

Vector Boson Scattering at a Multi-TeV Muon Collider

ParticleFace21 July Meeting
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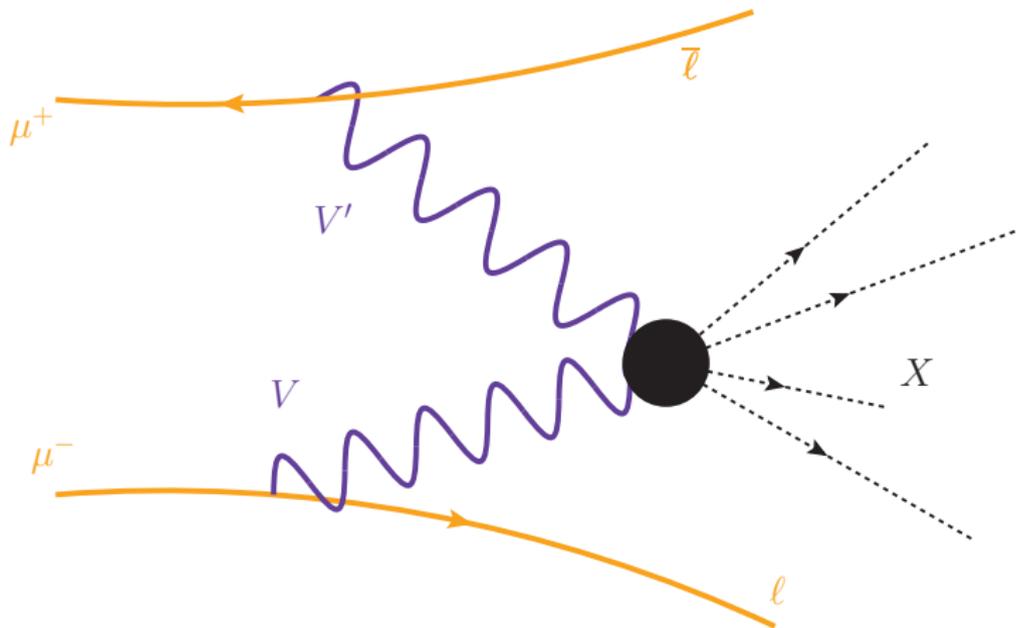


¹w/ A. Costantini, et. al., JHEP('20) [arXiv:2005.10289] + to appear

We made it!

Hardships continue but the outlook is encouraging

the big question: why a $\mu^+\mu^-$ collider?



Many motivations for a muon collider

- **Generically**, discovering laws of nature requires larger data sets and higher energies

See e.g., Al Ali, et al. [2103.14043]

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 - ▶ context: present flavor anomalies are arguably “ μ -flavor” anomalies

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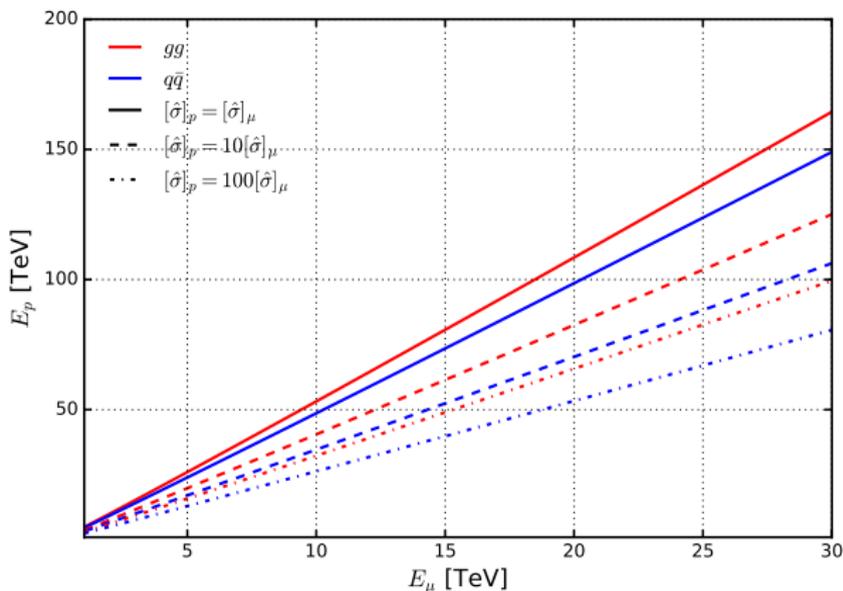
- **Excitingly**, *partonic* collisions at $Q \sim \mathcal{O}(10)$ TeV explore when electroweak (EW) symmetry is nearly restored, i.e., $M_{W/Z/H}^2/Q^2 \rightarrow 0$

Recent studies include Chen, et al [1611.00788] and Han, et al [2007.14300]

Like e^+e^- machines, $\mu^+\mu^-$ machines collide elementary particles

- up to rad. corrections, $\mu\mu$ collisions carry full energy $\implies \sqrt{\hat{s}} = \sqrt{s}$
- pp colliders, e.g., LHC and FCC-hh, need larger \sqrt{s} for same $\sqrt{\hat{s}}$

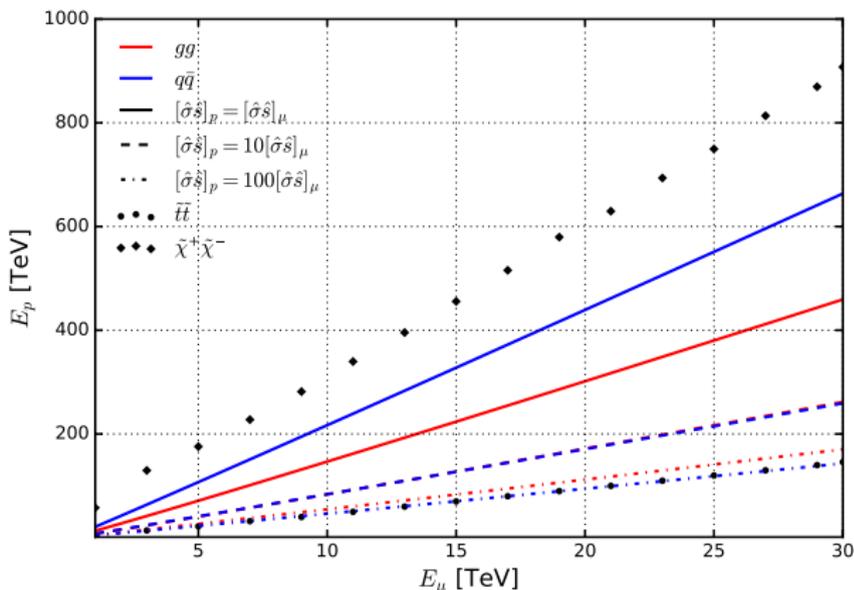
Plot: $\sqrt{s_{pp}}$ needed for $\hat{\sigma}_{pp} = \hat{\sigma}_{\mu\mu}$ in $2 \rightarrow 1$ processes at $\sqrt{s_{\mu\mu}}$



2 → 1 processes are only small subset of possibilities (and also special!)

• 2 → 2 process help give bigger picture (and variability!)

Plot: $\sqrt{s_{pp}}$ needed for $\hat{s}_{pp}\hat{\sigma}_{pp} = \hat{s}_{\mu\mu}\hat{\sigma}_{\mu\mu}$ in 2 → 2 processes at $\sqrt{s_{\mu\mu}}$



• Assumed that $2M = 0.9\sqrt{s_{\mu\mu}}$

• PDF and phase space impact 2 → 2 more than the 2 → 1 suggests

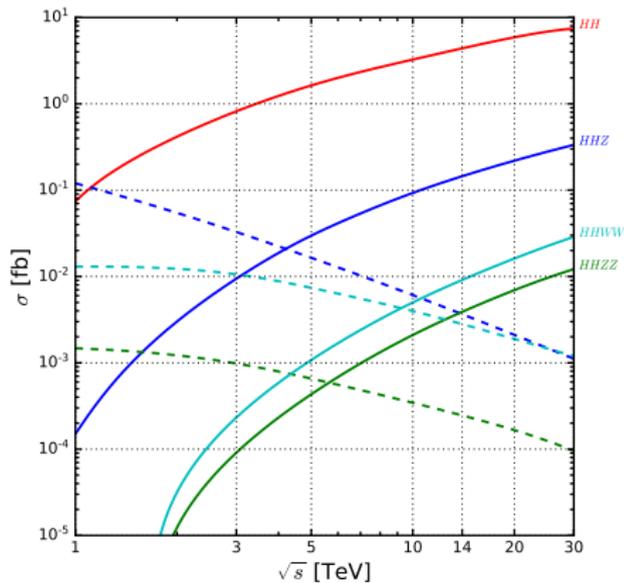
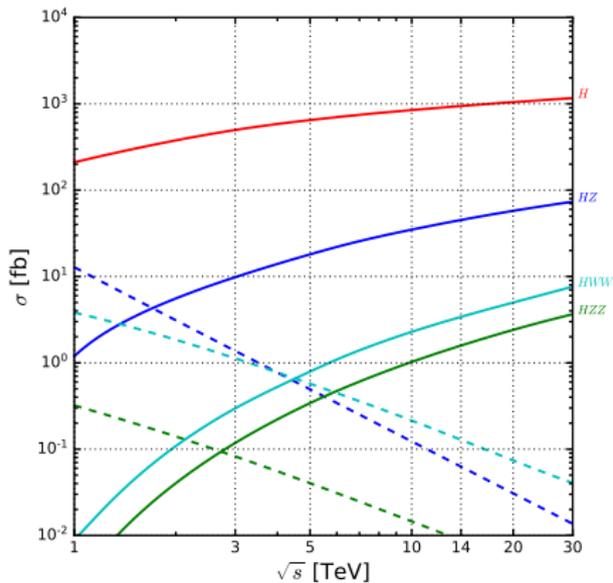
Running the numbers for a $\mu^+\mu^-$ scattering²³

²Out-of-the-box MadGraph5_aMC@NLO, except we upgraded the box to better handle (throw more die) phase space integration over t -channel momentum exchange

³For vector boson fusion/scattering (VBF/S) processes, we selected for VBF/VBS diagrams in a gauge-invariant manner

Higgs production

cross sections (σ) vs \sqrt{s} for
s-channel annihilation (dash) vs VBF (solid)

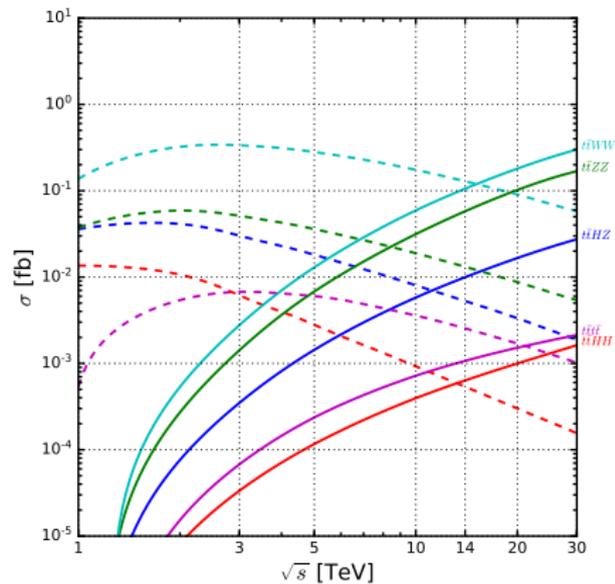
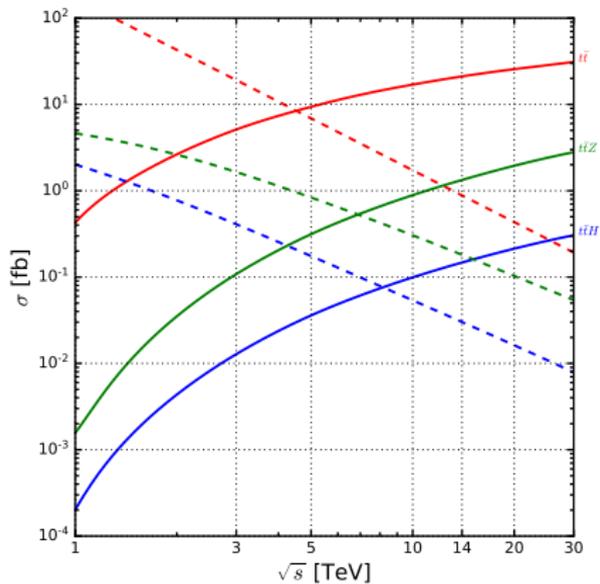


• $\sigma^{VBF} > \sigma^{s\text{-channel}}$ since

▶ $\sigma^{s\text{-channel}} \sim 1/s$

▶ $\sigma^{VBF} \sim \log^2(M_{VV}^2/M_V^2)/M_{VV}^2$ due to forward emission of $V = W/Z$

Top production

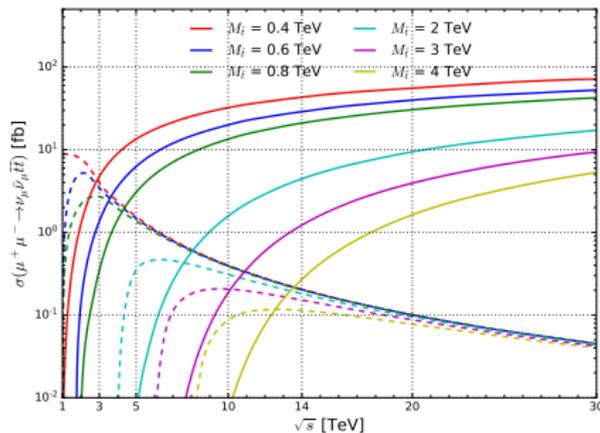
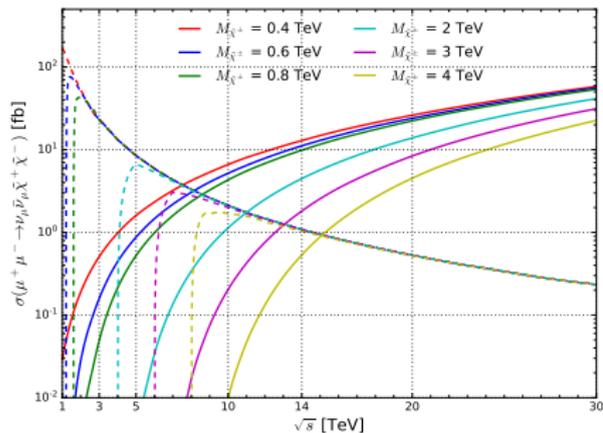


- Do you notice a pattern?

SUSY

(L) chargino pairs

(R) stop pairs

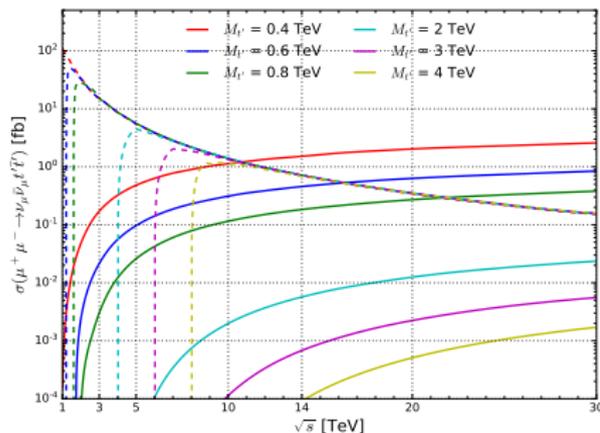
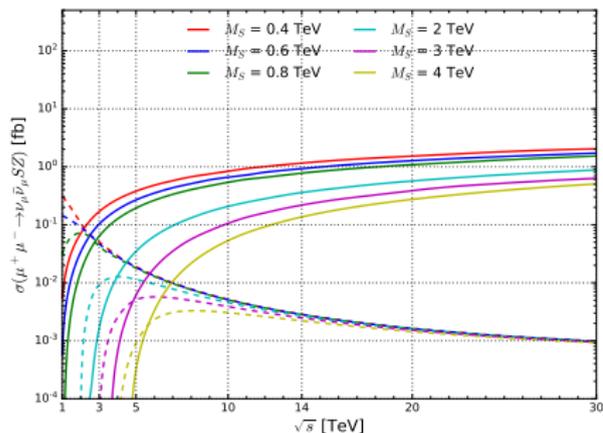


• And now?

Simple Extensions

(L) Singlet + Z production

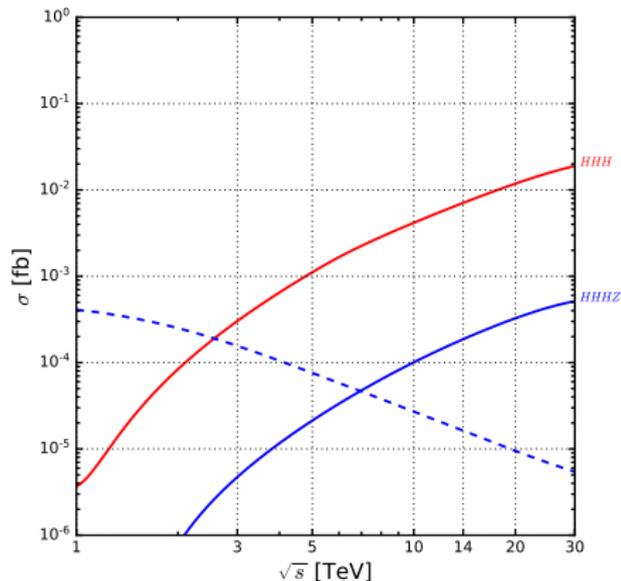
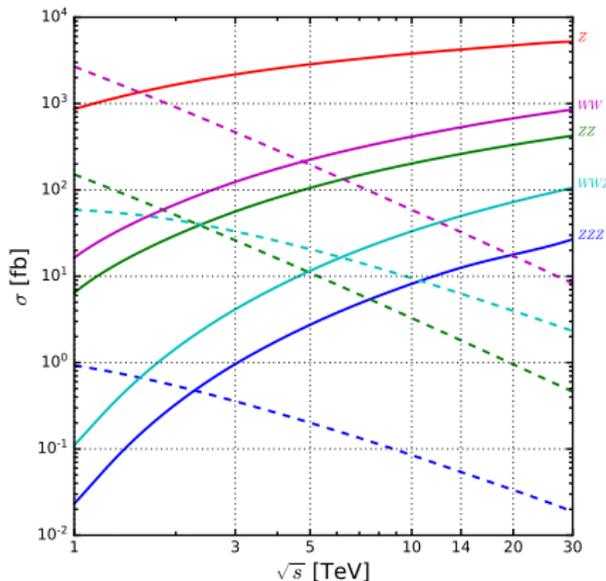
(R) vector-like top pair production



• ... a little different but a lot of the same

Many-boson production⁴

⁴ My favorite! I find these processes really neat!



- Eventually, **VBF becomes the dominant** production vehicle of many types of processes

When annihilation and VBS channels are driven by same physics, evidence that **dominance of VBS is universal** and occurs at \sqrt{s} for

w/ A. Costantini, et al [2005.10289]

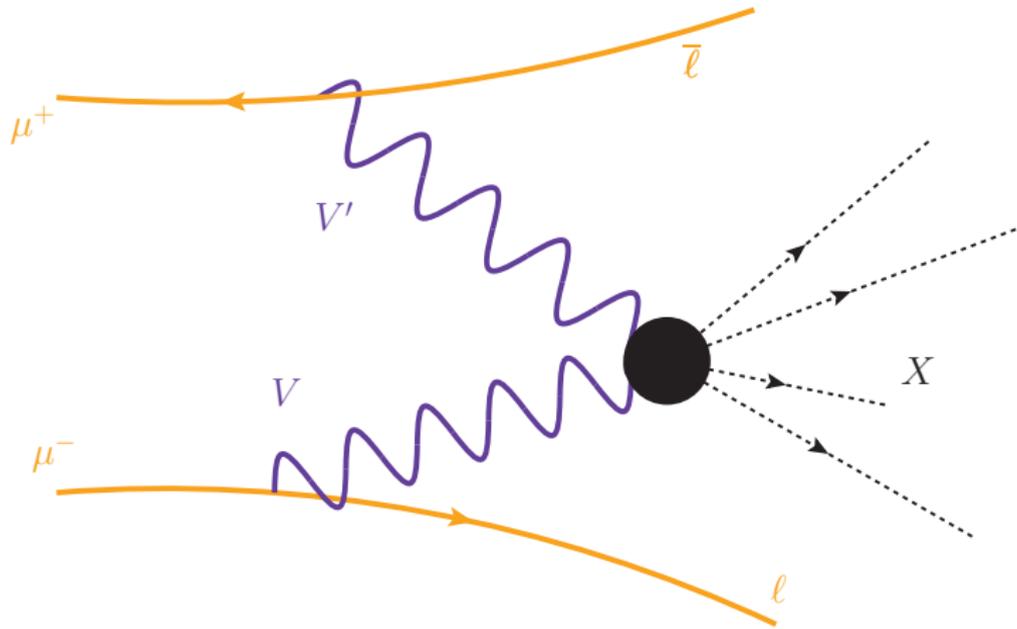
$$\frac{\sigma^{\text{VBF}}}{\sigma^{s\text{-}ch.}} \sim \mathcal{S} \left(\frac{g_W^2}{4\pi} \right)^2 \left(\frac{s}{M_X^2} \right) \log^2 \frac{s}{M_V^2} \log \frac{s}{M_X^2} > 1$$

Scaling estimate not so bad if $M_X \gg M_V$. Difference is about $\mathcal{O}(10\%)$

mass (M_X) [TeV]	SZ (Singlet)	H_2Z (2HDM)	$t'\bar{t}$ (VLQ)	$t\bar{t}$ (MSSM)	$\tilde{\chi}^0\tilde{\chi}^0$ (MSSM)	$\tilde{\chi}^+\tilde{\chi}^-$ (MSSM)	Scaling (Eq. 7.7)
400 GeV	2.1 TeV	2.1 TeV	11 TeV	2.9 TeV	3.2 TeV	7.5 TeV	1.0 (1.7) TeV
600 GeV	2.5 TeV	2.5 TeV	16 TeV	3.8 TeV	3.8 TeV	8.1 TeV	1.3 (2.4) TeV
800 GeV	2.8 TeV	2.8 TeV	22 TeV	4.3 TeV	4.3 TeV	8.5 TeV	1.7 (3.1) TeV
2.0 TeV	4.0 TeV	4.0 TeV	>30 TeV	7.8 TeV	6.9 TeV	11 TeV	3.7 (6.8) TeV
3.0 TeV	4.8 TeV	4.8 TeV	>30 TeV	10 TeV	9.0 TeV	13 TeV	5.3 (9.8) TeV
4.0 TeV	5.5 TeV	5.5 TeV	>30 TeV	13 TeV	11 TeV	15 TeV	6.8 (13) TeV

Table 9. For representative processes and inputs, the required muon collider energy \sqrt{s} [TeV] at which the VBF production cross section surpasses the s -channel, annihilation cross section, as shown in figure 17. Also shown are the cross over energies as estimated from the scaling relationship in equation (7.7) assuming a mass scale M_X ($2M_X$).

Question: For large enough \sqrt{s} , a $\mu^+\mu^-$ collider is effectively an “*EW boson collider*.” When do EW bosons become partons?



The Effective W Approximation⁵ (The Effective Vector Boson Approximation)

⁵Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

A brief interlude

At very high scales $Q \gg M_W, M_Z$, EW bosons can be treated as partons

a.k.a. the Effective W Approximation [Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)]

- Treatment of V_T identical to **gluons in QCD**; V_0 is novel complication
- $W/Z/\gamma/\nu_\ell$ PDFs for e^\pm, μ^\pm will be released soon in MadGraph5

https://code.launchpad.net/~maddevelopers/mg5amcnlo/2.9.x_eva_v2

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Implementation of EVA has been a multi-step process:

- **Automation** of matrix elements and cross sections for external partons with fixed helicity polarizations

D. Buarque Franzosi, O. Mattelaer, RR, S. Shil [1912.01725]

- ▶ Essentially enable $A_\lambda + B_{\lambda_B} \rightarrow C_{\lambda_C} + D_{\lambda_D} + \dots$ (λ_k =helicity)
- ▶ Theoretically easy (after reorganizing Collinear Fact. Thm and defining polarized PDF/parton shower)
- ▶ Dev. tricky since Lorentz invariance is lost (a ref. frame must be specified)

- **Development** of dPS integration routine (sde2) for t -channel mom.

K. Ostrolenk and O. Mattelaer [2102.00773]

W/Z/ γ / ν_ℓ PDFs for e^\pm , μ^\pm will be released soon in MadGraph5

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- Including **arbitrary** μ_f , standard μ_f variation, evo. by p_T and q

$$f_{V_+/f_L}(z, \mu_f^2) = \frac{g_V^2}{4\pi^2} \frac{g_L^2(1-z)^2}{2z} \log \left[\frac{\mu_f^2}{M_V^2} \right],$$

$$f_{V_-/f_L}(z, \mu_f^2) = \frac{g_V^2}{4\pi^2} \frac{g_L^2}{2z} \log \left[\frac{\mu_f^2}{M_V^2} \right],$$

$$f_{V_0/f_L}(z, \mu_f^2) = \frac{g_V^2}{4\pi^2} \frac{g_L^2(1-z)}{z},$$

$$f_{V_+/f_R}(z, \mu_f^2) = \left(\frac{g_R}{g_L} \right)^2 \times f_{V_-/f_L}(z, \mu_f^2)$$

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$$f_{V_0/f_R}(z, \mu_f^2) = \left(\frac{g_R}{g_L} \right)^2 \times f_{V_0/f_L}(z, \mu_f^2)$$

```
59 c /* *****  
60 c EVA (1/6) for f_L > v -  
61 double precision function eva_fl_to_vp(gg2,gL2,mv2,x,mu2,ievo)  
62 implicit none  
63 integer ievo ! evolution by q2 or pT2  
64 double precision gg2,gL2,mv2,x,mu2  
65 double precision coup2,split,xxlog,fourPi5q  
66 data fourPi5q/39.47841760435743d0/ ! = 4pi**2  
67  
68 c print*, 'gg2,gL2,mv2,x,mu2,ievo', gg2 !3,gL2,mv2,x,mu2,ievo  
69 coup2 = gg2*gL2/fourPi5q  
70 split = (1.00-x)**2 / 2.d0 / x  
71 if(ievo.eq.0) then  
72 | xxlog = dlog(mu2/mv2)  
73 | else  
74 | xxlog = dlog(mu2/mv2/(1.00-x))  
75 | endif  
76  
77 eva_fl_to_vp = coup2*split*xxlog  
78 return  
79 end  
80 c /* *****  
81 c EVA (2/6) for f_L > v -  
82 double precision function eva_fl_to_vm(gg2,gL2,mv2,x,mu2,ievo)  
83 implicit none  
84 integer ievo ! evolution by q2 or pT2  
85 double precision gg2,gL2,mv2,x,mu2  
86 double precision coup2,split,xxlog,fourPi5q  
87 data fourPi5q/39.47841760435743d0/ ! = 4pi**2  
88  
89 coup2 = gg2*gL2/fourPi5q  
90 split = 1.d0 / 2.d0 / x  
91 if(ievo.eq.0) then  
92 | xxlog = dlog(mu2/mv2)  
93 | else  
94 | xxlog = dlog(mu2/mv2/(1.00-x))  
95 | endif  
96  
97 eva_fl_to_vm = coup2*split*xxlog  
98 return  
99 end
```

some results on $V_\lambda V'_{\lambda'} \rightarrow X$ in $\mu^+ \mu^-$ collisions

PRELIMINARY⁶

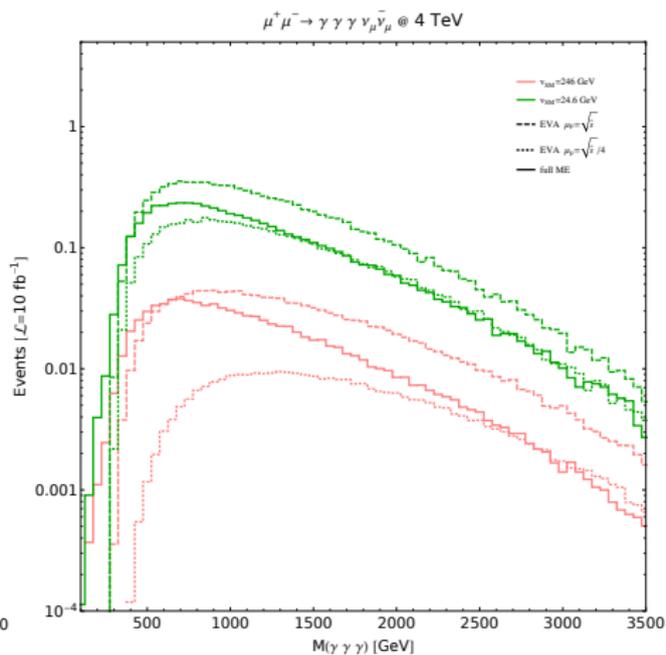
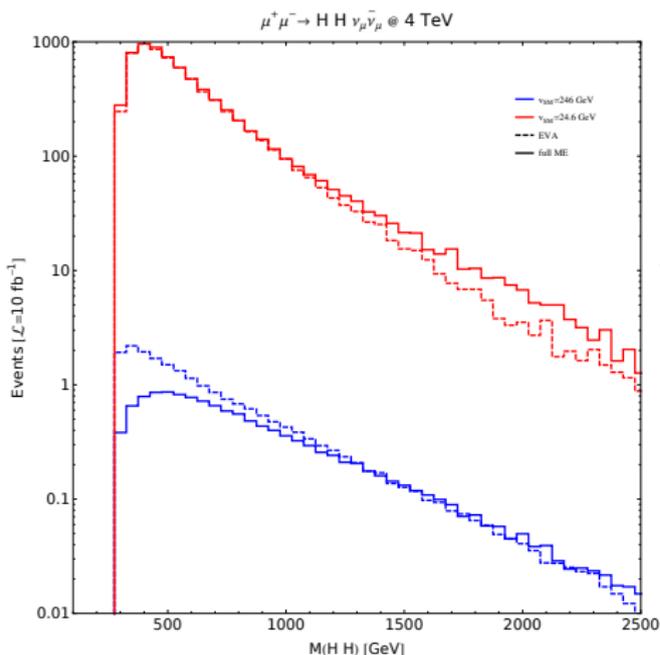
⁶ w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer [2108.?????]

the money plot

PRELIMINARY

Plot: M_{WW} for (L) $W_0 W_0 \rightarrow HH$ (R) $W_T W_T \rightarrow \gamma\gamma\gamma$

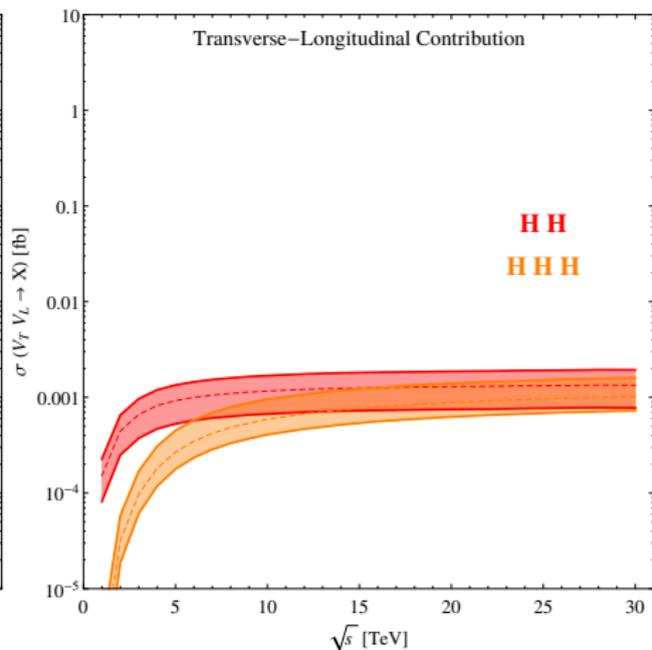
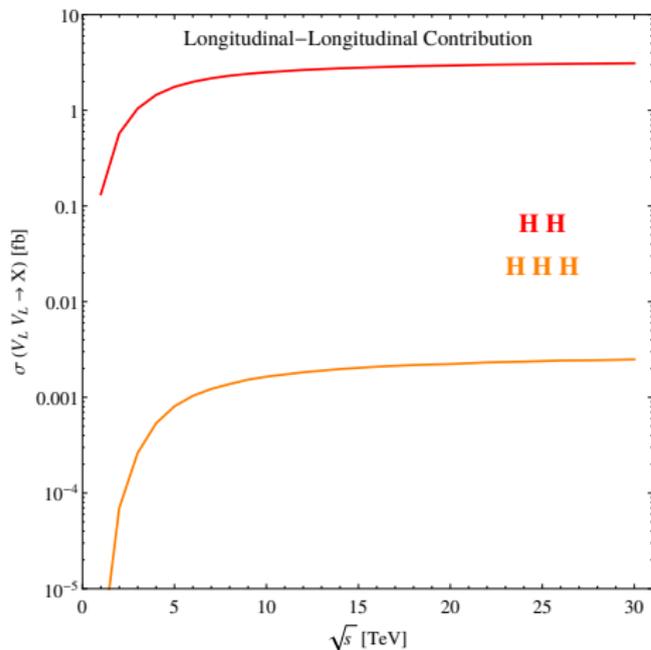
solid (dashed) = full ME (EVA); lower (upper) = $\sqrt{2}\langle\Phi\rangle = v_{EW} \left(\frac{v_{EW}}{10}\right)$



- EVA works within uncertainties when $(M_V/M_{VV}) \ll 1$. Fact of life: M_V is large $\implies M_{VV}$, and hence \sqrt{s} , must be that much larger!

Higgs production in EVA

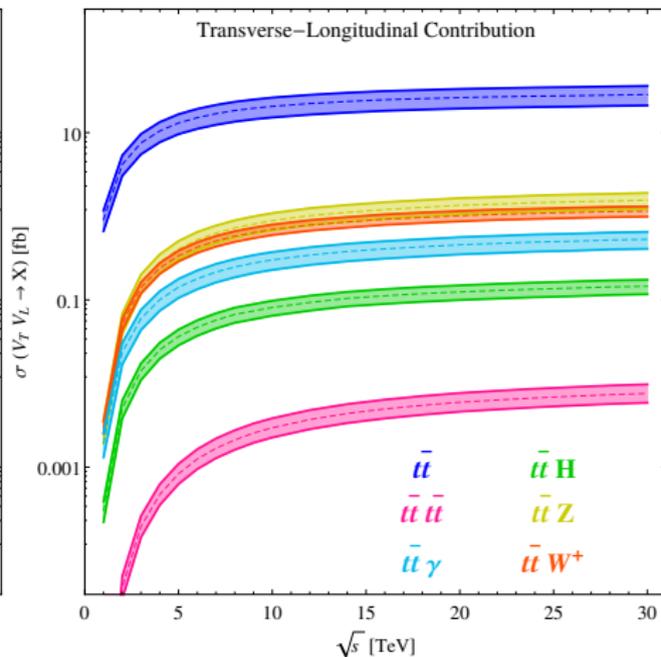
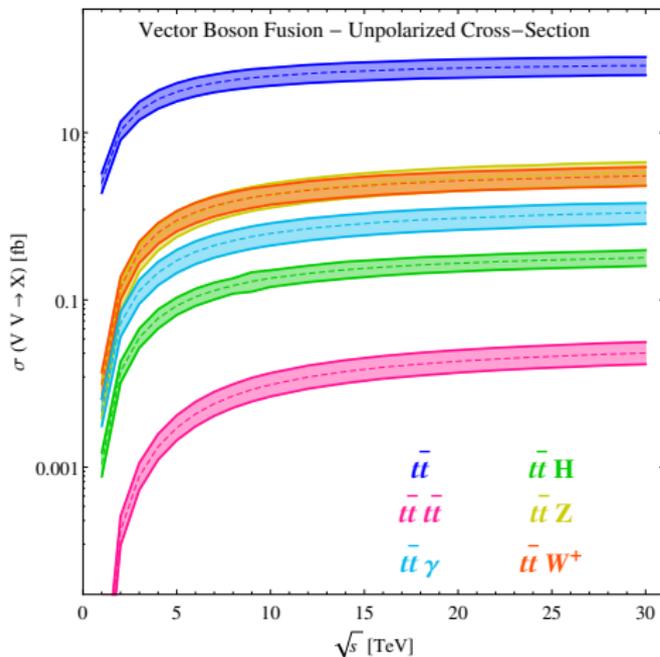
$$V_\lambda V'_\lambda \rightarrow nH: \quad (\text{L}) \quad V_0 V_0 \quad (\text{R}) \quad V_0 V_T + V_T V_0$$



- Importance of $V_\lambda V'_\lambda$ depends on number of H
- At $\mathcal{L} = 1 \text{ ab}^{-1}/\text{yr} \implies 10^3 \text{ HH}/\text{yr}$ (incredibly rich physics!)

Tops in EVA

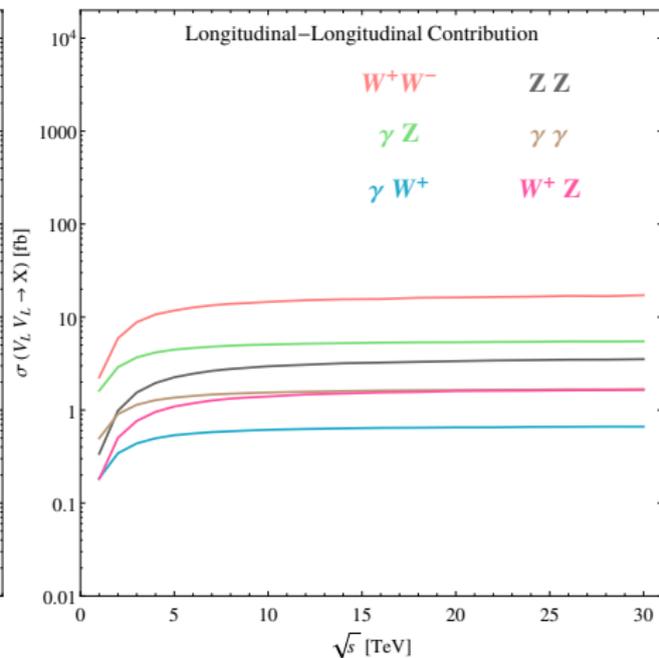
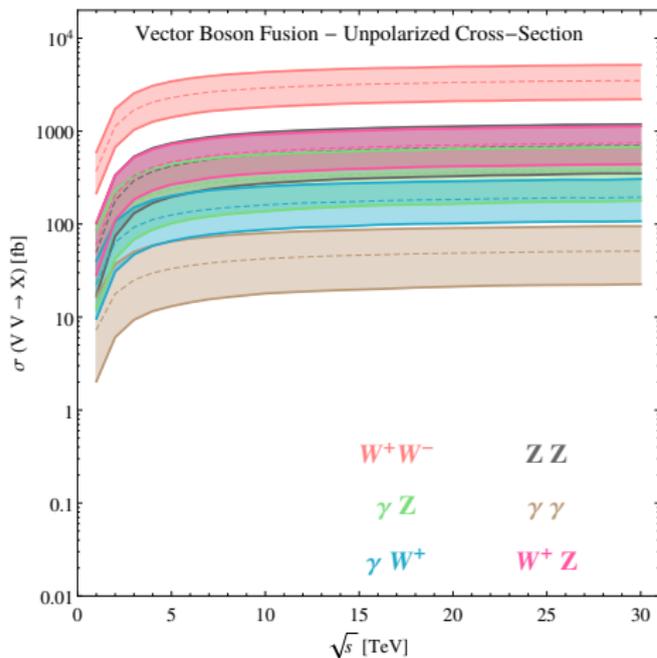
$V_\lambda V'_\lambda \rightarrow t\bar{t} + X$: (L) all polarizations (R) $V_T V_0 + V_0 V_T$



- Nontrivial contribution from all $V_\lambda V'_\lambda$ combinations
- Clear hierarchy of weak and EM couplings

Diboson in EVA

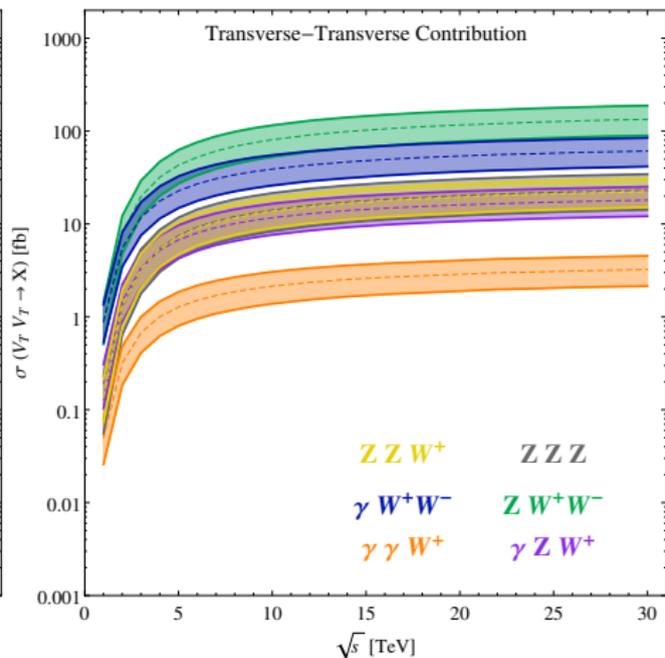
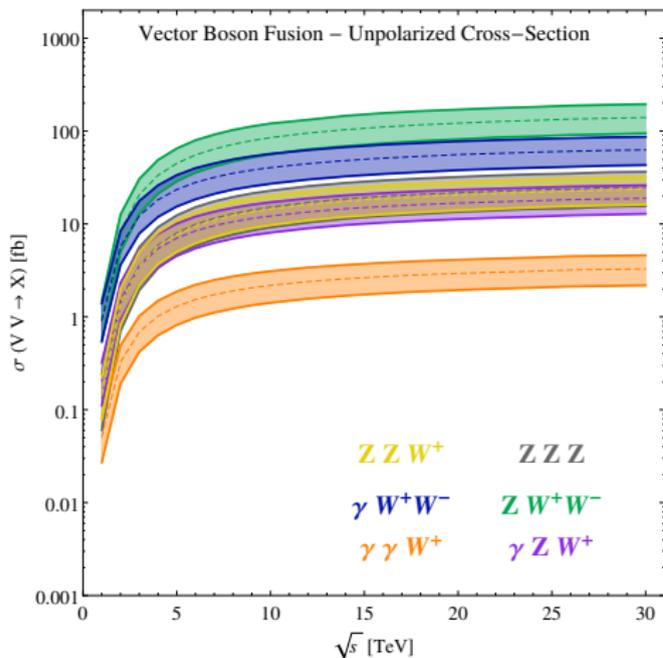
$V_\lambda V'_\lambda \rightarrow VV'$: (L) all polarizations (R) $V_0 V_0$



- Only minor role played by $V_0 V_0$ scattering
- At $\mathcal{L} = 1 \text{ ab}^{-1}/\text{yr} \implies 10^6 \text{ WW}/\text{yr}$ driven by non-Abelian couplings

Triboson in EVA

$V_\lambda V'_\lambda \rightarrow VV'V''$: (L) all polarizations (R) $V_T V_T$



- Major role played by $V_T V'_T$ scattering
- At $\mathcal{L} = 1 \text{ ab}^{-1}/\text{yr} \Rightarrow 10^3 \text{ } VV'V''/\text{yr}$

Multi-TeV-scale muon colliders provide new perspectives on the building blocks of nature, particularly the μ flavor sector!

- Using full matrix elements, we have found that multi-TeV-scale muon colliders are effectively EW boson colliders (eventually, $\sigma^{VBF} \gg \sigma^S$)

w/ A. Costantini, F. De Lillo, F. Maltoni, L. Mantani, O. Mattelaer, X. Zhao [2005.10289]

- Systematically exploring the validity of EVA, we have determined the sensitivity/strength of EVA's principal assumptions

w/ D. Buarque Franzosi, O. Mattelaer, S. Shil [1912.01725]

- Using EVA, $V_\lambda V'_\lambda$ scattering reveal a new picture of the EW sector at high energies (EVA in MadGraph5 will be released soon!)

w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer, et al [soon!]

