# Analytical study of the conditions of an electron beam steady transport in a plasma

Helen Barminova[1,2], Bushra Kak[2]
[1]*National Research Nuclear University MEPhI, Moscow, Russia*
[2]*RUDN University, Moscow, Russia*

## Introduction

The analytical model first proposed in [1,2] is under development now. This model describes the electron beam dynamics in a rarefied plasma. The model may be applied for the analysis of electron beam behavior in conventional and novel accelerators. Such analytical study of the peculiarities of electron beam behavior in a rarefied plasma may be useful for the development of the plasma and ion sources like EBIS too.
In whole the character of the beam interaction with a rarefied plasma depends on the relations between the densities of electron beam and plasma background, on the energy of the beam, the beam geometry, the beam phase configuration.
Here we consider the case of the beam current before the Alfven limit. Mathematical model is based on the method of Vlasov equation solution which considers the kinetic distribution function as a function of integrals of particle motion [1,2]. Modified model [3] is used in this report to study the conditions of steady beam transport.

## Numerical solution of the equation for the oscillations of electron beam radius

```
(Here is the part of the code)
f1=@(x,y,yy) yy;
f2=@(x,y,yy)((u^2)*(1+(1/2)*yy^2)-
2*I*y^2/((4+2*yy^2+2*u^2/(y^2))^(1/2)))/(y*(y^2+y^2*yy^2+(
u^2)/2));
a=c1;
b=c2;
h=c3;
n=(b-a)/h;
x(1)=a;
y(1)=c4;
yy(1)=0;
for i=1:n
   m1=h*f1(x(i),y(i),yy(i));
   k1=h*f2(x(i),y(i),yy(i));
   m2=h*f1(x(i)+0.5*h,y(i)+0.5*m1,yy(i)+0.5*k1);
   k2=h*f2(x(i)+0.5*h,y(i)+0.5*m1,yy(i)+0.5*k1);
   m3=h*f1(x(i)+0.5*h,y(i)+0.5*m2,yy(i)+0.5*k1);
   k3=h*f2(x(i)+0.5*h,y(i)+0.5*m2,yy(i)+0.5*k1);
   m4=h*f1(x(i)+h,y(i)+m3,yy(i)+k3);
   k4=h*f2(x(i)+h,y(i)+m3,yy(i)+k3);
   x(i+1)=x(i)+h;
   y(i+1)=y(i)+(1/6)*(m1+2*m2+2*m3+m4);
   yy(i+1)=yy(i)+(1/6)*(k1+2*k2+2*k3+k4);
  end
```

## Conclusions

The dynamics of an electron beam in a rarefied plasma is investigated by means of mathematical model based on exact analytical solution of the Vlasov equation. The case of axial-symmetric beam with various values of the current up to Alfven limit is considered. Dynamics of such a beam is essentially nonlinear. The equation for the beam radius is solved numerically with the help of MATLAB. In quasineutral regime the possibility of the beam propagation in plasma depends on the relation between the beam current and beam emittance. Depending on its initial parameters the beam transportation may be steady.
The analytical model applied allows to predict the beam behavior with most physical generality and scalability. The results obtained show the limits of the model applications as well.

Equation 1 describes the electron beam propagation in a rarefied plasma before the pinch. In the Equation all the variables and parameters are dimensionless.

$$\sqrt{1+\frac{R'^2}{2}+\frac{\varepsilon^2}{2R^2}}\,\frac{d}{dz}\left(R'\sqrt{1+\frac{R'^2}{2}+\frac{\varepsilon^2}{2R^2}}\right)+\frac{i}{R\sqrt{1+\frac{R'^2}{2}+\frac{\varepsilon^2}{2R^2}}}=\frac{\varepsilon^2}{R^3}. \qquad (1)$$

This equation corresponds to an axial-symmetric beam characterized by radius R, transverse emittance ε, and current i. The equation of the beam radius oscillations is essentially nonlinear and should be solved numerically. In our work the solution of the Eq.1 was obtained by means of MATLAB.

The Figures 1 and 2 show some results of numerical solution of Equation 1. Figure 1 illustrates the betatron oscillations of the beam radius for various values of initial beam parameters. Figure 2 illustrates the possibility of steady beam transportation under specific initial conditions.

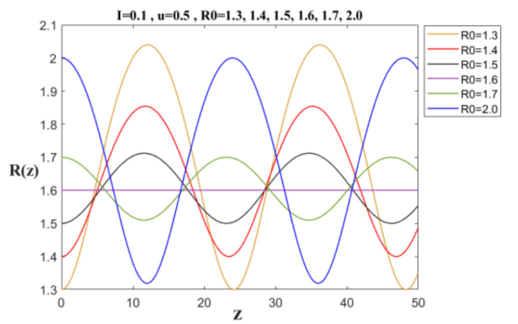


Fig.1 Dependence of the beam radius on the longitudinal coordinate z. Initial beam parameters are: R(0) = 1.3, 1.4, 1.5, 1.6, 1.7, R'(0) = 0, ε = 0.5 and 1.0, i = 0.1.

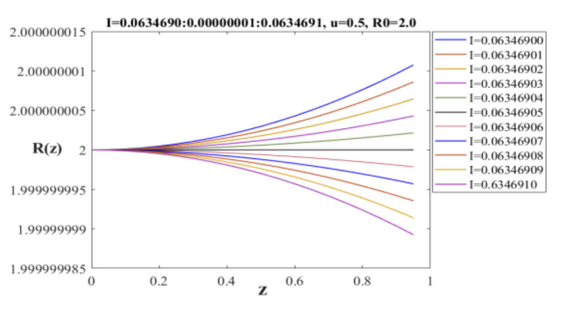


Fig.2. Dependence of the REB radius on the longitudinal coordinate z. Initial beam parameters are: R(0) = 2.0, R'(0)=0, ε = 0.5. The value of the current i lies in the range from 0.0634690 and 0.0634691.

### References

1. Barminova E.E., Chikhachev A.S. Journal of Communications Technologies and Electronics, 1993, v.38, p.122.
2. Barminova H. Journal of Plasma Physics, 2017, v.83(5), art. 705830501.
3. Barminova H.Y., Yeshchanova R. JPCS, 2020, v.1690, art.012049.