HEFT and the EW symmetry-breaking sector at high energy

Felipe J. Llanes-Estrada

People at Univ. Complutense de Madrid:

A. Dobado, J. Sanz-Cillero, A. Salas-Bernárdez, C. Quezada, J.R. Peláez...
(also J.A. Oller @ Murcia, R. L. Delgado @ Politécnica)

Presented at MBI, Aug. 26th 2021

1. A cute little Lagrangian

2. Some comments in comparing to *SM* and *SMEFT*

3. Unitarization of two-body $VV \to VV$ scattering

4. Systematic uncertainty estimate of IAM

Outline

1. A cute little Lagrangian

2. Some comments in comparing to *SM* and *SMEFT*

3. Unitarization of two-body $VV \to VV$ scattering

4. Systematic uncertainty estimate of IAM

And the Higgs was found...

- Explained the size of the atom and of all beautiful things

$$\frac{1}{\lambda_e v_{\rm Higgs}} \propto \frac{1}{m_e} \propto a_{\rm Bohr}$$

Bohr radius
gives us scale

25 SM+ν parameters

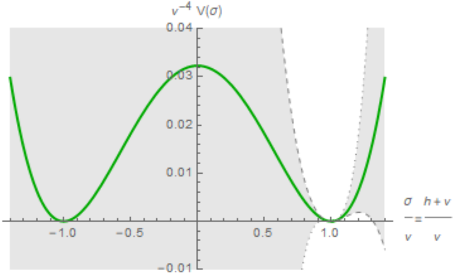
- 9 Yukawa couplings (6 q, 3 $l^{\pm}$ masses)
- 3 CKM mixing angles, 1 δ_{CP} phase
- 3 ν masses
- 3 PMNS mixing angles, 1 δ_{CP} phase
- 3 gauge couplings (α, α_s, θ_W)
- v, m_H

But EWSBS sector little known:
redundantly measure λ, μ, further unknown couplings?

Higgs self-couplings and couplings to Goldstone bosons

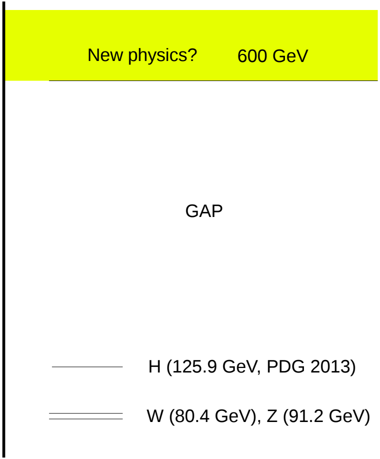
$\Phi \to W_L \sim \omega,\ \ Z_L \sim z,\ \ h$

Status of Higgs potential



(taken from J.J.Sanz-Cillero)

Mass Gap to new physics $\Longrightarrow$ EFT



Two formulations, SMEFT & HEFT

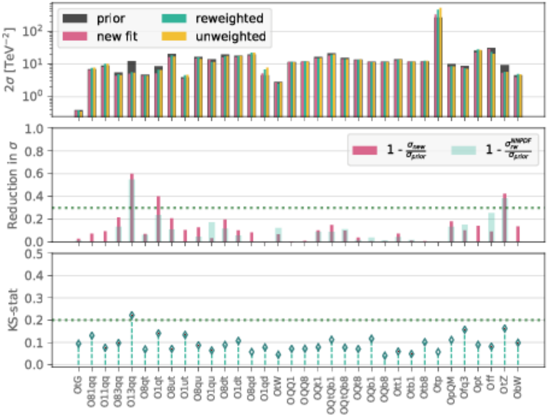
Number of operators

- SMEFT: 1350 CP-even operators to dimension 6
- HEFT: O(160) NLO operators with functions of h, ω

Alonso, Jenkins, Manohar & Trott JHEP 04 (2014) 159.

Buchalla & Catà, JHEP 07 (2012) 101, Nucl.Phys.B 880 (2014) 552.

Efforts to constrain Wilson coefficients AND pdfs ongoing



Beek, Nocera, Rojo, Slade, SciPost Phys. **7** (2019) 5, 070

Strategy 1: “No stone left unturned”, constrain them all.

Minimization suited for Artificial Intelligence

Too many terms = too much noise

Maybe enough scientists with enough coefficients will find separation from SM...

http://notapipe.biz/quality-quantity-and-infinite-monkeys/

LHC is unique for Electroweak Symmetry Breaking Sector

Cut to the chase: New physics in Vector Boson Scattering? (VBScan ✓)
Regime $m_h \ll E \ll \Lambda_{NP}$

- m_h, m_W, $m_Z \ldots$, $g_{\rm gauge} = 0$; only operators $\partial \propto E$

The VBS part becomes quite simple

- ~~9 Yukawa couplings (6 q, 3 $l^{\pm}$ masses)~~
- ~~3 CKM mixing angles, 1 δ_{CP} phase~~
- ~~3 ν masses~~
- ~~3 PMNS mixing angles, 1 δ_{CP} phase~~
- ~~3 gauge couplings (α, α_s, θ_W)~~
- ~~v, m_H~~

~~But EWSBS sector little known:~~
~~redundantly measure λ, μ, further unknown couplings?~~

(well, except v)

LHC is unique for Electroweak Symmetry Breaking Sector

- h additional scalar particle distinguished by symmetry breaking
- Relevant processes:
  $W_L W_L \to W_L W_L$, $W_L W_L \to (h)h$, $hh \to hh$, $W_L h \to W_L h$
- Advantage for expt.: use the highest energy data of the LHC!

(Under hypothesis of $SU(2)$ isospin custodial symmetry, only even numbers of W)

R.L. Delgado, A. Dobado, FJLE and others, series of Complutense papers

Equivalence theorem

- Scattering observables with W_L, Z_L approximately those of the Goldstone ω, z

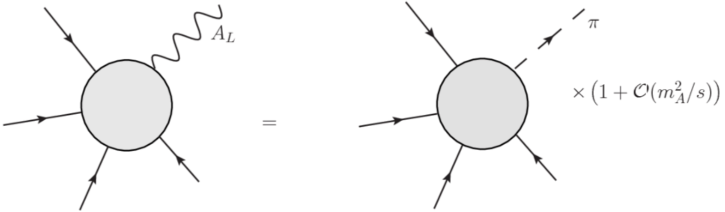


fig. by Anand and Cantrell

Lagrangian of HEFT for the SBS only:

$$\mathcal{L}_0 = \frac{v^2}{4} F(h) (D_\mu U)^\dagger (D^\mu U) + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h)$$

$U \equiv \sqrt{1 - \omega^2/v^2} + i\ \vec{\omega} \cdot \vec{\tau}/v$
(coordinates of the nonlinear sigma model)

$$F(h)_{HEFT} = 1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2 + \ldots$$

HEFT Lagrangian for electroweak symmetry breaking

Compact, seven-number version of agnostic TeV-scale new physics

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{1}{2}\left(1+2a\frac{h}{v}+b\left(\frac{h}{v}\right)^2\right)\partial_\mu\omega^a\partial^\mu\omega^b\left(\delta_{ab}+\frac{\omega^a\omega^b}{v^2}\right)\\
+\;& \frac{4a_4}{v^4}\partial_\mu\omega^a\partial_\nu\omega^a\partial^\mu\omega^b\partial^\nu\omega^b+\frac{4a_5}{v^4}\partial_\mu\omega^a\partial^\mu\omega^a\partial_\nu\omega^b\partial^\nu\omega^b\\
+\;& \frac{1}{2}\partial_\mu h\partial^\mu h+\frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2\\
+\;& \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^a\partial^\nu\omega^a+\frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^a\partial_\nu\omega^a.
\end{aligned}
$$

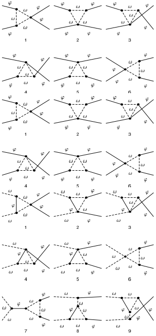
R. L. Delgado, A. Dobado and FJLE, PRD **91**, 075017 2015

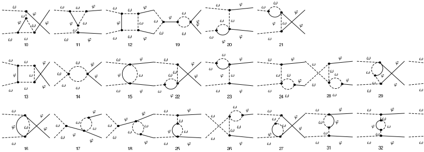
Derivative expansion ($\simeq$ ChPT)

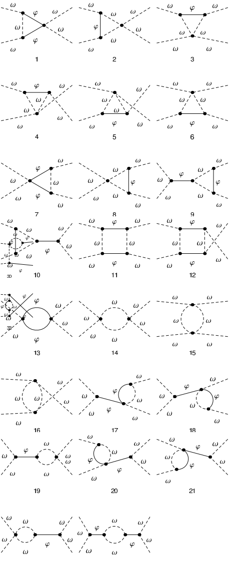
$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\left(1+2a\frac{h}{v}+b\left(\frac{h}{v}\right)^2\right)\boxed{\partial_\mu\omega^a\partial^\mu\omega^b}\left(\delta_{ab}+\frac{\omega^a\omega^b}{v^2}\right)\\
&+ \frac{4a_4}{v^4}\partial_\mu\omega^a\partial_\nu\omega^a\partial^\mu\omega^b\partial^\nu\omega^b+\frac{4a_5}{v^4}\boxed{\partial_\mu\omega^a\partial^\mu\omega^a\partial_\nu\omega^b\partial^\nu\omega^b}\\
&+ \frac{1}{2}\partial_\mu h\partial^\mu h+\frac{g}{v^4}\boxed{(\partial_\mu h\partial^\mu h)^2}\\
&+ \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^a\partial^\nu\omega^a+\frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^a\partial_\nu\omega^a.
\end{aligned}$$

R. L. Delgado, A. Dobado and FJLE, PRD **91**, 075017 2015

Renormalized one-loop computations







Delgado, Dobado and FLE, JHEP **02** (2014) 121

Couple additional channels

Top-antitop Castillo, Delgado, Dobado, FLE Eur.Phys.J.C **77** (2017) 7, 436

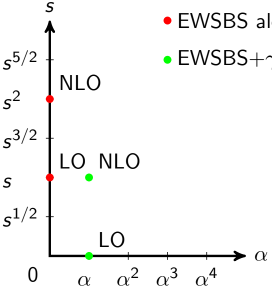
$$
\begin{aligned}
\mathcal{L}_4 \qquad &= -\left(1 + c_1\frac{h}{v} + c_2\frac{h^2}{v^2}\right)\left\{\left(1-\frac{\omega^2}{2v^2}\right)M_t t\bar{t}\right. \\
&\left. + \frac{i\sqrt{2}\omega^0}{v}M_t\bar{t}\gamma^5 t - i\sqrt{2}\frac{\omega^+}{v}M_t\bar{t}_R b_L + i\sqrt{2}\frac{\omega^-}{v}M_t\bar{b}_L t_R\right\}
\end{aligned}
$$

Photon-photon Delgado, Dobado, Herrero and Sanz-Cillero, JHEP **1407**, 149 (2014).

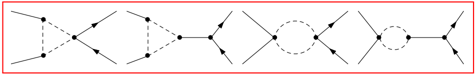
$$
\begin{aligned}
\mathcal{L}_4 = \frac{2e(a_2-a_3)}{v^2}A_{\mu\nu}\left[i\left(\partial^\nu\omega^+\partial^\mu\omega^- - \partial^\mu\omega^+\partial^\nu\omega^-\right)\right. \\
\left. + eA^\mu\left(\omega^+\partial^\nu\omega^- + \omega^-\partial^\nu\omega^+\right) - eA^\nu\left(\omega^+\partial^\mu\omega^- + \omega^-\partial^\mu\omega^+\right)\right] \\
-\frac{c_\gamma}{2}\frac{h}{v}e^2 A_{\mu\nu}A^{\mu\nu} + \frac{e^2 a_1}{2v^2}A_{\mu\nu}A^{\mu\nu}\left(v^2 - 4\omega^+\omega^-\right)
\end{aligned}
$$

Couple additional channels

- In pert. theory one more order



- But resonance poles can nonetheless appear



Outline

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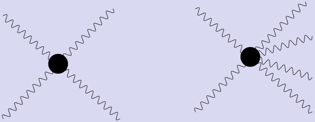
Cosmetic SMEFT-HEFT difference: Counting organization

- SMEFT by canonical dimension (independently of N_{loops})
- HEFT by number of derivatives (independently of $N_{\text{particles}}$)

Example 1



Example 2

$W_L W_L \partial^4 W_L W_L$

- NLO in HEFT (consider immediately after the SM $W_L \partial^2 W_L$)
- Dim. 8 In SMEFT (consider after Dim. 6 operators worked out)

Cosmetic SMEFT-HEFT difference: linear vs nonlinear

$$\mathcal{L}_{SM} = (D_\mu \Phi)^\dagger D^\mu \Phi - \underbrace{\lambda(\Phi^\dagger\Phi)^2 - \mu^2 \Phi^\dagger \Phi}_{V(\Phi)} + \mathcal{L}_{YK}$$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \omega_1 + i\omega_2 \\ (v + h) + i\omega_3 \end{pmatrix}$$

Linear representation: (SMEFT)

- ω_a and h fit in a left-$SU(2)$ doublet
- Higgs always in the combination: $(h + v)$
- Higher symmetry
- Typical when h is a fundamental field
- ET usually based in a cutoff Λ expansion: $O(d)/\Lambda^{d-4}$ ($d =$ operator dimension: 4,6,8 ...)

Non-linear representation: (HEFT)

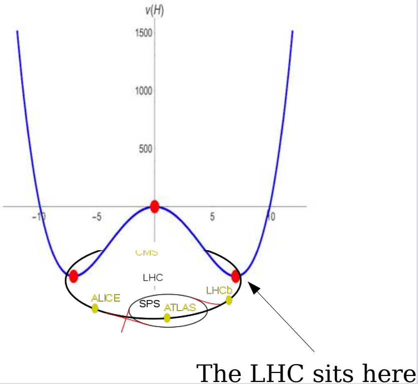
- h is a $SU(2)$ singlet and ω_a are coordinates on a coset:
  $SU(2)_L \times SU(2)_R / SU(2)_V \simeq SU(2) \simeq S3$
- Lesser symmetry and more independent higher dimension effective operators but less model dependent
- Derivative expansion
- EChL with $F(h)$ insertions
- Typical for composite models of the SBS (h as a GB)
  (Strongly interacting and consistent with the presence of the GAP)

Dobado and Espriu, Prog.Part.Nucl.Phys. 115 (2020) 103813

Cosmetic SMEFT-HEFT difference: linear vs nonlinear

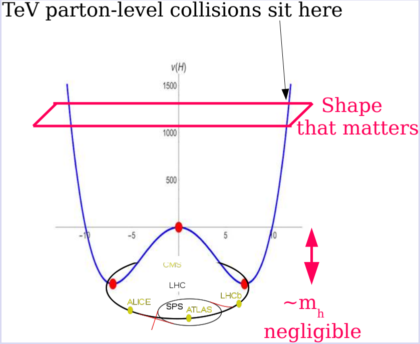
- But change coordinates like Cartesian to Spherical ones
- Use coordinate-independent approach (San Diego)

Note the “SM Higgs potential” might be a red herring



The LHC sits here

If there is new physics



TeV parton-level collisions sit here

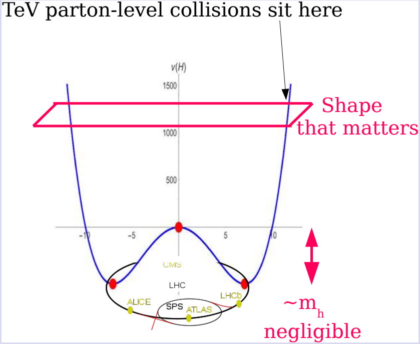
Shape that matters

$\sim m_h$ negligible

$$m_h \sim m_W \sim m_Z \ll \sqrt{s}$$

$$\Longrightarrow \quad V(h)_{\rm SM-like} \ll \sqrt{s}$$

If there is new physics



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$$\Longrightarrow \quad V(h)_{\rm SM-like} \ll \sqrt{s}$$

Seven-parameter EFT description
of what's important at LHC with new EWSBS physics

$F(h)$ multiplying Goldstone kinetic term wins at high E

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\boxed{\left(1+2a\frac{h}{v}+b\left(\frac{h}{v}\right)^2\right)}\partial_\mu\omega^a\partial^\mu\omega^b\left(\delta_{ab}+\frac{\omega^a\omega^b}{v^2}\right) \\
&+ \frac{4a_4}{v^4}\partial_\mu\omega^a\partial_\nu\omega^a\partial^\mu\omega^b\partial^\nu\omega^b+\frac{4a_5}{v^4}\partial_\mu\omega^a\partial^\mu\omega^a\partial_\nu\omega^b\partial^\nu\omega^b \\
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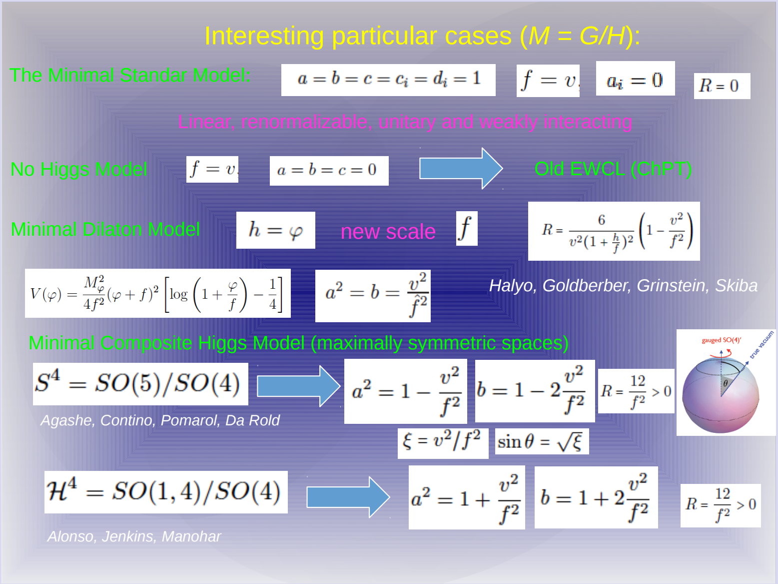
\end{aligned}$$

R. L. Delgado, A. Dobado and FJLE, PRD **91**, 075017 2015

Special cases



Interesting particular cases ($M = G/H$):

The Minimal Standar Model: $a=b=c=c_i=d_i=1$ $f=v$ $a_i=0$ $R=0$

Linear, renormalizable, unitary and weakly interacting

No Higgs Model $f=v$ $a=b=c=0$ → Old EWCL (ChPT)

Minimal Dilaton Model $h=\varphi$ new scale f $R=\frac{6}{v^2(1+\frac{h}{f})^2}\left(1-\frac{v^2}{f^2}\right)$

$V(\varphi)=\frac{M_\varphi^2}{4f^2}(\varphi+f)^2\left[\log\left(1+\frac{\varphi}{f}\right)-\frac{1}{4}\right]$ $a^2=b=\frac{v^2}{f^2}$

Halyo, Goldberber, Grinstein, Skiba

Minimal Composite Higgs Model (maximally symmetric spaces)

$S^4=SO(5)/SO(4)$ → $a^2=1-\frac{v^2}{f^2}$ $b=1-2\frac{v^2}{f^2}$ $R=\frac{12}{f^2}>0$

Agashe, Contino, Pomarol, Da Rold

$\xi=v^2/f^2$ $\sin\theta=\sqrt{\xi}$

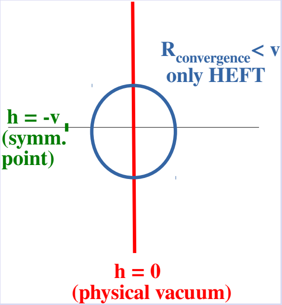
$\mathcal{H}^4=SO(1,4)/SO(4)$ → $a^2=1+\frac{v^2}{f^2}$ $b=1+2\frac{v^2}{f^2}$ $R=\frac{12}{f^2}>0$

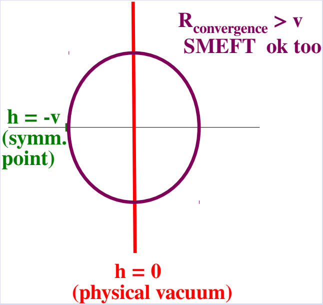
Alonso, Jenkins, Manohar

Courtesy of A. Dobado

San Diego criterion (geometric: indep. of field coordinates)

Convergence of $F(h)$ expansion in h field space
(not usual in experiment)





Problem: for now we have only the first one or two terms

San Diego criterion: symmetric point under custodial group

SMEFT is deployable if and only if (statement about the Lagrangian)

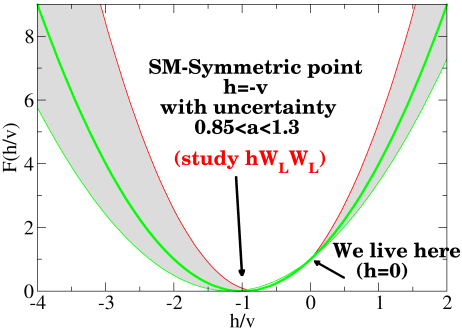
- There is a value $h = h^*$ where $F(h^*) = 0$, and
- F is analytic between our vacuum $h = 0$ and that point.

Note: The SM has just such point, $h = -v$ ($\Phi = 0$), where $F = (1 + h/v)^2 = 0$

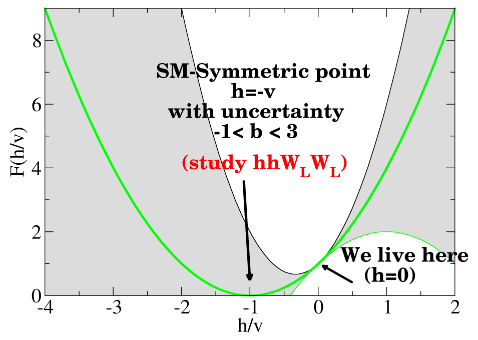
Alonso, Jenkins, Manohar JHEP **08** (2016) 101
Dobado, Espriu Prog.Part.Nucl.Phys. **115** (2020) 103813

Cohen, Craig, Lu, Sutherland JHEP **03** (2021) 237

Current status: vary *a* from 1

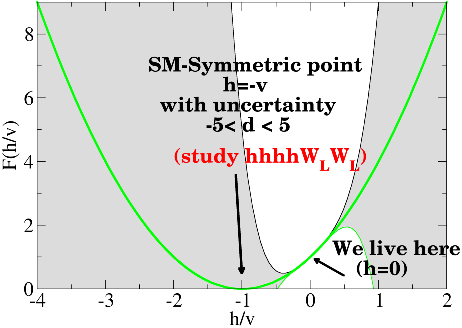


Current status: vary b from a^2

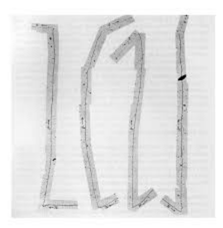


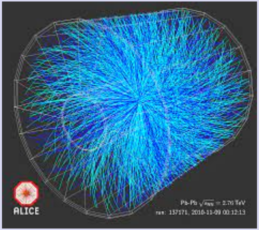
Delgado, Dobado, FLE Phys.Rev.Lett. **114** (2015) 22, 221803

Current status: vary h^4WW coefficient



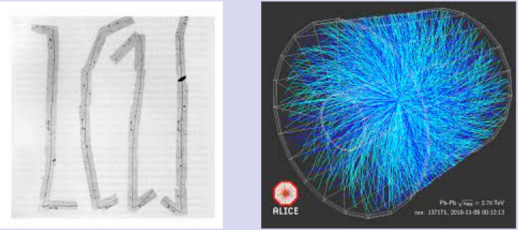
Can you produce multiboson final states?





60 years between two such pictures

Can you produce multiboson final states?



60 years between two such pictures

Ongoing project with A.Salas and J. Sanz Cillero at UCM and R. Gómez Ambrosio here at Milano.

Hints welcome

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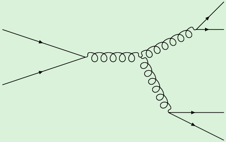
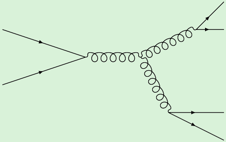
Is the LHC a high- or a low- energy machine?

- High energy scattering: $V << T$, Feynman diagrams, Madgraph, etc.

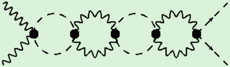


Is the LHC a high- or a low- energy machine?

- High energy scattering: $V << T$, Feynman diagrams, Madgraph, etc.



- Low energy respect to new physics (strongly interacting? $V \sim T$ requires resummation)



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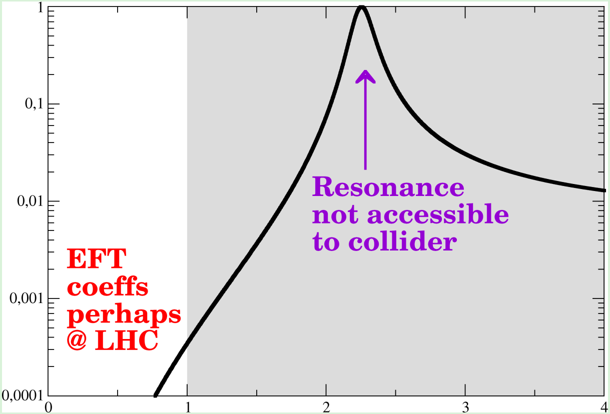

Expand partial wave amplitudes

$$T_I(s,t,u) = 16\eta\pi \sum_{J=0}^{\infty} (2J+1) t_{IJ}(s) P_J(\cos\theta_s)$$

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$$T_I(s,t,u) = 16\eta\pi \sum_{J=0}^{\infty} (2J+1) t_{IJ}(s) P_J(\cos\theta_s)$$

$$t_{IJ}(s) \simeq \underbrace{t_0}_{O(s)} + \underbrace{t_1}_{O(s^2)} + \dots$$

(typical HEFT expansion)

Inverse Amplitude Method

$$\frac{1}{t} \simeq \frac{1}{t_0 + t_1} \simeq \frac{1}{t_0} - \frac{t_1}{t_0^2} \implies \boxed{t^{IAM} \simeq \frac{t_0^2}{t_0 - t_1}}$$

Equivalent to $t_0 + t_1 + o(s^3)$

Inverse Amplitude Method

$$\frac{1}{t} \simeq \frac{1}{t_0 + t_1} \simeq \frac{1}{t_0} - \frac{t_1}{t_0^2} \implies \boxed{t^{IAM} \simeq \frac{t_0^2}{t_0 - t_1}}$$

Equivalent to $t_0 + t_1 + o(s^3)$

Advantage: for $s > s_{\text{th}}$,

$$\text{Im}\frac{1}{t_{IJ}(s)} = -\sigma(s) \simeq -1$$

Perturbative vs exact (elastic) unitarity

$$\text{Im}\, t_{IJ}(s) = \sigma(s) |t_{IJ}(s)|^2$$

Perturbative vs exact (elastic) unitarity

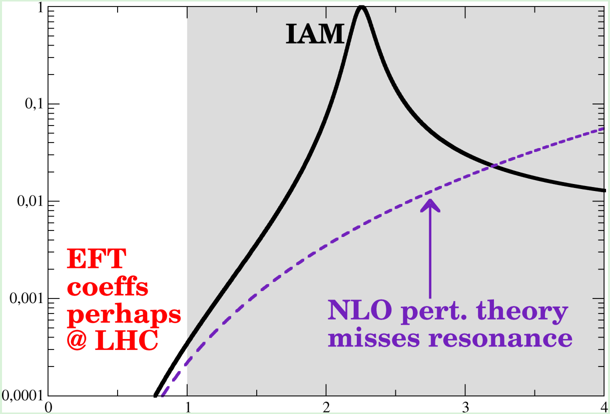
$$\text{Im}\, t_{IJ}(s) = \sigma(s)|t_{IJ}(s)|^2$$

- Exact in IAM
- Only order by order in EFT

$$\text{Im}\, t_1(s) = \sigma(s)|t_0(s)|^2$$

Why would anyone care?

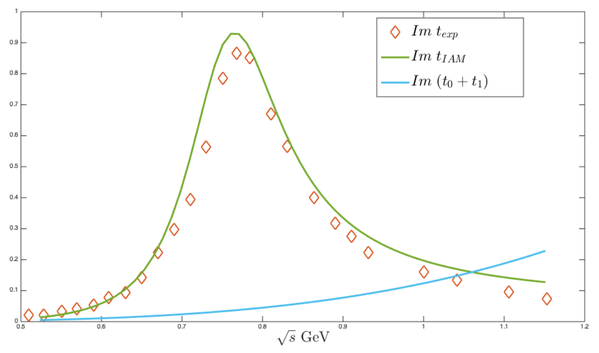
- EFT reliable only near threshold



Additionally, poor convergence

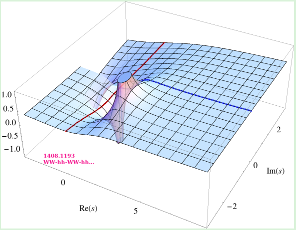
- Extreme case in *NN* interactions
- Recent work: Lang, Liebler, Schäfer-Siebert, Zeppenfeld EPJC **81** (2021) 7, 659

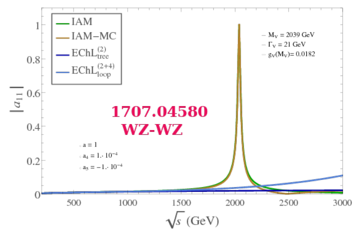
Much used in hadron physics to obtain resonances

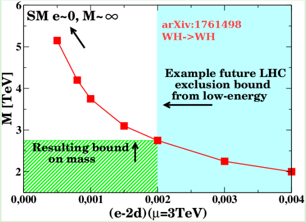


(This is an IAM prediction from threshold data, not a fit)

Prediction of resonances from HEFT





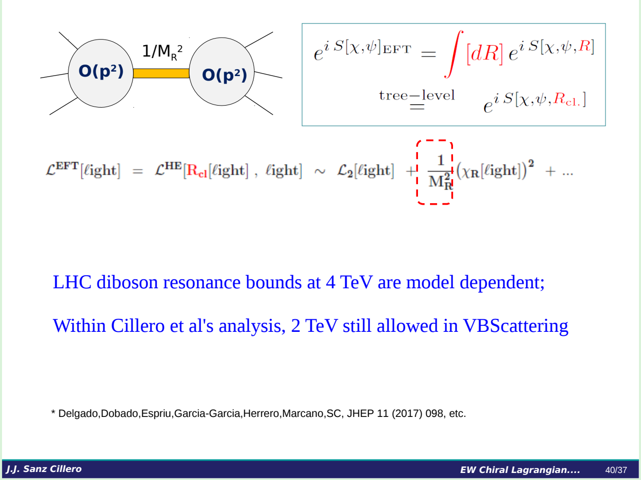


LHC bounds on HEFT coeffs $\Longrightarrow$ bounds on new physics scale

Bottom-to-Top extrapolation

An alternative to predict the scale of new physics is to use

Resonance saturation of EFT parameters



$$e^{i\,S[\chi,\psi]_{\rm EFT}} = \int [dR]\, e^{i\,S[\chi,\psi,R]} \overset{\rm tree-level}{=} e^{i\,S[\chi,\psi,R_{\rm cl.}]}$$

$$\mathcal{L}^{\rm EFT}[\ell{\rm ight}] = \mathcal{L}^{\rm HE}[R_{\rm cl}[\ell{\rm ight}]\,,\ \ell{\rm ight}] \sim \mathcal{L}_2[\ell{\rm ight}] + \boxed{\frac{1}{M_R^2}}(\chi_R[\ell{\rm ight}])^2 + \dots$$

LHC diboson resonance bounds at 4 TeV are model dependent;

Within Cillero et al's analysis, 2 TeV still allowed in VBScattering

* Delgado,Dobado,Espriu,Garcia-Garcia,Herrero,Marcano,SC, JHEP 11 (2017) 098, etc.

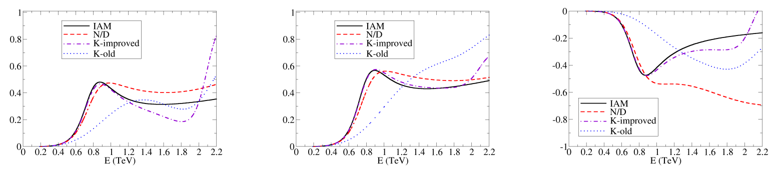
J.J. Sanz Cillero *EW Chiral Lagrangian....* 40/37

Several unitarization methods

- K-matrix (no good analyticity properties)
- N/D method (integral equations, not algebraic)
- Large N method (only approximate for $O(4) \to O(3)$)
- Inverse Amplitude Method $\Longrightarrow$
  control theory uncertainties (this work)

We have provided improved/simplified versions of all methods
R. L. Delgado, A. Dobado and FJLE, PRD **91**, 075017 2015; J.Phys.G 41 (2014) 025002.

Similar resonance mass for those with unitarity+analyticity



(Same 0^+ resonance seen in $\omega\omega \to \omega\omega$, $hh \to hh$, $\omega\omega \to hh$

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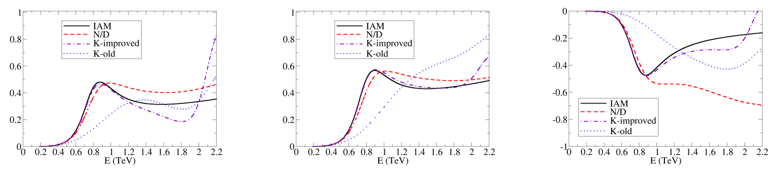


(Same 0^+ resonance seen in $\omega\omega \to \omega\omega$, $hh \to hh$, $\omega\omega \to hh$

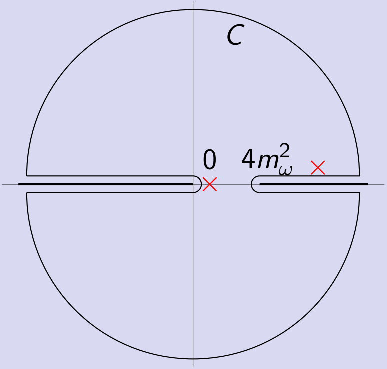
Can we constrain the uncertainty of one method ab initio?

Outline

1. A cute little Lagrangian
2. Some comments in comparing to *SM* and *SMEFT*
3. Unitarization of two-body $VV \to VV$ scattering
4. Systematic uncertainty estimate of IAM

Use its dispersive derivation: 2010.13709

- Causality $\Longrightarrow$ analiticity
- Large circumference convergence $G \propto e^{-s}$ (1912.08747)
- Can apply Cauchy's theorem



to the function $t_0^2(s')/t(s')(s-s')s'^3$.

Master formula is a dispersion relation for $G(s) \equiv \frac{t_0^2(s)}{t(s)}$

$$
\begin{aligned}
G(s) = G(0) + G'(0)s + \frac{1}{2}G''(0)s^2 + PC(G) + \\
+ \frac{s^3}{\pi}\int_{RC} ds' \frac{\mathrm{Im}\, G(s')}{s'^3(s'-s)} + \\
+ \frac{s^3}{\pi}\int_{LC} ds' \frac{\mathrm{Im}\, G(s')}{s'^3(s'-s)}
\end{aligned}
$$

Dispersion relation: approximations

$$\begin{aligned}G(s) = {}& \overbrace{G(0) + G'(0)s + \frac{1}{2}G''(0)s^2}^{\text{NLO subtraction constants}} + \overbrace{PC(G)}^{\text{Neglected}} + \\ &+ \frac{s^3}{\pi}\int_{RC} ds' \frac{\text{Im } G(s')}{s'^3(s'-s)} + \\ &\underbrace{+ \frac{s^3}{\pi}\int_{LC} ds' \frac{\text{Im } G(s')}{s'^3(s'-s)}}_{\text{NLO imaginary part Im G} \longrightarrow \text{-Im t1}}\end{aligned}$$

Gives $\boxed{t \simeq t_0^2/(t_0 - t_1) = t_{IAM}}$.

Sources of uncertainty

- Neglected pole contributions of t^{-1}:
  subthreshold Adler zeroes and CDD zeroes of t.

- Inelasticities due to KK (hh in HEFT), 4ω, etc.

- $\mathcal{O}(p^4)$ truncation of subtraction constants.

- Left cut approximation $Im\ G \simeq -Im\ t_1$.

The SM is a very specific theory

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(v+h)^2(\partial_\mu\vec{\omega})^2 \\ &+ \frac{1}{2}(\partial_\mu h)^2 + \dots\end{aligned}$$

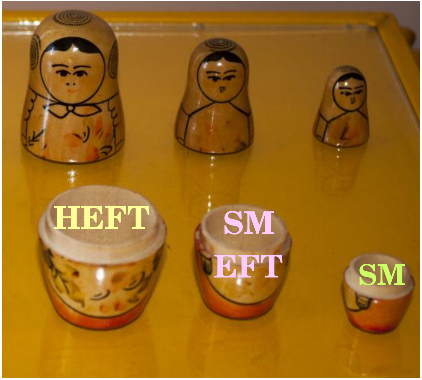
In terms of three Goldstone bosons $\vec{\omega}$ and Higgs h fields

SMEFT extension

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(v+h)^2 A\left(\frac{(v+h)^2}{\Lambda^2}\right)(\partial_\mu \vec{\omega})^2 \\ &+ \frac{1}{2}\left(1 + C\left(\frac{(v+h)^2}{\Lambda^2}\right)\right)(\partial_\mu h)^2 + \dots\end{aligned}$$

$$A(0) = 1, \;\; C(0) = 0$$

HEFT extension

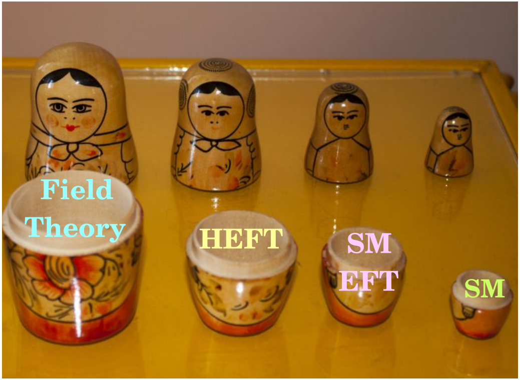


$$\mathcal{L} = \frac{1}{2}v^2 F\left(\frac{h}{v}\right)^2 (\partial_\mu \vec{\omega})^2 + \frac{1}{2}(\partial_\mu h)^2 + \dots$$

$$F(0) = 1$$

(n.b. field redefinition shows $SMEFT \subset HEFT$)

If different symmetry breaking pattern... still field theory

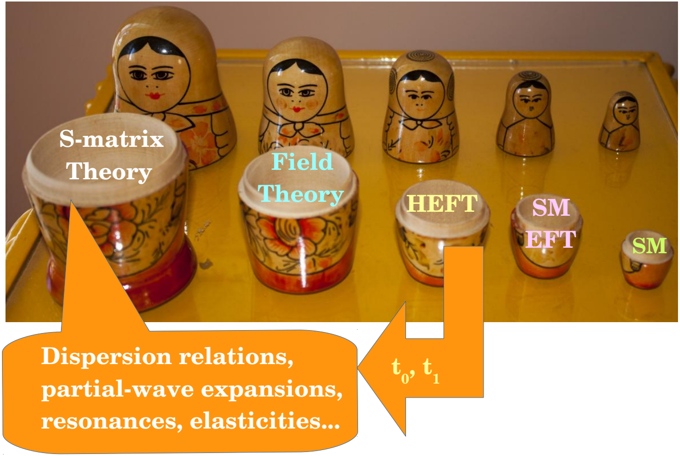


But *S*-matrix theory more general

Basic particle concepts part of S-matrix theory

Dispersion relations, partial-wave expansions, resonances, elasticities...

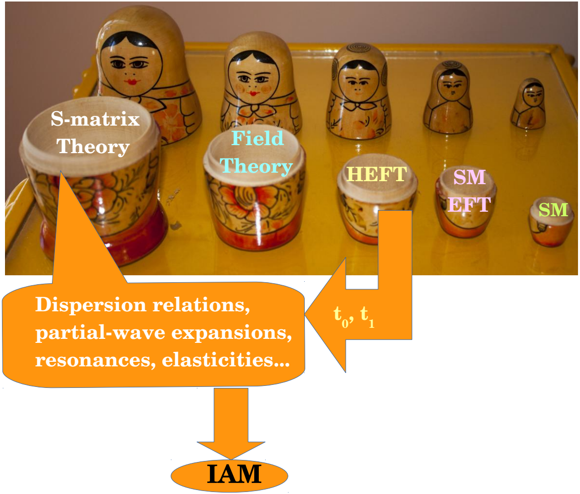
S-matrix too general: ambiguous $\Longrightarrow$ HEFT input



Dispersion relations, partial-wave expansions, resonances, elasticities...

t_0, t_1

More predictive Inverse Amplitude Method

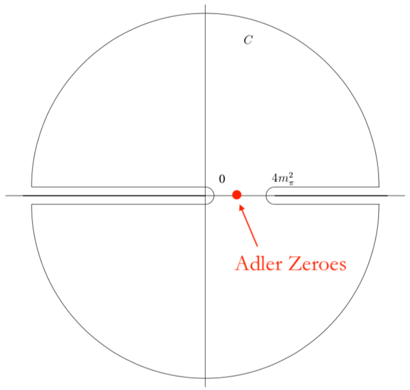


t_0, t_1

Dispersion relations,
partial-wave expansions,
resonances, elasticities...

IAM

Adler zeroes of t near threshold



EFT feature

$$t_0 + t_1 = a + bs + cs^2$$

vanishes near $s = -a/b$

Adler zeroes of t near threshold

Tiny uncertainty in resonance region because at/below threshold

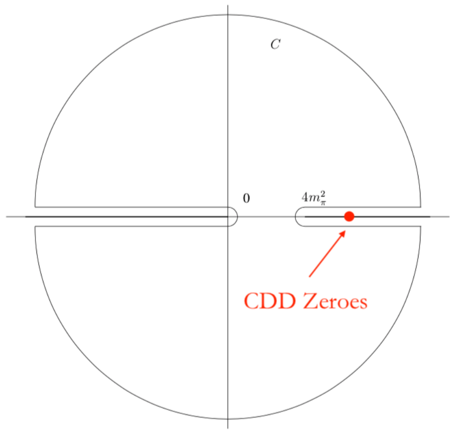
Adler zeroes of t near threshold

Tiny uncertainty in resonance region because at/below threshold

Uncertainty	Behavior	Displacement $\sqrt{s} = m_\rho$	improvable?
Adler zeroes of t	$(m_\omega/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM

0712.2763

CDD poles ($t = 0$ in resonance region: new physics)



("New" physics)

CDD poles ($t = 0$ in resonance region: new physics)

Can affect a resonance calculation dramatically

Need to

CDD poles ($t = 0$ in resonance region: new physics)

Can affect a resonance calculation dramatically

Need to

1. Check for CDD pole appearance: $t_0(s_C) + \mathrm{Re}\, t_1(s_C) = 0$

CDD poles ($t = 0$ in resonance region: new physics)

Can affect a resonance calculation dramatically

Need to

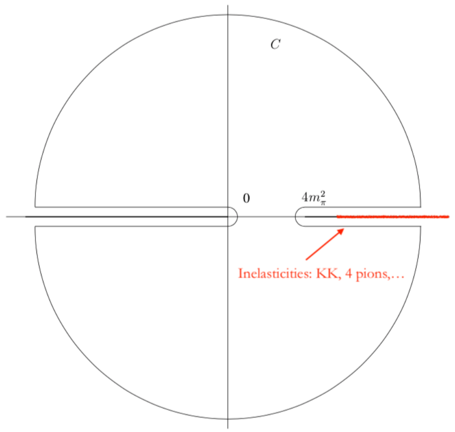
1. Check for CDD pole appearance: $t_0(s_C) + \mathrm{Re} t_1(s_C) = 0$
2. If present, modify

$$t_{\mathrm{IAM}} = \frac{t_0^2}{t_0 - t_1} \to \frac{t_0^2}{t_0 - t_1 + \frac{s}{s-s_c}\mathrm{Re}(t_1)} \; .$$

CDD poles ($t = 0$ in resonance region: new physics)

Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes

Inelastic 2-body channels



Inelastic 2-body channels

- Hadrons: $\pi\pi \to \pi\pi$ couples to KK

$$\mathrm{Im}\,\frac{1}{t_{\pi\pi}} \to -\sigma_{\pi\pi}\left(1 + \frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}\frac{\left|t_{\pi\pi\to K\bar{K}}\right|^2}{\left|t_{\pi\pi\to\pi\pi}\right|^2}\right)$$

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suppressed by phase-space $\frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}$ and low inelasticity in $t_{\pi\pi\to K\bar{K}}$

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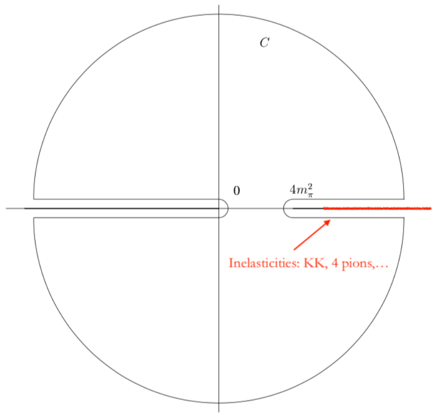
  suppressed by phase-space $\frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}$ and low inelasticity in $t_{\pi\pi\to K\bar{K}}$
- In HEFT only inelasticity in $\omega\omega - hh$ (actually zero in SM)
- We can use the coupled channel IAM directly or to estimate uncertainty in elastic IAM

Inelastic 2-body channels

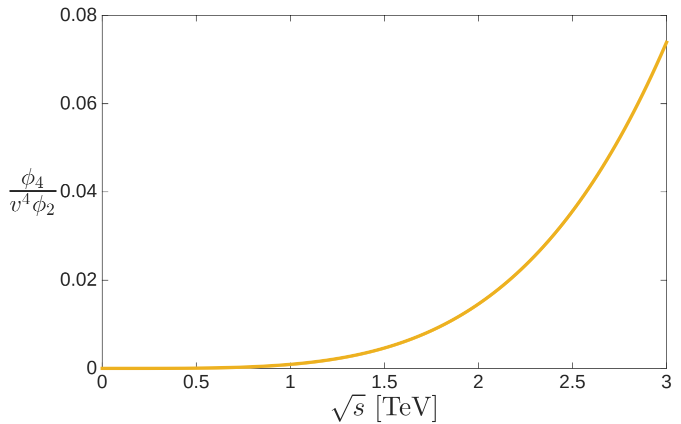
Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes
Inelastic 2-body	$(m_\rho/f_\pi)^4$	10^{-3}	Yes

Inelastic 4-body channels



- Difference with SMEFT: here, in ChPT or HEFT, additional particles *not* suppressed by the chiral counting.
  But phase space helps.

Inelastic 4-body channels

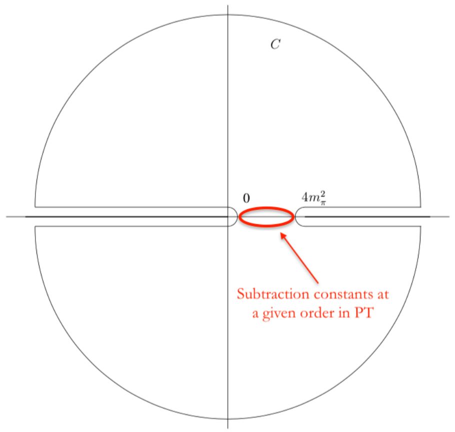


Inelastic 4-body channels

In hadron physics,
(with elastic and 4-π inelastic amplitudes taken as similar)

Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes
Inelastic 2-body	$(m_\rho/f_\pi)^4$	10^{-3}	Yes
Inelastic 4-body	$(m_\rho/f_\pi)^8$	10^{-4}	Partly

$O(p^4)$ truncation



Estimate based on size of NNLO counterterms ($\Longrightarrow$ subtraction constants) from Resonance Effective Field Theory

$O(p^4)$ truncation

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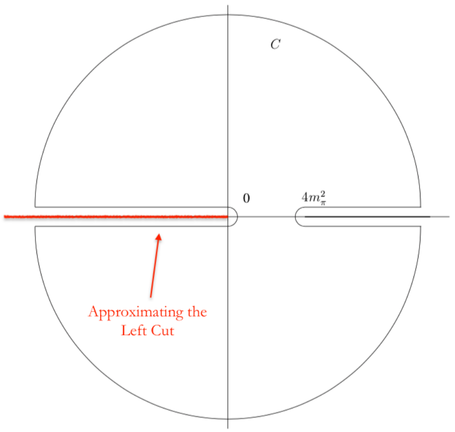
Inelastic 2-body	$(m_\rho/f_\pi)^4$	10^{-3}	Yes
Inelastic 4-body	$(m_\rho/f_\pi)^8$	10^{-4}	Partly
$O(p^4)$ truncation	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-2}	Yes

$$G(s) = \frac{t_0^2}{t} \simeq t_0 - t_1 - t_2 + \frac{t_1^2}{t_0}$$

Approximate left cut



Need to check

$$\int_{LC} ds' \frac{\text{Im } G + \text{Im } t_1}{s'^3(s'-s)} \ .$$

i.e., failure of IAM's

$$Im\ G = -Im\ t_1$$

over the left cut

Approximate left cut

Split interval in 3:

- Low-$|s|$ (ChPT/HEFT ✓) $|s|^{\frac{1}{2}} < 470\mathrm{MeV}$.
- Intermediate-$|s|$: Match to ChPT + natural-size counterterm + LC parameterizations from GKPY eqns.
- High -$|s|$: Sugawara-Kanazawa relates it to right cut: Regge asymptotics there. Far from RC anyway.

Approximate left cut

Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes
Inelastic 2-body	$(m_\rho/f_\pi)^4$	10^{-3}	Yes
Inelastic 4...-body	$(m_\rho/f_\pi))^8$	10^{-4}	Partly
$O(p^4)$ truncation	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-2}	Yes
Left Cut	$(m_\rho/f_\pi)^6$	0.17	Partly

Conclusion: mass gap to new physics

Use EFT for eventual $V_L V_L$ high-E data:
but if your Lagrangian has > 7 parameters,
little breeches, you are working too hard

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Conclusion: if you know your EFT...

- It often fails little above threshold $s \simeq 4m^2 + \epsilon$

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- Inverse Amplitude Method extends it to first resonance or $4\pi F$ or new: first zero (CDD-IAM). No clipping, no unitarity problem.

Conclusion: if you know your EFT...

- It often fails little above threshold $s \simeq 4m^2 + \epsilon$
- Inverse Amplitude Method extends it to first resonance or $4\pi F$ or new: first zero (CDD-IAM). No clipping, no unitarity problem.
- We have laid out Alex Salas-Bernárdez et al. (SciPost 2021) its systematic theory uncertainties

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