Anti-k_T jet function at next-to-next-to-leading order

Sven-Olaf Moch

Universität Hamburg

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Based on work done in collaboration with:

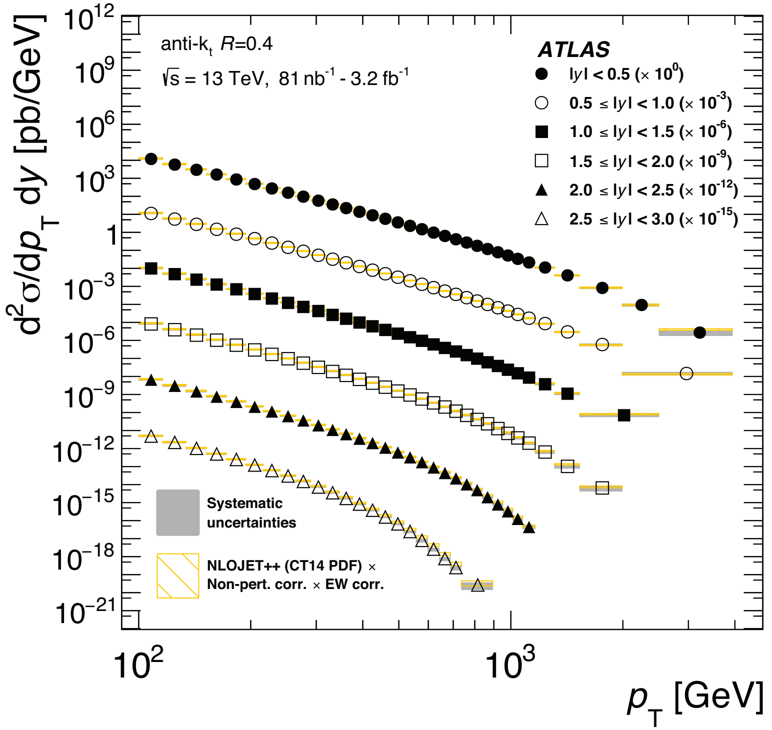
- *Anti-k_T jet function at next-to-next-to-leading order*
  Hao-Yu Liu, Xiaohui Liu, and S. M. arXiv:2103.08680

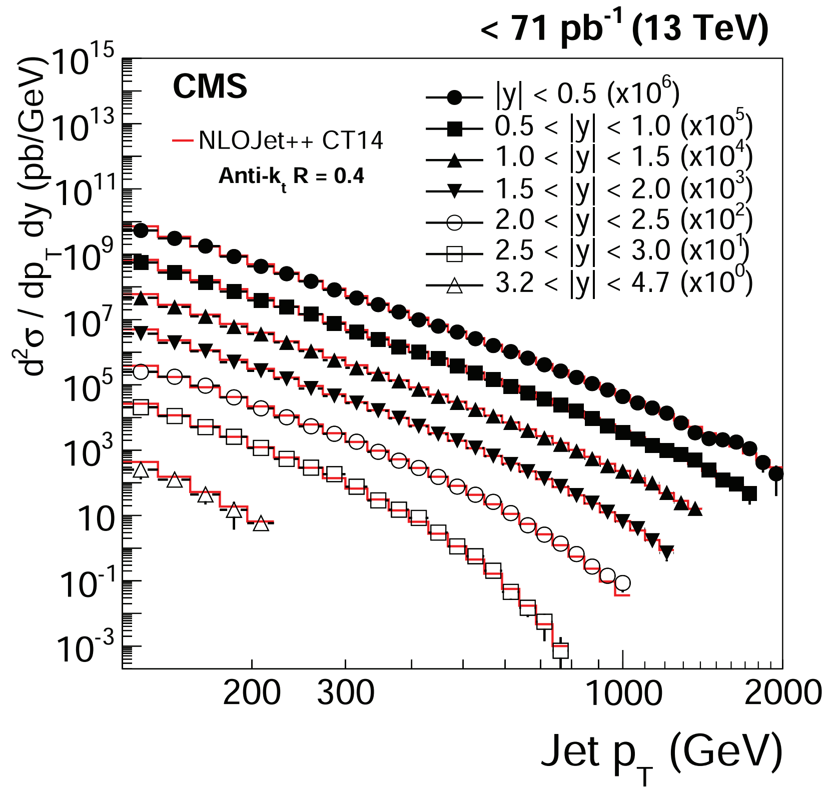
- *Threshold and jet radius joint resummation for single-inclusive jet production*
  S. M., Engin Eren, Katerina Lipka, Xiaohui Liu, and Felix Ringer
  PoS LL2018 (2018) 002 arXiv:1808.04574

- *Phenomenology of single-inclusive jet production with jet radius and threshold resummation*
  Xiaohui Liu, S. M. and Felix Ringer Phys.Rev.D 97 (2018) 5, 056026
  arXiv:1801.07284

- *Threshold and jet radius joint resummation for single-inclusive jet production*
  Xiaohui Liu, S. M. and Felix Ringer Phys.Rev.Lett. 119 (2017) no.21, 212001
  arXiv:1708.04641

Single-inclusive jet production

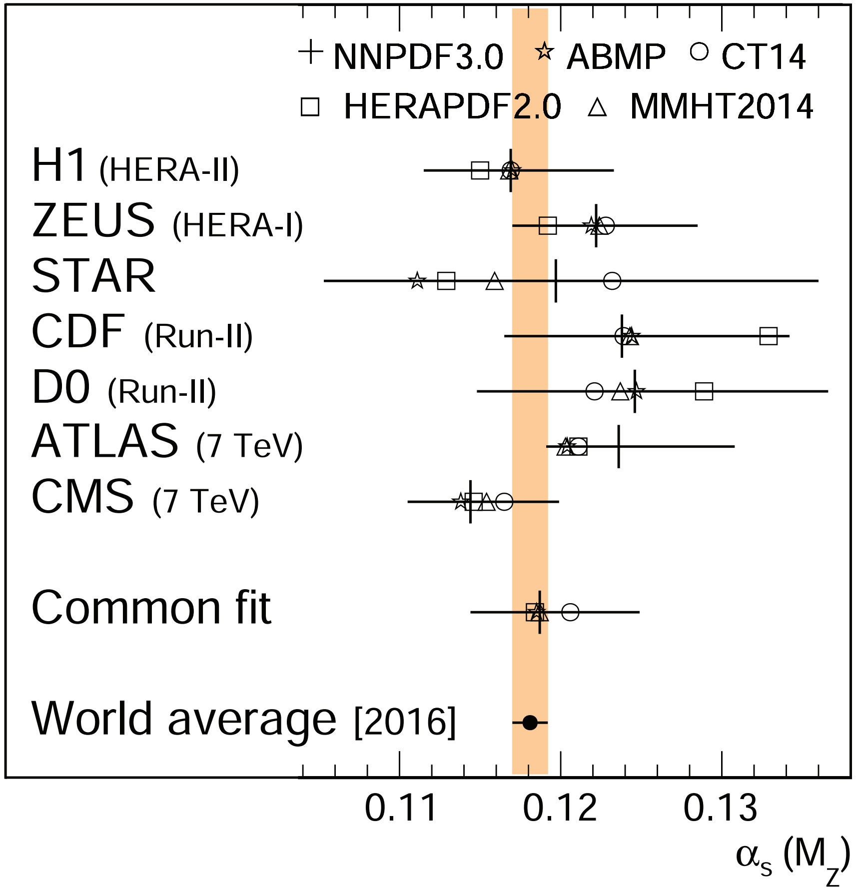




- Double differential cross section for $pp \to \text{jet} + X$ at $\sqrt{s} = 13$ TeV
  - transverse momentum p_T and rapidity y of signal-jet
  - ATLAS arXiv:1711.02692 (left), CMS arXiv:1605.04436 (right)
- Comparison with NLO perturbative QCD predictions (NLOJET++ Nagy)
  - impressive agreement over several orders of magnitude

Uses of inclusive jet data

- Dermination of $\alpha_s(M_Z)$ and PDFs (gluon at medium to large x)
  - partonic cross sections $\hat{\sigma}_{ij\to\mathrm{jet}} \propto \alpha_s^2(\mu)$
  - jet cross section $d\sigma_{pp\to\mathrm{jet}} = \alpha_s^2(\mu) \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes [\dots]$



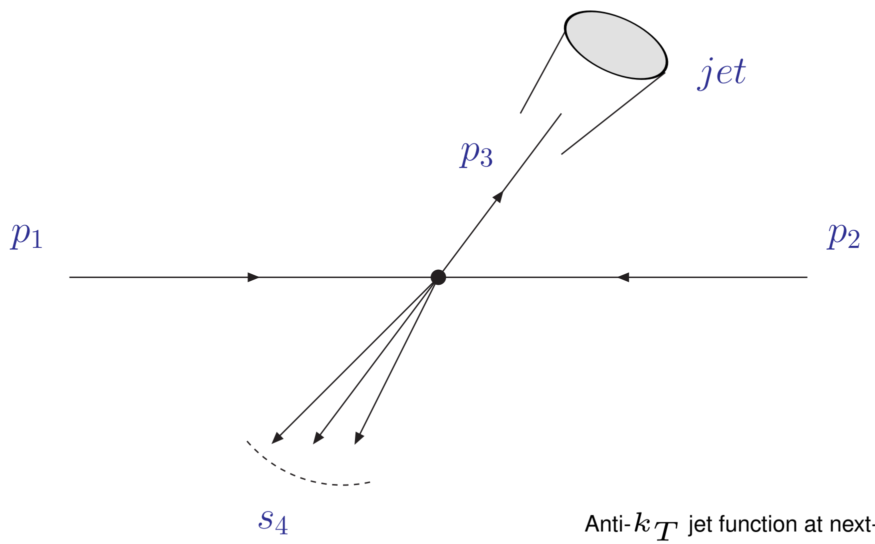
- $\alpha_s(M_Z)$ at NLO in QCD from inclusive jet cross section data
  Britzger, Rabbertz, Savoiu, Sieber '17
- correlations between PDFs and $\alpha_s(M_Z)$ are important

QCD factorization

- Double differential cross section for $pp \to \text{jet} + X$
  - transverse momentum p_T and rapidity η of signal-jet

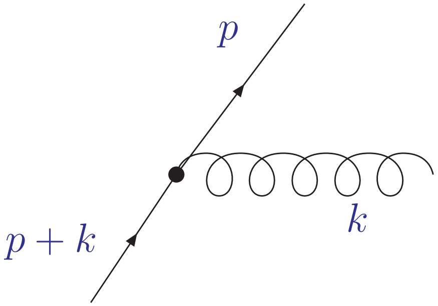
$$\frac{p_T^2 \mathrm{d}^2\sigma}{\mathrm{d}p_T^2 \mathrm{d}\eta} = \sum_{i_1 i_2} \int_0^{V(1-W)} \mathrm{d}z \int_{\frac{VW}{1-z}}^{1-\frac{1-V}{1-z}} \mathrm{d}v \, x_1^2 \, f_{i_1}(x_1) \, x_2^2 \, f_{i_2}(x_2) \, \frac{\mathrm{d}^2\hat{\sigma}_{i_1 i_2}}{\mathrm{d}v \, \mathrm{d}z}(v, z, p_T, R)$$

- PDFs f_i and variables $V = 1 - p_T e^{-\eta}/\sqrt{S}$, $VW = p_T e^{\eta}/\sqrt{S}$
- Partonic cross sections $\hat{\sigma}_{i_1 i_2}$ dependent on partonic kinematic variables on $s = x_1 x_2 S$, $v = u/(u+t)$ and partonic threshold $z = s_4/s \to 0$
- Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2$ with kinematics constraint $s + t + u = s_4$



Threshold logarithms

- Soft and collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: $E_g = 0$, soft $\theta_{qg} = 0$ collinear



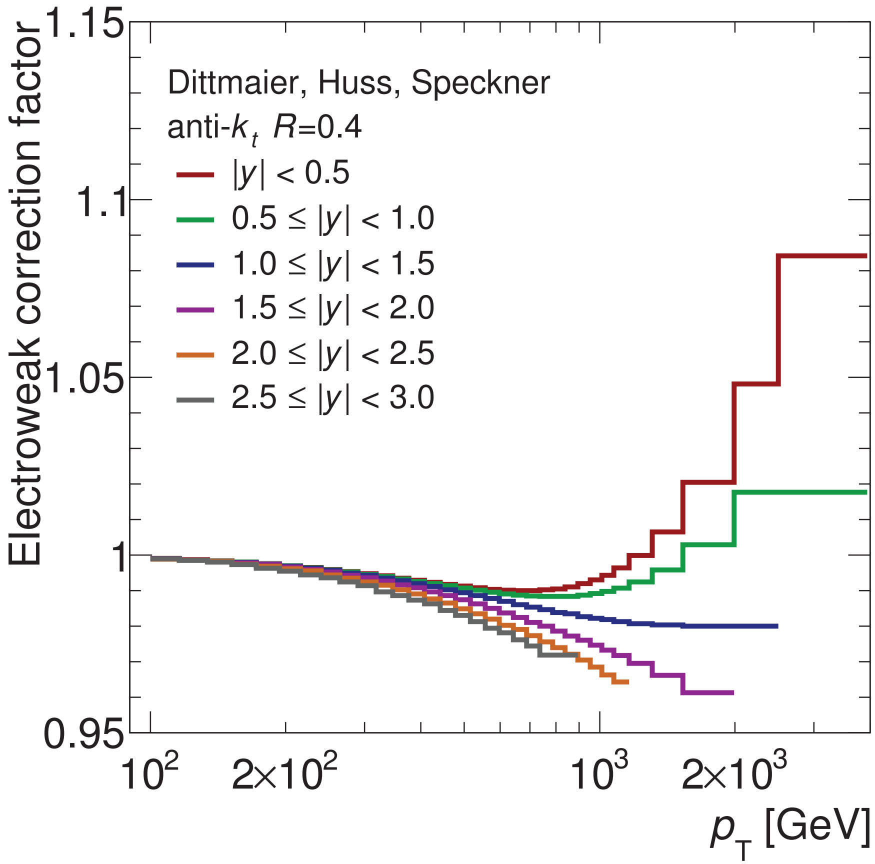
$$\frac{1}{(p+k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g \, d\sin\theta_{qg} \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \ln^2(\dots)$$

- Large double-logarithmic corrections $\ln(\dots) \gg 1$ near threshold
- Single-inclusive jet production with threshold logarithms $\alpha_s^n (\ln^{2n-1}(z)/z)_+$ for $z = s_4/s \to 0$
  - *positive* corrections enhance partonic cross sections $\hat{\sigma}_{i_1 i_2}$
  - long history of resummation Sterman ‘87; Catani, Trentadue ‘88; ...
- Same for weak radiative corrections: positive corrections for large $p_T \gg M_W$ Dittmaier, Huss, Speckner ‘12

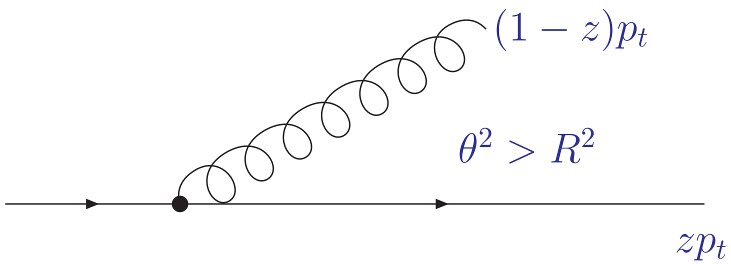
Electroweak corrections at NLO



- Electroweak correction factors for inclusive jet cross-section as function of jet p_T for all $|y|$ bins Dittmaier, Huss, Speckner '12
  - negative corrections $\mathcal{O}(1-3\%)$ for p_T in range $\mathcal{O}(\text{few } 100)$ GeV
  - sizeable (positive) effect $\mathcal{O}(10\%)$ at large $p_T \gg M_W$

Jet radius logarithms (I)

- Collinear singularity when the jet becomes very narrow
  - partons radiated outside of jet (not recombined with jet by chosen jet algorithm) become more and more collinear to emitter
- Example
  - loss of transverse momentum for leading jet
  - quasi-collinear branching of quark with transverse momentum p_T



$$\delta p_T = (1-z)p_T - p_T = -zp_T \text{ for } (1-z) > z$$
$$\delta p_T = (1-z)p_T - p_T = -zp_T \text{ for } z > (1-z)$$

- Perturbative radiation loss for average $\langle \delta p_T \rangle_{\mathrm{q}}$

$$\langle \delta p_T \rangle_{\mathrm{q}} = p_T \frac{\alpha_s}{2\pi} \int_{R^2}^{1} \frac{d\theta^2}{\theta^2} \int dz \big(\max[z, 1-z] - 1 \big) P_{qq}(z)$$

Jet radius logarithms (II)

- Leading order result for quark and gluon jets Dasgupta, Magnea, Salam '07

$$\langle \delta p_T \rangle_{\rm q} \;=\; C_F \frac{\alpha_s}{\pi}\, p_T \,\ln(R)\, \left(2\ln 2 - \frac{3}{8}\right) \;=\; 0.43\,\alpha_s\, \ln(R)$$

$$\langle \delta p_T \rangle_{\rm g} \;=\; \frac{\alpha_s}{\pi}\, p_T \,\ln(R)\, \left[C_A \left(2\ln 2 - \frac{43}{96}\right) + T_f\, n_f \frac{7}{48}\right] \;=\; 1.02\,\alpha_s\, \ln(R)$$

- Large single-logarithmic corrections $\ln(1/R) \gg 1$ for small R
  - *negative* corrections decrease partonic cross sections $\hat{\sigma}_{i_1 i_2}$
  - resummation ...

Joint resummation

SCET factorization

- Factorization in small-R and $z \to 0$ threshold limit
  - assume anti-k_t jet algorithm, $z \sim R$, and small finite mass of jet

$$\frac{\mathrm{d}^2\hat{\sigma}_{i_1 i_2}}{\mathrm{d}v\,\mathrm{d}z} = s\int \mathrm{d}s_X\,\mathrm{d}s_c\mathrm{d}s_G\,\delta(zs - s_X - s_G - s_c)$$
$$\times\mathrm{Tr}\left[\mathbf{H}_{i_1 i_2}(v, p_T\,, \mu_h\,, \mu)\,\mathbf{S}_G(s_G\,, \mu_{sG}\,, \mu)\right]\;J_X(s_X\,, \mu_X\,, \mu)$$
$$\times\sum_m \mathrm{Tr}\left[J_m(p_T R\,, \mu_J\,, \mu)\otimes_\Omega S_{c,m}(s_c R\,, \mu_{sc}\,, \mu)\right]$$

- Specific functions for individual kinematic regions
  - hard functions for $2 \to 2$ scattering $\mathbf{H}_{i_1 i_2}$ (known to 2-loops Broggio, Ferroglia, Pecjak, Zhang '14)
  - inclusive jet function $J_X(s_X)$ dependent on invariant mass s_X of the recoiling collimated radiation (known to order α_s^2 Becher, Neubert '06, Becher, Bell '10)
  - global soft function $\mathbf{S}_G$ accounts for wide-angle soft radiation which cannot resolve the small radius R (known to NLO Liu, S.M., Ringer '17)

Joint resummation

SCET factorization

- Factorization in small-R and $z \to 0$ threshold limit
  - assume anti-k_t jet algorithm, $z \sim R$, and small finite mass of jet

$$\frac{\mathrm{d}^2\hat{\sigma}_{i_1 i_2}}{\mathrm{d}v\,\mathrm{d}z} = s \int \mathrm{d}s_X\,\mathrm{d}s_c \mathrm{d}s_G\,\delta(zs - s_X - s_G - s_c)$$
$$\times \mathrm{Tr}\left[\mathbf{H}_{i_1 i_2}(v, p_T\,, \mu_h\,, \mu)\,\mathbf{S}_G(s_G\,, \mu_{sG}\,, \mu)\right]\; J_X(s_X\,, \mu_X\,, \mu)$$
$$\times \sum_m \mathrm{Tr}\left[J_m(p_T R\,, \mu_J\,, \mu) \otimes_\Omega S_{c,m}(s_c R\,, \mu_{sc}\,, \mu)\right]$$

- Specific functions for individual kinematic regions
  - signal-jet function $J(p_T\,R)$ accounts for energetic radiation inside jet Becher, Neubert, Rothen, Shao '15
  - soft-collinear ("coft") function $S_c(s_c R)$ captures soft radiation near jet boundary Becher, Neubert, Rothen, Shao '15, Chien, Hornig, Lee '15
- Sum runs over all collinear splittings and traces taken in color space
- '$\otimes_\Omega$' denotes associated angular integrals Becher, Neubert, Rothen, Shao '15

Phenomenology at NLL

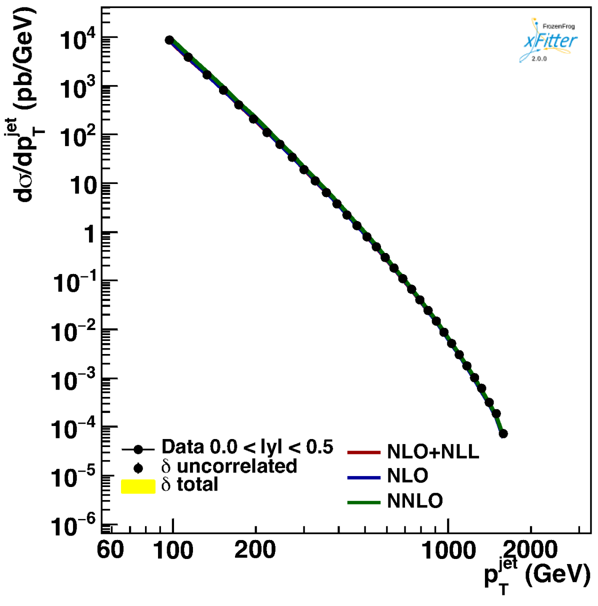
- Evaluation of cross section in SCET (resummation) with renormalization group equations
  - evolution of all functions from their natural scales μ_i to common hard scale $\mu = p_T^{\rm max}$
- Matching of NLL resummed results with full NLO calculation (need to avoid double counting)

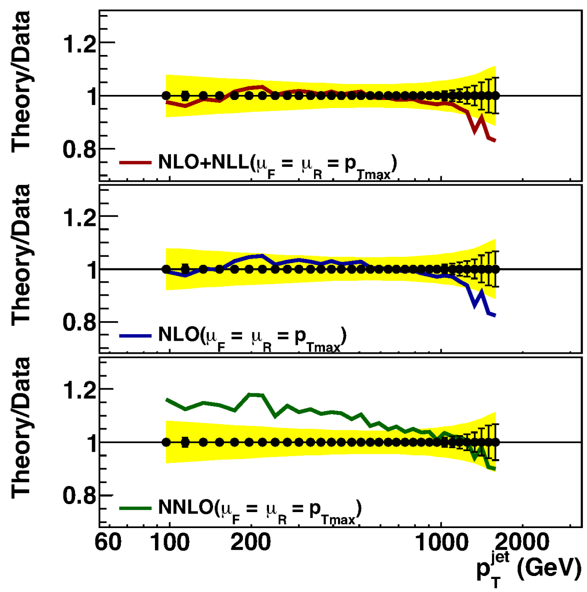
$$d\sigma = d\sigma_{\rm NLL} - d\sigma_{{\rm NLO}_{\rm sin}} + d\sigma_{\rm NLO}$$

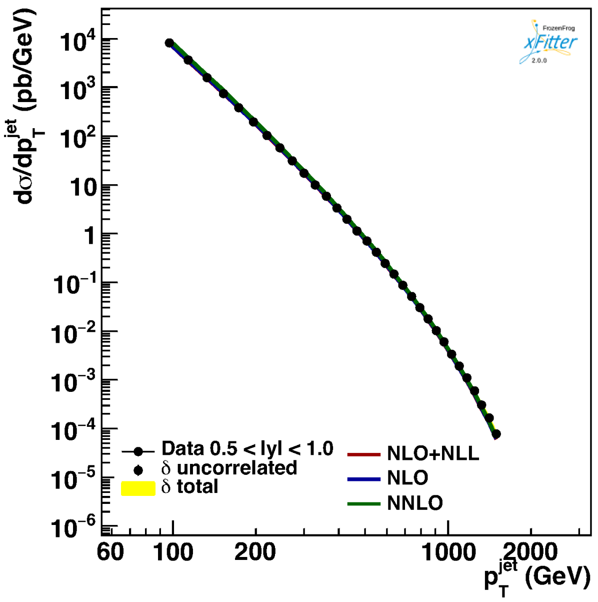
  - resummed cross section $d\sigma_{\rm NLL}$
  - fixed order NLO result in singular limit $d\sigma_{{\rm NLO}_{\rm sin}}$
  - complete fixed order NLO result $d\sigma_{\rm NLO}$

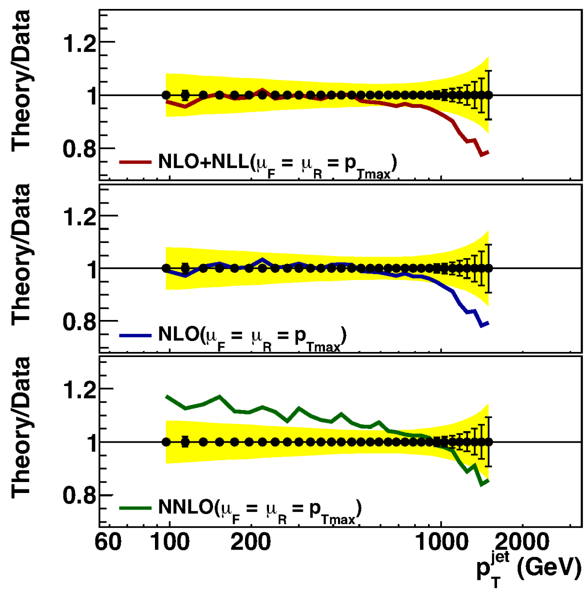
Comparision to data (I)

- Single-inclusive jet production $pp \to \text{jet} + \text{X}$
- Cross section as function of jet p_T at $\sqrt{S} = 8$ TeV
  CMS 1609.05331 [hep-ex]
- CT14 NNLO PDFs
- Jet radius $R = 0.7$, scale choice $\mu_R = \mu_F = p_T^{\max}$
- QCD theory predictions
  S.M., Eren, Lipka, Liu, Ringer '18
  - NLO (blue)
  - NNLO (green)
    Currie, Glover, Pires '16
  - NLO + NLL (red)
- Rapidity bins
  $0.0 \leq |y| < 0.5$ and
  $0.5 \leq |y| < 1.0$









Comparision to data (II)

- Single-inclusive jet production $pp \to \text{jet} + X$
- Cross section as function of jet p_T at $\sqrt{S} = 8$ TeV
  CMS 1609.05331 [hep-ex]
- CT14 NNLO PDFs
- Jet radius $R = 0.7$, scale choice $\mu_R = \mu_F = p_T^{\max}$
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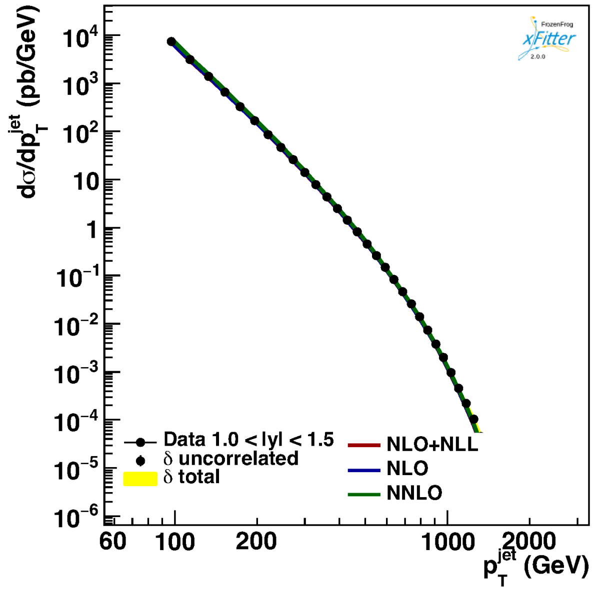
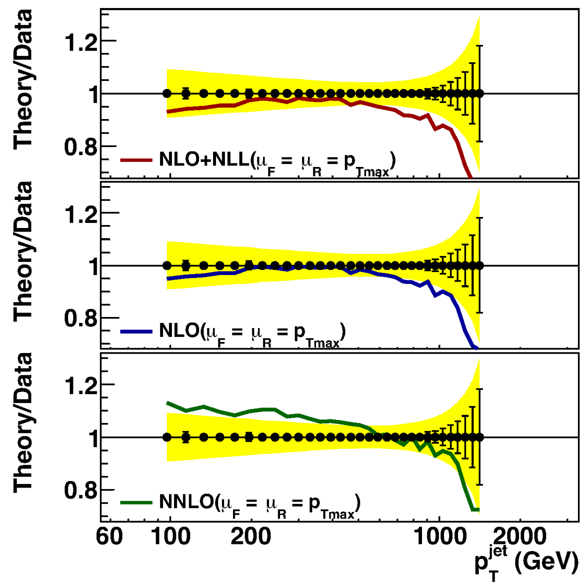
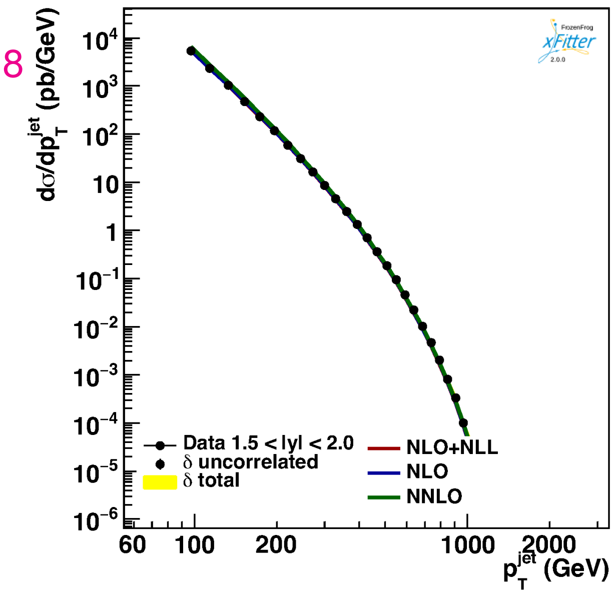
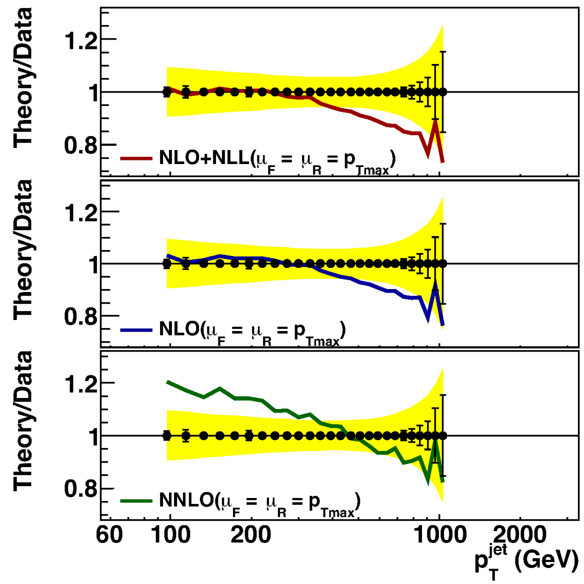
  - NLO (blue)
  - NNLO (green)
    Currie, Glover, Pires '16
  - NLO + NLL (red)
- Rapidity bins
  $1.0 \leq |y| < 1.5$ and
  $1.5 \leq |y| < 2.0$



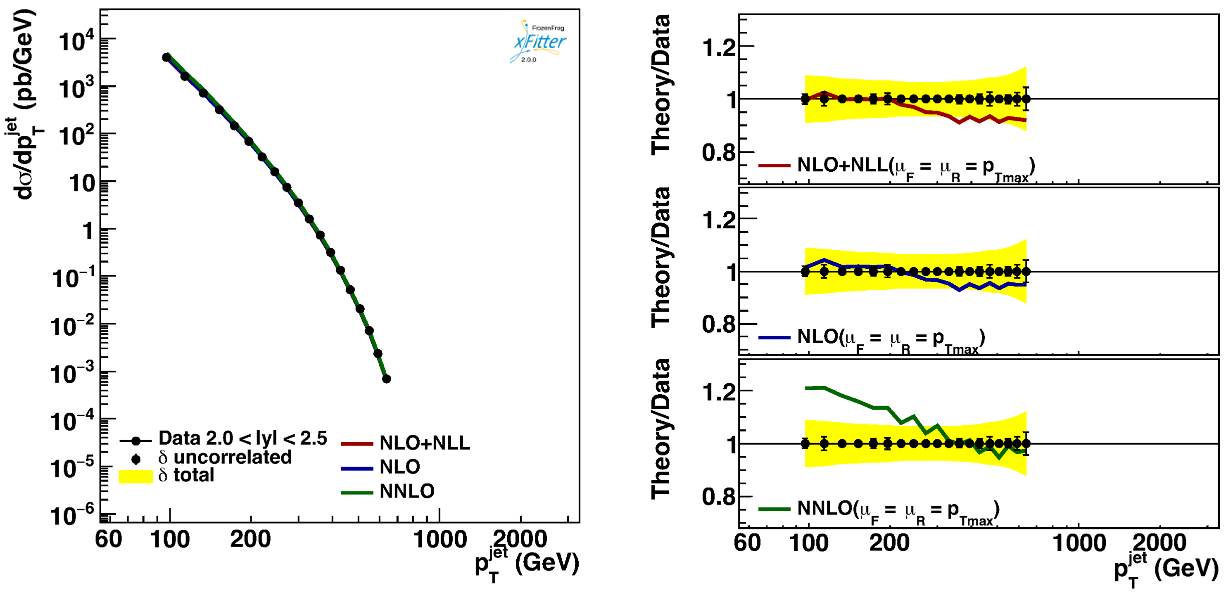


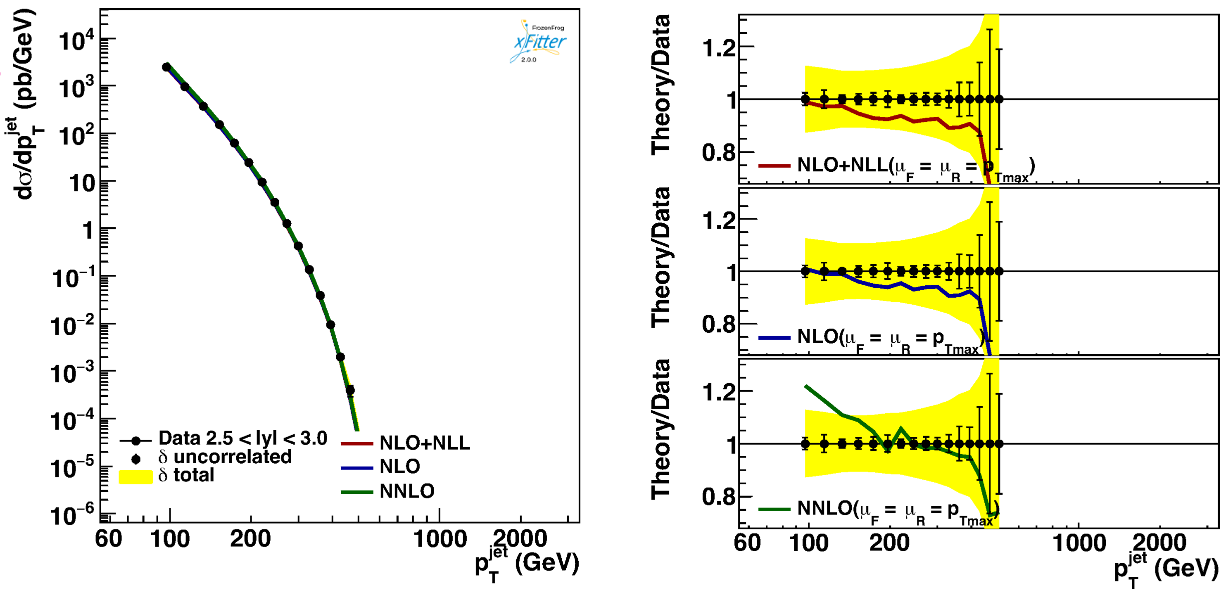




Comparision to data (III)

- Single-inclusive jet production $pp \to \text{jet} + \text{X}$
- Cross section as function of jet p_T at $\sqrt{S} = 8$ TeV
  CMS 1609.05331 [hep-ex]
- CT14 NNLO PDFs
- Jet radius $R = 0.7$, scale choice $\mu_R = \mu_F = p_T^{\max}$
- QCD theory predictions
  S.M., Eren, Lipka, Liu, Ringer '18
  - NLO (blue)
  - NNLO (green)
    Currie, Glover, Pires '16
  - NLO + NLL (red)
- Rapidity bins
  $2.0 \leq |y| < 2.5$ and
  $2.5 \leq |y| < 3.0$





Joint resummation beyond NLL (I)

Jet function

- Jet function containing m partons Becher, Neubert, Rothen, Shao '16

$$J_m^c \quad = \quad \int_{\sum p_i^{+,\perp}=p_J^{+,\perp}} \prod_{i=1}^{m} \frac{dE_i E_i^{d-3}}{2(2\pi)^{d-1}} \, \Theta_{\text{jet alg.}} \sum_{\text{spins}} |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$$

- Amplitude $|\mathcal{M}_m\rangle$ for collinear field
  - collinear momentum $p_J^\mu \approx \frac{p_J^+}{2} n_J^\mu$ to split into m particles with momenta $\{p_1 \, , \ldots \, , p_m\}$
  - light-like vector n_J along jet axis
- Condition $\Theta_{\text{jet alg.}}$ clusters all m collinear particles in one jet

Joint resummation beyond NLL (II)

SCET factorization simplified

- Angular integrals associated to signal jet function $\mathrm{Tr}\,[J_m(p_T R\,,\mu_J\,,\mu) \otimes_\Omega S_{c,m}(s_c R\,,\mu_{sc}\,,\mu)]$ are involved
- Simplified factorization used for predictions of jet measurements
  - decoupling of angular correlation between J_m and $S_{cs,m}$

$$d\sigma = \mathcal{F}_a \mathcal{F}_b \mathrm{Tr}[H S_G] \prod_c^N J^c\, S^c_{cs}\, e^{L_\mathrm{ngl}}$$

- Average of each function over its solid angle
  - $J = \sum_m \langle J_m \rangle_\Omega$ and $S_{cs} = \langle S_{cs,1} \rangle_\Omega$
- Consequence: non-global logarithms (NGLs) appear (factor e^{L_ngl})
  Dasgupta, Salam '01, Schwartz, Zhu '14, Caron-Huot '15 Larkoski, Moult, Neill '16, . . .

Task

- Direct NNLO calculation of $J = \sum_m \langle J_m \rangle_\Omega$
  - choose anti-k_T quark-jet function with E-scheme recombination

Jet function at NLO (I)

- NLO quark-jet function from integrated matrix element for $q_a \to q_i g_j$ over collinear two-body phase space Liu, Petriello '12

$$J^{(1)}_{bare} = \frac{1}{4} \frac{1}{(2\pi)^{d-1}} \frac{2\pi^{1-\epsilon}}{\Gamma(1-\epsilon)} \int \mathrm{d}z \mathrm{d}s_{ij}\, s_{ij}^{-\epsilon} (z(1-z))^{1-\epsilon}$$
$$\times \frac{8\pi\alpha_s Z_\alpha\, \mu^{2\epsilon} e^{\gamma_E \epsilon}}{(4\pi)^\epsilon s_{ij}} C_F \left[\frac{1+(1-z)^2}{z} - \epsilon z\right] \theta(R^2 - \Delta R_{ij}^2)$$

- Kinematics
  - $s_{ij} = 2p_i \cdot p_j$ with momenta p_i, p_j for partons i, j
  - z momentum fraction carried by gluon g_j
- $\theta(R^2 - \Delta R_{ij}^2)$ clusters two partons into one narrow jet with radius $R \ll 1$

$$\Delta R_{ij}^2 = \Delta\eta_{ij}^2 + \Delta\phi_{ij}^2 \approx \frac{2p_i \cdot p_j}{p_{i,T}\, p_{j,T}} = \frac{s_{ij}}{z(1-z)p_T^2}$$

  - $\Delta\eta_{ij}$ for rapidity difference between partons i, j
  - $\Delta\phi_{ij}$ for azimuthal angle difference between partons i, j
  - p_T jet transverse momentum

Jet function at NLO (II)

- Phase space integration with variables

$$x_1 \equiv \tilde{s}_{ij} = \frac{s_{ij}}{z\bar{z}(p_T R)^2} \leq 1\,, \quad x_2 \equiv z \leq 1\,,$$

- Bare quark-jet function at NLO with $L = \log \frac{\mu}{p_T R}$

$$J^{(1)}_{bare} = e^{2\epsilon L} \frac{\alpha_s C_F}{2\pi} \frac{Z_\alpha\, e^{\gamma_E \epsilon}}{\Gamma(1-\epsilon)} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 x_1^{-1-\epsilon} x_2^{-1-2\epsilon}$$
$$\times (1-x_2)^{-2\epsilon} \left[1 + (1-x_2)^2 - \epsilon\, x_2^2\right]$$

- Integration over angular variable $x_1 = \tilde{s}_{ij}$ provides

$$J^{(1)}_{bare} = e^{2\epsilon L} Z_\alpha \frac{\alpha_s}{2\pi} C_F \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{13}{2} - \frac{3\pi^2}{4} + \mathcal{O}(\epsilon) \right)$$

  - all k_T-type jet algorithms give same result at NLO

Jet function at NNLO (I)

Real-virtual contribution

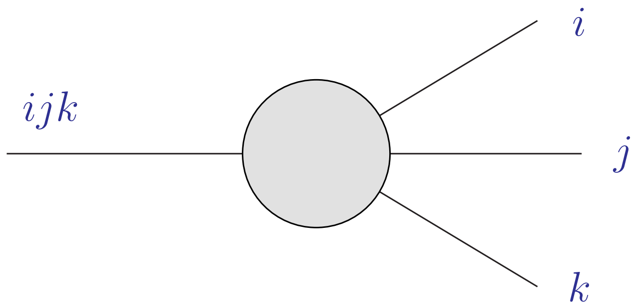
- Matrix element for the jet function is given by the one-loop correction to the splitting kernel $q_a \to q_i g_j$ over collinear two-body phase space
  Kosower '99, Kosower, Uwer '99, Bern, Del Duca, Kilgore, Schmidt '99, Sborlini, de Florian, Rodrigo '13
- Phase space for real-virtual corrections identical to NLO

$$
\begin{aligned}
J_{rv}^{(2)} \;=\; & \frac{\alpha_s^2}{(2\pi)^2}\, e^{4\epsilon L} \Bigg\{ C_F^2 \Bigg(\left(-\frac{5}{4} + \frac{\pi^2}{3} \right) \frac{1}{\epsilon^2} + \left(-\frac{31}{2} + \frac{\pi^2}{2} + 22\zeta_3 \right) \frac{1}{\epsilon} \\
& -\frac{575}{4} + \frac{137}{24}\pi^2 + 33\zeta_3 + \frac{10}{9}\pi^4 \Bigg) \\
& + C_F C_A \Bigg(-\frac{1}{4\epsilon^4} - \frac{3}{4\epsilon^3} + \left(-5 + \frac{11\pi^2}{24} \right) \frac{1}{\epsilon^2} \\
& + \left(-\frac{63}{2} + \frac{13\pi^2}{8} + \frac{26}{3}\zeta_3 \right) \frac{1}{\epsilon} - \frac{781}{4} + 11\pi^2 + \frac{85}{2}\zeta_3 - \frac{67}{1440}\pi^4 \Bigg) \Bigg\}
\end{aligned}
$$

Jet function at NNLO (II)

Real-real contribution

- Matrix element is given by the tree-level splitting kernel $q_a \to ijk$
  - for quark jet function $P_{\bar{q}'_1 q'_2 q_3}\,,\, P^{(\mathrm{id})}_{\bar{q}_1 q_2 q_3}\,,\, P^{(\mathrm{ab})}_{g_1 g_2 q_3}\,,\, P^{(\mathrm{nab})}_{g_1 g_2 q_3}$
- Integration of triple collinear kernel with anti-k_T clustering condition on partons ijk



Task

- Integration of kernel over collinear phase space $d\Phi_3^{\mathrm{c}}$

$$d\Phi_3^{\mathrm{c}} = ds_{123}\, ds_{12}\, ds_{13}\, ds_{23}\, \delta(s_{123} - s_{12} - s_{13} - s_{23})$$

$$\times dz_1\, dz_2\, dz_3\, \delta(1 - z_1 - z_2 - z_3) \times \frac{4\Theta(-\Delta)(-\Delta)^{-\frac{1}{2}-\epsilon}}{(4\pi)^{5-2\epsilon}\Gamma(1-2\epsilon)}$$

  - Gram determinant $\Delta = (z_3 s_{12} - z_1 s_{23} - z_2 s_{13})^2 - 4 z_1 z_2 s_{13} s_{23}$

Jet function at NNLO (III)

- Jet algorithm requires comparison of metrics

$$\rho_{ij} = \min\left[p_{T,i}^{-2\alpha}, p_{T,j}^{-2\alpha}\right] \frac{\Delta R_{ij}^2}{R^2}\,, \qquad \rho_i = p_{T,i}^{-2\alpha}$$

  - parameter $\alpha = 1$ defines anti-k_T algorithm
- Clustering of all partons in E-scheme recombination introduces ordering among integration variables s_{ij} and z_i
  - e.g. first $i + j \to \widetilde{ij}$, then $\widetilde{ij} + k \to \widetilde{ijk}$
- Account of all possible permutations necessary

Upshot

- Clustering conditions lead to constraints in phase space integrals
  - resolved by sector decomposition
  - numerical integration of phase space

Result (I)

- Bare quark-jet function at NNLO

$$
\begin{aligned}
J^{(2)}_{bare} \;=\; & \frac{\alpha_s^2}{(2\pi)^2}\, e^{4\epsilon L} \Bigg\{ C_F^2 \left(\frac{1}{2\epsilon^4} + \frac{3}{2\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{61}{8} - \frac{3\pi^2}{4}\right) - \frac{5.019(2)}{\epsilon} - 12.60(1) \right) \\
& + C_F C_A \left(\frac{11}{24\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{83}{36} - \frac{\pi^2}{8}\right) - \frac{12.348(2)}{\epsilon} - 103.77(1) \right) \\
& + C_F n_f T_f \left(-\frac{1}{6\epsilon^3} - \frac{7}{9\epsilon^2} + \frac{0.1067(3)}{\epsilon} + 16.688(5) \right) \Bigg\}
\end{aligned}
$$

- Poles up to $\mathcal{O}(\epsilon^{-2})$ can be checked with predictions from renormalization group equation of jet function
- Pole ϵ^{-2} in $C_F C_A$ term needs additional correction $-\frac{\pi^2}{12\epsilon^2}$ compared to renormalization group predictions $\longrightarrow$ non-global logarithm
  - same correction encountered in Sterman-Weinberg cone jet hemisphere mass distribution

Schwartz, Zhu '14, Becher, Neubert, Rothen, Shao '15

Result (II)

- Jet function J satisfies the renormalization group equation (up to non-global contributions)

$$\mu \frac{\mathrm{d}J}{\mathrm{d}\mu} = \left(2\Gamma[\alpha_s]L + \gamma_{J_q}\right) J$$

- Infrared structure of jet function similar to QCD form factor
  - cusp anomalous dimension Γ
  - anomalous dimension of quark-jet γ_{J_q} now known to two-loops

$$\begin{aligned} \gamma_0 &= 6C_F \\ \gamma_1 &= 17.14(3)\, C_F^2 - 171.10(3)\, C_F C_A - 7.916(5)\, C_F n_f T_f \end{aligned}$$

- Renormalized quark-jet function to $\mathcal{O}(\alpha_s^2)$

$$\begin{aligned} J &= 1 + \frac{\alpha_s}{2\pi} C_F \left(\frac{13}{2} - \frac{3\pi^2}{4}\right) \\ &\quad + \frac{\alpha_s^2}{(2\pi)^2} \left(1.55(1) C_F^2 - 95.08(1) C_A C_F + 13.530(5) C_F n_f T_f\right) \end{aligned}$$

  - boundary condition for resummation via renormalization group equation

Summary

- Joint resummation of threshold logarithms $\alpha_s^n (\ln^{2n-1}(z)/z)_+$ and small jet-radius logarithms $\alpha_s^n \ln^n(R)$
- Resummation at NLO $+$ NLL accuracy in good agreement with CMS data
- Large residual theoretical uncertainties from scale variation require resummed theoretical predictions at NNLO $+$ NNLL accuracy
- Computation of anti-k_t quark jet function at NNLO is first step towards NNLL resummation