QFT
QED, QCD
HEPP

- III

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Hadron physics is on the move. And so is QCD.

But the driving motivations have changed. No more "checks".

To apply, to precise , to go broader and deeper.

"Bump Hunting". Looking for unconventional objects/behaviour.

Putting QCD into extreme conditions. Modelling unknowns.

Physics of hadrons has never been simple. And will never be.

At the same time,

an explosive progress in analytic calculations of multi-leg QFT amplitudes and multi-loop corrections in recent years provides reappearing themes, motives, constructs, of **striking simplicity** !

Could it be that the deep structure of the underlying QFT dynamics is actually simpler than one dared to think ?

QCD made simpler?

R&D #1

N=4 SUSY :
a
CLASSICAL QFT ?

an inviting heresy

Low-Barnett-Kroll wisdom

and the story of

"classical gluons"

Low-Barnett-Kroll wisdom

Celebrated soft bremsstrahlung theorem was formulated by Francis Low in 1956 for scalar charged particles and later generalized by Barnett and Kroll to fermions.

The very classical nature of ***soft radiation*** makes it **universal** with respect to

intrinsic quantum properties of participating objects and
the nature of the underlying scattering process

- it is only ***classical movement*** of electromagnetic charges that matters!

$$d\sigma^{(1)}(p_i,\omega) \propto \frac{\alpha}{\pi}\frac{d\omega}{\omega}\left[\left(1-\frac{\omega}{E}\right)\cdot\sigma^{(0)}(p_i) + \left(\frac{\omega}{E}\right)^2\cdot\tilde{\sigma}(p_i,\omega)\right]$$

non-radiative ("Born") ***cross section***

Normally, for a particle production

$$|M|^2\cdot\frac{d^3k}{\omega} \propto \omega d\omega \qquad M=\mathcal{O}(1) \quad \text{in the } \omega\to 0 \text{ limit}$$

An enhanced matrix element, $M \propto \omega^{-1}$, characterizes classical field rather than particle

A dramatic consequence : **soft photons "*don't carry quantum numbers*"**

If the non-radiative process is for some reason forbidden (*parity, C-parity, angular momentum*) the ***veto cannot be lifted*** by emitting a **soft photon** !

LBK and QCD

This "drama" turns into "tragedy" in the QCD context :

soft gluons "*don't carry away no colour*" **either**

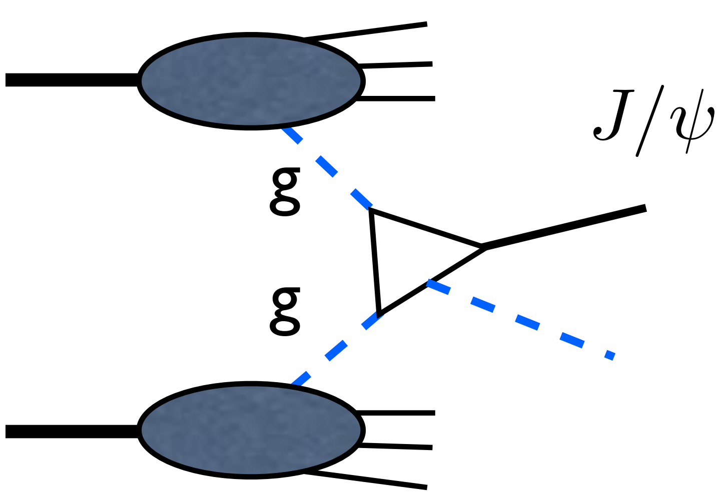
For many years pQCD practitioners were unable to describe the yields of heavy quarkonia in hadron collisions at Tevatron. For example, the measured yield of J/ψ at large transverse momenta was up to **50** times bigger than expected !

Pushed by this long-lasting failure, desperate theorists came up with a remedy :

the "***colour octet***" model for J/ψ ***production***

J/ψ is an S-state of $C\bar{C}$: $^{2s+1}L_J = {}^3S_1$ - a vector meson like photon : P=-1, C=-1

It can decay into 1(3) photon(s), a photon and 2 gluons, into 3 gluons (*in colour-symmetric state*)



two-body final state ⟹ ***smallness*** ...

... get free lunch by blaming **confinement** !

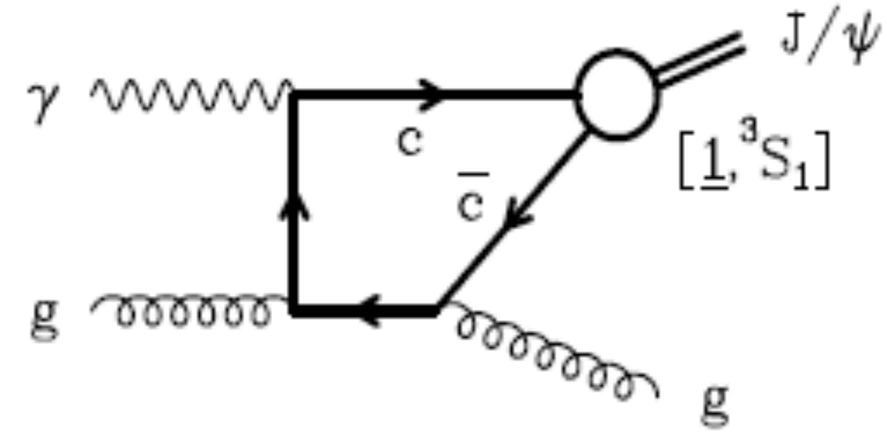
g+g ⟹ χ ⟹ J/ψ + *photon* (*40% of the yield*)

g+g ⟹ $J/\psi^{(8)}$ ⟹ J/ψ + *junk glue* : $\omega \sim \Lambda_{\rm QCD}$

LBK : the price to pay for "***colour evaporation***" = $\left(\frac{\Lambda_{\rm QCD}}{M_c}\right)^2 \ll 1$

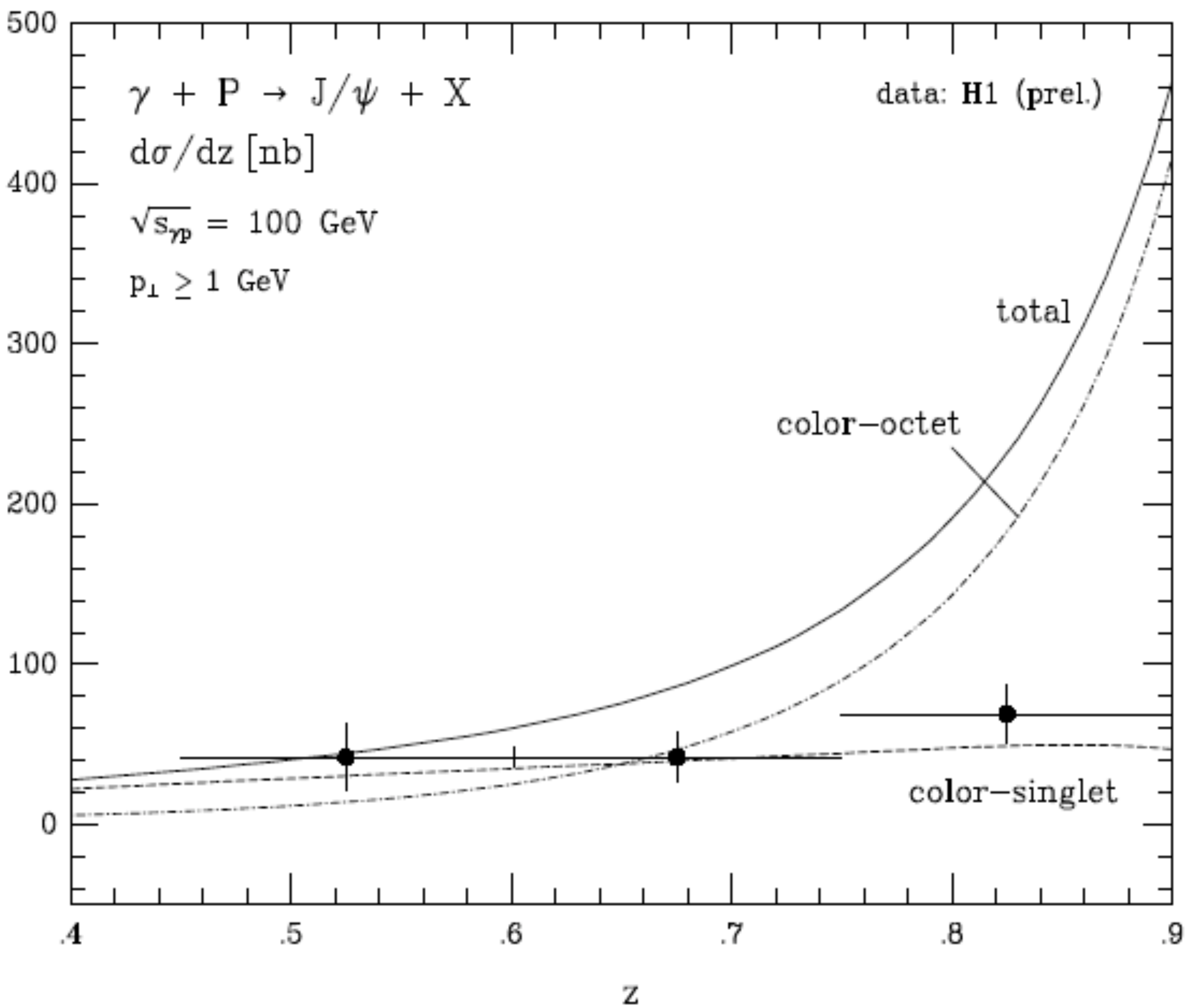
A key test : photon fragmentation into J/ψ in **e⁻ p** collisions (HERA)

photon - gluon fusion :



IF the final state shaken-off glue could be "junky"

then the J/ψ spectrum would have ***peaked*** at **z=1**



The J/ψ energy distribution $d\sigma/dz$ at the photon-proton centre of mass energy $\sqrt{s_{\gamma p}} = 100$ GeV integrated in the range $p_\perp \geq 1$ GeV.

Nowadays the "***colour octet model***" is dying away, slowly and peacefully, ***as it has to***

both from ***photo***- and ***electro***- production (HERA data)

and from ***hadron-hadron*** collisions

after M.Cacciari and P.Nason have explained the "mystery" of the "pQCD deficit" of heavy onia

How do soft gluons manifest themselves in parton dynamics ?

and what is has to do with SUSY ?

It is instructive to see how the LBK wisdom shows up in the QCD parton dynamics

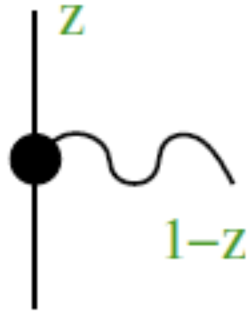
$$\frac{1+x^2}{1-x} \qquad \tilde{\gamma}_{q\to q(x)+g} = \frac{C_F\alpha_s}{\pi}\left[\frac{x}{1-x} + (1-x)\cdot\frac{1}{2}\right]$$

$$\frac{1+x^4+(1-x)^4}{x(1-x)} \qquad \tilde{\gamma}_{g\to g(x)+g} = \frac{C_A\alpha_s}{\pi}\left[\frac{x}{1-x} + (1-x)\cdot(x+x^{-1})\right]$$

classical ← ***gluons*** → quantum

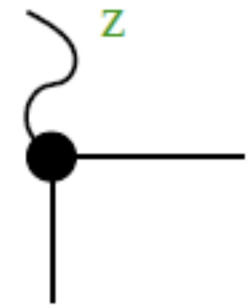
✗ Classical Field	✗ Quantum d.o.f.s (constituents)
✓ infrared singular, $d\omega/\omega$	✓ infrared irrelevant, $d\omega\cdot\omega$
✓ define the physical coupling	✓ make the coupling run
✓ responsible for	✓ responsible for conservation of
↪ DL radiative effects,	↪ P-parity, } in decays, production
↪ reggeization,	↪ C-parity, } in decays, production
↪ QCD/Lund string (gluers)	↪ colour
✓ play the major rôle in evolution	✓ minor rôle

Apparent and *Hidden* in **parton dynamics**

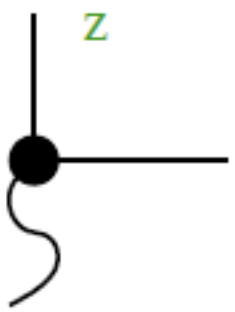


$$= C_F \cdot \frac{1+z^2}{1-z}$$

QED



$$= C_F \cdot \frac{1+(1-z)^2}{z}$$



$$= T_R \cdot \left[z^2 + (1-z)^2 \right]$$

$$= N_c \cdot \frac{1+z^4+(1-z)^4}{z(1-z)}$$

- Exchange the decay products : $z \rightarrow 1-z$
- Exchange the parent and the offspring : $z \rightarrow 1/z$

  Three (QED) "kernels" are inter-related; gluon self-interaction stays put
- The story continues, however : All four are related !

$$w_q(z) \; = \; {}^{q[g]}_{q}(z) + {}^{g[q]}_{q}(z) = {}^{q[\bar{q}]}_{g}(z) + {}^{g[g]}_{g}(z) \; = \; w_g(z)$$

***Colour factors** were **excluded** from the game !*

Super-Symmetric partner of QCD

+ infinite number of hidden invariants ! ..

$$C_F = T_R = C_A \; (= N_C)$$

Dynamics can be fully integrated if the system possesses
a sufficient number of conservation laws — integrals of motion.

Recall: Coulomb/Newton, Runge-Lentz and Focks's O(4)

Obviously, QFT has an infinite number of d.o.f.
Is an infinite # of invariants infinite enough to make QFT solvable ?

*In certain **QCD** problems (where QCD can be identified with a **SUSY** partner) the **integrability** feature does manifest itself !*

- ✓ the Regge behaviour (large N_c) — Lipatov; Faddeev & Korchemsky (1994)
- ✓ baryon wave function — Braun, Derkachov, Korchemsky, Manashov; Belitsky (1999)
- ✓ maximal helicity multi-gluon operators — Lipatov (1997); Minahan & Zarembo; Beisert & Staudacher (2003)

- ✗ It is clagons which dominate in all the *integrability cases*
- ✗ Tree multi-clagon (Parke–Taylor) amplitudes are *known exactly*

Parke–Taylor (1986) = Bassetto–Ciafaloni–Marchesini (1983)

*The higher the **symmetry**, the deeper **integrability** ...*

- ✗ Conformal theory $\beta(\alpha) \equiv 0$
- ✗ All order expansion for α_{phys}
- ✗ Full integrability via AdS/CFT

N=4 SYM — the extreme case!

Beisert, Eden, Staudacher (2006)

Maldacena; Witten,
Gubser, Klebanov, Polyakov (1998)

*What is so **special** about this theory ?*

look at the ***anomalous dimension*** :

(parton evolution "Hamiltonian")

Maximally super-symmetric YM field model:
Matter content = 4 Majorana fermions, 6 scalars;
everyone in the ajoint representation.

$$\gamma \Rightarrow \frac{x}{1-x}$$

$$\frac{C_A{}^{-1}\, d}{d \ln \mu^2}\left(\frac{\alpha(\mu^2)}{4\pi}\right)^{-1} = -\frac{11}{3} + \frac{4}{2}\cdot\int_0^1 dx\, 2[x^2+(1-x)^2] + \frac{6}{2!}\cdot\int_0^1 dx\, 2x(1-x) \;\mathbf{=0}$$

... makes one think of a *classical nature* (?) of the SYM-4 dynamics

Let us see what sort of functions the N=4 parton Hamiltonian is made of

In spite of having many states ($s = 0, \frac{1}{2}, 1$), the SYM-4 parton dynamics is built of a single "universal" anomalous dimension:

$$\gamma_+(N+2) = \tilde{\gamma}_+(N+1) = \gamma_0(N) = \tilde{\gamma}_-(N-1) = \gamma_-(N-2) \equiv \gamma_{\mathrm{uni}}(N)$$

$$\gamma_{\mathrm{uni}}^{(1)}(N) = -S_1(N) = -\int_0^1 \frac{dx}{x}\left(x^N - 1\right)\cdot \frac{x}{x-1} \equiv \mathbf{M}\left[\frac{x}{(1-x)_+}\right]$$

This is nothing but (**the Mellin image of**) ***the classical*** (**LBK**) ***gluon radiation spectrum !***

$$S_1(N) = \sum_{k=1}^{N} \frac{1}{k} = \psi(N+1) - \psi(1)$$

Euler Harmonic Sum

Beyond the 1st loop the answer is more complex.

New interesting functions show up

Euler -

- Zagier

harmonic sums

- In higher orders enter $m > 1$

$$S_m(N) = \sum_{k=1}^{N} \frac{1}{k^m} = \frac{(-1)^m}{\Gamma(m)} \int_0^1 dx\, x^N \frac{\ln^{m-1} x}{1-x} + \zeta(m)$$

- Starting from the 2nd loop, one encounters also *negative indices*

$$S_{-m}(N) = \sum_{k=1}^{N} \frac{(-1)^k}{k^m}$$

- multiple indices — *nested sums*

$$S_{m,\vec{\rho}}(N) = \sum_{k=1}^{N} \frac{S_{\vec{\rho}}(k)}{k^m} \qquad (\vec{\rho} = (m_1, m_2, \ldots, m_i))$$

$$\zeta_2 = \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \equiv S_2(\infty) \qquad \zeta_{2n} \equiv S_{2n}(\infty) \propto \pi^{2n}$$

more and more *transcendental*...

"Transcedentality" of a Harmonic Sum

= the sum of its indices

twist-2 anomalous dimension for N=4 SYM

Anatoly Kotikov - Lev Lipatov (2000)

$$\gamma_1 = -S_1$$

$$\gamma_2 = \frac{1}{2}S_3 + S_1 S_2 + \left(\frac{1}{2}S_{-3} + S_1 S_{-2} - S_{-2,1}\right)$$

Principle of
Maximal Transcedentality

***hypothesis*: sum of indices**
= 2L-1

This allowed them to predict the 3rd loop N=4 an. dim. without calculation - from that of QCD:

S. Moch, J.A.M. Vermaseren and A. Vogt (2004)

... that took some 20 years to calculate

Have a glance
at the simplest element
of the 3rd-loop
QCD parton anomalous dimension matrix :

to be compared with the corresponding
1st loop parton "Hamiltonian"

$$P_{qq}(x) = C_F \frac{1+x^2}{1-x}$$

$$+48\mathrm{H}_{-1,-1,2} + 40\mathrm{H}_{-1,0}\zeta_2 + 3\mathrm{H}_{-1,0,0} - 22\mathrm{H}_{-1,0,0,0} - 6\mathrm{H}_{-1,2} - 4\mathrm{H}_{-1,2,0} - 32$$

$$-\frac{3}{2}\mathrm{H}_0\zeta_2 - 13\mathrm{H}_0\zeta_3 - 14\mathrm{H}_{0,0}\zeta_2 - \frac{9}{2}\mathrm{H}_{0,0,0} + 6\mathrm{H}_{0,0,0,0} + 6\mathrm{H}_2\zeta_2 + 3\mathrm{H}_3 + 2\mathrm{H}_{3,0}$$

$$+(1-x)\Big[2\mathrm{H}_{-3,0} - \frac{31}{8} + 4\mathrm{H}_{-2,0,0} + \mathrm{H}_{0,0}\zeta_2 - 3\mathrm{H}_{0,0,0,0} + 35\mathrm{H}_1 + 6\mathrm{H}_1\zeta_2 - \mathrm{H}_{1,}$$

$$+(1+x)\Big[\frac{37}{10}{\zeta_2}^2 - \frac{93}{4}\zeta_2 - \frac{81}{2}\zeta_3 - 15\mathrm{H}_{-2,0} + 30\mathrm{H}_{-1}\zeta_2 + 12\mathrm{H}_{-1,-1,0} - 2\mathrm{H}_{-1,0}$$

$$-24\mathrm{H}_{-1,2} - \frac{539}{16}\mathrm{H}_0 - 28\mathrm{H}_0\zeta_2 + \frac{191}{8}\mathrm{H}_{0,0} + 20\mathrm{H}_{0,0,0} + \frac{85}{4}\mathrm{H}_2 - 3\mathrm{H}_{2,0,0} - 2\mathrm{H}_3$$

$$-\mathrm{H}_4\Big] + 4\zeta_2 + 33\zeta_3 + 4\mathrm{H}_{-3,0} + 10\mathrm{H}_{-2,0} + \frac{67}{2}\mathrm{H}_0 + 6\mathrm{H}_0\zeta_3 + 19\mathrm{H}_0\zeta_2 - 25\mathrm{H}_{0,0}$$

$$-2\mathrm{H}_2 - \mathrm{H}_{2,0} - 4\mathrm{H}_3 + \delta(1-x)\Big[\frac{29}{32} - 2\zeta_2\zeta_3 + \frac{9}{8}\zeta_2 + \frac{18}{5}{\zeta_2}^2 + \frac{17}{4}\zeta_3 - 15\zeta_5\Big]\Big)$$

$$-24\mathrm{H}_1\zeta_3 - 16\mathrm{H}_{1,-2,0} + \frac{2}{9}\mathrm{H}_{1,0} - 2\mathrm{H}_{1,0}\zeta_2 \ldots \mathrm{H}_{1,0,0} + 11\mathrm{H}_{1,0,0,0} + 8\mathrm{H}_{1,1,0,0}$$

well . . .

3rd loop for N=4 SYM

Loop # 1 : $\gamma_1 = -S_1$

Loop # 2 : $\gamma_2 = \frac{1}{2}S_3 + S_1 S_2 + \left(\frac{1}{2}S_{-3} + S_1 S_{-2} - S_{-2,1}\right)$

direct calculation by Kotikov & Lipatov, 2000

AK observation: γ_2 contains but the "most transcendental" structures !

Loop # 3 : since neither fermions nor scalars give rise to S_{2L-1}, pick out the *maximal transcedentality* pieces from the QCD an. dim.

$$
\begin{aligned}
\gamma_3 \quad = \quad & -\tfrac{1}{2}S_5 - \left[S_1^2 S_3 + \tfrac{1}{2}S_2 S_3 + S_1 S_2^2 + \tfrac{3}{2}S_1 S_4\right] \\
& - S_1\left[4S_{-4} + \tfrac{1}{2}S_{-2}^2 + 2S_2 S_{-2} - 6S_{-3,1} - 5S_{-2,2} + 8S_{-2,1,1}\right] \\
& - \left(\tfrac{1}{2}S_2 + 3S_1^2\right)S_{-3} - S_3 S_{-2} + \left(S_2 + 2S_1^2\right)S_{-2,1} + 12S_{-2,1,1,1} \\
& - 6\left(S_{-3,1,1} + S_{-2,1,2} + S_{-2,2,1}\right) + 3\left(S_{-4,1} + S_{-3,2} + S_{-2,3}\right) - \tfrac{3}{2}S_{-5}
\end{aligned}
$$

Compare parton Hamiltonians

N=4 SYM		QCD	
1st loop :	1 symbol	1st loop :	1 line
2nd loop :	1 line	2nd loop :	1 page
3rd loop :	1/2 page	3rd loop :	200 pages

Exploring another hidden symmetry – "Gribov-Lipatov reciprocity"

1 line

4 loops
5 loops
... ALL loops ?

D-r & Marchesini (2006)
Beccaria & Fiorini (2009)
Romuald Janik & Co (2010+)
someone (one day)

Morphology :
Euler-Zagier harmonic sums of "Maximal Transcedentality"

Genetics : "Maximal Transcedentality" = "classical gluons"

R&D #1

why care ?

QCD and SUSY-QCD share the gluon sector !

Importantly, the maximal transcedentality (*clagon*) structures constitute the bulk of the QCD anomalous dimensions.

Employ $\mathcal{N}=4$ SYM to simplify the essential part of the QCD dynamics

N =4 SYM dynamics is ***classical***, in (un)certain sense

No truly quantum effects are being seen

(look at the β-function and/or the anomalous dimension)

Classical does not necessarily mean ***simple.***

However, it has a good chance to be ***solvable.***

the QUEST

If this is true, the goal would be

to derive a one-line-all-orders expression for γ from $\gamma^{(1)}$ in $\mathcal{N}=4$ SYM
and then to export it into QCD,
to cover "90%" of the small-distance parton dynamics

R&D #2

Soft Gluon

QUEST

2- and 3-prong color antennae (up to 3 active colour partons) are "*trivial*" : coherence being taken care of, the answers turn out to be essentially additive

The case of 2 -> 2 hard parton scattering is more involved (4 emitters), especially so for ***gluon–gluon scattering***.

Here one encounters 6 (5 for SU(3)) colour channels which mix with each other under soft gluon radiation ...

A difficult quest of sorting out large angle gluon radiation in all orders in $(\alpha_s \log Q)^n$ was set up and solved by George Sterman and collaborators.

An additional look at the problem has revealed an intriguing ***puzzle***

(G.Marchesini & YLD, 2005)

Puzzle of large angle Soft Gluon radiation

Soft anomalous dimension for gluon-gluon scattering

$$\frac{\partial}{\partial \ln Q} M \propto \left\{ -N_c \ln\left(\frac{t\,u}{s^2}\right) \cdot \hat{\Gamma} \right\} \cdot M$$

6=3+3. Three eigenvalues are ”simple”.

Three ”***ain’t-so-simple***” ones were found to satisfy the *cubic equation*

$$\left[E_i - \frac{4}{3}\right]^3 - \frac{(1+3b^2)(1+3x^2)}{3}\left[E_i - \frac{4}{3}\right] - \frac{2(1-9b^2)(1-9x^2)}{27} = 0,$$

$$x = \frac{1}{N}, \qquad b \equiv \frac{\ln(t/s) - \ln(u/s)}{\ln(t/s) + \ln(u/s)}$$

Mark the ***mysterious symmetry*** w.r.t. to **x -> b**

*interchanging internal (**group rank**) and external (**scattering angle**) variables of the problem . . .*

Vows for “theoretical-theory” explanation !

SUSY

String Theory

AdS/CFT

. . .

R&D #3

Soft Photon

QUEST

Study of the Dependence of Direct Soft Photon Production on the Jet Characteristics in Hadronic Z^0 Decays

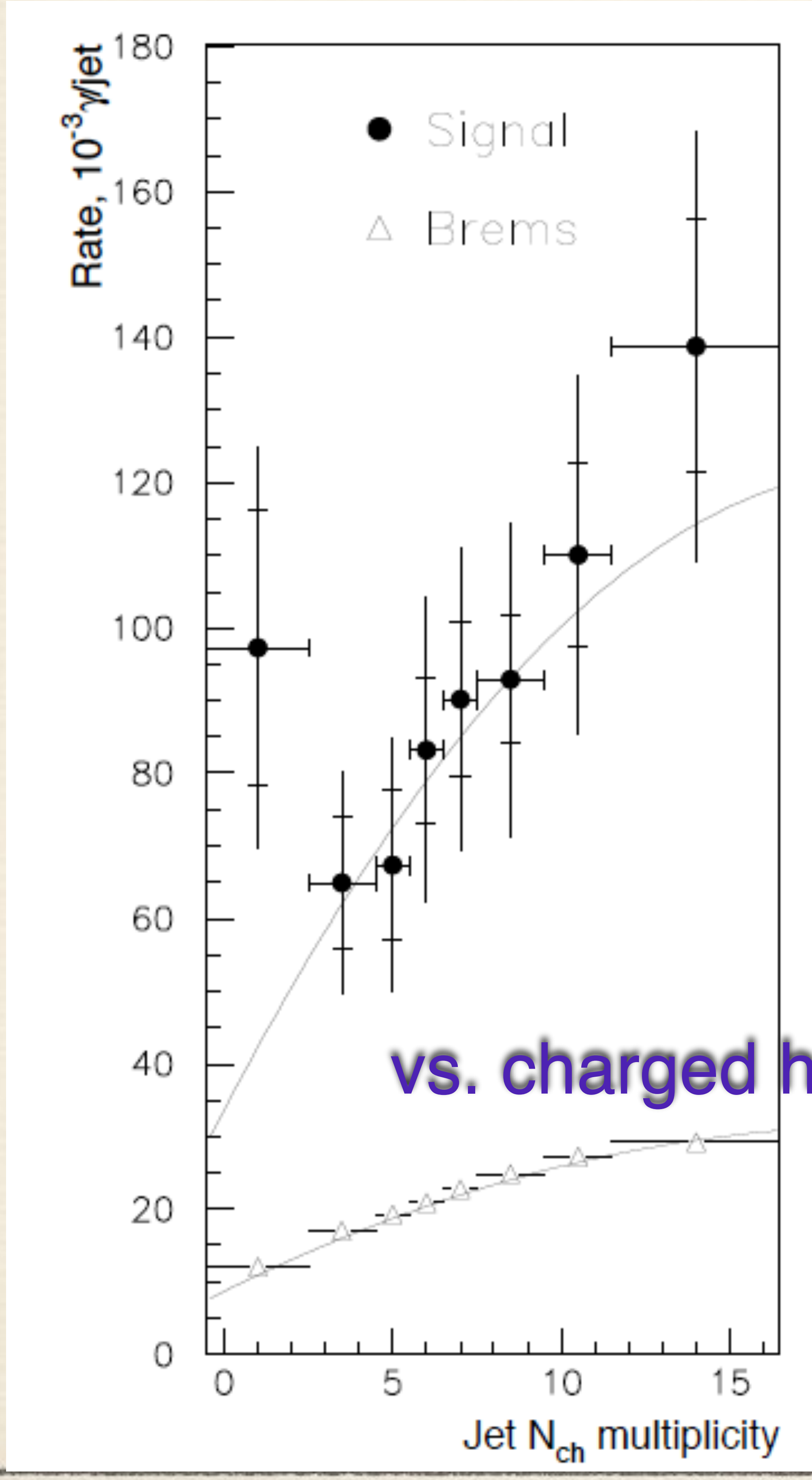
DELPHI Collaboration

$$\frac{dN_\gamma}{d^3\vec{k}} = \frac{\alpha}{(2\pi)^2}\frac{1}{E_\gamma}\int d^3\vec{p}_1 \ldots d^3\vec{p}_N \sum_{i,j}\eta_i\eta_j \frac{(\vec{p}_{i\perp}\cdot\vec{p}_{j\perp})}{(P_iK)(P_jK)}\frac{dN_{hadrons}}{d^3\vec{p}_1\ldots d^3\vec{p}_N}$$

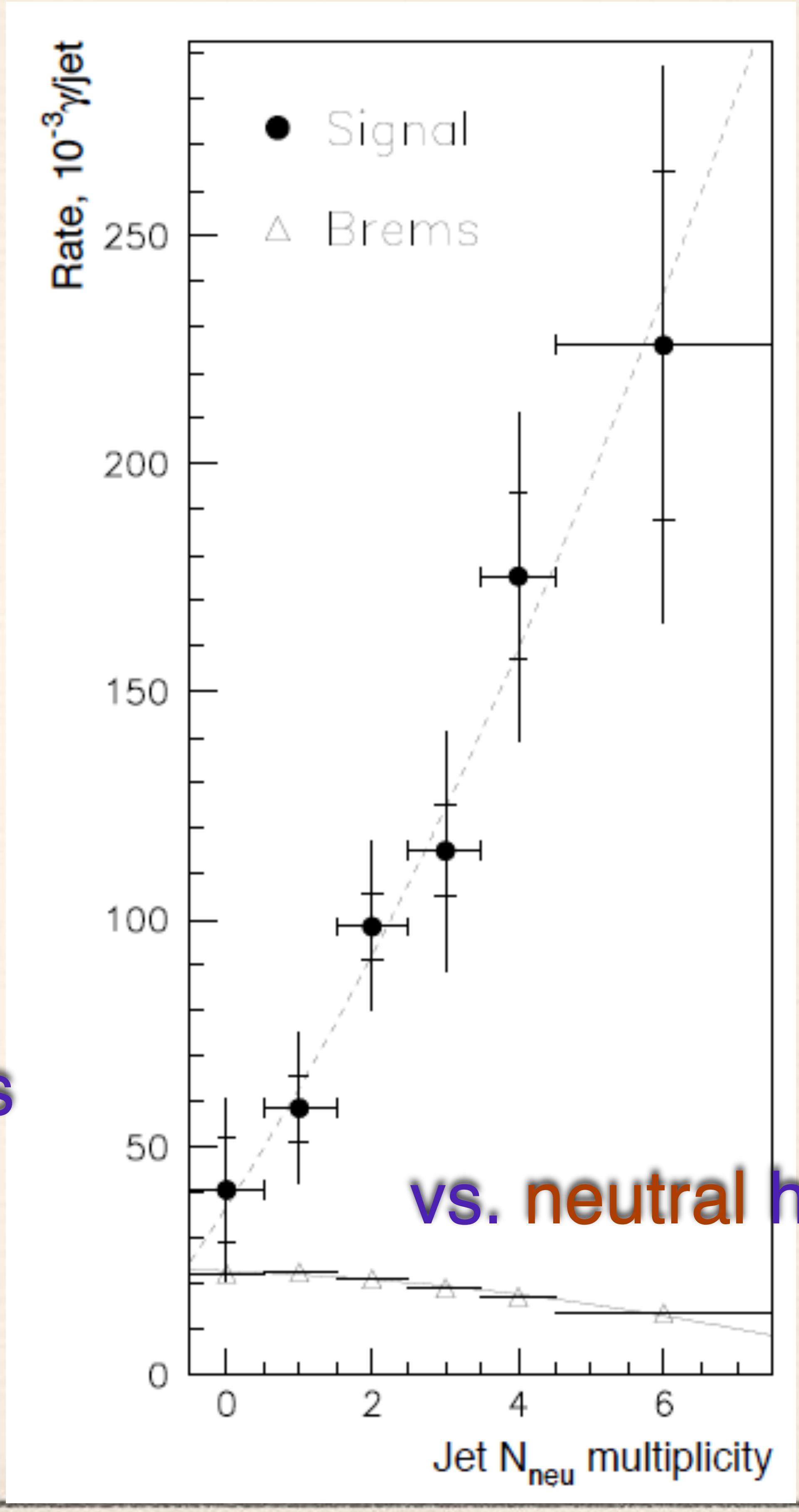
- ***calculate***
- ***compare with the data***
- ***say: "oh-la-la…"***

- $200\ \mathrm{MeV} \le E_\gamma \le 1\ \mathrm{GeV}$

DELPHI photons vs. hadron multiplicity



vs. charged hadrons



vs. neutral hadrons!

The yield of accompanying (softish) photons does not care much about the electric charges of produced hadrons.

Vapour stemming out of the “quark soup”
- prior to hadronization.

Direct witnesses of confinement !

NB: too little radiation off the quarks at the parton stage within the existing MCs (B. Webber, private communication)

Vows for qualitative and quantitative explanation !

warnings

40+ years later, we keep talking about puzzles and hints, about quests, about constructing, understanding QCD ...

What does **Unitarity** imply for a confined object?

How does **Causality** restrict quark and gluon Green functions and their interaction amplitudes?

What does the ***mass*** of an **INFO** mean?

([well] ***I****dentified* [but] ***N****on-****F****lying* ***O****bject*)

By 1958 Sidney Drell observed:

"Quantum electrodynamics (QED) has achieved a status of peaceful coexistence with its divergences...".

QCD is NOT yet in peace with its conceptual troubles...

Who to blame ? QED ***heritage*** → ***handicap***

warnings

An amazing success of the relativistic theory of electron and photon fields — QED — has produced a long-lasting negative impact: it taught the generations of physicists that came into the business in/after the 70's to "not to worry".

Indeed, today one takes a lot of things for granted :

- One rarely questions whether the alternative roads to constructing QFT — S*econdary Quantization*, *Functional Integral* and the *Feynman Diagram* approach — really lead to the ***same quantum theory*** of interacting fields.

- One feels ashamed to doubt an elegant, powerful, *but potentially deceiving,* Euclidean rotation technology translating QFT dynamics into that of a statistical system.

- One was taught to look upon the problems that arise with field-theoretical description of point-like objects and their interactions at very small distances (*ultraviolet divergences*) as purely technical : ***renormalize it and forget it***.

  The problem of ultraviolet regularization may be more than a technical trick in a QFT with apparently ***infrared-unstable dynamics*** :
  the ***ultraviolet*** and ***infrared*** regimes of the theory may be tightly linked.

- The Feynman diagram technique has to be reconsidered in QCD if one goes beyond trivial perturbative correction effects. ***(*** $i\epsilon$ *prescription =* ***stable perturbative vacuum)***

- To understand and describe a physical process in a ***confining theory***, it is necessary to learn how to take into consideration the ***response of the vacuum***, which should lead to essential modification of the quark and gluon Green functions.

GWS QED QCD

synchronized beauty of QFT performance

conclusions

The progress in QCD is slow and painful

On a positive note :

- A complete solution of the **N=4 SYM** QFT should provide us one day with a **one-line-all-order** description of the major part of QCD parton dynamics

- Understanding the anomalous production of "soft photons" in lepton-hadron and hadron-hadron interactions should teach us a thing or two about the nature of quark confinement

- Long live ***QFT***, and the ***QCD*** -its favourite naughty child!