Karlsruher Institut für Technologie

Particle Physics Phenomenology after the Higgs Discovery

Non-factorizable QCD corrections in WBF and WBF-like processes

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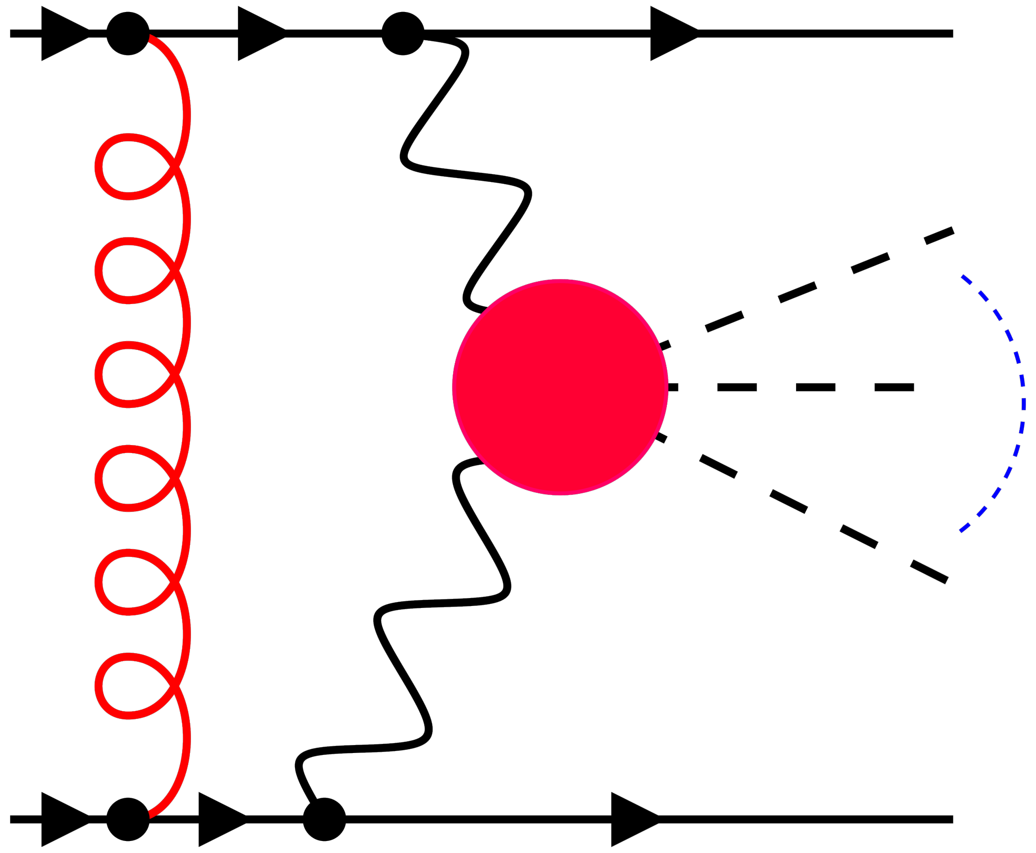
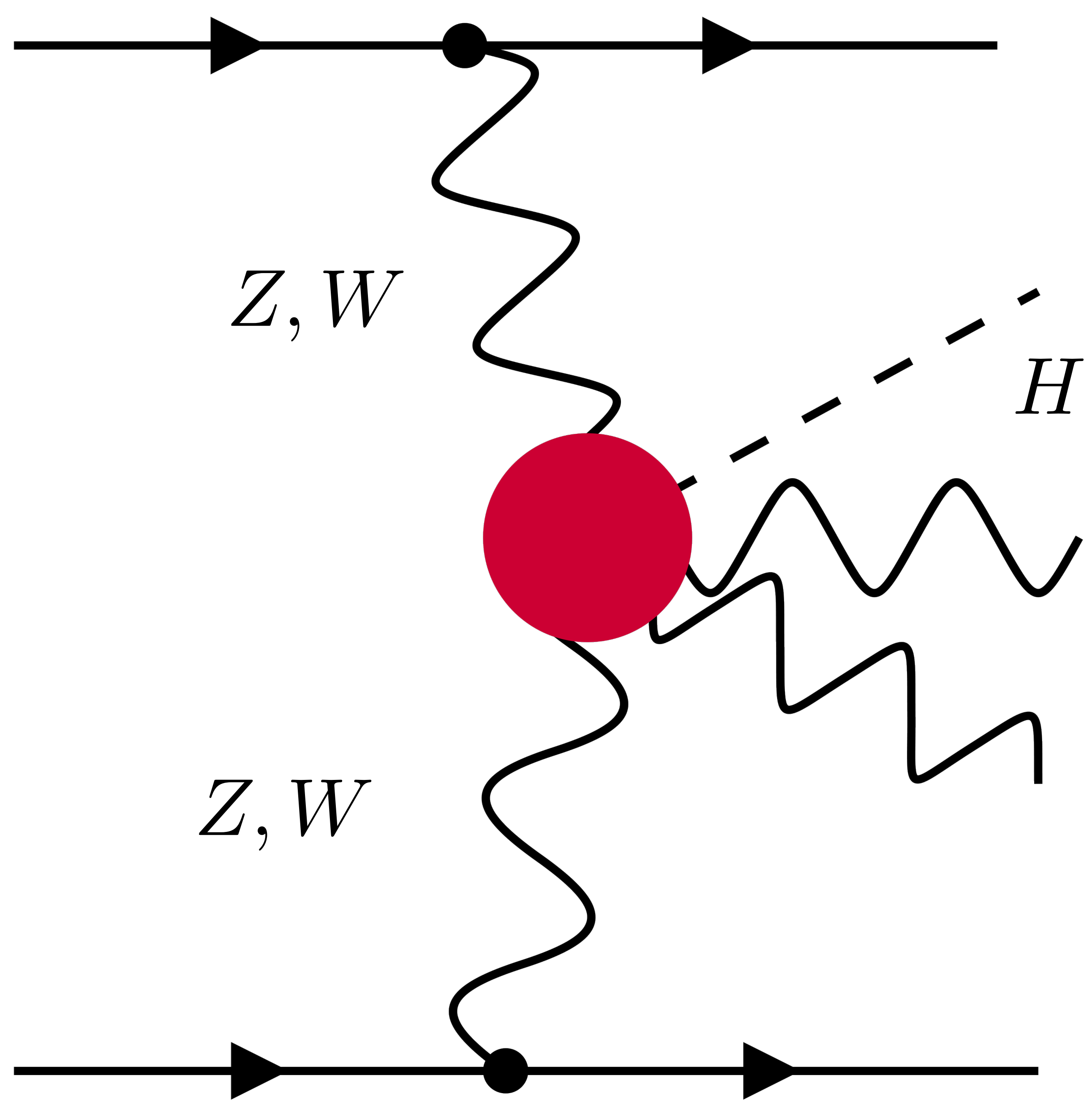
Multi-Boson Interaction 2021, Milan, August 2021

Based on collaboration with T. Liu and A. Penin



Weak boson fusion

Processes in which electroweak bosons fuse into electroweak final states are of great interest for studying electroweak symmetry breaking.

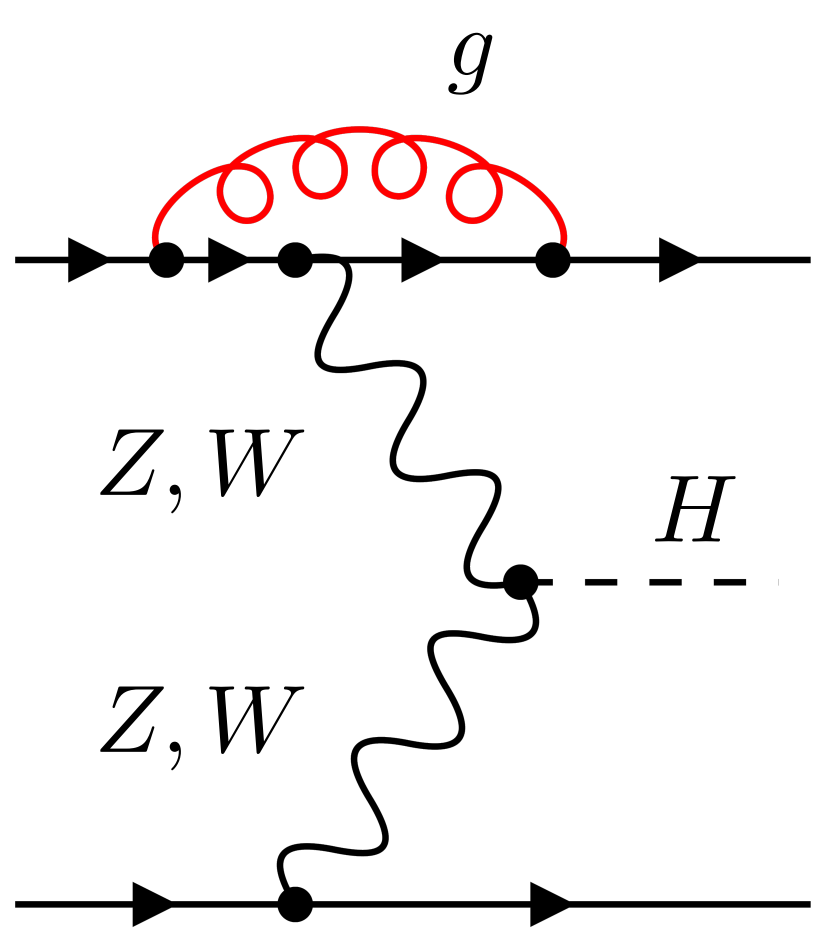


QCD corrections to weak boson fusion

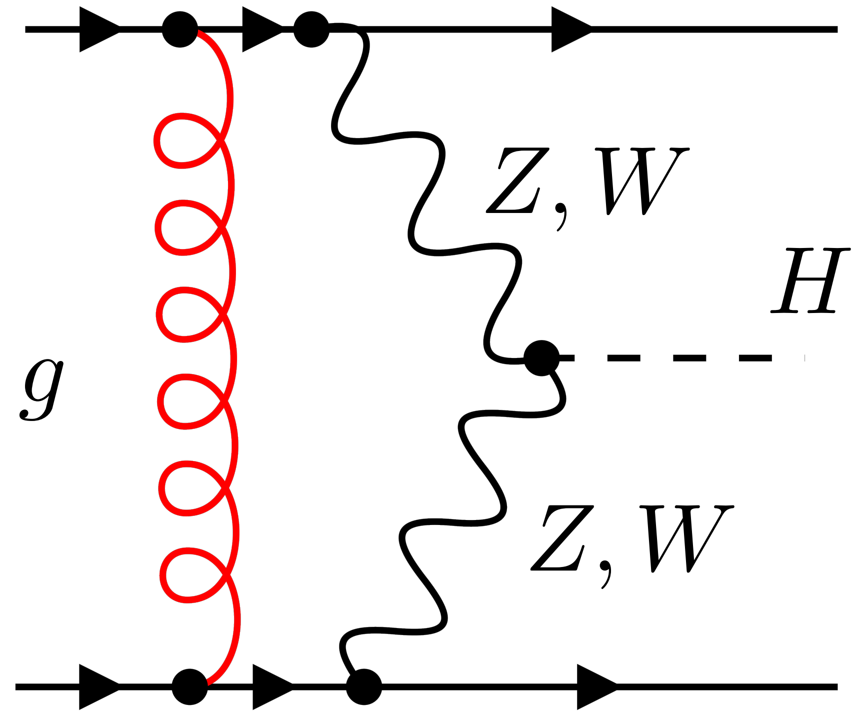
We need to provide precise description of such processes in the SM. This requires understanding QCD and electroweak corrections.

QCD corrections to such processes can be split into factorizable and non-factorizable corrections.

There are other contributions to Higgs production in WBF that do not fit into this template; we will discuss them later.



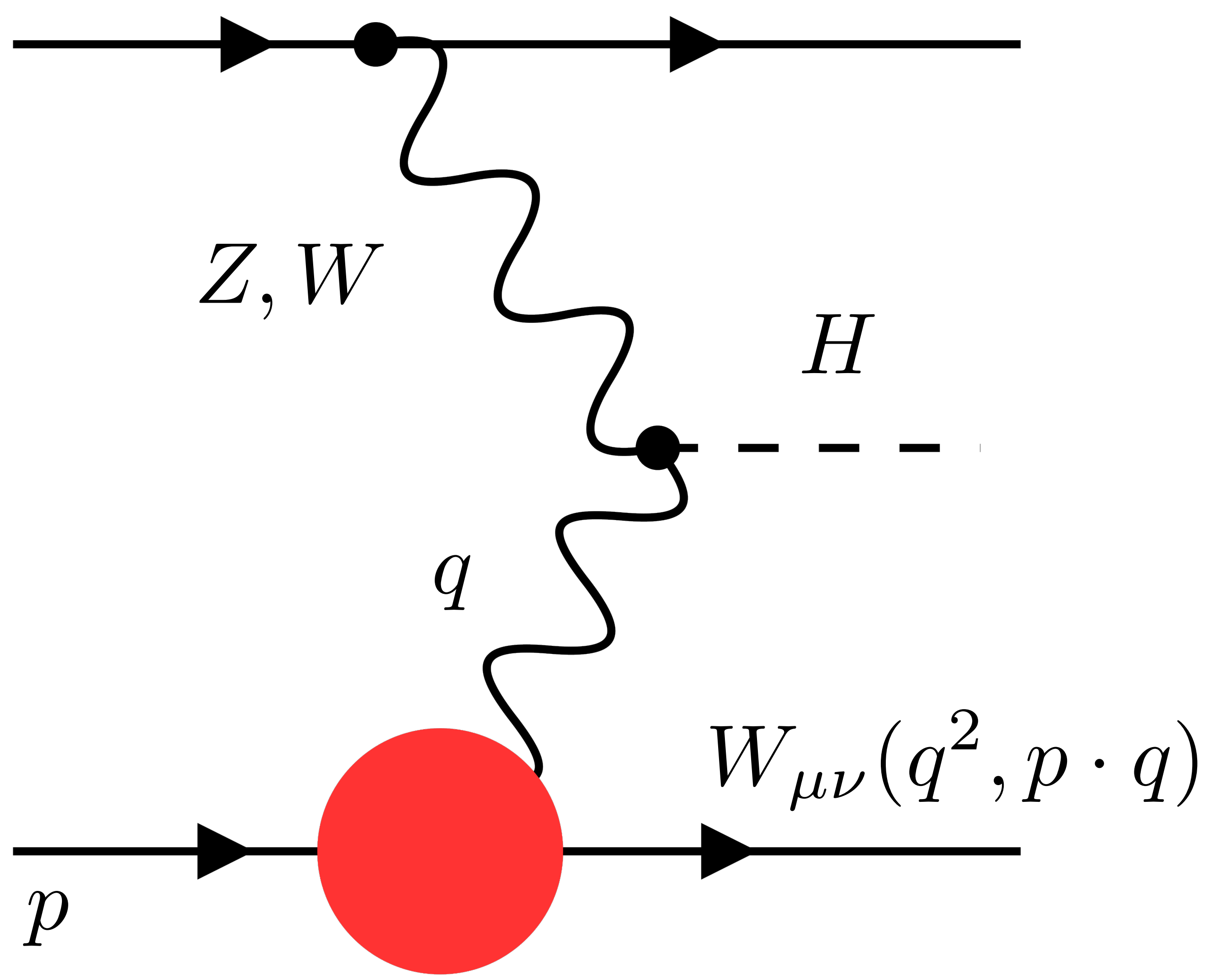
Factorizable QCD effects

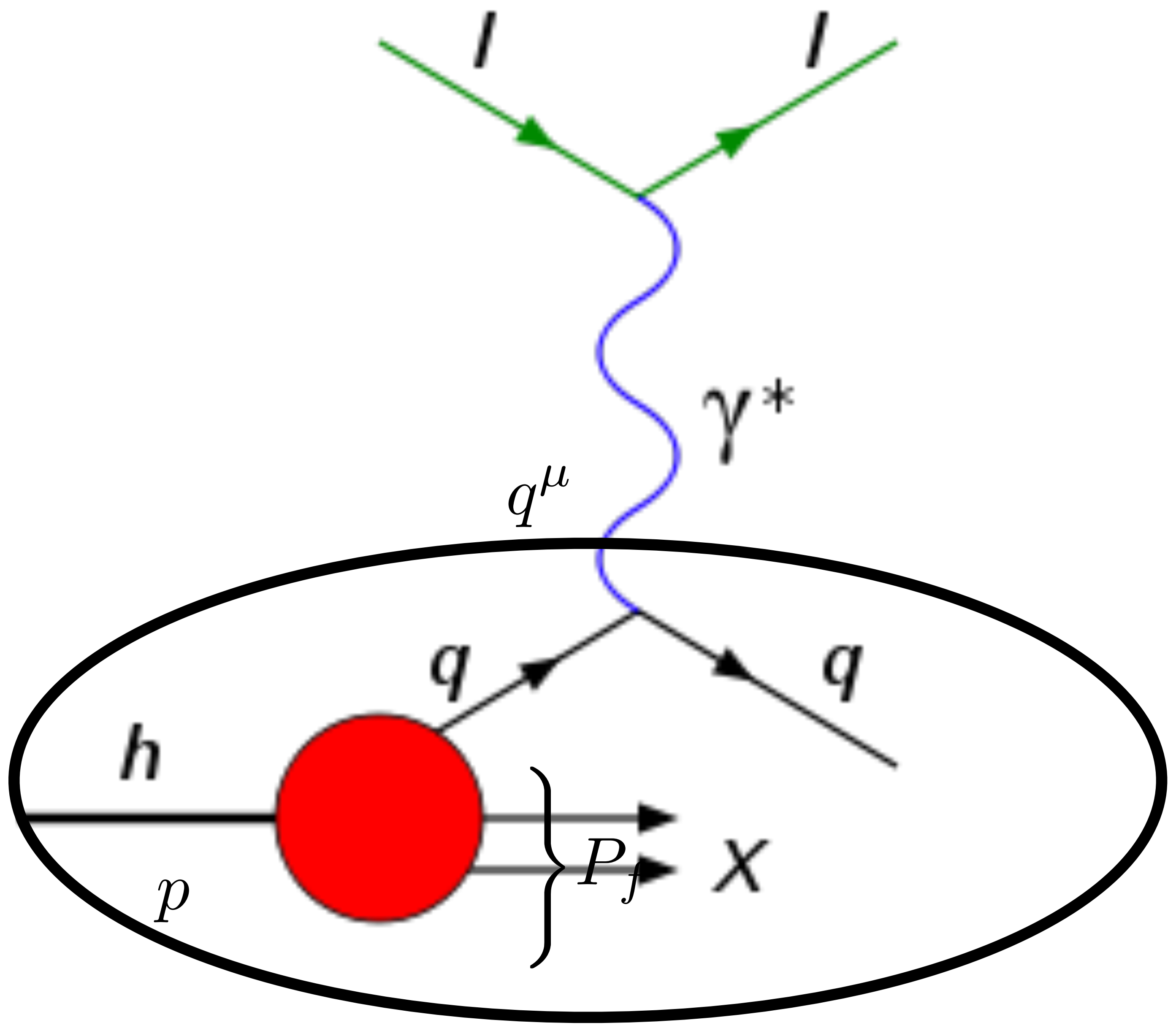


Non-factorizable QCD effects

Factorizable QCD corrections to weak boson fusion

Factorizable corrections to WBF are very well studied. One of the reasons is that factorizable contributions can be related to DIS coefficient functions.





$$W_{\mu\nu} = \frac{1}{2} \sum_{\lambda} \sum_{X_f} \int [\mathrm{d}P_f](2\pi)^d \delta^{(d)}(p - P_f - q) \, \langle p, \lambda | J^\mu | f \rangle \, \langle f | J^\nu | p, \lambda \rangle$$

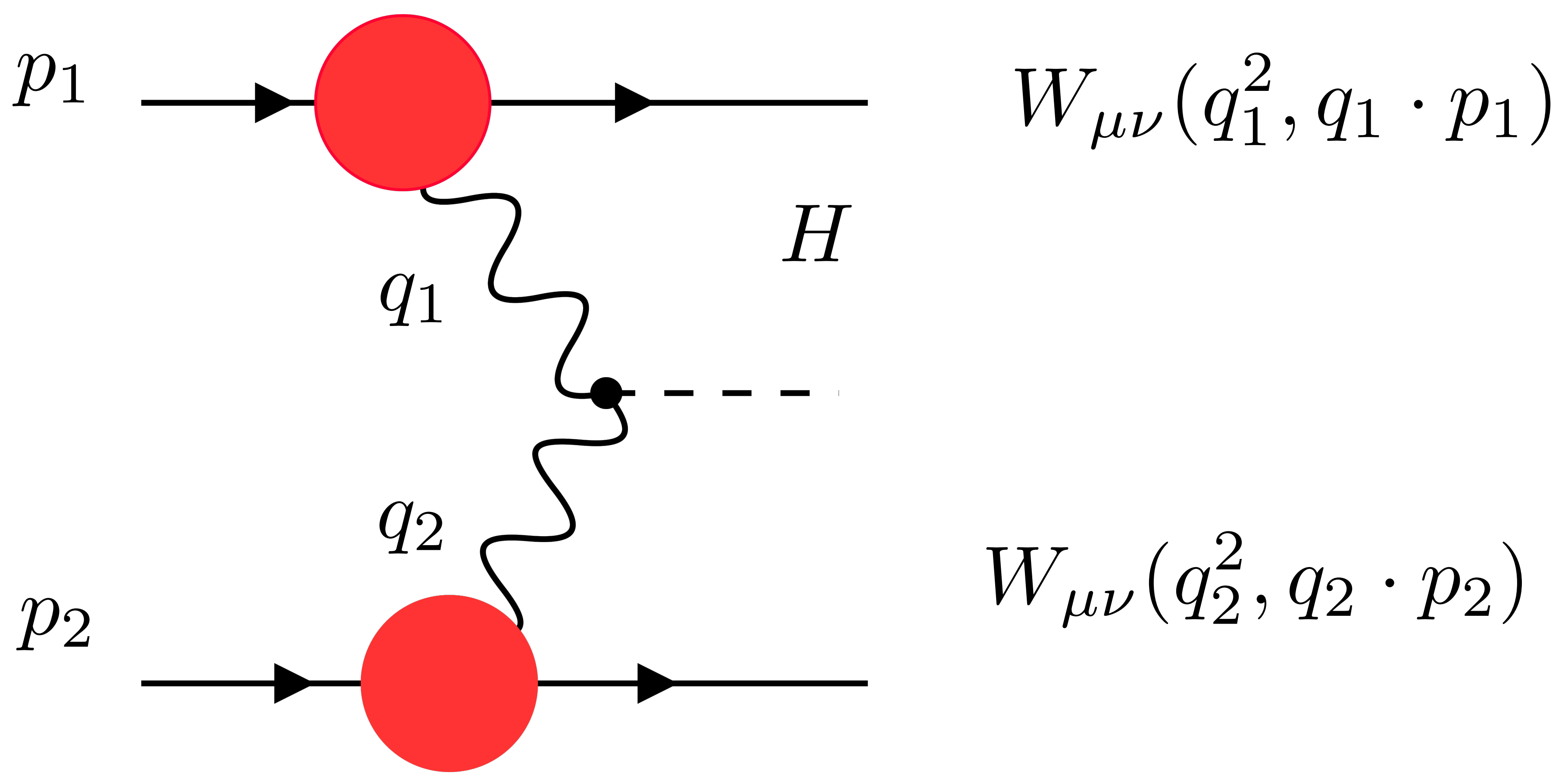
Factorizable QCD corrections to weak boson fusion

If the DIS structure functions are known up to a certain order in the perturbative expansion in QCD, the WBF cross section can be immediately computed through the same perturbative order. In practice, this has been done through N3LO in perturbative QCD.

van Neerven, Zijlstra; Moch, Vermaseren, Vogt
Boltoni, Maltoni, Moch, Zaro;
Karlberg, Dreyer

$$W_{\mu\nu} = \frac{1}{2}\sum_{\lambda}\sum_{X_f}\int [\mathrm{d}P_f](2\pi)^d\delta^{(d)}(p-P_f-q)\,\langle p,\lambda|J^\mu|f\rangle\,\langle f|J^\nu|p,\lambda\rangle$$

$$W^{\mu\nu} = W_1(q^2,p{\cdot}q)\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)+W_2(q^2,p{\cdot}q)\left(p^\mu-\frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu-\frac{p\cdot q}{q^2}q^\nu\right)-i\epsilon^{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{2p\cdot q}W_3(q^2,p\cdot q)$$



$$\mathrm{d}\sigma_{\mathrm{VBF}} \sim \frac{(2\pi)^4\delta^{(4)}(q_1+q_2-p_H)\mathrm{d}^4q_1\mathrm{d}^4q_2}{(q_1^2-m_V^2)(q_2^2-m_V^2)}\,W^{\mu\nu}(q_1^2,q_1{\cdot}p_1)W^{\mu'\nu'}(q_2^2,q_2{\cdot}p_2)\mathcal{M}_{VV\to H}^{\mu\mu',\nu\nu'}\,\frac{\mathrm{d}^3p_H}{2E_H(2\pi)^3}$$

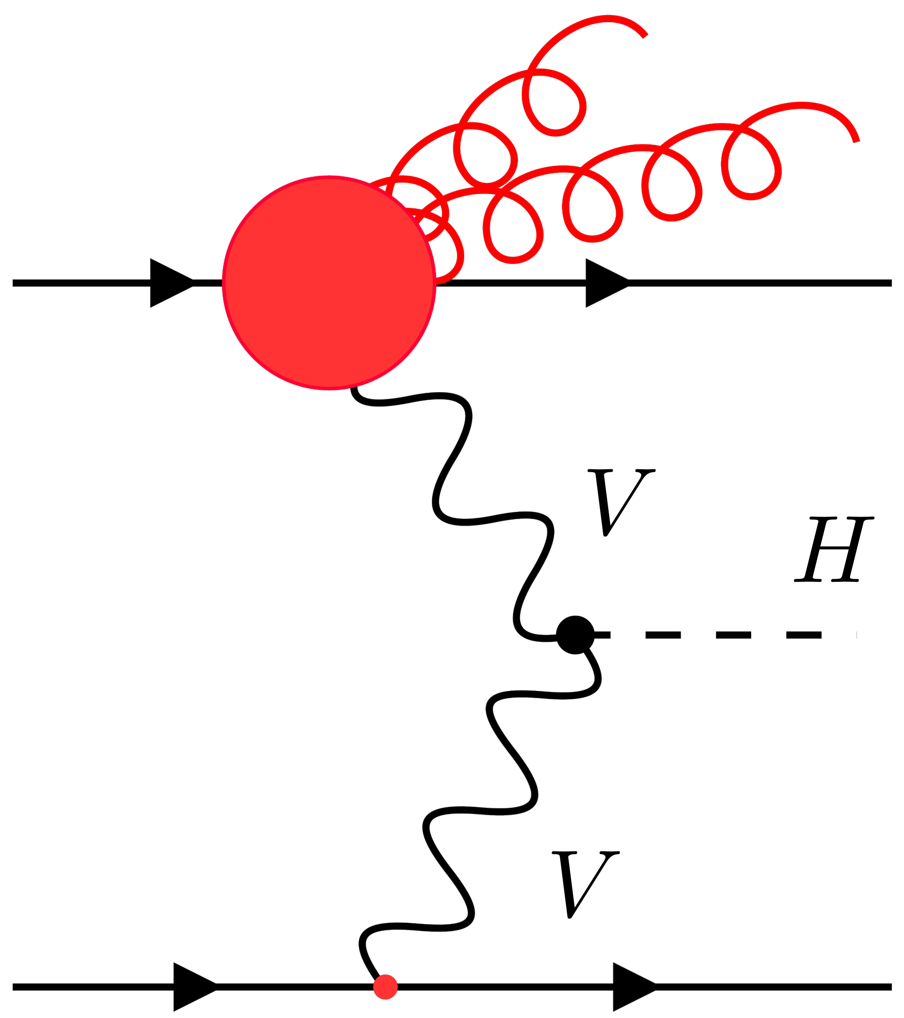
Factorizable QCD corrections to weak boson fusion

Within the structure functions approach, very moderate QCD corrections to Higgs production in WBF were found: O(-3%) at NLO, O(-1%) at NNLO, O(-0.1) % at N3LO.

However it is unclear if these results are relevant for weak boson fusion process as defined and studied at the LHC.

Indeed, the structure function approach involves integration over partons in the final state and does not allow us to impose constraints on QCD radiation. This is not ideal since WBF cuts are quite severe (the WBF cross section after cuts is only about 20 percent of the cross section without the WBF cuts) and involve forward tagging jets.

For this reason, it is important to perform a fully differential computation (even within the factorization approximation!) that accounts for WBF cuts on the tagging jets.



Typical WBF cuts

$$p_\perp^{j_{1,2}} > 25 \text{ GeV}, \quad |y_{j_{1,2}}| < 4.5,$$
$$\Delta y_{j_1,j_2} = 4.5, \quad m_{j_1,j_2} > 600 \text{ GeV},$$
$$y_{j_1} y_{j_2} < 0, \quad \Delta R > 0.4$$

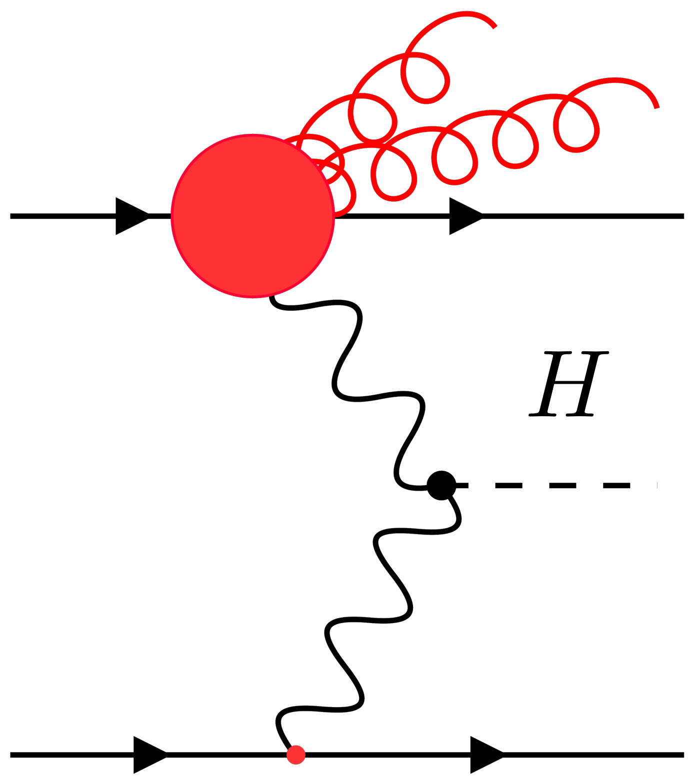
Fiducial NNLO QCD cross sections

A fully differential NNLO QCD computation within the factorization approximation has been performed using two different methods: projection-to-Born and antenna subtraction.

It was observed that the QCD corrections to the total cross section with WBF cuts are larger, by almost a factor of 3, than the QCD corrections computed in the structure function approximation.

Corrections to kinematic distributions can be even larger; they are also phase-space dependent.

	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032^{+0.057}_{-0.069}$	$0.957^{+0.066}_{-0.059}$
NLO	$3.929^{+0.024}_{-0.023}$	$0.876^{+0.008}_{-0.018}$
NNLO	$3.888^{+0.016}_{-0.012}$	$0.844^{+0.008}_{-0.008}$



$$p_\perp^{j_{1,2}} > 25\ \text{GeV}, \quad |y_{j_{1,2}}| < 4.5,$$
$$\Delta y_{j_1,j_2} = 4.5, \quad m_{j_1,j_2} > 600\ \text{GeV},$$
$$y_{j_1} y_{j_2} < 0, \quad \Delta R > 0.4$$

Cacciari, Dreyer, Karlberg, Salam, Zanderighi

Cruz-Martinez, Glover, Gehrmann, Huss

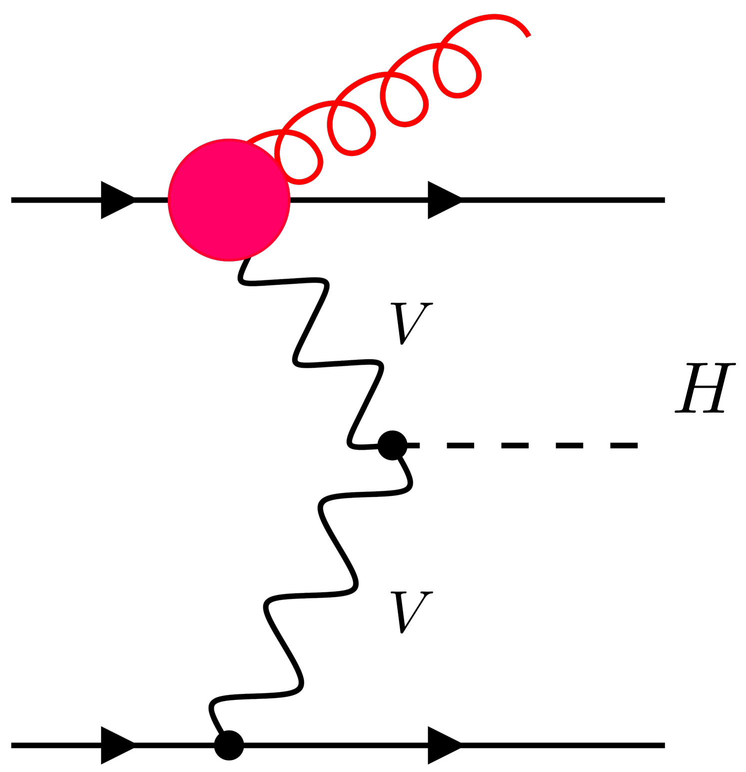
Towards N3LO QCD predictions for WBF

As we already mentioned, thanks to the connection between Higgs production in WBF and DIS, it is possible to estimate N3LO corrections to inclusive WBF cross section since N3LO DIS coefficient functions are known.

Karlberg, Dreyer

Moreover, although we do not quite know how to subtract real-emission singularities at N3LO QCD, the projection-to-Born method allows us to construct the fully differential N3LO cross section once the NNLO QCD corrections to single-jet production in DIS or WBF are known. Such calculation has been done and this implies that N3LO fully differential prediction for WBF is within reach.

Gehrmann et al.



An opportunity to make such high-order predictions for the WBF process is exciting and useful — it will enable the reduction of the theory uncertainty in the description of Higgs production in WBF to below a percent. However, it also forces us to think about other effects that are neglected currently.

Effects beyond the factorization (DIS) approximation

These effects are:

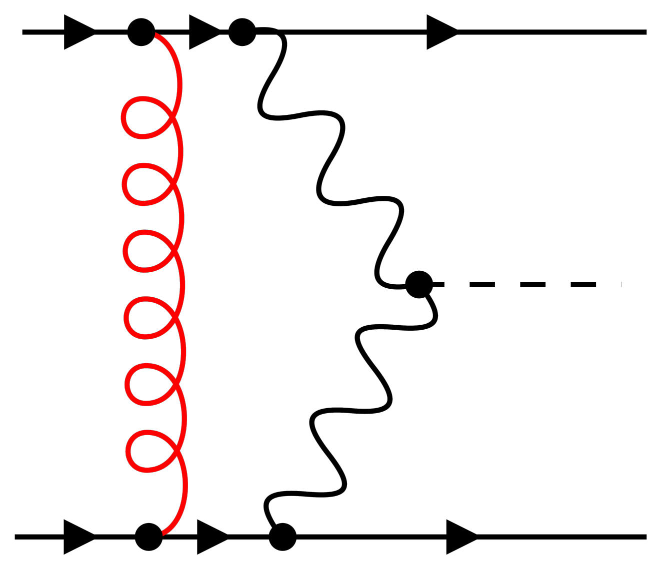
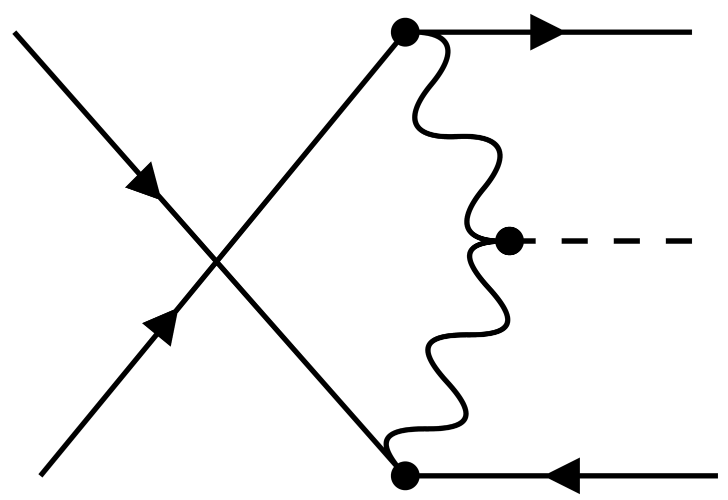
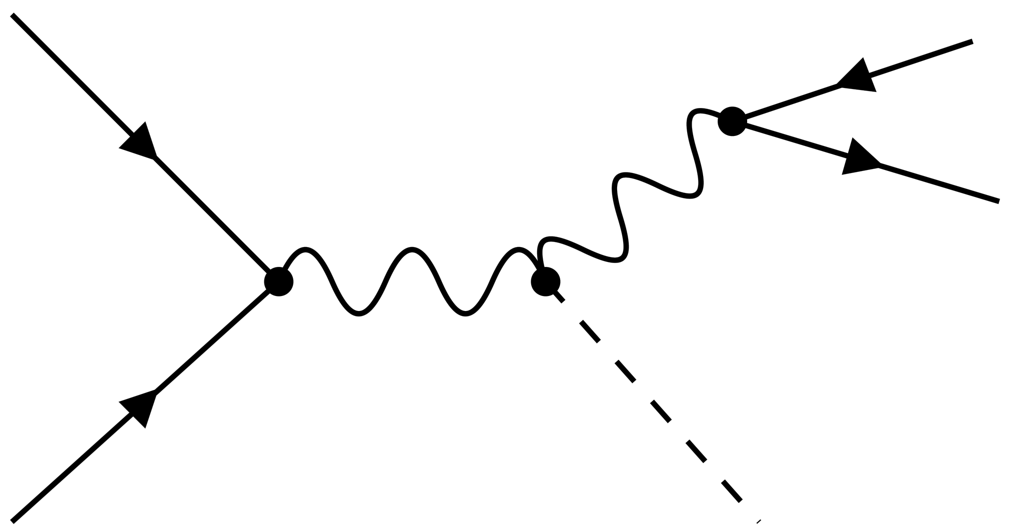
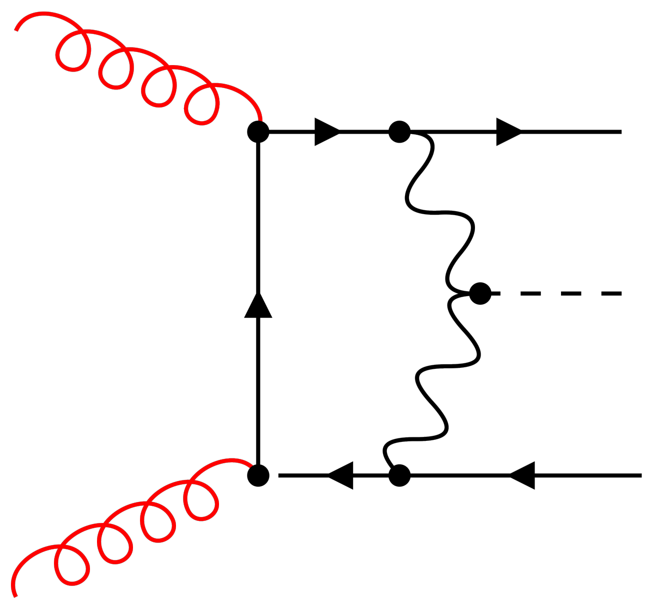
1) non-factorizable gluon exchanges between incoming quark lines;

2) t/u channel interferences (identical quarks); they contribute O(5%) at the inclusive level and O(0.5%) once the WBF cuts are applied;

3) HV(jj) final state contribution to WBF final states; they are negligible after WBF cuts;

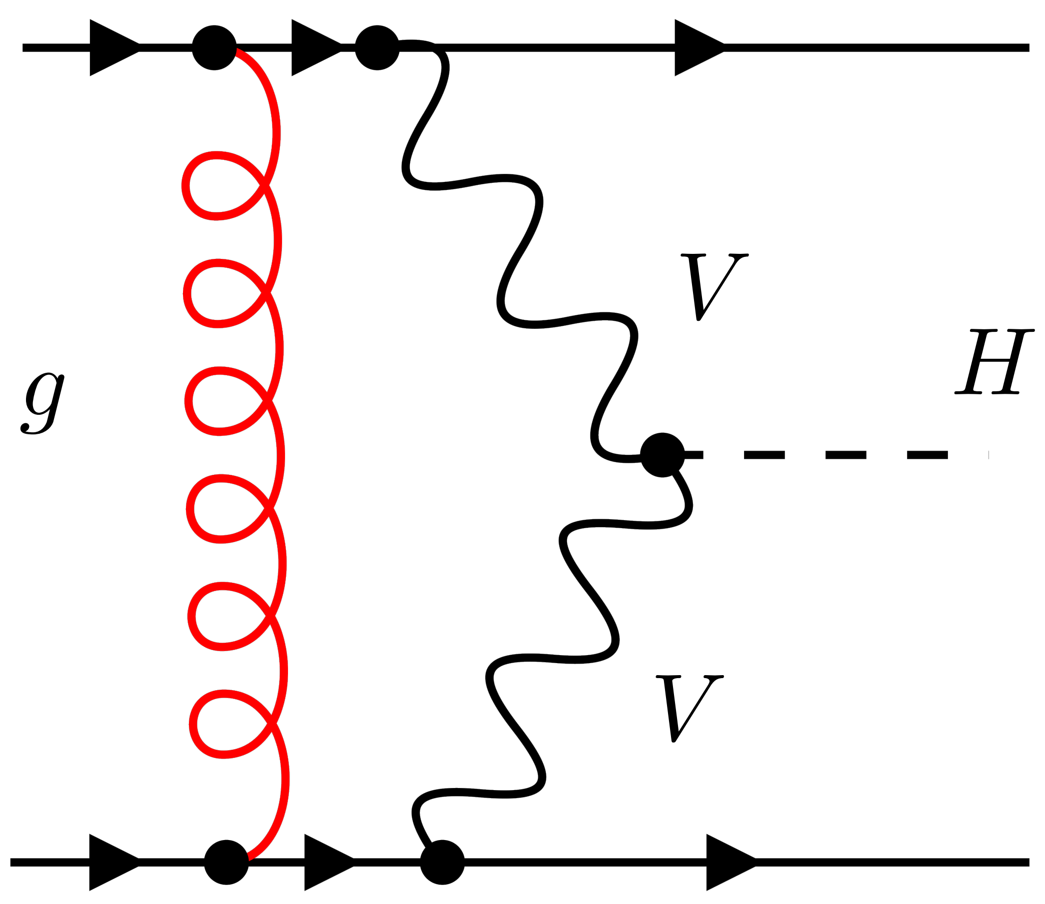
4) contribution of the "single-quark line processes" to WBF; less than a permille after the VBF cuts;

The majority of these effects can be (and has been) studied at lower-orders of perturbation theory and the above estimates are based on that; this cannot be done for non-factorizable corrections as we explain below. The non-factorizable contribution is, in a way, a very unique unknown.

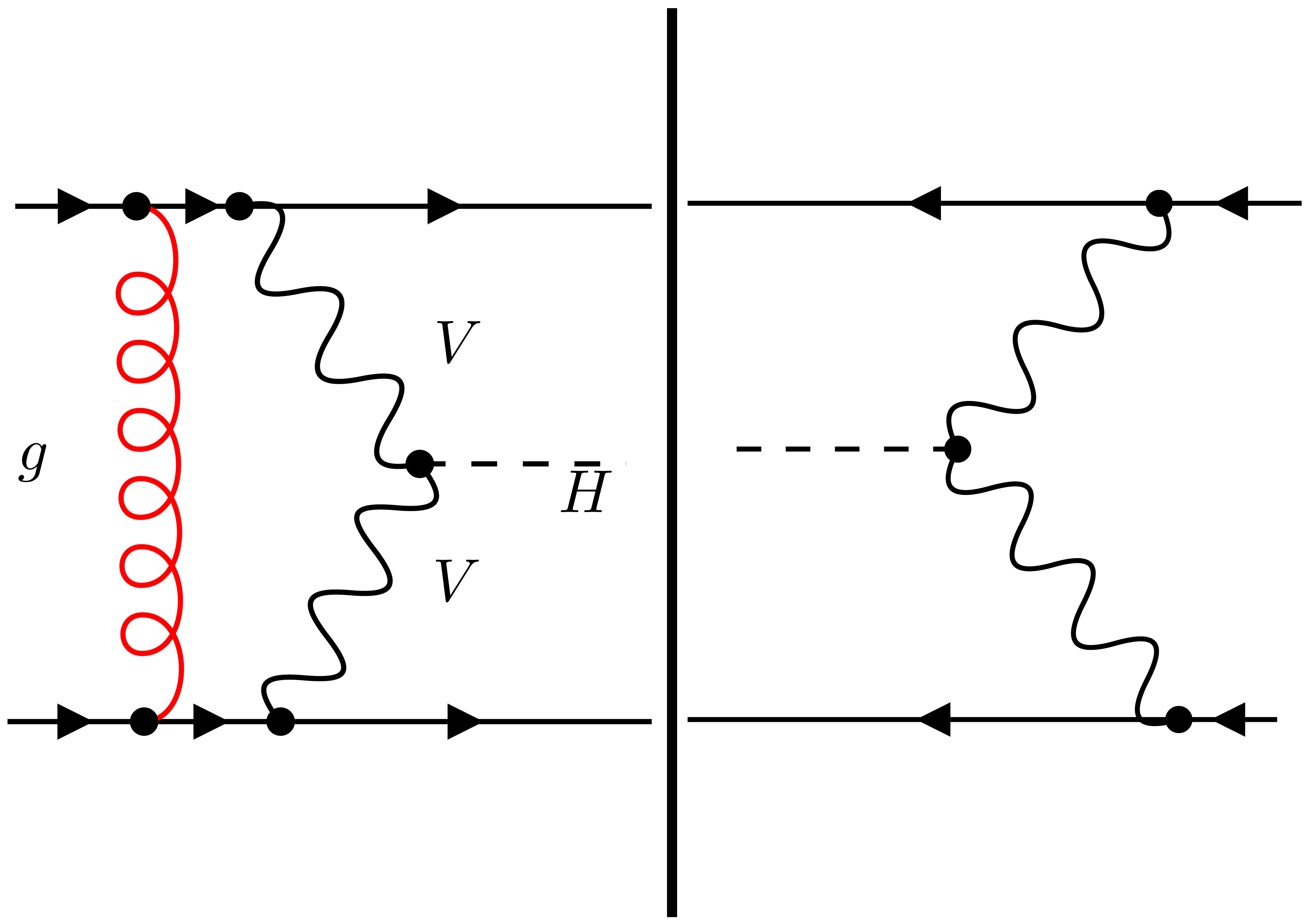
Non-factorizable contributions

It may appear that non-factorizable corrections contribute at next-to-leading order since the diagram below is definitely not zero.



Non-factorizable contributions

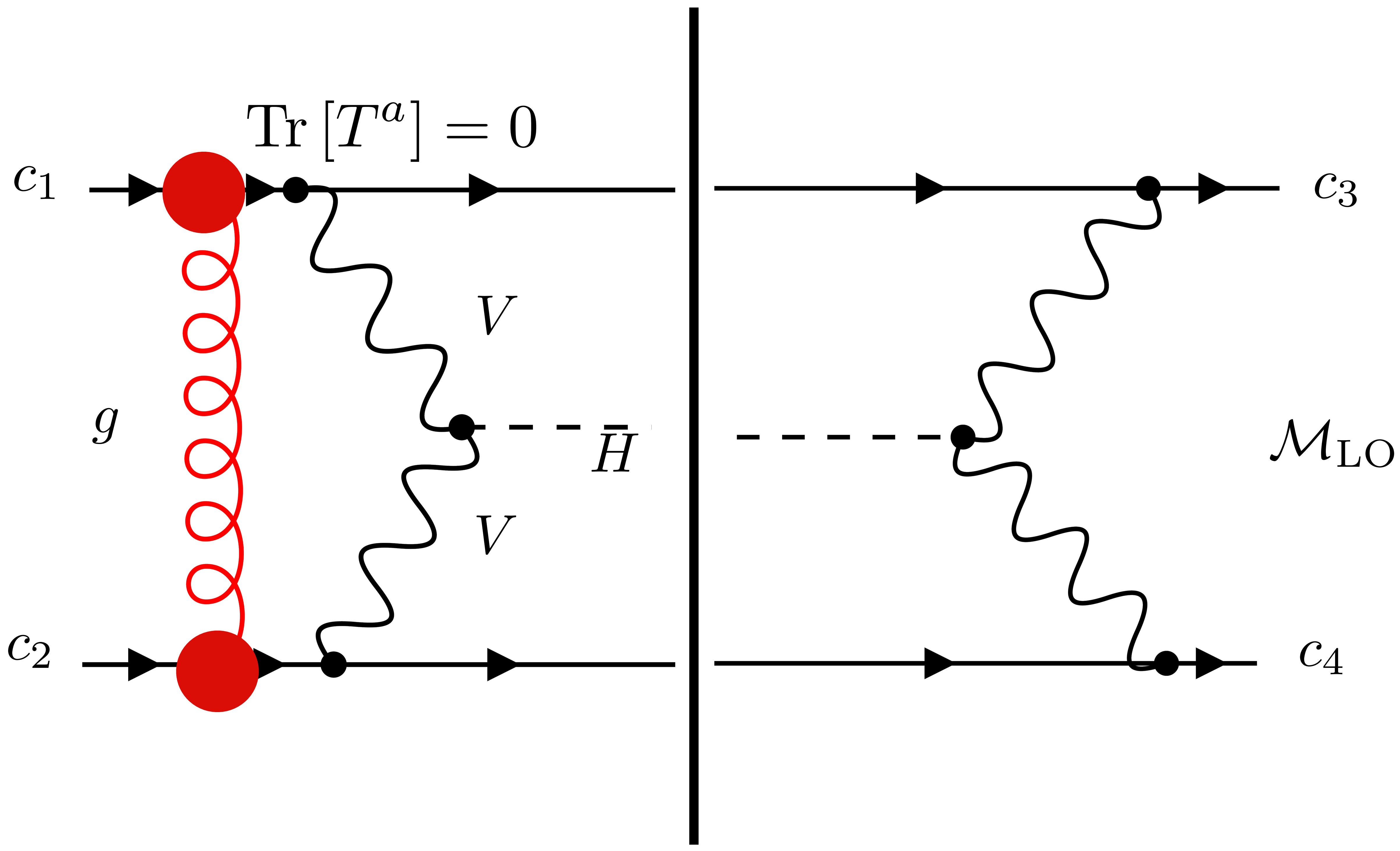
However, the cross section calculation. requires interference of the one-loop diagram with the tree level amplitude..



Non-factorizable contributions

…and this interference vanishes because of the color conservation. Although we illustrate this for virtual corrections, it is clear that this is the feature of real-emission diagrams as well.

$$\mathcal{M}_{\rm NLO} \sim T^a_{c_3 c_1}\, T^a_{c_4 c_2}$$



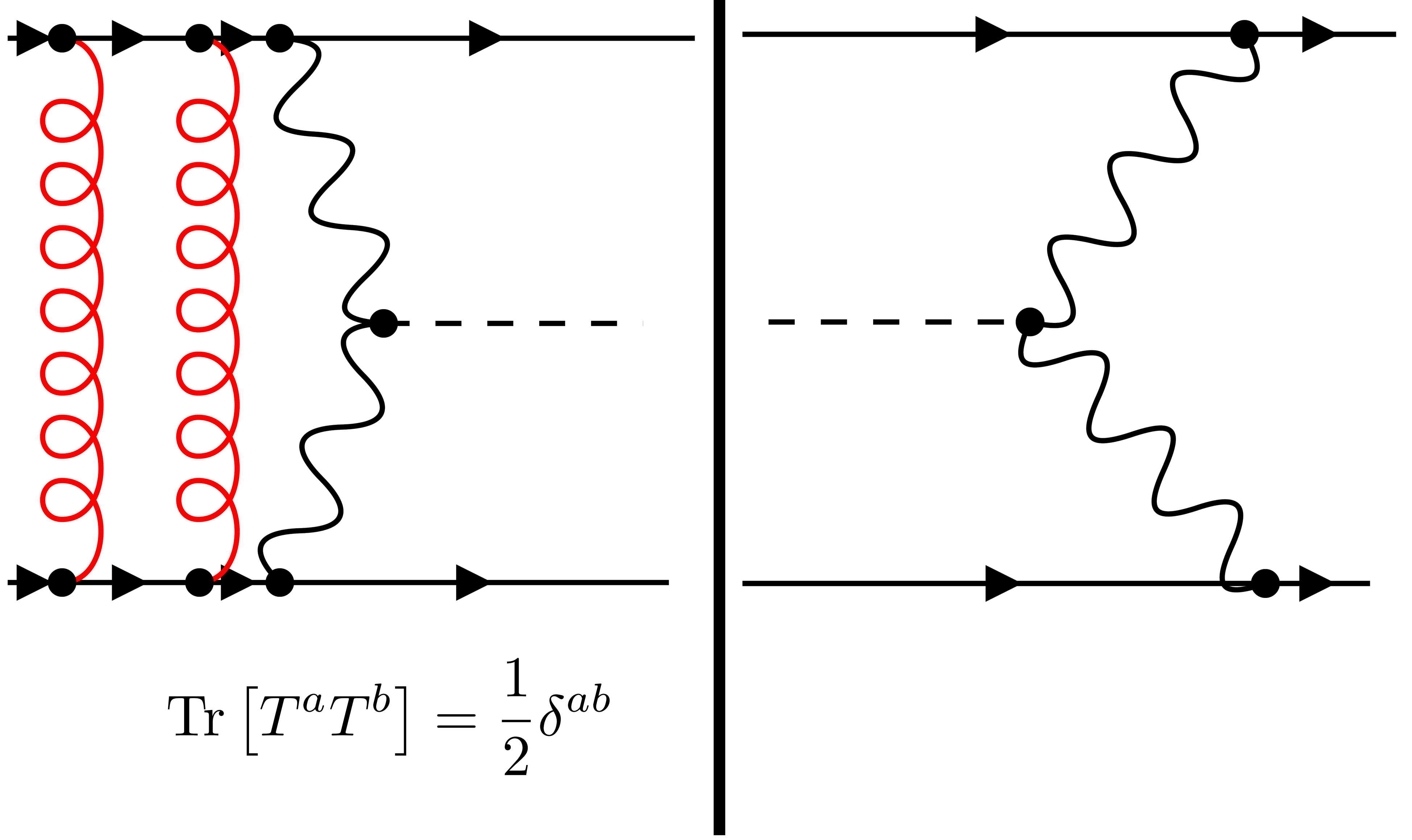
$$\mathcal{M}_{\rm LO} \sim \delta_{c_3 c_1}\, \delta_{c_4 c_2}$$

$$\mathcal{M}_{\rm NLO}\mathcal{M}^*_{\rm LO} \sim T^a_{c_3 c_1}\delta_{c_3 c_1}\; T^a_{c_4 c_2}\delta_{c_2 c_4} \sim \mathrm{Tr}[T^a]\;\mathrm{Tr}[T^a] = 0.$$

Non-factorizable contributions

At two loops, the situation changes. If the two gluons exchanged between quark lines are in a color-singlet state, the non-factorizable contribution is non-vanishing. However, it is color-suppressed relative to factorizable contributions.

$$\mathrm{Tr}\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$$



$$\mathrm{Tr}\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$$

$$\neq 0$$

$$\mathrm{fact}_{\mathrm{color}} = C_F^2 N_c^2 = \frac{(N_c^2-1)^2}{4}$$

$$\mathrm{non/fact}_{\mathrm{colour}} = \frac{\delta^{ab}\delta_{ab}}{4} = \frac{N_c^2-1}{4}$$

$$\boxed{\frac{\sigma_{\mathrm{non-fact}}}{\sigma_{\mathrm{VBF}}} \sim \alpha_s^2 \qquad \frac{\sigma_{\mathrm{non-fact}}}{\Delta\sigma_{\mathrm{VBF}}^{\mathrm{NNLO}}} \sim \frac{1}{N_c^2}}$$

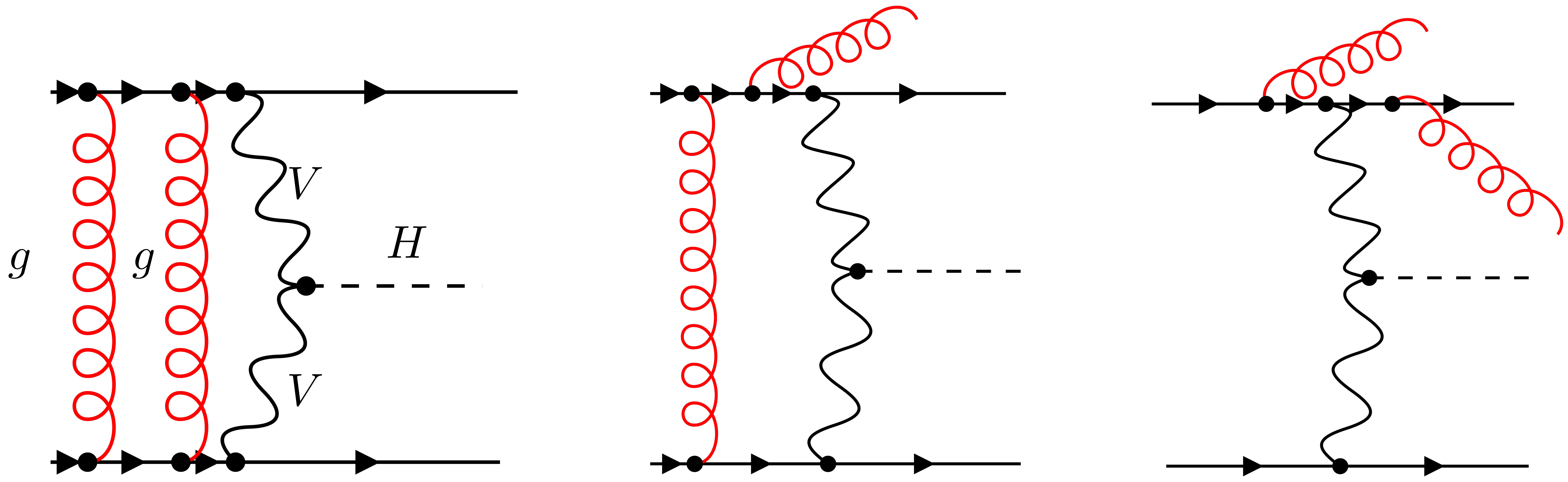
These estimates suggest that non-factorizable contributions should be significantly below 1%. Moreover, it was also argued that these contributions must be further suppressed because they are kinematically disfavoured. As we will see, the suppression issue is quite subtle.

Non-factorizable contributions

It is important to scrutinise these claims and to estimate the non-factorizable contributions in a convincing manner since we do not have much experience with such effects.

Since such corrections are of a NNLO type, understanding them at a fully-differential level requires us to compute the double-virtual, real-virtual and double-real contributions. It is clear that the double-real and the real-virtual contributions can be managed since they require (at most) one-loop computations supplemented with NNLO subtraction schemes.

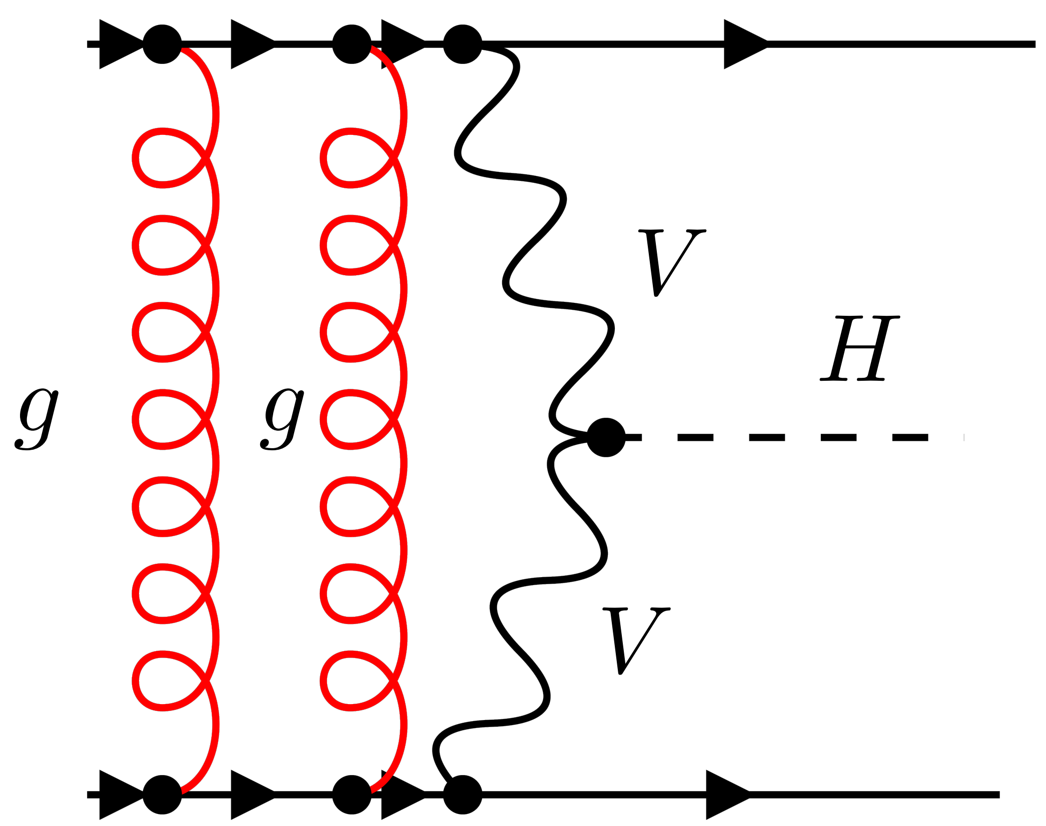
The double-virtual non-factorizable corrections are a problem since they require a two-loop five-point function with external (the Higgs) and internal (weak bosons) masses. It is impossible to compute such complicated diagrams/amplitudes using existing technology for multi-loop computations.



Non-factorizable contributions

However, WBF kinematics is particular — there are two high-energy jets flying in the opposite directions with relatively small transverse momenta. We also have rapidity gap between the jets, and between the jets and the Higgs. In other words, jets are energetic and forward and all (jet and Higgs) transverse momenta are comparable and (fairly) small.

Is it possible to construct an expansion of the virtual amplitudes by taking into account the smallness of jets transverse momenta relative to their energies?



$$\left| \frac{p_\perp}{\sqrt{s}} \right| \ll 1.$$

Typical WBF cuts

$$p_\perp^{j_1,j_2} > 25 \text{ GeV}, \quad |y_{j_1,j)2}| < 4.5$$
$$|y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1 j_2} > 600 \text{ GeV}$$

Non-factorizable contributions

The answer to this question is affirmative. In fact, the leading term in the required expansion is known as the high-energy scattering (Regge) limit.

We will only need the abelian version of the Regge limit that was extensively studied in the early days of physics at electron-positron colliders.

Translated to QCD language, these studies imply that virtual gluons are ``soft'' so that leading high-energy asymptotic is obtained by employing the following approximations:

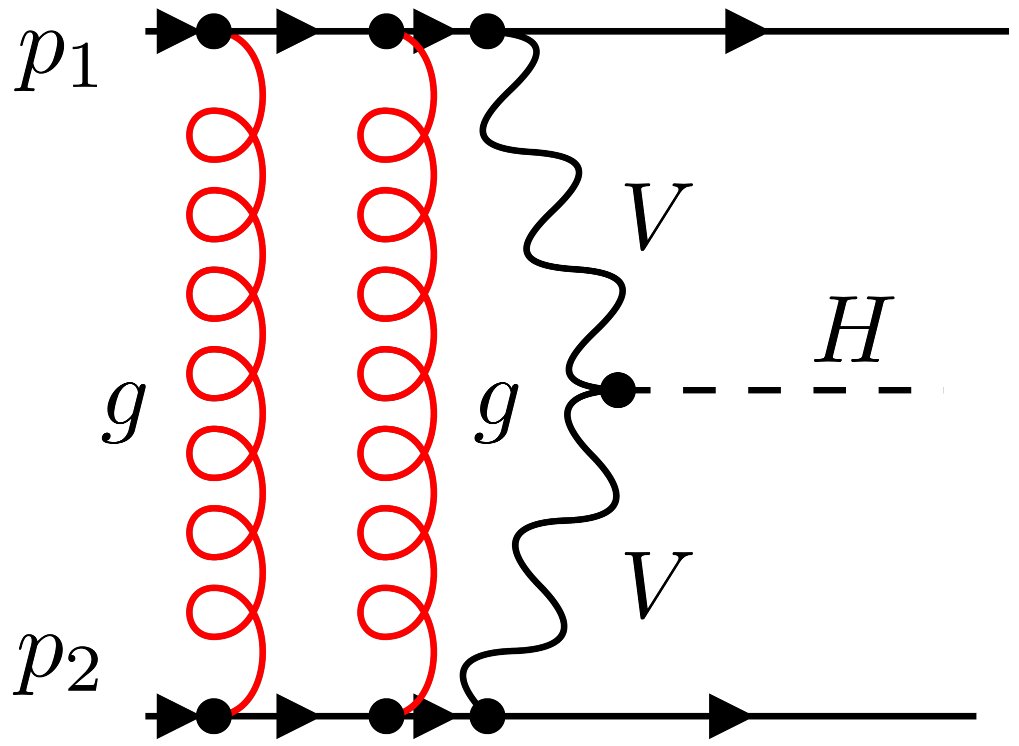
Sudakov, Lipatov, Gribov, Cheng, Wu, Chang, Ma

1) eikonal propagators for quark lines: $$\frac{1}{2pk + i0}$$

$$k = \alpha p_1 + \beta p_2 + k_\perp$$

2) eikonal couplings of quarks to gluons: $-2ie\, p_\mu$

3) no longitudinal momenta components in gluon and vector boson propagators: $$\frac{1}{k^2} \to -\frac{1}{k_\perp^2}$$



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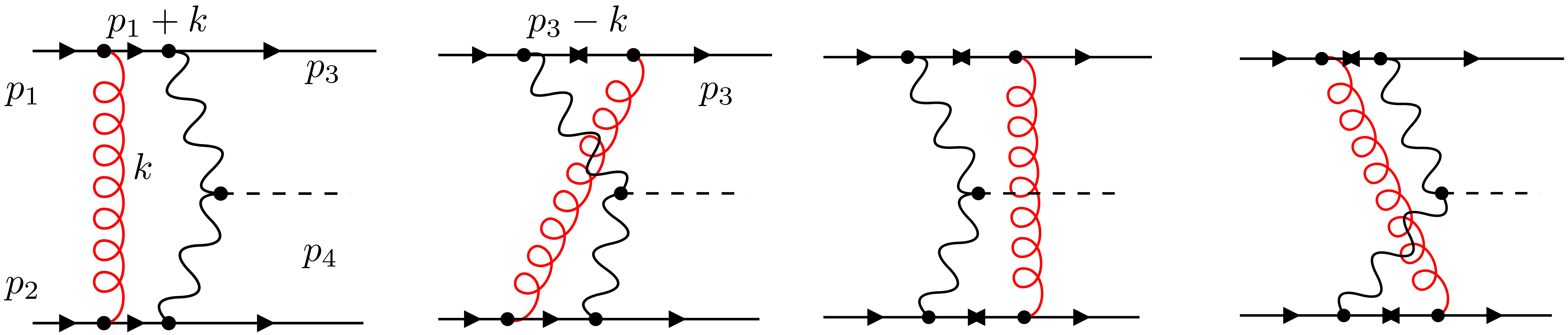
Non-factorizable contributions

Since we only need the interference of the two-loop amplitude with the leading order amplitude, the sum over colors makes the contribution abelian, i.e. QED-like. This effective abelianization, together with the eikonal approximation, makes one- and two-loop computations quite simple.

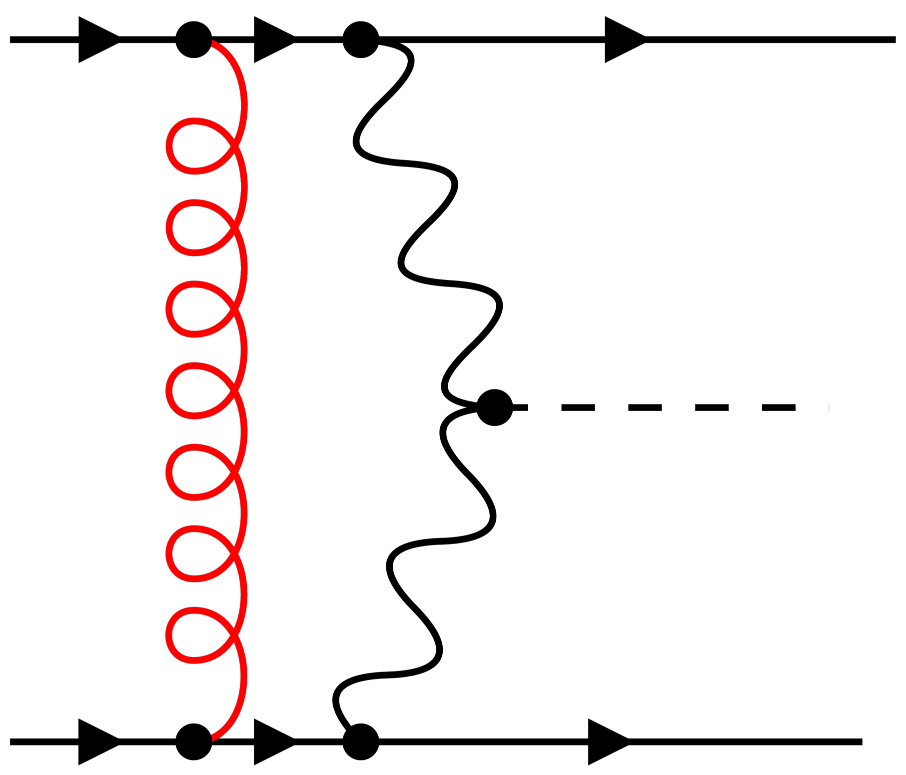


$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k_\perp^2} \frac{1}{(k_\perp - q_{3,\perp})^2 + M_V^2} \frac{1}{(k_\perp + q_{4,\perp})^2 + M_V^2} \left[\frac{1}{2p_1 k + i0} + \frac{1}{-2p_3 k + i0} \right] \left[\frac{1}{-2p_2 k + i0} + \frac{1}{2p_4 k + i0} \right]$$

$$\lim_{p_3 \to p_1} \left[\frac{1}{2p_1 k + i0} - \frac{1}{+2p_3 k + i0} \right] = -\frac{2i\pi}{s} \delta(\beta) \qquad \lim_{p_4 \to p_2} \left[\frac{1}{-2p_2 k + i0} + \frac{1}{2p_4 k + i0} \right] = -\frac{2i\pi}{s} \delta(\alpha)$$

$$k = \alpha p_1 + \beta p_2 + k_\perp \qquad \mathrm{d}^4 k = \frac{s}{2}\, \mathrm{d}\alpha\, \mathrm{d}\beta\, \mathrm{d}^2 k_\perp$$

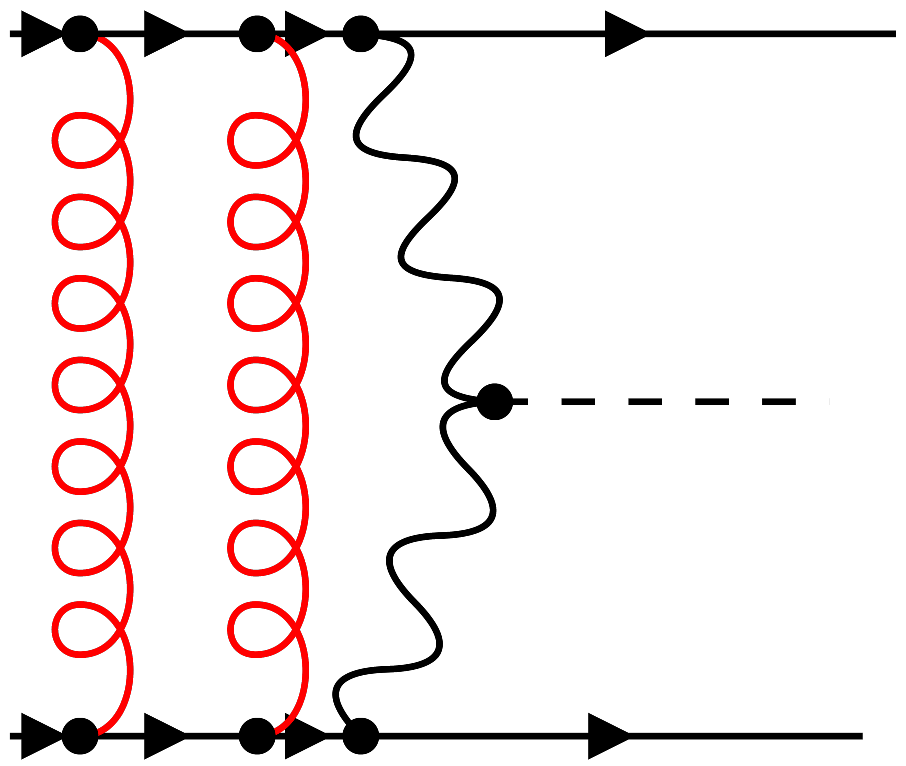
Non-factorizable contributions



$$\mathcal{M}^{(1)} = i\tilde{\alpha}_s \chi^{(1)}(\boldsymbol{q}_3, \boldsymbol{q}_4)\mathcal{M}^{(0)}$$

$$\chi^{(1)} = \frac{1}{\pi}\int \frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2+\lambda^2} \times \frac{\mathbf{q}_3^2+M_V^2}{(\mathbf{k}-\mathbf{q}_3)^2+M_V^2}\frac{\mathbf{q}_4^2+M_V^2}{(\mathbf{k}+\mathbf{q}_3)^2+M_V^2}$$

$$\chi^{(1)} = -\ln\left(\frac{\lambda^2}{M_V^2}\right) + f^{(1)}(\boldsymbol{q}_3, \boldsymbol{q}_4, M_V^2)$$



$$\mathcal{M}^{(2)} = -\frac{\tilde{\alpha}_s^2}{2!}\chi^{(2)}(\boldsymbol{q}_3, \boldsymbol{q}_4)\mathcal{M}^{(0)}$$

$$\chi^{(2)} = \frac{1}{\pi^2}\int \prod_{i=1}^{2} \frac{\mathrm{d}^2\mathbf{k}_i}{\mathbf{k}_i^2+\lambda^2} \times \frac{\mathbf{q}_3^2+M_V^2}{(\mathbf{k}_1+\mathbf{k}_2-\mathbf{q}_3)^2+M_V^2}\frac{\mathbf{q}_4^2+M_V^2}{(\mathbf{k}_1+\mathbf{k}_2+\mathbf{q}_4)^2+M_V^2}$$

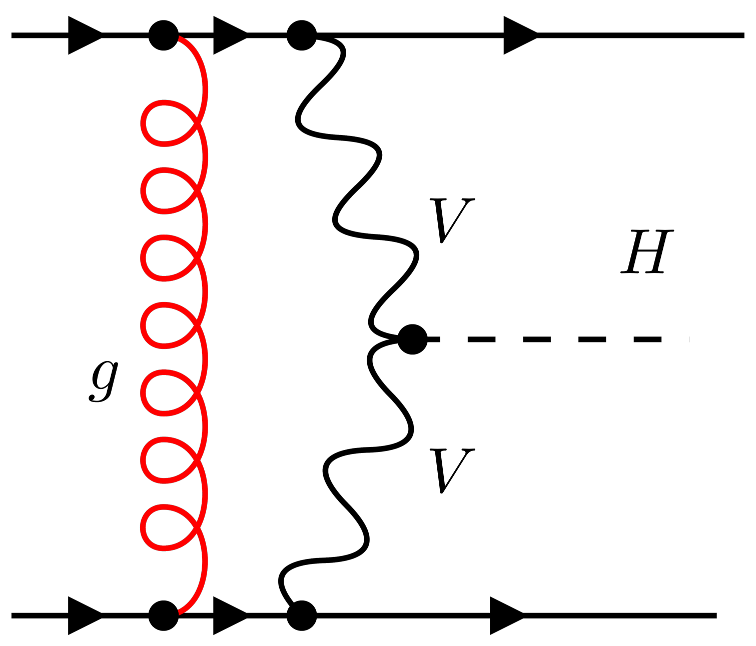
$$\chi^{(2)} = \ln^2\left(\frac{\lambda^2}{M_V^2}\right) - 2\ln\left(\frac{\lambda^2}{M_V^2}\right) f^{(1)}(\boldsymbol{q}_3, \boldsymbol{q}_4, M_V^2) + f^{(2)}(\boldsymbol{q}_3, \boldsymbol{q}_4, M_V^2)$$

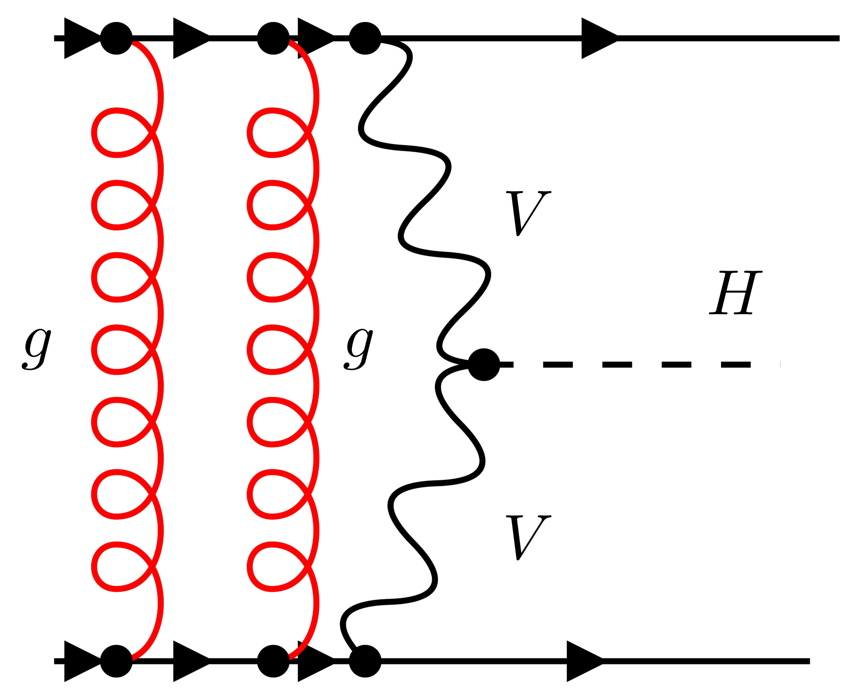
The infrared divergences that appear in non-factorizable contributions exponentiate into a Coulomb (Glauber) phase and cancel on their own, without any reference to real emission contributions.

$$\mathcal{M} = \mathcal{M}_0\, e^{-i\tilde{\alpha}_s \ln\frac{\lambda^2}{M_V^2}}\left[1 + i\tilde{\alpha}_s f^{(1)} - \frac{\tilde{\alpha}_s^2}{2} f^{(2)} + \dots\right].$$

Non-factorizable contributions

The final result is given by the sum of the one-loop amplitude squared and the interference of the two-loop and tree amplitudes. In this combination, the dependence on the gluon mass (i.e. the infra-red divergence) cancels out. Hence, we obtain a physical result that does not require us to account for the real emission contribution.





$$\boxed{\begin{aligned} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{NNLO}} &= \left(\frac{N_c^2-1}{4N_c^2}\right)\alpha_s^2\,\chi_{\mathrm{nf}}\,\mathrm{d}\sigma^{\mathrm{LO}} \\ \chi_{\mathrm{nf}}(\boldsymbol{q}_3,\boldsymbol{q}_4) &= [f^{(1)}(\boldsymbol{q}_3,\boldsymbol{q}_4)]^2 - f^{(2)}(\boldsymbol{q}_3,\boldsymbol{q}_4) \end{aligned}}$$

$$\begin{aligned} r_1 &= \boldsymbol{q}_3^2\,x + \boldsymbol{q}_4^2\,(1-x) - \boldsymbol{q}_H^2\,x(1-x)\,, \\ r_2 &= \boldsymbol{q}_H^2\,x(1-x) + M_V^2\,, \\ r_{12} &= r_1 + r_2\,, \\ \Delta_i &= \boldsymbol{q}_i^2 + M_V^2\,. \end{aligned}$$

$$f^{(1)} = \int\limits_0^1 \mathrm{d}x \frac{\Delta_3\Delta_4}{r_{12}^2}\left[\ln\left(\frac{r_{12}^2}{r_2 M_V^2}\right) + \frac{r_1-r_2}{r_2}\right],$$

$$\begin{aligned} f^{(2)} = \int\limits_0^1 \mathrm{d}x \frac{\Delta_3\Delta_4}{r_{12}^2}\Bigg[&\left(\ln\left(\frac{r_{12}^2}{r_2 M_V^2}\right) + \frac{r_1-r_2}{r_2}\right)^2 \\ &- \ln^2\left(\frac{r_{12}}{r_2}\right) - \frac{2r_{12}}{r_2}\ln\left(\frac{r_{12}}{r_2}\right) - 2\,\mathrm{Li}_2\left(\frac{r_1}{r_{12}}\right) \\ &- \left(\frac{r_1-r_2}{r_2}\right)^2 + \frac{\pi^2}{3}\Bigg], \end{aligned}$$

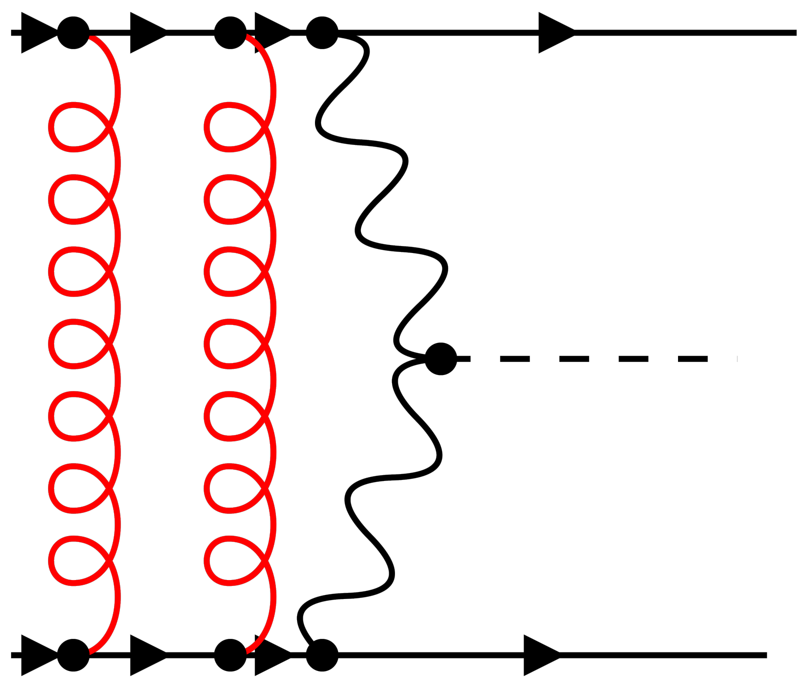
Non-factorizable contributions: some comments

Since non-factorizable corrections appear at NNLO for the very first time, there is no information about their renormalization scale dependence; one can expect though that the scale should be associated with jet transverse momenta and, therefore, should be low.

Furthermore, the non-factorizable corrections are π -enhanced; additional factors of π come from the residues of the eikonal propagators; this enhancement is important as it can compensate for the $1/N_c$ color suppression.

Finally, the non-factorizable effects are not included in parton showers, even approximately, since the contribution that we have discussed is fully-independent of any real emission and is IR-finite on its own.

$$\mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{NNLO}} = \left(\frac{N_c^2 - 1}{4 N_c^2} \right) \alpha_s^2(\mu) \, \chi_{\mathrm{nf}} \, \mathrm{d}\sigma^{\mathrm{LO}}$$



Fiducial cross section: non-factorizable corrections

Non-factorizable corrections to fiducial WBF cross section computed with cuts shown below are found to be -0.4 percent.

T.Liu, K.M., A. Penin

Compared to factorizable corrections, they are similar to the inclusive cross section case but smaller than corrections to the fiducial cross section.

	$\sigma^{(\text{no cuts})}$ [pb]		$\sigma^{(\text{VBF cuts})}$ [pb]	
LO	$4.032^{+0.057}_{-0.069}$	-2.6%	$0.957^{+0.066}_{-0.059}$	-8.4%
NLO	$3.929^{+0.024}_{-0.023}$	-1%	$0.876^{+0.008}_{-0.018}$	-3.6%
NNLO	$3.888^{+0.016}_{-0.012}$		$0.844^{+0.008}_{-0.008}$	

Cacciari, Dreyer, Karlberg, Salam, Zanderighi

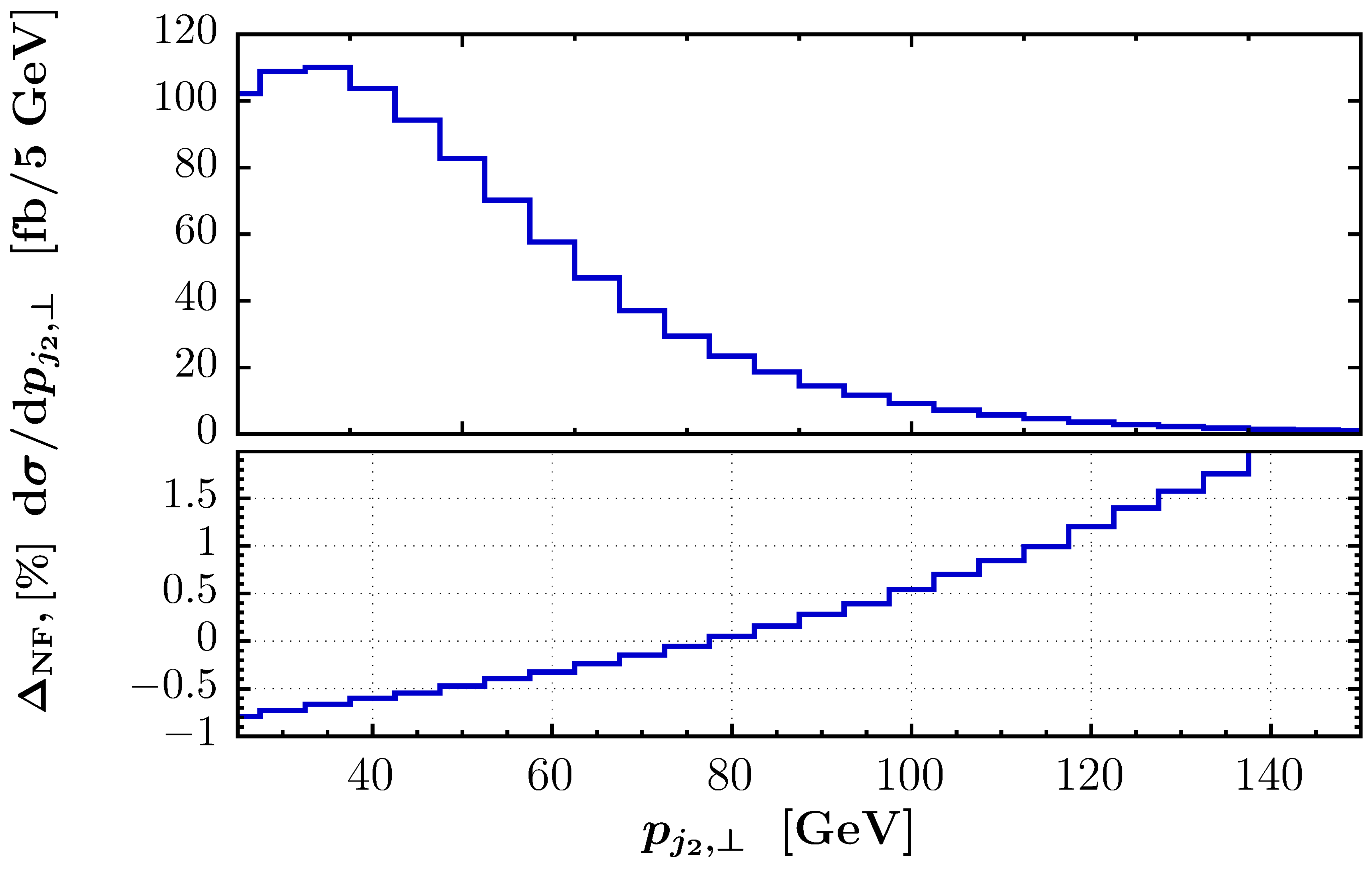
$$p_\perp^{j_1,j_2} > 25~\text{GeV}, \quad |y_{j_1,j)2}| < 4.5$$
$$|y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1 j_2} > 600~\text{GeV}$$

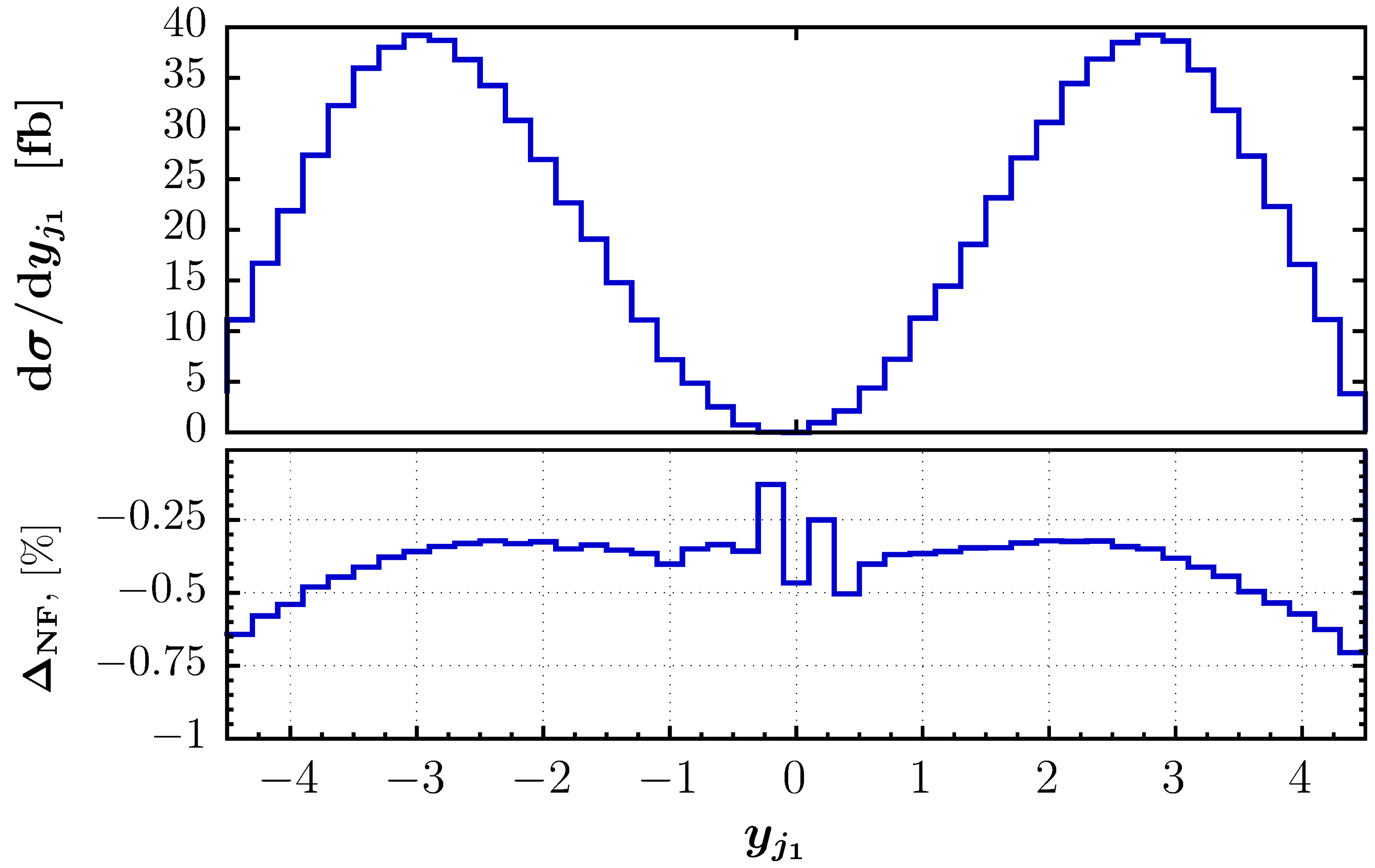
$$\mu_F = \left[\frac{m_H}{2} \sqrt{\frac{m_H^2}{4} + p_{\perp,H}^2} \right]^{1/2}$$

$$\mu_R = \sqrt{p_{\perp,j_1} p_{\perp,j_2}}$$

PDF set: NNPDF 3.0

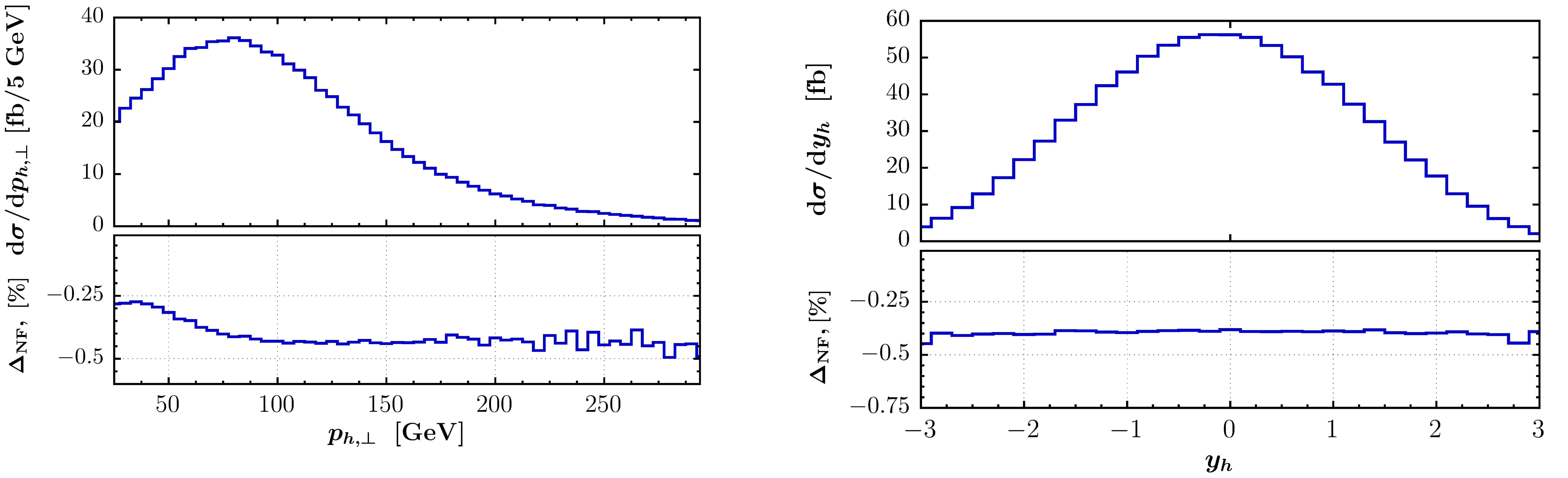
Kinematic distirbutions: non-factorizable corrections





We find percent level non-factorizable effects in kinematic distributions; corrections to transverse momentum distribution of a second jet change from negative to positive. Corrections to rapidity distributions are quite uniform.

Kinematic distirbutions: non-factorizable corrections



Corrections to the Higgs boson transverse momentum and rapidity distributions are rather constant. Both are about half a percent.

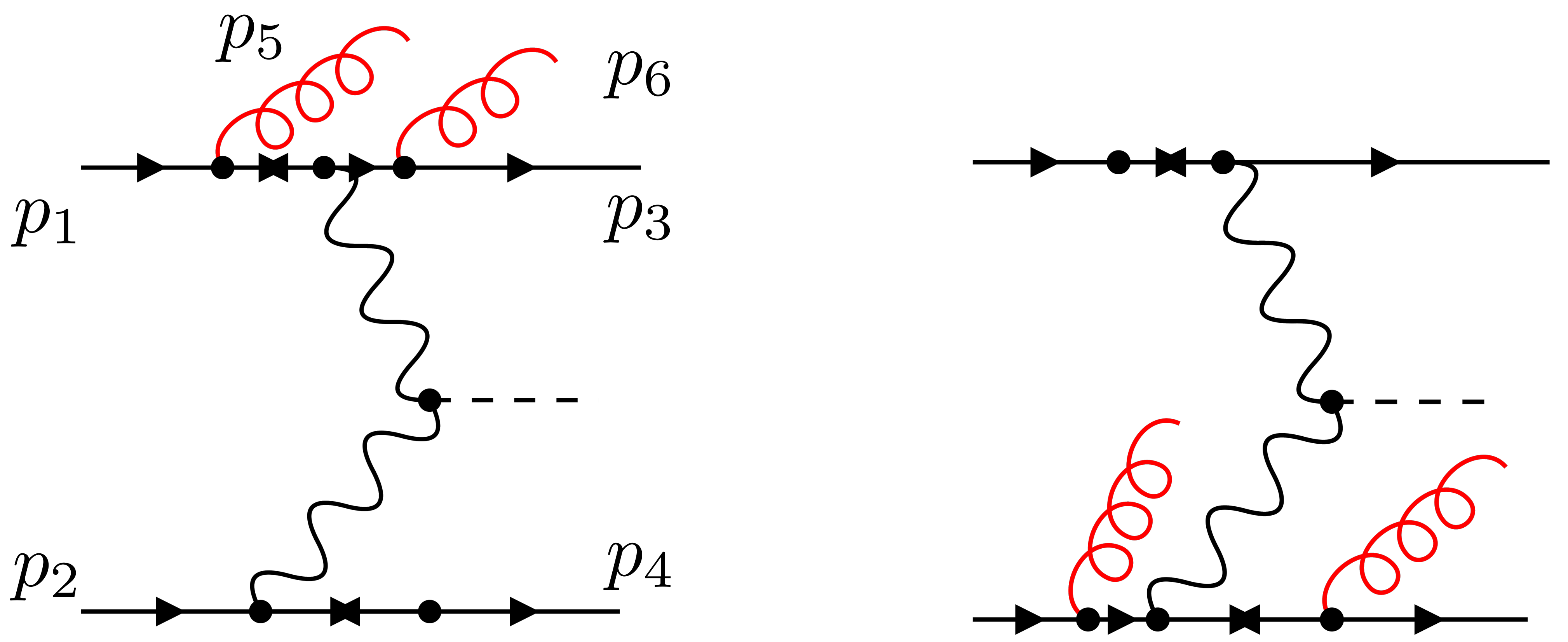
Non-factorizable corrections beyond eikonal approximation

Eikonal approximation (forward scattering) gives us the Coulomb phase; that is why infra-red singularities in this contribution cancel on their own. Because of the difference between a gluon and two weak bosons fusing into the Higgs boson, the result is different from zero.

Other effects, e.g. real emission contribution, require us to go beyond the forward scattering limit in computing both virtual and real emission contributions.

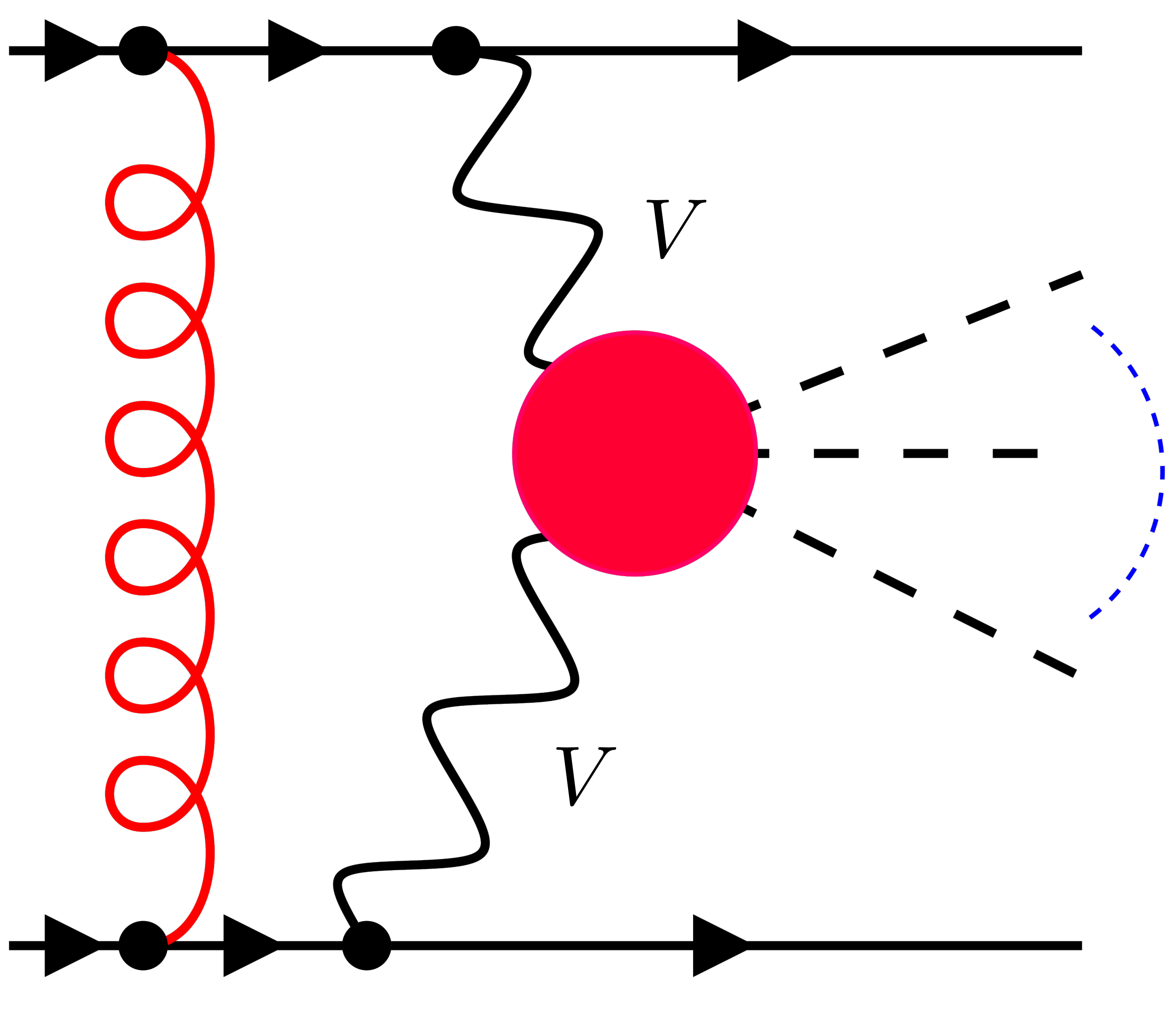
$$\lim_{p_5,p_6\to 0} |\mathcal{M}(1_q,2_Q,3_q,4_Q)|^2_{\rm nf} = (N_c^2-1)\mathrm{Eik}_{\rm nf}(p_5)\mathrm{Eik}_{\rm nf}(p_6)\ \mathcal{A}_0^2(1,2,3,4)$$

$$\mathrm{Eik}(p) = \sum_{i\in[1,3];j\in[2,4]} \lambda_{ij}\frac{p_i p_j}{(p_i p)(p_j p)} \qquad \lambda_{ij} = \begin{cases} 1 & i,j \text{ both incoming or outgoing} \\ -1 & \text{otherwise} \end{cases}$$



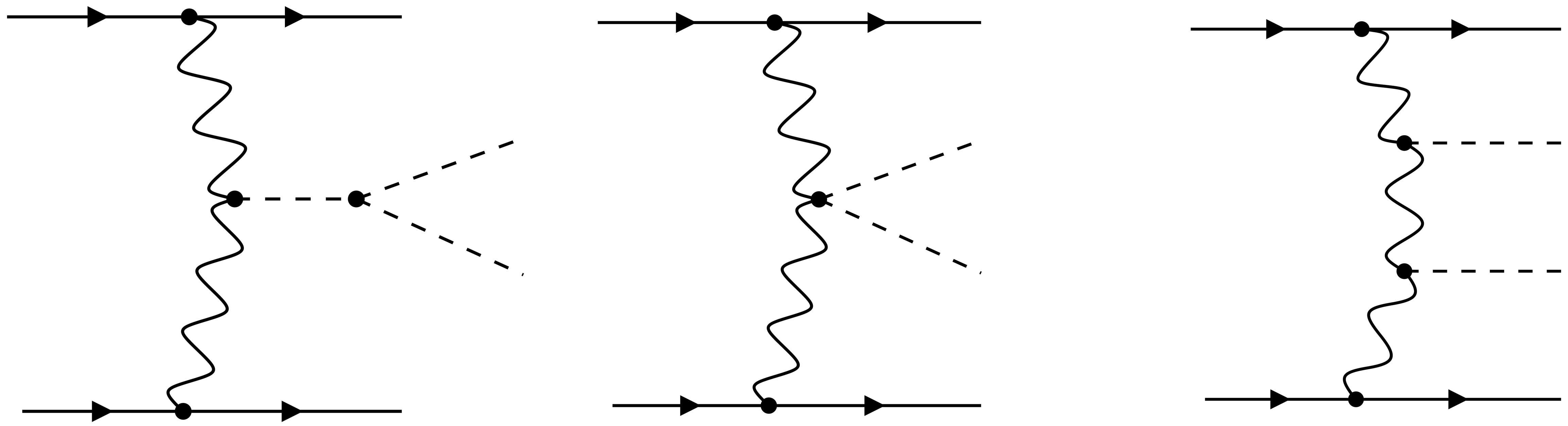
Other processes

The high-energy approximation is applicable for generic processes of the WBF type. However, the required computations becomes more complex since the blob may contain several Feynman diagrams.



$$k_i = \alpha_i p_1 + \beta_i p_2 + k_\perp$$

$$\sim \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{(2\pi)^2} \frac{B_{VV\to \mathrm{fin}}(q_{1,\perp}, q_{2,\perp}, k_\perp, ...)}{k_\perp^2 \; ((\mathbf{q}_{1,\perp} - \mathbf{k}_\perp)^2 + M_V^2)((\mathbf{q}_{2,\perp} + \mathbf{k}_\perp)^2 + M_V^2)}$$



$$\sigma_{HH} = 10.4|_{TT} + 14.2|_{BB} - 23.904_{TB} = 0.7 \text{ fb}$$

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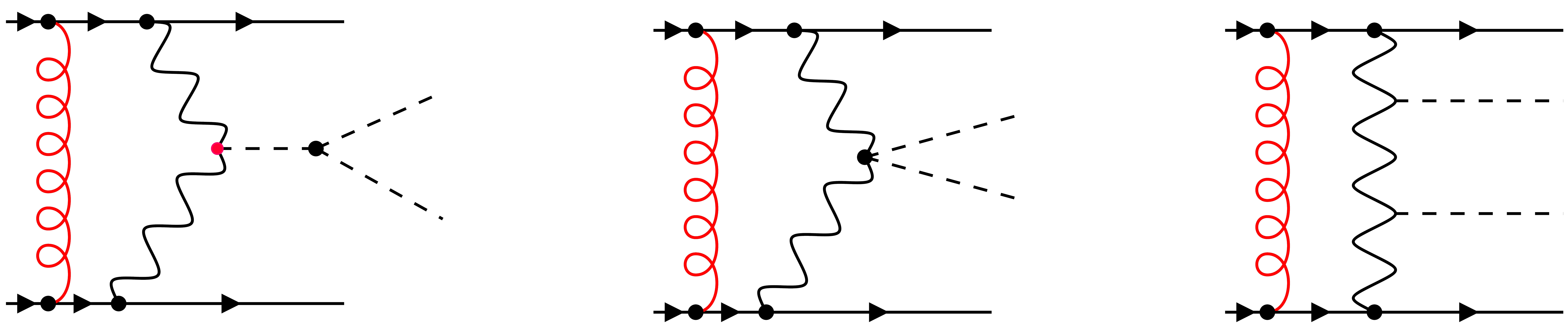
Di-Higgs production in WBF

The non-factorizable corrections were explicitly studied for the di-Higgs production in weak boson fusion.

In this case, very strong cancellations between various contributions to leading order cross section get destroyed by non-factorizable QCD corrections (but it is still respected by the factorizable ones).

As the result, non-factorizable corrections in such processes can be as large as the factorizable ones!



$\lambda = M_V$	σ_{TT}	σ_{BB}	σ_{TB}	Σ
Born	10.393 fb	14.172 fb	−23.904 fb	0.662 fb
1-loop NF	0.339%	0.518%	0.399%	2.03%
2-loop NF	−0.667%	−0.658%	−0.666%	−0.50%
Full NF	−0.327%	−0.139%	−0.267%	1.52%

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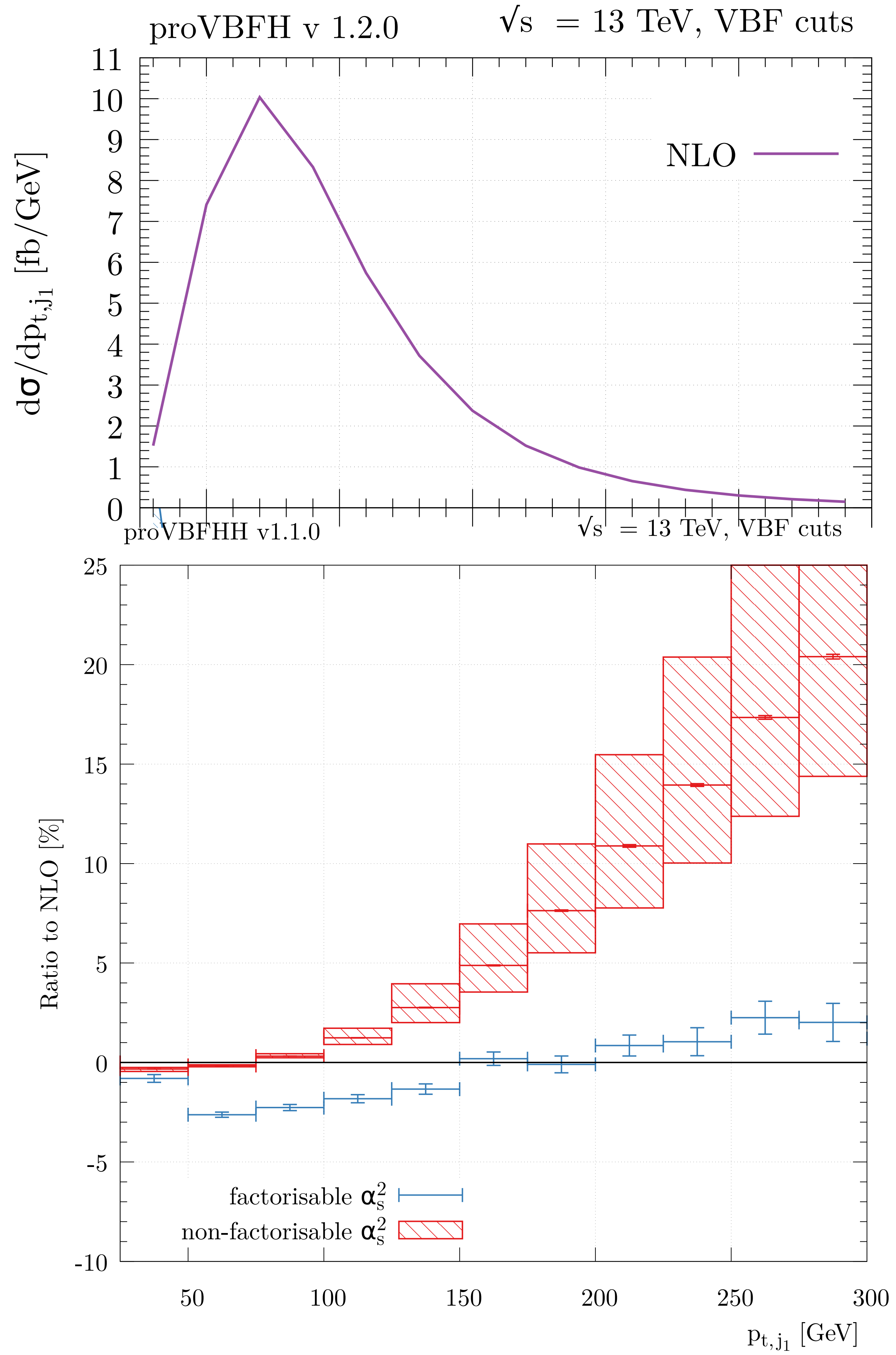
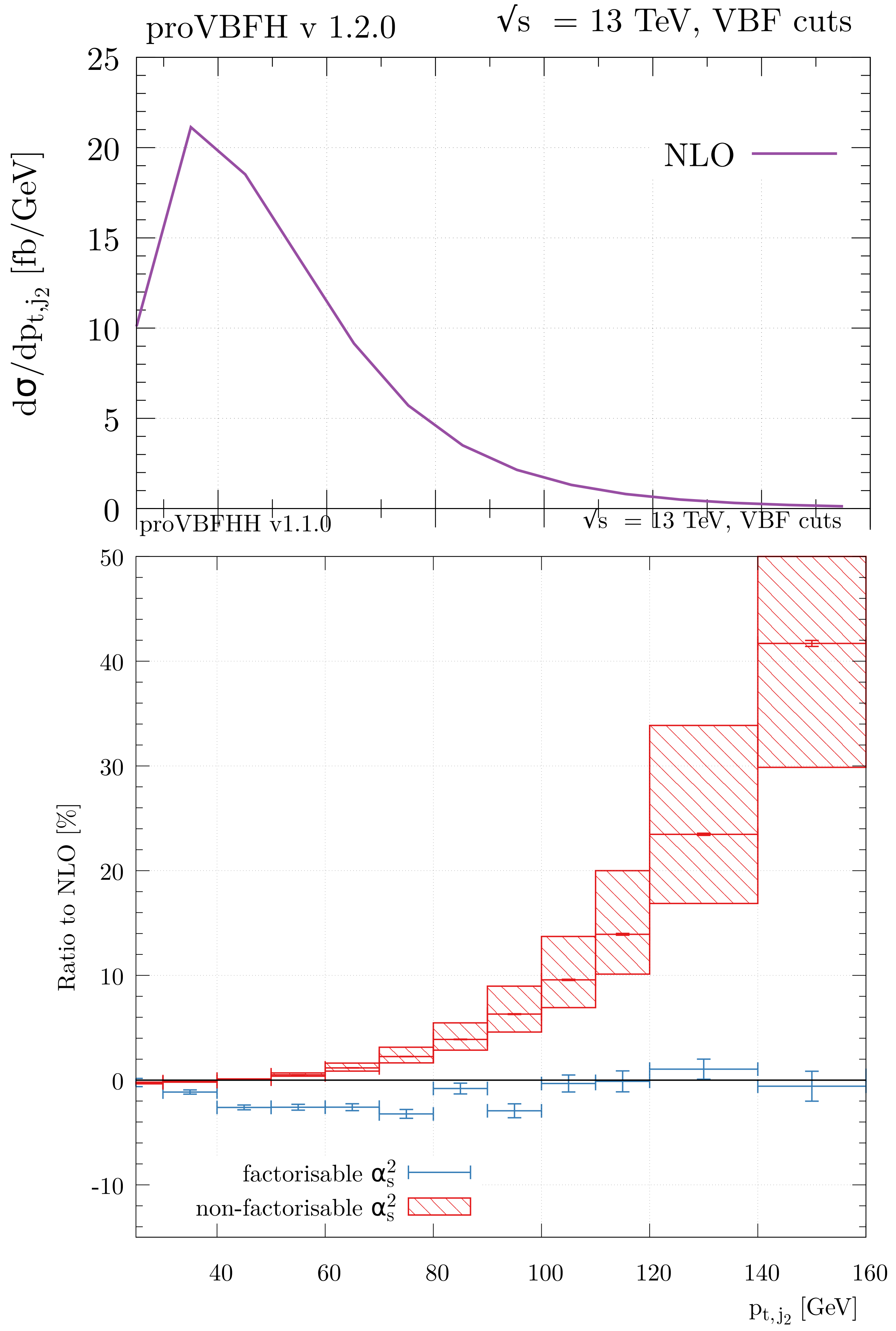
Di-Higgs production in WBF

For some kinematic distributions non-factorizable QCD corrections become dominant and reach 10-50 percent!





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Summary

We discussed the non-factorizable corrections to process of the weak boson fusion type Our interest to understand these effect is related to an impressive progress in computing factorizable corrections, where N3LO QCD corrections to inclusive WBF have been computed.

Non-factorizable corrections to WBF-like processes have the following properties:

1) they appear at next-to-next-to-leading order for the first time; thanks to WBF kinematic, they can be studied in the high-energy (eikonal) approximation;

2) they are colour-suppressed but π - enhanced; their origin is related to the Coulomb (Glauber) phase;

3) for Higgs production in WBF, these corrections can reach a percent level in kinematic distributions and they are strongly kinematic-dependent;

4) non-factorizable corrections seem to be even more important for more complex processes (e.g. double Higgs production in WBF) where they destroy subtle cancellations that can be present at LO;

5) non-factorizable corrections are definitely more important than factorizable N3LO corrections;

6) they cannot be estimated using parton showers, even approximately.