

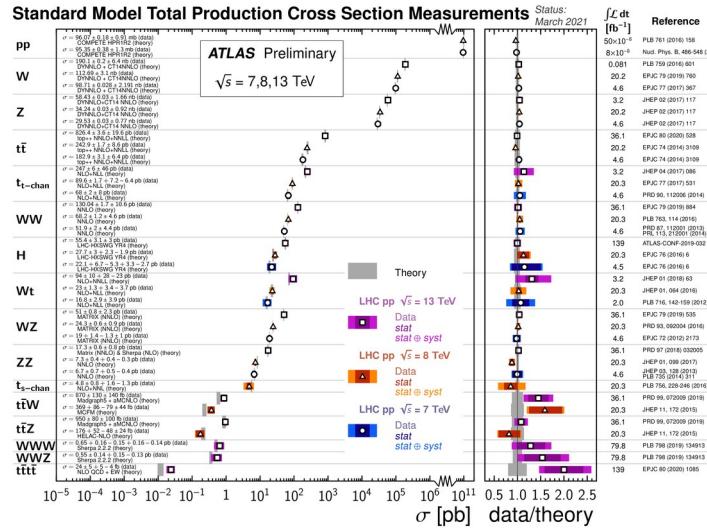
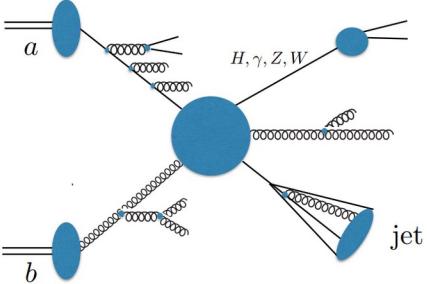
An easy way to obtain beta functions in gauge theories within Implicit Regularization: a two-loop study

Adriano Cherchiglia, D. C. Arias-Perdomo, B. Hiller, M. Sampaio, A. R. Vieira

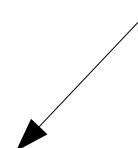
PARTICLEFACE 2021



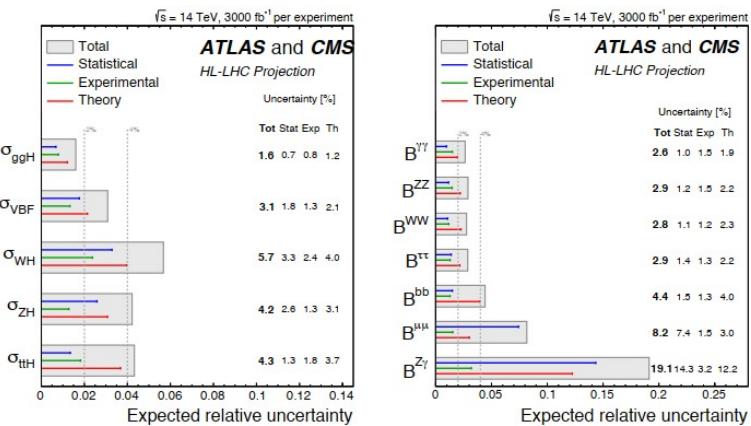
Motivation



$$d\sigma = \sum_{ab} \int dx_a \int dx_b f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \times d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_s(\mu_R^2))$$



Partonic higher loop corrections



Motivation

Eur. Phys. J. C (2017) 77:471
DOI 10.1140/epjc/s10052-017-5023-2

THE EUROPEAN
PHYSICAL JOURNAL C



Regular Article - Theoretical Physics

To d , or not to d : recent developments and comparisons of regularization schemes

C. Gnendiger^{1,a}, A. Signer^{1,2}, D. Stöckinger³, A. Broggio⁴, A. L. Cherchiglia⁵, F. Driencourt-Mangin⁶, A. R. Fazio⁷, B. Hiller⁸, P. Mastrolia^{9,10}, T. Peraro¹¹, R. Pittau¹², G. M. Pruna¹, G. Rodrigo⁶, M. Sampaio¹³, G. Sborlini^{6,14,15}, W. J. Torres Bobadilla^{6,9,10}, F. Tramontano^{16,17}, Y. Ulrich^{1,2}, A. Visconti^{1,2}

¹ Paul Scherrer Institut, 5232 Villigen, PSI, Switzerland

² Physik-Institut, Universität Zürich, 8057 Zürich, Switzerland

³ Institut für Kern- und Teilchenphysik, TU Dresden, 01062 Dresden, Germany

⁴ Physik Department T31, Technische Universität München, 85748 Garching, Germany

⁵ Centro de Ciências Naturais e Humanas, UFABC, 09210-170 Santo André, Brazil

⁶ Instituto de Física Corpuscular, UVEG-CSIC, Universidad de Valencia, 46980 Paterna, Spain

⁷ Departamento de Física, Universidad Nacional de Colombia, Bogotá D.C., Colombia

⁸ CFisUC, Department of Physics, University of Coimbra, 3004-516 Coimbra, Portugal

⁹ Dipartimento di Fisica ed Astronomia, Università di Padova, 35131 Padua, Italy

¹⁰ INFN, Sezione di Padova, 35131 Padua, Italy

¹¹ Higgs Centre for Theoretical Physics, The University of Edinburgh, Edinburgh EH9 3FD, UK

¹² Dep. de Física Teórica y del Cosmos and CAFPE, Universidad de Granada, 18071 Granada, Spain

¹³ Departamento de Física, ICEX, UFMG, 30161-970 Belo Horizonte, Brazil

¹⁴ Dipartimento di Fisica, Università di Milano, 20133 Milan, Italy

¹⁵ INFN, Sezione di Milano, 20133 Milan, Italy

¹⁶ Dipartimento di Fisica, Università di Napoli, 80126 Naples, Italy

¹⁷ INFN, Sezione di Napoli, 80126 Naples, Italy

Eur. Phys. J. C (2021) 81:250
<https://doi.org/10.1140/epjc/s10052-021-08996-y>

THE EUROPEAN
PHYSICAL JOURNAL C



Review

May the four be with you: novel IR-subtraction methods to tackle NNLO calculations

W. J. Torres Bobadilla^{1,2,a}, G. F. R. Sborlini³, P. Banerjee⁴, S. Catani⁵, A. L. Cherchiglia⁶, L. Cieri⁵, P. K. Dhani^{5,7}, F. Driencourt-Mangin², T. Engel^{4,8}, G. Ferrera⁹, C. Gnendiger⁴, R. J. Hernández-Pinto¹⁰, B. Hiller¹¹, G. Pellizzetti¹², J. Pires¹³, R. Pittau¹⁴, M. Rocco¹⁵, G. Rodrigo⁶, M. Sampaio⁶, A. Signer^{4,8}, C. Signorile-Signorile^{16,17}, D. Stöckinger¹⁸, F. Tramontano¹⁹, Y. Ulrich^{4,8,20}

¹ Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, 80805 Munich, Germany

² Instituto de Física Corpuscular, UVEG-CSIC, 46980 Paterna, Spain

³ Deutsches Elektronensynchrotron DESY, 15738 Zeuthen, Germany

⁴ Paul Scherrer Institute (PSI), 5232 Villigen, Switzerland

⁵ INFN, Sezione di Firenze, 50019 Sesto Fiorentino, Italy

⁶ CCNH, Universidade Federal do ABC, Santo André 09210-580, Brazil

⁷ INFN, Sezione di Genova, 16146 Genoa, Italy

⁸ Physik-Institut, Universität Zürich, 8057 Zurich, Switzerland

⁹ Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano, 20133 Milan, Italy

¹⁰ Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Sinaloa, 80000 Culiacán, Mexico

¹¹ CFisUC, Department of Physics, University of Coimbra, 3004-516 Coimbra, Portugal

¹² Institut für Theoretische Physik und Astrophysik, Universität Würzburg, 97074 Würzburg, Germany

¹³ Laboratório de Instrumentação e Física de Partículas LIP, 1649-003 Lisbon, Portugal

¹⁴ Dep. de Física Teórica y del Cosmos and CAFPE, Universidad de Granada, 18071 Granada, Spain

¹⁵ Università di Milano-Bicocca and INFN, Sezione di Milano, 20126 Milan, Italy

¹⁶ Institut für Theoretische Teilchenphysik, Karlsruher Institut für Technologie, 76128 Karlsruhe, Germany

¹⁷ Dipartimento di Fisica e Arnold-Regge Center, Università di Torino and INFN, 10125 Torin, Italy

¹⁸ Institut für Kern- und Teilchenphysik, TU Dresden, 01062 Dresden, Germany

¹⁹ Università di Napoli and INFN, Sezione di Napoli, 80126 Naples, Italy

²⁰ Institute for Particle Physics Phenomenology, Durham DH1 3LE, UK

Regularization methods in 4D

Implicit Regularization

Mod. Phys. Lett. A 13, 1597 (1998)

Four-Dimensional Regularization

JHEP 1211, 151(2012)

Four-Dimensional Unsubtraction

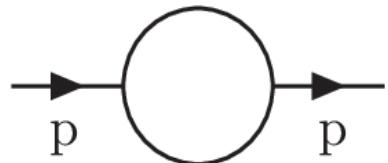
JHEP 1608 (2016) 160

- tailored to extract UV divergences
- complies with BPHZ (unitarity, locality, Lorentz invariance)
[A. C, Sampaio, Nemes \(2011\)](#)
- complies with abelian gauge invariance to all-orders
[Ferreira, A.C, Nemes, Hiller, Sampaio \(2012\)](#)
[Vieira, A.C, Sampaio\(2016\)](#)
- non-abelian gauge invariance working examples
[A. C, Arias-Perdomo, Vieira, Sampaio, Hiller \(2020\)](#)
- IR divergences under study (1 and 2 loop)
[Eur. Phys. J. C \(2017\) 77:471](#)
[Eur. Phys. J. C \(2021\) 81:250](#)

BPHZ at work

A. C, Sampaio, Nemes (2011)

Arias-Perdomo, A. C, Hiller, Sampaio(2021)



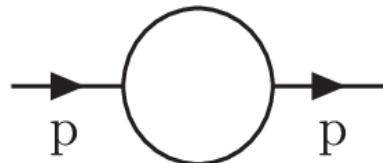
$$\Xi^{(1)} \equiv \frac{g^2}{2} \int_k \frac{1}{k^2} \frac{1}{(k-p)^2} = \lim_{\mu^2 \rightarrow 0} \frac{g^2}{2} \int_k \frac{1}{(k^2 - \mu^2)} \frac{1}{[(k-p)^2 - \mu^2]}, \quad \int_k \equiv \int \frac{d^6 k}{(2\pi)^6}$$

$$\Xi^{(1)} = \frac{g^2}{2} \int_k \frac{1}{(k^2 - \mu^2)} \left[\sum_{l=0}^{2(n^{(k)}-1)} f_l^{(k, p)} + \bar{f}^{(k, p)} \right]$$

BPHZ at work

A. C, Sampaio, Nemes (2011)

Arias-Perdomo, A. C, Hiller, Sampaio(2021)



$$\Xi^{(1)} = -\frac{g^2 p^2}{6} \left[I_{\log}(\lambda^2) - b_6 \ln \left(-\frac{p^2}{\lambda^2} \right) + \frac{8b_6}{3} \right].$$

2n dimensions

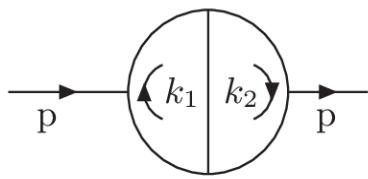
Renormalization group scale

$$I_{\log}(\lambda^2) \equiv \int_k \frac{1}{(k^2 - \lambda^2)^n}$$

BPHZ at work

A. C, Sampaio, Nemes (2011)

Arias-Perdomo, A. C, Hiller, Sampaio(2021)



$$\frac{\Xi_A^{(2)}}{ig^4} = \frac{1}{2} \int_{k_1 k_2} \Delta(k_1) \Delta(k_1 - p) \Delta(k_1 - k_2) \Delta(k_2) \Delta(k_2 - p), \quad \Delta(k_i) \equiv \frac{1}{k_i^2 - \mu^2}$$

$$\int_{k_1 k_2} \Delta(k_1) \Delta(k_1 - k_2) \Delta(k_2) \left[\sum_{l=0}^{2(n^{(k_1)}-1)} f_l^{(k_1, p)} + \bar{f}^{(k_1, p)} \right] \left[\sum_{m=0}^{2(n^{(k_2)}-1)} f_m^{(k_2, p)} + \bar{f}^{(k_2, p)} \right].$$

$k_1 \rightarrow \infty$ and k_2 fixed

$k_2 \rightarrow \infty$ and k_1 fixed

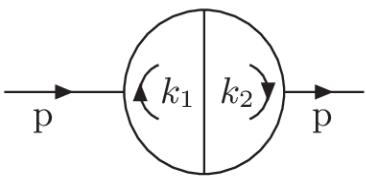
$k_1 \rightarrow \infty$ and $k_2 \rightarrow \infty$

Classify divergent terms according to

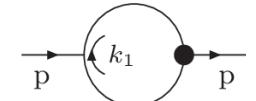
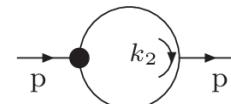
BPHZ at work

A. C, Sampaio, Nemes (2011)

Arias-Perdomo, A. C, Hiller, Sampaio(2021)



Counterterms automatically identified



$$\frac{\bar{\Xi}_A^{(2)}}{ig^4} \equiv \frac{b_6 p^2}{6} \left[2I_{\log}^{(2)}(\lambda^2) - \frac{13}{3} I_{\log}(\lambda^2) + \text{finite} \right].$$

● = $(-1) \times$ Divergence of

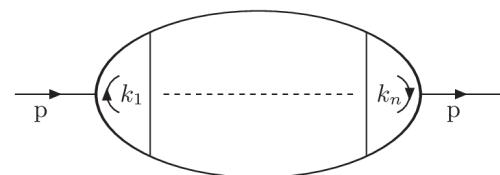
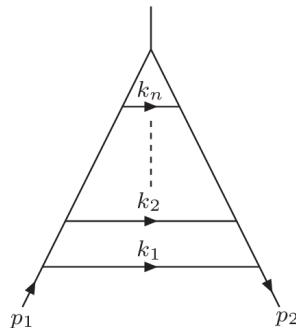
↙
2n dimensions

$$I_{\log}^{(l)}(\lambda^2) \equiv \int \frac{1}{(k_l^2 - \lambda^2)^n} \ln^{l-1} \left(-\frac{k_l^2 - \lambda^2}{\lambda^2} \right),$$

BPHZ at work

A. C, Sampaio, Nemes (2011)

Arias-Perdomo, A. C, Hiller, Sampaio(2021)



$$\frac{\bar{\Lambda}^{(n)}}{i^{n+1} g^{2n+1}} \equiv \sum_{m=1}^n c_m^{(n-1)} I_{log}^{(m)}(\lambda^2) + \text{finite},$$



Recursively obtained

Gauge theories

A. C, Arias-Perdomo, Vieira, Sampaio, Hiller (2020)

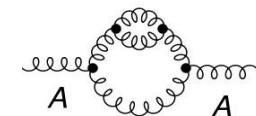
tailored to extract UV divergences

→ β - function

$$Z_g = Z_A^{-1/2}$$

Background field method

$$\int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} G(k, q, p) \rightarrow \begin{aligned} & \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - \lambda^2)^2} \quad (k \rightarrow q) \\ & \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - \lambda^2)^2} \ln \left(-\frac{k^2 - \lambda^2}{\lambda^2} \right) \quad (k \rightarrow q) \\ & \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{(k^2 - \lambda^2)^2} \frac{1}{(q^2 - \lambda^2)^2} \end{aligned}$$



Gauge theories

A. C, Arias-Perdomo, Vieira, Sampaio, Hiller (2020)

tailored to extract UV divergences

→ β - function

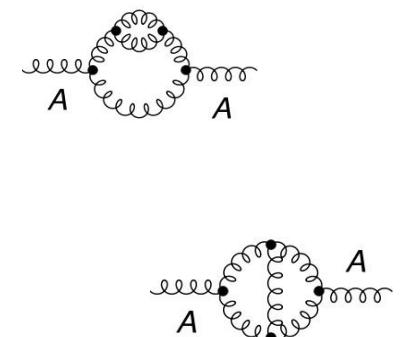
$$Z_g = Z_A^{-1/2}$$

Background field method

$$\int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} G(k, q, p) \rightarrow \begin{aligned} & \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - \lambda^2)^2} \quad (k \rightarrow q) \\ & \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - \lambda^2)^2} \ln \left(-\frac{k^2 - \lambda^2}{\lambda^2} \right) \quad (k \rightarrow q) \\ & \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{(k^2 - \lambda^2)^2} \frac{1}{(q^2 - \lambda^2)^2} \end{aligned}$$

$$\beta = -g\lambda \frac{\partial \ln Z_g}{\partial \lambda}$$

RGE scale



Gauge theories

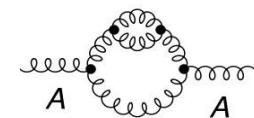
A. C, Arias-Perdomo, Vieira, Sampaio, Hiller (2020)

tailored to extract UV divergences

→ β - function

$$\beta = -g_s \left[\left(11 - \frac{2}{3} n_f \right) \left(\frac{g_s}{4\pi} \right)^2 + \left(102 - \frac{38}{3} n_f \right) \left(\frac{g_s}{4\pi} \right)^4 \right]$$

Background field method



- (UV part) comply with non-abelian gauge invariance
- Connection with dimensional methods (DReg and DRED)

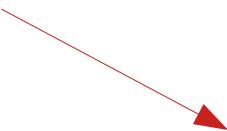
Gauge theories

A. C, Arias-Perdomo, Vieira, Sampaio, Hiller (2020)

$$\int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} G(k, q, p) \quad \longrightarrow \quad \mathcal{I}_{\text{IREG}}^{\text{div}} = c_0 I_{\log}^{(2)}(\lambda^2) + c_1 I_{\log}(\lambda^2) + c_3 I_{\log}^2(\lambda^2)$$



$$\mathcal{I}_{\text{DRED}}^{\text{div}} \Big|_d = \frac{a_0}{\epsilon^2} + \frac{\hat{a}_1}{\epsilon}$$



$$\mathcal{I}_{\text{IREG}}^{\text{div}} \Big|_d = \frac{a_0}{\epsilon^2} + \frac{a_1}{\epsilon}$$

- Connection with dimensional methods (DReg and DRED)

Implicit Regularization - chiral

Viglioni, **A.C**, Vieira, Hiller, Sampaio (2016)

Bruque, **A.C**, Pérez-Victoria (2018)

$$\{\gamma_\mu, \gamma_5\} = 0$$

Example – 2D (euclidean space)

$$\text{tr}(\{\gamma_5, \gamma_\mu\}\gamma_\nu\gamma_\rho\gamma_\sigma) = \text{tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) + \text{tr}(\gamma_\mu\gamma_5\gamma_\nu\gamma_\rho\gamma_\sigma) = -4(g_{\mu\nu}\epsilon_{\rho\sigma} - g_{\mu\rho}\epsilon_{\nu\sigma} + g_{\mu\sigma}\epsilon_{\nu\rho})$$

$$[(\text{tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) + \text{tr}(\gamma_\mu\gamma_5\gamma_\nu\gamma_\rho\gamma_\sigma))I^{\mu\sigma}]_R = 4\pi\epsilon_{\rho\nu} \neq 0$$

Implicit Regularization - chiral

Viglioni, A.C, Vieira, Hiller, Sampaio (2016)

Bruque, A.C, Pérez-Victoria (2018)

$$\{\gamma_\mu, \gamma_5\} = 0$$

Example – 2D (euclidean space)

Even in 4D methods, chiral theories must be dealt with care!

$$[(\text{tr}(\gamma_5 \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma) + \text{tr}(\gamma_\mu \gamma_5 \gamma_\nu \gamma_\rho \gamma_\sigma)) I^{\mu\sigma}]_R = -4 [(g_{\mu\nu} \epsilon_{\rho\sigma} - g_{\mu\rho} \epsilon_{\nu\sigma} + \text{circled } g_{\mu\sigma} \epsilon_{\nu\rho}) I^{\mu\sigma}]_R$$

$$g_{\mu\sigma} [I^{\mu\sigma}]_R = g_{\mu\sigma} \left[\int d^2 k \frac{k^\mu k^\sigma}{(k^2 + m^2)^2} \right] \neq \left[\int d^2 k \frac{k^2}{(k^2 + m^2)^2} \right] = [g_{\mu\sigma} I^{\mu\sigma}]_R$$

Implicit Regularization - chiral

Viglioni, A.C, Vieira, Hiller, Sampaio (2016)

Bruque, A.C, Pérez-Victoria (2018)

$$\gamma_5 \in GnS$$

$$QnS = GnS \oplus X$$

IReg

$$QdS = GnS \oplus Q(-2\epsilon)S$$

DReg

$$g_{\mu\sigma}[I^{\mu\sigma}]_R \neq [g_{\mu\sigma}I^{\mu\sigma}]_R$$

$$\bar{g}_{\mu\sigma}[I^{\mu\sigma}]_R = [\bar{g}_{\mu\sigma}I^{\mu\sigma}]_R$$

$$QnS = QdS \oplus Q(2\epsilon)S = GnS \oplus Q(-2\epsilon)S \oplus Q(2\epsilon)S$$

DRed

- BRST symmetry broken – need symmetry-restoring counterterms

Implicit Regularization - chiral

A.C, (2021)

- Toward two-loop level
 - abelian left-model

$$GnS \longrightarrow \mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}_L \not{\partial} \psi_L + e\bar{\psi}_L \not{A} \psi_L$$

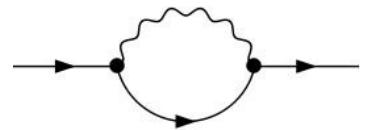
$$QnS = GnS \oplus X \longrightarrow \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi} \not{\partial} \psi + e\bar{\psi}_L \not{A} \psi_L$$

↓
gauge breaking term $i \left(\bar{\psi}_L \hat{\not{\partial}} \psi_R + \bar{\psi}_R \hat{\not{\partial}} \psi_L \right)$

Implicit Regularization - chiral

A.C, (2021)

$$\Sigma(p)|_{\text{div}} = ie^2 \left[\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - \lambda^2)^2} \right] \not{p} P_L$$



$$\Sigma(p)|_{\text{fin}} = \frac{e^2}{(4\pi)^2} \left[\log \left(-\frac{\bar{p}^2}{\lambda^2} \right) - 2 \right] \not{p} P_L$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{\partial} \psi + e\bar{\psi}_L \not{A} \psi_L$$

Implicit Regularization - chiral

A.C, (2021)

$$(p_1 + p_2)^\mu \sim \text{Diagram} \stackrel{?}{=} e \left[\text{Diagram}_1 - \text{Diagram}_2 \right]$$

The diagram on the left shows a vertex with two incoming lines and one outgoing wavy line. The wavy line has arrows indicating flow from left to right. The two incoming lines have arrows pointing towards the vertex. The right side of the equation shows the definition of the vertex function $\Sigma(p)$ as the sum of two terms, each represented by a loop diagram with two external lines labeled p_1 or p_2 .

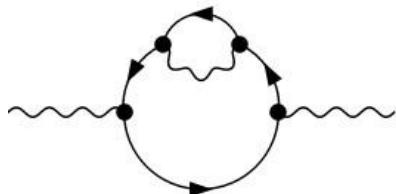
$$(p_1 + p_2)_\mu \Gamma^\mu(p_1, p_2) = e [\Sigma(p_1) - \Sigma(-p_2)] - \frac{e^3}{(4\pi)^2} (p_1 + p_2)_\mu \bar{\gamma}^\mu P_L$$

GnS

Implicit Regularization - chiral

A.C, (2021)

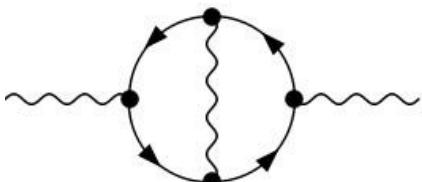
- tailored to extract UV divergences



$$\mathcal{A}_{\mu\nu}|_{\text{div}} = \frac{ie^4}{(4\pi)^4} [g_{\mu\nu}p^2 - p_\mu p_\nu] \mathcal{F}$$

transverse

QED



$$\beta = e \left[\frac{2}{3} \left(\frac{e}{4\pi} \right)^2 + 2 \left(\frac{e}{4\pi} \right)^4 \right]$$

$$\beta = e \left[\frac{4}{3} \left(\frac{e}{4\pi} \right)^2 + 4 \left(\frac{e}{4\pi} \right)^4 \right]$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{\partial} \psi - e\bar{\psi}_L \not{A} \psi_L$$

$$\frac{e}{2} \bar{\psi} \gamma_\mu (1 - \gamma_5) \psi A^\mu$$

naive scheme $\{\gamma_\mu, \gamma_5\} = 0$

Implicit Regularization - chiral

A.C, (2021)

naive scheme $\{\gamma_\mu, \gamma_5\} = 0$

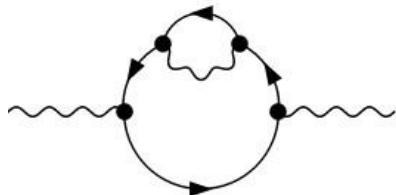


UV counterterms cancel
(subdivergences)

$$\beta = e \left[\frac{2}{3} \left(\frac{e}{4\pi} \right)^2 + 2 \left(\frac{e}{4\pi} \right)^4 \right]$$

Implicit Regularization - chiral

A.C, (2021)

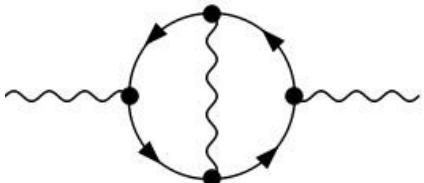


$$QnS = GnS \oplus X$$

transverse

$$\mathcal{A}'_{\mu\nu}|_{\text{div}} = \frac{ie^4}{(4\pi)^4} [\bar{g}_{\mu\nu}\bar{p}^2 - \bar{p}_\mu\bar{p}_\nu] \mathcal{F}'$$

QED

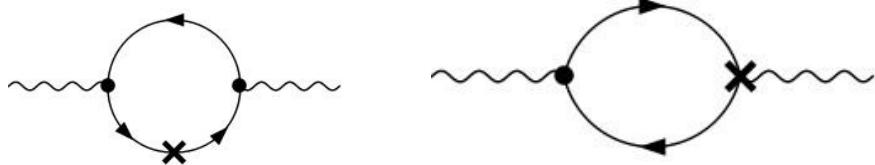


$$\beta = e \left[\frac{4}{3} \left(\frac{e}{4\pi} \right)^2 + 4 \left(\frac{e}{4\pi} \right)^4 \right]$$

$$\beta = e \left[\frac{2}{3} \left(\frac{e}{4\pi} \right)^2 + \frac{10}{3} \left(\frac{e}{4\pi} \right)^4 \right]$$

Implicit Regularization - chiral

A.C, (2021)



- UV counterterms DO NOT cancel (subdivergences)

$$QnS = GnS \oplus X$$

$$\Sigma(p)|_{\text{div}} = ie^2 \left[\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - \lambda^2)^2} \right] \not{p} P_L$$

\downarrow

GnS



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{\partial} \psi + e\bar{\psi}_L \not{A} \psi_L$$

QnS

23 / 32

Implicit Regularization - chiral

A.C, (2021)



$$QnS = GnS \oplus X$$

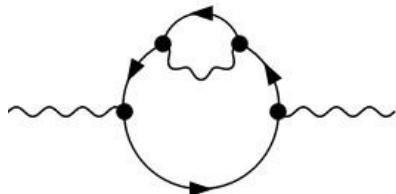
- Finite restoring-symmetry counterterms

$$(p_1 + p_2)_\mu \Gamma^\mu(p_1, p_2) = e [\Sigma(p_1) - \Sigma(-p_2)] - \frac{e^3}{(4\pi)^2} (p_1 + p_2)_\mu \bar{\gamma}^\mu P_L$$

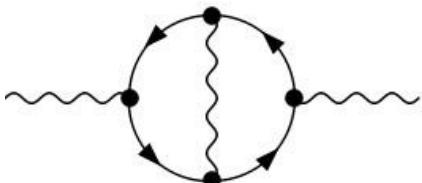
GnS

Implicit Regularization - chiral

A.C, (2021)



$$\beta = e \left[\frac{2}{3} \left(\frac{e}{4\pi} \right)^2 + 2 \left(\frac{e}{4\pi} \right)^4 \right]$$



- Same result of naive scheme

$$QnS = GnS \oplus X$$

QED

$$\beta = e \left[\frac{4}{3} \left(\frac{e}{4\pi} \right)^2 + 4 \left(\frac{e}{4\pi} \right)^4 \right]$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{\partial} \psi - e\bar{\psi}_L \not{A} \psi_L$$

$$\frac{e}{2} \bar{\psi} \not{A} (1 - \gamma_5) \psi \quad 25 / 32$$

Standard Model – 2 loop

A.C, (2021)

$$\beta_1 = \frac{\alpha_1^2}{(4\pi)^2} \left[\frac{2}{5} + \frac{16}{3} n_f \right] + \frac{\alpha_1^2}{(4\pi)^3} \left[\frac{18\alpha_1}{25} + \frac{18\alpha_2}{5} - \frac{34\text{tr}\hat{T}}{5} - 2\text{tr}\hat{B} - 6\text{tr}\hat{L} \right. \\ \left. + n_f \left(\frac{76\alpha_1}{15} + \frac{12\alpha_2}{5} + \frac{176\alpha_3}{15} \right) \right]$$

$$\beta_2 = \frac{\alpha_2^2}{(4\pi)^2} \left[-\frac{86}{3} + \frac{16}{3} n_f \right] + \frac{\alpha_2^2}{(4\pi)^3} \left[\frac{6\alpha_1}{5} - \frac{518\alpha_2}{3} - 6\text{tr}\hat{T} - 6\text{tr}\hat{B} - 2\text{tr}\hat{L} \right. \\ \left. + n_f \left(\frac{4\alpha_1}{5} + \frac{196\alpha_2}{3} + 16\alpha_3 \right) \right]$$

$$\beta_3 = \frac{\alpha_3^2}{(4\pi)^2} \left[-44 + \frac{16}{3} n_f \right] + \frac{\alpha_3^2}{(4\pi)^3} \left[-408\alpha_3 - 8\text{tr}\hat{T} - 8\text{tr}\hat{B} \right. \\ \left. + n_f \left(\frac{22\alpha_1}{15} + 6\alpha_2 + \frac{304\alpha_3}{3} \right) \right]$$

$$Z_g = Z_A^{-1/2}$$

Background field method

Conclusions

- Given the prospects for future years, it is a necessity to increase precision of electroweak radiative corrections;
- It is well-known that the treatment of chiral theories is tricky in dimensional methods;
- Regularization methods in 4D share similar problems. However, the setup may be simpler;
- Among 4D regularization methods, Implicit Regularization provides an algorithmic to extract UV divergent parts of Feynman amplitudes, in a way compatible to BPHZ;
- Some applications:
 - simple abelian chiral model at two-loop level as a working example
 - Standard Model gauge coupling beta function to two-loop level

Backup

Backup

$$\frac{1}{(k - p_i)^2 - \mu^2} = \sum_{j=0}^{n_i^{(k)} - 1} \frac{(-1)^j (p_i^2 - 2p_i \cdot k)^j}{(k^2 - \mu^2)^{j+1}} + \frac{(-1)^{n_i^{(k)}} (p_i^2 - 2p_i \cdot k)^{n_i^{(k)}}}{(k^2 - \mu^2)^{n_i^{(k)}} [(k - p_i)^2 - \mu^2]},$$

$$I_{\log}^{\nu_1 \dots \nu_{2r}}(\mu^2) \equiv \int_k \frac{k^{\nu_1} \dots k^{\nu_{2r}}}{(k^2 - \mu^2)^{r+2}} \int_k \frac{\partial}{\partial k_\mu} \frac{k^\nu}{(k^2 - \mu^2)^n} = 4 \left[\frac{g_{\mu\nu}}{4} I_{\log}(\mu^2) - I_{\log}^{\mu\nu}(\mu^2) \right] = 0,$$

$$I_{\log}(\mu^2) = I_{\log}(\lambda^2) + \frac{i}{(4\pi)^2} \ln \frac{\lambda^2}{\mu^2}$$

Backup

$$QnS \quad g_{\mu\sigma}[I^{\mu\sigma}]_R \neq [g_{\mu\sigma}I^{\mu\sigma}]_R$$

$$GnS \quad \bar{g}_{\mu\sigma}[I^{\mu\sigma}]_R = [\bar{g}_{\mu\sigma}I^{\mu\sigma}]_R$$

$$i\Pi_{\mu\nu}(p) = (-)(-ie)^2 \int_k \text{Tr} \left\{ \gamma_\mu \frac{i}{(\not{k})} \gamma_\nu \frac{i}{(\not{k} - \not{p})} \right\}.$$

$$i\Pi_{\mu\nu}(p) = (-e^2) \text{Tr} \left\{ \gamma_\mu \gamma_\alpha \gamma_\nu \gamma_\beta (I_{\alpha\beta} - I_\alpha p_\beta) \right\} \quad \text{where} \quad I_{\alpha_1 \dots \alpha_n} = \int_k \frac{k_{\alpha_1} \cdots k_{\alpha_n}}{k^2(k-p)^2}.$$

$$i \frac{\Pi_{\mu\nu}}{(-e^2)} = \frac{4}{3} \left[I_{\log}(\lambda^2) - b \ln \left(-\frac{p^2}{\lambda^2} \right) + \frac{5}{3} b \right] (g_{\mu\nu} p^2 - p_\mu p_\nu) + \frac{2b}{3} g_{\mu\nu}$$

$$\int_k \frac{k^2}{k^2(k-p)^2} = \int_k \frac{1}{(k-p)^2} = 0 \neq g^{\alpha\beta} \int_k \frac{k_\alpha k_\beta}{k^2(k-p)^2} = -\frac{bp^2}{6}$$

Backup

$$f_{\mu\nu} = \int d^2k \frac{\partial}{\partial k_\mu} \frac{k_\nu}{k^2 + m^2} = \int d^2k \left(\frac{\delta_{\mu\nu}}{k^2 + m^2} - 2 \frac{k_\mu k_\nu}{(k^2 + m^2)^2} \right)$$

$$[I_{\mu\nu}]^R = \frac{1}{2} \delta_{\mu\nu} \left[\int d^2k \frac{1}{k^2 + m^2} \right]^R = \frac{1}{2} \delta_{\mu\nu} \left(\left[\int d^2k \frac{k^2}{(k^2 + m^2)^2} \right]^R + \left[\int d^2k \frac{m^2}{(k^2 + m^2)^2} \right]^R \right) = \frac{1}{2} \delta_{\mu\nu} \left([I_{\alpha\alpha}]^R + \pi \right)$$

$$\begin{aligned} [I_{\mu\nu}]^R &= \left[\int d^d k \frac{k_\mu k_\nu}{(k^2 + m^2)^2} \right]^S = \left[\int d^d k \frac{1}{d} \delta_{\mu\nu} \frac{k^2}{(k^2 + m^2)^2} \right]^S = \left[\int d^d k \left(\frac{1}{2} + \frac{\varepsilon}{4} + O(\varepsilon^2) \right) \delta_{\mu\nu} \frac{k^2}{(k^2 + m^2)^2} \right]^S \\ &= \left[\frac{1}{2} \delta_{\mu\nu} \int d^d k \frac{k^2}{(k^2 + m^2)^2} + \left(\frac{\varepsilon}{4} + O(\varepsilon^2) \right) \delta_{\mu\nu} \left(2\pi \frac{1}{\varepsilon} + O(\varepsilon^0) \right) \right]^S = \frac{1}{2} \delta_{\mu\nu} \left([I_{\alpha\alpha}]^R + \pi \right), \end{aligned}$$

Backup

$$\mathcal{L}_{amend} = i \frac{\delta_L}{2} \left(\bar{\psi}_L \hat{\partial} \psi_R + i \delta_L \bar{\psi}_R \hat{\partial} \psi_L \right) + i \delta_L \bar{\psi}_R \bar{\partial} \psi_R$$