14 July 2021
PARTICLEFACE 2021

Composite Higgs states in the top-bottom condensation model

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Supported by

Fundação para a Ciência e Tecnologia (FCT) through the Grant No. CERN/FIS-COM/0035/2019,

the European Cooperation in Science and Technology organisation through the COST Action CA16201 program.
Published in
Symmetry 2021, 13, 1130. https://doi.org/10.3390/sym13071130.

Why is the top quark so heavy?

mass:	2.16 MeV	1.27 GeV	172.76 GeV
	u	**c**	**t**
	up	charm	top
mass:	4.67 MeV	93 MeV	4.18 GeV
	d	**s**	**b**
	down	strange	bottom

The heavy mass of the top quark necessarily implies that it couples more strongly to the electroweak symmetry breaking sector than any other quark or lepton, and suggests that the top quark itself may play a role in electroweak symmetry breaking [Terazava (1980), Nambu (1988)].

Bardeen, Hill and Lindner (1990)

Bardeen, Hill and Lindner, (BHL), inspired by the work of Nambu subsequently gave a technically complete implementation of the top-condensate idea and obtained the first realistic predictions in a minimal scheme. BHL straightforwardly implemented a BCS or Nambu—Jona-Lasinio (NJL) mechanism in which a new fundamental interaction associated with a high energy scale Λ is used to trigger the formation of a low-energy condensate $\langle \bar{t}\, t \rangle$. In this scheme only the known dynamics of the Standard Model (SM) is incorporated.

That time we knew from CDF that $m_t \geq 89\,GeV$.

A mechanism for dynamically breaking the symmetries of the electroweak interactions which relies upon the formation of top-condensate yields

$$m_{H^0} = 2\,m_t \geq 178\,GeV\,.$$

The full RG effects cause $m_H \neq 2\,m_t$, in particular, for Λ at the GUT scale, BHL have found

$$m_t = 229\,GeV\,,\, m_{H^0} = 256\,GeV\,.$$

Presently, it is well established that

$$m_t = 172.76\,GeV\,,\, m_{H^0} = 125.1\,GeV\,.$$

Does it mean that the Nambu mechanism is not responsible for dynamic symmetry breaking of electroweak interactions?

We think that there is still some room for the Nambu scenario. To demonstrate this, let us consider the Miransky, Tanabashi, and Yamawaki model (1989) which, in contrast with the BHL model, contains two Higgs doublets.

The model of Miransky, Tanabashi, and Yamawaki

$$\boxed{L = \overline{\psi_L} i \gamma^\mu D_\mu \psi_L + \sum_{a=1}^{2} \overline{\psi_R^a} i \gamma^\mu D_\mu \psi_R^a + L_{YM} + L_{4\psi},} \qquad \psi^a = \begin{pmatrix} t \\ b \end{pmatrix},$$

$$L_{4\psi} = g_1 \left(\overline{\psi_L^a}\, \psi_R^b\right)\left(\overline{\psi_R^b}\, \psi_L^a\right) + g_2 \left(\overline{\psi_L^a}\, \psi_R^b\right)\left(i\,\tau_2\right)^{ac}\left(i\,\tau_2\right)^{be}\left(\overline{\psi_L^c}\, \psi_R^e\right)$$
$$+ g_3 \left(\overline{\psi_L^a}\, \psi_R^b\right) \tau_3^{bc} \left(\overline{\psi_R^c}\, \psi_L^a\right) + h.c.$$

The couplings “g” are considered to be real and positive, and the symmetry content associated to the corresponding interactions is

$$g_1 : SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A.$$

$$g_2 : SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_V.$$

$$g_3 : SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_V \times U(1)_A.$$

Functional integral and bosonic variables

$$Z=\int d\psi\, d\bar{\psi}\exp\left(i\int d^4x\, L\right)=\int d\sigma_\alpha\, d\pi_\alpha\, d\psi\, d\bar{\psi}\exp\left(i\int d^4x\, L'\right)$$

$$L'=L_f+L_{YM}+L_{\pi,\sigma}$$

$$L_f=\bar{\psi}\left(i\gamma^\mu D_\mu+\sigma+i\gamma_5\pi\right)\psi,\qquad \sigma=\sigma_\alpha\tau_\alpha,\pi=\pi_\alpha\tau_\alpha,\qquad \alpha=0,1,2,3$$

$$L_{\pi,\sigma}=-\frac{1}{\bar{g}^2}\left[(g_1+g_2)(\pi_0^2+\sigma_i^2)+(g_1-g_2)(\pi_i^2+\sigma_0^2)-2g_3(\pi_0\pi_3+\sigma_0\sigma_3-\sigma_1\pi_2+\sigma_2\pi_1)\right]$$

$$\bar{g}^2=g_1^2-g_2^2-g_3^2.$$

$$2\sigma_0=(g_1+g_2)\bar{\psi}\tau_0\psi+g_3\bar{\psi}\tau_3\psi,\qquad 2\pi_0=(g_1-g_2)\bar{\psi}i\gamma_5\tau_0\psi+g_3\bar{\psi}i\gamma_5\tau_3\psi,$$

$$2\sigma_1=(g_1-g_2)\bar{\psi}\tau_1\psi-g_3\bar{\psi}i\gamma_5\tau_2\psi,\qquad 2\pi_1=(g_1+g_2)\bar{\psi}i\gamma_5\tau_1\psi+g_3\bar{\psi}\tau_2\psi,$$

$$2\sigma_2=(g_1-g_2)\bar{\psi}\tau_2\psi+g_3\bar{\psi}i\gamma_5\tau_1\psi,\qquad 2\pi_2=(g_1+g_2)\bar{\psi}i\gamma_5\tau_2\psi-g_3\bar{\psi}\tau_1\psi,$$

$$2\sigma_3=(g_1-g_2)\bar{\psi}\tau_3\psi+g_3\bar{\psi}\tau_0\psi,\qquad 2\pi_3=(g_1+g_2)\bar{\psi}i\gamma_5\tau_3\psi+g_3\bar{\psi}i\gamma_5\tau_0\psi.$$

Two doublets of auxiliary fields

$$\Phi_1 = \begin{pmatrix} \pi_2 + i\,\pi_1 \\ \sigma_0 - i\,\pi_3 \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \sigma_1 - i\,\sigma_2 \\ -\sigma_3 + i\,\pi_0 \end{pmatrix}.$$

Under the infinitesimal SU(2)xU(1) transformations they behave like fundamental SU(2)-doublets

$$\delta\,\Phi_{1,2} = i\,\omega_i \frac{\tau_i}{2} \Phi_{1,2} - i\,\beta_0 \Phi_{2,1}$$

with the following quark content

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} g_1 + g_2 & -g_3 \\ -g_3 & g_1 - g_2 \end{pmatrix} \begin{pmatrix} \Phi_1^0 \\ \Phi_2^0 \end{pmatrix},$$

$$\Phi_1^0 = \begin{pmatrix} \bar{b}_R t_L - \bar{b}_L t_R \\ \bar{b}_R b_L + \bar{t}_L t_R \end{pmatrix}, \qquad \Phi_2^0 = \begin{pmatrix} \bar{b}_R t_L + \bar{b}_L t_R \\ \bar{b}_R b_L - \bar{t}_L t_R \end{pmatrix}.$$

On the way to the Higgs Lagrangian

For low-energy phenomena $\mu \ll \Lambda$ we may wish to integrate out the short-distance components of quark fields. As a result, at scales below the cutoff Λ the auxiliary boson fields develop induced, fully gauge-invariant, kinetic terms and quartic interaction contributions in the effective action (this is how the Higgs sector of the model is generated).

$$L' \Rightarrow L' + L_{sd}$$

$$L_{sd} = \frac{1}{32\pi^2} tr\left[C_1 Y + C_2 \left(\frac{F_{\mu\nu}^2}{12} - \frac{Y^2}{2}\right)\right]$$

$$C_1 = \int_{1/\Lambda^2}^{1/\mu^2} \frac{dt}{t^2} = \Lambda^2 - \mu^2, \qquad C_2 = \int_{1/\Lambda^2}^{1/\mu^2} \frac{dt}{t} = \ln\frac{\Lambda^2}{\mu^2},$$

$$Y = \sigma^2 + \pi^2 + i\gamma_5[\sigma, \pi] - i\nabla_\mu \gamma^\mu(\sigma + i\gamma_5\pi) - \frac{i}{4}[\gamma_\mu, \gamma_\nu]F^{\mu\nu}$$

Higgs potential

$$
\begin{aligned}
V_H = & -\bar{C}_1\left(\Phi_1^+\Phi_1+\Phi_2^+\Phi_2\right)\\
& +2\bar{C}_2\left[\frac{1}{4}\left(\Phi_1^+\Phi_1+\Phi_2^+\Phi_2\right)^2+\left(\Phi_1^+\Phi_1\right)\left(\Phi_2^+\Phi_2\right)-\left(\Im\left(\Phi_1^+\Phi_2\right)\right)^2\right]\\
& +\frac{1}{\bar{g}^2}\left[\left(g_1-g_2\right)\Phi_1^+\Phi_1+\left(g_1+g_2\right)\Phi_2^+\Phi_2+2g_3\Re\left(\Phi_1^+\Phi_2\right)\right]
\end{aligned}
$$

At this stage, one can compare with the general form of the 2HDM potential and identify in their notation three mass terms and five real-valued and independent quartic couplings

$$
m_{11}^2=\frac{g_1-g_2}{\bar{g}^2}-\bar{C}_1, \quad m_{22}^2=\frac{g_1+g_2}{\bar{g}^2}-\bar{C}_1, \quad m_{12}^2=-\frac{g_3}{\bar{g}^2},
$$

$$
\lambda_1=\lambda_2=\frac{1}{3}\lambda_3=-\lambda_4=\lambda_5=\bar{C}_2.
$$

It shows that the potential of the MTY model is quite restrictive, it has only four real independent parameters instead of eight in the most general case for such class of theories.

The gap equation

The model considered displays of critical behavior, giving origin to the quark masses. This occurs when the neutral scalar fields acquire non-vanishing vacuum expectation values $\langle\sigma_0\rangle=-m_0, \langle\sigma_3\rangle=-m_3$. These scalar fields must be redefined such that the minimum configuration of the potential in the new variables corresponds to vanishing of their vacuum expectation values. This is achieved by shifting

$$\sigma_0 \to \sigma_0 - m_0, \qquad \sigma_3 \to \sigma_3 - m_3.$$

The gap equations then result in the removal from the effective potential of linear terms in these new fields

$$\left(g_1 - g_3\right) m_t - g_2 m_b = \bar{g}^2 m_t \left(\bar{C}_1 - m_t^2 \bar{C}_2\right)$$
$$\left(g_1 + g_3\right) m_b - g_2 m_t = \bar{g}^2 m_b \left(\bar{C}_1 - m_b^2 \bar{C}_2\right)$$

where

$$m_t = m_0 + m_3, \qquad m_b = m_0 - m_3.$$

Quark condensates

The solutions of the gap equations can be studied in terms of the quark condensates

$$m_t = -(g_1 + g_3)\langle \bar{t}\, t \rangle - g_2 \langle \bar{b}\, b \rangle$$

$$m_b = -(g_1 - g_3)\langle \bar{b}\, b \rangle - g_2 \langle \bar{t}\, t \rangle$$

These show that the absence of the bottom quark condensate does not necessarily mean that the bottom quark mass is zero: in the case that $g_2 \neq 0$, that is, when the $U(1)_A$ symmetry is violated, the top quark condensate also provides mass to the bottom quark.

Diagonalization

The quadratic part of the effective potential must be diagonalized. This is a convenient place to do it, as the gap equations carry information on the two diagonalization angles: the angle θ that diagonalizes the charged sector, and θ′ for the neutral one. They are related to each other and to ratios involving the quark masses and the couplings g_2, g_3 through

$$\tan\theta = \frac{m_3}{m_0}, \qquad \tan 2\theta' = 3\tan 2\theta - 2\frac{g_3}{g_2}$$

$$H_1 = \begin{pmatrix} \phi_2 + i\phi_1 \\ \chi'_0 - i\phi_3 \end{pmatrix}, \qquad H_2 = \begin{pmatrix} \chi_1 - i\chi_2 \\ -\chi'_3 + i\phi_0 \end{pmatrix},$$

$$\begin{pmatrix} \chi'_0 \\ \chi'_3 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_3 \end{pmatrix}, \qquad \alpha = \theta - \theta'$$

Masses of Higgs states and Nambu sum rules

$$m_{\phi_i}^2 = 0,$$

$$m_{\chi_0}^2 = 4m^2 + \frac{2g_2}{\bar{g}^2 \bar{C}_2}\left(\frac{1}{\cos 2\theta} - \frac{1}{\cos 2\theta'}\right), \quad m^2 = m_0^2 + m_3^2 = \frac{1}{2}\left(m_t^2 + m_b^2\right),$$

$$m_{\chi_3}^2 = 4m^2 + \frac{2g_2}{\bar{g}^2 \bar{C}_2}\left(\frac{1}{\cos 2\theta} + \frac{1}{\cos 2\theta'}\right), \quad m_{\chi^\pm}^2 = \frac{4g_3}{\bar{g}^2 \bar{C}_2 \sin 2\theta'}, \quad m_{\phi_0}^2 = \frac{4g_2}{\bar{g}^2 \bar{C}_2 \cos 2\theta'}.$$

From the mass formulas, we have

$$m_{\chi_0}^2 + m_{\chi_3}^2 = m_{\phi_0}^2 + 4\left(m_t^2 + m_b^2\right),$$

$$m_{\chi^+}^2 + m_{\chi^-}^2 = 2m_{\phi_0}^2 + 4\left(m_t^2 + m_b^2\right),$$

which points out the non-zero mass of the ϕ_0 meson as the origin of the Nambu sum rule violation. When there is no interaction in the coupling constant g_2, the model has an additional $U(1)_A$ global Peccei–Quinn-like symmetry.

Specific case

If we set $g_2 = 0$, the Higgs particle masses become:

$$m_{\chi_0} = 2m_b, \quad m_{\chi_3} = 2m_t, \quad m_{\chi^{\pm}} = 2m, \quad m_{\phi_0} = 0.$$

These relations agree with the Nambu sum rule. In other words, the $U(1)_A$ breaking $\propto g_2$ accounts for the deviation from the canonical Nambu sum rule. Hence, a light composite Higgs boson is built mainly of $\bar{b}b$ condensates with some proportion of $\bar{t}t$ due to the interaction $\propto g_2$. Accordingly an increase in its mass occurs in the interval

$$2m_b \leqslant m_{\chi_0} \leqslant m_t,$$

Numerical results

The model we considered here has five free parameters: $g_1, g_2, g_3, \Lambda, \mu$. They will be fixed at the SM scale,

$$\mu = \Lambda_{EW} = 246\, GeV\,.$$

The cutoff Λ is fixed from the vacuum expectation value v

$$v = m\sqrt{\bar{C}_2} = 254.6\, GeV\,.$$

Given that, we may calculate the ratio Λ/μ:

$$\frac{\Lambda}{\mu} = \exp\left[\frac{(2\pi v)^2}{N_c (m_t^2 + m_b^2)}\right] = 2.345 \times 10^{12}.$$

And, as a result,

$$\Lambda = 0.58 \times 10^{15}\, GeV\,.$$

Moreover, we assume that g_1, g_2, g_3 are chosen so as to obtain the phenomenologically consistent solutions to gap equations. In other words, one chooses the free parameters to get the experimental values of quark masses, $m_t = 173 \pm 0.4\, GeV$, $m_b = 4.18^{+0.04}_{-0.03}$. We also require the Higgs mass state to be $m_{\chi_0} = 125\, GeV$. We may verify that this can be accomplished under compatible conditions.

The Higgs state spectrum as displayed in equations above depends on three independent parameters, namely m_t and m_b, and the ratio g_3/g_2 which we replace by the dimensionless parameter a

$$\frac{g_3}{g_2}=a\tan 2\theta$$

The other parameters in mass formulas are eliminated using the gap equations. Moreover, the angle θ depends on the quark masses

$$\tan 2\theta=\frac{m_t^2-m_b^2}{2m_t m_b}$$

which yields θ = 43.6°, while θ′ can be written in terms of the quark masses and the parameter a only. In addition, we have the following relation between two mixing angles

$$\tan 2\theta'=(3-2a)\tan 2\theta\,,$$

we see that θ′ < 0 if a > 3/2. The mass formulas can then be written as

$$m_{\chi_0}^2=\frac{2m^2}{a-1}(2a-1-\Delta),\quad m_{\chi_3}^2=\frac{2m^2}{a-1}(2a-1+\Delta),\quad m_{\phi_0}^2=\frac{4m^2}{a-1},\quad m_{\chi^\pm}^2=\frac{4m^2a}{a-1},$$

where $$\Delta=\sqrt{\cos^2 2\theta+(3-2a)^2\sin^2 2\theta}\,.$$

By fixing the parameter a with mass of the standard Higgs state, we obtain a=4.84, and the following estimates:

$$m_{\chi_3}=346\,GeV\,,$$
$$m_{\chi^{\pm}}=275\,GeV\,,$$
$$m_{\phi_0}=125\,GeV\,,$$

and

$$\theta' = -44.8^{\circ}.$$

The values of four-Fermi couplings follow in a straightforward manner. These equations show, however, that the coupling constants of the model must be extremely fine-tuned when $\mu^2 \ll \Lambda^2$. Explicitly,

$$\frac{g_1}{g_c}:\frac{g_2}{g_c}:\frac{g_3}{g_c}=1:10^{-26}:10^{-24},$$

where

$$g_c=\frac{4\pi^2}{N_c\Lambda^2}=3.9\times10^{-29}\,GeV^{-2}.$$

Finally, in spite of a good agreement in the estimates of both the quark masses and the ground Higgs state, the values for the neutral, ϕ_0, mass and for the mass of the charged, $\chi^{\pm}$ Higgs states are likely experimentally disfavoured. More work needs to be done in refining the calculation of the mass spectrum using the RG-approach.

Conclusions

The model has a series of interesting consequences:

1) It leads to phenomenological values for the mass of the SM composite Higgs and heavy quarks. Top-condensation models usually yield significantly overestimated values.

2) As a result, the standard Higgs is not a pure $\bar{t}t$ bound state, but has an essential part associated with the light bottom quarks. The underlying mechanism has been clarified.

3) The neutral boson $\phi 0$ violates the standard Nambu sum rules but only at next to the leading order of the 1/N expansion. Such a violation leads to a degeneracy in mass for the main Higgs χ_0 and ϕ_0, $m_{\chi_0} \simeq m_{\phi_0}$. This degeneracy is of a random nature.

4) The model is extremely fine-tuned. This is the known hierarchy problem of the SM. The top condensation models cannot clarify this question. However, the fine-tuning problem is isolated in the gap equations.

Thank you for your attention