

Z-boson decay at the NNNLO level

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Outline

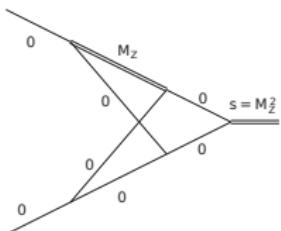
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4. Numerical calculations - Methods and tools
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1. Introduction

| Γ_i [MeV] | $\Gamma_e, \Gamma_\mu, \Gamma_\tau$ | $\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$ | Γ_d, Γ_s | Γ_u, Γ_c | Γ_b | Γ_Z |
|--------------------------------------------------------------------------------------------|-------------------------------------|-------------------------------------------------------|----------------------|----------------------|--------------|--------------|
| Born | 81.142 | 160.096 | 371.141 | 292.445 | 369.56 | 2420.2 |
| $\mathcal{O}(\alpha)$ | 2.273 | 6.174 | 9.717 | 5.799 | 3.857 | 60.22 |
| $\mathcal{O}(\alpha\alpha_s)$ | 0.288 | 0.458 | 1.276 | 1.156 | 2.006 | 9.11 |
| $\mathcal{O}(N_f^2\alpha^2)$ | 0.244 | 0.416 | 0.698 | 0.528 | 0.694 | 5.13 |
| $\mathcal{O}(N_f\alpha^2)$ | 0.120 | 0.185 | 0.493 | 0.494 | 0.144 | 3.04 |
| $\mathcal{O}(\alpha_{\text{bos}}^2)$ | 0.017 | 0.019 | 0.058 | 0.057 | 0.167 | 0.505 |
| $\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t^3)$ $\alpha_t\alpha_s^3, \alpha_t^2\alpha_s)$ | 0.038 | 0.059 | 0.191 | 0.170 | 0.190 | 1.20 |

- 2016, estimation, bosonic NNLO $\sim 0 \pm 0.1$ MeV
- 2018, exact result: 0.505 MeV

1. Introduction



Euclidean results (constant part, $(p_1 + p_2)^2 = m_Z^2 = 1$):

| | |
|--------------|---------------------|
| Analytical : | -0.4966198306057021 |
| MB(Vegas) : | -0.4969417442183914 |
| MB(Cuhre) : | -0.4966198313219404 |
| FIESTA : | -0.4966184488196595 |
| SecDec : | -0.4966192150541896 |

Minkowskian results (constant part, $-(p_1 + p_2)^2 = m_Z^2 = 1$):

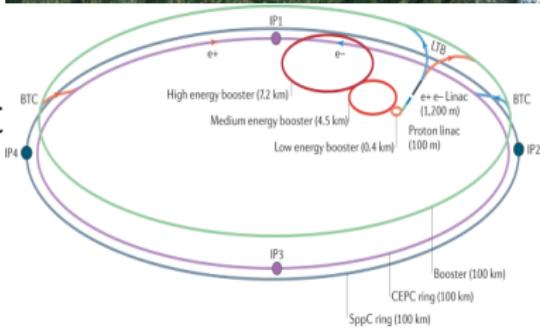
| | |
|------------------------|-------------------------------------------------------------------------|
| Analytical : | -0.778599608979684 - 4.123512593396311 · i |
| MBnumerics : | -0.778599608324769 - 4.123512600516016 · i |
| MB(QMC) : | -0.7785242512636401 - 4.123512600516016 · i |
| SecDec : | big error [2016], -0.77 - i · 4.1 [2017], -0.778 - i · 4.123 [2019] |
| pySecDec + rescaling : | -0.778598 - i · 4.123512 [2020] |

1. Introduction

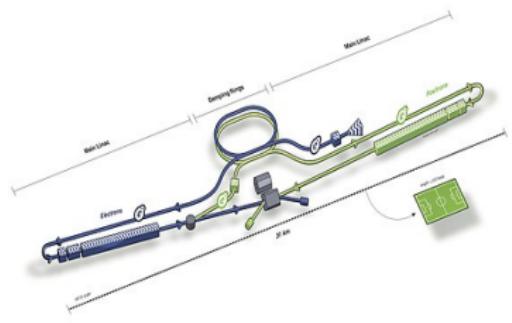
FCC



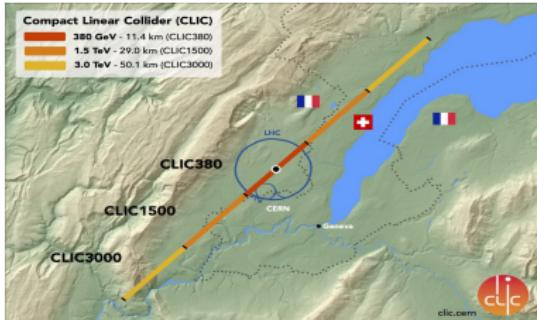
CEPC



ILC



CLIC

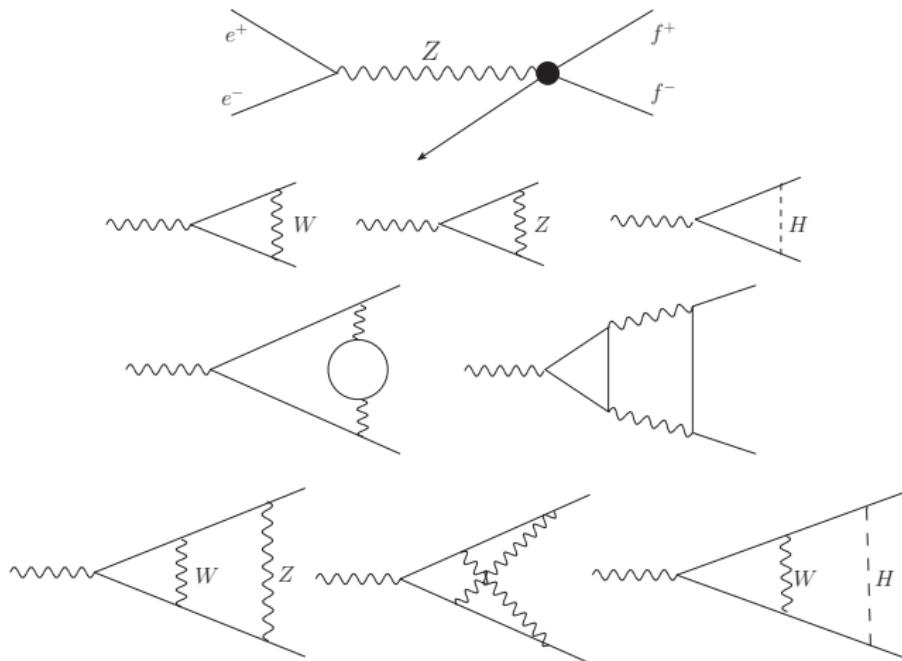


2. Motivation

| | $\delta\Gamma_Z$ [MeV] | δR_l [10^{-4}] | δR_b [10^{-5}] | $\delta \sin_{\text{eff}}^{2,l} \theta$ [10^{-6}] |
|-------------------------------------------------------|------------------------|----------------------------|----------------------------|-------------------------------------------------------|
| Present EWPO theoretical uncertainties | | | | |
| EXP-2018 | 2.3 | 250 | 66 | 160 |
| TH-2018 | 0.4 | 60 | 10 | 45 |
| EWPO theoretical uncertainties when FCC-ee will start | | | | |
| EXP-FCC-ee | 0.1 | 10 | $2 \div 6$ | 6 |
| TH-FCC-ee | 0.07 | 7 | 3 | 7 |

Table: Comparison for selected precision observables of present experimental measurements (EXP-2018), current theory errors (TH-2018), FCC-ee precision goals at the end of the Tera-Z run (EXP-FCC-ee) and rough estimates of the theory errors assuming that electroweak 3-loop corrections and the dominant 4-loop EW-QCD corrections $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^3)$ are available at the start of FCC-ee (TH-FCC-ee). Based on discussion in 1809.01830.

2. Motivation



$$R = \sum_n r_n \alpha^n$$

3. The task

| $Z \rightarrow b\bar{b}$ | | | |
|--------------------------|--------|-----------|----------------|
| Number of topologies | 1 loop | 2 loops | 3 loops |
| | 1 | 5 | 50 |
| Number of diagrams | 15 | 1114 | 120187 |
| Fermionic loops | 0 | 150 | 17580 |
| Bosonic loops | 15 | 964 | 102607 |
| QCD / EW | 1 / 14 | 98 / 1016 | 10405 / 109782 |

Table: The number of Z decay Feynman diagrams needed to be calculated to meet FCC-ee experimental accuracy. Tadpoles, products of lower loop diagrams and symmetrical diagrams are not included.

$\mathcal{O}(10^3)$ Self-energy integrals to be calculated as a warm-up.

Automation, cross-checks and precision of calculations are crucial.

4. Numerical calculations - Methods and tools

- Sector Decomposition method
 - FIESTA
 - (py)SecDec
- Mellin-Barnes method
 - PlanarityTest
 - AMBRE
 - MB, MBresolve
 - MBnumerics
 - QMB (MB+quasiMC)

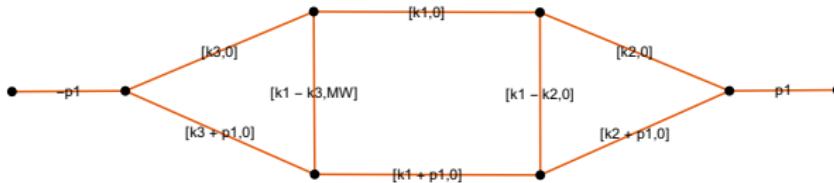
NNLO Z-pole SM completed: 10^{-8} accuracy achieved for most of Feynman integrals and at least 10^{-6} for the few worst integrals with one of the methods.

Numerical calculations - Methods and tools

Many groups present rapid progress:

- Analytical/numerical solutions for Master Integrals (MIs) by DEqs and uniform transcendentality (UT)
- Reductions at the integrand level
- Expansions: by regions; Taylor expansion in Feynman parameters (TayInt)
- NNNLO massive self-energies: TVID-2
- Loop-tree duality
- Four-dimensional unsubtraction

5. Examples of calculations - pySecDec

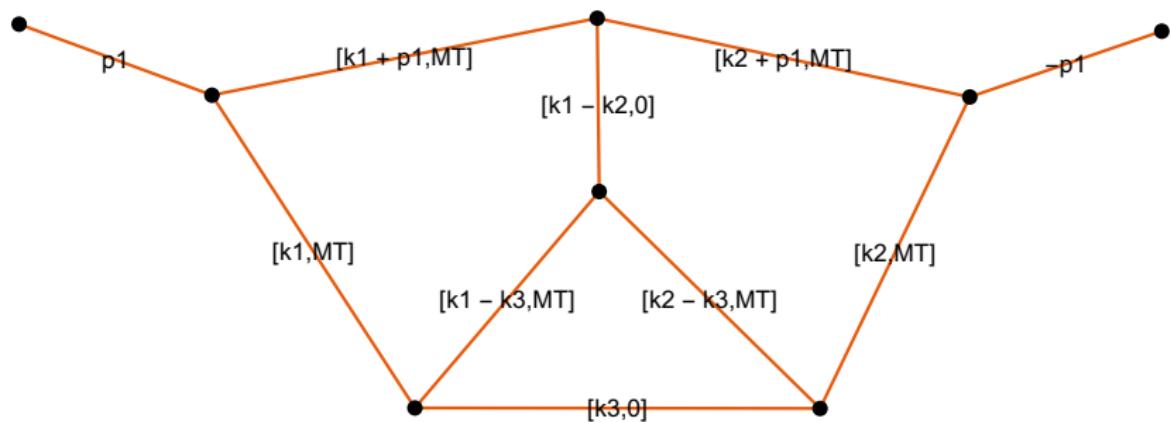


| Number of points | Integrator | Result | Absolute error |
|------------------|------------|-----------------|------------------------|
| 10^7 | QMC | 8.62988528 | 4.99×10^{-6} |
| | Divonne | 8.62995472 | 5.31×10^{-5} |
| 10^9 | QMC | 8.6298878517237 | 3.72×10^{-11} |
| | Divonne | 8.6298944179328 | 2.56×10^{-6} |

Table: Comparison of Quasi-Monte Carlo and Divonne integrators for calculation in Euclidean point ($M_Z^2 = 1 = -s$).

5. Examples of calculations - pySecDec

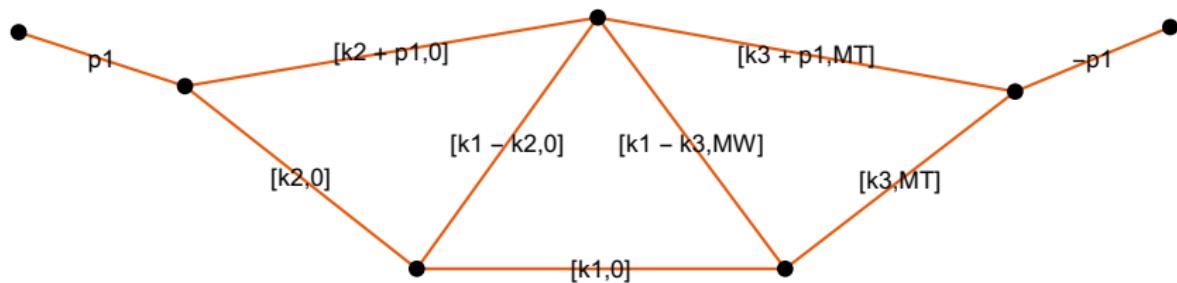
- Minkowskian kinematics ($M_Z^2 = 1 = s$), below threshold



$$0.025510249860 \pm 6.23 \times 10^{-10}.$$

5. Examples of calculations - pySecDec

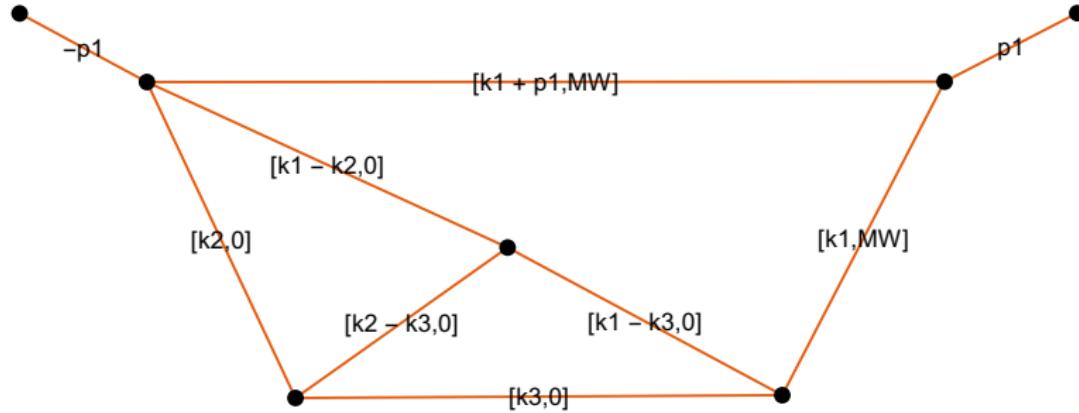
- Minkowskian kinematics ($M_Z^2 = 1 = s$), above threshold



$$-1.977\textcolor{red}{90} - 3.170\textcolor{red}{70} \cdot i \pm (0.000\textcolor{red}{70} + 0.000\textcolor{red}{56} \cdot i).$$

5. Examples of calculations - MB

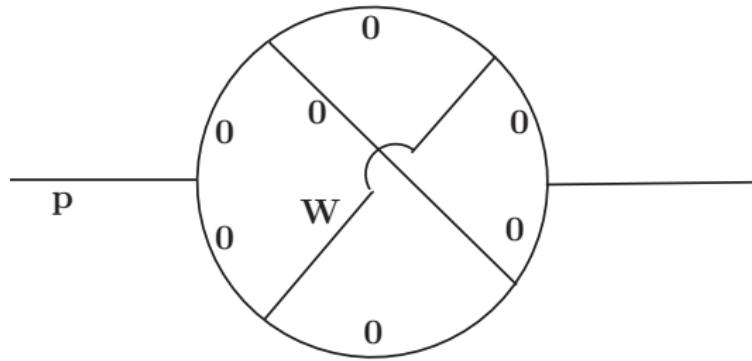
- Minkowskian kinematics ($M_Z^2 = 1 = s$), above threshold



| Method | Result | Absolute error |
|------------|-------------------------------------------------------------------|-------------------------------------------------------|
| MBnumerics | $-18.7794069\textcolor{red}{62} - 6.3907850\textcolor{red}{27} i$ | $10^{-9} + 10^{-9} i$ |
| pySecDec | $-18.78\textcolor{red}{7167067} - 6.38\textcolor{red}{4327811} i$ | $0.00\textcolor{red}{93} + 0.00\textcolor{red}{97} i$ |

5. Examples of calculations - DEqs

- Minkowskian kinematics ($M_Z^2 = 1 = s$), above threshold



$$I_{DEqs} = -(0. - 19.1262302988\textcolor{red}{13844} \cdot I)$$

$$I_{pySecDec} = 0.\textcolor{red}{460} - 19.\textcolor{red}{164} \cdot I \pm (0.\textcolor{red}{298} + 0.\textcolor{red}{281} \cdot I)$$

for more details see the following talk by M. Hidding

6. Summary and outlook

- Enormous task to be done
- Automation
- Testing and development of methods and tools
- Cross-checks
- Promising view for the future

Thank You for your attention