The $(g-2)_\mu$ in the Standard Model: review of the calculation of hadronic contributions

Gilberto Colangelo

UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

National Center for Scientific Research "Δ*HMOKPITO*Σ"
Colloquium, April 20, 2021

Outline

- Introduction: $(g-2)_\mu$
  - Early history
  - Present status
- Hadronic Vacuum Polarization contribution to $(g-2)_\mu$
- Hadronic light-by-light contribution to $(g-2)_\mu$
- The $(g-2)_\mu$ after the Fermilab measurement
- Conclusions and Outlook

Outline

Introduction: $(g-2)_\mu$
- Early history
- Present status

Hadronic Vacuum Polarization contribution to $(g-2)_\mu$

Hadronic light-by-light contribution to $(g-2)_\mu$

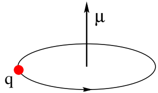
The $(g-2)_\mu$ after the Fermilab measurement

Conclusions and Outlook

$g = 2$ and the discovery of spin

- In classical and quantum mechanics

$$\mathcal{H}_{\rm magn} = -\vec{\mu}\cdot\vec{B}$$



$$\vec{\mu} = \frac{q}{2m}\vec{L}$$

- Uhlenbeck and Goudsmit (26): data on the anomalous Zeeman effect are explained by a “self-rotating” electron

$$\vec{\mu}_e = g_e \frac{e}{2m_e}\vec{S}_e \qquad S_e = \frac{1}{2} \qquad g_e = 2$$

- Soon after, Pauli provided the correct quantum-mechanical theory of spin: “classically indescribable two-valuedness”

- Dirac (28): relativistic quantum mechanical equation
  $\Rightarrow$ spin$= 1/2$ $\quad g_e = 2$

Deviations from $g_e = 2$

Twenty years later a more precise experiment showed a deviation from $g_e = 2$

$$a_e \equiv \frac{g_e - 2}{2} = 0.00118 \pm 0.00003 \quad \text{Kusch and Foley (47)}$$

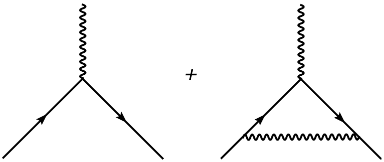
Deviations from $g_e = 2$

Twenty years later a more precise experiment
showed a deviation from $g_e = 2$

$$a_e \equiv \frac{g_e - 2}{2} = 0.00118 \pm 0.00003 \quad \text{Kusch and Foley (47)}$$

which can be understood in quantum electrodynamics (QED)

$$a_e = \frac{\alpha}{2\pi} = 0.00116 \qquad \text{Schwinger (48)}$$



Deviations from $g_e = 2$

Twenty years later a more precise experiment
showed a deviation from $g_e = 2$

$$a_e \equiv \frac{g_e - 2}{2} = 0.00118 \pm 0.00003 \quad \text{Kusch and Foley (47)}$$

which can be understood in quantum electrodynamics (QED)

$$a_e = \frac{\alpha}{2\pi} = 0.00116 \qquad \text{Schwinger (48)}$$

and provided one of the first strong confirmations of QED

Deviations from $g_e = 2$

On Quantum-Electrodynamics and the Magnetic Moment of the Electron

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts

December 30, 1947

ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu = (\frac{1}{2}\pi)e^2/\hbar c = 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium[1] have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.[2]

J. Schwinger, Letter to the editor, Phys. Rev. (1948)

Deviations from $g_e = 2$

On Quantum-Electrodynamics and the Magnetic Moment of the Electron

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
December 30, 1947

ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu = (\frac{1}{2}\pi)e^2/\hbar c = 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium[1] have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.[2]

J. Schwinger, Letter to the editor, Phys. Rev. (1948)

Photo by Jacob Bourjaily

Current status of the $(g-2)_e$

Experimental value: Hanneke, Fogwell, Gabrielse 2008

$$a_e = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \qquad [0.24\text{ppb}]$$

Theoretical calculation of QED contributions up to $\mathcal{O}(\alpha^5)$ completed Aoyama, Hayakawa, Kinoshita, Nio 2012-18

[Hadronic contributions $\sim 2 \cdot 10^{-12}$, EW below the exp. uncertainty]

Comparison between theory and experiment $\Rightarrow$ determine α

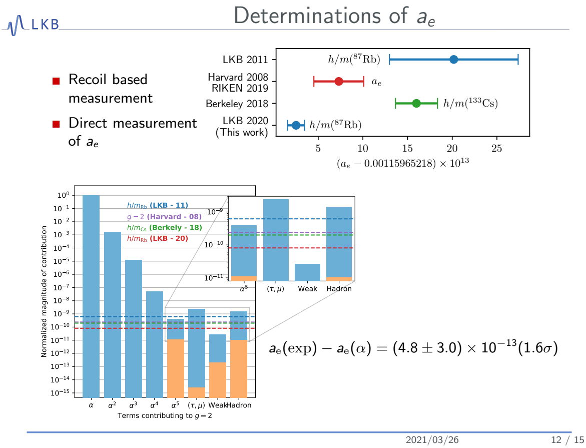
$$\alpha^{-1}(a_e) = 137.035\,999\,150(33) \qquad [0.24\text{ppb}]$$

$$\alpha^{-1}(Cs) = 137.035\,999\,045(28) \qquad [0.20\text{ppb}]$$ Parker et al.; Berkeley 2018

$$\alpha^{-1}(Rb) = 137.035\,999\,206(11) \qquad [0.08\text{ppb}]$$ Morel et al.; LKB 2020

$1.6\,\sigma$ discrepancy with a_e, but $\sim 5\,\sigma$ Cs vs Rb!

Current status of the $(g-2)_e$



Talk by P. Cladé, Moriond EW 2021

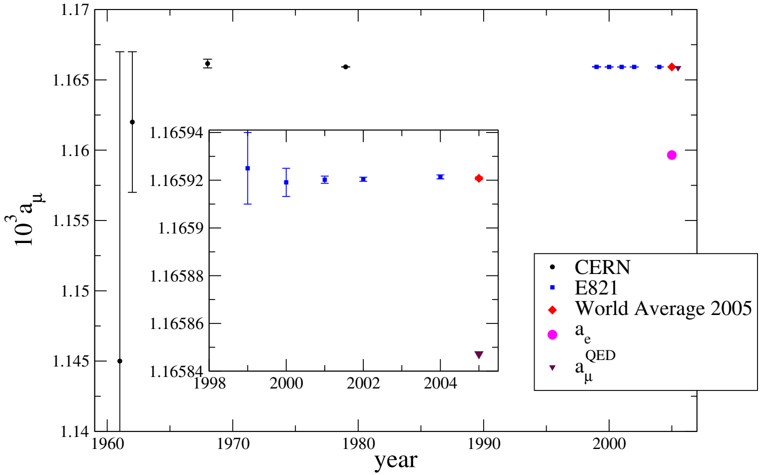
The muon $(g-2)$

- Berestetskii *et al.* (1956) pointed out that a_μ is more sensitive to the behaviour of QED at higher energy scales (shorter distances) than a_e
- Schwinger (1957) suggested to use a_μ to search for a field whose different coupling to μ and e could explain their mass difference
- In 1961 the first measurement of a_μ was carried out by Charpak, Farley, Garwin, Muller, Sens, Telegdi and Zichichi at CERN

$$a_\mu = 0.001145 \pm 0.000022$$

in good agreement with Schwinger's calculation:
the leading correction is mass independent

History of a_μ measurements



a_μ, QED and the SM

World Average (before FNAL)

$$a_\mu^{\rm exp} = (116\,592\,089 \pm 63) \times 10^{-11}$$

a_μ, QED and the SM

World Average (before FNAL)

$$a_\mu^{\rm exp} = (116\,592\,089 \pm 63) \times 10^{-11}$$

- The bulk of the difference between a_e and a_μ is due to QED and originates from large logs of m_μ/m_e

$$a_\mu^{\rm QED} - a_e^{\rm QED} \quad = \quad 619\,500.2 \times 10^{-11}$$

a_μ, QED and the SM

World Average (before FNAL)

$$a_\mu^{\exp} = (116\,592\,089 \pm 63) \times 10^{-11}$$

- The bulk of the difference between a_e and a_μ is due to QED and originates from large logs of m_μ/m_e

$$\begin{aligned} a_\mu^{\mathrm{QED}} - a_e^{\mathrm{QED}} &= 619\,500.2 \times 10^{-11} \\ a_\mu^{\exp} - a_\mu^{\mathrm{QED}} &= (7360 \pm 63) \times 10^{-11} \end{aligned}$$

a_μ, QED and the SM

World Average (before FNAL)

$$a_\mu^{\mathrm{exp}} = (116\,592\,089 \pm 63) \times 10^{-11}$$

- The bulk of the difference between a_e and a_μ is due to QED and originates from large logs of m_μ/m_e

$$a_\mu^{\mathrm{QED}} - a_e^{\mathrm{QED}} = 619\,500.2 \times 10^{-11}$$
$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} = (7360 \pm 63) \times 10^{-11}$$

- Hadronic contributions are large

$$a_\mu^{\mathrm{had}} \simeq 7000 \times 10^{-11}$$

"Seen" at the 5σ level already in 1979

a_μ, QED and the SM

World Average (before FNAL)

$$a_\mu^{\text{exp}} = (116\,592\,089 \pm 63) \times 10^{-11}$$

- The bulk of the difference between a_e and a_μ is due to QED and originates from large logs of m_μ/m_e

$$\begin{aligned} a_\mu^{\text{QED}} - a_e^{\text{QED}} &= 619\,500.2 \times 10^{-11} \\ a_\mu^{\text{exp}} - a_\mu^{\text{QED}} &= (7360 \pm 63) \times 10^{-11} \end{aligned}$$

- Hadronic contributions are large

$$a_\mu^{\text{had}} \simeq 7000 \times 10^{-11}$$

  "Seen" at the 5σ level already in 1979
- Weak contributions to a_μ

$$a_\mu^{\text{EW}} = 154 \times 10^{-11} \simeq 2.5 \Delta a_\mu^{\text{exp}}$$

White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice , *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment (E821)	116 592 089(63)
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	279(76)

White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice BMW(20), *udsc*)	7075(55)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment (E821)	116 592 089(63)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)

White Paper (2020): $(g-2)_\mu$, experiment vs SM

White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g-2$ Theory Initiative
Steering Committee:
GC
Michel Davier
Simon Eidelman
Aida El-Khadra (co-chair)
Martin Hoferichter
Christoph Lehner (co-chair)
Tsutomu Mibe (J-PARC E34 experiment)
(*Andreas Nyffeler* until summer 2020)
Lee Roberts (Fermilab E989 experiment)
Thomas Teubner
Hartmut Wittig

White Paper (2020): $(g-2)_\mu$, experiment vs SM

White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

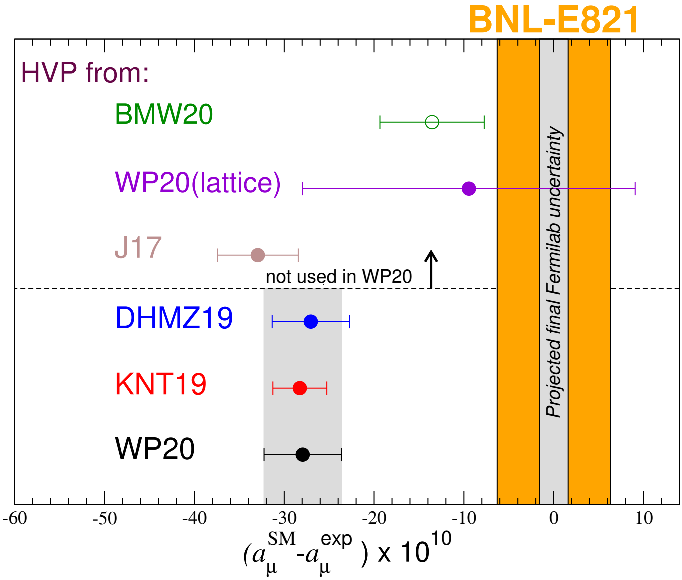
Muon $g-2$ Theory Initiative
Workshops:

- First plenary meeting, Q-Center (Fermilab), 3-6 June 2017
- HVP WG workshop, KEK (Japan), 12-14 February 2018
- HLbL WG workshop, U. of Connecticut, 12-14 March 2018
- Second plenary meeting, Mainz, 18-22 June 2018
- Third plenary meeting, Seattle, 9-13 September 2019
- Lattice HVP workshop, virtual, 16-20 November 2020
- Fourth plenary meeting, KEK (virtual), 28 June-02 July 2021

White Paper executive summary (my own)

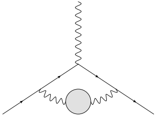
- QED and EW known and stable, negligible uncertainties
- HVP dispersive: consensus number, conservative uncertainty (KNT19, DHMZ19, CHS19, HHK19)
- HVP lattice: consensus number, $\Delta a_\mu^{\rm HVP,latt} \sim 5\,\Delta a_\mu^{\rm HVP,disp}$
  (Fermilab-HPQCD-MILC18,20, BMW18, RBC/UKQCD18, ETM19,SK19, Mainz19, ABTGJP20)
- HVP BMW20: central value $\to$ discrepancy $< 2\sigma$;
  $\Delta a_\mu^{\rm HVP,BMW} \sim \Delta a_\mu^{\rm HVP,disp}$ not yet published $\to$ not in WP
- HLbL dispersive: consensus number, w/ recent improvements $\Rightarrow$ $\Delta a_\mu^{\rm HLbL} \sim 0.5\,\Delta a_\mu^{\rm HVP}$
- HLbL lattice: single calculation, agrees with dispersive ($\Delta a_\mu^{\rm HLbL,latt} \sim 2\,\Delta a_\mu^{\rm HLbL,disp}$) $\to$ final average (RBC/UKQCD20)

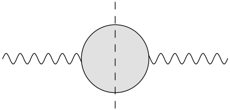
Status of $(g-2)_\mu$, experiment vs SM (as of April 6, 2021)



Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$



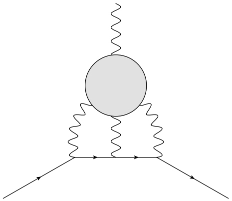


  - unitarity and analyticity $\Rightarrow$ dispersive approach
  - $\Rightarrow$ direct relation to experiment: $\sigma_{\rm tot}(e^+e^- \to \text{hadrons})$
  - e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
  - alternative approach: lattice, becoming competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$
- Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)



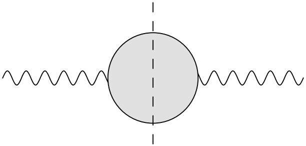
- earlier: "*it cannot be expressed in terms of measurable quantities*"
- recently: dispersive approach $\Rightarrow$ data-driven, systematic treatment
- lattice QCD is becoming competitive

(Mainz, RBC/UKQCD)

Outline

HVP contribution: Master Formula

Unitarity relation: simple, same for all intermediate states



$$\mathrm{Im}\Pi(q^2) \propto \sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

Analyticity $\Rightarrow$ Master formula for HVP

Bouchiat, Michel (61)

$$a_\mu^{\text{hvp}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}K(s)R(s)$$

$K(s)$ known, depends on m_μ and $K(s) \sim \frac{1}{s}$ for large s

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	-0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

2π: comparison with the dispersive approach

The 2π channel can itself be described dispersively $\Rightarrow$ more constrained theoretically Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18)

Energy range	ACD18	CHS18	DHMZ19	KNT19
$\leq 0.6\,\mathrm{GeV}$		110.1(9)	110.4(4)(5)	108.7(9)
$\leq 0.7\,\mathrm{GeV}$		214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
$\leq 0.8\,\mathrm{GeV}$		413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
$\leq 0.9\,\mathrm{GeV}$		479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
$\leq 1.0\,\mathrm{GeV}$		495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
$[0.6, 0.7]\,\mathrm{GeV}$		104.7(7)	104.2(5)(5)	104.4(5)
$[0.7, 0.8]\,\mathrm{GeV}$		198.3(9)	199.8(0.9)(1.2)	198.9(7)
$[0.8, 0.9]\,\mathrm{GeV}$		66.6(4)	67.5(4)(6)	66.6(3)
$[0.9, 1.0]\,\mathrm{GeV}$		15.3(1)	15.5(1)(2)	15.3(1)
$\leq 0.63\,\mathrm{GeV}$	132.9(8)	132.8(1.1)	132.9(5)(6)	131.2(1.0)
$[0.6, 0.9]\,\mathrm{GeV}$		369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$\left[\sqrt{0.1}, \sqrt{0.95}\right]\,\mathrm{GeV}$		490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

WP(20)

Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been so combined:

- central values are obtained by simple averages (for each channel and mass range)
- the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- $1/2$ difference DHMZ$-$KNT (or BABAR$-$KLOE in the 2π channel, if larger) is added to the uncertainty

Final result:

$$a_\mu^{\text{HVP, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10}$$
$$= 693.1(4.0) \times 10^{-10}$$

WP(20)

The BMW result

Borsanyi et al. Nature 2021

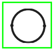
State-of-the-art lattice calculation of $a_\mu^{\text{HVP, LO}}$ based on

- current-current correlator, summed over all distances, integrated in time with appropriate kernel function
- using staggered fermions on an $L \sim 6$ fm lattice ($L \sim 11$fm used for finite volume corrections)
- at (and around) physical quark masses
- including isospin-breaking effects

The BMW result

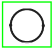
Borsanyi et al. Nature 2021

Isospin-symmetric



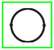
Connected light

$633.7(2.1)_{\text{stat}}(4.2)_{\text{syst}}$



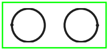
Connected strange

$53.393(89)_{\text{stat}}(68)_{\text{syst}}$



Connected charm

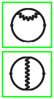
$14.6(0)_{\text{stat}}(1)_{\text{syst}}$



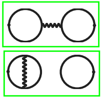
Disconnected

$-13.36(1.18)_{\text{stat}}(1.36)_{\text{syst}}$

QED isospin breaking: valence

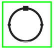


Connected $-1.23(40)_{\text{stat}}(31)_{\text{syst}}$



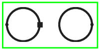
Disconnected $-0.55(15)_{\text{stat}}(10)_{\text{syst}}$

Strong–isospin breaking



Connected

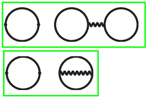
$6.60(63)_{\text{stat}}(53)_{\text{syst}}$



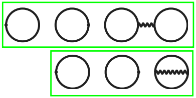
Disconnected

$-4.67(54)_{\text{stat}}(69)_{\text{syst}}$

QED isospin breaking: sea



Connected $0.37(21)_{\text{stat}}(24)_{\text{syst}}$



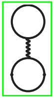
Disconnected $-0.040(33)_{\text{stat}}(21)_{\text{syst}}$

Other

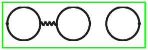
Bottom; higher-order; perturbative

$0.11(4)_{\text{tot}}$

QED isospin breaking: mixed



Connected $-0.0093(86)_{\text{stat}}(95)_{\text{syst}}$



Disconnected $0.011(24)_{\text{stat}}(14)_{\text{syst}}$

Finite-size effects

Isospin-symmetric

$18.7(2.5)_{\text{tot}}$

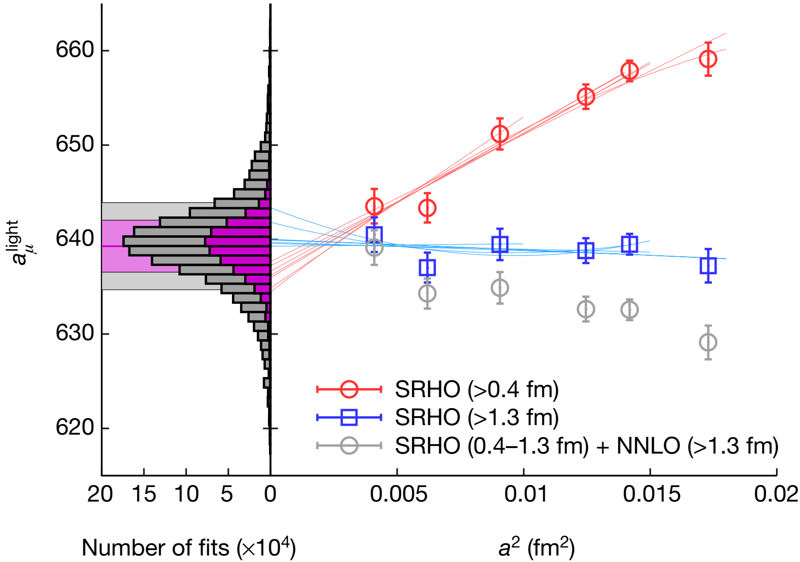
Isospin-breaking

$0.0(0.1)_{\text{tot}}$

$$a_\mu^{\text{LO-HVP}}\,(\times 10^{10}) = 707.5(2.3)_{\text{stat}}(5.0)_{\text{syst}}(5.5)_{\text{tot}}$$

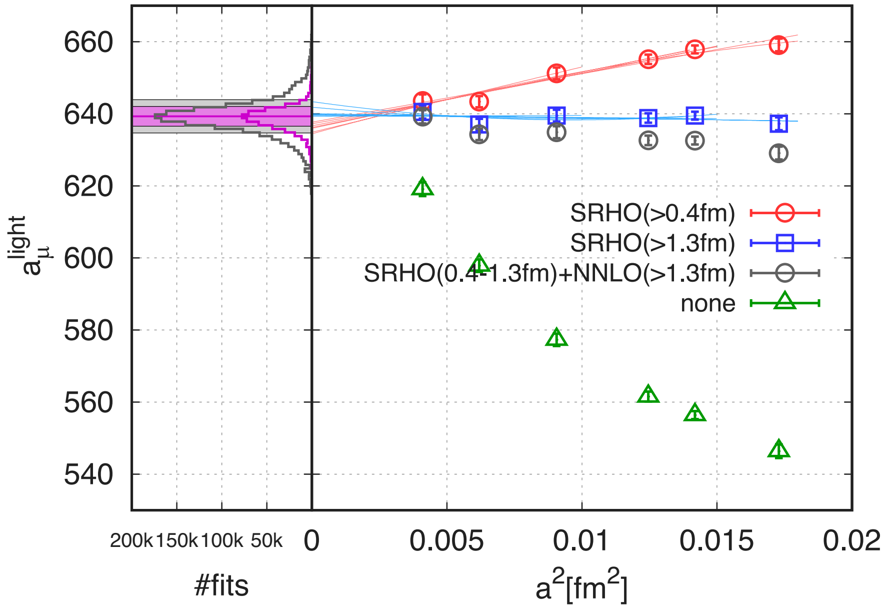
The BMW result

Borsanyi et al. Nature 2021



The BMW result

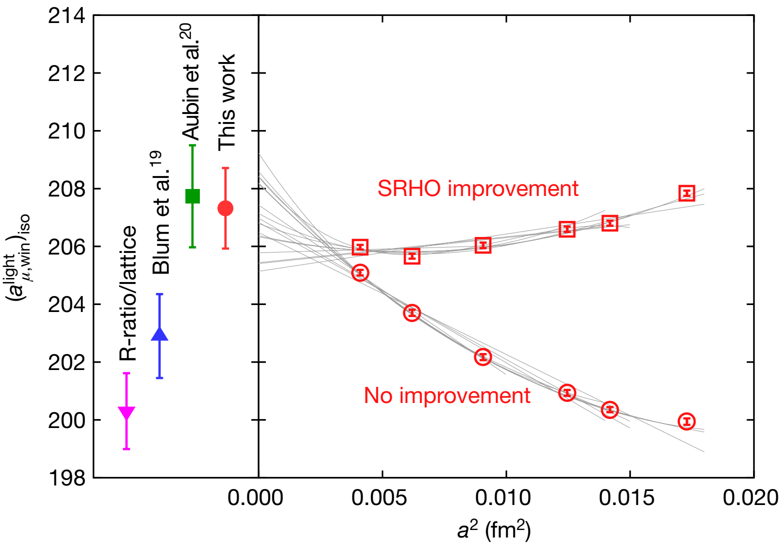
Borsanyi et al. Nature 2021



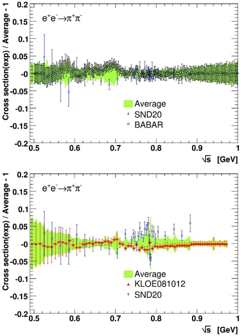
The BMW result

Borsanyi et al. Nature 2021

Article



Quantifying level of discrepancy in data in view of BMW20 result



- Taking SND20 as referee
- Small discrepancy 0.65 - 0.7 GeV BABAR
- Large discrepancy > 0.7 GeV KLOE

- Comparing $10^{10} \times a_\mu^{\mathrm{LO\ had}}$ results:

BMW20	$708.7 \pm 2.8 \pm 4.5$ (5.3)	0.75%	--
DHMZ19 all	$694.0 \pm 1.0 \pm 2.5 \pm 0.7 \boxed{\pm 2.8}$ (4.0) stat syst QCD BABAR-KLOE	0.58%	2.2 σ
DHMZ19 $-$KLOE	$696.8 \pm (3.1)$	0.44%	1.9 σ
DHMZ19 $-$BABAR	$691.2 \pm (3.1)$	0.44%	2.9 σ
WP20 all	$693.1 \pm 2.8 \pm 0.7 \pm 2.8$ (4.0) exp QCD BABAR-KLOE	0.58%	2.3 σ

- BABAR/KLOE discrepancy results in a 30% loss in precision
- But does not account for the difference with BMW20
- However difference reduced for BABAR compared to KLOE

Slide by Michel Davier, HVP Lattice Workshop, November 2020

New data sets (SND20 and BESIII) are being analyzed. Both are between KLOE and BaBar.

Consequences of the BMW result

A shift in the value of $a_\mu^{\text{HVP, LO}}$ would have consequences:

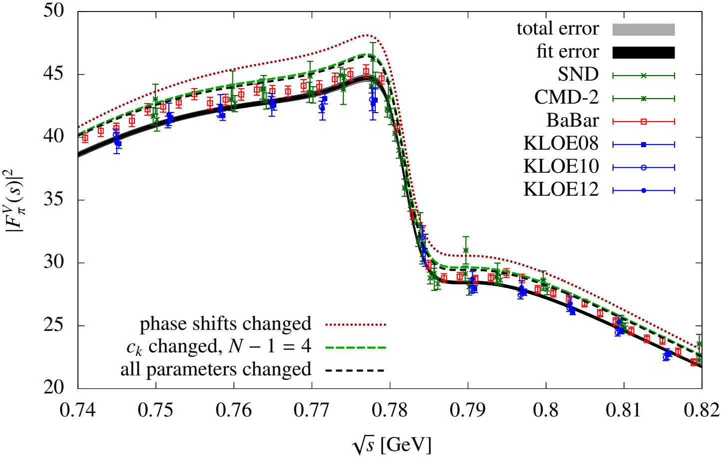
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad \Delta\sigma(e^+e^- \to \text{hadrons})$
- $\Delta\alpha_{\text{had}}(M_Z^2)$ is determined by an integral of the same $\sigma(e^+e^- \to \text{hadrons})$ (more weight at high energy)
- changing $a_\mu^{\text{HVP, LO}}$ necessarily implies a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$: size depends on the energy range of $\Delta\sigma(e^+e^- \to \text{hadrons})$
- a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$ has an impact on the EW-fit
- to save the EW-fit $\Delta\sigma(e^+e^- \to \text{hadrons})$ must occur below ~ 1 (max 2) GeV

  Crivellin, Hoferichter, Manzari, Montull (20)/Keshavarzi, Marciano, Passera, Sirlin (20)/Malaescu, Schott (20)
- or the need for BSM physics would be moved elsewhere

Changes in $\sigma(e^+e^- \to \mathrm{hadrons})$ below 1 GeV?

- Below 1 – 2 GeV only one significant channel: $\pi^+\pi^-$
- Strongly constrained by analyticity and unitarity ($F_\pi^V(s)$)
- $F_\pi^V(s)$ parametrization which satisfies these
  $\Rightarrow$ small number of parameters — GC, Hoferichter, Stoffer (18)
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad$ shifts in these parameters
  analysis of the corresponding scenarios — GC, Hoferichter, Stoffer (21)

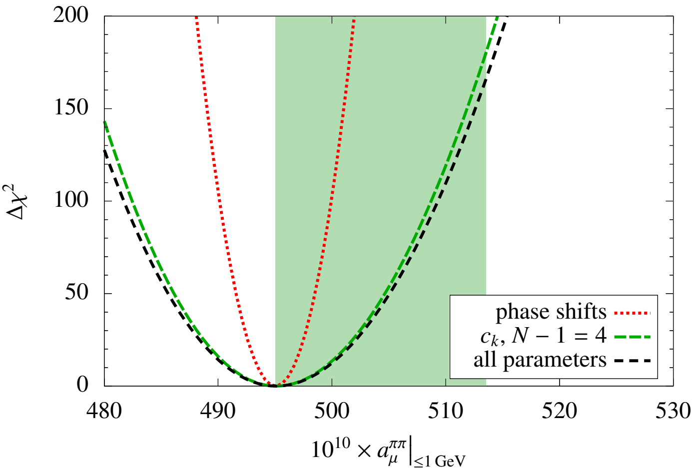
Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

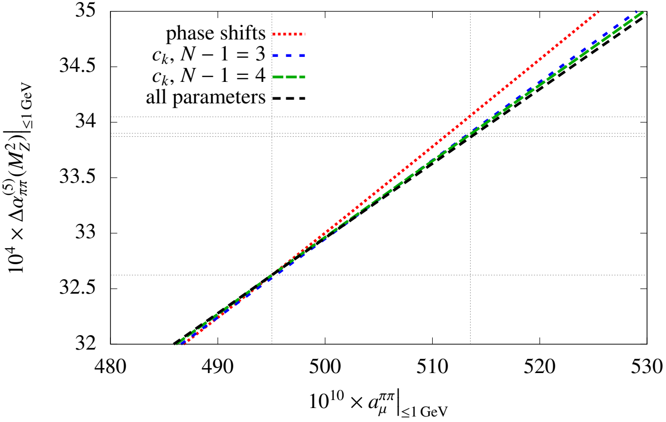
Tension [BMW20 vs e^+e^- data] stronger for KLOE than for BABAR

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

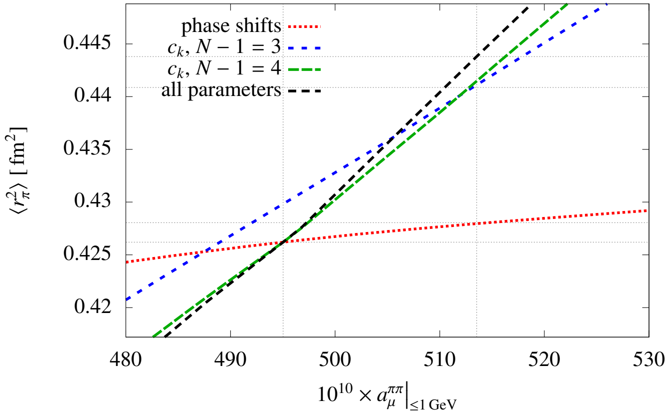
Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

$$10^4 \Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \begin{cases} 272.2(4.1) & \text{EW fit} \\ 276.1(1.1) & \sigma_{\text{had}}(s) \end{cases}$$

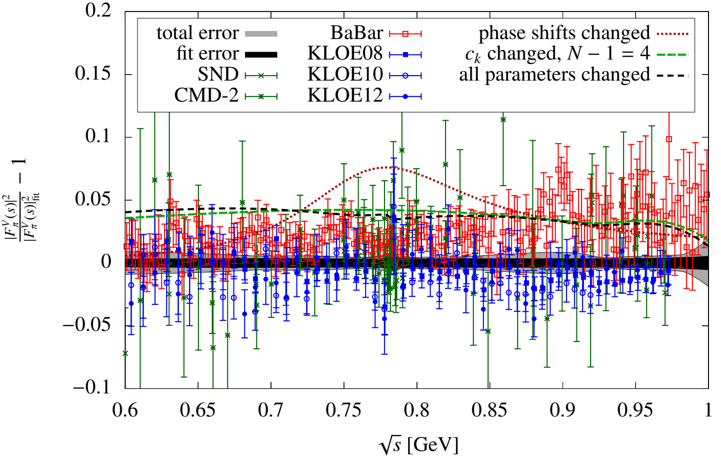
Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

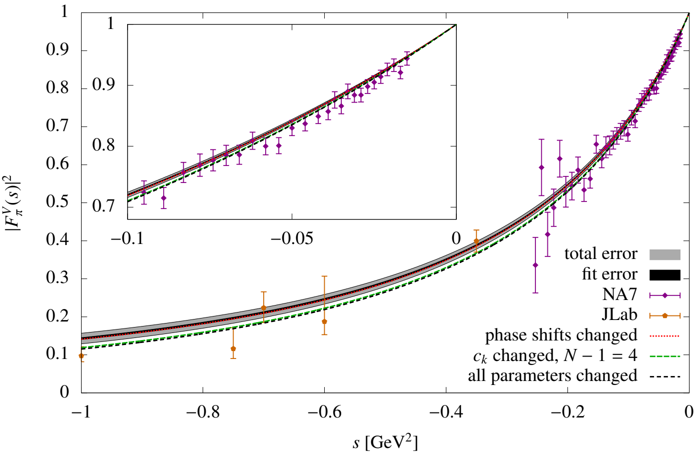
$$\langle r_\pi^2 \rangle = \begin{cases} 0.429(4)\text{fm}^2 & \text{CHS(18)} \\ 0.436(5)(12)\text{fm}^2 & \chi\text{QCD(20)} \end{cases}$$

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

Outline

HLbL contribution: Master Formula

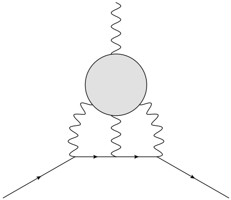
$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{48\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau)\bar{\Pi}_i(Q_1, Q_2, \tau)$$

Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta:

$$Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$$

The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$. CHPS (15)

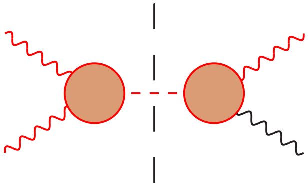
- T_i: known kernel functions
- $\bar{\Pi}_i$ are amenable to a dispersive treatment: their imaginary parts are related to measurable subprocesses



Setting up the dispersive calculation

We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$



Pion pole: imaginary parts = δ-functions

Projection on the BTT basis: easy ✓

Our master formula=explicit expressions in the literature ✓

Input: pion transition form factor — Hoferichter et al. (18)

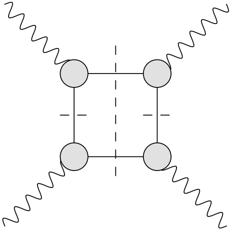
First results of direct lattice calculations — Gerardin, Meyer, Nyffeler (16,19)

Setting up the dispersive calculation

We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$

π-box with the BTT set:



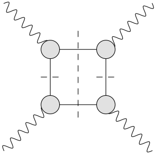
– we have constructed a Mandelstam representation for the contribution of the 2-pion cut with LHC due to a pion pole

– we have explicitly checked that this is identical to sQED multiplied by $F_V^\pi(s)$ (FsQED)

Setting up the dispersive calculation

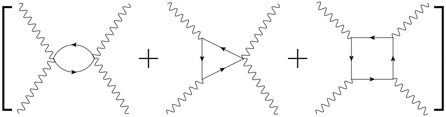
We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$



$$\equiv F_\pi^V(q_1^2) F_\pi^V(q_2^2) F_\pi^V(q_3^2)$$

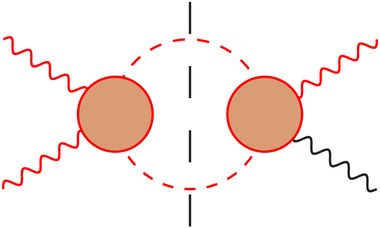
$\times$



Setting up the dispersive calculation

We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$



The "rest" with 2π intermediate states has cuts only in one channel and will be
calculated dispersively after partial-wave expansion

Setting up the dispersive calculation

We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$

Contributions of cuts with anything else other than one and two pions in intermediate states are neglected in first approximation

of course, the η, η' and other pseudoscalars pole contribution, or the kaon-box/rescattering contribution can be calculated within the same formalism

Improvements obtained with the dispersive approach

Contribution	PdRV(09) *Glasgow consensus*	N/JN(09)	J(17)	WP(20)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	− 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

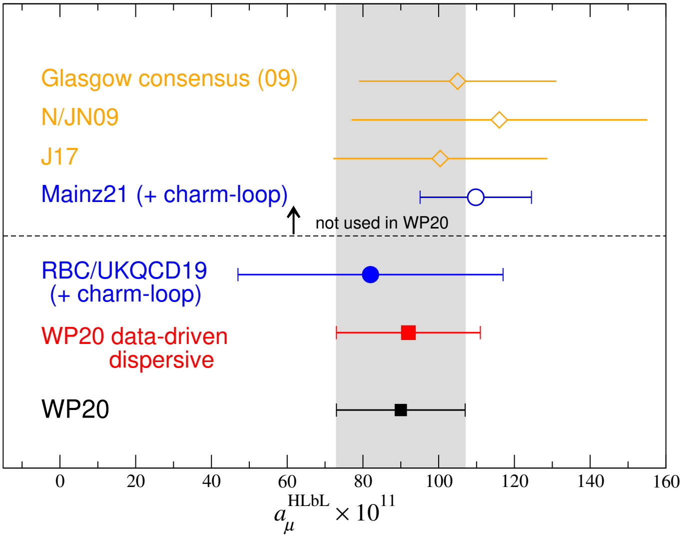
- significant reduction of uncertainties in the first three rows: low-energy region well constrained by a dispersive approach

  CHPS (17), Masjuan, Sánchez-Puertas (17) Hoferichter, Hoid, Kubis, Leupold, Schneider (18)

- $1-2$ GeV and asymptotic region (short distance constraints) have been improved, but still work in progress (see WP(20))

  Melnikov, Vainshtein (04), (.......), Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez (20,21)

Situation for HLbL



Outline

Fermilab Muon $g-2$ experiment

Fermilab Muon $g-2$ experiment

Credit: Brookhaven National Laboratory

Fermilab Muon $g-2$ experiment

Photo: Darin Clifton/Ceres Barge

Fermilab Muon $g-2$ experiment

Credit: Fermilab

Fermilab Muon $g-2$ experiment

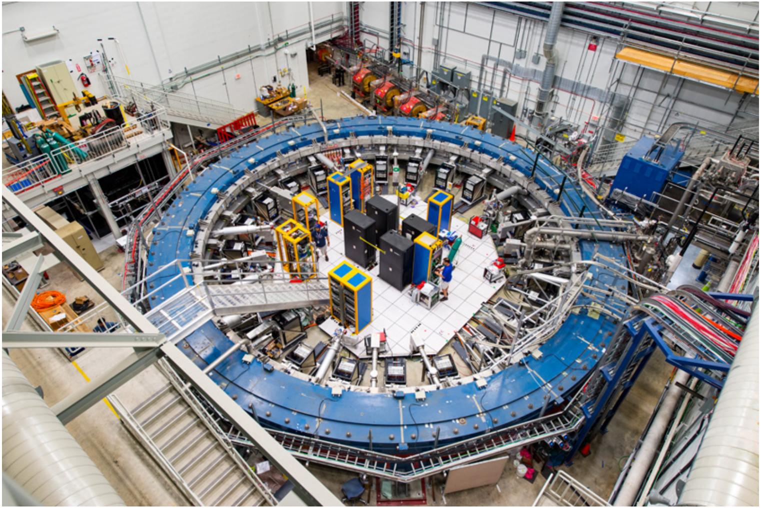


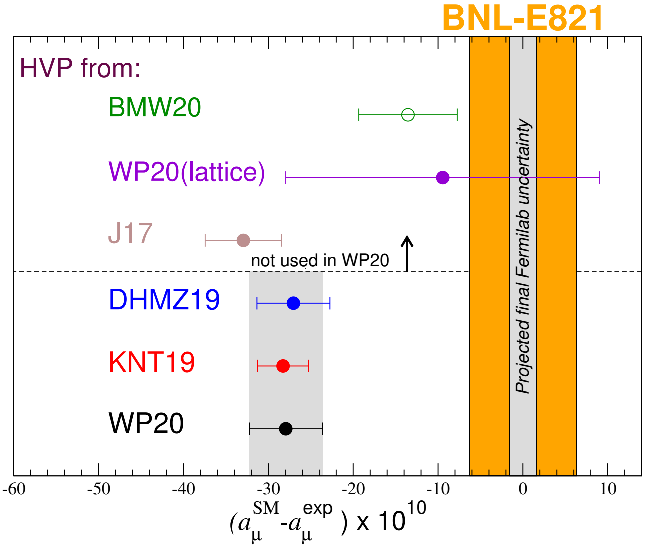
Photo: Reidar Hahn, Fermilab

Status of $(g-2)_\mu$, experiment vs SM

$$a_\mu(BNL) = 116\,592\,089(63) \times 10^{-11}$$
$$a_\mu(FNAL) = 116\,592\,040(54) \times 10^{-11}$$
$$a_\mu(Exp) = 116\,592\,061(41) \times 10^{-11}$$

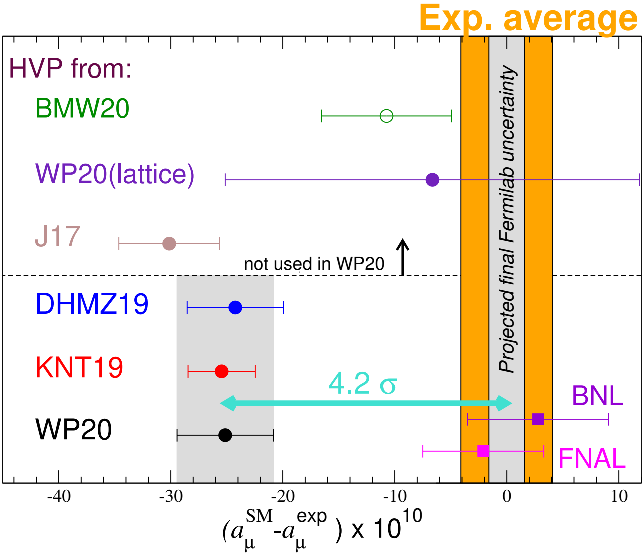
Status of $(g-2)_\mu$, experiment vs SM

Before the Fermilab result



Status of $(g-2)_\mu$, experiment vs SM

After the Fermilab result



Outline

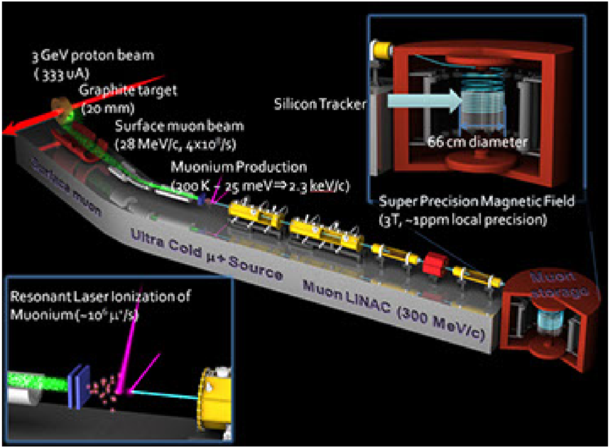
Conclusions

- The WP provides the current status of the SM evaluation of $(g-2)_\mu$: 4.2σ discrepancy with experiment (w/ FNAL)
- Evaluation of the HVP contribution based on the dispersive approach: 0.6% error $\Rightarrow$ dominates the theory uncertainty
- Recent lattice calculation [BMW(20)] has reached a similar precision but differs from the dispersive one (=from e^+e^- data).
  If confirmed $\Rightarrow$ discrepancy with experiment $\searrow$ below 2σ
- Evaluation of the HLbL contribution based on the dispersive approach: 20% accuracy. Two recent lattice calculations [RBC/UKQCD(20), Mainz(21)] agree with it

Outlook

- The Fermilab experiment aims to reduce the BNL uncertainty by a factor four $\Rightarrow$ potential 7σ discrepancy
- Improvements on the theory side:
  - HVP data-driven:
    Other e^+e^- experiments are available or forthcoming:
    SND, BESIII, CMD3, BaBar $\Rightarrow$ Further error reductions
  - HVP lattice:
    BMW result must be confirmed (or refuted) by others.
    Difference to data-driven evaluation must be understood
  - HLbL data-driven: goal of $\sim$ 10% uncertainty within reach
  - HLbL lattice: RBC/UKQCD $\Rightarrow$ similar precision as Mainz.
    Good agreement with data-driven evaluation.

Future: Muon $g-2$/EDM experiment @ J-PARC



Credit: J-PARC