Gravitational Fragmentation in the Primordial Dark Age

BSM PANDEMIC Seminar

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2021 04 13

Outline

ΛCDM Cosmology & Inflation

Reheating

Slow Reheating and the Schrödinger-Poisson Equations

Future & Summary

ΛCDM Cosmology

The universe seems well described by only a few ingredients:

- General relativity
- Standard model of particle physics
- Dark energy (Λ)
- Cold dark matter (CDM)

ΛCDM Cosmology

Open questions

- How do general relativity and quantum mechanics work together?
- What is dark matter?
- What is dark energy?
  Why does Λ have its present value?
- Why is there more matter than anti-matter?
- The **Horizon Problem** or **Homogeneity Problem**
- The **Flatness Problem**
- What seeds the **initial perturbations** that source present day structure?

Inflation

Exponential growth in the early universe solves cosmology's initial conditions problems

- Explains homogeneity and flatness
- Predicts a spectrum of perturbations in the cosmic microwave background
- Consistent with observations

Inflation

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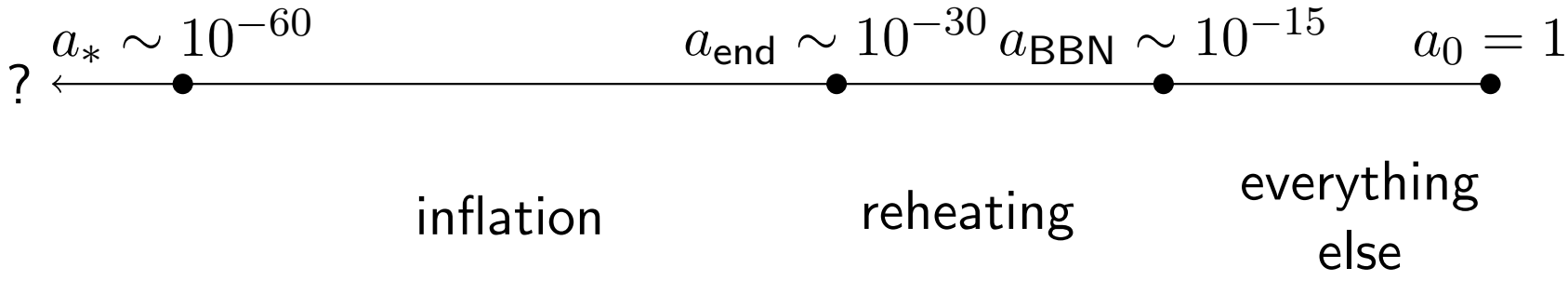
- Explains homogeneity and flatness
- Predicts a spectrum of perturbations in the cosmic microwave background
- Consistent with observations
- What was the mechanism for inflation?
  Hundreds of models have been proposed and most have not been ruled out by observations.
  - Generally some slowly varying scalar field(s)
  - Potential and couplings determine predictions

A. H. Guth, Adv. Ser. Astrophys. Cosmol. **3**, 139-148 (1987) doi:10.1103/PhysRevD.23.347
A. D. Linde, Adv. Ser. Astrophys. Cosmol. **3**, 149-153 (1987) doi:10.1016/0370-2693(82)91219-9
A. Albrecht and P. J. Steinhardt, Adv. Ser. Astrophys. Cosmol. **3**, 158-161 (1987) doi:10.1103/PhysRevLett.48.1220

Reheating



- At the end of inflation the universe is full of an almost homogeneous condensate
- **How does inflation transition into big bang nucleosynthesis?**
  The way and speed with which reheating happens affects the predictions of inflation
  - Model's viability
  - Required amount of inflation
  - Coupling to dark matter

Reheating

Scenarios:

- Self-interactions or couplings to other fields
  - Resonance
  - Rapid reheating
  - Formation of solitons or oscillons
- **Weak couplings**
  - Slow reheating
  - Less understood

L. F. Abbott, E. Farhi and M. B. Wise, Phys. Lett. B **117**, 29 (1982) doi:10.1016/0370-2693(82)90867-X
A. Albrecht, P. J. Steinhardt, M. S. Turner and F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982) doi:10.1103/PhysRevLett.48.1437
L. Kofman, A. D. Linde and A. A. Starobinsky, arXiv:hep-th/9405187
Y. Shtanov, J. H. Traschen and R. H. Brandenberger, arXiv:hep-ph/9407247
L. Kofman, A. D. Linde and A. A. Starobinsky, arXiv:hep-ph/9704452

Slow Reheating

What happens when couplings are weak?

- The universe can expand for a long time without significant reheating
- Perturbations in the inflaton field grow gravitationally

$$\delta \equiv \frac{\rho}{\langle \rho \rangle} \sim a$$

- Perturbation theory breaks down when $\delta \sim 1$ at $a/a_{\mathsf{end}} \sim 10^6$
- The scales with the most growth are those on the horizon at the end of inflation

K. Jedamzik, M. Lemoine and J. Martin, arXiv:1002.3039
R. Easther, R. Flauger and J. B. Gilmore, arXiv:1003.3011

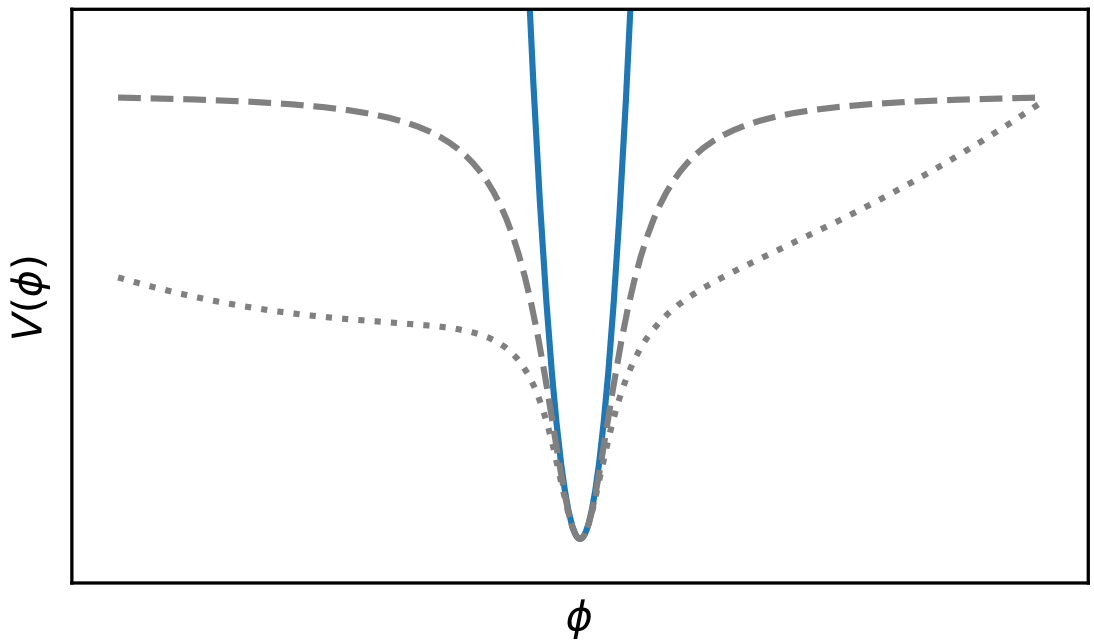
Slow Reheating

Possible consequences

- Nonlinear phenomena in even the simplest models of inflation
- Decay of the inflaton enhanced in overdensities
- Collapse to primordial black holes
  - Hawking radiation as reheating mechanism
- Gravitational radiation sourced by overdensities and/or primordial black holes
- Remnants could be dark matter

Scenario

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \mathcal{O}(\phi^3)$$



NM, S. Hotchkiss and R. Easther, arXiv:1909.11678, Phys. Rev. Lett. **124**, no.6, 061301 (2020)

Scenario

- The inflaton has coherent oscillations

$$\phi \sim \frac{1}{t}\sin(mt)$$

- The density grows with the scale factor

$$\delta \sim a$$

- These are on different timescales

$$\frac{t_H}{t_\phi} = \sqrt{3}\left(\frac{a}{a_{\mathsf{end}}}\right)^{3/2}$$

The Klein-Gordon Equations

ϕ obeys the Klein-Gordon and Einstein equations

$$\nabla_\mu \nabla^\mu \phi - \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu} = 8\pi \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{4} g_{\mu\nu} \left(\partial_\kappa \partial^\kappa \phi - V(\phi) \right) \right)$$

The Schrödinger-Poisson Equations

- Make a transformation

$$\phi = \frac{1}{ma^{3/2}}\left(\psi e^{-imt} + \psi^* e^{imt}\right)$$

- The Klein-Gordon equations become the Schrödinger-Poisson equations

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2ma^2}\nabla^2\psi + m\psi\Phi$$
$$\frac{1}{a^2}\nabla^2\Phi = \frac{4\pi G}{a^3}\left(\psi\psi^* - \left\langle|\psi|^2\right\rangle\right)$$

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- The Schrödinger-Poisson equations are already of interest
  - Ultralight dark matter

The Schrödinger-Poisson Equations

Constraints

- The largest scales must be sub-horizon

$$L_{\text{box}} \ll \frac{1}{H}$$

- The field should have small derivatives

$$|\ddot{\psi}| \ll m|\dot{\psi}| \ll m^2|\psi|$$
$$\left|\frac{1}{a^2}\nabla^2\psi\right| \ll m|\psi|$$

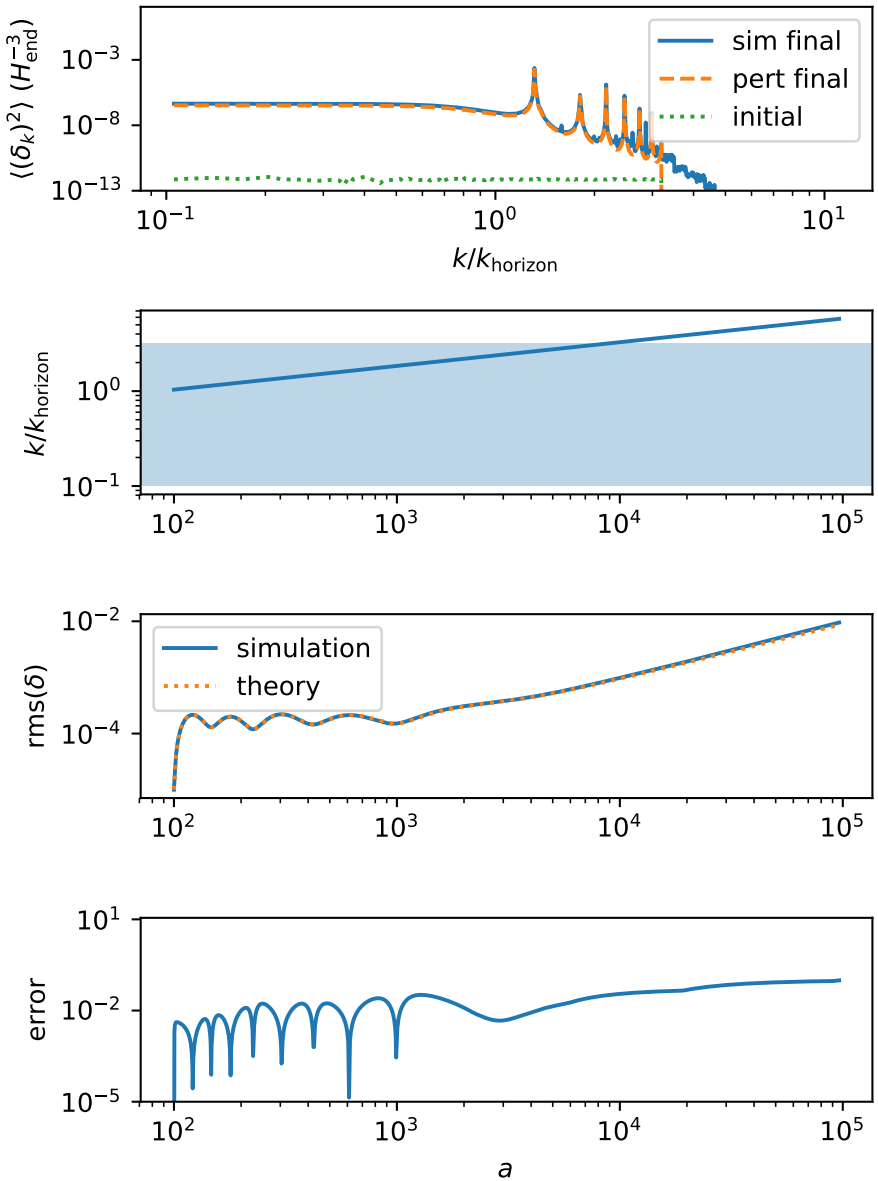
- Expansion should not be too fast

$$H \equiv \frac{\dot{a}}{a} \ll m$$
$$\dot{H} \ll mH$$

PyUltraDark

- Based on `PyUltraLight`, an ultralight dark matter code
- Added
  - expansion with $a \propto t^{2/3}$
  - adaptive time steps
  - cosmological initial conditions
  - consistency checks on phase
- Can't handle:
  - large velocities
  - formation of black holes, etc.
  - multinode simulations
- Look for a new code soon: `JultraDark`

PyUltraDark



Initial Conditions

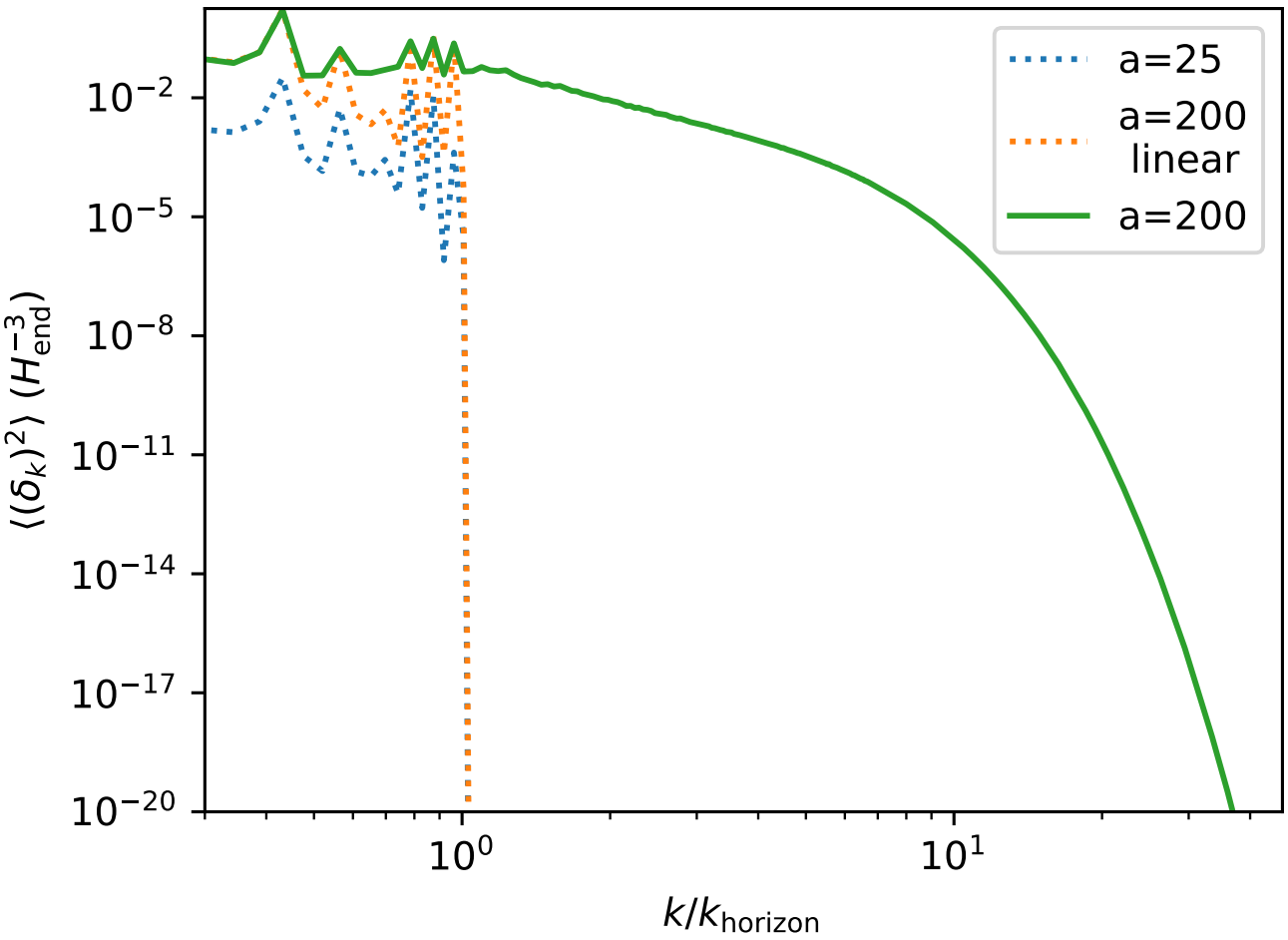
- Initial perturbation spectrum

$$\delta_k(a) = \alpha \frac{\delta_{\text{end},k}}{\delta_{g,k}(a=1)} \delta_{g,k}(a) G_k \,.$$

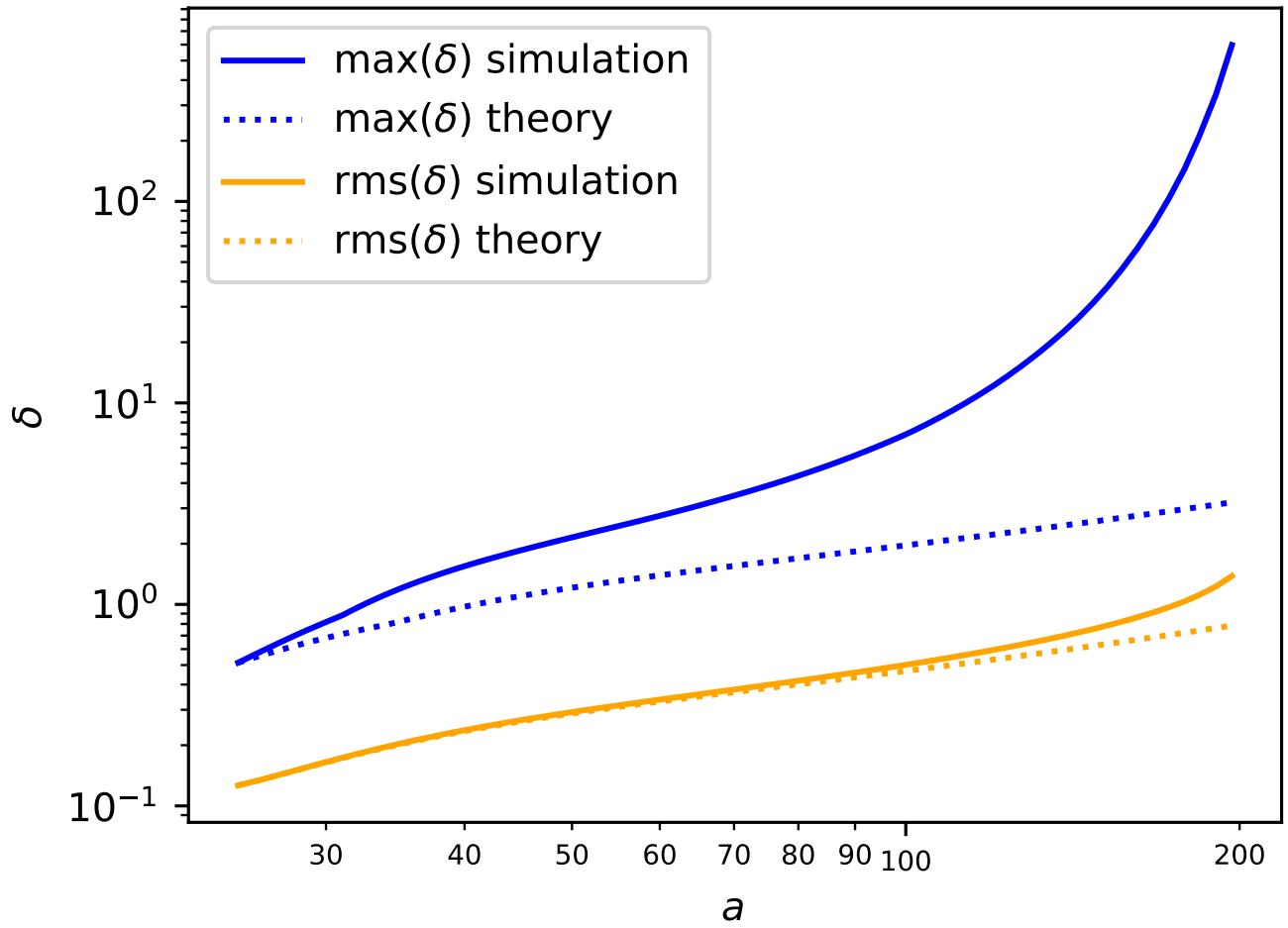
$$\delta_{\text{end},k} = \begin{cases} 0 & \text{for } k = 0 \\ \frac{1}{\sqrt{3}} M_{\text{Pl}} m k^{1/2} & \text{for } k < k_{\text{Horizon}} \\ 0 & \text{for } k > k_{\text{Horizon}} \end{cases} .$$

- Boost spectrum to get non-linearity sooner
- Fast forward to $\delta \lesssim 1$ in perturbation theory

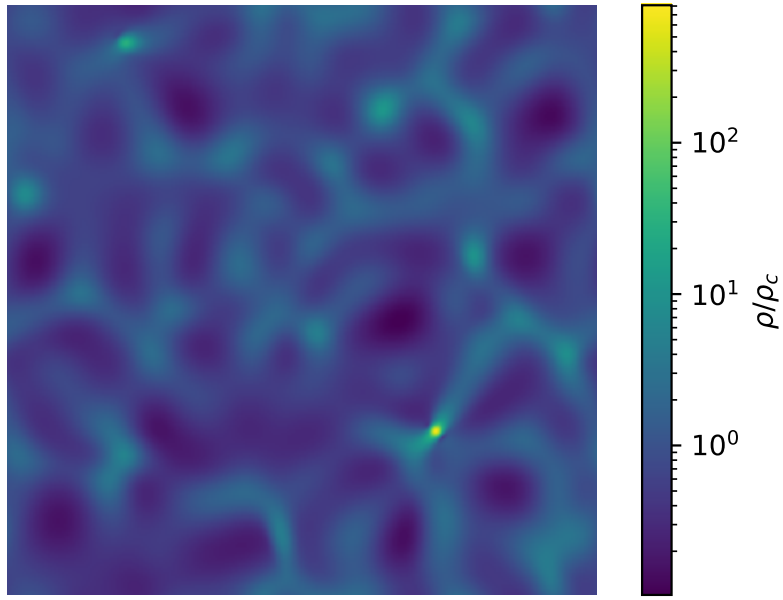
Results

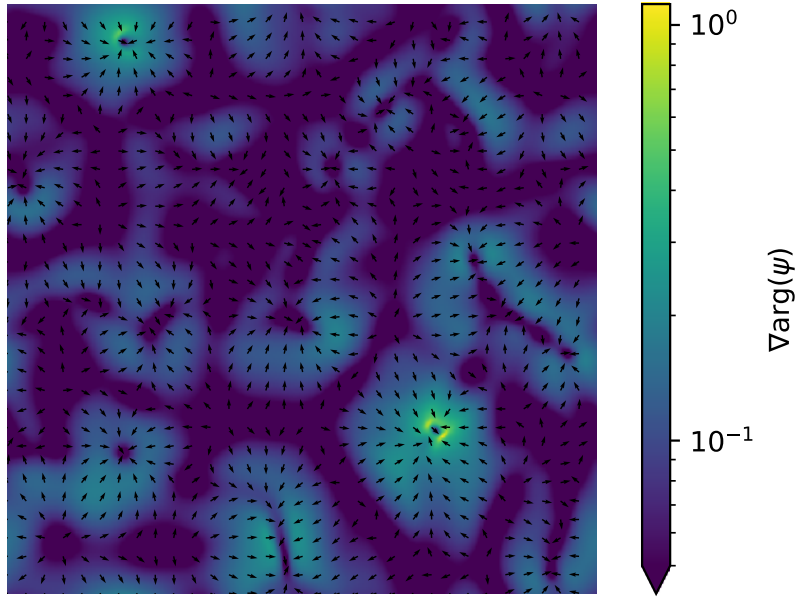

a=25
a=200 linear
a=200
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
10^{-20}
10^{0}
10^{1}
$\langle(\delta_k)^2\rangle\ (H_{\mathrm{end}}^{-3})$
k/k_{horizon}

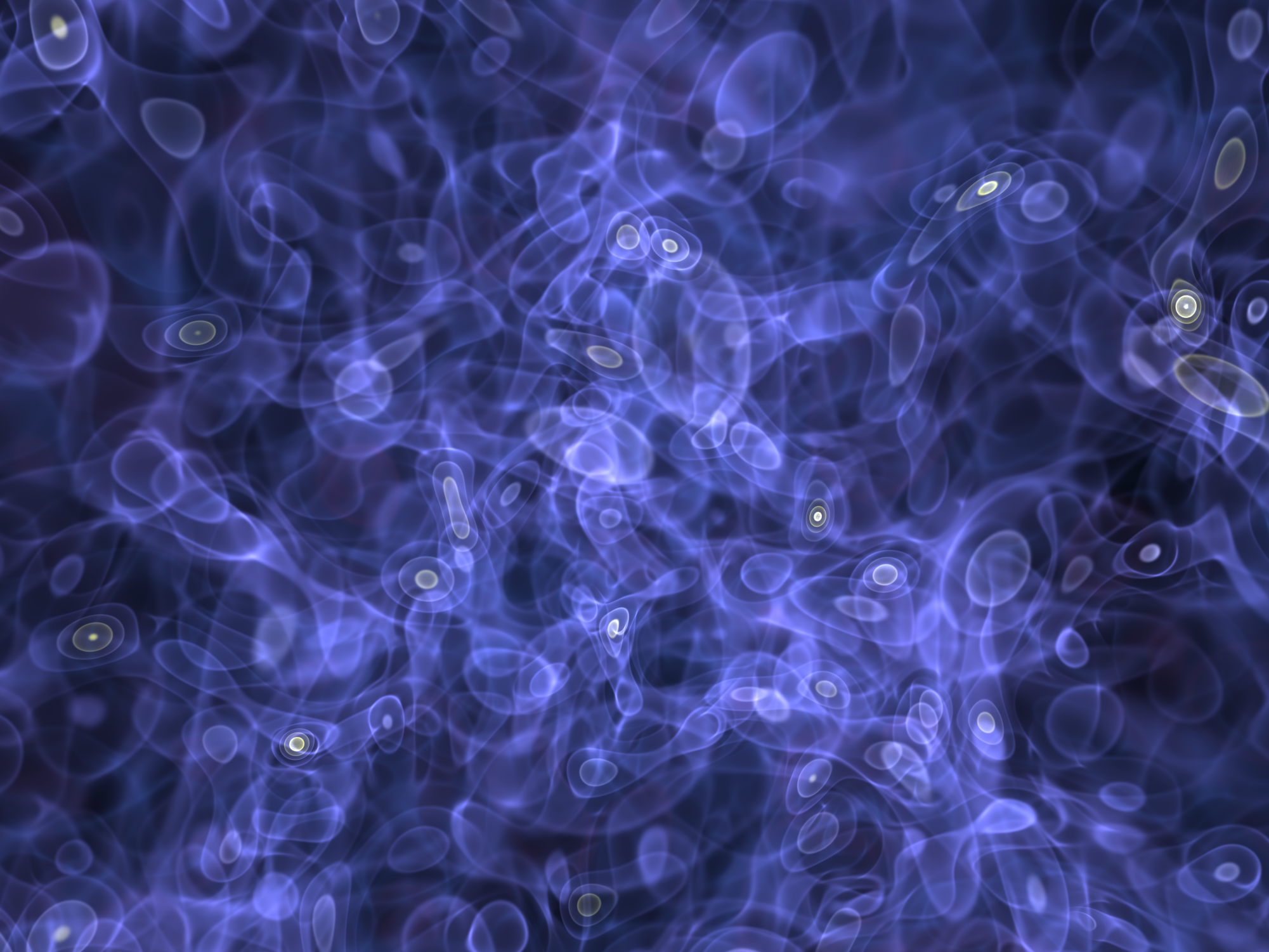
Results


max(δ) simulation
max(δ) theory
rms(δ) simulation
rms(δ) theory
10^2
10^1
10^0
10^{-1}
δ
30
40
50
60
70
80
90
100
200
a

Results







Future

- More detailed simulations
  - More realistic initial conditions
  - Compare models
  - What is the end state of the collapse?
- Further application of techniques from dark matter and structure formation
- Add couplings to standard model
- Similar use of Schrödinger-Poisson equations to simulate gravitational interactions of structures formed during preheating

Summary

- The post-inflation dynamics of the inflaton are described by the Schrödinger-Poisson equations
- Gravitational collapse in the early universe can be analogous to structure formation in the late universe
- First simulations of the gravitational growth of perturbations in the inflaton field during reheating
- Confirmed formation of large overdensities
- More advanced codes will go further and make observational predictions

Matching Equation

- The reheating temperature is useful for constraining models of inflation
- The *matching equation* determines how much inflation is needed

$$\log \frac{a_{\text{end}}}{a_*} = N = 56.12 - \log \frac{k}{k_*} + \frac{1}{3(1+w)} \log \frac{2}{3} + \log \frac{V_k^{1/4}}{V_{\text{end}}^{1/4}} + \frac{1-3w}{3(1+w)} \log \frac{\rho_{\text{reheat}}^{1/4}}{V_{\text{end}}^{1/4}} + \log \frac{V_k^{1/4}}{10^{16}\ \text{GeV}}$$

  - $w =$ equation of state during reheating
  - $\rho_{\text{reheat}} =$ density at end of reheating

Perturbation Theory

$$\psi = \sqrt{\rho_0}\sqrt{1+\delta}\exp(iS)$$

$$\delta_{d,k} = +\left(\frac{3}{x^2} - 1\right)\sin x - \frac{3}{x}\cos x \qquad \text{decaying for } x^2 < 6$$

$$\delta_{g,k} = -\left(\frac{3}{x^2} - 1\right)\cos x - \frac{3}{x}\sin x \qquad \text{growing for } x^2 < 6$$

$$S_{d,k} = \left(-\frac{6}{x^3} + \frac{3}{x}\right)\sin x + \left(\frac{6}{x^2} - 1\right)\cos x$$

$$S_{g,k} = \left(+\frac{6}{x^3} + \frac{3}{x}\right)\cos x + \left(\frac{6}{x^2} - 1\right)\sin x$$

$$x = \frac{k^2}{mH_0\sqrt{a}}$$