# VBF & MULTIPLE BOSONS AT FUTURE LEPTON COLLIDERS

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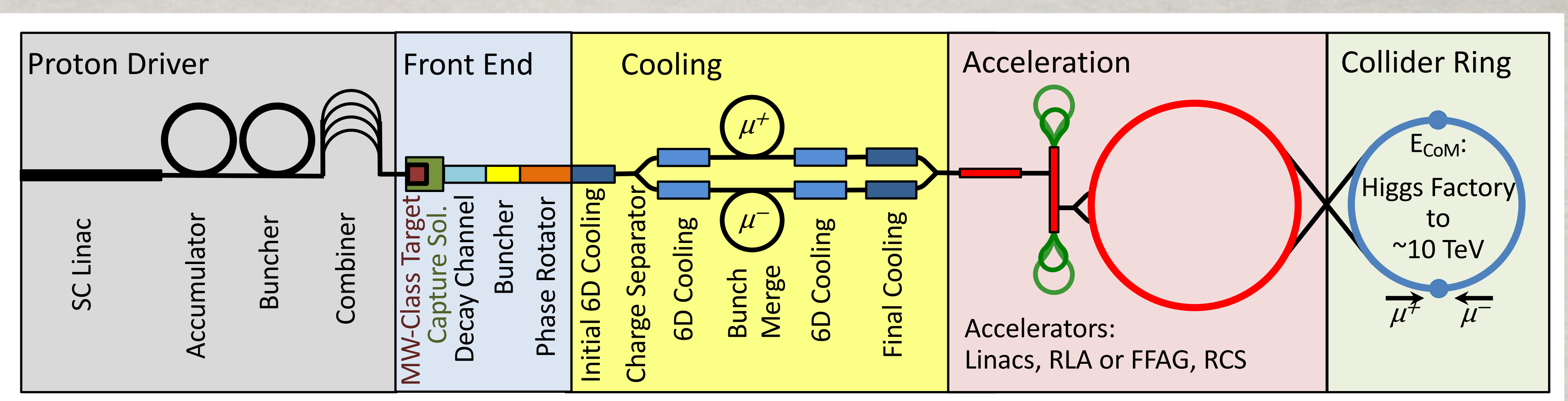
Multi-Boson Interactions 2021

23-27 August 2021

1. High energy $\mu^+\mu^-$ Collisions
2. EW Physics @ Ultra-high Energies
3. Precision Higgs Physics
4. Multiple Boson Production

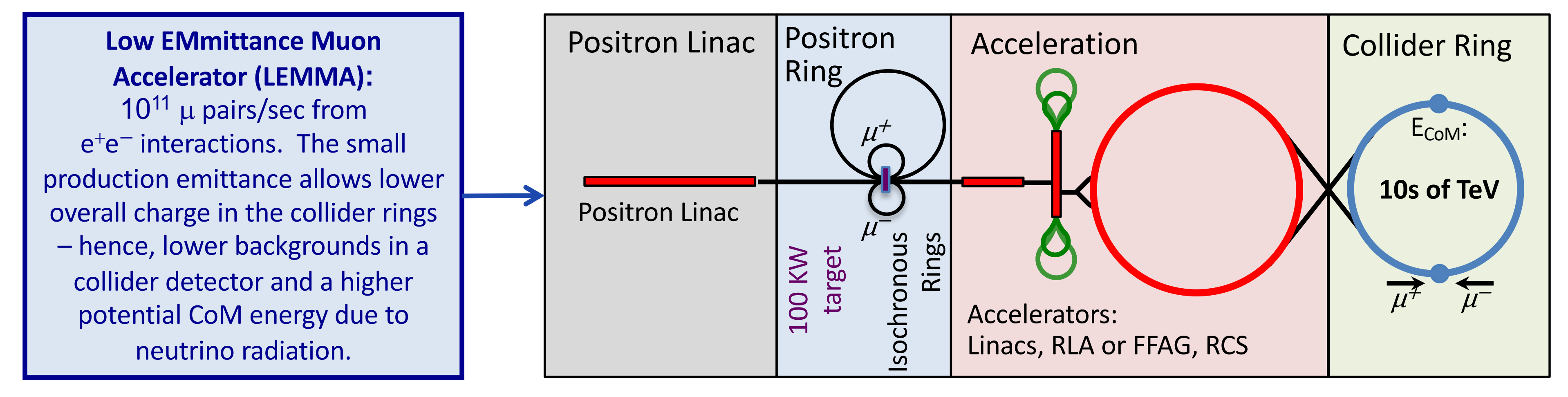
# Proton-Driver:



Muon Accelerator Program
map.fnal.gov

New results on $\mu$ cooling by MICE collaboration
Nature 508(2020)53

# LEMMA: $e^+e^-$ (at rest) → $\mu^+\mu^-$ (at threshold)


**Low EMmittance Muon Accelerator (LEMMA):**
$10^{11}$ $\mu$ pairs/sec from $e^+e^-$ interactions. The small production emittance allows lower overall charge in the collider rings – hence, lower backgrounds in a collider detector and a higher potential CoM energy due to neutrino radiation.

Low EMittance Muon Accelerator
web.infn.it/LEMMA

J.P. Delahauge et al., arXiv:1901.06150

# Collider benchmark points:

- The Higgs factory:

$E_{cm} = m_H$

$L \sim 1\ \mathrm{fb}^{-1}/\mathrm{yr}$

$\Delta E_{cm} \sim 5\ \mathrm{MeV}$

| **Parameter** | **Units** | **Higgs** |
|---|---|---|
| CoM Energy | TeV | 0.126 |
| Avg. Luminosity | $10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ | 0.008 |
| Beam Energy Spread | % | 0.004 |
| Higgs Production/$10^7$ sec | | 13’500 |
| Circumference | km | 0.3 |

- Multi-TeV colliders:

Lumi-scaling scheme: **σ** $L \sim$ const.

1 $\mathrm{ab}^{-1}$ /yr

$$L \gtrsim \frac{5\ \text{years}}{\text{time}} \left( \frac{\sqrt{s}_\mu}{10\ \text{TeV}} \right)^2 2 \cdot 10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

The aggressive choices:

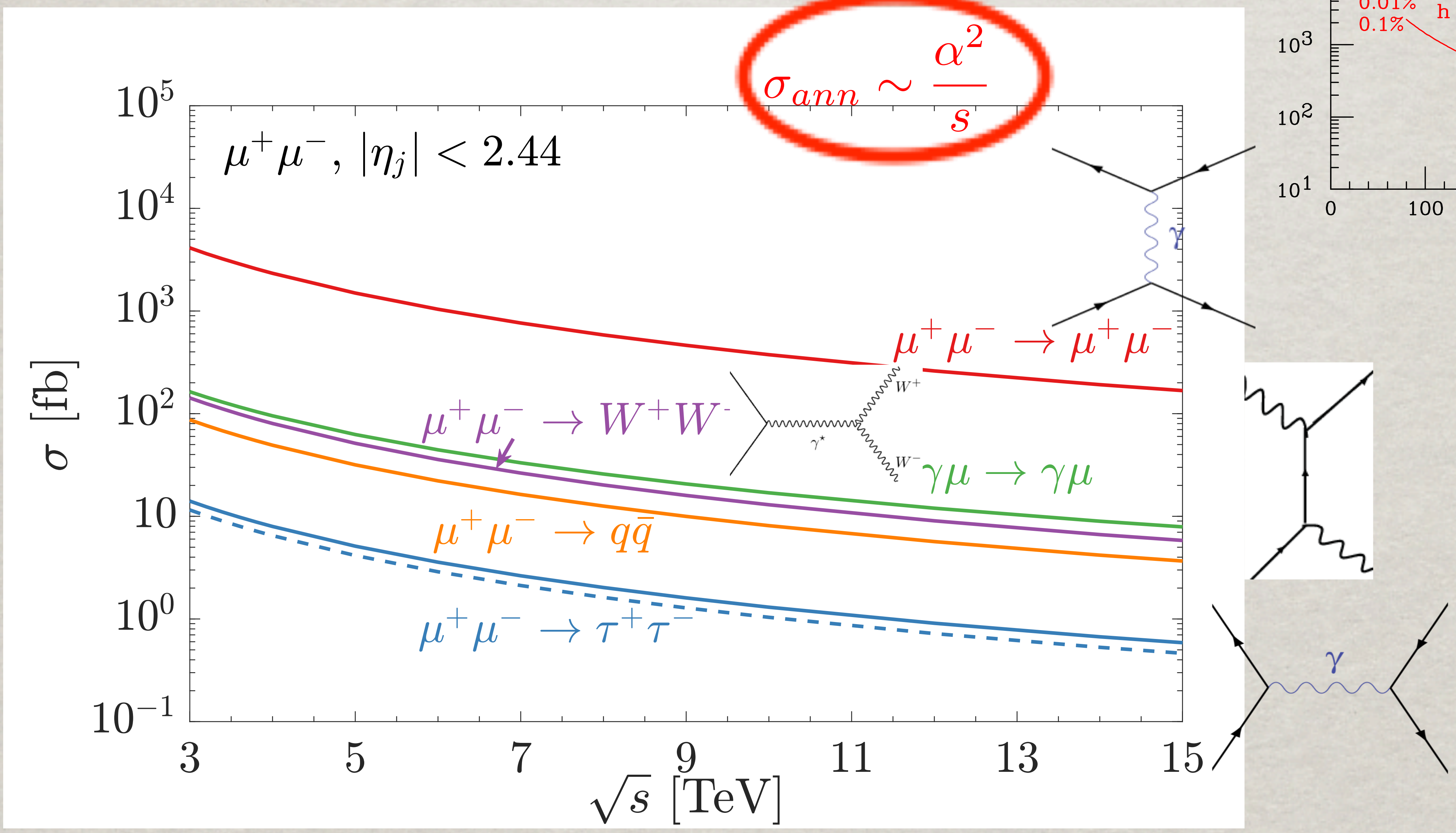
$$\sqrt{s} = 3,\ 6,\ 10,\ 14,\ 30 \text{ and } 100 \text{ TeV}, \quad \mathcal{L} = 1,\ 4,\ 10,\ 20,\ 90, \text{ and } 1000\ \mathrm{ab}^{-1}$$

European Strategy, arXiv:1910.11775; arXiv:1901.06150; arXiv:2007.15684.

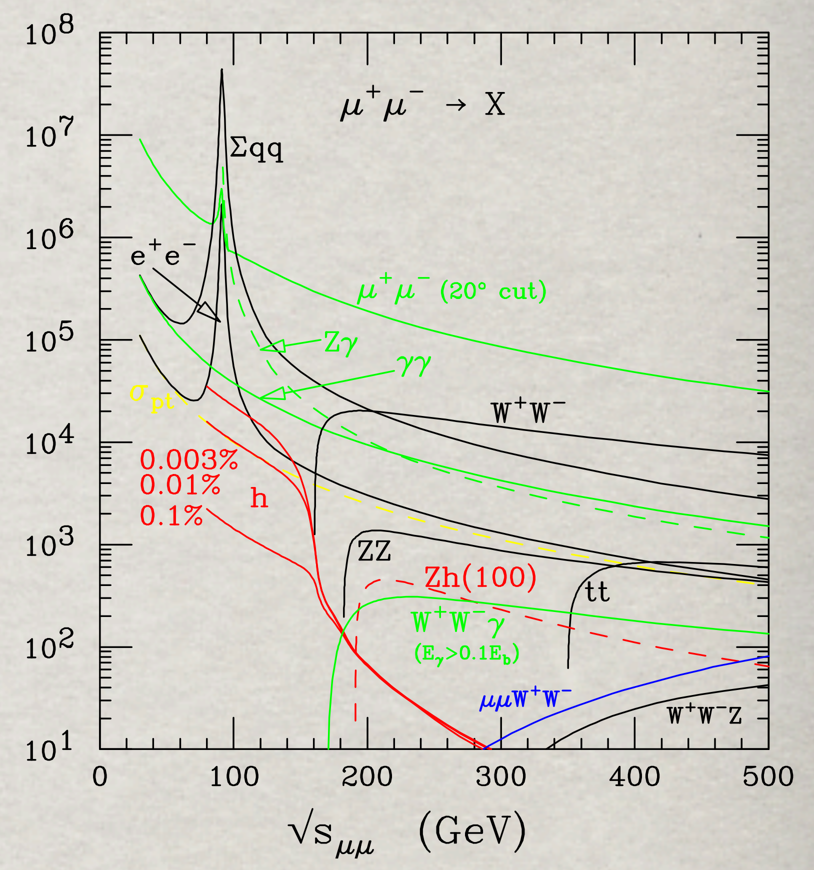
# 1. $\mu^+\mu^-$ Collisions at High Energies:

## What will happen when you turn on a $\mu^+\mu^-$ Smasher?

Leading-order $\mu^+\mu^-$ annihilation:

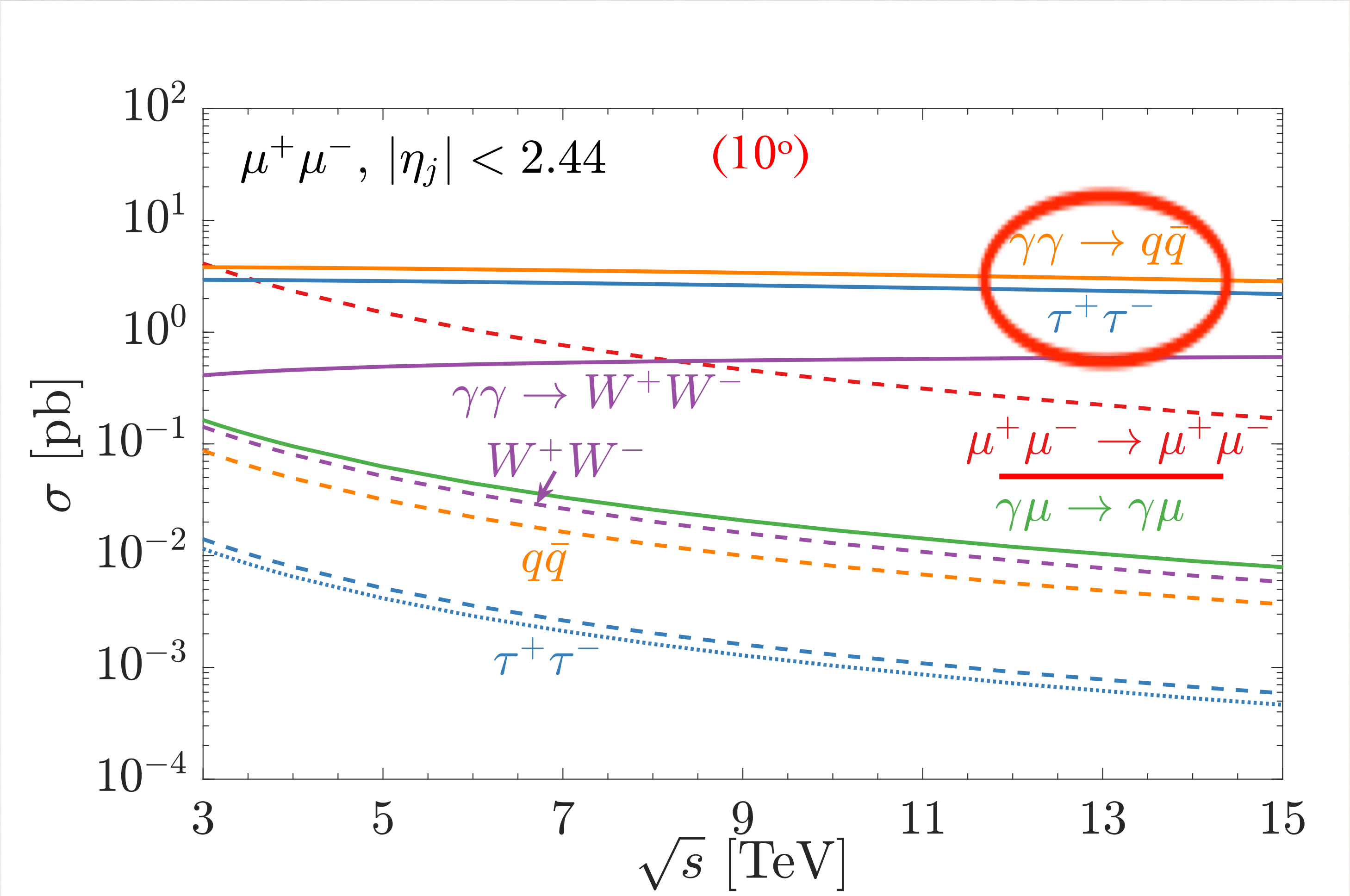


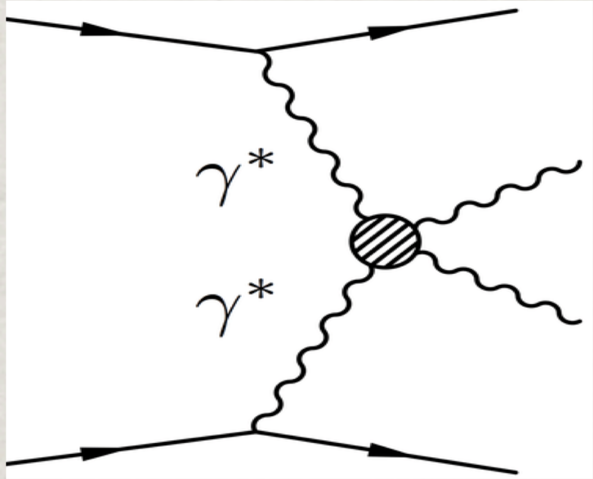
$$\sigma_{ann} \sim \frac{\alpha^2}{s}$$



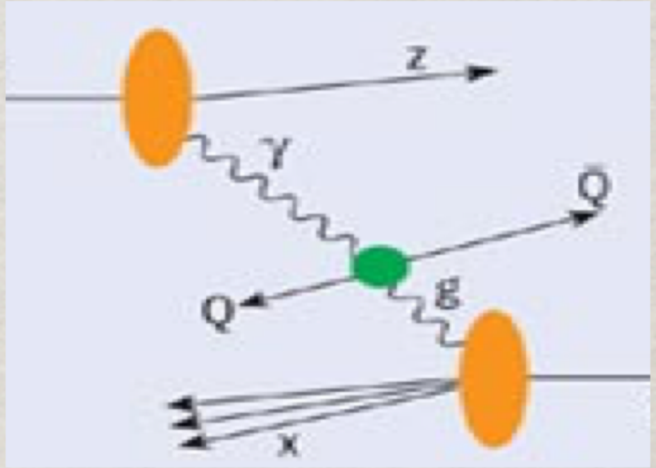
# Photon-induced QED cross sections have larger rates

$$\sigma_{fusion} \sim \frac{\alpha^2}{m_{jj}^2}\log^2\left(\frac{Q^2}{m^2}\right)$$





$m_{jj}$

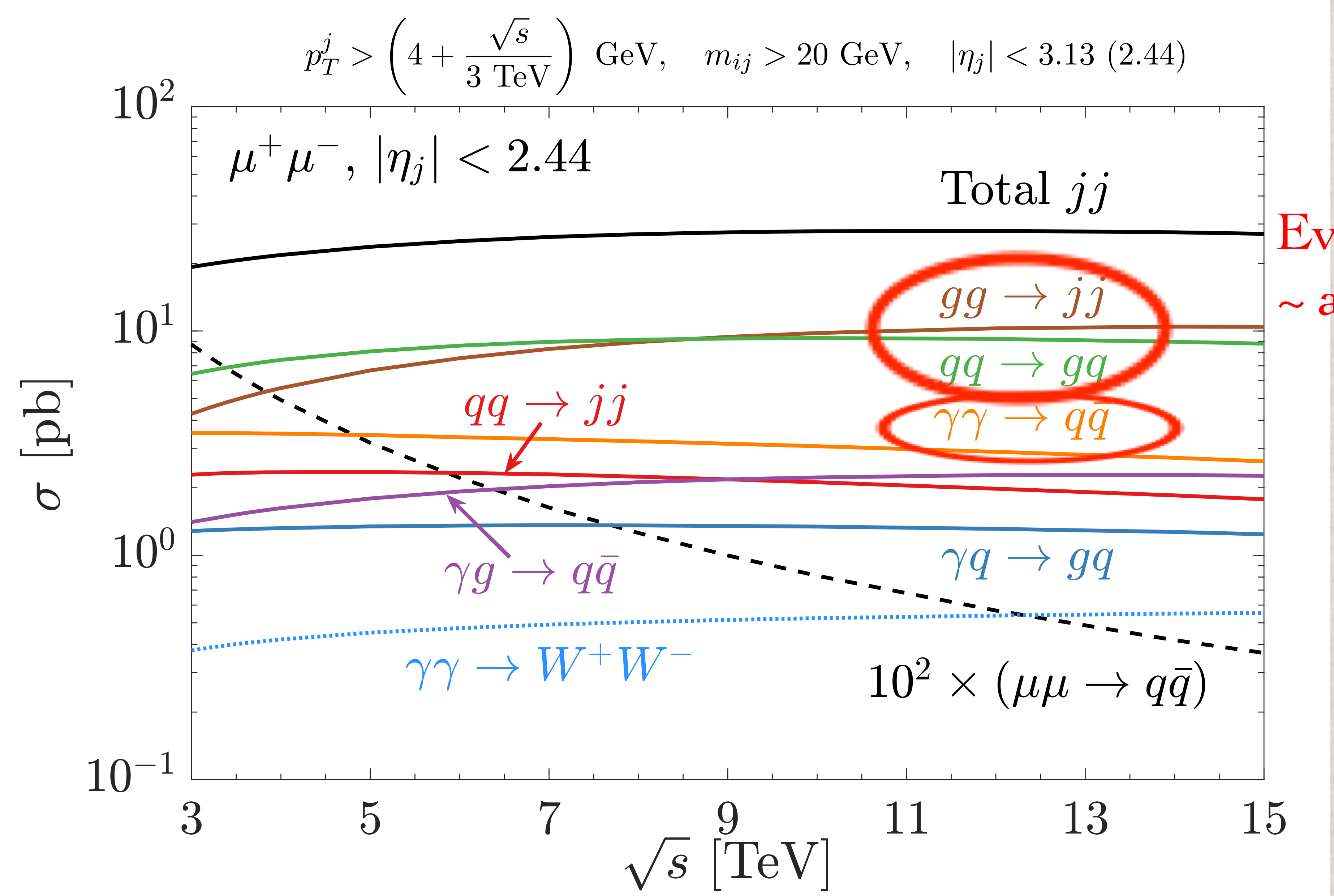


$$p_T^j > \left(4+\frac{\sqrt{s}}{3\text{ TeV}}\right)\text{ GeV},\quad m_{ij} > 20\text{ GeV},\quad |\eta_j| < 3.13\ (2.44)$$

Quarks/gluons come into the picture via SM DGLAP:

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2}\begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}$$

# Di-jet production: $\gamma\gamma \to q\bar{q},\ \gamma g \to q\bar{q},\ \gamma q \to gq,$ $qq \to qq(gg),\ gq \to gq,$ and $gg \to gg(q\bar{q})$

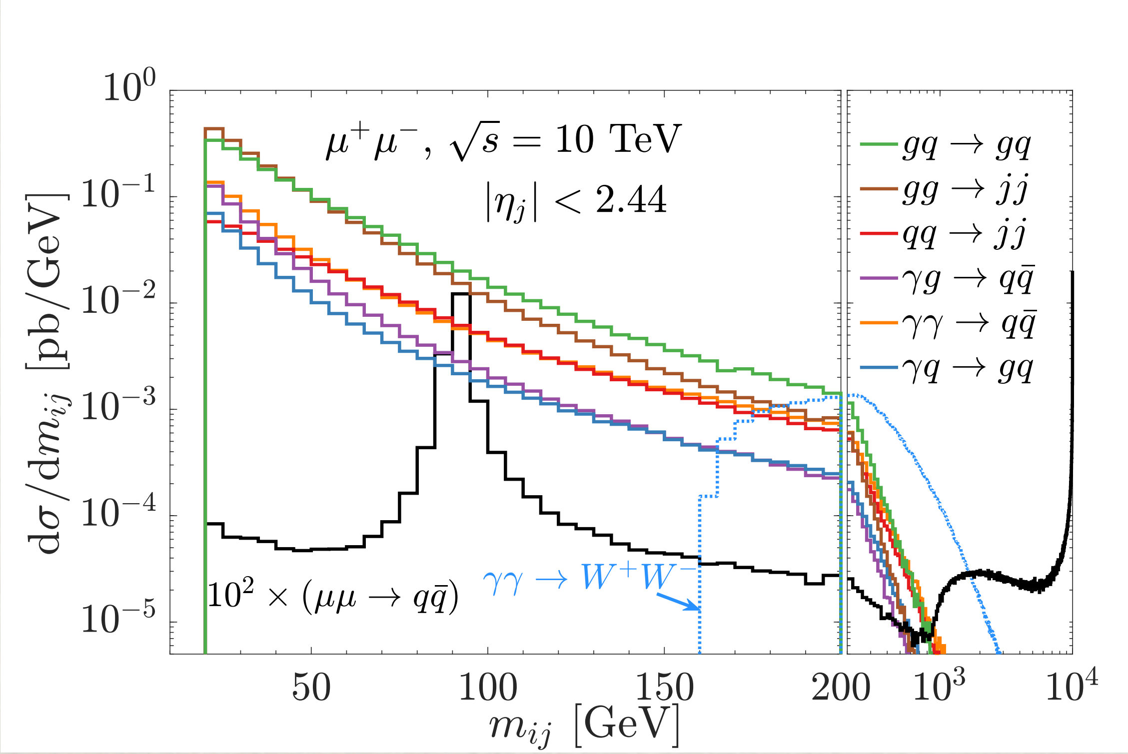


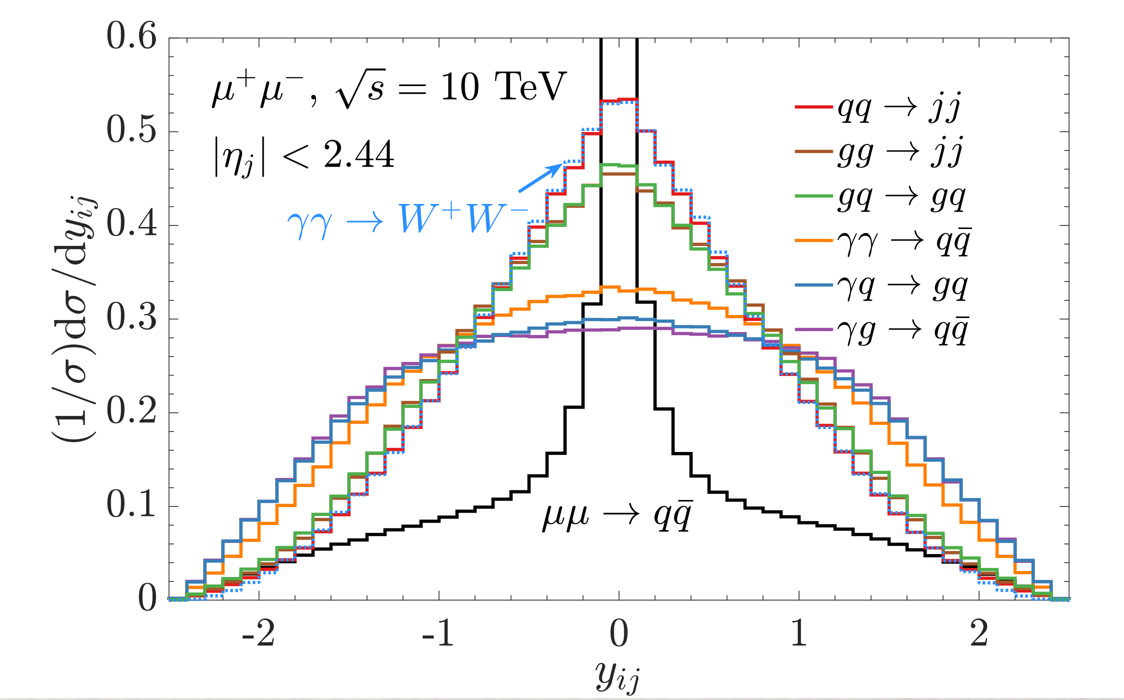
Event rate
~ a few Hz

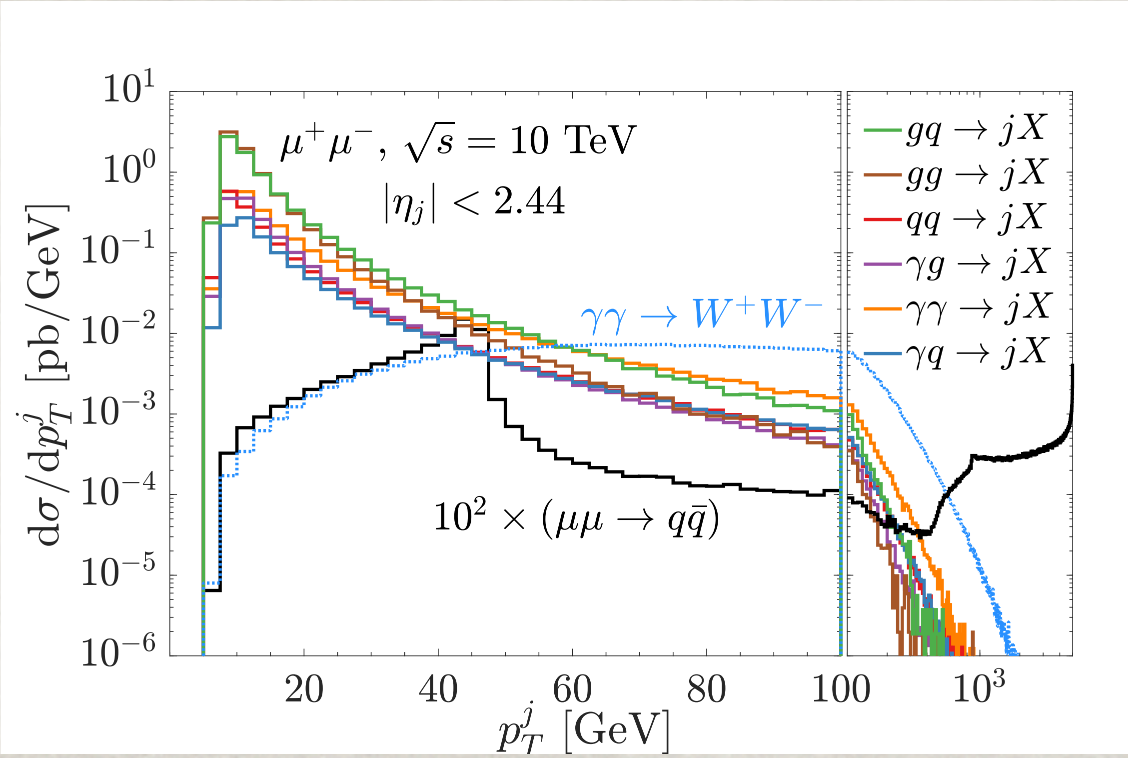
→ Jet production dominates at low energies

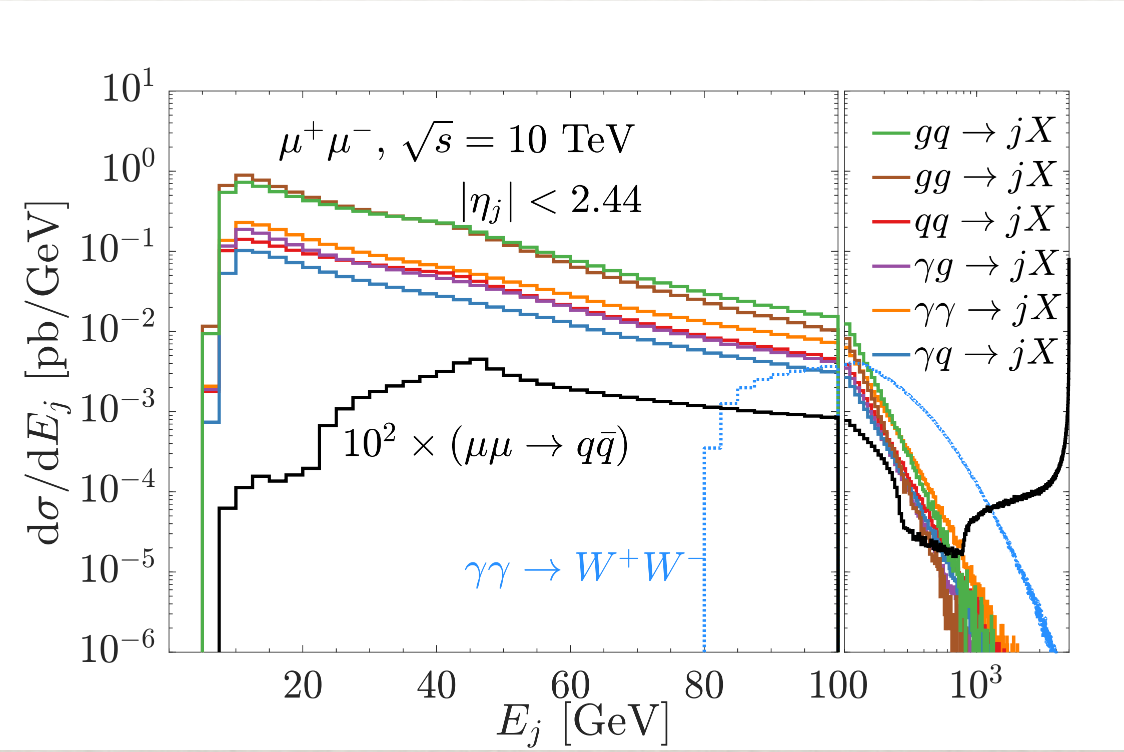
TH, Yang Ma, Keping Xie, arXiv:2103.09844.

# Di-jet kinematical features









To effectively separate the QCD backgrounds:

$p_T > 60$ GeV

# 2. EW physics at ultra-high energies:

$$\frac{v}{E}: \ \frac{v\ (250\ \mathrm{GeV})}{10\ \mathrm{TeV}} \approx \frac{\Lambda_{QCD}\ (300\ \mathrm{MeV})}{10\ \mathrm{GeV}}$$

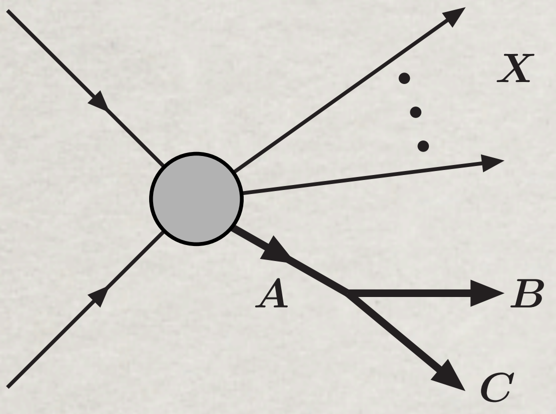
$$v/E,\ m_t/E,\ M_W/E \to 0!$$

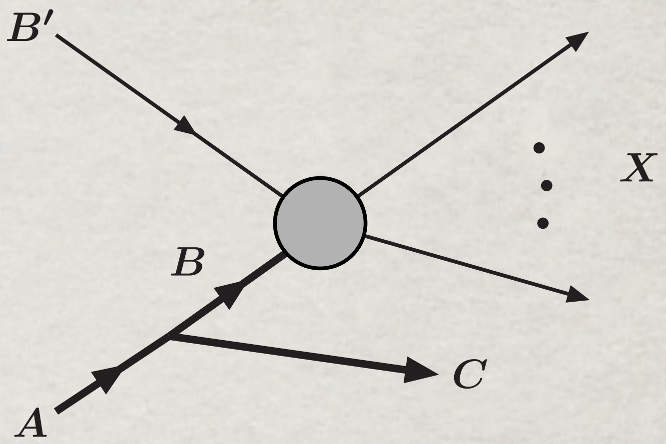
- A massless theory:
  → splitting phenomena dominate!
- EW symmetry restored:
  → $SU(2)_L \times U(1)_Y$ unbroken gauge theory
- v/E as power corrections
  → Higher twist effects.

J. Chen, TH, B. Tweedie, arXiv:1611.00788;
G. Cuomo, A. Wulzer, arXiv:1703.08562; 1911.12366.

Ciafaloni et al., hep-ph/0004071; 0007096; A. Manohar et al., 1803.06347.
C. Bauer, Ferland, B. Webber et al., arXiv:1703.08562; 1808.08831.

# EW splitting physics: EW PDFs & showering





$$d\sigma_{X,BC} \simeq d\sigma_{X,A} \times d\mathcal{P}_{A\to B+C}$$

$$E_B \approx zE_A, \quad E_C \approx \bar{z}E_A, \quad k_T \approx z\bar{z}E_A\theta_{BC}$$

$$\frac{d\mathcal{P}_{A\to B+C}}{dz\, dk_T^2} \simeq \frac{1}{16\pi^2} \frac{z\bar{z}\,|\mathcal{M}^{(\mathrm{split})}|^2}{(k_T^2 + \bar{z}m_B^2 + zm_C^2 - z\bar{z}m_A^2)^2}$$

- On the dimensional ground: $|\mathcal{M}_{split}|^2 \sim k_T^2$ or $m^2$
- When SU(2) quantum numbers not summed/averaged, factorized formalism may NOT be valid:
→ Bloch-Nordsieck theorem violation

Ciafaloni et al., hep-ph/0004071; 0007096
C. Bauer, Ferland, B. Webber et al., arXiv:1703.08562; 1808.08831.
A. Manohar et al., 1803.06347, J. Chen, TH, B. Tweedie, arXiv:1611.00788.

# EW splitting functions:

## Start from the unbroken phase – all massless.

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_{\phi} + \mathcal{L}_{f} + \mathcal{L}_{Yuk}$$

Chiral fermions: $f_s$, gauge bosons: $B, W^0, W^\pm$; $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(h - i\phi^0) \end{pmatrix}$

## e.g.: fermion splitting:

| | $\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{1+\bar{z}^2}{z}\right)$ | | $\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{z}{2}\right)$ |
|---|---|---|---|
| | $\to\ V_T\ f_s^{(\prime)}$ | $[BW]^0_T\ f_s$ | $H^{0(*)}\ f_{\text{-}s}$ or $\phi^\pm\ f'_{\text{-}s}$ |
| $f_{s=L,R}$ | $g_V^2(Q^V_{f_s})^2$ | $g_1 g_2 Y_{f_s} T^3_{f_s}$ | $y^2_{f_R^{(\prime)}}$ |

Ciafaloni et al., Hep-ph/0505047.

Infrared & collinear singularities ($P_{gq}$)

Collinear singularity, Chirality-flip, Yukawa

# EW Symmetry breaking & Goldstone-boson Equivalence Theorem (GET):

Lee, Quigg, Thacker (1977); Chanowitz & Gailard (1984)

At high energies $E >> M_W$, the longitudinally polarized gauge bosons behave like the corresponding Goldstone bosons.
(They remember their origin!)

"Scalarization" to implement the Goldstone-boson Equivalence Theorem (GET):

$$\epsilon(k)^{\mu}_{L} = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^{\mu}}{m_W} + O(m_W/E)$$

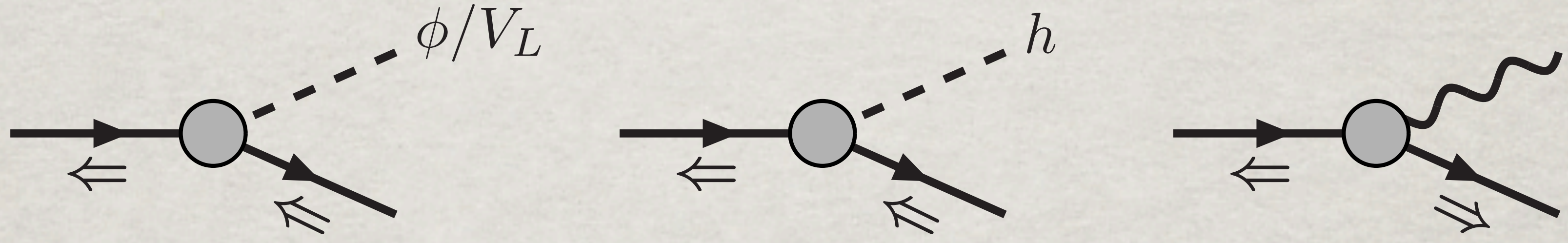
GET violation as power corrections $v/E$.
Like in QCD: higher-twist effects $\Lambda_{QCD}/E$.

J. Chen, TH, B. Tweedie, arXiv:1611.00788;
G. Cuomo, A. Wulzer, arXiv:1703.08562; 1911.12366.

# Splitting in a broken gauge theory:

New fermion splitting: $\frac{v^2}{k_T^2}\ \frac{dk_T^2}{k_T^2} \sim (1 - \frac{v^2}{Q^2})$

$V_L$ is of IR, h no IR



$$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}\left(\frac{1}{z}\right)$$

$$\to\ V_L\ f_s^{(\prime)}\ (V\neq\gamma)$$

$$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}$$

$$h\ f_s$$

$$\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}$$

$$V_T\ f_{\text{-}s}^{(\prime)}$$

Chirality conserving:
Non-zero for massless f

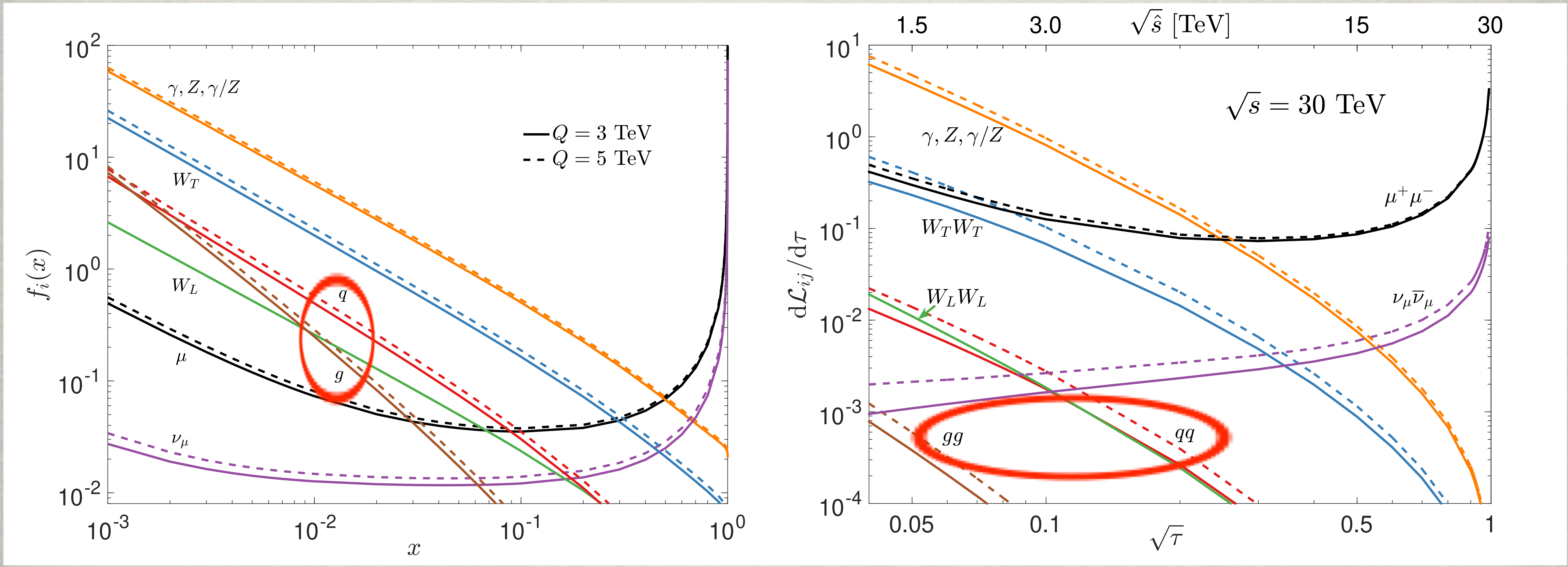
Chirality flipping:
~$m_f$

The DPFs for $W_L$ thus don't run at leading log:
"Bjorken scaling" restored (higher-twist effects)!

# • EW PDFs at a muon collider:

“partons” dynamically generated

$$\frac{\mathrm{d}f_i}{\mathrm{d}\ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j$$



$\mu^{\pm}$: the valance. $\ell_R,\ \ell_L,\ \nu_L$ and $B, W^{\pm,3}$: LO sea.

Quarks: NLO; gluons: NNLO.

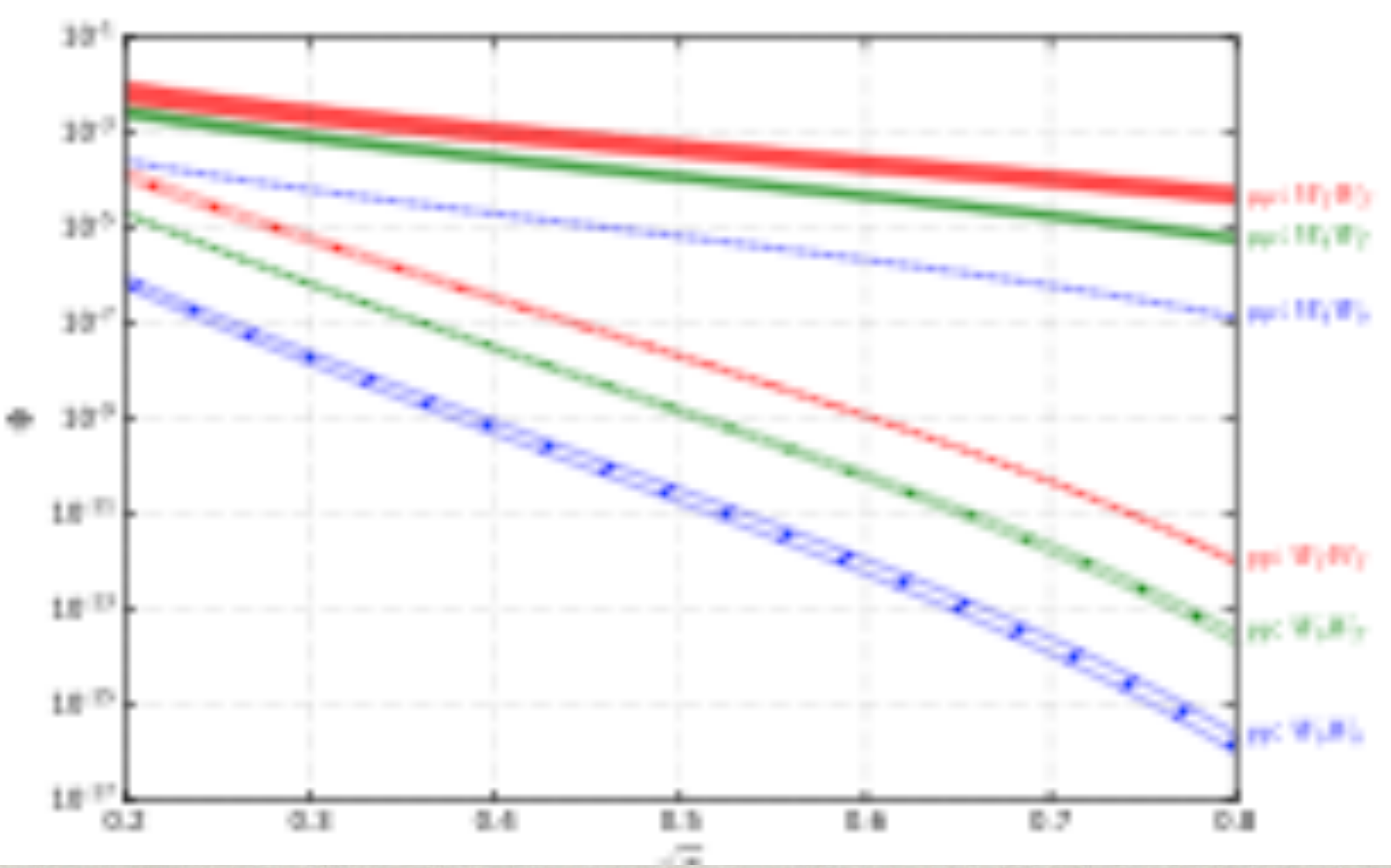
TH, Yang Ma, Keping Xie, arXiv:2007.14300

# VBF luminosities: $\mu$-C versus pp

$$\Phi_{W^+_{\lambda_1} W^-_{\lambda_2}}(\tau, \mu_f) = \int_\tau^1 \frac{d\xi}{\xi} \; f_{W_{\lambda_1}/\mu}(\xi, \mu_f) \; f_{W_{\lambda_2}/\mu}\left(\frac{\tau}{\xi}, \mu_f\right)$$

$$\Phi_{V_\lambda V'_{\lambda'}}(\tau, \mu_f) = \frac{1}{1 + \delta_{V_\lambda V'_{\lambda'}}} \int_\tau^1 \frac{d\xi}{\xi} \int_{\tau/\xi}^1 \frac{dz_1}{z_1} \int_{\tau/\xi/z_1}^1 \frac{dz_2}{z_2} \sum_{q,q'} \qquad (3.18)$$

$$\left[f_{V_\lambda/q}(z_2) f_{V'_{\lambda'}/q'}(z_1) f_{q/p}(\xi) f_{q'/p}\left(\frac{\tau}{\xi z_1 z_2}\right) + f_{V_\lambda/q}(z_2) f_{V'_{\lambda'}/q'}(z_1) f_{q/p}\left(\frac{\tau}{\xi z_1 z_2}\right) f_{q'/p}(\xi)\right].$$

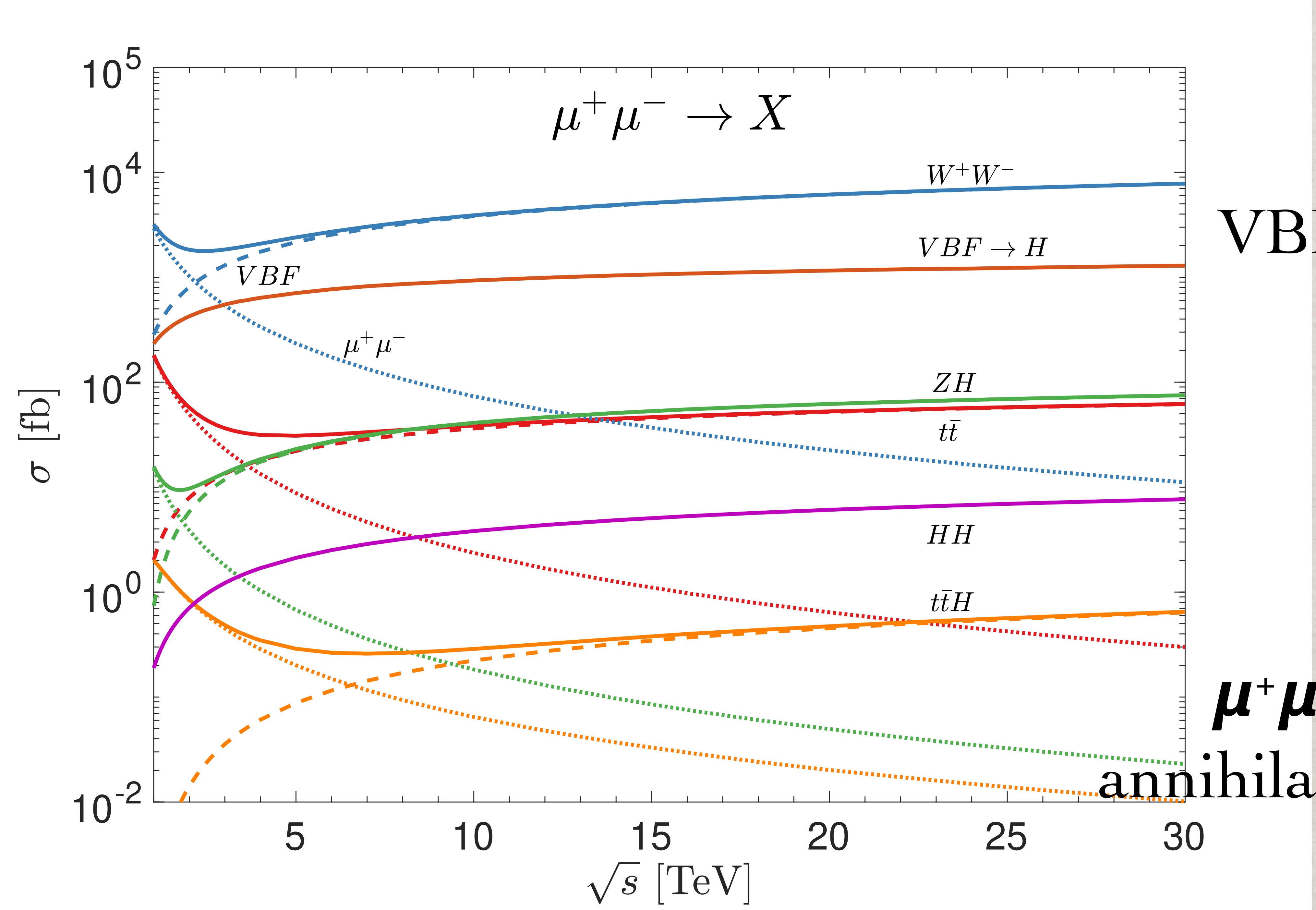


F. Maltoni, R. Ruiz et al., arXiv:2005.10289

# • “Semi-inclusive” processes

## Just like in hadronic collisions:

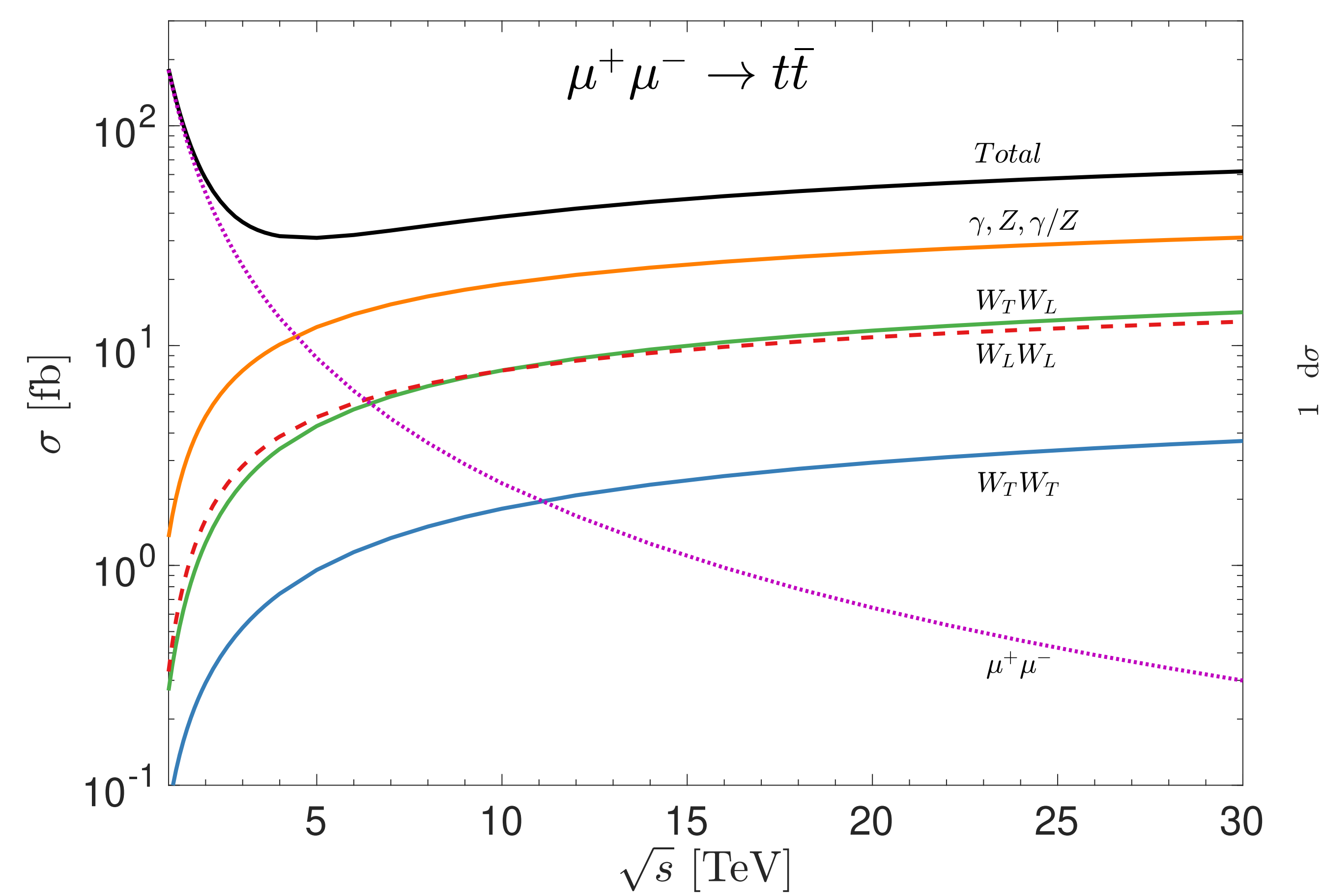
## $\mu^+\mu^-$ → exclusive particles + remnants

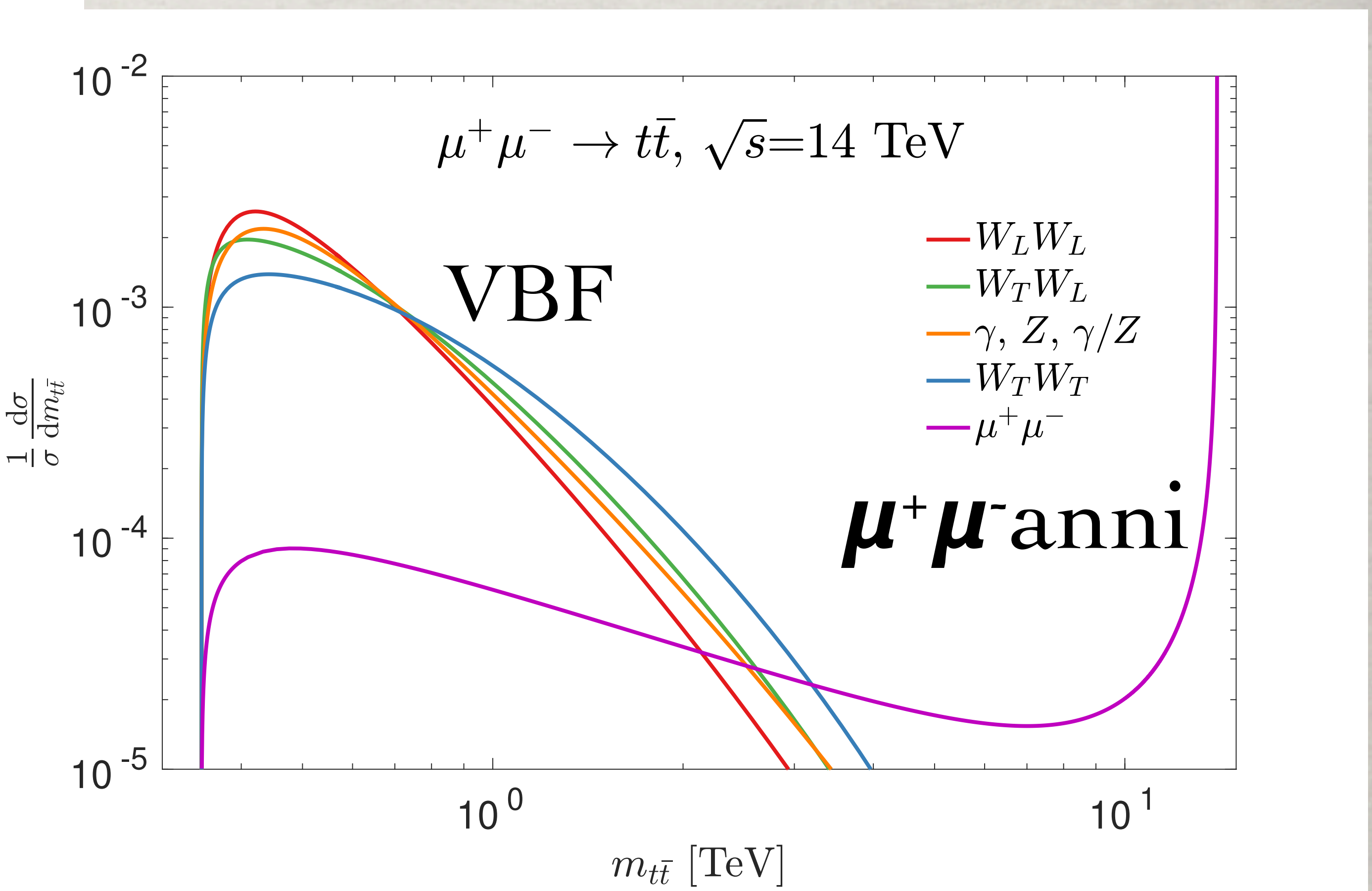


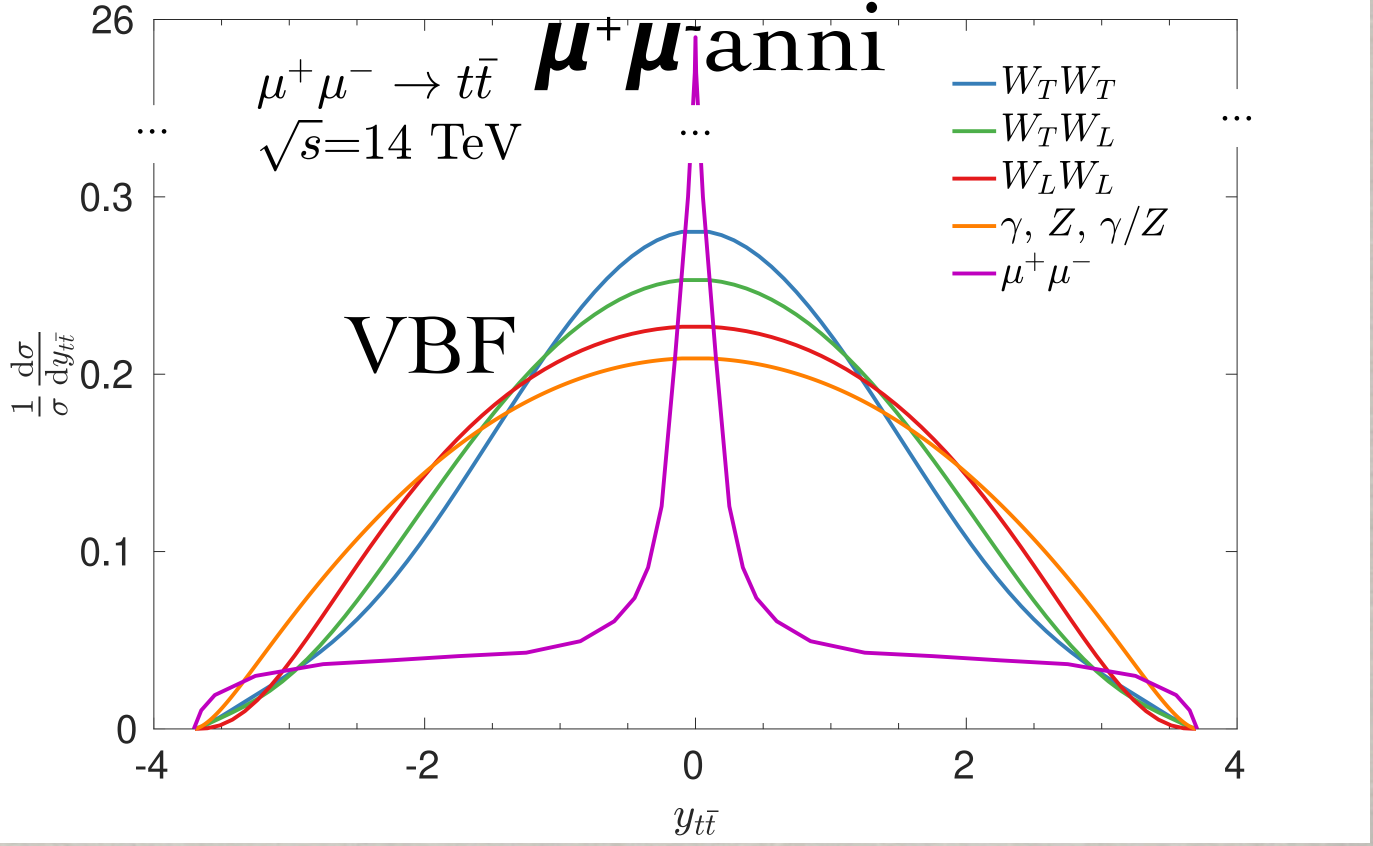
VBF

**$\mu^+\mu^-$** annihilations

# Underlying sub-processes:





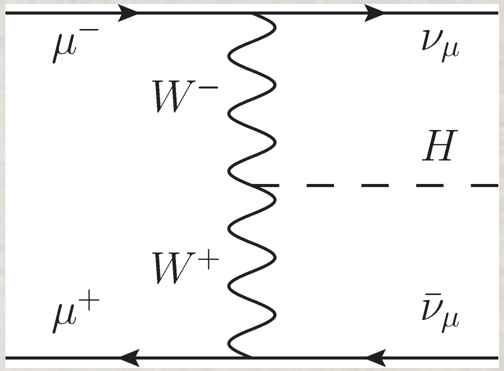


Partonic contributions

**$\mu^+\mu^-$ Collider:**
**“Buy one, get one free”**
**Annihilation +VBF**

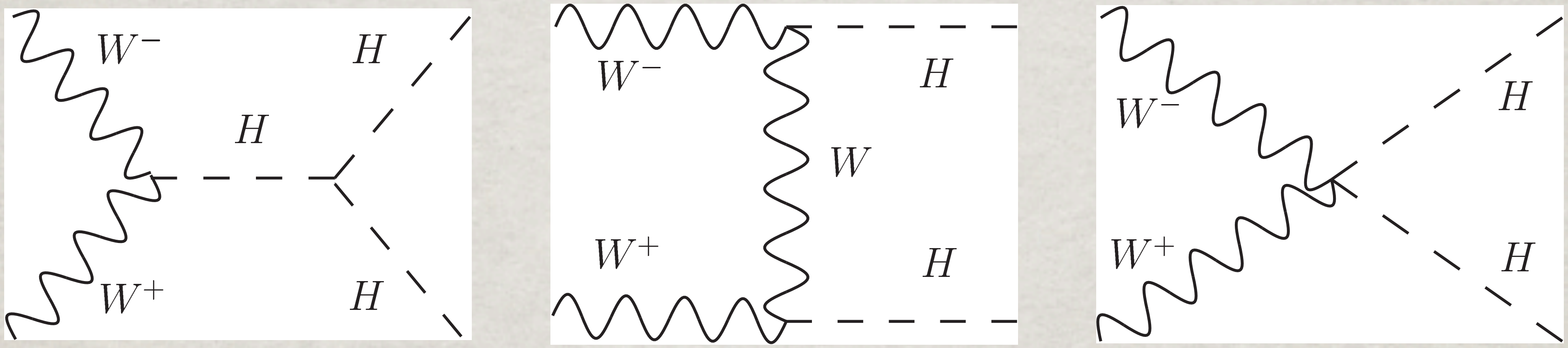
# 3. Precision Higgs Physics

$$\mu^+\mu^- \to \nu_\mu \bar{\nu}_\mu \; H \qquad (WW \text{ fusion}),$$
$$\mu^+\mu^- \to \mu^+\mu^- \; H \qquad (ZZ \text{ fusion}).$$



WWH / ZZH couplings

HHH / WWHH couplings:



(a) (b) (c)

| $\sqrt{s}$ (TeV)<br>benchmark lumi (ab$^{-1}$) | 3<br>1 | 6<br>4 | 10<br>10 | 14<br>20 | 30<br>90 |
|---|---|---|---|---|---|
| $\sigma$ (fb): $WW \to H$ | 490 | 700 | 830 | 950 | 1200 |
| $ZZ \to H$ | 51 | 72 | 89 | 96 | 120 |
| $WW \to HH$ | 0.80 | 1.8 | 3.2 | 4.3 | 6.7 |
| $ZZ \to HH$ | 0.11 | 0.24 | 0.43 | 0.57 | 0.91 |
| $WW \to ZH$ | 9.5 | 22 | 33 | 42 | 67 |
| $WW \to t\bar{t}H$ | 0.012 | 0.046 | 0.090 | 0.14 | 0.28 |
| $WW \to Z$ | 2200 | 3100 | 3600 | 4200 | 5200 |
| $WW \to ZZ$ | 57 | 130 | 200 | 260 | 420 |

10M H

500k HH

TH, D. Liu, I. Low,
X. Wang, arXiv:2008.12204

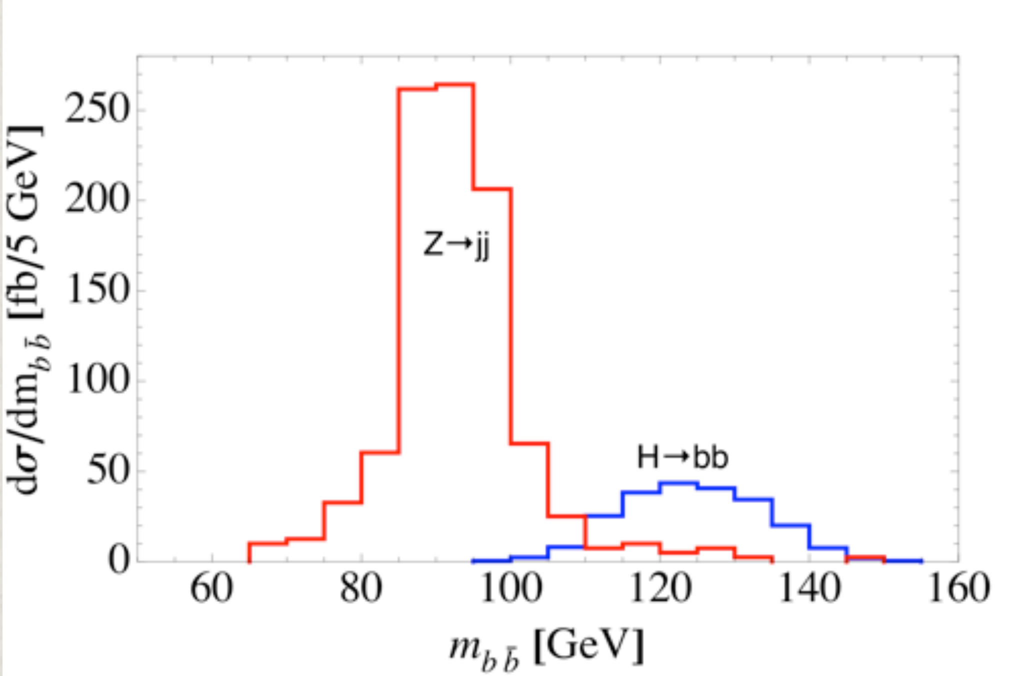
# Achievable accuracies

$$\mathcal{L} \supset \left(M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu\right)\left(\kappa_V \frac{2H}{v} + \kappa_{V_2} \frac{H^2}{v^2}\right) - \frac{m_H^2}{2v}\left(\kappa_3 H^3 + \frac{1}{4v}\kappa_4 H^4\right)$$

## Leading channel H → bb:

$$\Delta E/E = 10\%.$$

$$10^\circ < \theta_{\mu^\pm} < 170^\circ.$$



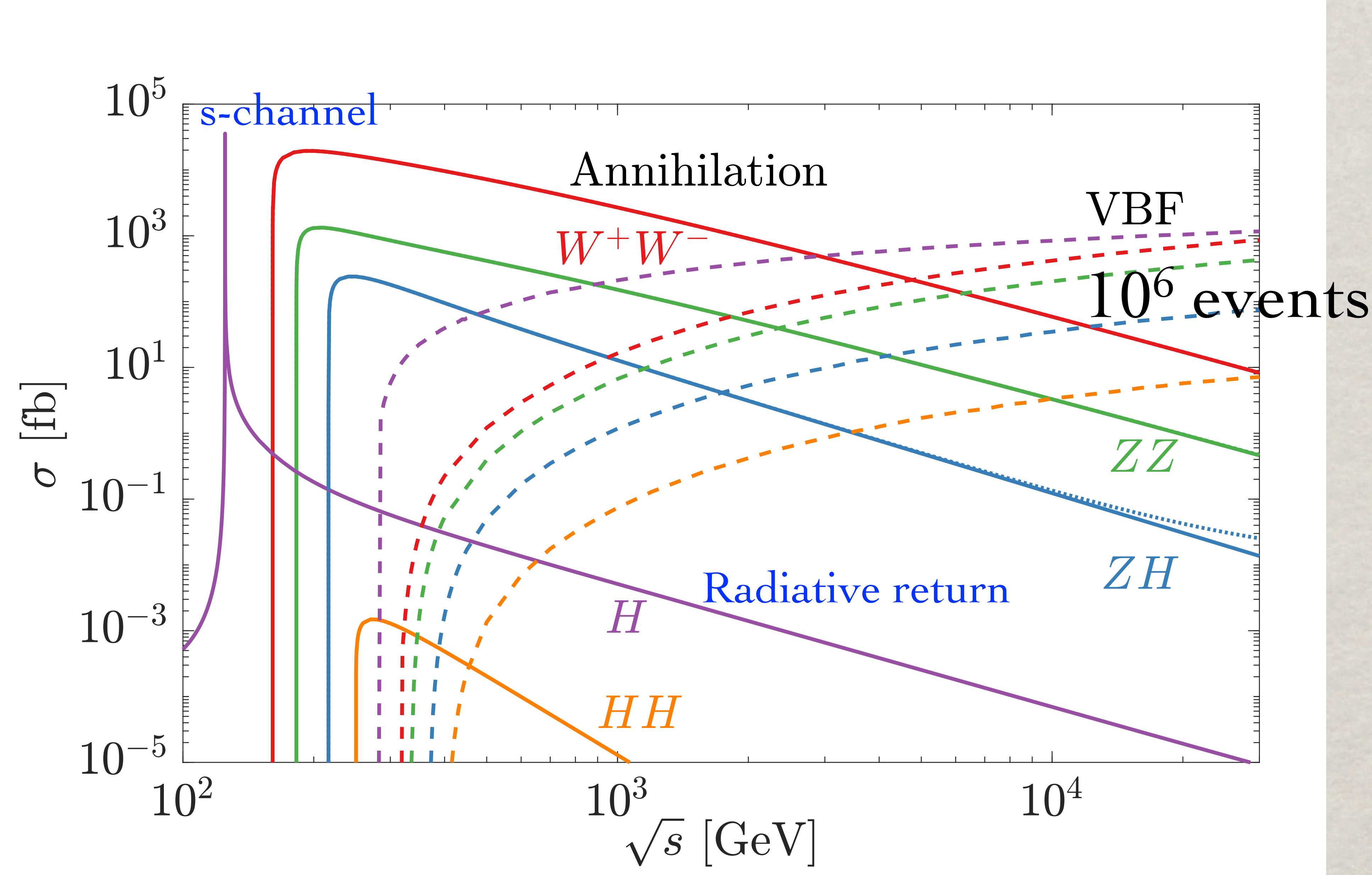
| $\sqrt{s}$ (lumi.) | 3 TeV (1 ab$^{-1}$) | 6 (4) | 10 (10) | 14 (20) | 30 (90) | Comparison |
|---|---|---|---|---|---|---|
| $WWH$ ($\Delta\kappa_W$) | 0.26% | 0.12% | 0.073% | 0.050% | 0.023% | 0.1% [41] |
| $\Lambda/\sqrt{c_i}$ (TeV) | 4.7 | 7.0 | 9.0 | 11 | 16 | (68% C.L.) |
| $ZZH$ ($\Delta\kappa_Z$) | 1.4% | 0.89% | 0.61% | 0.46% | 0.21% | 0.13% [17] |
| $\Lambda/\sqrt{c_i}$ (TeV) | 2.1 | 2.6 | 3.2 | 3.6 | 5.3 | (95% C.L.) |
| $WWHH$ ($\Delta\kappa_{W_2}$) | 5.3% | 1.3% | 0.62% | 0.41% | 0.20% | 5% [36] |
| $\Lambda/\sqrt{c_i}$ (TeV) | 1.1 | 2.1 | 3.1 | 3.8 | 5.5 | (68% C.L.) |
| $HHH$ ($\Delta\kappa_3$) | 25% | 10% | 5.6% | 3.9% | 2.0% | 5% [22, 23] |
| $\Lambda/\sqrt{c_i}$ (TeV) | 0.49 | 0.77 | 1.0 | 1.2 | 1.7 | (68% C.L.) |

**Table 7**: Summary table of the expected accuracies at 95% C.L. for the Higgs couplings at a variety of muon collider collider energies and luminosities.

TH, D. Liu, I. Low, X. Wang, arXiv:2008.12204

# 4. Multiple Boson Production

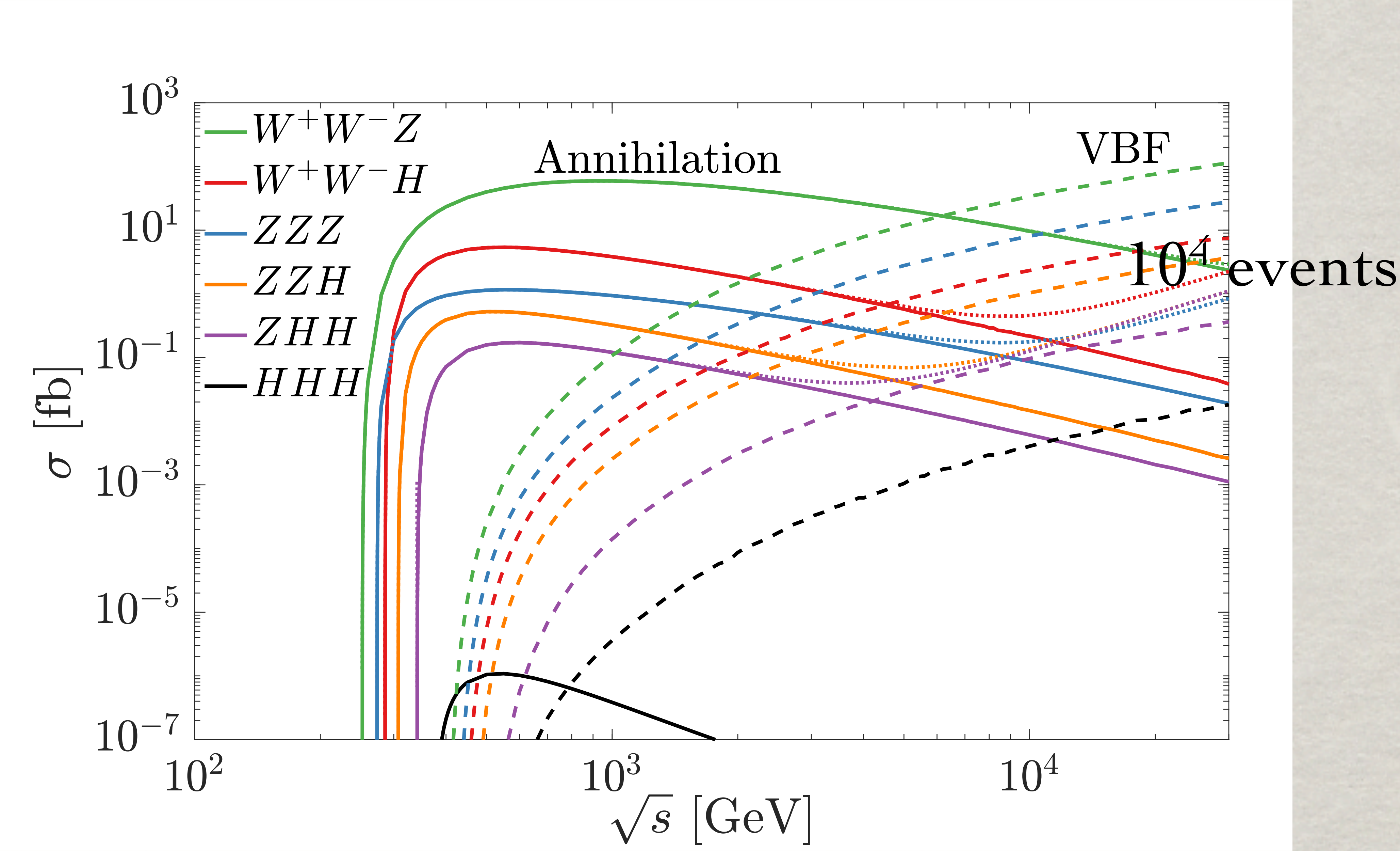
## Pair Boson Production



VBFs take over ~ 2 - 3 TeV.

TH, W. Kilian, N. Kreher, Y.Ma, J. Reuter, T. Striegl and K. Xie: arXiv:2108.05362

# Triple Boson Production

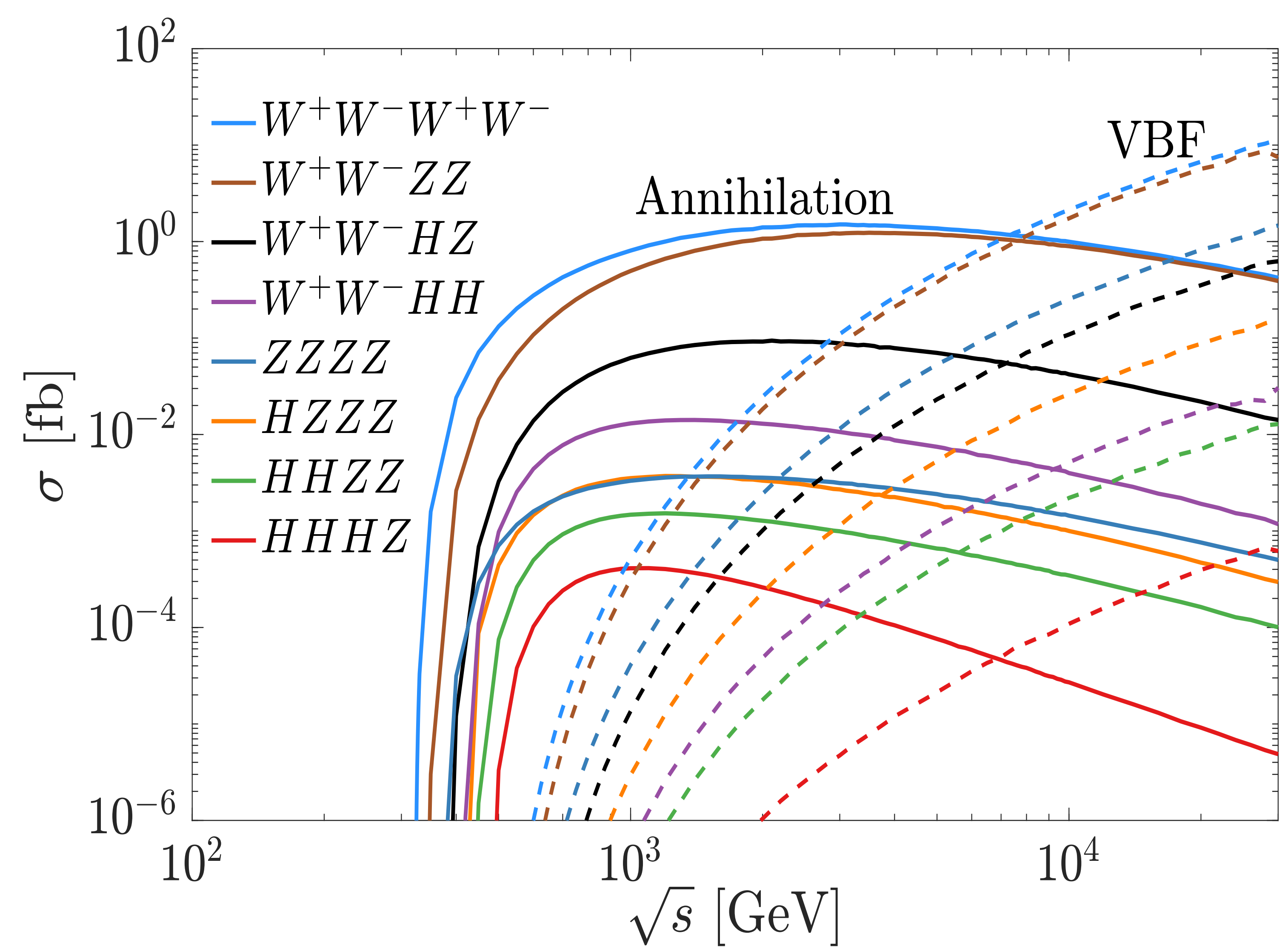


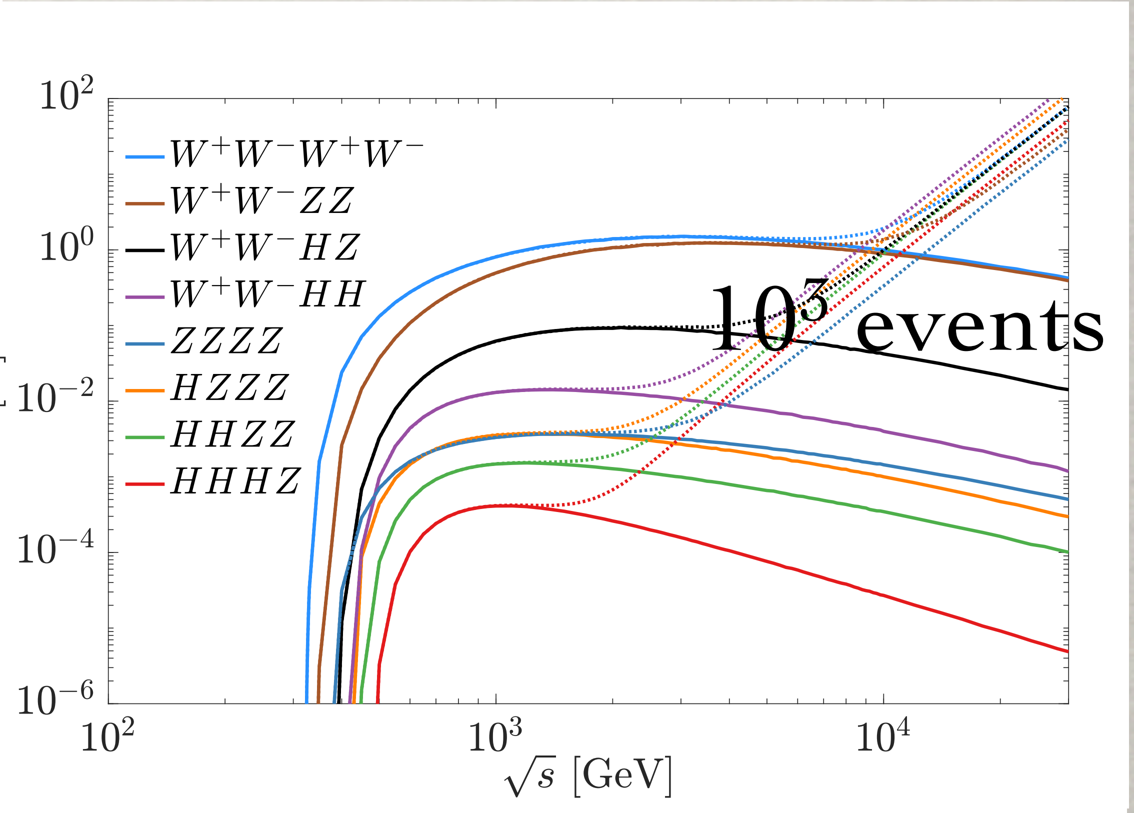
- VBFs take over ~ 4 - 6 TeV.
- Sensitive to Higgs-**μ** coupling: $\Lambda > 10\ \text{TeV}\sqrt{\frac{g}{\Delta\kappa_\mu}}$

  ***Δκ*** $_{\mu}$ ~ 1% - 10%

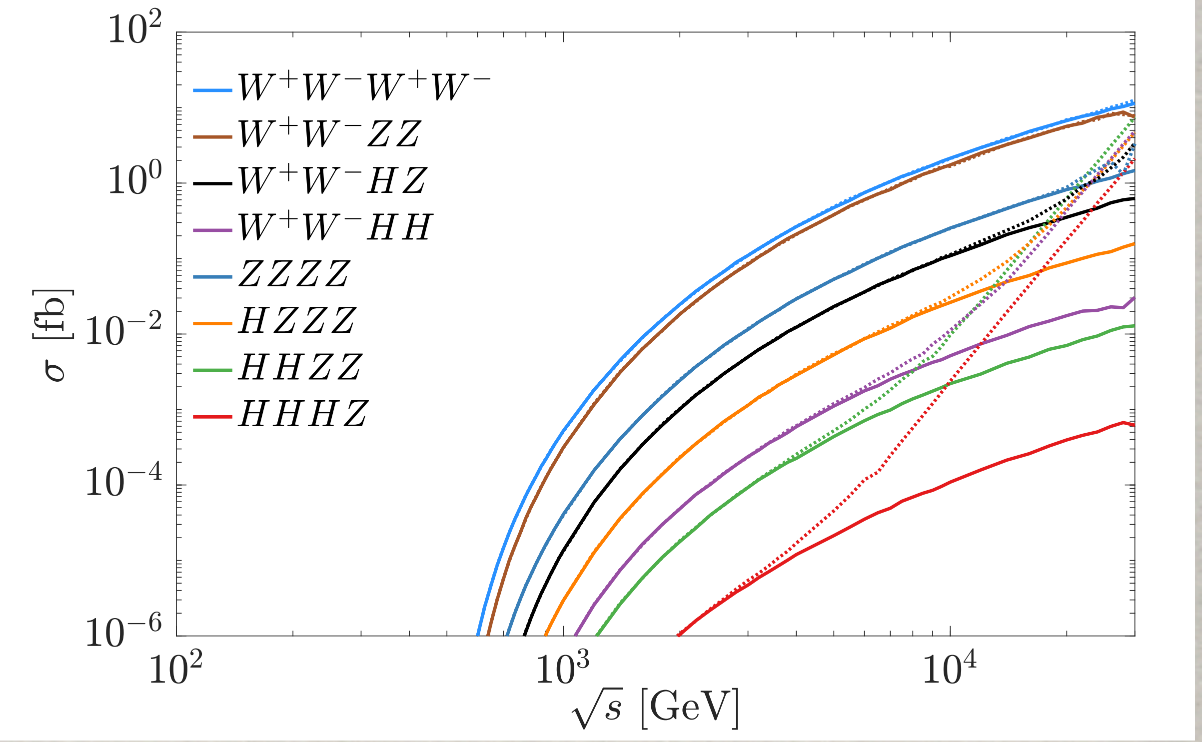
TH, W. Kilian, N. Kreher, Y.Ma, J. Reuter, T. Striegl and K. Xie: arXiv:2108.05362

# Quadruple Boson Production





$10^{3}$ events



- VBFs take over ~ 8 - 10 TeV.
- Sensitive to H-**μμ** coupling.

TH, W. Kilian, N. Kreher, Y.Ma, J. Reuter, T. Striegl and K. Xie: arXiv:2108.05362

# Summary

## Multi-TeV lepton colliders:

- Unprecedented accuracies for WWH, WWHH, $H^3$, $H^4$
- Bread & butter SM EW physics in the new territory:
  EW factorization theorem violation;
  Goldstone boson equivalence

- Multiple boson processes sensitive to new physics:
  muon-Higgs coupling

- New particle (Q,H…) mass coverage $M_H \sim (0.5 - 1)E_{cm}$
- Decisive coverage for minimal WIMP DM $M \sim 0.5\ E_{cm}$

**Muon collider an interesting option to pursue!**