The muon $g-2$ in the standard model and a lattice QCD calculation of the HVP contribution

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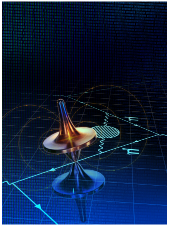
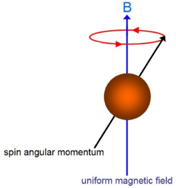


Budapest-Marseille-Wuppertal collaboration [BMWc] – Borsanyi, Fodor, Guenther, Hoelbling, Katz, LL, Lippert, Miura, Szabo, Parato, Stokes, Toth, Torok, Varnhorst

Nature 593 (2021) 51, online 7 April 2021 → BMWc '20
PRL 121 (2018) 022002 (Editors' Selection) → BMWc '17
Aoyama et al., Phys. Rep. 887 (2020) 1-166 → WP '20

Introduction and motivation

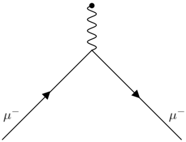
- Muon behaves like a tiny magnet with dipole moment



$$\vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu}\vec{S}$$

Leading order SM:

$$g_\mu = 2$$

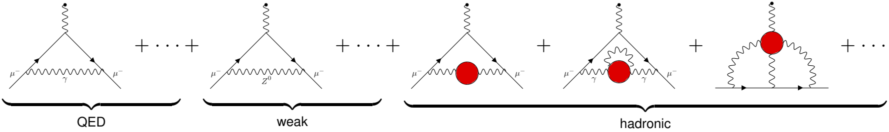


- Quantity of interest is the *anomalous* contribution

$$a_\mu = \frac{g_\mu - 2}{2}$$

$\rightarrow$ given quantum corrections (loops)

- a_μ can be measured very precisely [Chris Polly]
- a_μ can be computed equally precisely in the SM [this talk]

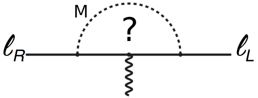


Introduction and motivation

Big question:

$$a_\mu^{\text{exp}} = a_\mu^{\text{SM}}?$$

- YES $\rightarrow$ another success for the SM *(at given level of precision)*
- NO $\rightarrow$ some new fundamental physics must be contributing to a_μ^{exp}, e.g.



- Complementarity w/ *direct* searches (e.g. LHC): may be sensitive to dofs that are too massive or too weakly coupled to be produced or measured directly today
- Complementarity w/ other *indirect* searches (FCNCs (e.g. in s and b decays), EDMs, . . .)

  $\rightarrow$ a_μ is flavor & CP conserving and chirality flipping ($L \leftrightarrow R$)

  $\Rightarrow$ probes mass generating mechanism of the theory

Introduction and motivation

- Chirality-flipping transition associated w/ contribution of particle w/ $M \gg m_\mu$ is generically

$$a_\mu^{\mathrm{M}} = C \left(\frac{\Delta_{LR}}{m_\mu}\right) \left(\frac{m_\mu}{M}\right)^2$$

- In EW theory, $M = M_W$, chirality flipping from Yukawa, i.e.

$$\Delta_{LR} = m_\mu \qquad \text{and} \qquad C \sim \frac{\alpha}{4\pi \sin^2\theta_W}$$

- In BSM, can have chiral enhancement:
  - SUSY: $C \sim \alpha/\left(4\pi \sin^2\theta_W\right)$, $M = M_{\mathrm{SUSY}}$ & $\Delta_{LR} = (\mu/M_{\mathrm{SUSY}}) \times \tan\beta \times m_\mu$
  - Radiative m_μ models: $\Delta_{LR} \simeq m_\mu$, $C \sim 1$ and $M = M_{\mathrm{N}\Phi}$
  - ...

[Chris Polly →] $a_\mu^{\mathrm{exp}} = 0.00116592061(41)$ [0.35 ppm] [BNL'06 & Fermilab '21]

Reference standard model calculation of a_μ

[WP '20 = Aoyama et al., Phys. Rep. 887 (2020) 1-166]

At needed precision: all three interactions and all SM particles

$$\begin{aligned} a_\mu^{\text{SM}} &= a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{EW}} \\ &= O\left(\frac{\alpha}{2\pi}\right) + O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) + O\left(\left(\frac{e}{4\pi \sin\theta_W}\right)^2 \left(\frac{m_\mu}{M_W}\right)^2\right) \\ &= O\left(10^{-3}\right) + O\left(10^{-7}\right) + O\left(10^{-9}\right) \end{aligned}$$

QED contributions to a_ℓ

Loops with only photons and leptons: can expand in $\alpha = e^2/(4\pi) \ll 1$

$$a_\ell^{\rm QED} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

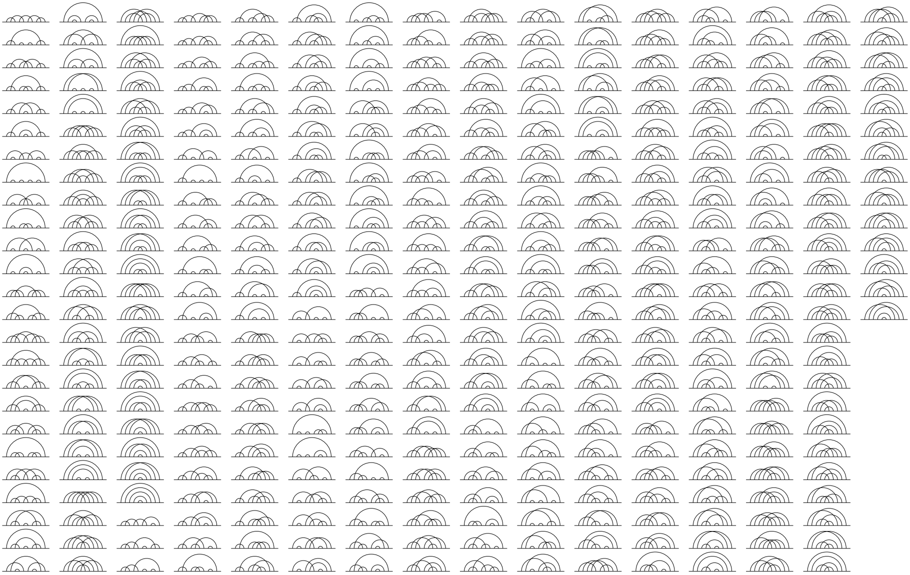
$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

- $A_1^{(2)}, A_1^{(4)}, A_1^{(6)}, A_2^{(4)}, A_2^{(6)}, A_3^{(6)}$ known analytically (Schwinger '48; Sommerfield '57, '58; Petermann '57; ...)
- $O((\alpha/\pi)^3)$: 72 diagrams (Laporta et al '91, '93, '95, '96; Kinoshita '95)
- $O((\alpha/\pi)^4; (\alpha/\pi)^5)$: 891;12,672 diagrams (Laporta '95; Aguilar et al '08; Aoyama, Kinoshita, Nio '96-'18)
  - Automated generation of diagrams
  - Numerical evaluation of loop integrals
  - Not all contributions are independently checked
  - One recent check (S. Volkov '19) gives for contribution w/ no lepton loops (nll)

$$A_1^{(10)}|^{\rm nll}_{\rm AKN} = 7.668(159) \to A_1^{(10)}|^{\rm nll}_{\rm SV} = 6.793(90)$$

$\to$ impact on $a_{e,\mu}$ negligible at present

5-loop QED diagrams



(Aoyama et al '15)

QED contribution to a_μ

From Cs recoil measurement [Mueller et al '18]:

$$\alpha = 137.035\,999\,046(27) \;\; [0.2 \text{ ppb}]$$

Then:

$$
\begin{array}{rrlrl}
 & & & \text{\% of } a_\mu & \text{order} \\
a_\mu^{\text{QED}} \times 10^{10} = & & 11\,614\,097.3321 \quad (23) & 99.6133\% & \alpha \\
 & + & 41\,321.7626 \quad (7) & 0.3544\% & \alpha^2 \\
 & + & 3\,014.1902 \quad (33) & 0.0259\% & \alpha^3 \\
 & + & 38.1004 \quad (17) & 0.0003\% & \alpha^4 \\
 & + & 0.5078 \quad (6) & 4\cdot 10^{-6} & \alpha^5 \\
\hline
 = & & 11\,658\,471.8931 & \multicolumn{2}{l}{(7)_{m_\tau}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_\alpha \;\; [0.9 \text{ ppb}]}
\end{array}
$$

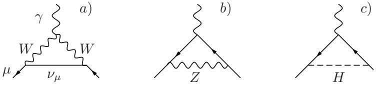
(Aoyama et al '12, '18, '19)

99.994% of a_μ are due to QED contributions!

$$
\begin{aligned}
a_\mu^{\text{exp}} - a_\mu^{\text{QED}} &= 734.2(4.1) \times 10^{-10} \\
&\stackrel{?}{=} a_\mu^{\text{EW}} + a_\mu^{\text{had}}
\end{aligned}
$$

Electroweak contributions to a_μ: Z, W, H, etc. loops

1-loop

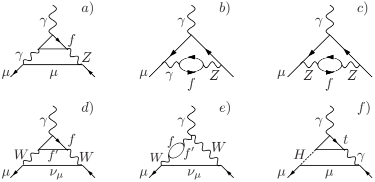


$$a_\mu^{\mathrm{EW},(1)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\right)$$

$$= 19.479(1)\times 10^{-10}$$

(Gnendiger et al '15, Aoyama et al '20 and refs therein)

2-loop



$$a_\mu^{\mathrm{EW},(2)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\,\frac{\alpha}{\pi}\right)$$

$$= -4.12(10)\times 10^{-10}$$

(Gnendiger et al '15 and refs therein)

$$a_\mu^{\mathrm{EW}} = 15.36(10)\times 10^{-10}$$

Hadronic contributions to a_μ: quark and gluon loops

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} = 718.9(4.1) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{had}}$$

- Clearly right order of magnitude:

$$a_\mu^{\text{had}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

(already Gourdin & de Rafael '69 found $a_\mu^{\text{had}} = 650(50) \times 10^{-10}$)

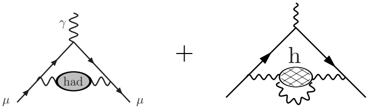
- However, must be determined to sub-percent accuracy & involves quarks and gluons at low energies

  $\Rightarrow$ must be able to describe the highly nonlinear dynamics of confinement
  $\Rightarrow$ cannot rely on the perturbative methods used for QED and weak corrections
  $\Rightarrow$ need methods that allow a fully non-perturbative determination

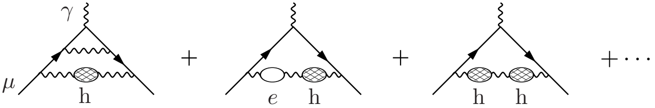
- Decompose:

$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbyL}} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

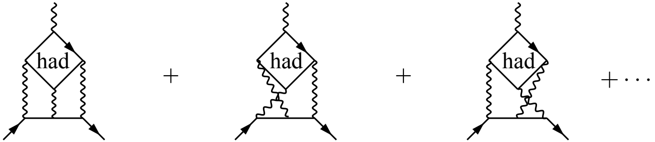
Hadronic contributions to a_μ: diagrams



$$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$$

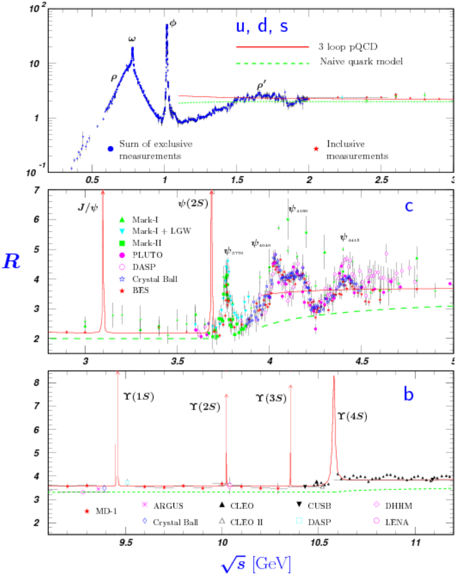


$$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$$



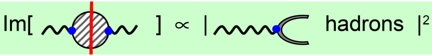
$$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$$

HVP from $e^+e^- \to$ had (or $\tau \to \nu_\tau +$ had)



(PDG compilation)

Use [Bouchiat et al 61] optical theorem **(unitarity)**



$$\mathrm{Im}\Pi(s) = -\frac{R(s)}{12\pi}, \quad R(s) \equiv \frac{\sigma(e^+e^- \to \text{had})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

and a once subtracted dispersion relation **(analyticity)**

$$\hat{\Pi}(Q^2) = \int_{s_{th}}^{\infty} ds \, \frac{Q^2}{s(s+Q^2)} \frac{1}{\pi} \mathrm{Im}\Pi(s)$$

$$\Rightarrow a_\mu^{\text{LO-HVP}} = \frac{\alpha^2}{3\pi^2} \int_{M_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

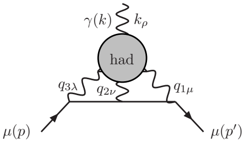
$\Rightarrow \hat{\Pi}(Q^2)$ & $a_\mu^{\text{LO-HVP}}$ from data: sum of exclusive $\pi^+\pi^-$ etc. channels from CMD-2, SND, BES, KLOE '08,'10&'12, BABAR '09, etc.

$$a_\mu^{\text{LO-HVP}} = 694.0(1.0)(3.9) \times 10^{-10} \; [0.6\%]$$

[DHMZ'19, also KNT'19, CHHKS'19, ...] (sys. domin.)

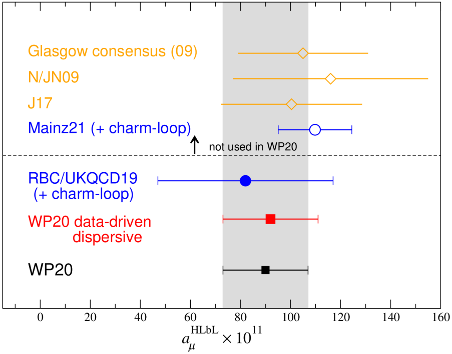
Can also use $I(J^{PC}) = 1(1^{--})$ part of $\tau \to \nu_\tau +$ had and isospin symmetry + corrections

Hadronic light-by-light



- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq$ 10% instead of $\simeq$ 0.2%
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09): $a_\mu^{\text{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - → Phenomenology: rigorous data driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,. . . '15-'20]
  - → Lattice: first two solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improve
- Agreed upon average and conservative error estimate [WP '20]

$$a_\mu^{\text{HLbL}} = 9.0(1.7) \times 10^{-10}$$



[Colangelo '21]

Reference standard model prediction and comparison to experiment

Reference SM result vs experiment

SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$	Ref.
QED [5 loops]	11658471.8931 ± 0.0104	[Aoyama '19, WP '20]
EW [2 loops]	15.36 ± 0.10	[Gnendiger '15, WP '20]
HVP Tot. (R-ratio)	684.5 ± 4.0	[WP '20]
HLbL Tot.	9.2 ± 1.8	[WP '20]
SM [0.37 ppm]	11659181.0 ± 4.3	[WP '20]
Exp [0.35 ppm]	11659206.1 ± 4.1	[BNL '06 + FNAL '21]

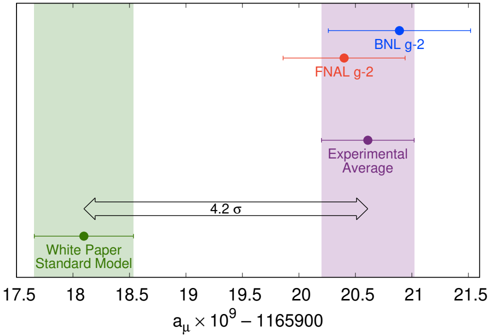
- Exp$-$SM $= (25.1 \pm 5.9) \times 10^{-10}$ [4.2σ]

  $\Rightarrow$ evidence for BSM physics [FNAL'21]

  $\rightarrow \sim 2 \times$ EW contribution

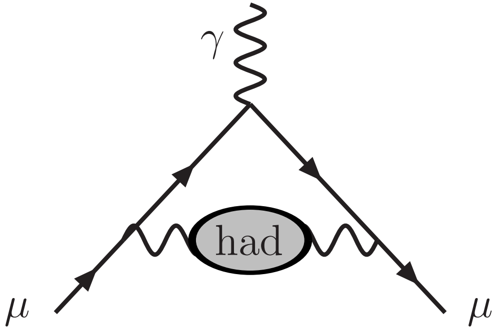
Important to check most uncertain contributions (HVP & HLbL) w/ fully independent methods

$\rightarrow$ *ab initio* calculations of contribution using **lattice quantum chromodynamics (QCD)**



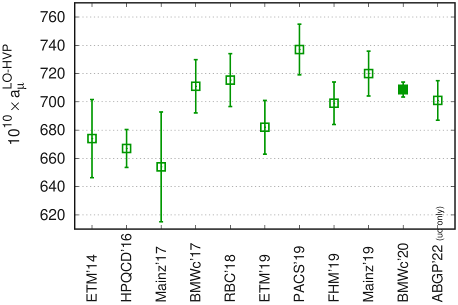
Lattice QCD calculation of $a_\mu^{\text{LO-HVP}}$

[BMWc ’20 = BMWc, Nature 593 (2021) 51]
[BMWc ’17 = BMWc, PRL 121 (2018) 022002]



All quantities related to a_μ will be given in units of 10^{-10}

$a_\mu^{\text{LO-HVP}}$ from lattice QCD



Several onging lattice QCD efforts

Significant improvements required after BMWc '17 to be competitive w/ reference approach
$\rightarrow$ new methods were needed

BMWc '17 $\rightarrow$ BMWc '20

711.0(18.9) [2.7%] $\rightarrow$ 707.5(5.5) [0.8%]

statistical : (7.5) $\rightarrow$ (2.3)

physical point : (5.5) $\rightarrow$ (1.1)

finite-size : (13.5) $\rightarrow$ (2.5)

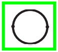
continuum extrapolation : (8.0) $\rightarrow$ (4.1)

QED & $(m_d - m_u)$ corrections : (5.1) $\rightarrow$ (1.4)

(Nature paper has 95 pp "Supplementary Information" detailing methods)

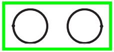
Summary of contributions to $a_\mu^{\text{LO-HVP}}$

Isospin symmetric


connected light
633.7(2.1)(4.2)


connected strange
53.393(89)(68)

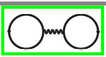
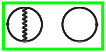

connected charm
14.6(0)(1)


disconnected
-13.36(1.18)(1.36)

QED isospin-breaking: valence

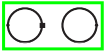


connected -1.23(40)(31)

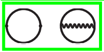


disconnected -0.55(15)(10)

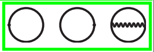
Strong isospin-breaking


connected
6.60(63)(53)


disconnected
-4.67(54)(69)

QED isospin-breaking: sea

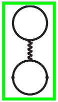


connected 0.37(21)(24)



disconnected -0.040(33)(21)

Etc.

bottom; higher order; perturbative
0.11(4)

QED isospin-breaking: mixed


connected -0.0093(86)(95)


disconnected 0.011(24)(14)

Finite-size effects

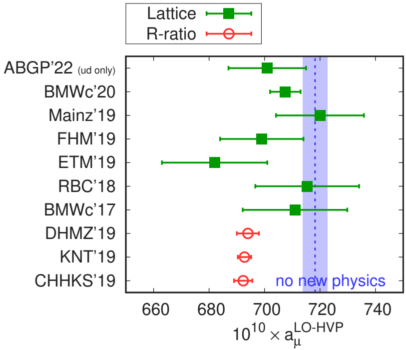
isospin-symmetric
18.7(2.5)

isospin-breaking
0.0(0.1)

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

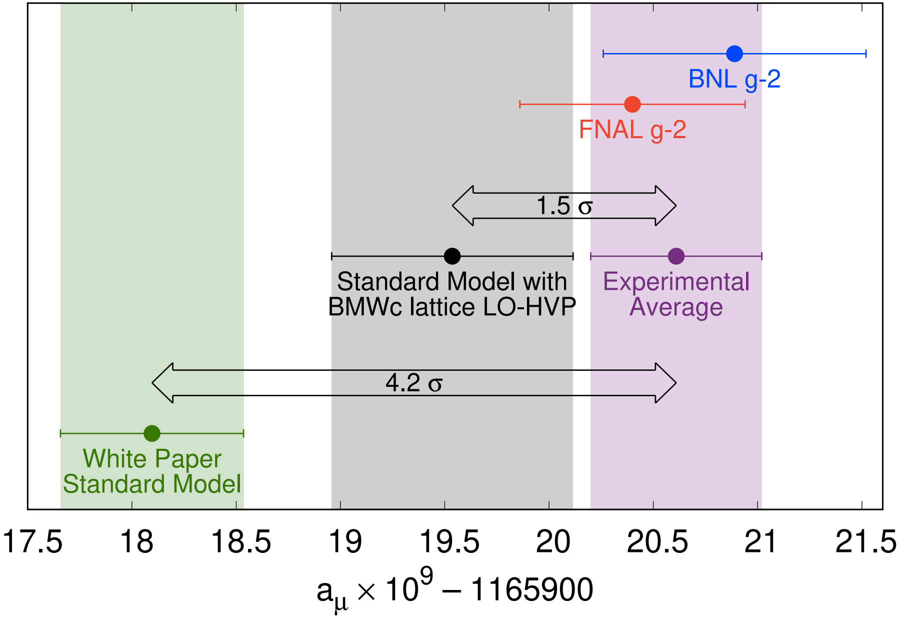
Comparison and outlook

Comparison



- Consistent with other lattice results
- Total uncertainty is divided by $3 \div 4$ . . .
- . . . and comparable to R-ratio and $(g-2)$ experiment
- Consistent w/ a_μ measurement @ 1.5σ level ("no new physics" scenario) !
- But 2.1σ larger than R-ratio average value [WP '20]

Fermilab plot w/ BMWc result

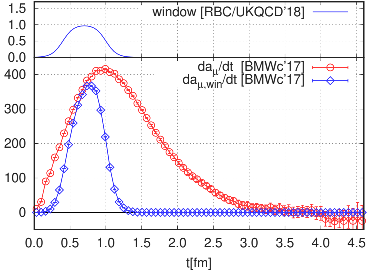


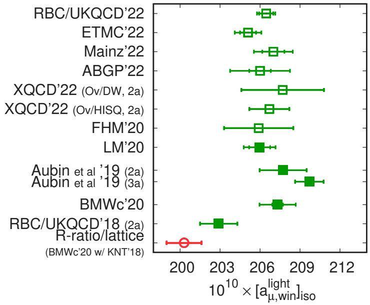
Intermediate window results

- Less challenging than full $a_\mu^{\text{LO-HVP}}$
  - much better signal/noise
  - much smaller FV effects
  - much smaller discretization effects (long & short distance)

$\rightarrow$ other LQCD groups have comparable errors

- Accounts for $\sim 1/3$ of total $a_\mu^{\text{LO-HVP}}$
- 3.7σ tension w/ R-ratio [BMWc'20]
- 7.0 out of 14.4 lattice vs R-ratio excess in $10^{10} \times a_\mu^{\text{LO-HVP}}$
- 7.0 out of 25.9(5.9) expt vs reference SM excess in $10^{10} \times a_\mu$
- Summer/Fall '22: Mainz, ETMC and RBC/UKQCD confirm BMWc'20 result for $a_{\mu,\text{win}}^{\text{LO-HVP}}$ & $a_{\mu,ud,\text{win}}^{\text{LO-HVP}}$ using $\neq$ fermion discretizations and fine lattices





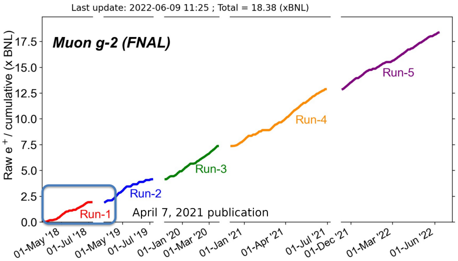
Conclusions and outlook

- a_μ is measured to 0.35 ppm and predicted in SM to 0.37 ppm
- BMWc'20's lattice QCD calculation of $a_\mu^{\text{LO-HVP}}$ reaches precision comparable to reference $e^+e^- \to$ hadrons approach for first time
- While reference SM prediction [WP'20] gives $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 25.1(5.9) \times 10^{-10}$, i.e. a 4.2σ indication of new physics, . . .
- . . . lattice QCD calculation reduces this difference to 1.5σ, $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 10.7(1.7) \times 10^{-10}$ . . .
- . . . at expense of 2.1σ tension w/ $a_\mu^{\text{LO-HVP}}$ & 3.7σ tension w/ $a_{\mu,\text{win}}^{\text{LO-HVP}}$ from $e^+e^- \to$ hadrons
- BMWc'20's $a_{\mu,\text{win}}^{\text{LO-HVP}}$ tension fully confirmed by Mainz'22, ETMC'22 and RBC/UKQCD'22

  $\to$ must be understood
- Of course, need confirmation of BMWc'20's high lattice value for much more challenging total value of $a_\mu^{\text{LO-HVP}}$

Conclusions and outlook

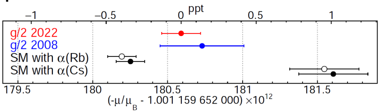
- Upcoming experimental progress at FNAL:
  - Results of Run 2/3 expected early 2023
    $\rightarrow \delta_{\mathrm{rel}} a_\mu \sim 0.23\,\mathrm{ppm}$
  - W/ ongoing (low priority) Run 6 should reach $> 20\times$ BNL
    $\rightarrow \delta_{\mathrm{rel}} a_\mu \lesssim 0.14\,\mathrm{ppm}$ ca. '25



- Must reduce error on HLbL by $1.5 \div 2 \ldots$
- $\ldots$ & lattice HVP error by ~ 4!
  $\rightarrow$ new methods and simulations are needed again
- Must also reduce share of *systematic* error on HVP
- Whole picture can still change!

Conclusions and outlook

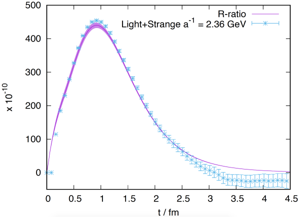
- If disagreement between LQCD and data driven approach can be understood and fixed
  $\rightarrow$ combine to improve overall uncertainty [RBC/UKQCD '18]
- Important to pursue $e^+e^- \rightarrow$ hadrons measurements [BaBar, CMD-3, BES III, Belle II, . . .]
- $\mu e \rightarrow \mu e$ experiment MUonE very important for experimental crosscheck and complementarity w/ LQCD
- Important to pursue J-PARC $g_\mu - 2$ and pursue a_e experiments



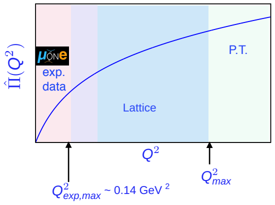
[Fan et al '22]

$\rightarrow \delta_{\rm rel} a_e = 0.11\,\text{ppb} \Rightarrow \delta_{\rm rel} a_e \frac{m_\mu^2}{m_e^2} = 4.8\,\text{ppm}$ vs

$\delta_{\rm rel} a_\mu|_{\rm FNAL} \simeq 0.14\,\text{ppm}$



[RBC/UKQCD '18]



[Marinkovic et al '19]