

Chiral Theories in DimReg & Breitenlohner-Maison / 't Hooft-Veltman scheme: Application to χ QED at 2 loops

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Based on [[arXiv:2004.14398 \(JHEP 08 \(2020\) 08, 024\)](#)] and [[Work In Progress](#)],
with Amon Ilakovac, Marija Mađor-Božinović (PMF Zagreb),

Paul Kühler and Dominik Stöckinger (IKT, TU Dresden)

Motivation: Dimensional Regularization and γ_5 (1/2)

- Theory: Divergent multi-loop Feynman integrals; Regularization.
Experiment: Running of parameters; Renormalization.
Fundamental QFT properties: Unitarity, Causality. Renormalizable gauge(-fixed) theories → **BRST symmetry** must **remain preserved**.
- **Dimensional Regularization** (DReg, [['t Hooft,Veltman–1972](#)]...): $\mu^{4-d} \int d^d x$ widely used in calculations / literature / automated codes, etc.: doesn't break gauge and Lorentz symmetries (**as long as NO** γ_5 , e.g. QCD).

$d = 4 - 2\epsilon$ “dimensions”: $\mathbb{M}_d = \mathbb{M}_4 \oplus \mathbb{M}_{-2\epsilon}$.

Small $\epsilon > 0$ regularizes UV divergences ($\epsilon < 0$ for IR divs.).

Lorentz objects (metrics, ...): $X_{\mu\dots} = \overline{X}_{\mu\dots} + \widehat{X}_{\mu\dots}$

$\widehat{X}_{\mu\dots}$: **evanescent** objects.

$$g_{\mu\nu} g^{\nu\mu} = d, \quad \bar{g}_{\mu\nu} \bar{g}^{\nu\mu} = 4, \quad \hat{g}_{\mu\nu} \hat{g}^{\nu\mu} = -2\epsilon.$$



[1] Robbert 't Hooft and Martin Veltman

Motivation: Dimensional Regularization and γ_5 (2/2)

- Observable nature **chiral** \Rightarrow Realistic 4D models contain **chiral** fermions (e.g. Standard Model, ...) $\rightsquigarrow \mathbb{P}_{R/L} = (1 \pm \gamma_5)/2$. \rightsquigarrow chiral anomaly, pion decay...
 - ▶ DimReg and Dirac γ^μ matrices? [Collins-1986] Intrinsic 4D objects $\gamma_5, \epsilon_{\mu\nu\rho\sigma}$?
- In 4D: $\{\gamma_5, \gamma^\mu\} = 0$, $\text{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) = 4i\epsilon_{\mu_1\mu_2\mu_3\mu_4}$, $\text{Tr}(ab) = \text{Tr}(ba)$.
Inconsistent in d -D: $\text{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) \propto (d-4) \text{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) \underset{d \rightarrow 4}{=} 0$.
- Semi-“naive” γ_5 + manual traces fixes, syms. restoration (using Ward IDs, ...):

$$\{\gamma_5, \gamma^\mu\} = 0, \quad \text{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) \neq 4i\epsilon_{\mu_1\mu_2\mu_3\mu_4}, \quad \text{Tr}(ab) = \text{Tr}(ba),$$
- Non-cyclicity schemes [Kreimer-1990,'94] (“reading-point prescription”, ...):

$$\{\gamma_5, \gamma^\mu\} = 0, \quad \text{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) = 4i\epsilon_{\mu_1\mu_2\mu_3\mu_4}, \quad \text{Tr}(ab) \neq \text{Tr}(ba).$$
- \exists numerous other γ_5 schemes (see e.g. the reviews [Gnendiger...-2017, Bruque...-2018], and [Larin-1993, Trueman-1995, Jegerlehner-2000]).

Consistency wrt. unitarity/causality *not always clear* at high orders...

DimReg, BMHV scheme. Our goals.

't Hooft-Veltman-Breitenlohner-Maison ("BMHV") scheme

[Breitenlohner,Maison–1975, Breitenlohner,Maison–1977]

$$\gamma_5 = (i/4!) \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma, \{ \gamma_5, \bar{\gamma}^\mu \} = 0, \text{ but } [\gamma_5, \hat{\gamma}^\mu] = 0,$$

and: $\{ \gamma_5, \gamma^\mu \} = \{ \gamma_5, \hat{\gamma}^\mu \}$, $[\gamma_5, \gamma^\mu] = [\gamma_5, \bar{\gamma}^\mu]$.

Cyclic trace, and $\text{Tr}(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_4}) = 4i \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4}$.



Proven axiomatically consistent (*unitarity/causality*) at all orders.

1/ ϵ -pole (e.g. MS(bar)) subtraction \Rightarrow “**Dimensional renormalization**” (DimRen).

- ▶ Take DReg/DimRen+BMHV **seriously** w/o compromises. Computer-algebraic progress makes this more tractable.
- ▶ Apply our work [[arXiv:2004.14398](#)]+[[in progress](#)], based on [[Martin,Sanchez-Ruiz–1999](#)] to the **massless chiral QED** (χ QED) at 1 and 2 loops.
- ▶ **Catalogue of 1 and 2 loops** singular and **BRST-restoring finite counterterms**, as a basis for **consistent higher-order calculations** in the BMHV scheme.

Outline

1 Chiral Right-handed QED (χ QED)

- χ QED action S_0 ; extension to d dimensions
- Completed χ QED in d dimensions
- BRST invariance of χ QED? Restore @ loop-level?

2 Calculations @ 1 loop

- 1-loop singular counterterm action
- Restore BRST invariance @ 1 loop
- Results: 1-loop finite counterterm action

3 Calculations @ 2 loops

- 2-loop singular counterterm action
- Restore BRST invariance @ 2 loops
- Results: 2-loop finite counterterm action

4 Discussion & Summary

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Chiral Right-handed QED (χ QED) in $d = 4$

$$\begin{aligned}
 S_0^{(4D)} &= \int d^4x \left(i\overline{\psi_R}_i \not{D}_{ij} \psi_{Rj} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 - \bar{c} \partial^2 c \right. \\
 &\quad \left. + \rho^\mu s A_\mu + \bar{R}^i s \psi_{Ri} + s \overline{\psi_R}_i \bar{R}^i \right) \\
 &= (\overline{S_{\bar{\psi}\psi_R}} + \overline{S_{\psi_R A \psi_R}}) + \overline{S_{AA}} + S_{\text{g-fix}} + \overline{S_{\bar{c}c}} + \overline{S_{\rho c A}} + \overline{S_{\bar{R}c\psi}} + \overline{S_{\bar{\psi}cR}}.
 \end{aligned}$$

- ▶ RH fermions $\psi_{Ri} \equiv \mathbb{P}_R \psi_i$, “Generators” \mathcal{Y}_{Rij} .
- ▶ Covariant derivative $D_{ij}^\mu = \partial^\mu \delta_{ij} - ie_A \mathcal{Y}_{Rij} A^\mu$,
Field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
- ▶ **R_ξ gauge-fixing** $-\frac{1}{2\xi} (\partial_\mu A^\mu)^2 \sim \frac{\xi}{2} B^2 + B \partial^\mu A_\mu$; **Ghost field** c .
- ▶ **External BRST sources K_ϕ :** ρ^μ, \bar{R}^i, R^i , sourcing the BRST transformations

$$\begin{aligned}
 sA_\mu &= \partial_\mu c, & s\psi_{Ri} &= ie_A c \mathcal{Y}_{Rij} \psi_{Rj}, \\
 s\bar{c} &= B \equiv -\frac{1}{\xi} \partial A, & s\overline{\psi_R}_i &= ie_A \overline{\psi_R}_j c \mathcal{Y}_{Rji}.
 \end{aligned}$$

(Similar to **Batalin-Vilkovisky** “antifields” [Batalin,Vilkovisky–1977,’81,’84].)

Extension to d dimensions

Bosonic fields in d -dims. Chiral fermions introduce **two problems**:

- 1 Kinetic term is **chiral** \Rightarrow non-regularized propagator $\propto 1/\bar{p}$ in d -D [Bilal-2008]

\Rightarrow Need an actual d -D kinetic term:

\approx “left-handed inert” fermion component.

(Inert because removed in interactions due to $\mathbb{P}_{R/L}$.)

$$i\bar{\psi}_i \not{\partial} \psi_i$$

- 2 How to promote in d -D the $\bar{\psi} \mathbb{P}_L \not{\partial} \mathbb{P}_R \psi$ interaction term $\propto \bar{\psi} \gamma^\mu \mathbb{P}_R \psi$?

$\gamma_\mu \mathbb{P}_R = \mathbb{P}_L \gamma_\mu = \mathbb{P}_L \gamma_\mu \mathbb{P}_R$ only in 4D, not in d -D.

$\bar{\psi} \gamma^\mu \mathbb{P}_R \psi$, $\bar{\psi} \mathbb{P}_L \gamma^\mu \psi$, $\bar{\psi} \mathbb{P}_L \gamma^\mu \mathbb{P}_R \psi$

\Rightarrow NO unique d -dimensional extension!

\Rightarrow Use the interaction term that makes calculations **the most simple**:

$$\bar{\psi} \mathbb{P}_L \gamma^\mu \mathbb{P}_R \psi$$

“symmetric chiral-projection”

(Explicitly conveys the fact that fermions are chiral.)

\equiv Larin symmetrization prescription $\frac{1}{2} (\gamma^\mu - \gamma_5 \gamma^\mu \gamma_5) \mathbb{P}_R$.

The completed χ QED in d dimensions

The complete defining χ QED action in d dimensions:

$$S_0 = \int d^d x \left(i\bar{\psi}_i \not{D}_{ij} \psi_j - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 - \bar{c} \partial^2 c + \mathcal{L}_{\text{ext}} \right),$$

with:

$$i\bar{\psi}_i \not{D}_{ij} \psi_j = i\bar{\psi}_i \not{\partial} \psi_i + e_A \mathcal{Y}_{Rij} \bar{\psi}_i \not{P_L} \not{A} \not{P_R} \psi_j,$$

and the external BRST-source terms:

$$\mathcal{L}_{\text{ext}} = \rho^\mu s_d A_\mu + \bar{R}^i s_d \psi_{Ri} + s_d \overline{\psi_{Ri}} R^i.$$

$$S_0 = (\overline{S_{\bar{\psi}\psi}} + \widehat{\mathbf{S}_{\bar{\psi}\psi}} + \overline{S_{\bar{\psi}_R A \psi_R}}) + S_{AA} + S_{\text{g-fix}} + S_{\bar{c}c} + S_{\rho c A} + S_{\bar{R}c\psi} + S_{\bar{\psi}cR}.$$

All-loop order BRST invariance?

Aim: Verifying/enforcing BRST invariance \forall orders of perturbation.

Algebraic renormalization framework.

BRST invariance for **quantum effective action** Γ (up to $\mathcal{O}(\hbar^n)$):

Functional Slavnov-Taylor Identities (STI) (\sim Ward IDs (WTI) with gauge transfos.):

$$\mathcal{S}(\Gamma) \equiv \int dx \left(\sum_{\Phi=A,\psi,\bar{\psi},c} \text{Tr} \frac{\delta\Gamma}{\delta K_\Phi(x)} \frac{\delta\Gamma}{\delta\Phi(x)} + B(x) \frac{\delta\Gamma}{\delta\bar{c}(x)} \right) \stackrel{?}{=} 0.$$

($S\Gamma_{\text{ren}}$: in 4 dims on renormalized Γ_{ren} ; $S_d\Gamma_{\text{DReg}}$: in DimReg on Γ_{DReg} .)

Quantum Action Principle [Lowenstein–1971, Piguet, Sorella–1995], [Piguet, Rouet–1981]
 \Rightarrow BRST/ST breaking as a *local operator insertion* Δ in Γ :

$$\mathcal{S}(\Gamma) = \Delta \cdot \Gamma.$$

BRST restoration really matters only at the renormalized level (in 4D).

BRST invariance/breaking @ tree-level?

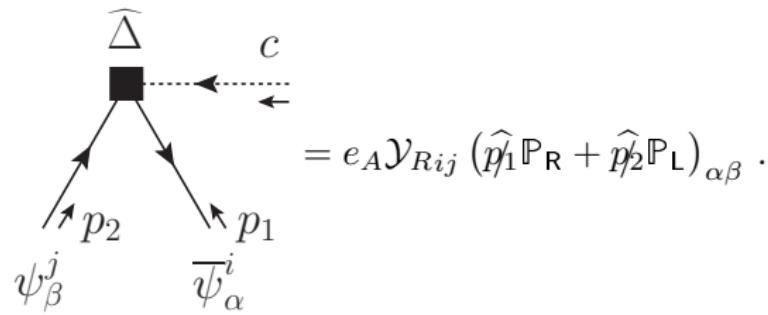
- χ QED is BRST-invariant at tree-level in 4D due to gauge symmetry:

$$\mathcal{S}_4 S_0^{(4D)} = 0.$$

- Is it still so in d -dimensions? \Rightarrow No! \exists BRST breaking $\widehat{\Delta}$ at tree-level:

$$\mathcal{S}_d S_0 = \textcolor{red}{s_d \widehat{S}_{\bar{\psi}\psi}} = \int d^d x e_A \mathcal{Y}_{Rij} c \left\{ \bar{\psi}_i \left(\overleftarrow{\widehat{\partial}} \mathbb{P}_R + \overrightarrow{\widehat{\partial}} \mathbb{P}_L \right) \psi_j \right\} \equiv \widehat{\Delta}.$$

Interpreted as an interaction vertex whose Feynman rule is:



“Loop”-level BRST restoration; Renormalized action (1/2)

$\mathcal{S}\Gamma = \Delta \cdot \Gamma$ generalized for Γ_{DReg} with Regularized QAP [Breitenlohner,Maison–1977]:

using $\Delta_d \equiv \widehat{\Delta} + \Delta_{\text{ct}}$,

$$\mathcal{S}_d \Gamma_{\text{DReg}} = \Delta_d \cdot \Gamma_{\text{DReg}} \quad \underset{d \rightarrow 4}{\rightsquigarrow} \quad \mathcal{S}\Gamma_{\text{ren}} = \text{LIM}_{d \rightarrow 4} (\mathcal{S}_d \Gamma_{\text{DReg}}) = \Delta \cdot \Gamma_{\text{ren}}.$$

($\text{LIM}_{d \rightarrow 4}$: take $d \rightarrow 4$ and cancel evanescent structures. $\Gamma_{\text{ren}} \equiv \text{LIM}_{d \rightarrow 4} (\Gamma_{\text{DReg}})$.)

At $\mathcal{O}(\hbar^{n+1})$:

$$\begin{aligned} (\mathcal{S}\Gamma_{\text{ren}})^{(n+1)} &= \text{LIM}_{d \rightarrow 4} \{ \Delta_d^{(\leq n)} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(n+1)} + \Delta_{\text{sct}}^{(n+1)} \} \\ &\quad + N[\Delta_d^{(\leq n)}] \cdot \Gamma_{\text{ren}}|^{(n+1)} + \Delta_{\text{fct},4}^{(n+1)}, \end{aligned}$$

$$\Delta_{\text{ct}} = \Delta_{\text{sct}} + \Delta_{\text{fct}} \equiv s_d S_{\text{sct}} + s_d S_{\text{fct}}, \quad \Delta_{\text{fct},4} \equiv s_4 S_{\text{fct},4} = \text{LIM}_{d \rightarrow 4} (s_d S_{\text{fct}}).$$

S_{fct} : such that $\Delta_{\text{fct},4}$ cancels the irrelevant anomalies from $N[\Delta_d] \cdot \Gamma_{\text{ren}}$.

Remove irrelevant anomalies if possible, with **Finite CT action** S_{fct} .

Relevant anomalies cannot be removed: ~~BRST symmetry, renormalizability~~.

“Loop”-level BRST restoration; Renormalized action (2/2)

$$\Delta_d \equiv \widehat{\Delta} + \Delta_{\text{ct}} ; \quad (S\Gamma_{\text{ren}})^{(n+1)} = \lim_{d \rightarrow 4} \{ \Delta_d^{(\leq n)} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(n+1)} + \Delta_{\text{sct}}^{(n+1)} \} \\ + N[\Delta_d^{(\leq n)}] \cdot \Gamma_{\text{ren}}|^{(n+1)} + \Delta_{\text{fct},4}^{(n+1)} .$$

General Procedure at a fixed \hbar^{n+1} order:

- 1 Do the procedure at the previous order \hbar^n .
- 2 Evaluate $S_{\text{sct}}^{(n+1)}$ and $\Delta_{\text{sct}}^{(n+1)} = s_d S_{\text{sct}}^{(n+1)}$.
- 3 Evaluate $\Delta_d^{(\leq n)} \cdot \Gamma_{\text{DReg}}|^{(n+1)}$: loop diagrams with insertion of $\Delta_d^{(\leq n)}$.
- 4 Check whether their divergent part cancels with $\Delta_{\text{sct}}^{(n+1)}$ (breaking is finite).
Evaluate their finite 4-dimensional part: $N[\Delta_d^{(\leq n)}] \cdot \Gamma_{\text{ren}}|^{(n+1)}$.
- 5 Define $S_{\text{fct},4}^{(n+1)}$ such that $\Delta_{\text{fct},4}^{(n+1)} = s_4 S_{\text{fct},4}^{(n+1)} \stackrel{\text{def.}}{=} -N[\Delta_d^{(\leq n)}] \cdot \Gamma_{\text{ren}}|^{(n+1)}$ (“irrelevant anomalies”), and verify the *absence of relevant anomalies*.

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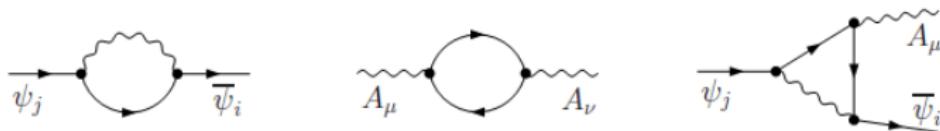
3 Calculations @ 2 loops

- 2-loop singular counterterm action
- Restore BRST invariance @ 2 loops
- Results: 2-loop finite counterterm action

4 Discussion & Summary

1-loop singular counterterm $S_{\text{sct}}^{(1)}$ action

1-loop SCT action evaluated from 1-loop diagrams (self-energies, vertices):



$$S_{\text{sct}}^{(1)} = -\Gamma^{(1)}|_{\text{div}}^{\text{BMHV}} = S_{\text{sct},\text{inv}}^{(1)} + S_{\text{sct},\text{evan}}^{(1)}.$$

$S_{\text{sct},\text{inv}}^{(1)}$ arises from usual renormalization transformation, $S_{0,\text{inv}} \rightarrow S_{0,\text{inv}} + S_{\text{ct},\text{inv}}$:

$$S_{\text{sct},\text{inv}}^{(1)} = \frac{-\hbar e_A^2}{16\pi^2\epsilon} \left(\frac{2 \text{Tr}[\mathcal{Y}_R^2]}{3} S_{AA} + \xi_A \sum_j (\mathcal{Y}_R^j)^2 \left(\overline{S_{\bar{\psi}\psi_R}^j} + \overline{S_{\bar{\psi}_RA\psi_R}^j} \right) \right).$$

Second term specific to BMHV scheme, arises from fermion loops, & evanescent:

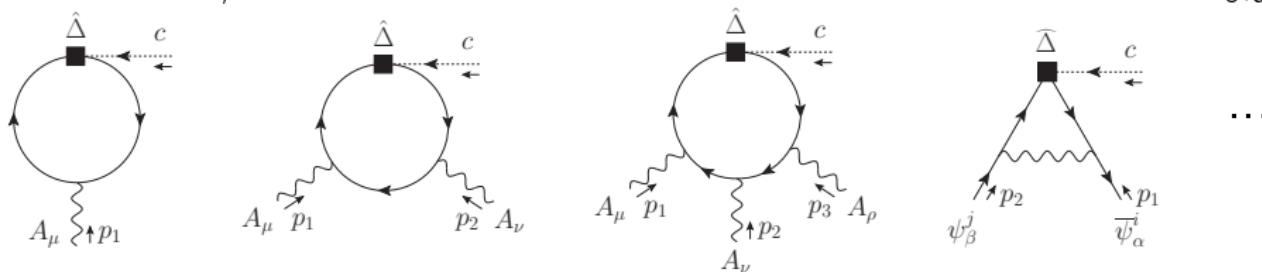
$$S_{\text{sct},\text{evan}}^{(1)} = \frac{-\hbar}{16\pi^2\epsilon} \frac{e_A^2 \text{Tr}[\mathcal{Y}_R^2]}{3} \int d^d x \frac{1}{2} \left(F_{\mu\nu} F^{\mu\nu} - \overline{F_{\mu\nu} F^{\mu\nu}} + \bar{A}_\mu \hat{\partial}^2 \bar{A}^\mu \right).$$

(Specific case of our [[arXiv:2004.14398](#)])

Evaluation of $\Delta_{\text{sct}}^{(1)}$ – Cancellation with $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(1)}$

$$(\mathcal{S}\Gamma_{\text{ren}})^{(1)} = \underset{d \rightarrow 4}{\text{LIM}} \{ \widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(1)} + \Delta_{\text{sct}}^{(1)} \} + N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} + \Delta_{\text{fct},4}^{(1)}.$$

$\Delta_{\text{sct}}^{(1)} = s_d S_{\text{sct,even}}^{(1)} \neq 0$. Calculate the non-zero terms contributing to $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(1)}$.



| Terms from $s_d S_{\text{sct}}^{(1)} \times \left(\frac{\hbar}{16\pi^2\epsilon}\right)^{-1}$ | Terms from $\widehat{\Delta} \cdot \Gamma_{\text{DReg}} _{\text{div.}}^{(1)} \times \left(\frac{\hbar}{16\pi^2\epsilon}\right)^{-1}$ |
|--|---|
| $-e_A^2 \frac{\text{Tr}[y_R^2]}{3} s_d \int d^d x \frac{1}{2} \bar{A}_\mu \widehat{\partial}^2 \bar{A}^\mu =$ $-e_A^2 \frac{\text{Tr}[y_R^2]}{3} \int d^d x (\bar{\partial}_\mu c)(\widehat{\partial}^2 \bar{A}^\mu)$ -0 | $i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}_{Ac}^\mu]_{\text{div.}}^{(1)} + i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}_{AAc}^{\nu\mu}]_{\text{div.}}^{(1)} \Rightarrow$ $S_{cA} = e_A^2 \frac{\text{Tr}[y_R^2]}{3} \int d^d x (\bar{\partial}_\mu c)(\widehat{\partial}^2 \bar{A}^\mu)$ $+ S_{cAA} = 0$ |
| 0 | $i[\widehat{\Delta} \cdot \Gamma_{\text{DReg}}_{\psi\bar{\psi}c}^{ji}]_{\text{div.}}^{(1)} \Rightarrow S_{c\bar{\psi}\psi} = 0$ |

Evaluation of the finite part: $N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)}$: Bonneau IDs

$$(\mathcal{S}\Gamma_{\text{ren}})^{(1)} = \underset{d \rightarrow 4}{\text{LIM}} \{ \widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(1)} + \Delta_{\text{sct}}^{(1)} \} + N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} + \Delta_{\text{fct},4}^{(1)}.$$

$$N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} = \underset{d \rightarrow 4}{\text{LIM}} [\widehat{\Delta} \cdot \Gamma^{(1)}]_{\text{fin}},$$

finite part of $\widehat{\Delta} \cdot \Gamma_{\text{DReg}}$ after renormalization (removal of divs. and taking $\underset{d \rightarrow 4}{\text{LIM}}$).

@ Fixed \hbar order: **limited finite number** of UV-singular diagrams.

Shown by interpreting $N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)}$ using **Bonneau Identities** [Bonneau–1980]:

$$\text{At } \mathcal{O}(\hbar): \quad N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} = N \left[-\text{r.s.p.} \left[N[\check{\Delta}] \cdot \Gamma \right]_{\check{g}=0}^{(1)} \right] \equiv \underset{d \rightarrow 4}{\text{LIM}} \left(-\text{r.s.p.} \left[\check{\Delta} \cdot \Gamma \right]_{\check{g}=0}^{(1)} \right).$$

“r.s.p.”: residue of simple pole in $\nu = 4 - d = 2\epsilon$. $\check{\Delta}$: $\widehat{\Delta}$ and formally replace $\hat{g}_{\mu\nu} \rightsquigarrow \check{g}_{\mu\nu}$ with:

$$\check{g}_{\mu\nu} g^{\nu\rho} = \check{g}_{\mu\nu} \hat{g}^{\nu\rho} = \check{g}_\mu^\rho, \quad \check{g}_{\mu\nu} \bar{g}^{\nu\rho} = 0, \quad \check{g}_\mu^\mu = 1.$$

⊕ no residual finite evanescent terms \Rightarrow *Main advantage of this method.*

Results: Anomalies = 0; Finite counterterms $S_{\text{fct}}^{(1)}$

$$(\mathcal{S}\Gamma_{\text{ren}})^{(1)} = \underset{d \rightarrow 4}{\text{LIM}} \{ \widehat{\Delta} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(1)} + \Delta_{\text{sct}}^{(1)} \} + N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} + \Delta_{\text{fct},4}^{(1)}.$$

Relevant anomalies? $\frac{-\hbar}{16\pi^2} \frac{e_A^3}{3} \int d^4x \epsilon^{\mu\nu\rho\sigma} c d_\psi (\partial_\rho A_\mu) (\partial_\sigma A_\nu)$, with: $d_\psi = 2 \text{Tr}[\mathcal{Y}_R^3]$ the anomaly coefficient, **that we choose = 0**, e.g. SM with correct hypercharges.

Finite $\mathcal{O}(\hbar)$ counterterms $S_{\text{fct}}^{(1)}$ such that $\Delta_{\text{fct},4}^{(1)} = s_4 S_{\text{fct}}^{(1)} = -N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)}$:

$$S_{\text{fct}}^{(1)} = \frac{\hbar}{16\pi^2} \frac{e_A^2}{6} \left\{ \int d^4x \left(-\text{Tr}[\mathcal{Y}_R^2] \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e_A^2 \text{Tr}[\mathcal{Y}_R^4]}{2} \bar{A}_\mu \bar{A}^\mu \bar{A}_\nu \bar{A}^\nu \right) + (5 + \xi) (\mathcal{Y}_R^j)^2 \overline{S_{\bar{\psi}\psi_R}} \right\}$$

+ any BRST-invariant term .

(Specific case of our [arXiv:2004.14398])

- ▶ **Not gauge invariant!** (\equiv –breaking) and **non-vanishing**.
- ▶ No $S_{\bar{R}c\Psi_R}$, $S_{Rc\overline{\Psi_R}}$ terms: $U(1)$ abelian, $\propto C_2(G) = 0$.

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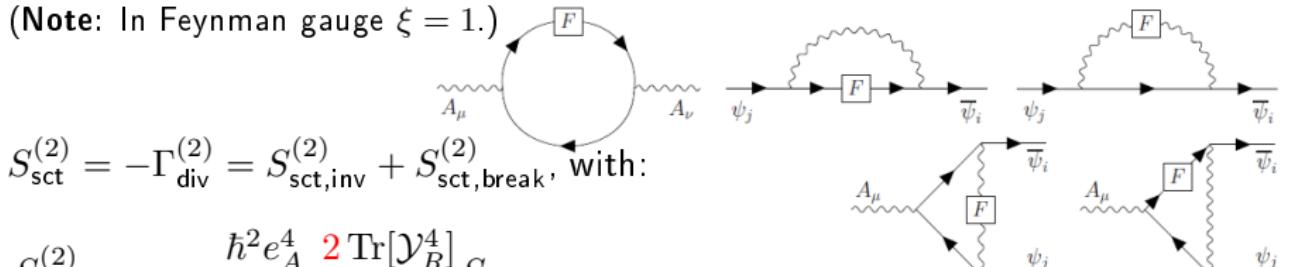
- 2-loop singular counterterm action
- Restore BRST invariance @ 2 loops
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2-loop singular counterterm $S_{\text{sct}}^{(2)}$ action

Photon, fermion SE and fermion-photon vertex. Additional diagrams:

(Note: In Feynman gauge $\xi = 1$.)



$$S_{\text{sct}}^{(2)} = -\Gamma_{\text{div}}^{(2)} = S_{\text{sct,inv}}^{(2)} + S_{\text{sct,break}}^{(2)}, \text{ with:}$$

$$\begin{aligned} S_{\text{sct,inv}}^{(2)} = & -\frac{\hbar^2 e_A^4}{256\pi^4} \frac{2 \text{Tr}[\mathcal{Y}_R^4]}{3\epsilon} S_{AA} \\ & + \frac{\hbar^2 e_A^4}{256\pi^4} \frac{(\mathcal{Y}_R^j)^2}{\epsilon} \left[\left(\frac{1}{2\epsilon} + \frac{17}{12} \right) (\mathcal{Y}_R^j)^2 - \frac{1}{9} \text{Tr}[\mathcal{Y}_R^2] \right] \left(\overline{S_{\bar{\psi}\psi_R}^j} + \overline{S_{\psi_R A \psi_R}^j} \right), \end{aligned}$$

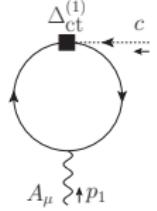
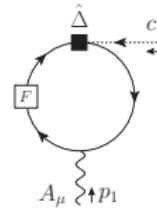
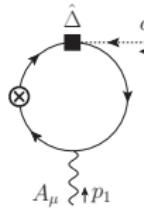
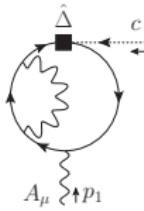
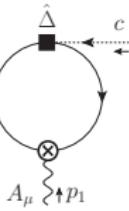
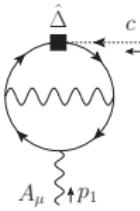
and:

$$\begin{aligned} S_{\text{sct,break}}^{(2)} = & -\frac{\hbar^2 e_A^4}{256\pi^4} \frac{\text{Tr}[\mathcal{Y}_R^4]}{3\epsilon} \left[2 \left(\overline{S_{AA}} - S_{AA} \right) + \left(\frac{1}{2\epsilon} - \frac{17}{24} \right) \int d^d x \frac{1}{2} \bar{A}_\mu \partial^2 \bar{A}^\mu \right] \\ & - \frac{\hbar^2 e_A^4}{256\pi^4} \frac{(\mathcal{Y}_R^j)^2}{3\epsilon} \left(\frac{5}{2} (\mathcal{Y}_R^j)^2 - \frac{2}{3} \text{Tr}[\mathcal{Y}_R^2] \right) \overline{S_{\bar{\psi}\psi_R}^j}. \end{aligned}$$

Evaluation of $\Delta_{\text{sct}}^{(2)}$ – Cancellation with $\Delta_d^{(1)} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(2)}$

$$(\mathcal{S}\Gamma_{\text{ren}})^{(2)} = \underset{d \rightarrow 4}{\text{LIM}} \{ \Delta_d^{(1)} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(2)} + \Delta_{\text{sct}}^{(2)} \} + N[\Delta_d^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)} + \Delta_{\text{fct},4}^{(2)},$$

$$\Delta_d^{(1)} \equiv \hat{\Delta} + \Delta_{\text{ct}}^{(1)}.$$



...

$$\begin{aligned} \Delta_{\text{sct}}^{(2)} = s_d S_{\text{sct}}^{(2)} &= \frac{-\hbar^2 e_A^4}{256\pi^4} \frac{\text{Tr}[\mathcal{Y}_R^4]}{6} \left(\frac{1}{\epsilon^2} - \frac{17}{12\epsilon} \right) \int d^d x (\bar{\partial}_\mu c) \hat{\partial}^2 \bar{A}^\mu \\ &\quad - \frac{\hbar^2 e_A^5}{256\pi^4} \frac{(\mathcal{Y}_R^j)^3}{3\epsilon} \left(\frac{5}{2} (\mathcal{Y}_R^j)^2 - \frac{2}{3} \text{Tr}[\mathcal{Y}_R^2] \right) \int d^d x c \bar{\partial}_\mu (\bar{\psi} \bar{\gamma}^\mu \mathbb{P}_R \psi). \end{aligned}$$

- Compared to 1-loop case, $\Delta_{\text{sct}}^{(2)}$ contains a non-evanescent contribution.
- Compute the divergent part $\Delta_d^{(1)} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(2)}$: it exactly cancels with $\Delta_{\text{sct}}^{(2)}$: ensures no divergent contribution to the STI.

Evaluation of the finite part: $N[\Delta_d^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)}$: Bonneau IDs

$$(\mathcal{S}\Gamma_{\text{ren}})^{(2)} = \underset{d \rightarrow 4}{\text{LIM}} \{ \Delta_d^{(1)} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(2)} + \Delta_{\text{sct}}^{(2)} \} + \textcolor{red}{N[\Delta_d^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)}} + \Delta_{\text{fct},4}^{(2)}.$$

$$N[\Delta_d^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)} = N[\widehat{\Delta} + \Delta_{\text{ct}}^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)} = N[\widehat{\Delta} + \Delta_{\text{fct}}^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)} + \underbrace{N[\Delta_{\text{sct}}^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)}}_{= 0 \text{ in } U(1) \text{ because non-1PI-insertable}}.$$

Interpret $N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(2)}$ using **Bonneau Identities** [Bonneau–1980]: at $\mathcal{O}(\hbar^2)$:

$$[N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}]^{(2)} = \underset{d \rightarrow 4}{\text{LIM}} \left(-\text{r.s.p.} \left[N[\check{\Delta}] \cdot \Gamma_{\text{DReg}} \right]_{\tilde{g}=0}^{(2)} \right) + \underbrace{N \left[-\text{r.s.p.} \left[N[\check{\Delta}] \cdot \Gamma_{\text{DReg}} \right]_{\tilde{g}=0}^{(1)} \right]}_{\equiv N[\widehat{\Delta}] \cdot \Gamma_{\text{ren}}|^{(1)} = -N[\Delta_{\text{fct}}^{(1)}]} \cdot \Gamma_{\text{ren}}^{(1)}.$$

© Fixed \hbar^2 order: **limited finite number** of UV-singular diagrams.

Hence:

$$N[\Delta_d^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)} = \underset{d \rightarrow 4}{\text{LIM}} \left(-\text{r.s.p.} \left[N[\check{\Delta}] \cdot \Gamma_{\text{DReg}} \right]_{\tilde{g}=0}^{(2)} \right) = \underset{d \rightarrow 4}{\text{LIM}} \left([(\widehat{\Delta} + \Delta_{\text{ct}}^{(1)}) \cdot \Gamma_{\text{DReg}}]_{\text{fin}}^{(2)} \right).$$

Results: $N[\Delta_d^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)} - \text{Finite counterterms } S_{\text{fct}}^{(2)}$

$$(\mathcal{S}\Gamma_{\text{ren}})^{(2)} = \underset{d \rightarrow 4}{\text{LIM}} \{ \Delta_d^{(1)} \cdot \Gamma_{\text{DReg}}|_{\text{div.}}^{(2)} + \Delta_{\text{sct}}^{(2)} \} + N[\Delta_d^{(1)}] \cdot \Gamma_{\text{ren}}|^{(2)} + \Delta_{\text{fct},4}^{(2)}.$$

From the requirement $\mathcal{S}\Gamma_{\text{ren}} = 0$:

$$\Delta_{\text{fct},4}^{(2)} = s_4 S_{\text{fct}}^{(2)} = - \underset{d \rightarrow 4}{\text{LIM}} \left\{ \left([\widehat{\Delta} + \Delta_{\text{ct}}^{(1)}] \cdot \Gamma_{\text{DReg}} \right)^{(2)} + \Delta_{\text{sct}}^{(2)} \right\},$$

from which:

$$\begin{aligned} S_{\text{fct}}^{(2)} = & \left(\frac{\hbar}{16\pi^2} \right)^2 \int d^4x e_A^4 \left\{ \text{Tr}[\mathcal{Y}_R^4] \frac{11}{48} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + e_A^2 \frac{\text{Tr}[\mathcal{Y}_R^6]}{8} \bar{A}_\mu \bar{A}^\mu \bar{A}_\nu \bar{A}^\nu \right. \\ & \left. - (\mathcal{Y}_R^j)^2 \left(\frac{127}{36} (\mathcal{Y}_R^j)^2 - \frac{1}{27} \text{Tr}[\mathcal{Y}_R^2] \right) \bar{\psi}_j i \not{\partial} \mathbb{P}_{\text{R}} \psi_j \right\} \\ & + \text{any BRST-invariant term.} \end{aligned}$$

- Same structure as $S_{\text{fct}}^{(1)}$. Is it always true @ any order?

Details + Checks

Calculations: Mathematica. Loop amplitudes with FeynCalc

[Mertig...–1990, Shtabovenko...–2016]; ϵ -“expansion” with interface FeynHelpers

[Shtabovenko–2016] to Package-X [Patel–2017] (**1 loop**), or TARCER (**2 loops**)

IRD method [Misiak...–1994, Chetyrkin...–1997], external momentum derivative, ...

Checks: In $U(1)$, WTI \equiv STI. Structural behaviour, 1-PI amplitudes relations.

- 1 Photon SE transversality: $p_\nu \Gamma_{A_\mu(p) A_\nu(-p)} = 0$
- 2 Symmetry/coeff. relations of multi-photon vertices ($p_{1+2+3} \equiv p_1 + p_2 + p_3$):
 $(p_{1+2+3})_\sigma \Gamma_{A_\rho(p_3) A_\nu(p_2) A_\mu(p_1) A_\sigma(-p_{1+2+3})} = 0$
- 3 Relation fermion SE / fermion-photon vertex:

$$(p_{1+2})_\mu \Gamma_{\psi_j(p_2) \bar{\psi}_i(p_1) A_\mu(-p_{1+2})} = e_A \mathcal{Y}_{Rij} \left(\Gamma_{\psi_j(-p_1) \bar{\psi}_i(p_1)} - \Gamma_{\psi_j(p_2) \bar{\psi}_i(-p_2)} \right).$$

At given \hbar^n , always satisfied for divergences. Finite parts: only with $S_{\text{fct}}^{(n)}$.

Outline

1 Chiral Right-handed QED (χ QED)

- χ QED action S_0 ; extension to d dimensions
- Completed χ QED in d dimensions
- BRST invariance of χ QED? Restore @ loop-level?

2 Calculations @ 1 loop

- 1-loop singular counterterm action
- Restore BRST invariance @ 1 loop
- Results: 1-loop finite counterterm action

3 Calculations @ 2 loops

- 2-loop singular counterterm action
- Restore BRST invariance @ 2 loops
- Results: 2-loop finite counterterm action

4 Discussion & Summary

Discussion (1/2)

Dimensional Regularization/Renormalization has some **freedom** in definitions:

- When extending the model to d dimensions:
⇒ Different possible fermion-gauge-boson chiral interactions;
- Any additional **finite BRST-invariant** terms in the $S_{\text{fct}}^{(1,2,\dots)}$:
⇒ Different choices would modify calculations at higher-orders.

⇒ Different dimensional BMHV “schemes”!

Each of these points **needs to be explicitly mentioned** for completeness!

Very **small set** of finite counterterms needed to restore BRST symmetry at any given order. (Compare with manual BRST/Ward ID restoration for individual processes in e.g. NDR...)

Discussion (2/2)

** Under Investigation **

- $S_{\text{sct,even}}^{(1,2)}$ and $S_{\text{fct}}^{(1,2)}$ are $\neq 0$. No effect on 1-loop-level RGEs. However they matter for renormalization at higher orders (≥ 2 loops), from insertion in loop diagrams. Compare with literature (e.g. RGEs: [Machacek, Vaughn–1983, '84, '85], ...)
- $S_{\text{sct,even}}^{(1,2)} \neq 0 \longrightarrow$ We cannot use straightforwardly the Z multiplicative renormalization factors for defining RGEs: β_{e_A} , γ_φ .
 - ▶ RGEs for the DimReg theory: define $\beta_{\widehat{\mathcal{O}}}$ for the (non-physical) evanescent operators \Longrightarrow All β -functions need to be considered for consistency.
 - ▶ RGEs for the renormalized 4D theory: the effects of the evanescent operators dilute into the other non-evanescent ones [Schubert (Nucl.Phys.B323, 1989)]. Can be consistently described using Bonneau-like IDs for $\mu \partial_\mu \Gamma_{\text{ren}}$ [Bonneau–1980].

Summary

- Our 1-loop results [[arXiv:2004.14398](#)] on γ_5 in DimReg + BMHV applied to the **massless χ QED** at 1-loop and extended at 2-loop order.
- Systematic consistent treatment of γ_5 in d dimensions, backed by all-loop orders rigorously proven BMHV scheme in perturbative QFT and algebraic renormalization framework. **Finite local CTs** \rightarrow BRST invariance **restored**.
- Explicit formulae for 1 and 2-loop **singular, evanescent, and BRST-restoring finite counterterms** for χ QED in DimReg. Necessary for consistent higher-order calculations.

For the future:

- Generalization for application to 2-loop Standard Model?
- Massive case, non-zero VEV? (1-loop Abelian-Higgs by [[Sanchez-Ruiz–2002](#)].)
- Higher-order results?

Summary

- Our 1-loop results [[arXiv:2004.14398](#)] on γ_5 in DimReg + BMHV applied to the **massless χ QED** at 1-loop and extended at 2-loop order.
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Thank you!

Supplement

BRST symmetry



BRST symmetry: Residual symmetry after fixing the gauge

(≈ “generalized” version of gauge symmetry). [Becchi,Rouet,Stora–1975,Tyutin–1975]

Infinitesimal gauge transfo. of fields: $\varphi_i \rightarrow \delta_\alpha \varphi_i$ linear in the (small) gauge parameter α

$\stackrel{\alpha^a \rightarrow \theta c^a}{\Rightarrow}$ θ : Grassmann parameter;
 c^a : (anticommuting) ghost.
BRST transformation of φ :
 $\delta_{\text{BRST}} \varphi = \theta s \varphi \equiv \delta_\alpha \varphi|_{\alpha^a \rightarrow \theta c^a}.$

Effective action Γ : Interpretation & notation (1/2)

Effective action: Generating functional for 1-particle irreducible (1PI) Green's functions [Weinberg-1996]:

$$\begin{aligned}\Gamma[\Phi] &= \sum_{n \geq 2} \frac{1}{|n|!} \int \left(\prod_{i=1}^n d^4 x_i \phi_i(x_i) \right) \Gamma_{\phi_n \dots \phi_1}(x_1, \dots, x_n) \\ \left(\stackrel{\text{Fourier}}{\text{transform}} \right) &= \sum_{n \geq 2} \frac{1}{|n|!} \int \left(\prod_{i=1}^n \frac{d^4 p_i}{(2\pi)^4} \tilde{\phi}_i(p_i) \right) \Gamma_{\phi_n \dots \phi_1}(p_1, \dots, p_n) \underbrace{(2\pi)^4 \delta^4(\sum_{j=1}^n p_j)}_{\text{Momentum conservation}} ,\end{aligned}$$

$\Gamma_{\phi_n \dots \phi_1}$ are the 1PI Green's functions defined by:

$$\begin{aligned}i\Gamma_{\phi_n \dots \phi_1}(x_1, \dots, x_n) &= \left. \frac{i\delta^n \Gamma[\Phi]}{\delta \phi_n(x_n) \dots \delta \phi_1(x_1)} \right|_{\phi_i=0} = \langle \Omega | \mathbb{T}[\phi_n(x_n) \dots \phi_1(x_1)] | \Omega \rangle^{\text{1PI}} \\ &\equiv \langle \phi_n(x_n) \dots \phi_1(x_1) \rangle^{\text{1PI}} ,\end{aligned}$$

and $i\Gamma_{\phi_n \dots \phi_1}(p_1, \dots, p_n) \equiv \langle \tilde{\phi}_n(p_n) \dots \tilde{\phi}_1(p_1) \rangle^{\text{1PI}}$ is defined similarly.

Effective action Γ : Interpretation & notation (2/2)

$$\Gamma[\Phi] = \sum_{n \geq 2} \frac{-i}{|n|!} \int \left(\prod_{i=1}^n d^4 x_i \phi_i(x_i) \right) \langle \phi_n(x_n) \cdots \phi_1(x_1) \rangle^{1\text{PI}}$$

$$= \sum_{n \geq 2} \frac{-i}{|n|!}$$

Field-Operator insertion in $\Gamma[\Phi]$ [Piguet,Rouet-1981]:
 (e.g. counterterm insertions in loop diagrams...)

$$\mathcal{O}(x) \cdot \Gamma[\Phi] = \sum_{n \geq 2} \frac{-i}{|n|!} \int \left(\prod_{i=1}^n d^4 x_i \phi_i(x_i) \right) \langle \mathcal{O}(x) \phi_n(x_n) \cdots \phi_1(x_1) \rangle^{1\text{PI}}$$

$$= \sum_{n \geq 2} \frac{-i}{|n|!}$$

Notation:
 $\mathcal{O} \cdot \Gamma[\Phi] = \int dx \mathcal{O}(x) \cdot \Gamma[\Phi].$

Notation: “Normal Products” $N[\mathcal{O}(x)]$

Introduced by Zimmermann [Zimmermann–1973].
(See also [Lowenstein–1971].)

For a field-product operator $\mathcal{O}(x)$, a normal product $N[\mathcal{O}(x)]$ is defined as the “finite part” of $\mathcal{O}(x)$, i.e. via the finite part of the time-ordered Green’s functions of $\mathcal{O}(x)$:

$$\langle N[\mathcal{O}] \prod_i \phi_i(x_i) \rangle^{\text{1PI}} = \text{Fin.} \left(\langle \mathcal{O} \prod_i \phi_i(x_i) \rangle^{\text{1PI}} \right).$$

[Piguet, Rouet–1981]



They depend on the chosen renormalization scheme:

- ▶ In BPHZ renormalization (original): done by subtracting the first terms of a Taylor expansion of loop integrands up to a given order (“degree” of subtraction). → \exists different normal products associated to the choice of the “degree” of subtraction. [Piguet, Rouet–1981]
- ▶ In dimensional renormalization (DimRen): the normal products are defined with respect to the ϵ -pole subtraction. [Collins–1974]

Bonneau Identities, graphical interpretation (1/2)

In DimRen, normal products $N[\hat{\mathcal{O}}]$ of evanescent operators $\hat{\mathcal{O}}$ of the form $\hat{\mathcal{O}} \equiv (\hat{g}_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu})\mathcal{O}_{\mu\nu\rho\dots}$ are interpreted [Bonneau-1980] as the difference between two ways of performing a “subtraction” in this renormalization scheme.
⇒ “Zimmermann-like” identities: Bonneau Identities.

$$N[\hat{\mathcal{O}}] \cdot \Gamma_{\text{ren}} = - \sum_{n=2}^{n_{\max}=4} \sum_{\substack{J=\{j_1, \dots, j_n\}, \{i_1, \dots, i_r\}/ \\ 0 \leq r \leq \delta(J)}} \sum_{\substack{1 \leq i_j \leq n}} \frac{(-i)^r}{r!} \frac{\partial^r}{\partial p_{i_1}^{\mu_1} \cdots \partial p_{i_r}^{\mu_r}} \cdot (-i\hbar) \text{r.s.p.} \left. \overline{\left\langle \prod_{i=1}^n \widetilde{\phi}_{j_i}(p_i) N[\check{\mathcal{O}}](q = -\sum p_i) \right\rangle}^{\text{1PI}} \right|_{\substack{p_i=0 \\ \check{g}=0}}$$
$$\times N \left[\frac{1}{n!} \prod_{k=n}^1 \left(\prod_{\{\alpha / i_\alpha = k\}} \partial_{\mu_\alpha} \right) \phi_{j_k} \right] \cdot \Gamma_{\text{ren}} + \text{similar with additional BV sources insertions.}$$

r.s.p.: residue of simple pole in $\nu = 2\epsilon = 4 - d$. Overline: 1PI minimally subtracted (“subrenormalized”).
 $\check{g} \sim \hat{g}/\nu$, where this ν is not submitted to Laurent ν -expansion for the r.s.p..

$$N[\hat{\mathcal{O}}] \cdot \Gamma_{\text{ren}} = \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i} N[\overline{\mathcal{O}}_i] \cdot \Gamma_{\text{ren}}.$$

Expands evanescent operators $\hat{\mathcal{O}}$ on a basis of quantum 4D operators of the renormalized 4D theory.

Bonneau Identities, graphical interpretation (2/2)

$$N[\widehat{\mathcal{O}}] \cdot \Gamma_{\text{ren}} = \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i} N[\overline{\mathcal{O}}_i] \cdot \Gamma_{\text{ren}}$$

$$\left(\begin{array}{c} \text{Diagram 1: A shaded loop with a red dot at the top. Labels: } \nu \cdot \mathcal{O} \text{ or } -\hat{g}_{\mu\nu} \mathcal{O}_{\mu\nu} \dots \\ \text{Diagram 2: Similar to Diagram 1, with a red dot at the top. Labels: } \nu \text{ or } \mathcal{O}_{\mu\nu} \dots \\ \text{Diagram 3: Similar to Diagram 1, with a red dot at the top. Labels: } \mathcal{O} \text{ or } \mathcal{O}_{\mu\nu} \dots \\ \text{Diagram 4: A shaded loop with a blue square at the top. Labels: } \Phi_1(x_1) \dots \Phi_n(x_n) \end{array} \right) = \sum_{\Gamma_i} \Gamma_i$$

$$= \sum_{\{\overline{\mathcal{O}}\}_i} c_{\overline{\mathcal{O}}_i} \text{Diagram 4}$$

All sub-loops are subrenormalized, including the loop containing the “special vertex” $\widehat{\mathcal{O}}$.

Calculation machinery

- Calculations performed with Mathematica.
- Model programmed using FeynRules [Christensen...–2009,Alloul...–2014] (**with no BRST sources since unsupported**). Manually patched for supporting arbitrary $SU(N)$ (N not limited to numerical values).
- Loop diagrams (**w/o BRST sources**) generated using FeynArts [Hahn–2000]. Amplitudes evaluated using FeynCalc [Mertig...–1990,Shtabovenko...–2016]; ϵ -“expansion” obtained using the FeynCalc’s interface FeynHelpers [Shtabovenko–2016] to Package-X [Patel–2017] (**1 loop**), or TARCER (**2 loops**)
WARNING! Using development version of FeynCalc that includes needed fixes (versions up to 17th June 2019 are OK).
- Diagrams with sources manually generated, then evaluated using FeynCalc as described above.
- Semi-automated (manually and computer) evaluation of group-structure invariants, using notations similarly defined as those in Machacek & Vaughn [Machacek,Vaughn–1983,'84,'85].

The R-Model defining action S_0

Model with generic gauge group \mathcal{G} (usually $SU(N)$; can be something else...) with right-handed (RH) fermions in “right” (R) rep. of \mathcal{G} and scalars in S rep. of \mathcal{G} , both coupling to gauge bosons.

Originally defined in 4 dimensions, using either Weyl, or Dirac fermions with projectors $\mathbb{P}_{R/L} = (1 \pm \gamma_5)/2$.

$$S_0^{(4D)} = \int d^4x (\mathcal{L}_{\text{YM}}^{(4D)} + \mathcal{L}_{\Psi}^{(4D)} + \mathcal{L}_{\Phi}^{(4D)} + \mathcal{L}_{\text{Yuk}}^{(4D)} + \mathcal{L}_{\text{gh}}^{(4D)} + \mathcal{L}_{\text{g-fix}}^{(4D)}) ,$$

with:

$$\mathcal{L}_{\text{YM}}^{(4D)} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}, \quad \mathcal{L}_{\Phi}^{(4D)} = \frac{1}{2} (D_\mu \Phi_m)^2 - \frac{\lambda_H^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p ,$$

$$\mathcal{L}_{\Psi}^{(4D)} = i \bar{\Psi}_i \not{\partial} \mathbb{P}_R \Psi_i + g_S T_{Rij}^a \bar{\Psi}_i \not{G}^a \mathbb{P}_R \Psi_j \equiv i \bar{\Psi}_i \not{D}_R^{ij} \Psi_j ,$$

$$\mathcal{L}_{\text{Yuk}}^{(4D)} = -(Y_R)_{ij}^m / 2 \Phi_m \overline{\Psi}_i^C \mathbb{P}_R \Psi_j + \text{h.c.} ,$$

$$\mathcal{L}_{\text{gh}}^{(4D)} = \partial_\mu \bar{c}_a \cdot D^{ab\mu} c_b, \quad \mathcal{L}_{\text{g-fix}}^{(4D)} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G_\mu^a .$$

Note the Yukawa interaction with charge-conjugated fermion (\neq Dirac model where left component couples to right component).

Nota about charge conjugation

While it is clear how to define the charge conjugation operation in 4D with e.g. an explicit construction: numerically $C = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \sim i\gamma^0\gamma^2$ with the good properties,

In d -D we can define a similar operation only by its action on the fermions – such that it turns fermions to their charge-conjugate and back: $\Psi^C = C\bar{\Psi}^T$ –, and its action on Dirac 4-spinor bilinears:

$$(\Psi^C)^C = \Psi, C^T = -C;$$

$$\bar{\Psi}_i^C \Gamma \Psi_j^C = -\Psi_i^T C^{-1} \Gamma C \bar{\Psi}_j^T = \bar{\Psi}_j C \Gamma^T C^{-1} \Psi_i = \eta_\Gamma \bar{\Psi}_j \Gamma \Psi_i,$$

$$\text{with: } \eta_\Gamma = \begin{cases} +1 & \text{for } \Gamma = \mathbb{1}, \gamma_5, \gamma^\mu \gamma_5, \\ -1 & \text{for } \Gamma = \gamma^\mu, \sigma^{\mu\nu}, \sigma^{\mu\nu} \gamma_5. \end{cases}$$

(See e.g. Appendix A of [Tsai–2011].)

BRST transformations of fields of R-Model

The d -dimensional BRST transformations on the fields are as follows:

$$s_d G_\mu^a = D_\mu^{ab} c^b = \partial_\mu c^a + g_S f^{abc} G_\mu^b c^c ,$$

$$s_d \Psi_i = s_d \Psi_{Ri} = i c^a g_S T_{Rij}^a \Psi_{Rj} , \quad s_d \bar{\Psi}_i = s_d \bar{\Psi}_{Ri} = +i \bar{\Psi}_{Rj} c^a g_S T_{Rji}^a ,$$

$$s_d \Phi_m = i c^a g_S \theta_{mn}^a \Phi_n ,$$

$$s_d c^a = -\frac{1}{2} g_S f^{abc} c^b c^c \equiv i g_S c^2 ,$$

$$s_d \bar{c}^a = B^a , \quad s_d B^a = 0 \iff (\bar{c}^a, B^a) \text{ is a BRST doublet} ,$$

with a similar form (noted s in what follows) in 4D.

The BRST operator s_d is nilpotent: $s_d(s_d \phi) = 0$, similarly to its 4D counterpart.

The completed R-Model defining action S_0 in d -D

Our complete defining action in d dimensions, including the antifields, reads:

$$S_0 = \int d^d x (\mathcal{L}_{\text{YM}} + \mathcal{L}_{\Psi} + \mathcal{L}_{\Phi} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{gh}} + \mathcal{L}_{\text{g-fix}} + \mathcal{L}_{\text{ext}}),$$

with: $\mathcal{L}_{\text{YM}} = \frac{-1}{4} F_{\mu\nu}^a F^{a\mu\nu}$, $\mathcal{L}_{\Phi} = \frac{1}{2} (D_\mu \Phi^m)^2 - \frac{\lambda_H^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p$,

$$\mathcal{L}_{\Psi} \Rightarrow i \bar{\Psi}_i \not{D}_R^{ij} \Psi_j = i \bar{\Psi}_i \not{\partial} \Psi_i + g_S T_{Rij}^a \bar{\Psi}_R \not{\mathbb{P}}_{\text{L}} \not{\mathbb{G}}^a \not{\mathbb{P}}_{\text{R}} \Psi_{Rj},$$

$$\mathcal{L}_{\text{Yuk}} = -(Y_R)_{ij}^m / 2 \Phi_m \bar{\Psi}_{Ri}^C \not{\mathbb{P}}_{\text{R}} \Psi_{Rj} + \text{h.c.},$$

$$\mathcal{L}_{\text{gh}} = \partial_\mu \bar{c}_a \cdot D^{ab}{}^\mu c_b, \quad \mathcal{L}_{\text{g-fix}} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G_\mu^a,$$

$$\mathcal{L}_{\text{ext}} = \rho_a^\mu s_d G_\mu^a + \zeta_a s_d c^a + \bar{R}^i s_d \Psi_{Ri} + s_d \bar{\Psi}_R \not{R}^i + \mathcal{Y}^m s_d \Phi_m.$$

Quantum numbers (mass dimension, ghost number and (anti)commutativity):

| | G_μ^a | $\bar{\Psi}_i, \Psi_i$ | Φ_m | c^a | \bar{c}^a | B^a | ρ_a^μ | ζ_a | R^i, \bar{R}^i | \mathcal{Y}^m | ∂_μ | s |
|-----------|-----------|------------------------|----------|-------|-------------|-------|--------------|-----------|------------------|-----------------|----------------|-----|
| mass dim. | 1 | 3/2 | 1 | 0 | 2 | 2 | 3 | 4 | 5/2 | 3 | 1 | 0 |
| ghost # | 0 | 0 | 0 | 1 | -1 | 0 | -1 | -2 | -1 | -1 | 0 | 1 |
| comm. | + | - | + | - | - | + | - | + | + | - | + | - |

What about a L-Model?

How do the results modify for left-handed (LH) fermions? Two approaches:

- 1 Either note that $\mathbb{P}_R \leftrightarrow \mathbb{P}_L$, corresponding to the change $\gamma_5 \leftrightarrow -\gamma_5$, and related change $\epsilon^{\mu\nu\rho\sigma} \leftrightarrow -\epsilon^{\mu\nu\rho\sigma}$.
- 2 Or, view LH fermions in a “left” (L) representation of \mathcal{G} , as being the charge-conjugate of corresponding RH fermions that would belong to the conjugate representation of the “left” ones: $\mathbb{P}_L \Psi_L \equiv (\mathbb{P}_R \Psi_R)^C$, and $T_L \leftrightarrow T_R \equiv T_{\bar{L}}$.

NOTE: Possible mixings between RH and LH fermions (in the Yukawa sector...)!

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