- I

QFT
QED, QCD
HEPP

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INTRO

Wouldn't it be great to learn everything about **QFT and particle physics** in one week? Sure it would.
Alas: the field was born about 100 years ago, and it *explosive* phase is at least 60 years old.

What could one take home?
- *a fresh look at known phenomena and ideas,*
- *a taste of unfamiliar links between familiar things,*
- *open problems to think about and work on*.

PLAN

1. **3P**, **3F** & **3G** of QFT
2. Feynman diagrams, **!!** and **??**
3. Renormalization and **QED** Running Couplings

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4. **QCD** an Autopsy of AF
5. Hard Processes and QCD Partons
6. QCD Radiophysics & LPHD
7. LPHD inside jets

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8. Parton Dynamics and **SUSY**
9. 3 mysteries = **3 R&D** projects

QFT: 3 Pillars

3P
3F
3G

Quantum: taught us to study, interpret, predict ***bizarre phenomena*** involving ***unimaginable*** objects

Relativistic: *7* *μsec/day* *∂GPS=SR*

Antiparticles, CPT

Probing e^-, ***encounter*** $e^- \; e^+ e^-$

Many-body: **1, 2, *too many***

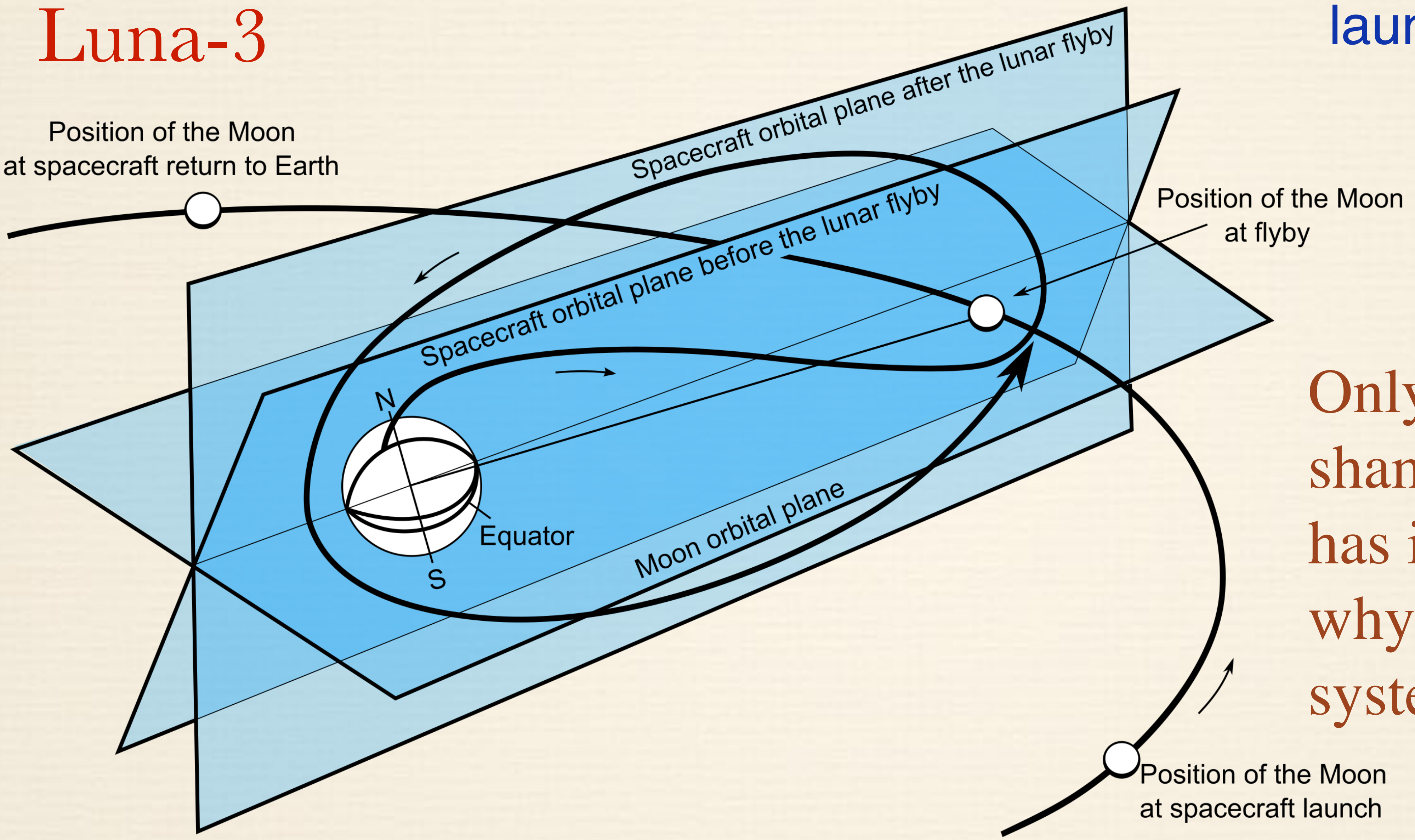
3-body dynamics virtually unknown

even classically

1, 2, many

Luna-3

launched 4.10.1959



Only after this shameful experience has it become clear why is our Solar system ***planar***

"an **eternal** manmade satellite!"

(a few days later fell on the Earth...)

*"vertical Moon" - lifetime **52 days***

QFT: 3 Formulations

3P
3F
3G

Secondary Quantisation

Operator language. Lagrangian. Fock space.

Field = sum of basic states with definite occupation numbers.

Creation and Annihilation operators. (Anti)Commutation relations.

Equations of motion from Variational Principle.

Functional Integral

Integration over *trajectories* in the space of field wave functions.

Variational Principle = action's extremum in the exponent of FI.

Both formulations yield the same rules for an ergonomic graphic technique that organises perturbative expansion - Feynman Diagrams.

Feynman Diagrams

can be looked upon as an independent way to constructing QFT.

Gribov formulation of QED without mentioning the word "*Lagrangian*"!

Secondary Quantization

Functional Integral

3P
3F
3G

Feynman Diagrams

So, at the *perturbation theory* level, all 3 Formulations are equivalent.

Beyond *PT* - not so clear…

Especially so in QCD with its bizarre fundamental fields (quarks and gluons) that do not show up in the physical spectrum. And, as a consequence, with non-trivial structure of the vacuum.

Lagrangian-based formulations are better suited for exposing *gauge symmetries*.

Equipped with SQF it is easier to handle Causality and its consequences.

The best for finding symmetry factors of complicated FDs.

Also, it is natural to use for parametrizing vacuum expectation values of composite QCD field operators ("ITEP sumrules").

Indispensable for studying potential role of large field configurations that provide non-trivial action extremums (solitons, instantons, lipatons,…).

non-PT

FDF is not insensitive to non-PT physics either. Here non-PT physics can be triggered by examining how does PT series diverge! ("*renormalons*")

QFT: 3 Generalities

3P
3F
3G

Analyticity ⟵ Causality
Crossing ⟵ Relativity
Unitarity ⟵ Quantum dynamics

Scattering amplitude is an ***analytic*** function of energy $\Longleftarrow$ Causality

Scattering amplitude is analytic in **s**, but also in **t,u** $\Longleftarrow$ Crossing.

An analytic function is identified by its ***singularities*** in the complex plane.

Position and Character of singularities $\Longleftarrow$ Unitarity

In the Born approximation (tree graphs) -
only poles (bound states; particle exchange)

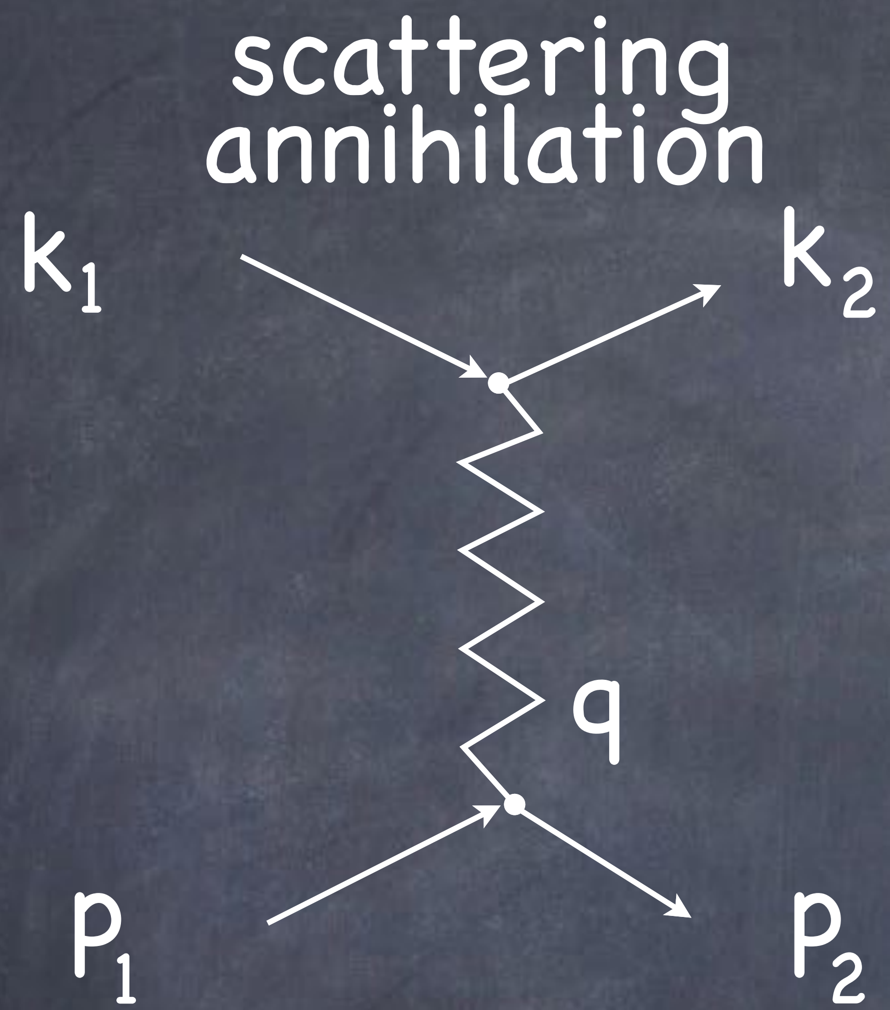
Example of amplitude satisfying the General Principles in all channels :

"Veneziano Amplitude" — birth of String Theory.

Beyond the Born approximation (diagrams with loops) -
LANDAU RULES for determining singularities of *arbitrary* Feynman diagrams.

Feynman Diagrams satisfy these general properties automatically!

relativistic crossing



$s=(p_1+k_1)^2$ invariant energy

$t=(p_1-p_2)^2$ momentum transfer

one and the same amplitude as a function of its *invariants* A(s,t) describes three physically different processes related by *crossing*

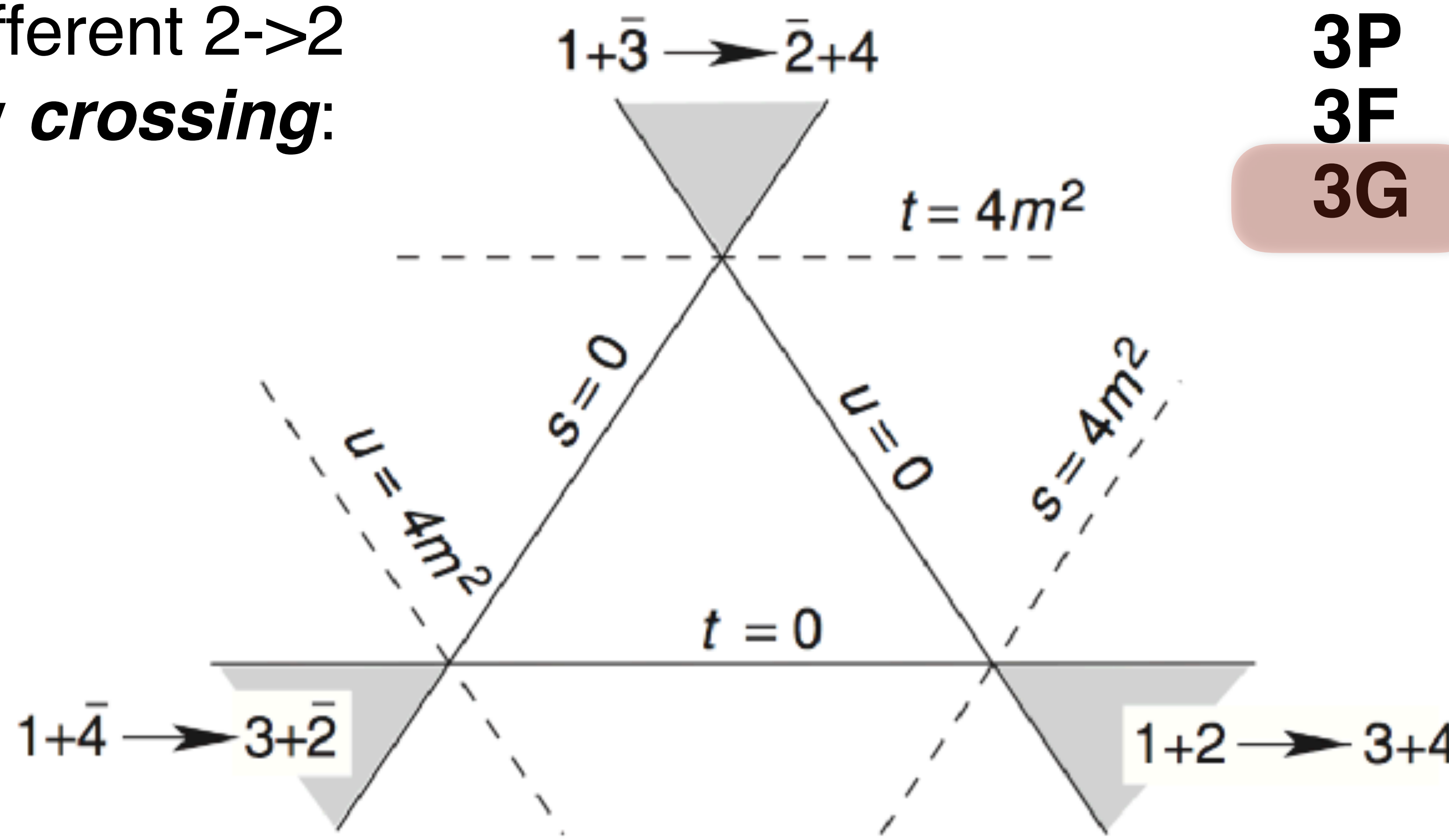
A(s,t) is an **analytic function** of energy s (causality) and of the momentum transfer t (crossing)

One function describes three different 2->2 interaction processes related by ***crossing***:

3P
3F
3G

$$s\text{-channel}: \quad 1+2 \to 3+4,$$
$$t\text{-channel}: \quad 1+\bar{3} \to \bar{2}+4,$$
$$u\text{-channel}: \quad 1+\bar{4} \to 3+\bar{2},$$

$$s = (p_1 + p_2)^2 \geq (m_1 + m_2)^2;$$
$$t = (p_1 + \bar{p}_3)^2 \geq (m_1 + m_3)^2;$$
$$u = (p_1 + \bar{p}_4)^2 \geq (m_1 + m_4)^2.$$



Physical regions of 3 crossing-related reactions on the Mandelstam plane

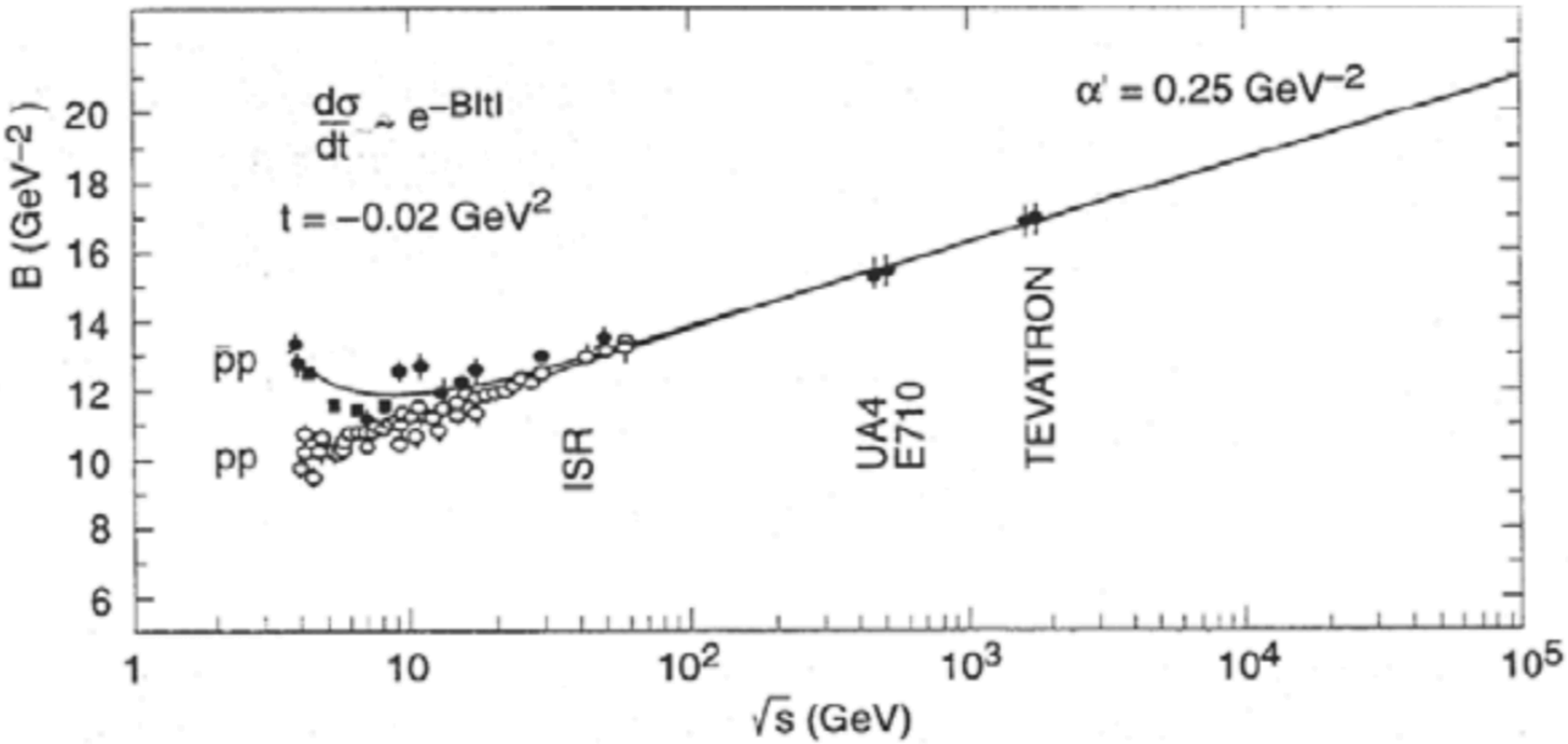
It is important to remember that the *unitarity* seriously restricts the scattering amplitude. Moreover, these restrictions are different in each of the three crossing channels. Thus, one function has to satisfy three specific unitarity relations in complementary physical regions on the Mand plane.

Example of combining use of General Principles —

Growth of interaction radius with energy ("shrinkage of diffractive cone").

shrinkage of diffractive cone

3P
3F
3G



The forward elastic cross-section shrinks as ln s in an enormous energy range: $30 \leq \sqrt{s} \leq 2000$ GeV

In other words, the hadron ***swells*** with increasing energy: $R^2 \simeq \alpha' \ln s$

Maximal growth of the hadron radius (*allowed by 3Gs*): $R \propto \ln s$

In accord with the ***Froissart-Martin bound*** $\sigma_{\rm tot} = 2\pi R^2 \leq C \ln^2 s$

The aim of QFT - multi-body quantum systems, especially with *variable number of particles*.

To do it better than multi-body Schroedinger equations!

Automatically gives totally symmetric/anti-symmetric multi-body wave functions (recall Slater determinants in QM).

Ensures Lorentz covariant description for relativistic systems.

Provides transparent way of organising PT series (FDs).

Describes how does the number of particles actually change in the system (production, absorption, radiation, annihilation,…)

Apart from *elementary particle* and *nuclear physics*,

QFT is widely used in *condensed matter physics*, *quantum optics*,

in some cases - in *atomic & molecular physics*,

even in *"financial physics"*…

abstractions and shortcomings of QFT

point-like objects
engaged in local (point) interactions

Prise to pay - **divergencies :**
mass and interaction constant (**charge**)
not calculable

UV Divergencies

(QED)

Energy of Coulomb electric field surrounding a pointlike electron :

$$\mathscr{E} = \frac{1}{4\pi}\int d^3r\, \vec{E}^2 \propto e^2 \int \frac{d^3r}{r^4} = \infty$$

The integral diverges at small distances, that is at large frequencies

Therefore, "Ultraviolet" divergence

No wonder that characteristics on an object *change* and have to be "***renormalized***", when you make it interact with environment.

E.g. put the object in the medium (QED vacuum in our case)

Probability to find an electron in a bare state (stripped of photons) is smaller than 1

⟹ Wave function Renormalization

Electron mass is different from the mass of a bare electron field that we insert in the QED Lagrangian

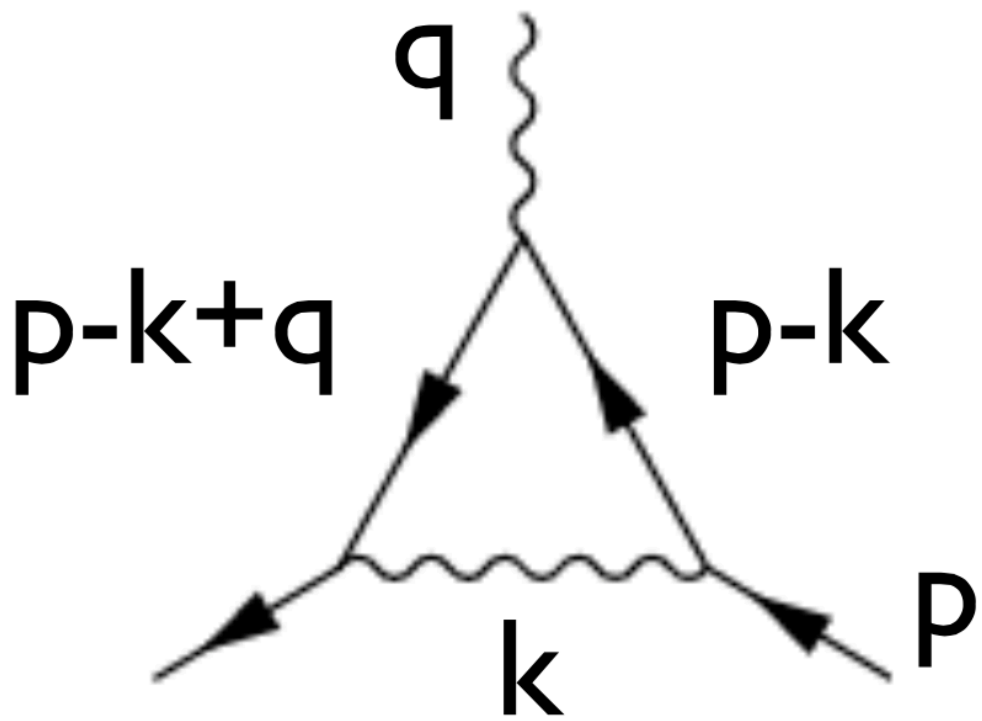
⟹ Mass Renormalization

Interaction strength (electron charge) also acquires correction

⟹ Coupling Renormalization

What **is** worrying that these corrections turn to be in our case **infinite...**

The one-loop contribution to the vertex function Γ.



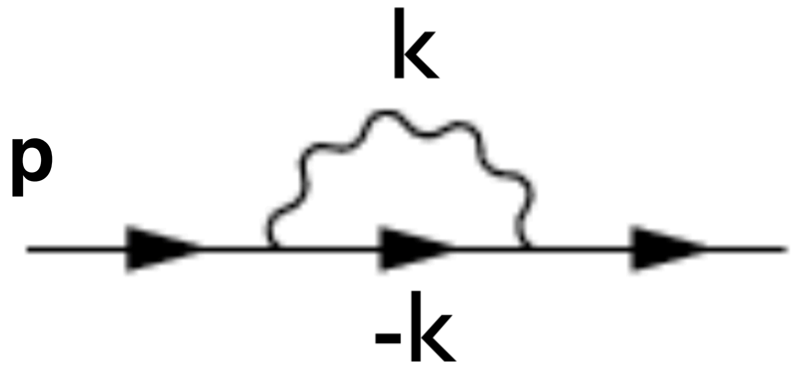
For large k

$$\int \frac{1}{k}\frac{1}{k}\frac{1}{k^2}d^4k$$

log divergence

The one-loop contribution to the electron self-energy function Σ.

$\Sigma(p)$ =



$= \delta_m \cdot m_e + \delta_Z \cdot \hat{p}$

Naively $\int \frac{1}{k}\frac{1}{k^2}d^4k$ linear divergence

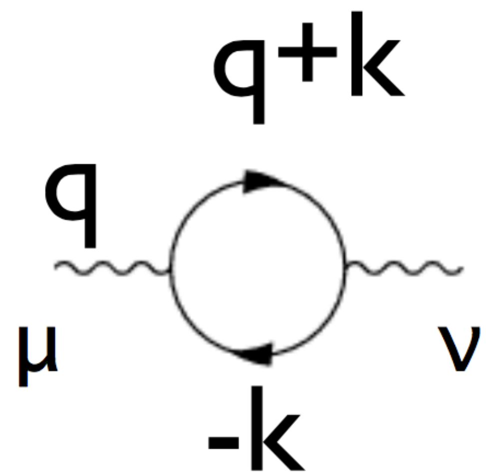
actual divergence is logarithmic

NB: renormalization of the **electron wave function** intimately related with that of the **vertex** (Ward Identity)
Cancellation of the two is a consequence of ***Current conservation*** **(GAUGE INVARIANCE).**

Otherwise, $e_{\text{Hydrogen}} = e_e + e_p \neq 0.$

The most confident experimental result, ever!

The one-loop contribution to the vacuum polarization function Π.

$I_{\mu\nu}(q)=$ 

naively $\int \frac{1}{k}\frac{1}{k} d^4k$ quadratic divergence

GAUGE INVARIANCE: $I_{\mu\nu}(q) = A(q^2)g_{\mu\nu}q^2 + B(q^2)q_\mu q_\nu,$ $A = -B$

actual divergence is logarithmic

NB: GAUGE INVARIANCE *does not imply* **zero photon mass !**

Another highly confident experimental result!

A sneaky way out:

cannot calculate in a sensible manner? - don't!

Try to express everything in terms of physical (measurable) ***masses*** and ***charges***.

If/when you succeed - your theory is

renormalizable

ready for making high-accuracy predictions.

and what if you do not?

Strictly speaking, there is nothing wrong with

NON-RENORMALIZABLE QFTS

Non-renormalizable QFTs: good for dealing with ***specific phenomena*** in a ***limited range*** of parameters

Effective QFTs

- small-energy $\pi - \mathrm{N}$ interactions — "Chiral Lagrangian"
- critical phenomena — 2nd order phase transitions
- effective d.o.f. (e.g. phonons) — in solid state physics

However, *particle physics* is more ambitious than that:

we'd like to know *everything* about *everything*, *everywhere*!

(that is, dynamics of *all* particle interactions, at *all* scales)

Strangely, we manage to (*or were allowed to*) satisfy our ambitions!

*and in a **quite non-trivial way**, too ...*

Now and then it ain't enough to ***renormalize*** mass(es) and charge(s).

In the **SM** a ***finer tuning*** turned out to be necessary …

Three examples of a prayer graciously answered:

To ensure suppression of FCNC (*GIM mechanism*) — 4th quark (charm)

To allow for *CP violation* in the SM — 3rd quark generation (t,b)

To cure *axial anomaly* (#quarks = #leptons)

To make Z,W massive without ruining *renormalizability* — the Higgs boson

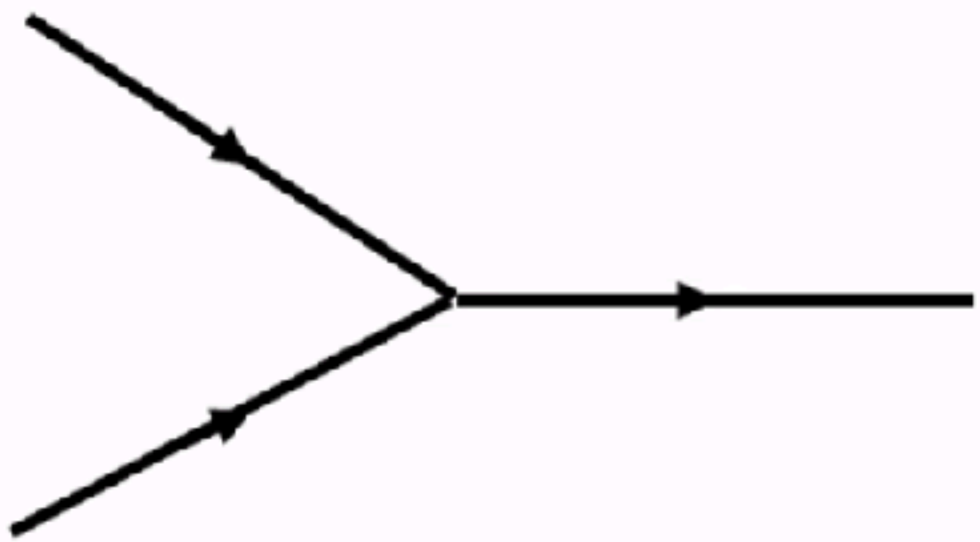
Do we have ***all*** our ambitions satisfied? Well, ***almost…***

The example of **non**-renormalizable dynamics - **Quantum Gravity**

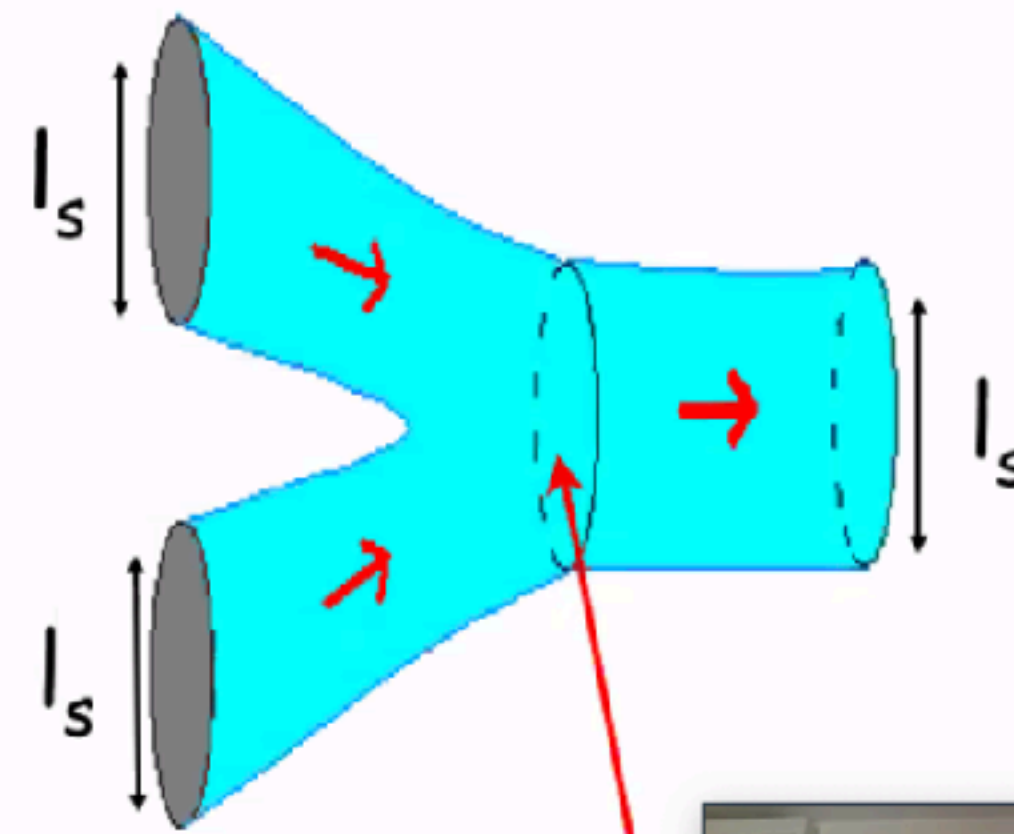
With interaction directly proportional to energies (masses), loop integrals diverge terribly in the UV - at small distances.

… Quantum Strings ?

Field Theory



String Theory



Interactions smeared over regions of order l_s

• Quantum String Theory (QST), with its magic, could be such a sought-for completion, but:

• QST is a package, you can't just use what you like about it and throw the rest.

• QST comes already equipped with SUSY, but also with extra dimensions, with dangerous massless scalars, and with a whole landscape of possible vacua.

• It is already ruled out at the perturbative level, but so is QCD...

Otherwise, all QFTs we need today for SM *are* renormalizable

Quantum Electrodynamics

Feynman-Schwinger-Tomonaga

Electro-Week Interactions

or GWS theory

Glashow-Weinberg-Salam

Quantum Chromodynamics

Gross-Wilczek-Politzer

NB: NP not for QCD but for

"**Asymptotic Freedom**" -

the most unexpected and marvellous property of quark-gluon interactions

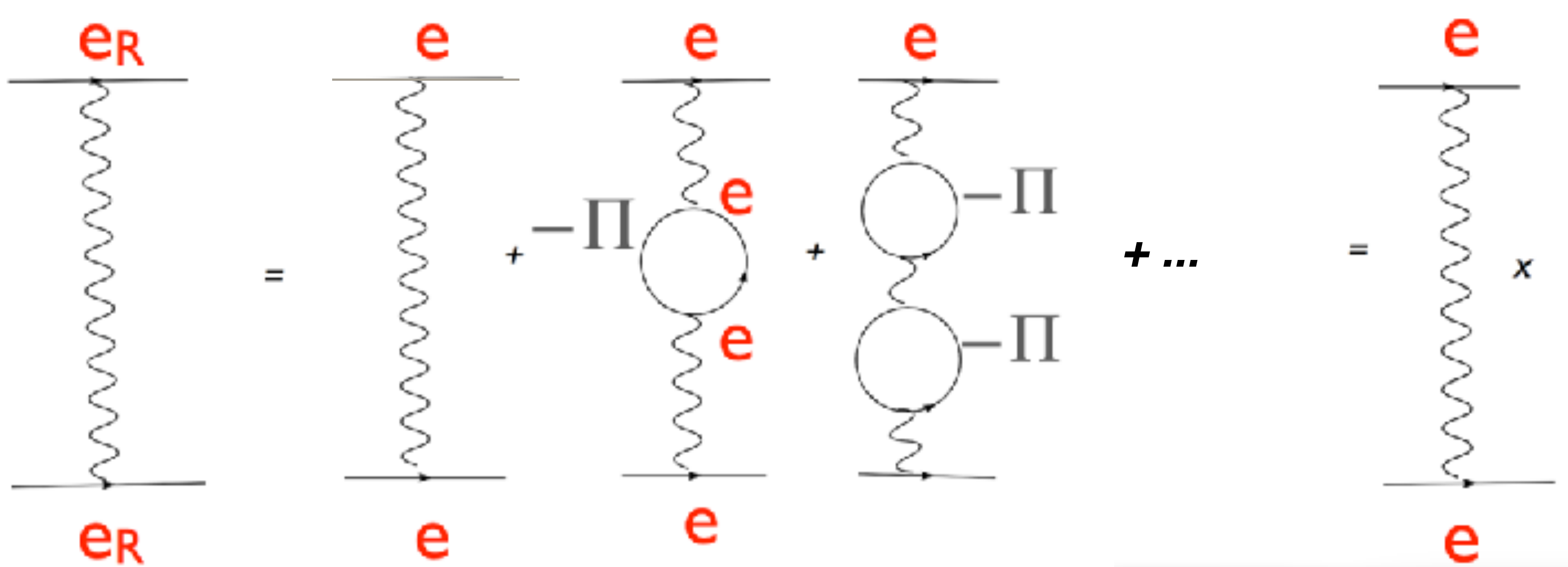
enter
Running Coupling

(QED)

$$\Pi(q^2) = \frac{\alpha}{3\pi}\log\frac{M^2}{m_e^2} + \frac{\alpha}{15\pi}\frac{q^2}{m_e^2}$$ for small q

$$\left(\alpha = \frac{e^2}{4\pi}\right)$$

$$\Pi(q^2) = \frac{\alpha}{3\pi}\log\frac{M^2}{-q^2} \qquad |q^2| \gg m_e^2$$



$$\frac{e^2}{1-(-\Pi)}$$

Invert:

$$\frac{1}{e_R^2} = \frac{1}{e^2} + \frac{1}{12\pi^2}\ln\frac{M^2}{m_e^2} + \frac{1}{60\pi^2}\frac{q^2}{m_e^2}$$

Invert: $$\frac{1}{e_R^2} = \frac{1}{e^2} + \frac{1}{12\pi^2}\ln\frac{M^2}{m_e^2} + \frac{1}{60\pi^2}\frac{q^2}{m_e^2}$$

Rutherford amplitude for q ⟶ 0 : physical electric charge from $$A = \frac{e_R^2(0)}{q^2}$$

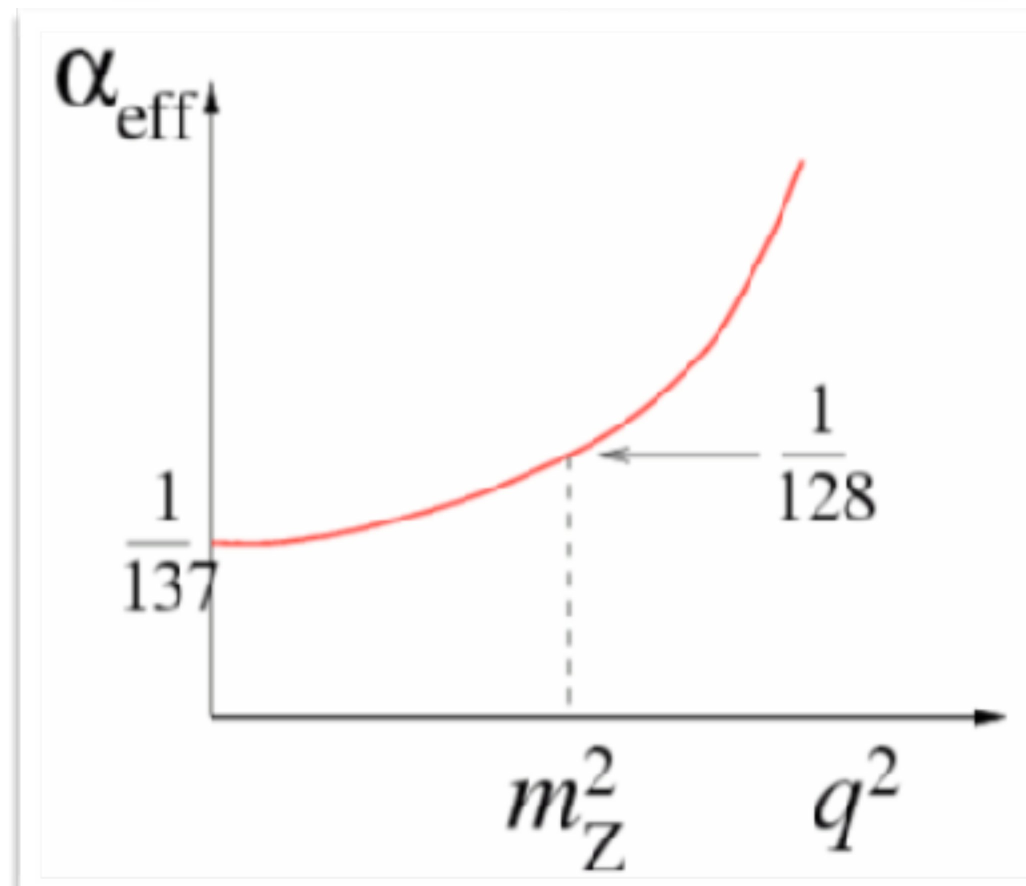
Absorb an infinite log into redefinition of the coupling!

$$\frac{1}{e_R^2(0)} \equiv \frac{1}{e^2} + \frac{1}{12\pi^2}\ln\frac{M^2}{m_e^2}$$

Running e.m. Coupling!

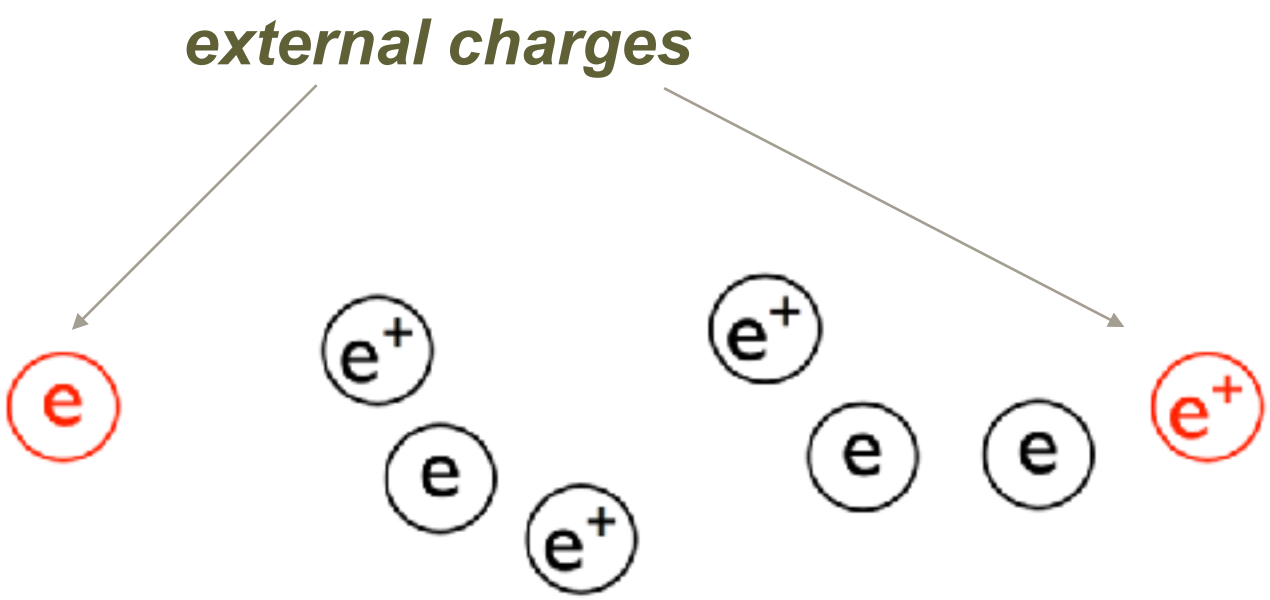
Then for large momentum transfer,

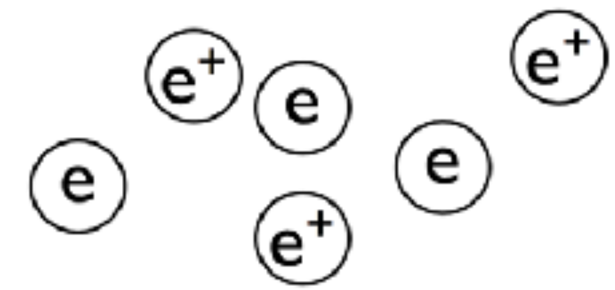
$$\frac{1}{e_R^2(q^2)} = \frac{1}{e^2(0)} - \frac{1}{12\pi^2}\ln\frac{-q^2}{m_e^2}$$



Physical reason - Why?

Screening





Pairs in the vacuum

Vacuum is polarized due to the presence of charges → Screening of the Coulomb interaction → Drop of effective charge

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log(Q^2/\mu^2)}$$

in addition to electron - muon,... quarks

extra factor $N = \sum_{leptons, quarks} e_i^2$

$$\alpha(Q^2) \to \infty \ at \ Plank \ scale$$

★ The "***Landau pole***" occurs if we keep the physical coupling $e_R^2(0)$ fixed (as we should)

★ If we decided instead to keep the bare coupling finite, then our theory (QED) would have had a ***trivial continuum limit*** ($M \to \infty$), that is, $e_R = 0$

Known as "NULLIFICATION" of the theory, or "MOSCOW ZERO"

$$\alpha \longrightarrow \alpha(\mathbf{k}^2)$$

L.Landau 1954

An initial calculation contained *a* ***wrong sign*** - a QCD-ish β-function **!!**

For a couple of weeks L.Landau and I.Pomeranchuk enthusiastically discussed with their pupils a beautiful physical picture of charge disappearing when you probe the electron closer and closer to its "core"...

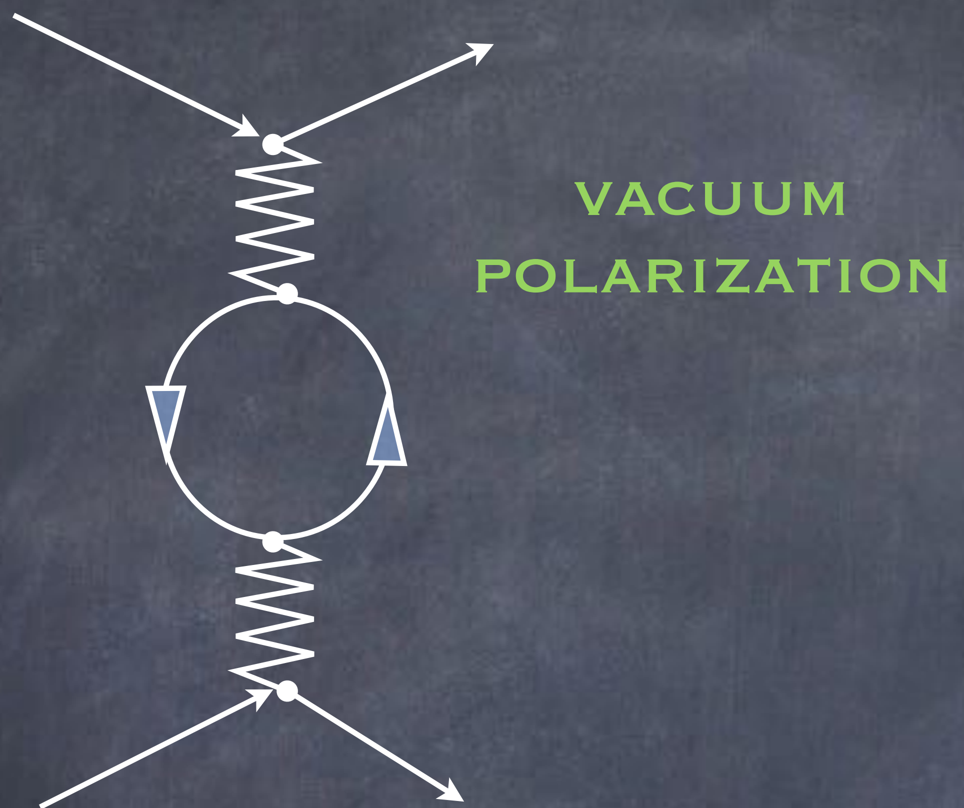
-The picture that we know today under the name of ***asymptotic freedom***

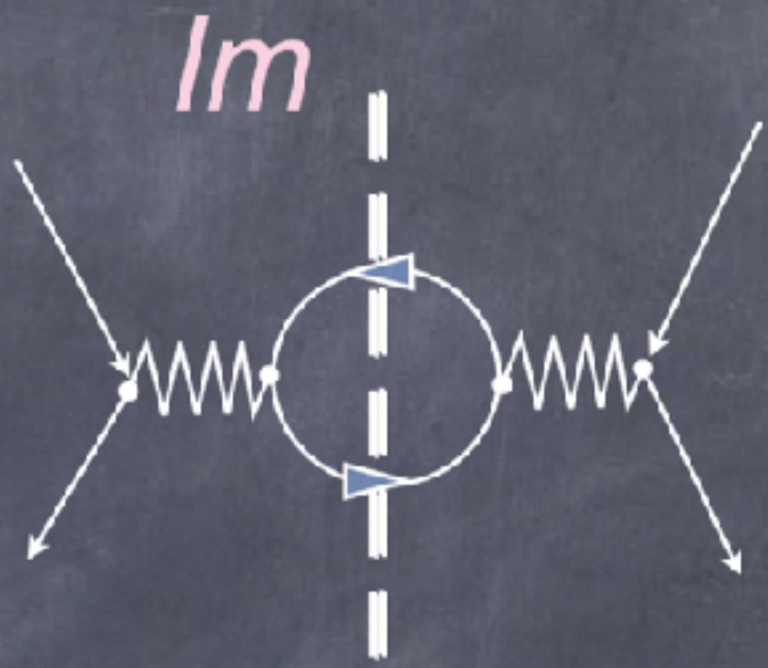
... The error was found (B.Ioffe and A.Galanin); the published result is correct

L.D. Landau, A.A. Abrikosov, and I.M. Khalatnikov, Dokl. Akad. Nauk SSSR 95, 497, 773, 1177 (1954)

A profound study undertaken by I.Pomeranchuk (1955-58) has lead to conclusion that QED-ish behaviour of the coupling as a general, inevitable property of any QFT...

As any QFT amplitude, the ***vacuum polarization*** *loop is* *analytic* *in* $\boldsymbol{k^2}$.





$\mathrm{Im}\, A = BB^{*} > 0$

Since in the ***crossing*** channel, the Im part of the loop amplitude is proportional to the cross section of pair production (***unitarity***), it got to be ***positive***.

This determines the ***sign*** of the logarithm in the running coupling thus making the *asymptotically free* behaviour of the effective coupling look ***impossible*** !

Direct consequence of ***causality*** (analyticity), relativistic ***crossing*** and ***unitarity***

This finding has deeply traumatised particle physics theory.

A 3-D landscape of the 50's-60's:

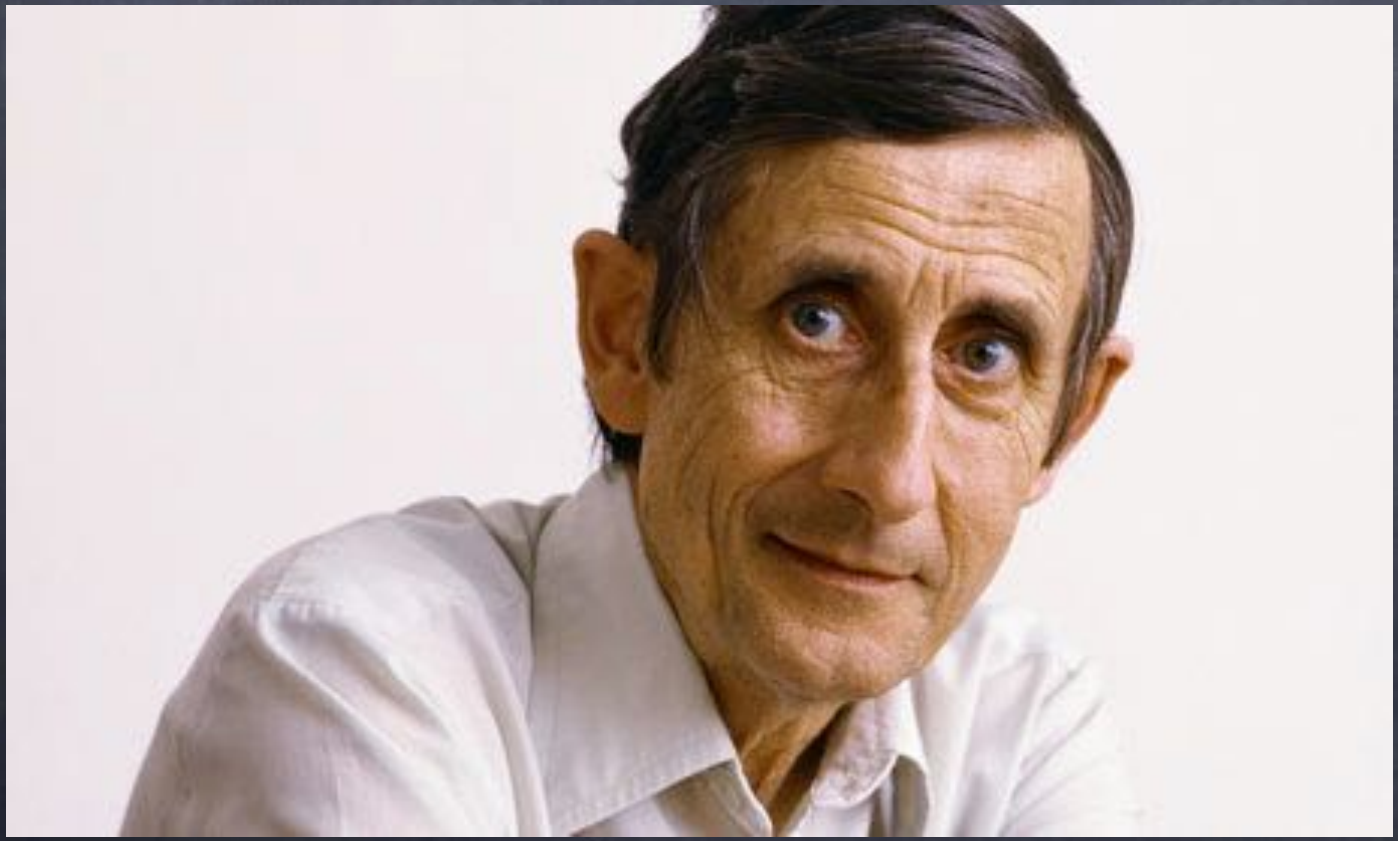
Despair

Distrust

Diverticula (Diversions)

Despair

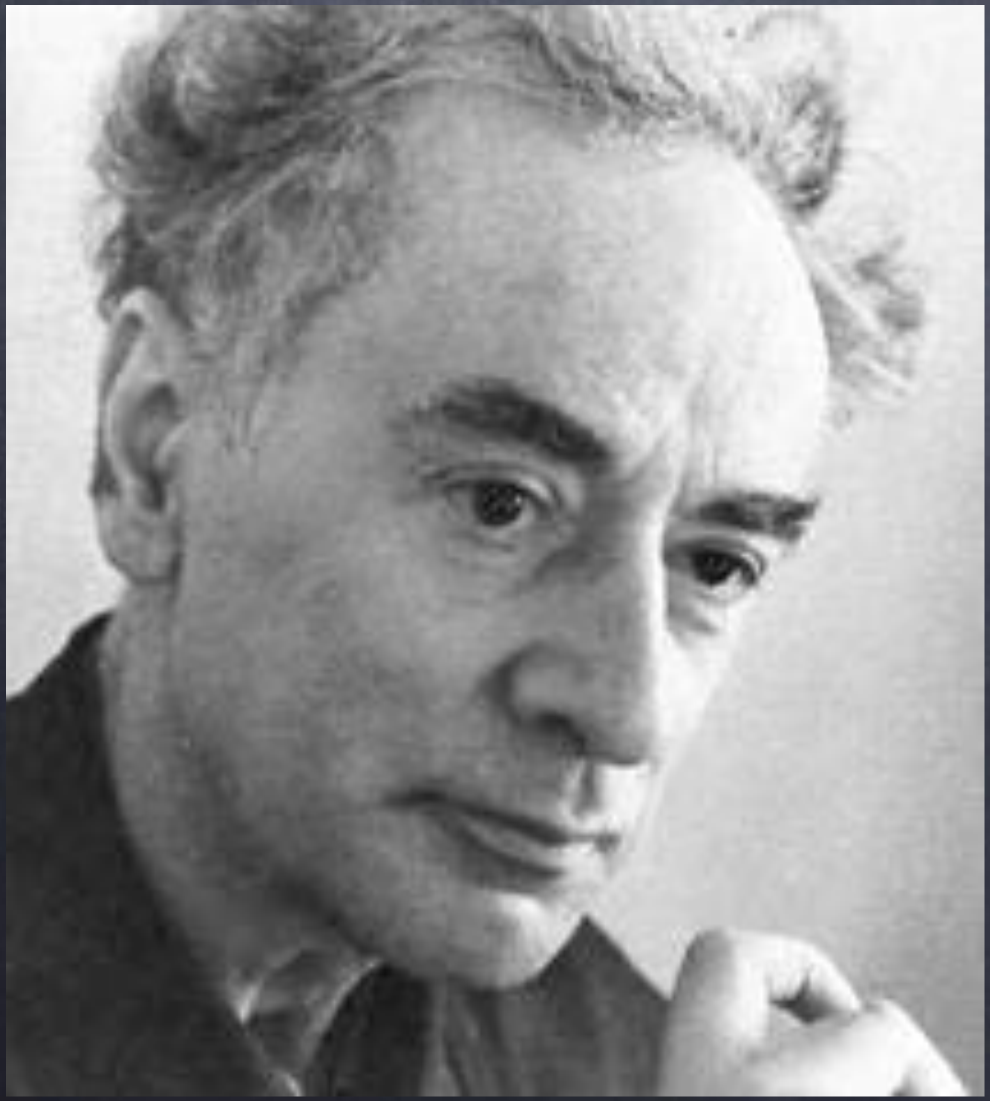
1958 **Freeman Dyson :** *“ the correct meson theory will not be found in the next **hundred years** ”*



(Freeman Dyson has always argued that it is better to be wrong than to be vague)

Distrust

1960 **Lev Landau :**



*“ the **Hamiltonian method** for strong interactions is **dead** and must be buried, although of course with deserved honour ”*

Diversions

Distrust has triggered :

Profound studies of general features of the relativistic scattering theory

Pomeranchuk theorem.   Froissart bound

Exploration of Analytic properties of scattering amplitudes

Dispersion relations

Crossing as specific feature of relativistic theory

"Bootstrap" and birth of the String Theory

Veneziano amplitude

Unitarity and its analytic continuation into crossing channels

Mandelstam

Growth of the interaction radius with collision energy
- an inevitable consequence of Unitarity + Causality + Relativity

Gribov

Analytic continuation of “partial wave” amplitudes onto complex angular momentum values.

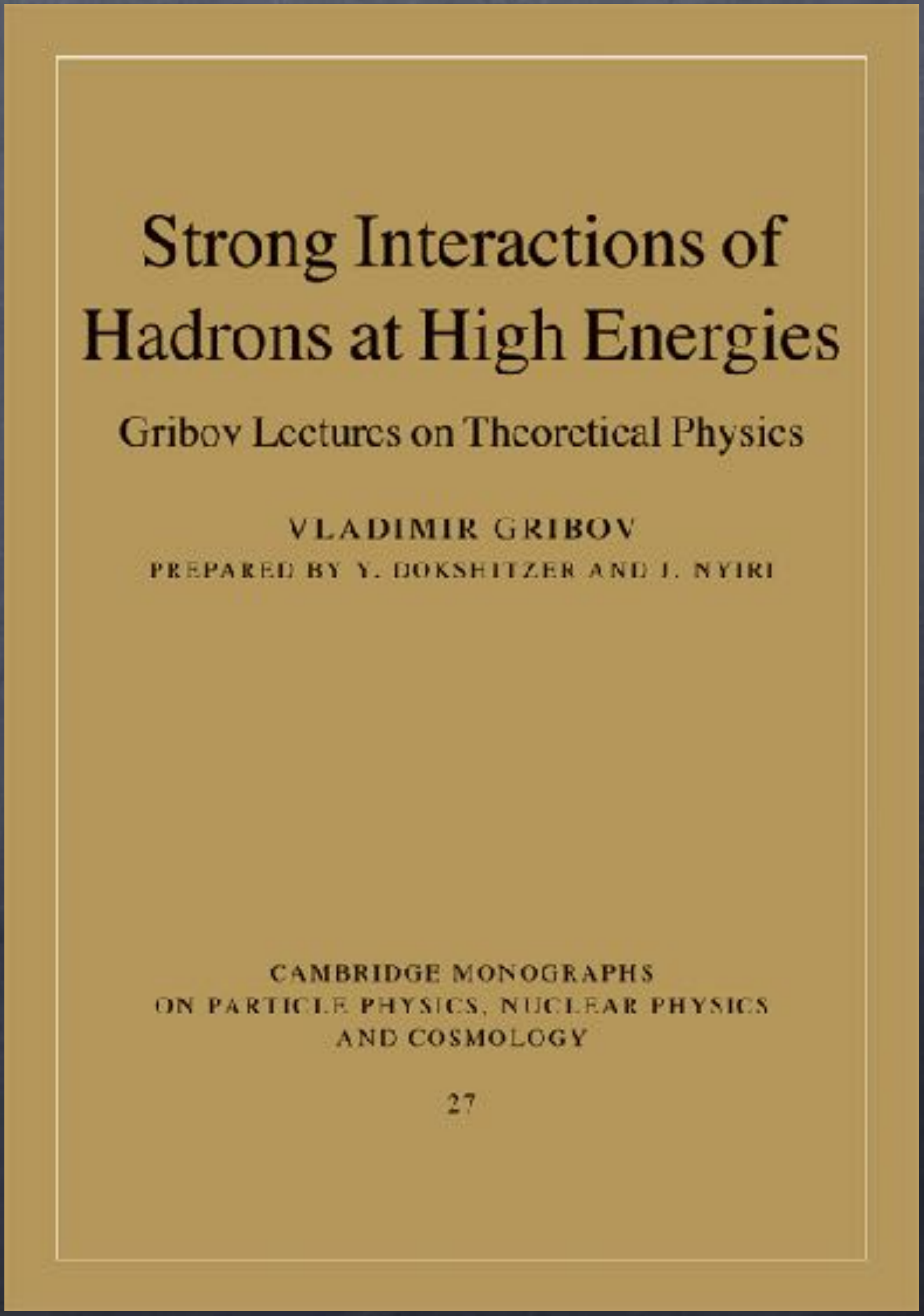
Singularities of these partial waves driving the high energy behaviour of scattering amplitudes in the crossing channel.

“Pomeron” as the leading singularity in the vacuum channel.

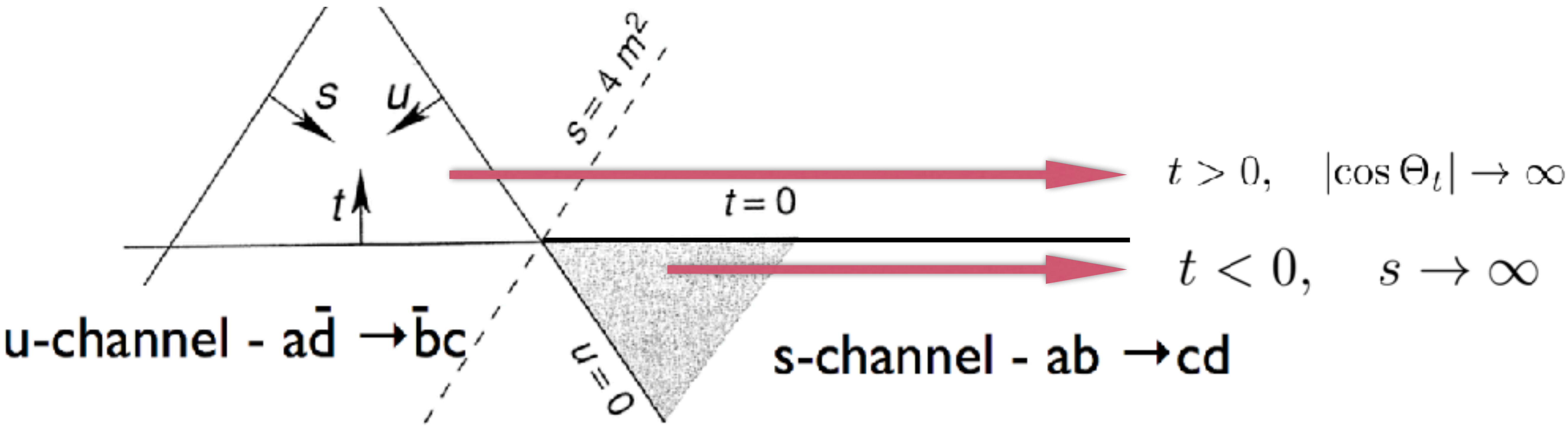
Interacting Pomerons as the first example of intrinsic dynamical instability “in the infrared”.

“Scaling” regime, and the breakthrough in the theory of second order phase transitions.

Gribov-Regge theory of high energy hadron interactions



crossing, analyticity and high-energy behaviour



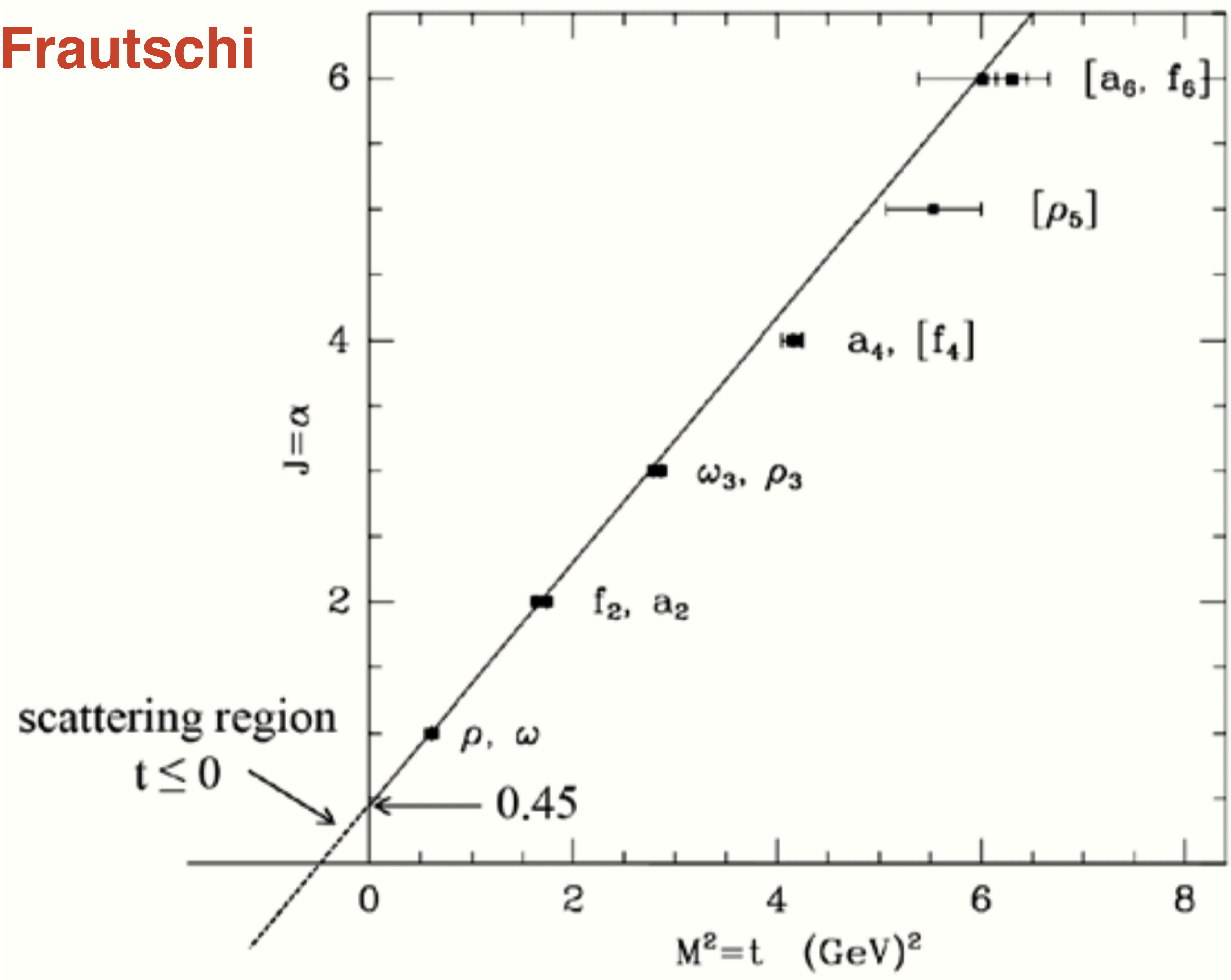
Unphysical limit $|\cos\Theta_t| \to \infty$ is determined by spectrum of t-channel resonances!

Complex angular momenta in non-relativistic QM (Regge, 1959).

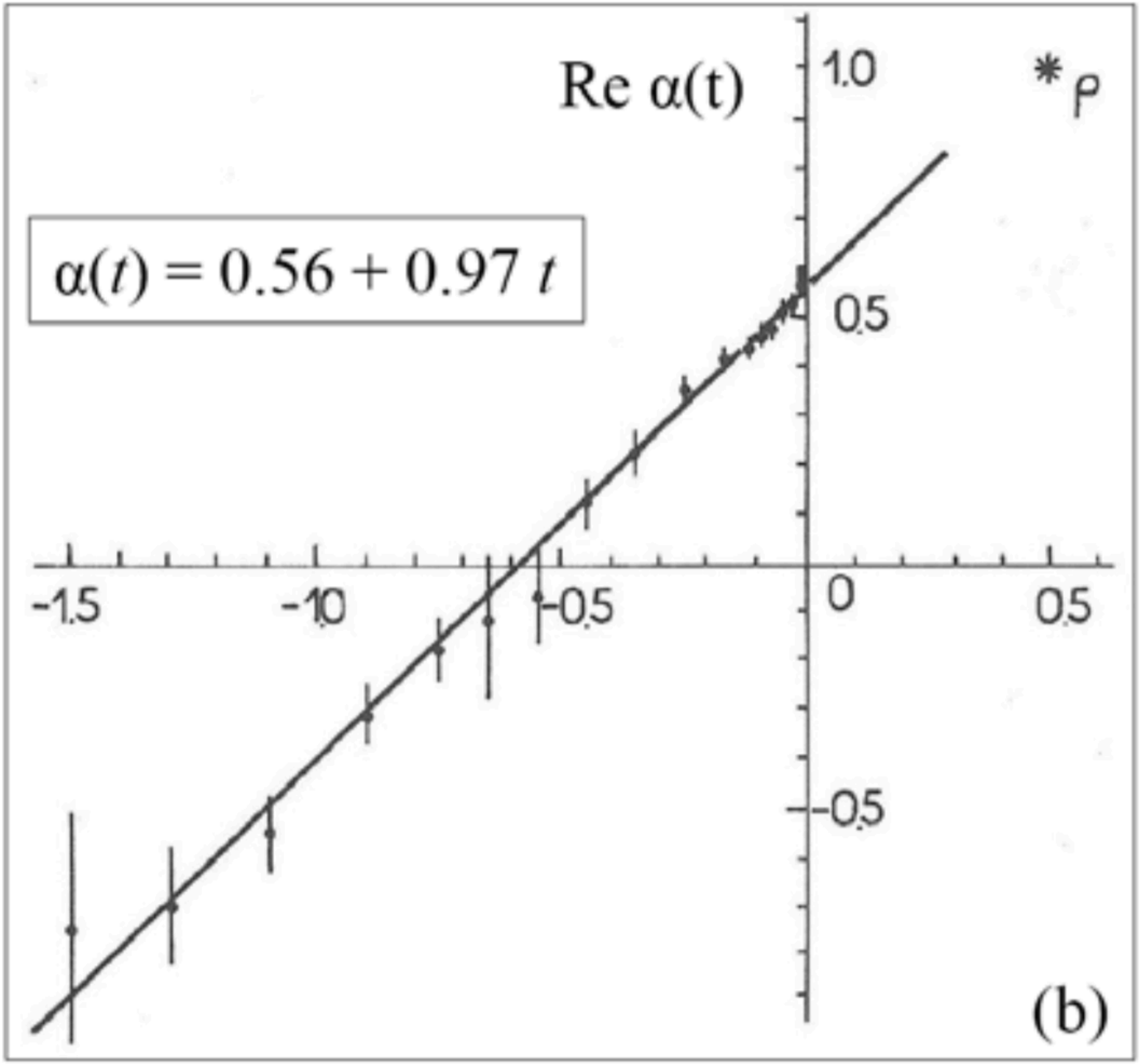
"Regge trajectory" combines resonances with increasing spin.

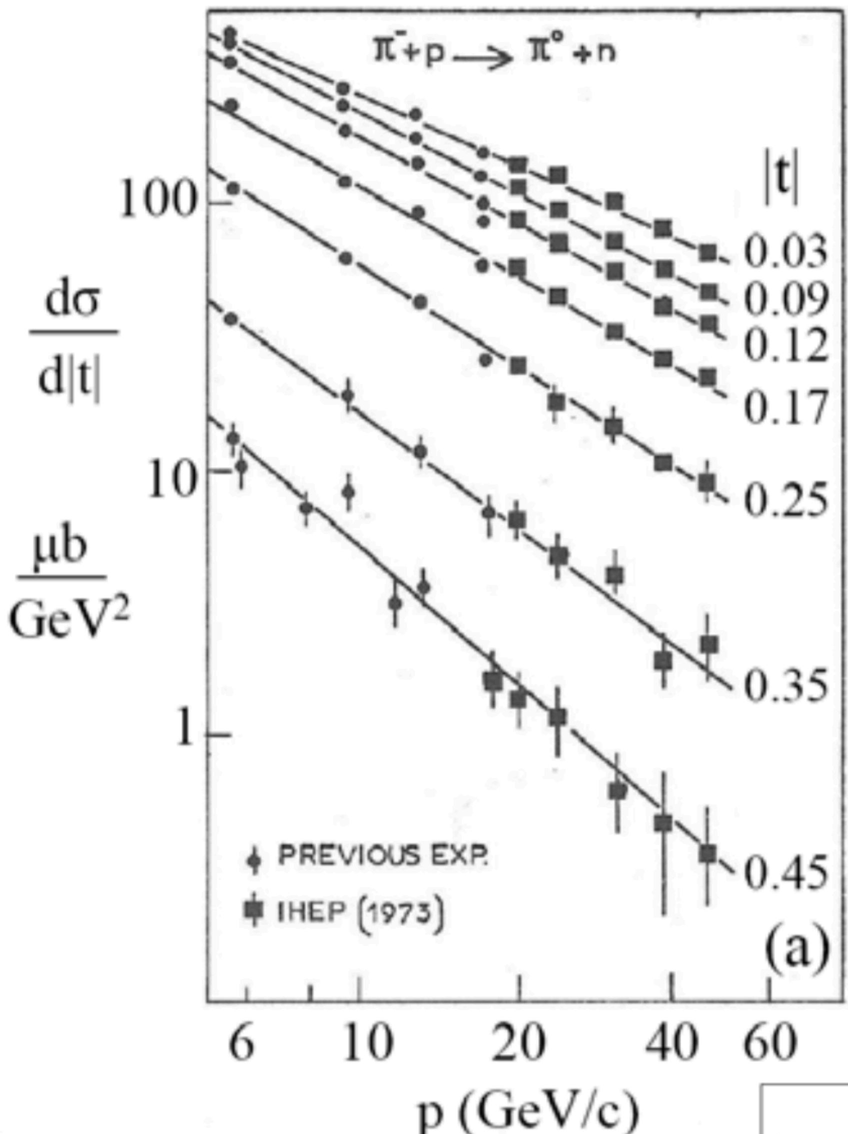
In **relativistic theory** (Gribov & Froissart, 1961) - a handle for high energy finite-t scattering.

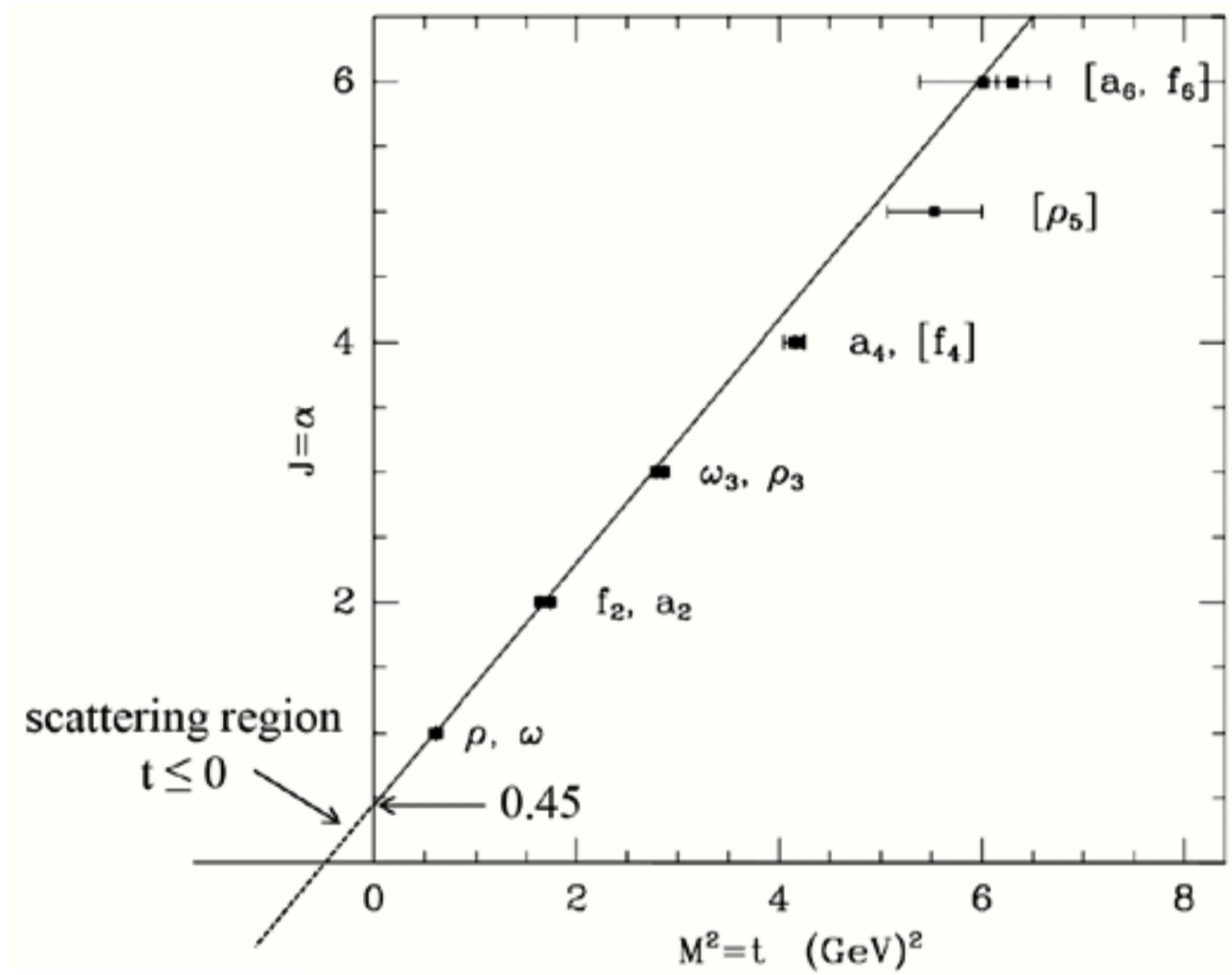
Chew-Frautschi plot

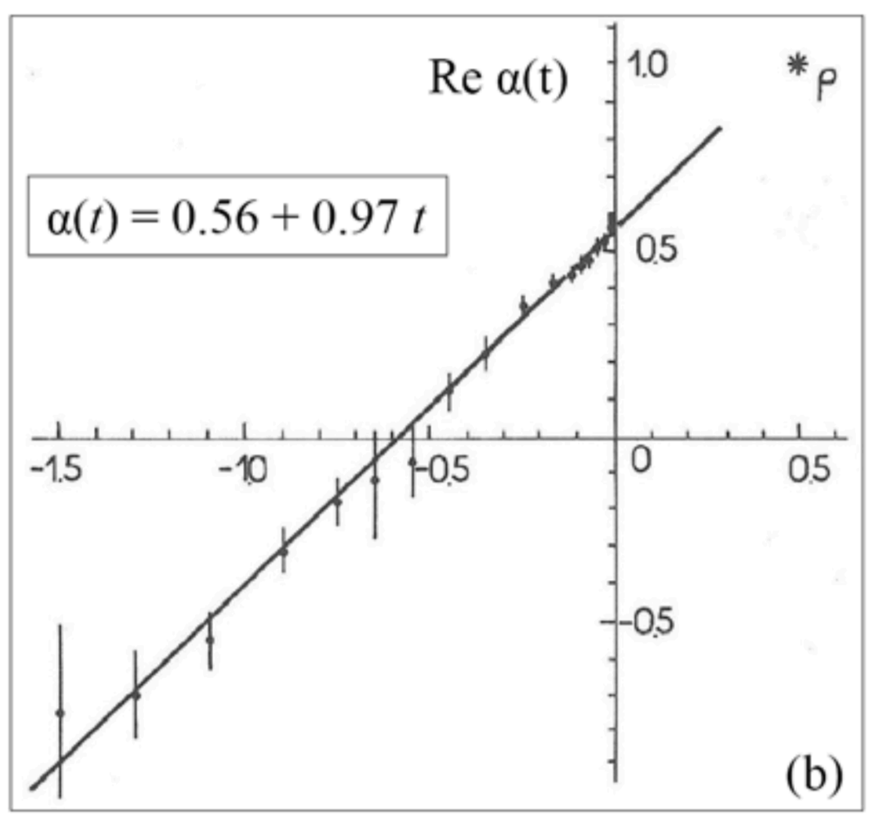


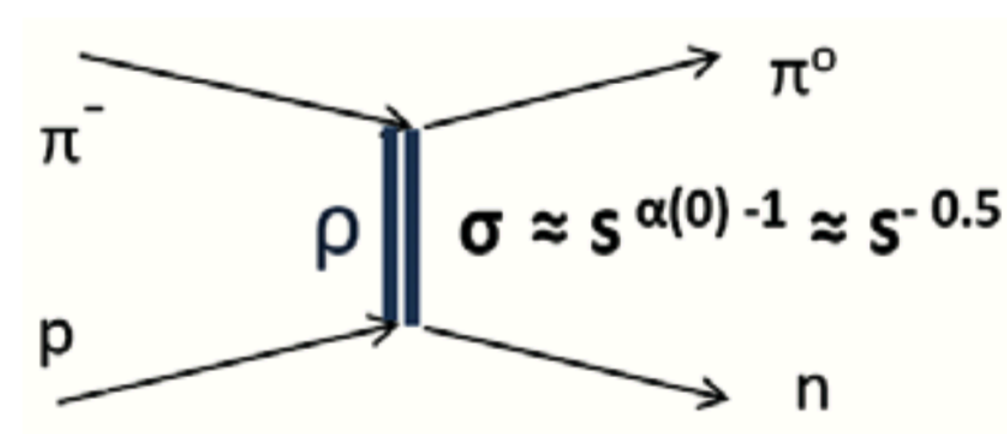
power of s-behaviour of "*charge-exchange*" 2->2 amplitude




π⁻+p → π°+n
|t|
0.03
0.09
0.12
0.17
0.25
0.35
0.45
dσ/d|t|
μb/GeV²
100
10
1
PREVIOUS EXP.
IHEP (1973)
(a)
6
10
20
40
60
p (GeV/c)


[a6, f6]
[ρ5]
a4, [f4]
ω3, ρ3
f2, a2
ρ, ω
J=α
scattering region
t ≤ 0
0.45
M²=t (GeV)²
0
2
4
6
8


Re α(t)
*ρ
α(t) = 0.56 + 0.97 t
1.0
0.5
-0.5
-1.5
-1.0
0
(b)


π⁻
p
π°
n
ρ
σ ≈ s^{α(0) -1} ≈ s^{- 0.5}

punchline #1

back to the running coupling

Thus, the QED coupling - effective electric charge - increases (eventually, catastrophically) with momentum transfer (= at small space-time intervals).

For many years such a behaviour was believed to be general, common for all QFTs, as it follows from the basic properties: ***relativity*** (crossing), ***causality*** (cause vs effect) and ***unitarity*** (probability)

In QCD, on the contrary, effective charge was found to ***fall*** with increase of momentum transfer!

How did the QCD coupling manage to do so without violating the "General Principles"?

T.B.C.