch to follow...

Magnetic moments...classical

- Magnetic moments have been an invaluable tool for probing basic physics for a very long time!
- For a system of classical charged particles...

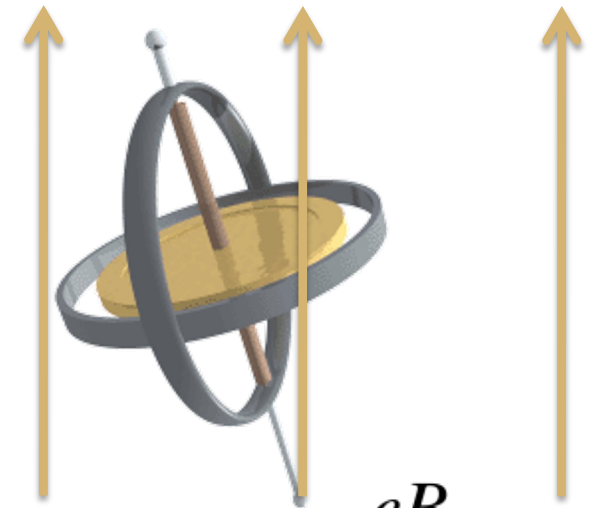


$$\vec{\mu} = \sum_i \frac{q_i}{2m_i} \vec{L}_i \quad \vec{\tau} = \vec{\mu} \times \vec{B}, \quad U = -\vec{\mu} \cdot \vec{B}$$

- For particles with spin

$$\vec{\mu} = g \frac{q\hbar}{4mc} \vec{\sigma}, \quad \vec{S} = \frac{\hbar}{2} \vec{\sigma}$$

- The g-factor is a proportionality constant parameterizing the strength
 - Can determine g by measuring the Larmor precession frequency

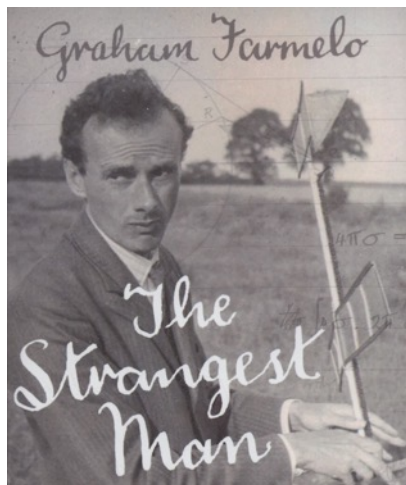


$$\omega_s = g \frac{eB}{2mc}$$

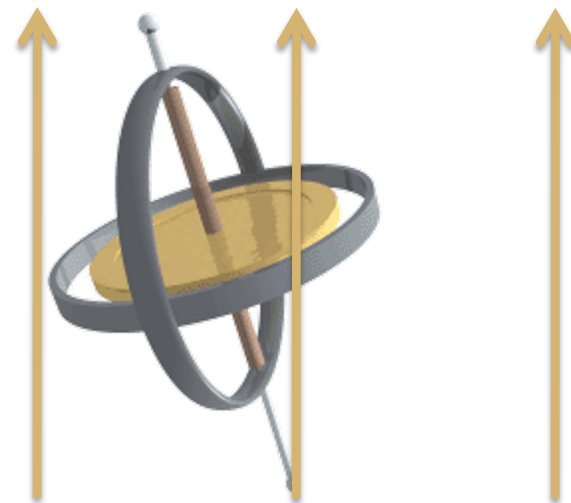
Magnetic moments...classical to quantum

- For a spin $\frac{1}{2}$ point particle the classical expectation is $g = 1$
- With Stern-Gerlach and atomic spectroscopy experiments in the 1920s, it became apparent that $g_e = 2$.
- An electron (or muon) precesses twice as fast
- Solution to the g problem appeared in 1926 with a relativistic treatment by Thomas
- Incorporated in Dirac's famous equation by 1928

$$\left(\frac{1}{2m} (\vec{P} + e\vec{A})^2 + \frac{e}{2m} \vec{\sigma} \cdot \vec{B} - eA^0 \right) \psi_A = (E - m) \psi_A$$



So, for an elementary spin $\frac{1}{2}$ particle in Dirac's theory, $g=2$!



$$\omega_s = g \frac{eB}{2mc}$$

Moments have been testing BSM for decades

- The success of Dirac's theory got experimentalists excited about making a measurement of the g-factor for the proton
- Stern and Estermann set out to make the measurement in 1933

“Don't you know the Dirac theory? It is obvious that $g_p=2$.”, Pauli to Stern

$$g_p \approx 5.6$$



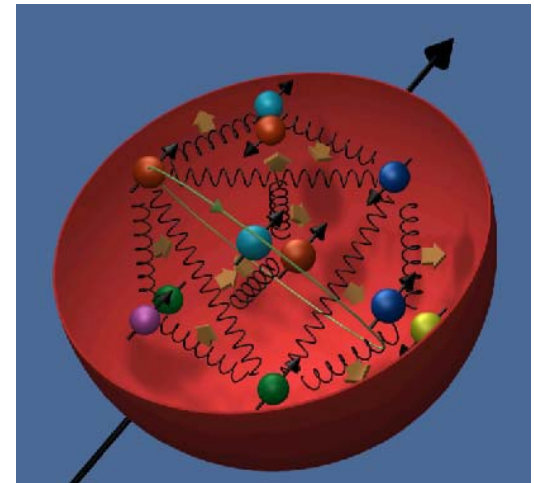
The first 'anomalous' magnetic moment!

- Same year, Rabi inferred $g_n = -3.8$ from measurements on the deuteron



How does a neutral particle develop a magnetic moment?

30 more years to develop quark model



Can see how powerful magnetic moments are for exploring new physics!

Proof that nature abhors a vacuum

- At least for the electron, things were in good shape with Dirac's new theory until 1948 when gains in precision revealed another 'anomaly'
- Kusch and Foley employed atomic spectroscopy to precisely measure g_e



$$g_e = 2.00238(6)$$

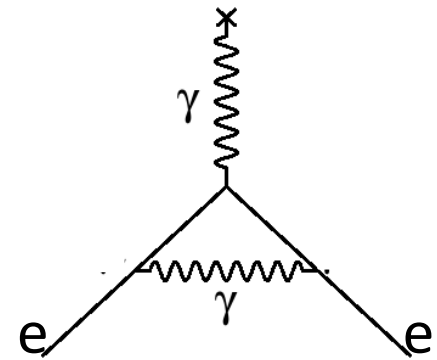
Thus, the anomalous magnetic moment was discovered, fractionally g differs from 2 by $(g-2)/2 = 0.1\%$

QED discovered



- Schwinger takes one look at the anomaly in the g-factor and immediately knows what's up

$$g_e = 2.00238(6)$$



Schwinger term describing 1st order electron self-interaction

$$g_e \approx 2\left(1 + \frac{\alpha}{2\pi}\right) \approx 2.00232$$

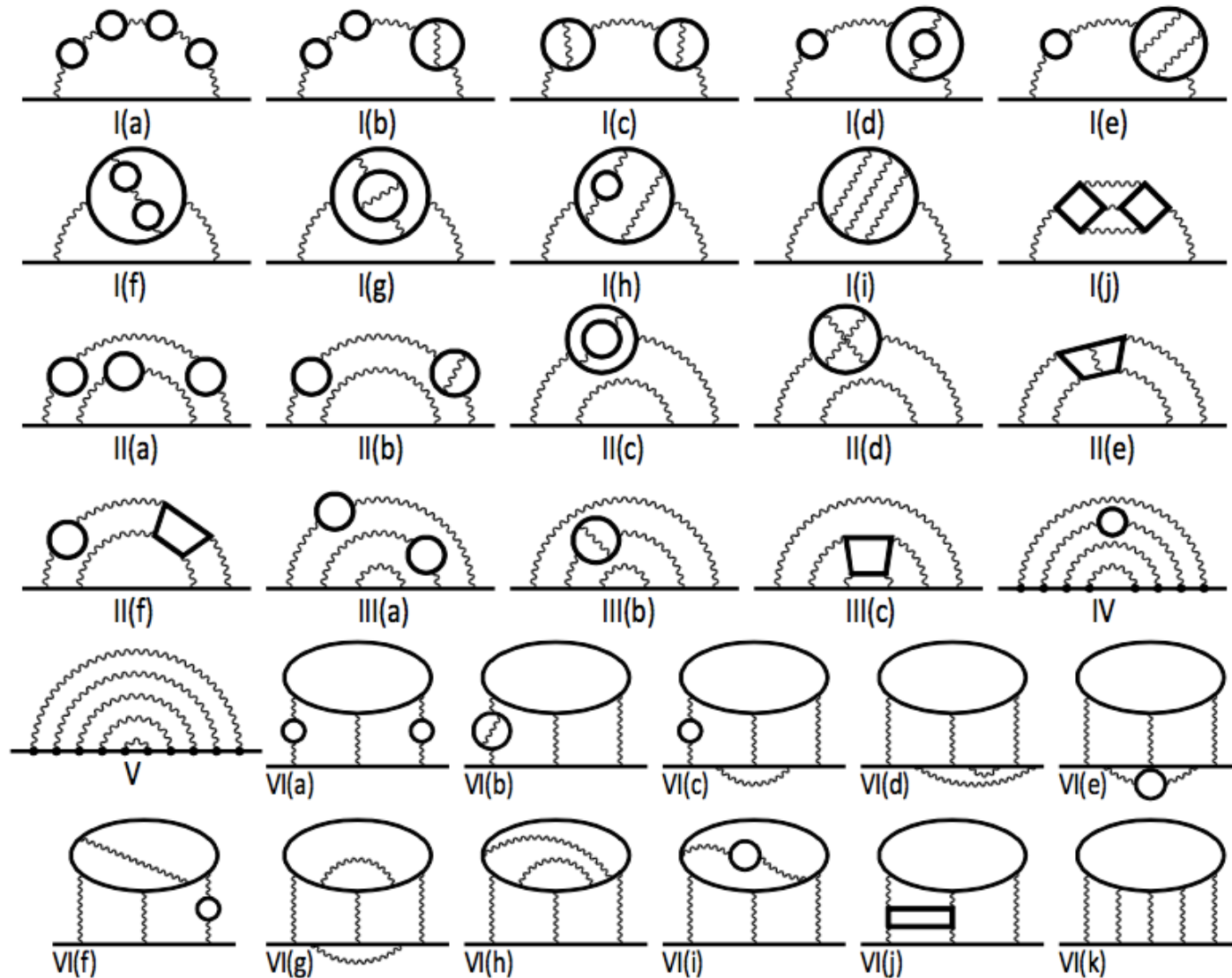
Calculation agrees well with experiment,
and that is how we build confidence in
new physics models!



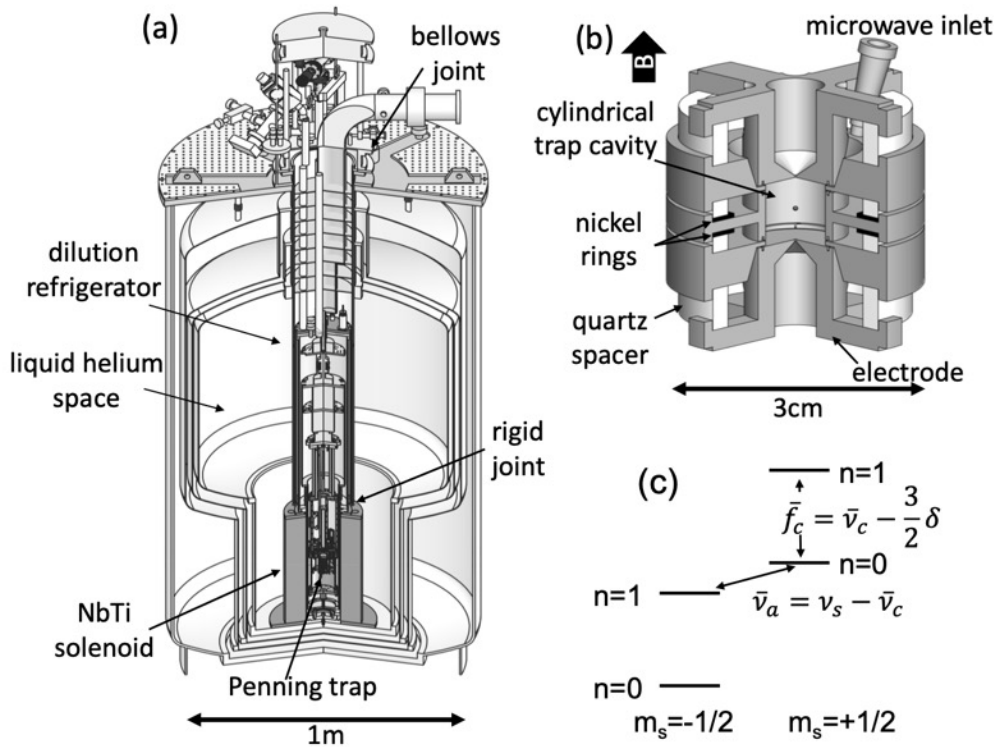
QED calculation now out to 5 loops

T. Aoyama, M. Hayakawa,
T. Kinoshita, M. Nio (PRLs, 2012)

12672
diagrams

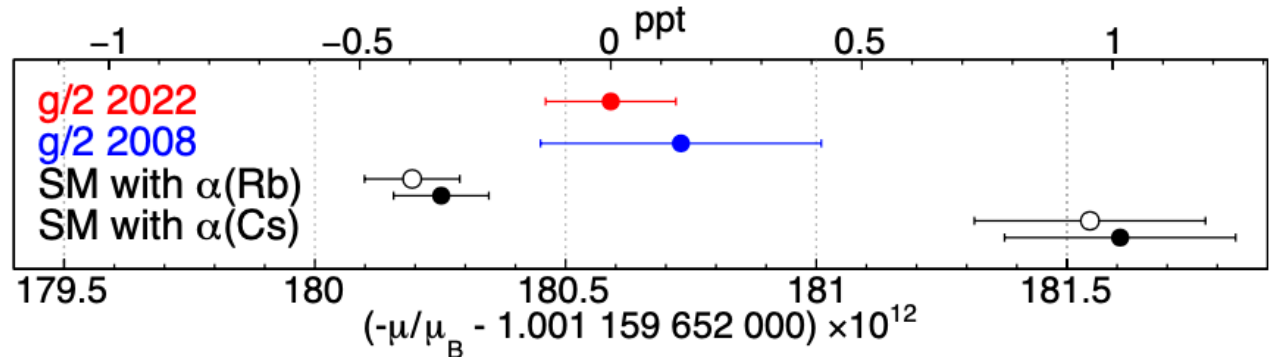


Latest measurement of electron g-2

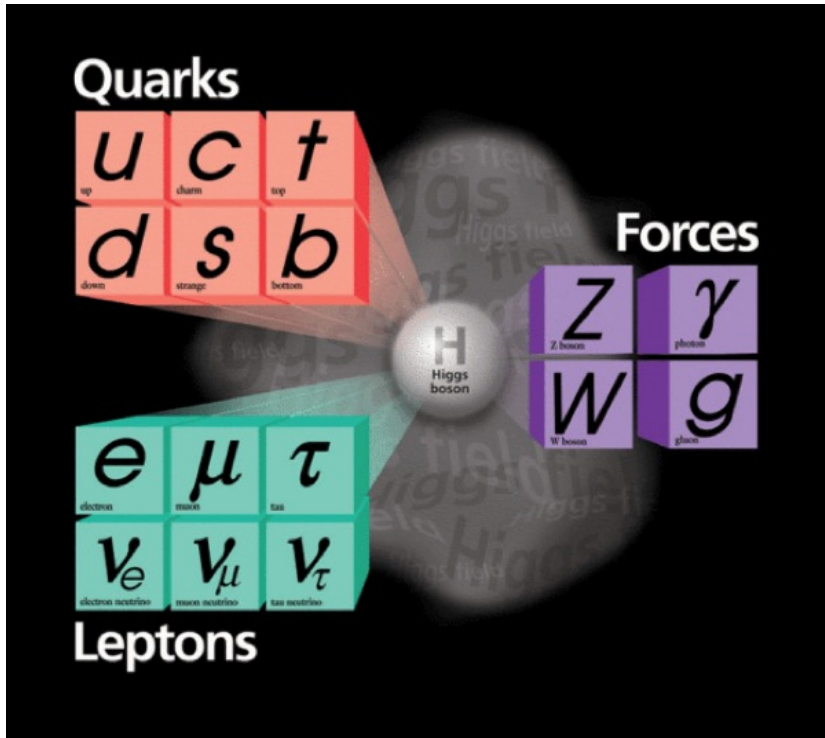


- New measurement achieved 0.13 ppt uncertainty on g
- $g_e = 2.002\,319\,304\,36(26)$
- Most stringent test of any fundamental particle parameter in the SM
- Beautiful test of QED, but sensitivity to new physics limited by uncertainty on α

Fan, Myers, Sukra,
& Gabrielse
arXiv:2209.18084



Muon g-2 for other charged leptons

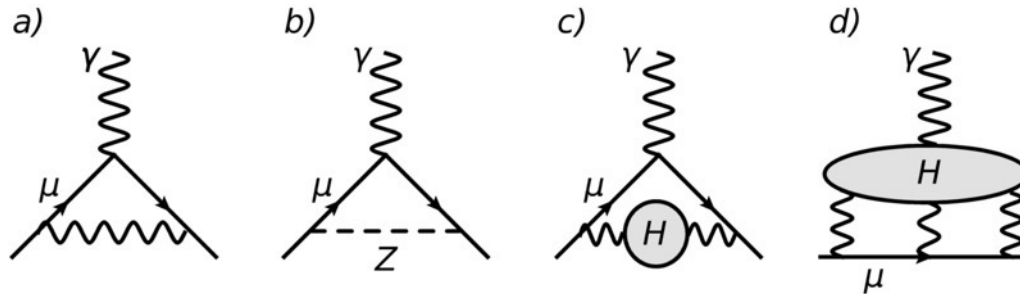


- Muon and tau are also good candidate for g-2 tests
- Sensitivity to higher masses entering loops, goes as m^2
- Muons are particularly good
 - Can be produced copiously

$$\text{BR}(\pi^\pm \rightarrow \mu^\pm \nu) = 99.9877\%$$
 - Relatively long lifetime $2.2 \mu\text{s}$

$$\lambda_{\text{sens}} \propto \left(\frac{m_\mu}{m_e} \right)^2 \approx 40,000$$

SM calculation observations on a_μ vs a_e



Source	$a_\mu \times 10^{-11}$	$a_e \times 10^{-11}$
a) QED	116 584 718.9 (0.1)	115 965 218.090 (0.070)*
b) EW	154 (1)	N/A
c) HVP	6845 (40)	0.16576 (0.00125)
d) HLBL	92 (18)	N/A

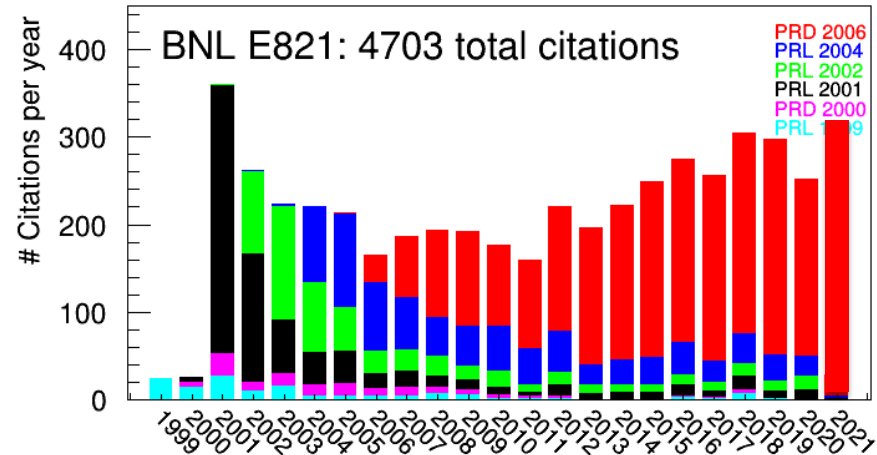
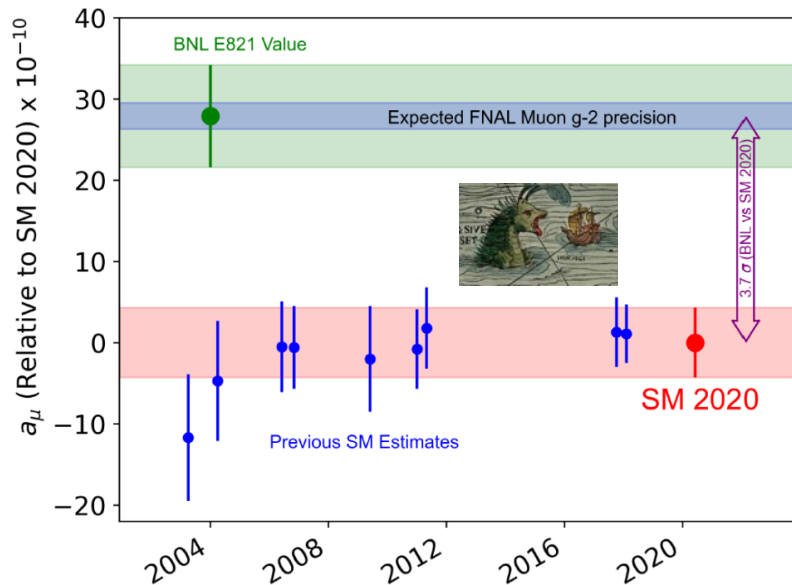
*Quoting the simple midpoint and range of Rb/Cs results for α

- $a_\mu(\text{HVP})/a_e(\text{HVP}) \simeq 41,300 \rightarrow$ very close to the m^2 scaling
- $a_e(\text{HVP})/\delta a_e(\text{exp}) \simeq 12.7 \leftrightarrow a_\mu(\text{HVP})/\delta \mu(\text{exp}) \simeq 167$
 \rightarrow shows why muon $g-2$ is so powerful for new physics
- Uncertainty in α negligible for g_μ (and g_τ)

A hint of new physics from BNL

- a_μ last measured 20 years ago at Brookhaven National Lab (BNL) where an interesting 2.7σ hint of new physics was discovered
 - Over time it grew to 3.7σ with improvements in theory

$$a_\mu = \frac{g - 2}{2}$$



Plus, >1000 FNAL PRL citations since April 2022 publication

- For scale, the difference is $\sim 270 \times 10^{-11}$, or $\sim 2x$ EW contribution

Bringing g-2 to Fermilab

- Goal: Bring the container used to hold the muons from BNL and couple it to Fermilab's powerful accelerator beam
- Reduce the overall error by a factor of 4 to **140 ppb**
 - 20x the muons → **100 ppb** stat error from (4.5x better)
 - systematics at the same **100 ppb** level (3x better)

Brookhaven Muon Storage Ring



Parts of the 50' diameter storage ring could not come apart!!

Storage ring transported by land/sea in 2013



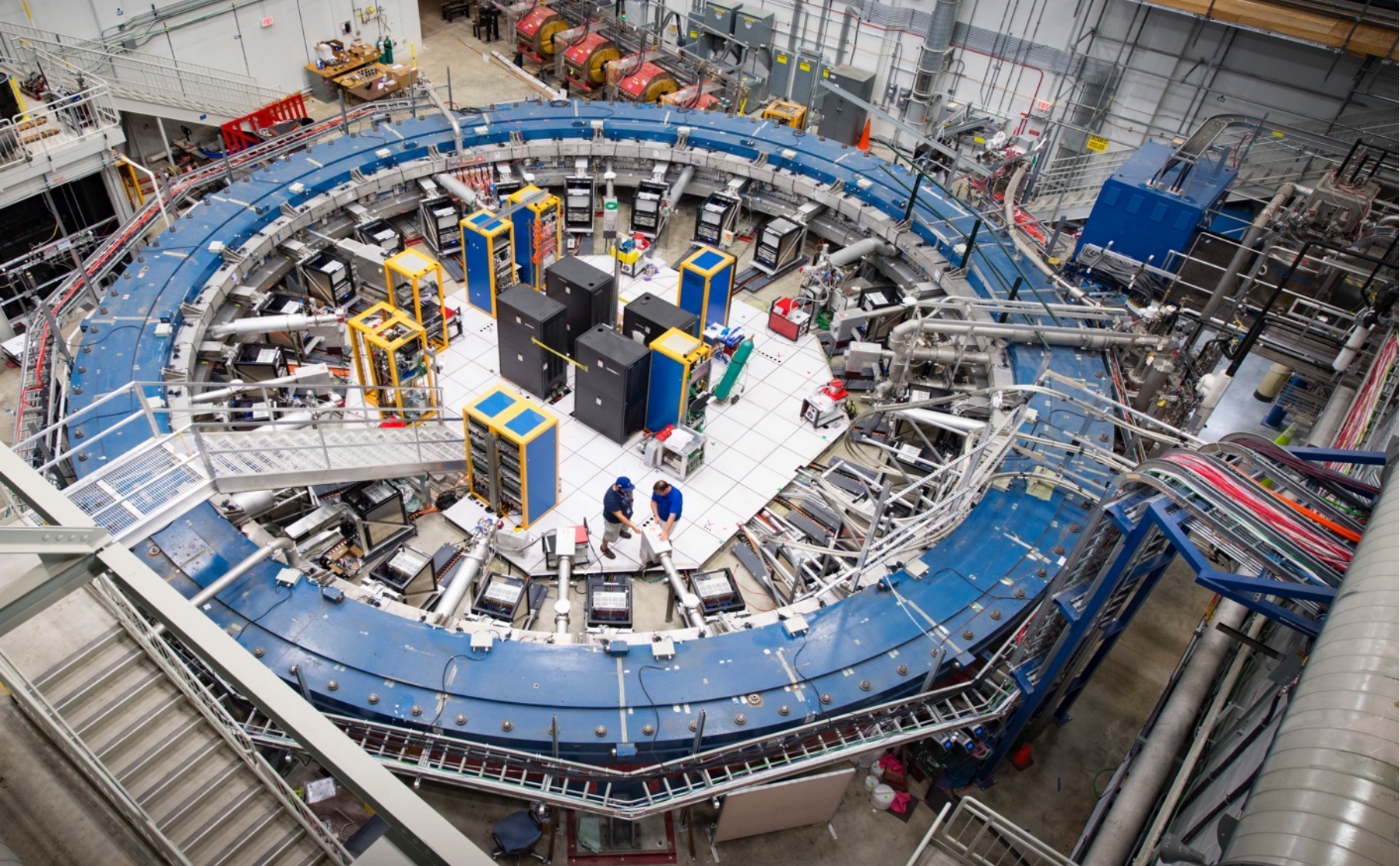
Including 30 miles of Chicago suburbs!



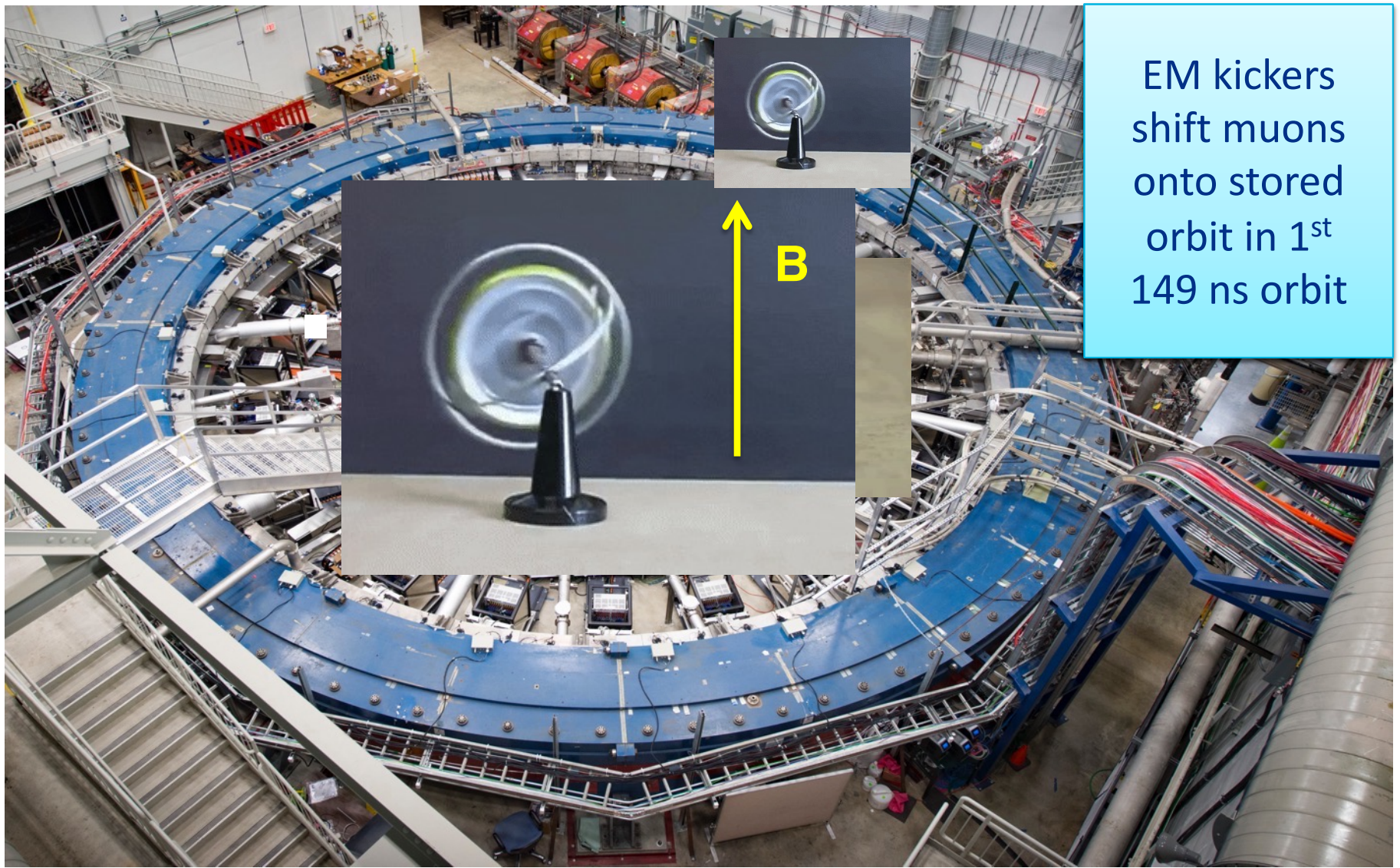
Amazing photo ops



All put back together at Fermilab!



All put back together at Fermilab!



EM kickers
shift muons
onto stored
orbit in 1st
149 ns orbit

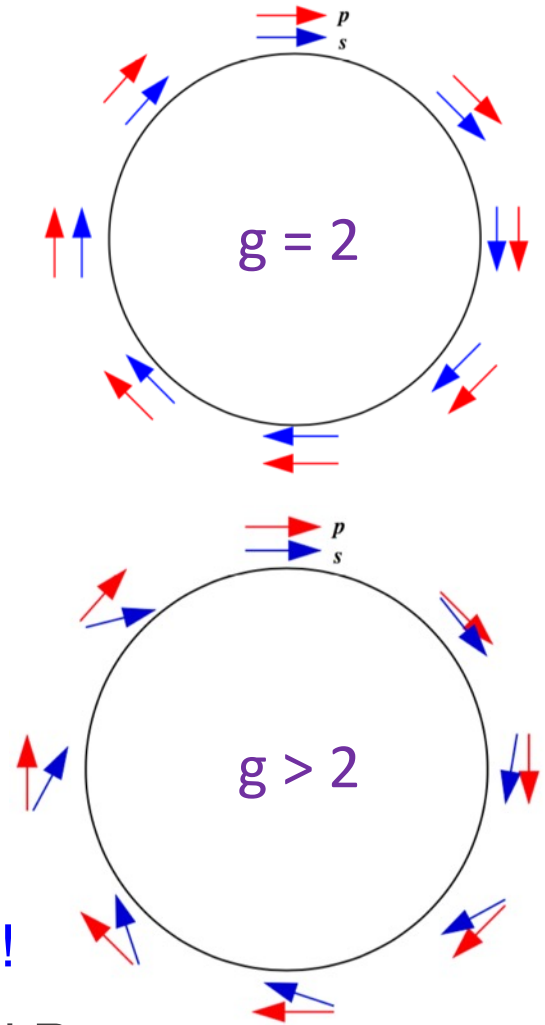
Why use a storage ring?

- The rate that the muon spin rotates, ω_s , with respect to the cyclotron frequency, ω_c , is given by

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = - \left(\frac{g_\mu - 2}{2} \right) \frac{q\vec{B}}{m} = -a_\mu \frac{q\vec{B}}{m}$$

- If $g = 2$ exactly, the spin and momentum vectors remain locked together $\rightarrow \omega_a = 0$
 - But $g = 2.0023\dots$ & $(g-2)/2 = 0.00116\dots$
- ω_a is directly proportional to a_μ
 - \rightarrow 800x more sensitive than expt at rest!
- To extract a_μ , we need to determine ω_a and B

$$a_\mu = \left(\frac{e}{m} \right)^{-1} \frac{\omega_a}{B}$$



Not quite as simple as $a_\mu = \left(\frac{e}{m}\right)^{-1} \frac{\omega_a}{B}$

- Full BMT equation \rightarrow spin precession modified by E-fields and non-perpendicular motion relative to B

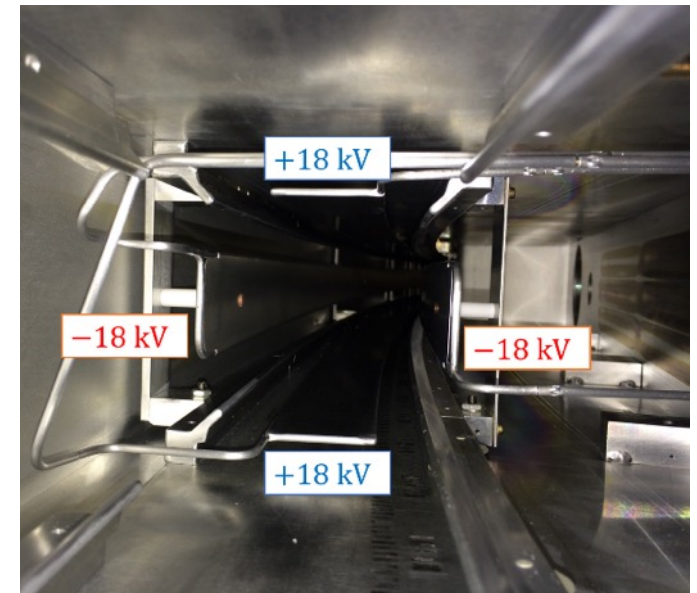
$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -\frac{q}{m_\mu} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

Pitch correction

Electric field correction

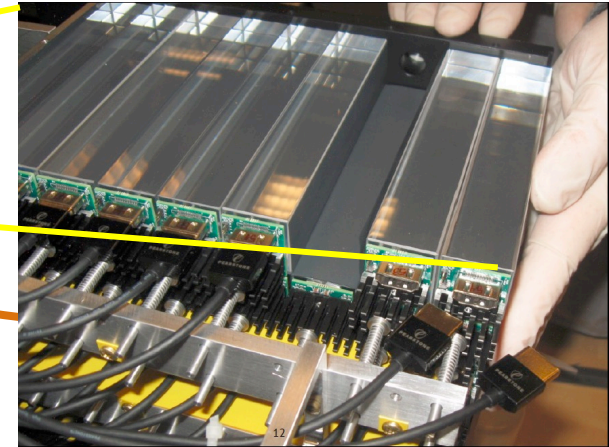
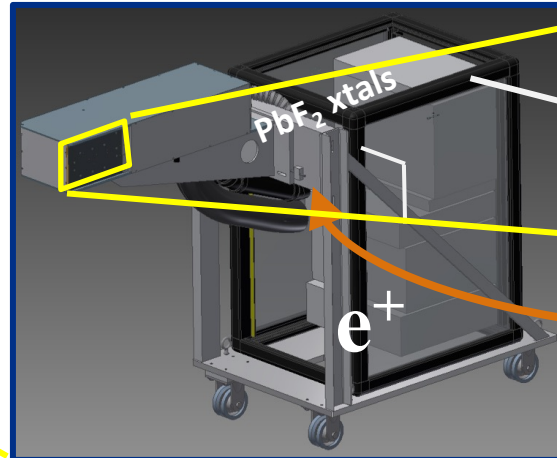
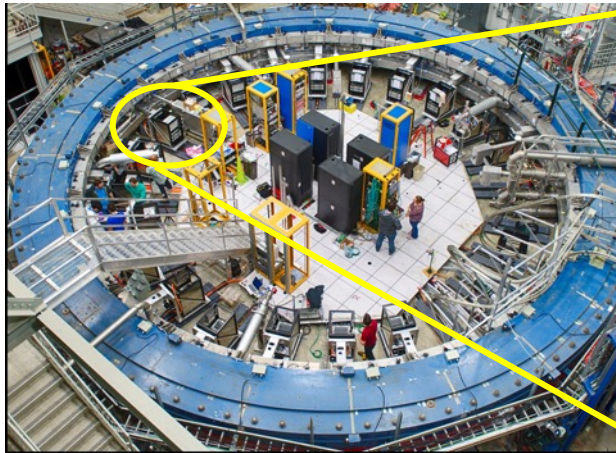
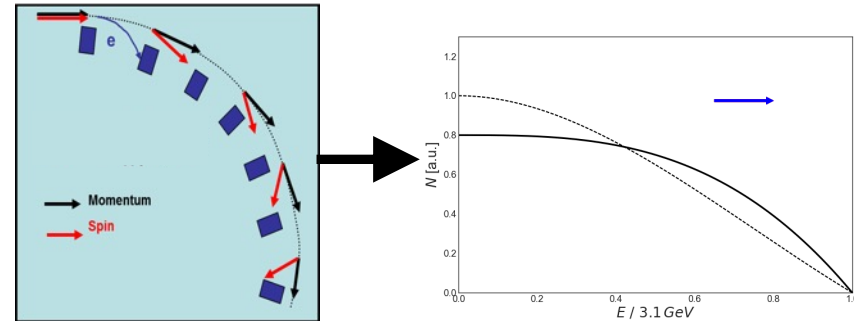
0, for $\gamma = 29.3$,
 $v = 99.94c$

- Experiment requires a quadrupole E-field to keep muon vertically confined \rightarrow horizontal and vertical harmonic coherent betatron oscillation (CBO)
- Choosing to run at the 'magic momentum' minimizes impact of E-field



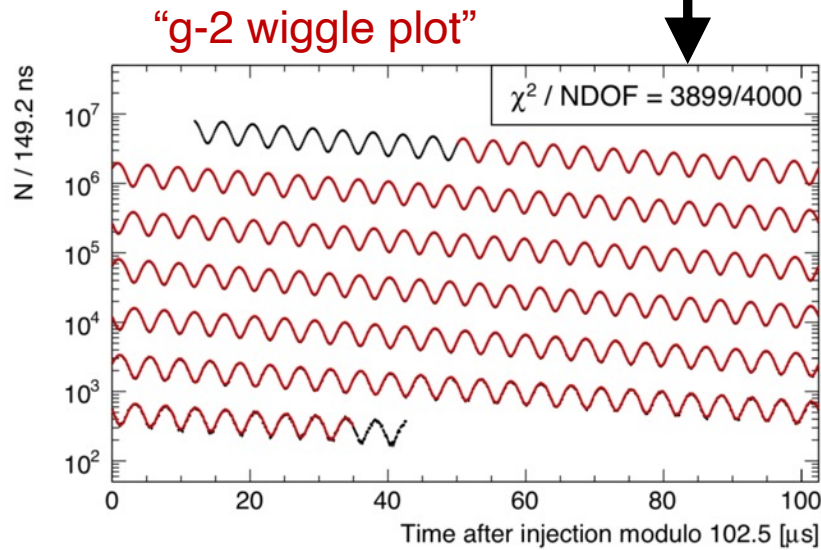
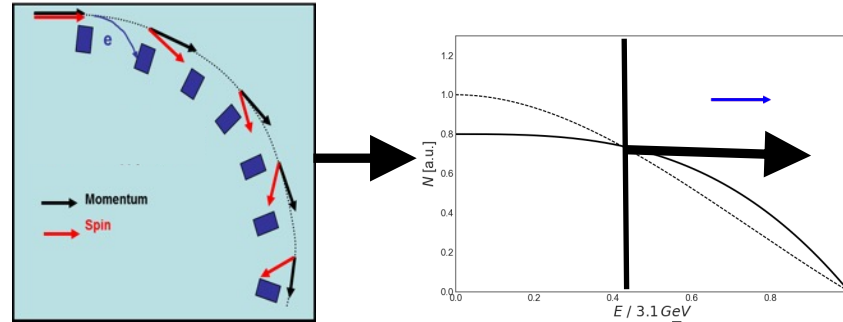
How do we measure a_μ ?

- Parity violation in muon decay \rightarrow high energy decay positrons are preferentially emitted in the muon spin direction
- Measure the energy spectrum with detectors around the inside of the ring



- Major upgrades from BNL:
 - 6x9 array to PbF₂ crystals allows us to spatially separate pileup with better Cerenkov timing relative to the PbSciFi monolithic BNL calorimeters
 - 800 MHz waveform digitizers sample at twice the rate
 - Modern computing bandwidth allows us to keep data down to 0 threshold (1 GeV at BNL)
 - Sophisticated laser systems allows us to monitor gain changes at 1 part in 10⁴

Generating the 'wobble plot'



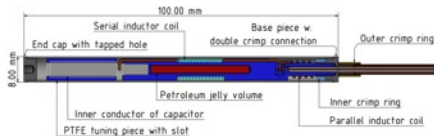
$$f(t) \simeq N_0 e^{-\lambda t} [1 + A \cos(\omega_{at} + \phi)]$$

We also need B at < 100 ppb to determine a_μ

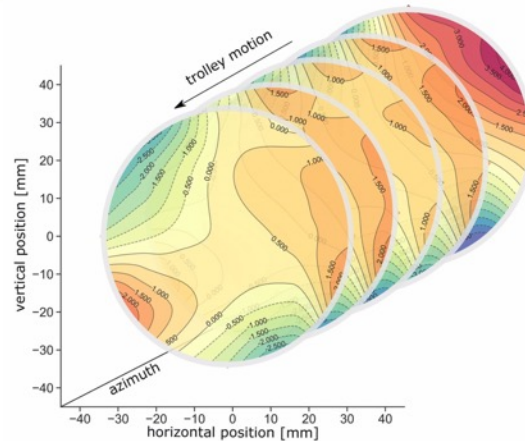
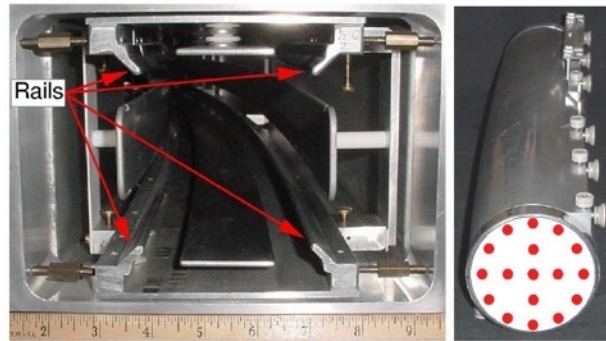
$$\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{mc}$$

- Use NMR to find B-field in terms of proton precession frequency ω_p (comagnetometer)

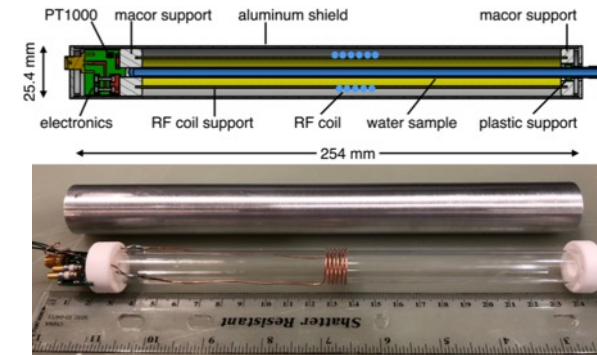
378 fixed probes monitor 24/7



NMR trolley maps field every 3 days



Trolley cross-calibrated to absolute probes

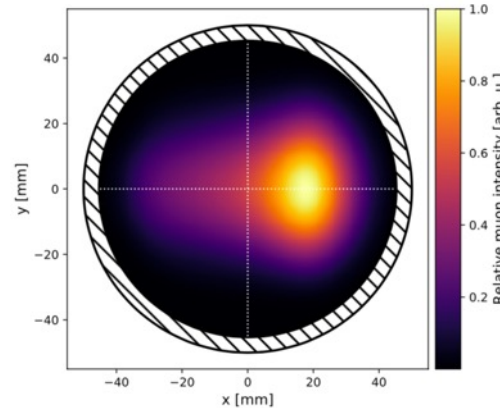
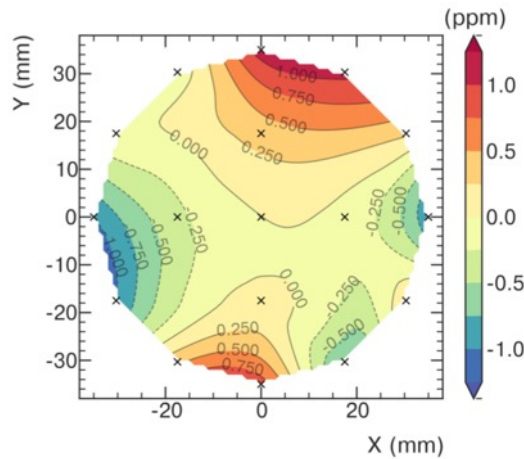
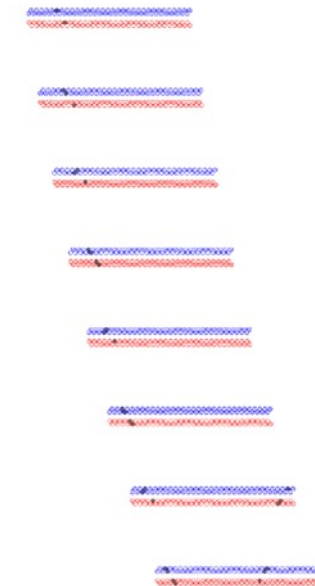
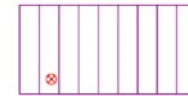
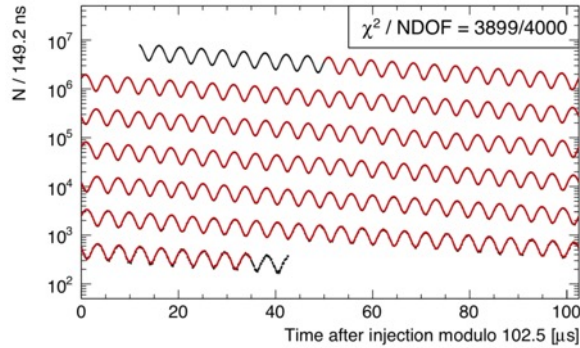


Absolute probes all cross-calibrated at ANL test magnet

- Shimmed field 3x better, hall temp to +/- 1C, all field systems improved

The analysis 'big' picture

$$\frac{\omega_a}{\omega_p \otimes \rho(r)} \Rightarrow$$



*All plots actual Run 1 data

- *In vacuo* straw trackers tell us the spatial distribution and many other muon beam properties (CBO, p-dist)
 - Also a major addition compared with BNL

Systematic error progress - ω_a

Run 1 stat
error 434 ppb

	BNL actual [ppb]	FNAL TDR [ppb]	FNAL Run 1 [ppb]
Gain + residuals	120	20	19
Pileup	80	40	37
Lost muons	90	20	5
CBO	70	30	40
E-field/pitch	50	30	55
Phase acceptance	N/A	N/A	75
Total	180	70	108

*Run 1 ω_a systematics are simple averages over 4 data sets, correlations approximate, BNL \leftrightarrow FNAL mapping not perfect but close enough

- 1st publication (Run 1) \sim 6% total stats, many systematics errors scale down with stats
- **CBO** driven by increased amplitude due to poor kick in Run 1&2, reduced x2 with kicker upgrades by Run 3
- **E-field/pitch** driven by impact of time/momentum correlations of the muon bunch at injection, will improve with better simulation and measurements
- **Phase acceptance** primarily due to failed quad resistors that led to beam instability, fixed in Run 2 and beyond
- **On track to beat 70 ppb goal from Run 2 and beyond!**

Systematic error progress - B

Run 1 stat
error 434 ppb

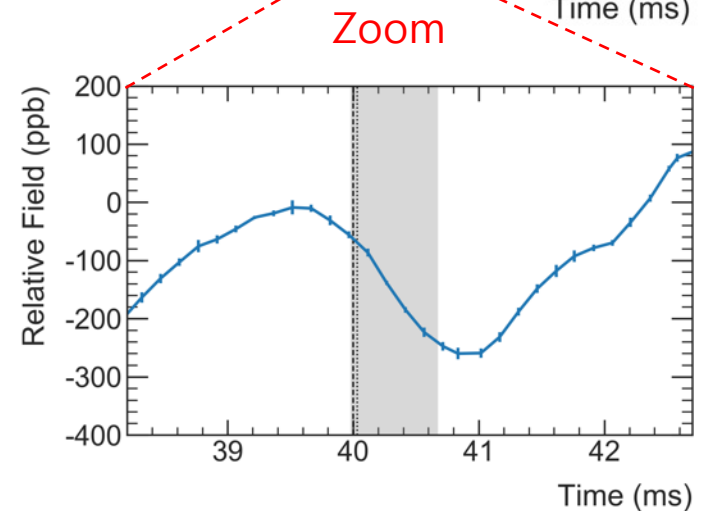
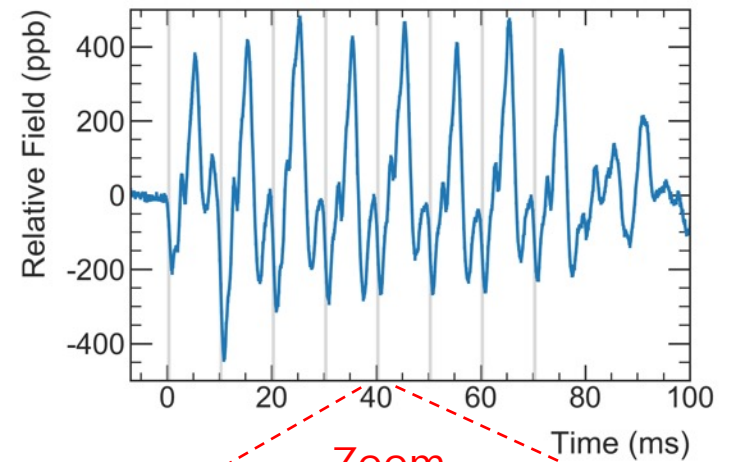
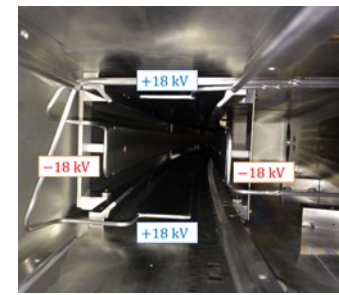
	BNL actual [ppb]	FNAL TDR [ppb]	FNAL Run 1 [ppb]
Trolley calibration	90	30	32
Trolley B measurements	50	30	25
Fixed probe tracking	70	30	23
Muon weighting	30	10	20
Absolute calibration	50	35	19
Configuration	Under other	Under other	23
Kicker transients	Under other	Under other	37
Quad transients	Under other	Under other	92
Other	100	50	negligible
Total	170	70	114

*BNL \leftrightarrow FNAL mapping not perfect but close enough

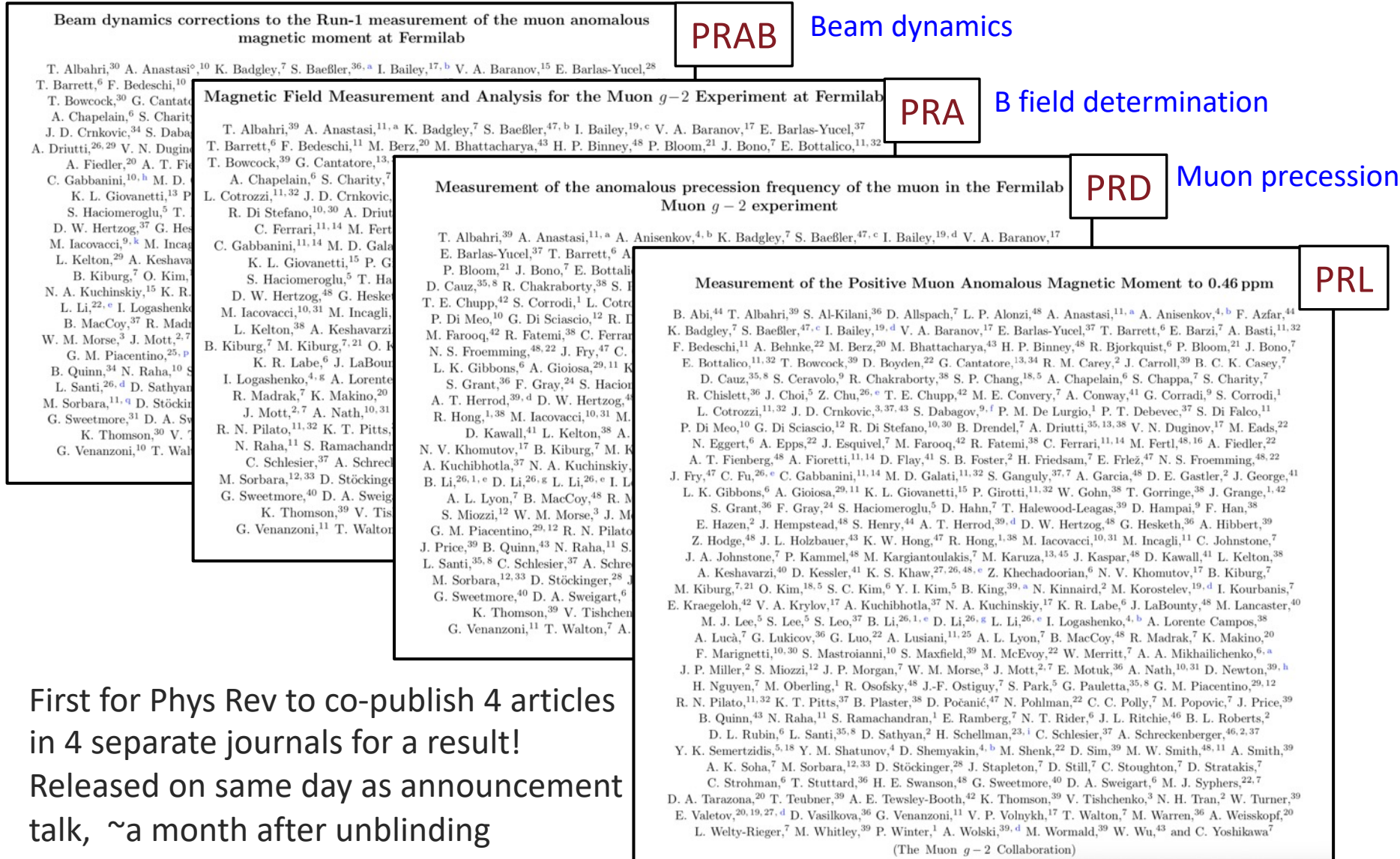
- **Trolley calibration** improves with more calibrations, trolley NMR sample temperature dependence better determined
- **Muon weighting** will improve due to better centered beam (kicker upgrade)
- **Kicker/quad transients** reduced to < 30 ppb with better mapping for Run 2 and beyond
- On track to beat 70 ppb goal from Run 2 and beyond!

B_q – Quad transients

- Recall, E- field keeps muons vertically confined
- Quads pulsed \rightarrow induces mech. vibrations \rightarrow oscillating conductor perturbs B field
 - Deliver 8 muon bunches with 10 ms spacing \rightarrow 3x closer to 100 Hz natural resonance than BNL
- Built special NMR probes to map the effect
 - Long process to make measurements
- Overall correction is 17 ppb
 - Only matters in window when muons are present, averaged over 8 bunches, averaged over 43% of ring with quad coverage
- 92 ppb Run 1 uncertainty is dominated by not having a complete map for Run 1
 - Analysis of more complete map is nearly done
 - Expect uncertainty to be reduced x3 for Run 2 and beyond



Four articles on arXiv and published in Phys Rev



- First for Phys Rev to co-publish 4 articles in 4 separate journals for a result!
- Released on same day as announcement talk, ~a month after unblinding



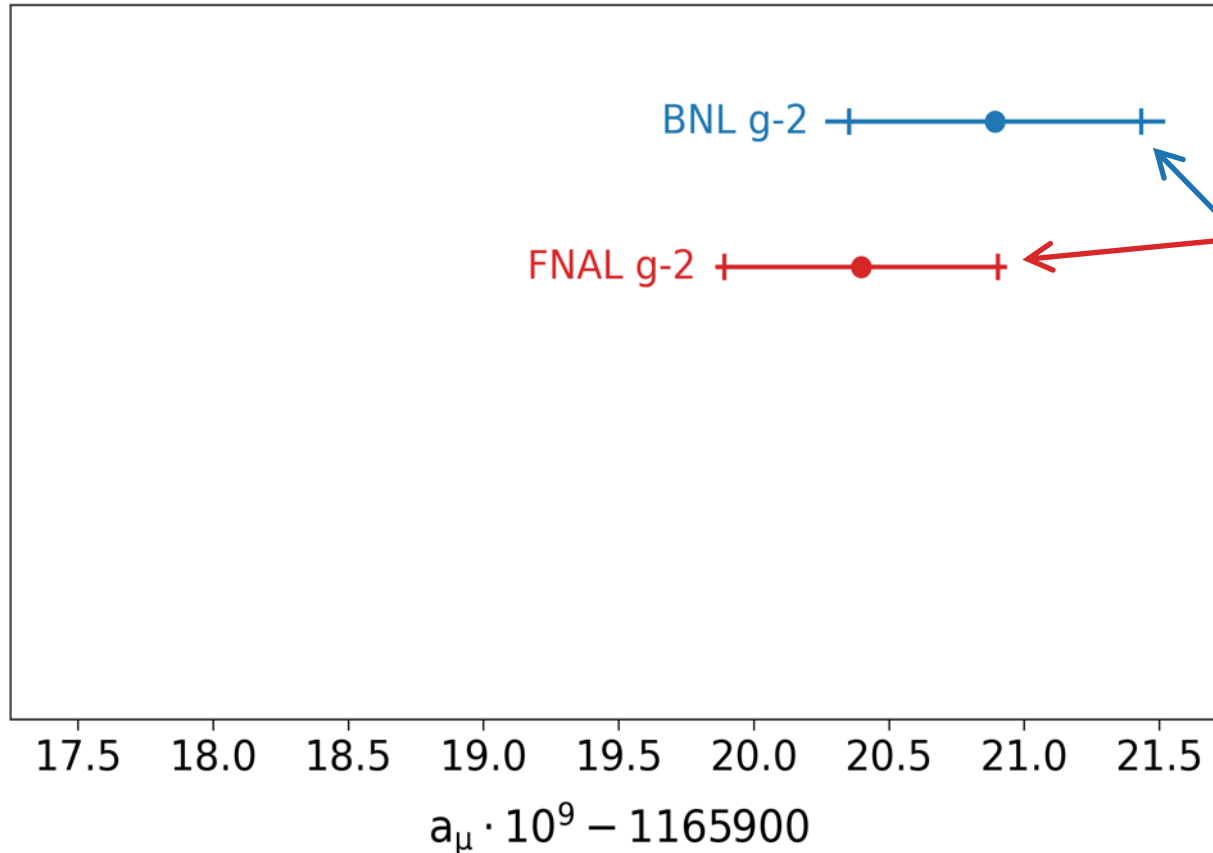
Final uncertainties from Run 1

Quantity	Correction Terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	–	434
ω_a^m (systematic)	–	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}} \langle \omega_p(x, y, \phi) \times M(x, y, \phi) \rangle$	–	56
B_k	-27	37
B_q	-17	92
$\mu'_p(34.7^\circ)/\mu_e$	–	10
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	157
Total fundamental factors	–	25
Totals	544	462

- 462 ppb overall error
 - 434 ppb statistical
 - 157 ppb systematic
 - 25 ppb CODATA inputs
- Results for Run 1 are vastly dominated by statistical error
- At 157 ppb systematic error
 - Nearly half of BNL
 - Not quite to 100 ppb goal
- Project getting to < 80 ppb systematic control with next publication, with stat error nearly reduced by x2

Run 1 result

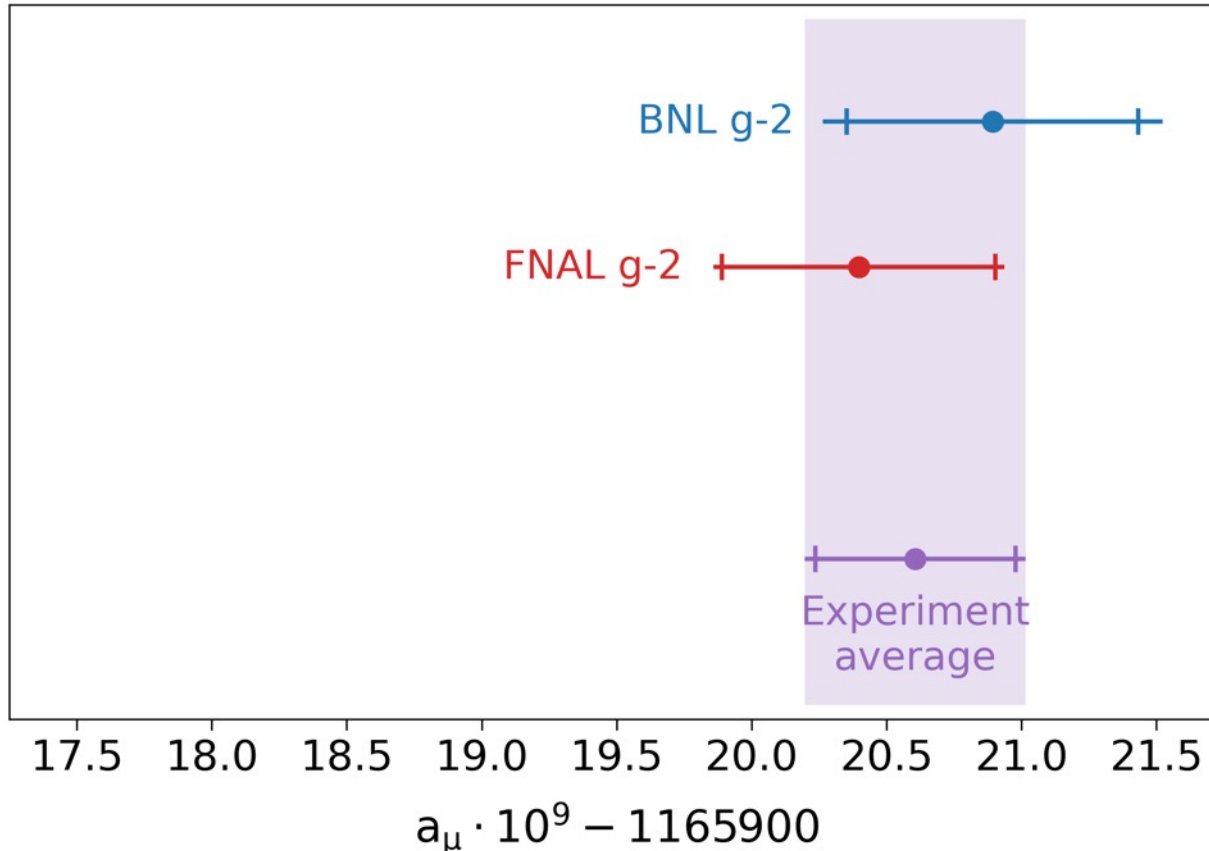
$$a_\mu(\text{FNAL g-2; Run 1}) = 0.00116592040(54) \rightarrow 463 \text{ ppb}$$



- 15% smaller error than BNL
- Both experiments dominated by statistical error
- Good agreement \rightarrow safe to combine

Experimental combination

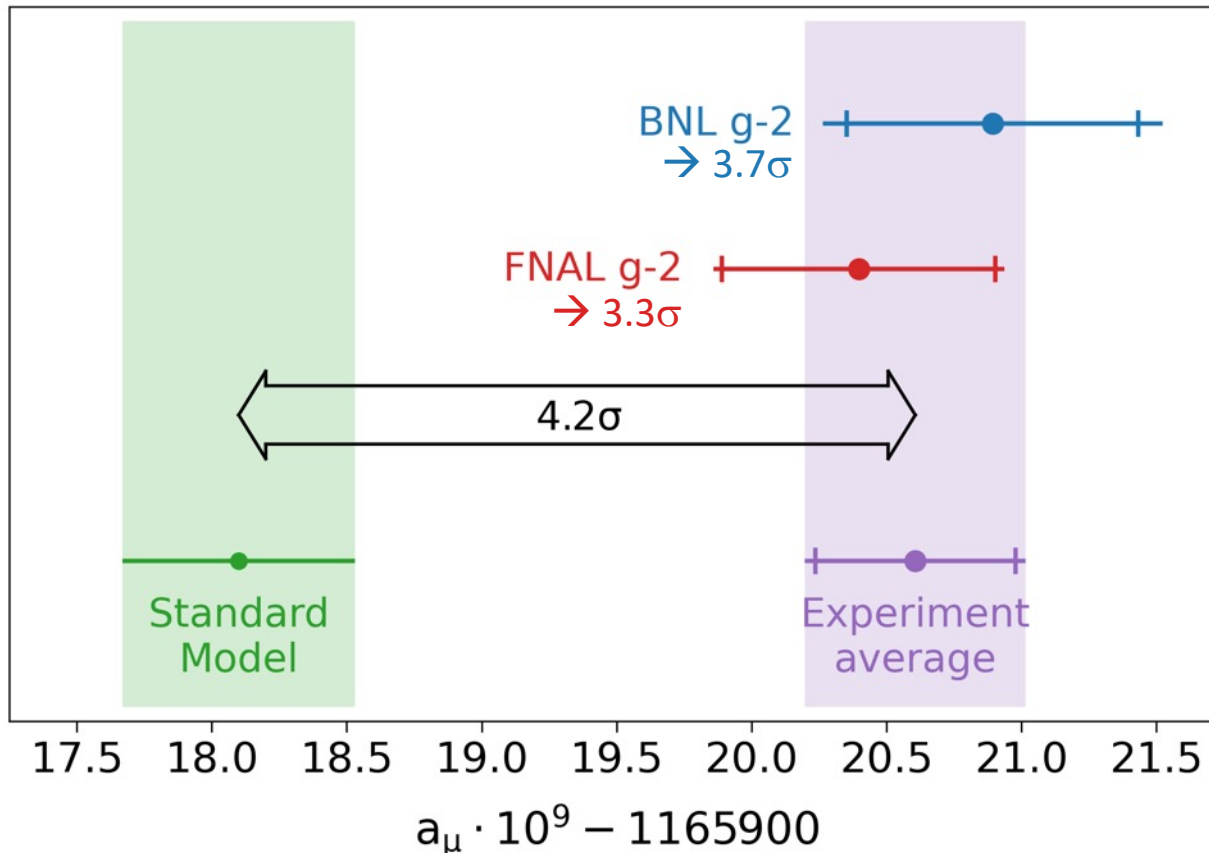
$$a_\mu(\text{Exp}) = 0.00116592061(41) \rightarrow 350 \text{ ppb}$$



- 15% smaller error than BNL
- Both experiments dominated by statistical error
- Good agreement \rightarrow safe to combine

Comparison to SM ($e^+e^- \rightarrow \text{hadrons}$)

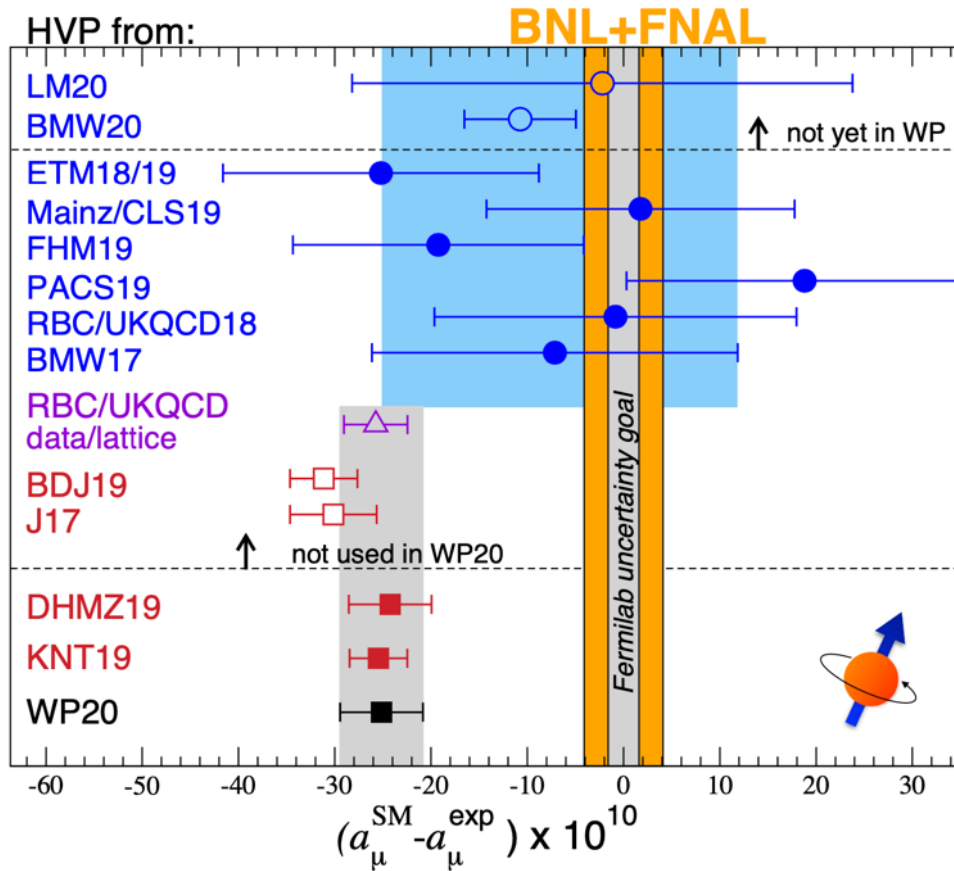
$$a_\mu(\text{SM}) = 0.00116591810(43) \rightarrow 368 \text{ ppb}$$



- Individual tension with SM
 - BNL: 3.7σ
 - FNAL: 3.3σ

$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = 0.00000000251(59) \rightarrow 4.2\sigma$$

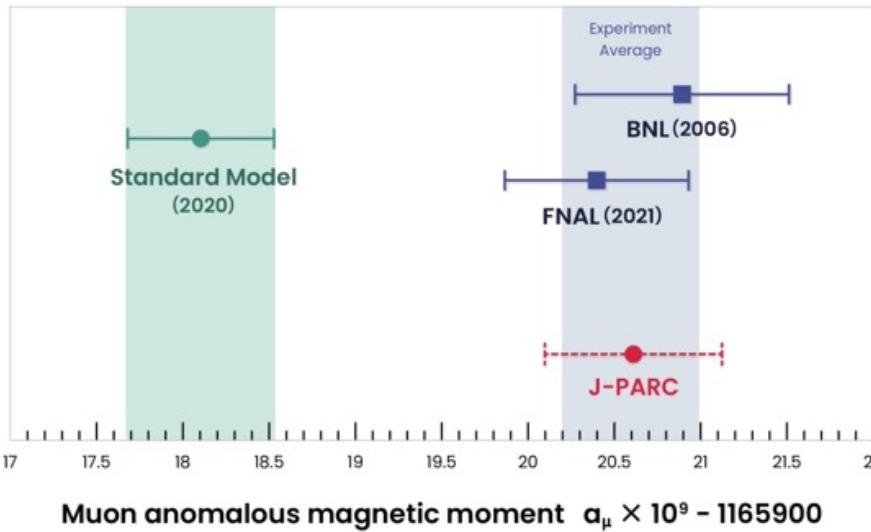
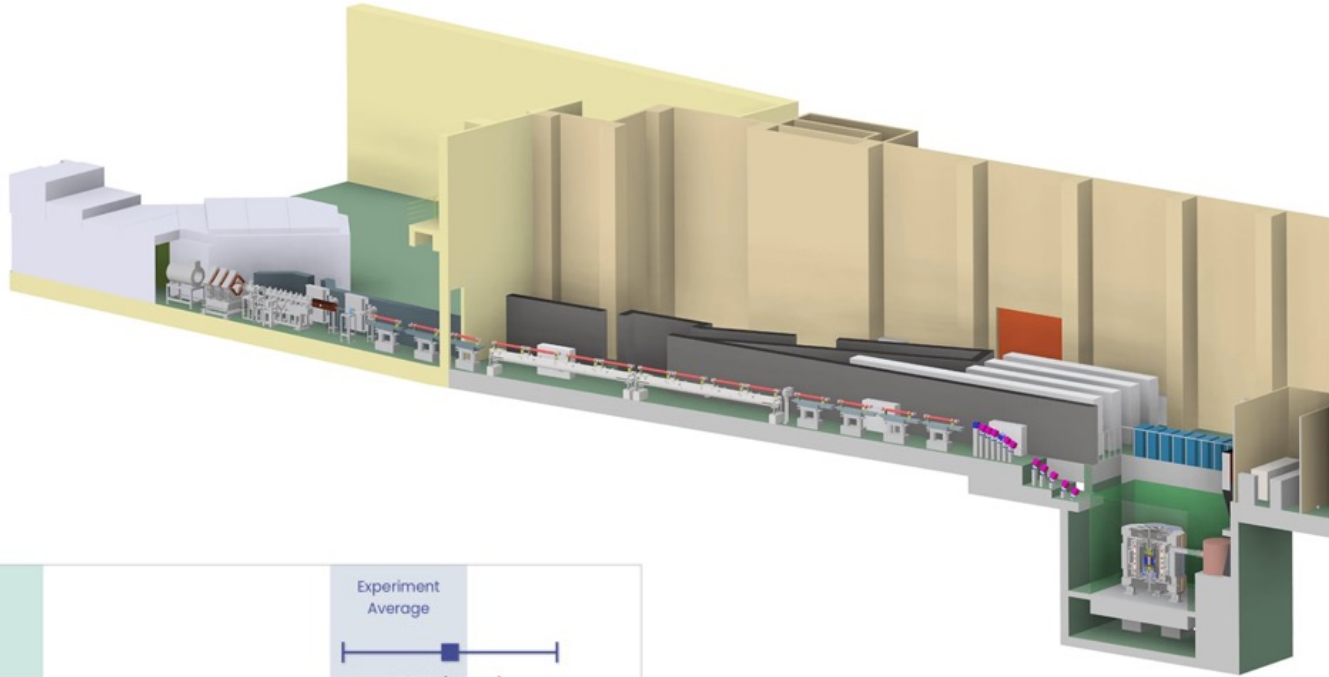
Next steps for Muon g-2



- Lattice calculations making rapid progress and show tension with data-driven determination of a_{μ} (HVP)
- FNAL expt in final run this year and project meeting or exceeding 140 ppb goal

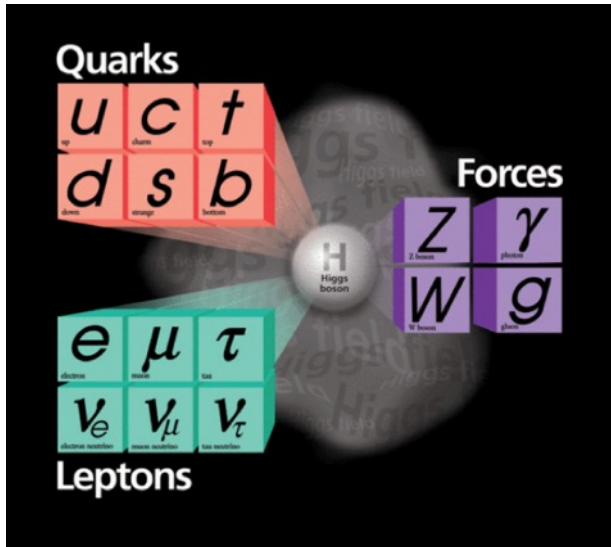
Plot from Muon g-2 Theory Initiative
(consortium of 100+ theorists)

Muon g-2 at J-PARC



- Complementary technique
 - μ beam accelerated from rest
 - no E fields
 - smaller magnet
- Aiming for a result comparable to current results towards the end of the decade

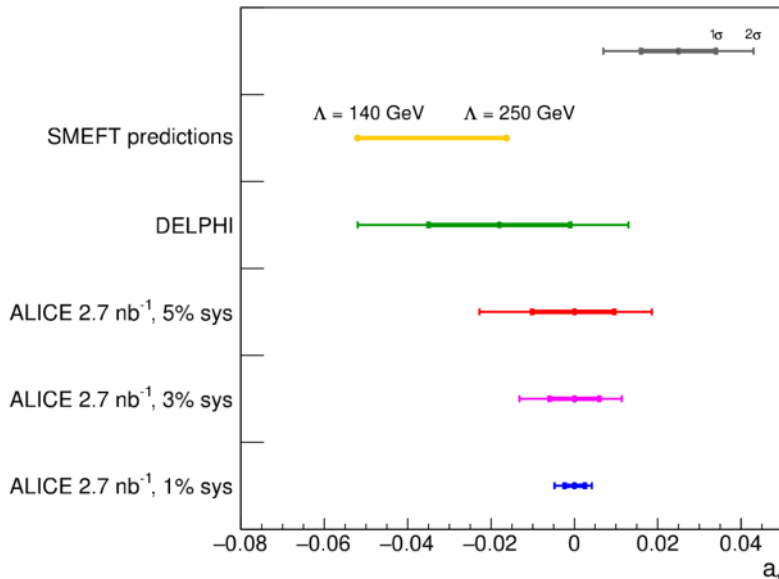
Muon $g-2$ from tau



- m^2 scaling makes $\tau \sim 280x$ more sensitive to heavy particles appearing in loops
 - Need 40 ppm a_τ to compete w/ 140 ppb a_μ
- But 290 ps lifetime precludes standard spin precession technique
 - Relies on measuring coupling strength to photons and looking for a deviation in cross-section

- Best limits from DELPHI

$$-0.052 < a_\tau < 0.013 \text{ (95\%C.L.)}$$



EPJ Web of Conferences **262**, 01021 (2022)

Conclusions

- Magnetic moments are fantastic laboratories for testing our Standard Model
- Charged leptons provide unique sensitivity
- Recent FNAL muon $g-2$ results confirm the BNL experiment at 450 ppb precision
- FNAL experiment nearly has statistics on tape needed to reach final 140 ppb goal and is in final (6th) year of running
 - Next result with 3-4x stats and < 100 ppb systematics expected this spring

(The Muon $g - 2$ Collaboration)

¹Argonne National Laboratory, Lemont, IL, USA

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Thank you!

We rely on others for e/m and absolute H₂O calib

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

ω_a : the muon spin precession frequency

$\tilde{\omega}'_p(T_r)$: precession of protons in water sample mapping the field and weighted by the muon distribution

Goal: 140 ppb =
100 ppb (stat) \oplus 100 ppb (syst)

$\tilde{\omega}'_p(T)$

Proton Larmor precession frequency in a spherical water sample. Temperature dependence known to < 1 ppb/°C.
[Metrologia 13, 179 \(1977\)](#), [Metrologia 51, 54 \(2014\)](#),
[Metrologia 20, 81 \(1984\)](#)

$\frac{\mu_e(H)}{\mu'_p(T)}$

Measured to 10.5 ppb accuracy at T = 34.7°C
[Metrologia 13, 179 \(1977\)](#)

$\frac{\mu_e}{\mu_e(H)}$

Bound-state QED (exact)
[Rev. Mod. Phys. 88 035009 \(2016\)](#)

$\frac{m_\mu}{m_e}$

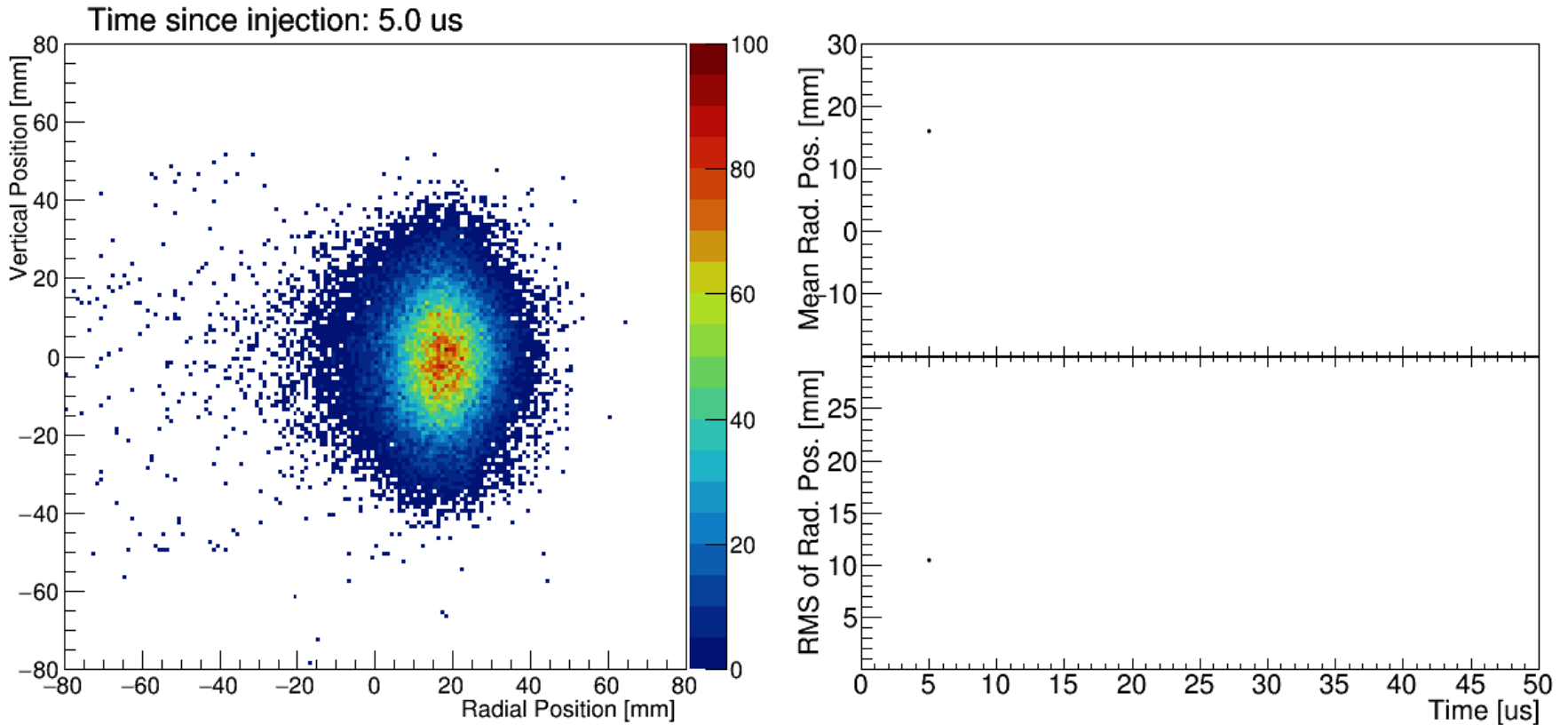
Known to 22 ppb from muonium hyperfine splitting
[Phys. Rev. Lett. 82, 711 \(1999\)](#)

$\frac{g_e}{2}$

Measured to 0.28 ppt
[Phys. Rev. A 83, 052122 \(2011\)](#)

All < 22 ppb

Imaging CBO with the trackers



- The *in vacuo* straw trackers give us a much better understanding of beam-related systematic than BNL.

But wait, there's more...

$$\frac{\omega_a}{\tilde{\omega}_p} = \left(\frac{f_{\text{clock}} \omega_a (1 + C_e + C_p + C_{\text{ml}} + C_{\text{pa}})}{(1 + B_{\text{QT}} + B_{\text{Eddy}}) f_{\text{field}} \omega_p \otimes \rho(\mathbf{r})} \right)$$

E-field & pitch corrections
Muon loss & phase acceptance corrections

Field transients
Field calibration

- Every one of these terms has been studied in extraordinary detail. How much?

Systematics (numerator)

Source	Uncertainty
Frequency Standard	1 ppt
Frequency Synthesizers	0.1 ppb
Digitization Frequency	2 ppb
Total Systematic	2 ppb

Data Set	Run-1a	Run-1b	Run-1c	Run-1d
C_{pa}	-184	-165	-117	-164
Stat. uncertainty	23	20	15	14
Tracker & CBO	73	43	41	44
Phase maps	52	49	35	46
Beam dynamics	27	30	22	45
Total uncertainty	96	74	60	80

$R(\omega_a)$ with detailed systematics categories [ppb]				
Total systematic uncertainty	65.2	70.5	54.0	48.8
Time randomization	14.8	11.7	9.2	6.9
Time correction	3.9	1.2	1.1	1.0
Gain	12.4	9.4	8.9	4.8
Pileup	39.1	41.7	35.2	30.9
Pileup artificial dead time	3.0	3.0	3.0	3.0
Muon loss	2.2	1.9	5.2	2.4
CBO	42.0	49.5	31.5	35.2
Ad-hoc correction	21.1	21.1	22.1	10.3

*Run 1 ω_a data analyzed in four subsets

	1a	1b	1c	1d
C_p (ppb)	176	199	191	166
Statistical uncertainty	<0.1	<0.1	<0.1	<0.1
Tracker alignment/reco.	11.0	12.3	12.0	10.7
Tracker res. & acc. removal	3.3	3.9	3.7	3.0
Azimuthal avg. & calo. acc.	1.0	1.3	2.2	1.1
Amplitude fit	1.2	0.4	1.0	2.9
Quad alignment/voltage	4.4	4.4	4.4	4.4
Systematic uncertainty	12.4	13.7	13.6	12.3

Data Set	Run-1a	Run-1b	Run-1c	Run-1d
C_{ml}	-14	-3	-7	-17
Phase-momentum	2	0	1	3
Form of $l(t)$	2	0	1	1
f_{loss} function	2	1	2	2
Linear sum ($\sigma_{C_{ml}}$)	6	2	4	6

	1a	1b	1c	1d
C_e (ppb)	471	464	534	475
Statistical uncertainty	0.4	0.5	0.4	0.2
Fourier method	8.4	13.4	14.4	3.9
Momentum-time correlation	52	52	52	52
Quad alignment/voltage	6.4	6.4	6.4	6.4
Field index	1.7	1.5	1.7	4.0
Systematic uncertainty	53	54	54	53

Systematics (denominator)

run-1 (substructure)	77.4 ppb
azimuthal shape*	7.6 ppb
skin depth	12.6 ppb
frequency extraction (0.4/1ms)	4.6 ppb
Q3L: fit, position	1.5 ppb
repeatability	13.3 ppb
drift	10.2 ppb
radial dependency	4.4 ppb
2 nd 8-pulses	14.0 ppb
total -15.0 ppb	81.7 ppb

Source	Uncertainty (ppb)
Temperature	15 – 28
Configuration	22
Trolley	25
Fixed Probe Production	<1
Fixed Probe Baseline	8
Tracking Drift	22 – 43
Total	43 – 62

PROBE	Calibration Coefficients		
	Value (Hz)	Stat (Hz)	Syst (Hz)
1	90.81	0.38	2.02
2	84.21	0.65	1.18
3	95.02	0.53	2.19
4	86.03	0.25	1.28
5	92.96	0.51	1.10
6	106.24	0.46	1.35
7	116.64	0.96	1.61
8	76.39	0.60	1.21
9	83.52	0.23	1.64
10	24.06	1.39	1.26
11	177.55	0.22	1.99
12	110.85	0.44	1.73
13	122.89	2.08	1.93
14	77.11	0.53	1.88
15	74.82	1.06	1.59
16	20.35	0.44	2.94
17	172.12	1.23	1.96
AVG		0.70	1.70

Quantity	Symbol	Value	Unit
Diamagnetic Shielding T dep	$(1/\sigma)d\sigma/dT$	-10.36(30)	ppb/°C
Bulk Susceptibility	δ_b	-1504.6 ± 4.9	ppb
Material Perturbation	δ_s	15.2 ± 13.3	ppb
Paramagnetic Impurities	δ_p	0 ± 2	ppb
Radiation Damping	δ_{RD}	0 ± 3	ppb
Proton Dipolar Fields	δ_d	0 ± 2.3	ppb

Run-1 Estimate:
 $B_k = -27.4 \pm 37$ ppb

Dataset	correction [ppb]				uncertainty [ppb]			
	1a	1b	1c	1d	1a	1b	1c	1d
1. Tracker and calo effects	-	-	-	-	9.2	13.3	15.6	19.7
2. COD effects	1.6	1.5	1.7	1.4	5.2	4.7	5.2	4.9
3. In-fill time effects	-1.9	-2.3	-1.2	-4.1	-	-	-	-
Total	-0.3	-0.8	0.5	-2.7	10.6	14.1	16.5	20.3