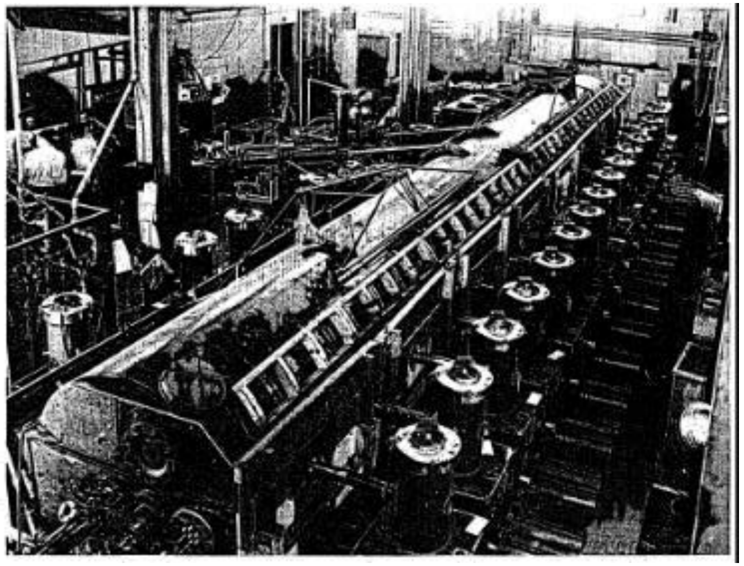
Maurizio Vretenar CERN

LECTURE 2: Linear Accelerators

August 2021

Introduction to Particle Accelerator Science and Technology

After WWII, accelerator technology goes fast!



Luis Alvarez and the Drift Tube Linac

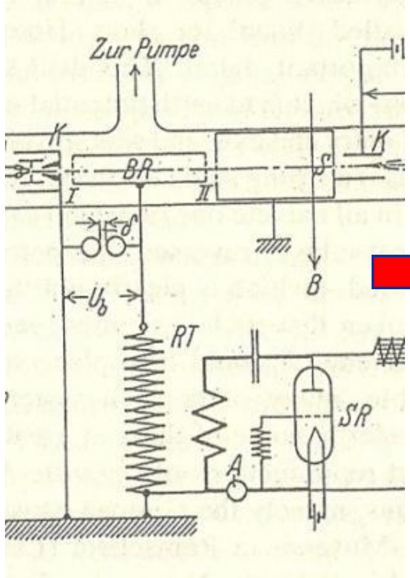
The war effort gave the competences and the components to go to higher frequencies (in the MHz – GHz range) and to try acceleration of a proton beam to the MeV range using the Wideröe principle.

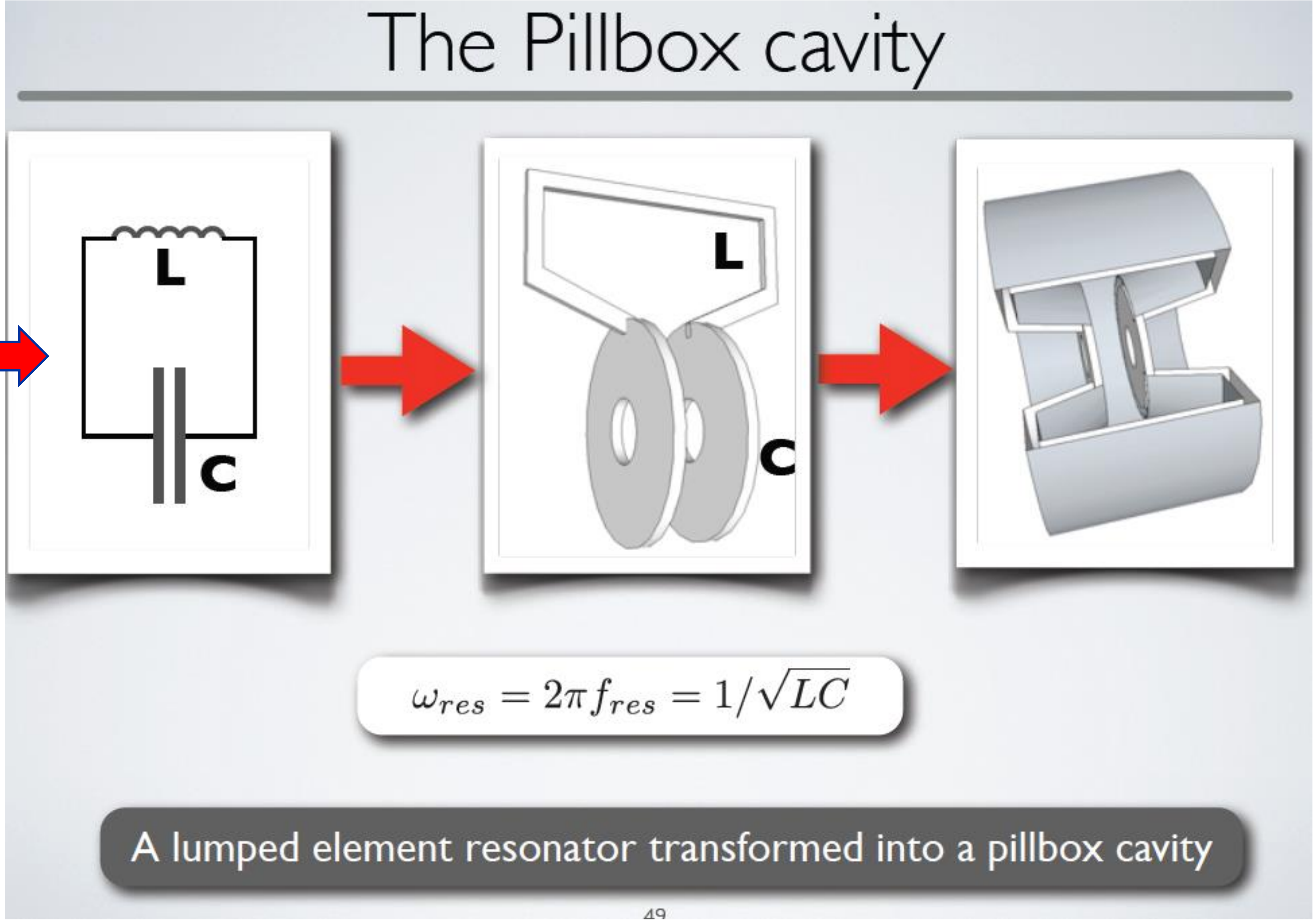
The 1st **Drift Tube Linac (DTL)** by L. Alvarez and his team at Berkeley, reaches 32 MeV in 1947.

Alvarez, an experimental physicists, worked at MIT on radar during the war. In 1945 had the tools and the competences to build his own accelerator.

1. The "drift tubes" are inside a cavity resonator.
2. Frequency : Alvarez receives from the US Army a stock of 2'000 (!) surplus **202.56 MHz** transmitters, built for a radar surveillance system. 26 were installed to power the DTL with a total of 2.2 MW. They were soon replaced because unreliable, but this frequency remained as a standard linac frequency.

From the Wideröe gap to the linac cell





The Pillbox cavity

$$\omega_{res} = 2\pi f_{res} = 1/\sqrt{LC}$$

A lumped element resonator transformed into a pillbox cavity

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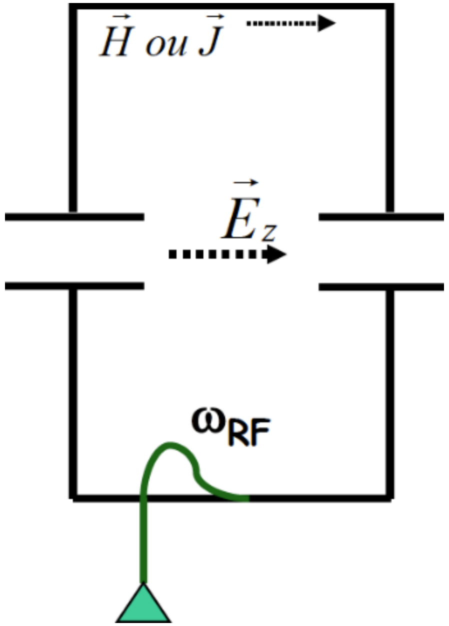
Courtesy F. Gerigk

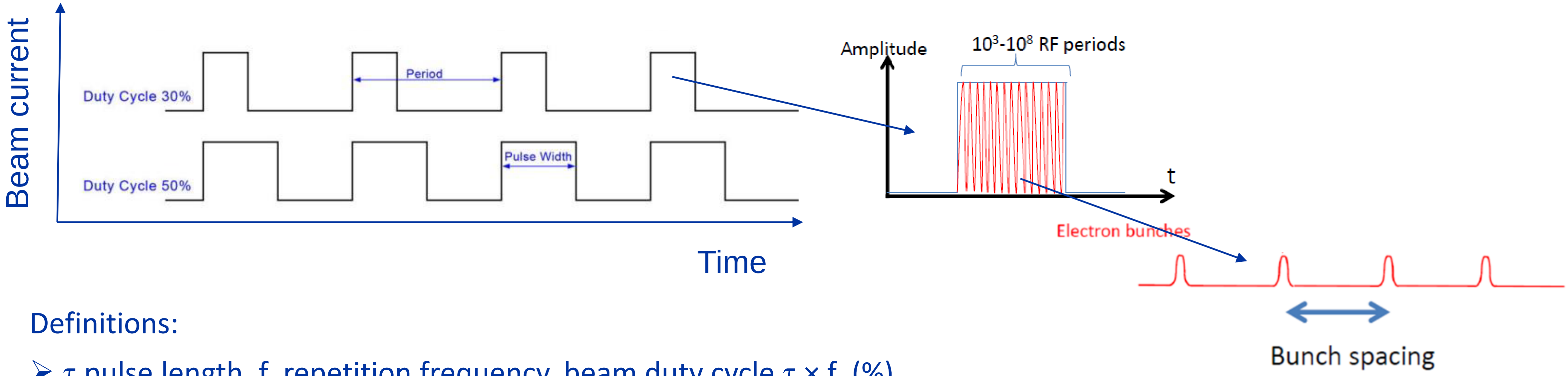


An external RF generator injects via a «coupler» some RF power that builds an electromagnetic field. The RF power is finally dissipated on the metallic walls of the cavity.

2. Introduction to linacs:
main concepts, building blocks, synchronicity

Beam time structure in an accelerator – some definitions

The beam current delivered by an accelerator is usually **pulsed** and **bunched**



Definitions:

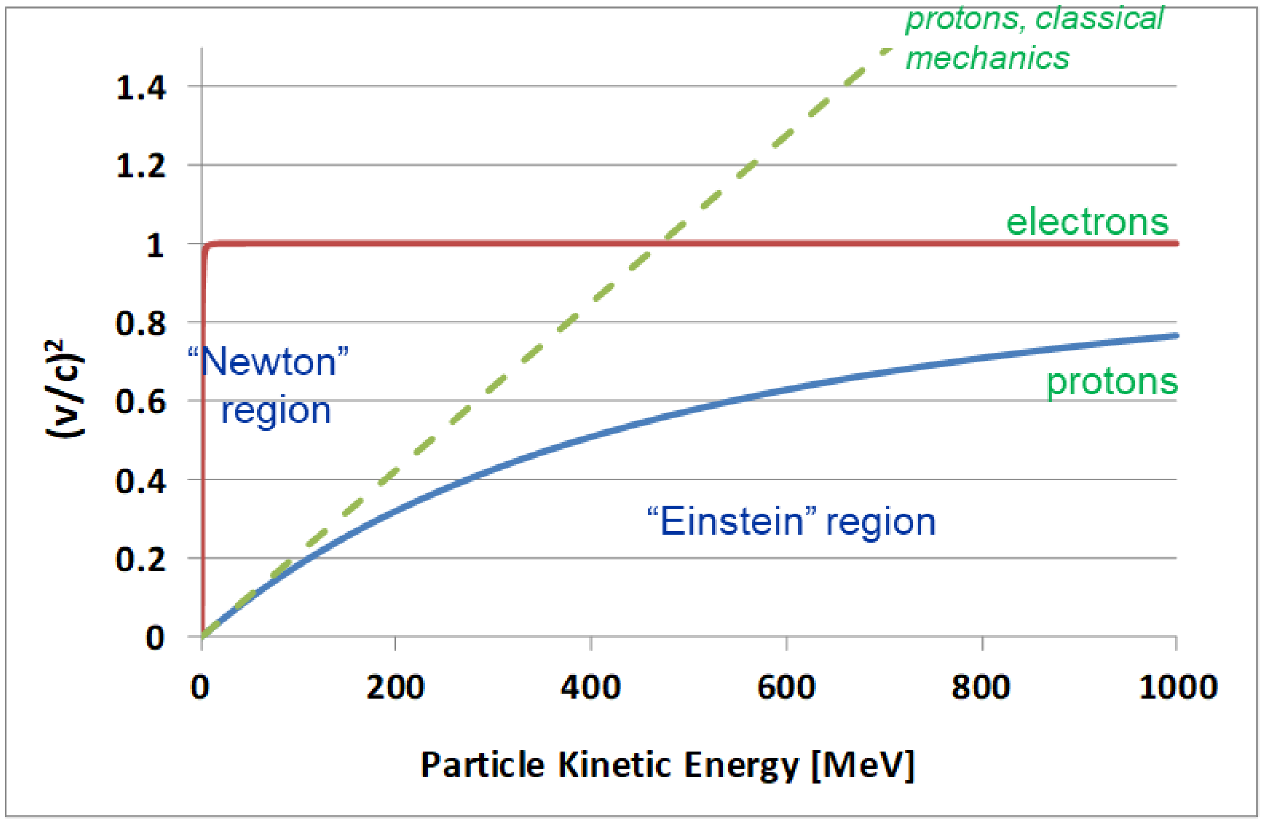
- τ pulse length, f_r repetition frequency, beam duty cycle $\tau \times f_r$ (%).
- Main parameters: *E* kinetic energy of the particles coming out of the linac [MeV]

  I average current during the beam pulse [mA] (different from *average current* and *bunch current* !)

  P beam power = electrical power transferred to the beam during acceleration:

$$P\ [\mathrm{W}] = V_{tot} \times I = E\ [\mathrm{eV}] \times I\ [\mathrm{A}] \times \text{duty cycle}$$

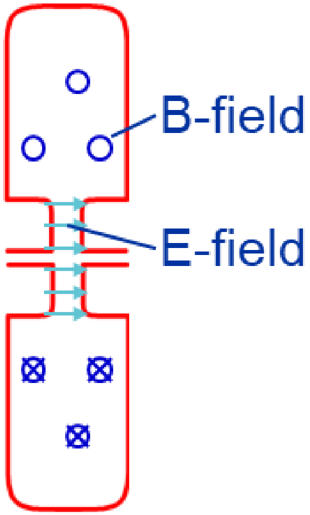
Proton and Electron Velocity

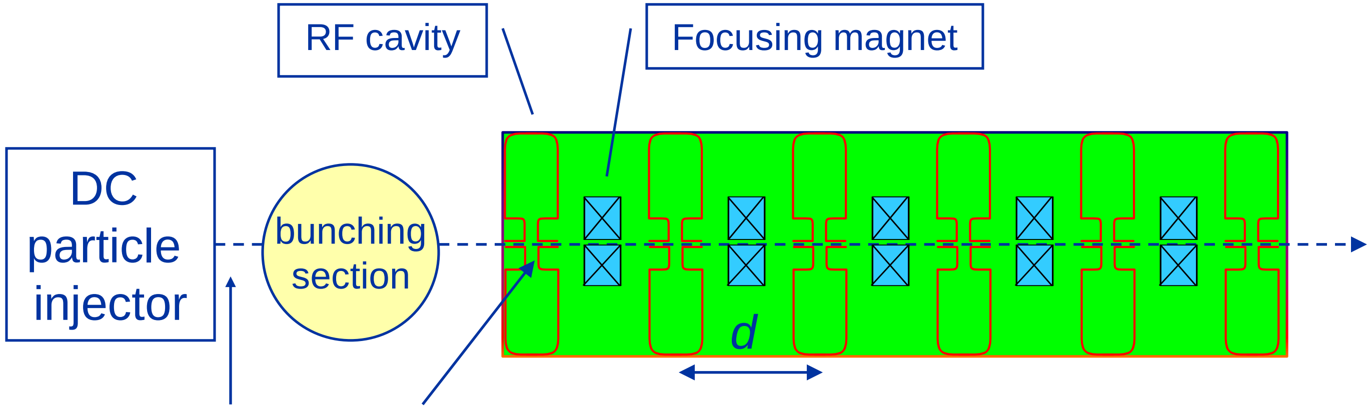


→ Protons (rest energy 938.3 MeV): follow "Newton" mechanics up to some tens of MeV ($\Delta v/v < 1\%$ for $W < 15$ MeV) then slowly become relativistic ("Einstein" region). From the GeV range velocity is nearly constant (v~0.95c at 2 GeV) → linacs can cope with the increasing particle velocity, synchrotrons cover the range where v nearly constant.

→ Electrons (rest energy 511 keV, 1/1836 of protons): relativistic from the keV range (v~0.1c at 2.5 keV) then increasing velocity up to the MeV range (v~0.95c at 1.1 MeV) → v~c after few meters of acceleration (typical gradient 10 MeV/m).

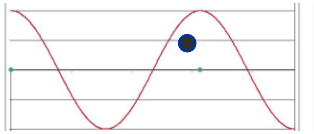
Basic linear accelerator structure





Protons: energy ~20-100 keV
β= v/c ~ 0.01

Accelerating gap: $E = E_0 \cos(\omega t + \phi)$
en. gain $\Delta W = eV_0 T\cos\phi$



Acceleration → the beam has to pass in each cavity on a phase ϕ near the crest of the wave ⇨

1. The beam must to be bunched at frequency ω
2. distance between cavities and phase of each cavity must be correlated

Phase change from cavity *i* to *i*+1 is $\Delta\phi = \omega\tau = \omega\frac{d}{\beta c} = 2\pi\frac{d}{\beta\lambda}$

⇨ For the beam to be synchronous with the RF wave (“ride on the crest”) phase must be related to distance by the relation: $\boxed{\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}}$

… and on top of acceleration, we need to introduce in our “linac” some focusing elements

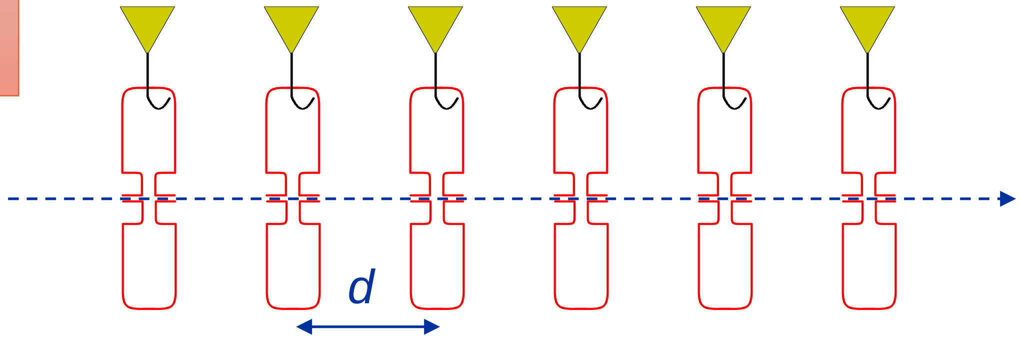
… and on top of that, we will couple a number of gaps in an “accelerating structure”

Accelerating structure architecture

When β increases during acceleration, either the phase difference between cavities *Δφ* decreases or their distance *d* increases.

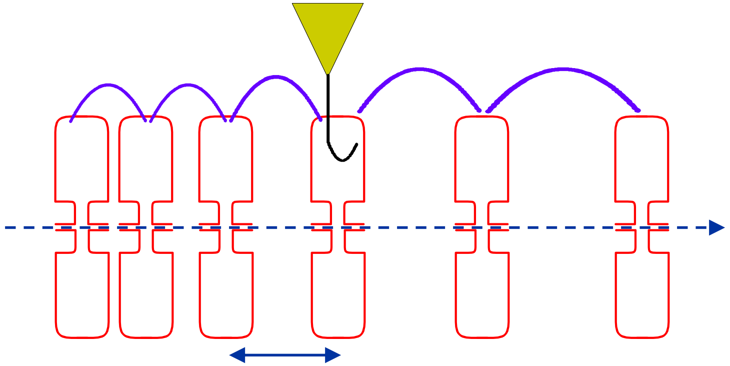
$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

d = const.
ϕ variable



Individual cavities – *distance between cavities constant, each cavity fed by an individual RF source, phase of each cavity adjusted to keep synchronism, used for linacs required to operate with different ions or at different energies. Flexible but expensive!*

ϕ = const.
d variable



$d \propto \beta\lambda$

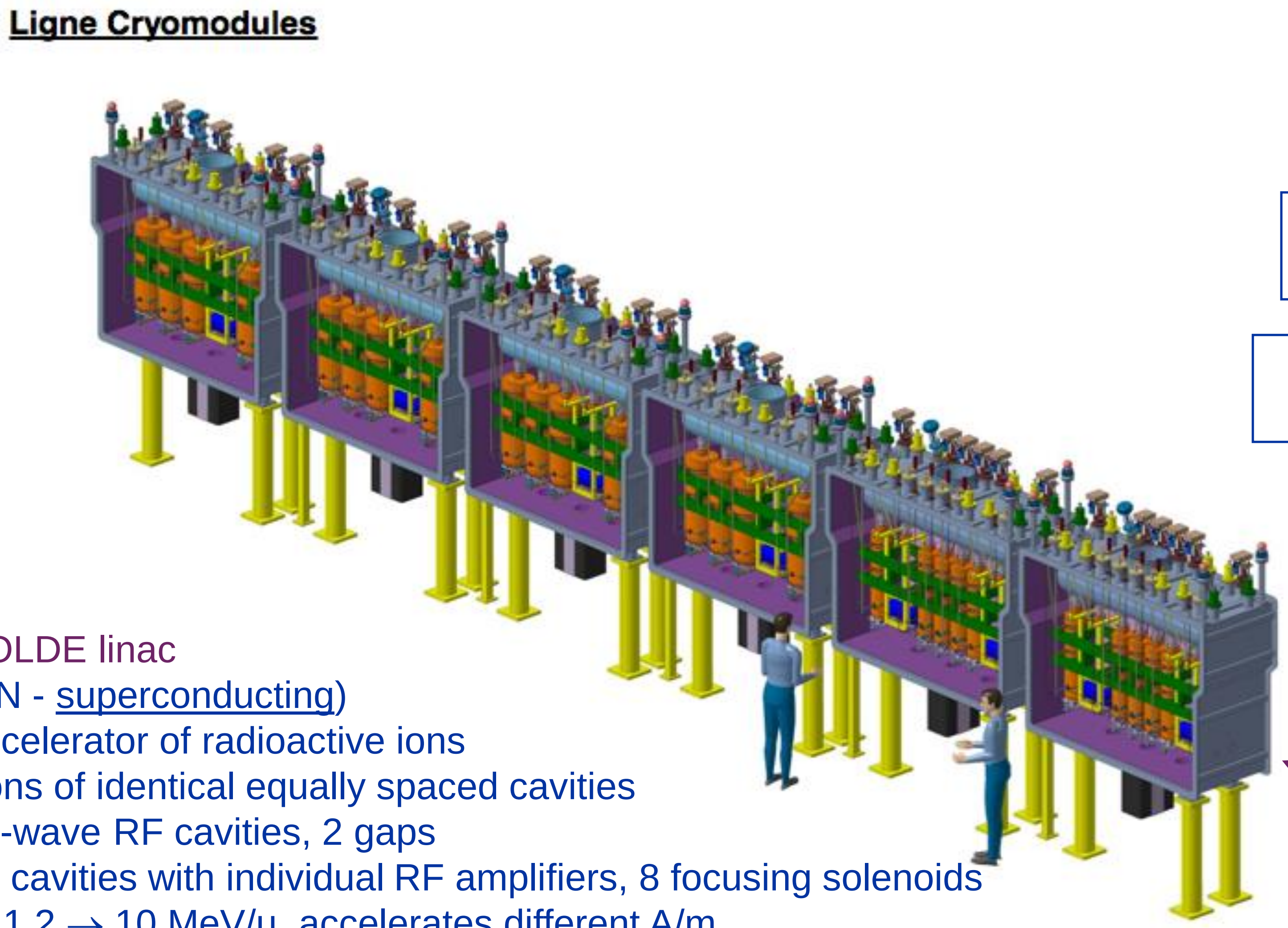
Better, but 2 problems:

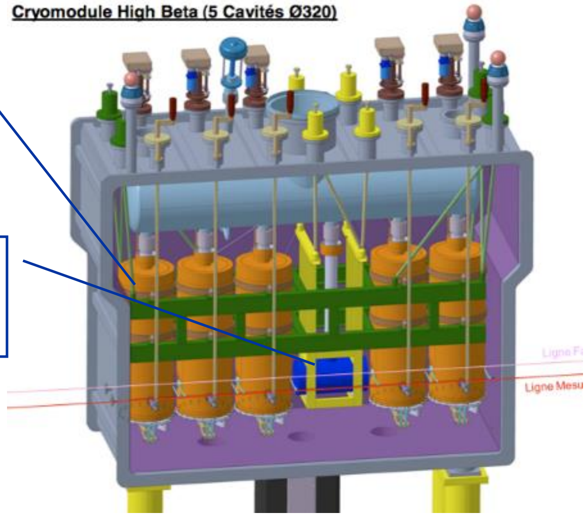
1. *create a “coupling”;*
2. *create a mechanical and RF structure with increasing cell length.*

Coupled cell cavities - *a single RF source feeds a large number of cells (up to ~100!) - the phase between adjacent cells is defined by the coupling and the distance between cells increases to keep synchronism . Once the geometry is defined, it can accelerate only one type of ion for a given energy range. Effective but not flexible.*

A simple case: an individual cavity linac

The goal is flexibility: acceleration of different ions (e/m) at different energies → need to change phase relation for each ion-energy





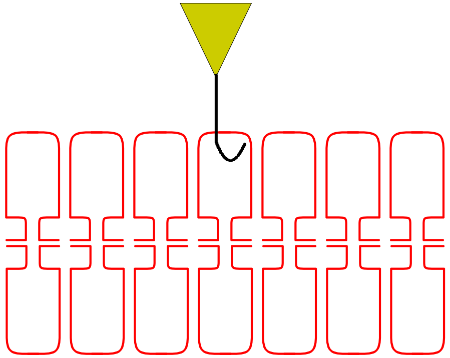
RF cavity

focusing solenoid

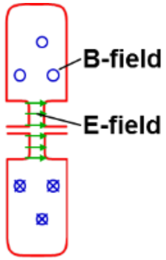
beam

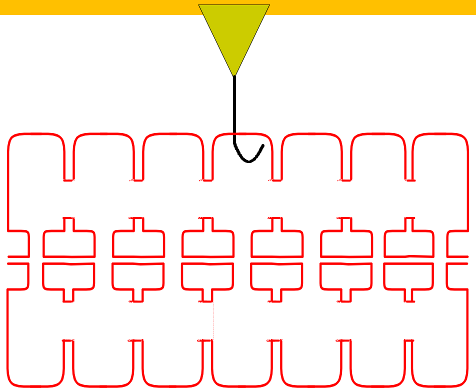
HIE-ISOLDE linac
at CERN - superconducting)
Post-accelerator of radioactive ions
2 sections of identical equally spaced cavities
Quarter-wave RF cavities, 2 gaps
12 + 20 cavities with individual RF amplifiers, 8 focusing solenoids
Energy 1.2 → 10 MeV/u, accelerates different A/m

Coupling accelerating cells



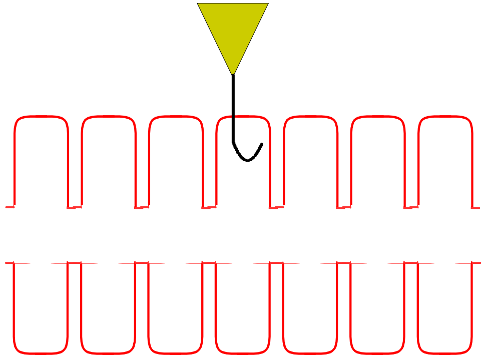
How can we couple together a chain of n accelerating cavities ?





1. Magnetic coupling:

open “slots” in regions of high magnetic field → B-field can couple from one cell to the next



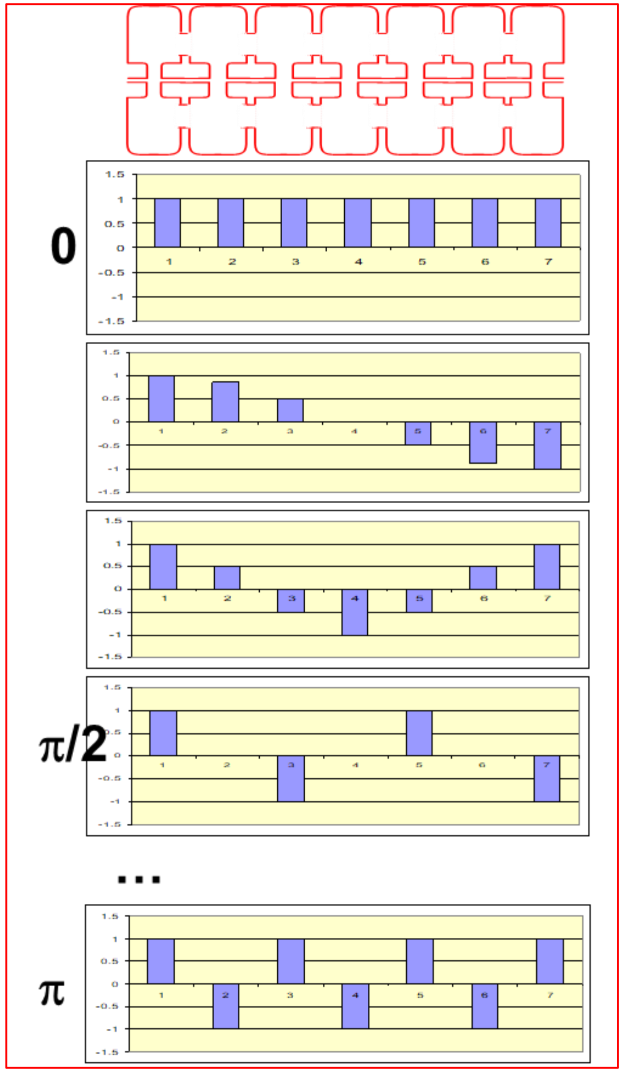
2. Electric coupling:

enlarge the beam aperture → E-field can couple from one cell to the next

The effect of the coupling is that the cells no longer resonate independently but will have common resonances with well defined field patterns.

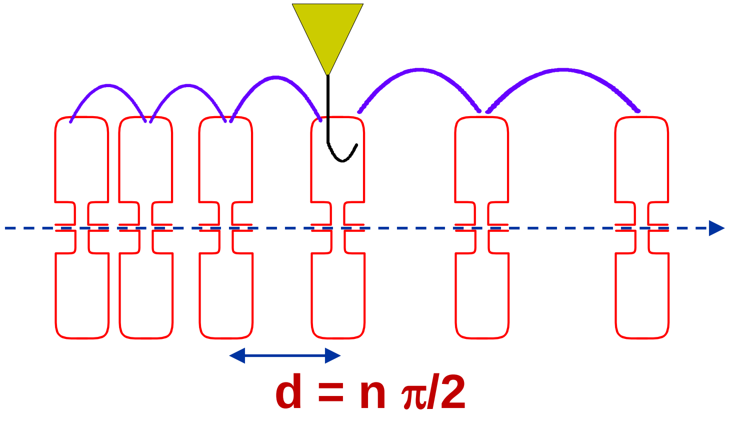
STANDING WAVE modes, corresponds to the modes in a vibrating string.

0 mode, π mode, and sometimes π/2 modes can be used for acceleration

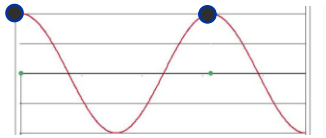


Phase relation in coupled cell systems

$$\boxed{\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}}$$



To keep the particle on the crest (or near the crest) in every gap, the phase difference between gaps must be a **multiple of π** (was exactly π in the Wideröe)



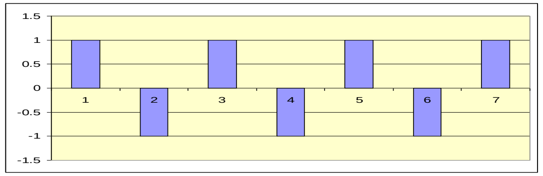
$$\boxed{\Delta\varphi = n\pi}$$

The first 2 «modes» are useful for acceleration:

π mode

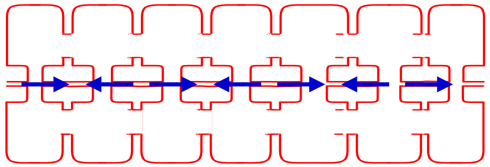
$$\boxed{\Delta\varphi = \frac{\pi}{2}}$$

π



$$\boxed{d = \frac{\beta\lambda}{2}}$$

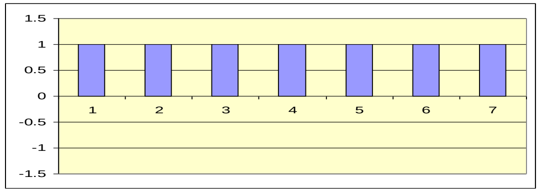
"Wideröe" mode, is the most efficient



2π(0) mode

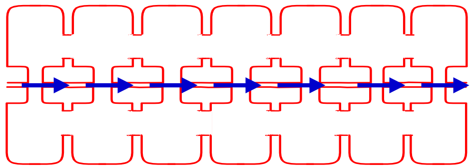
$$\boxed{\Delta\varphi = \pi}$$

0

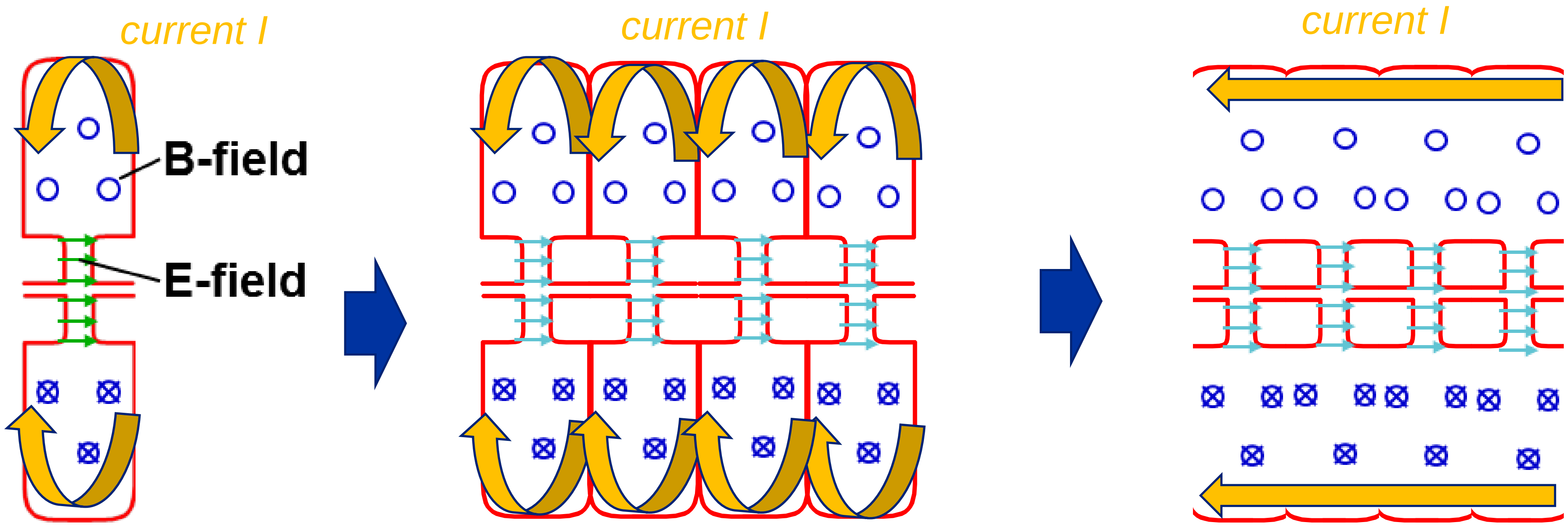


$$\boxed{d = \beta\lambda}$$

"Alvarez" mode, less efficient (less gaps/length), but DTL structure can reduce power loss



The Alvarez Drift Tube Linac – an efficient 0 mode



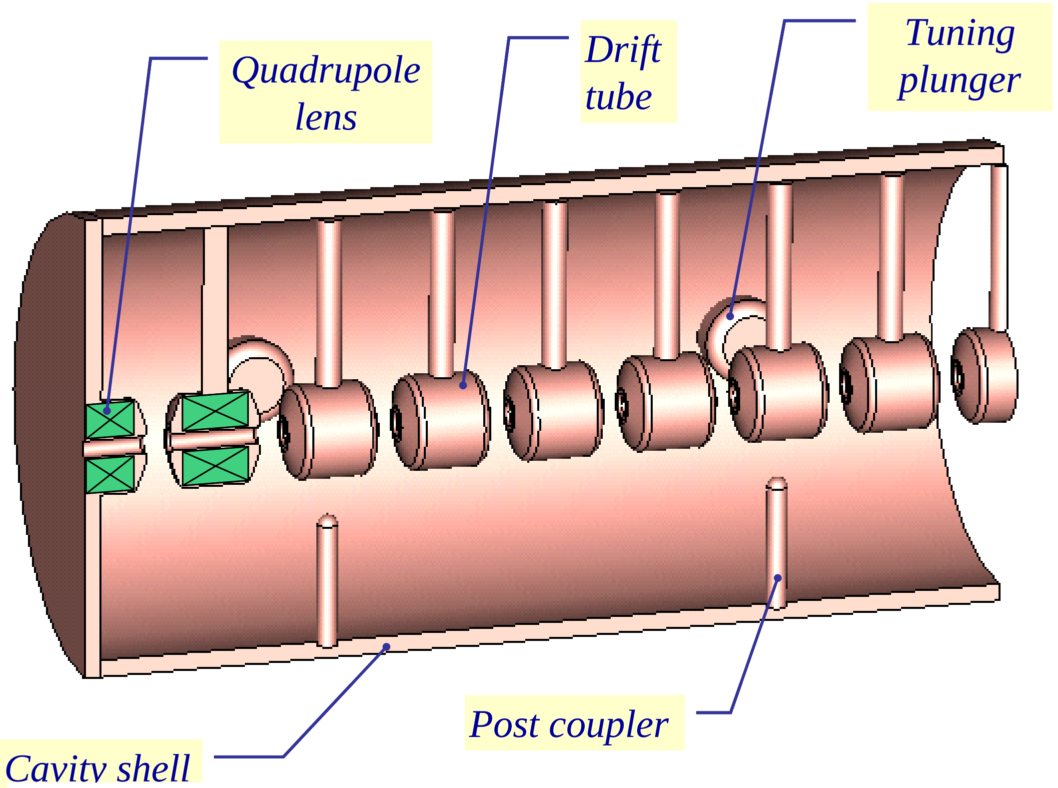
Electric current flows on cavity walls, dissipating power

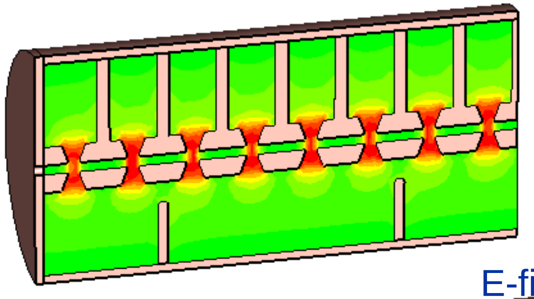
On the walls between cells, the current has opposite sign on adjacent walls → cancels out, we can remove the walls without changing the electromagnetic field

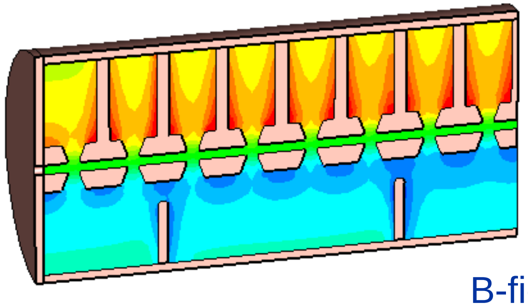
The Alvarez DTL is an open structure with maximum coupling between cells (clean mode spectrum) and minimum power loss on the walls (high shunt impedance corresponding to high efficiency)

Drift Tubes cannot «float» in air → they will be connected to the walls by stems

The Drift Tube Linac (DTL)




E-field


B-field

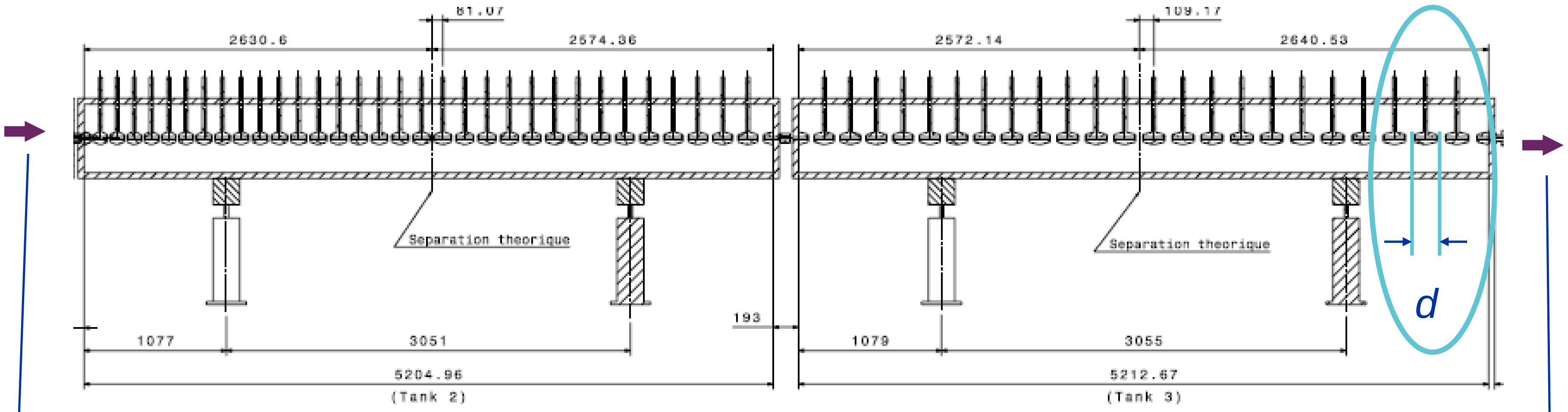
Standing wave linac structure for protons and ions, β=0.1-0.5, f=20-400 MHz

Drift tubes are suspended by stems (no net RF current on stem)

Coupling between cells is maximum (no slot, fully open !)

The 0-mode allows a long enough cell (d=$\beta\lambda$) to house focusing quadrupoles inside the drift tubes!

An example of Drift Tube Linac



10 MeV,
β = 0.145

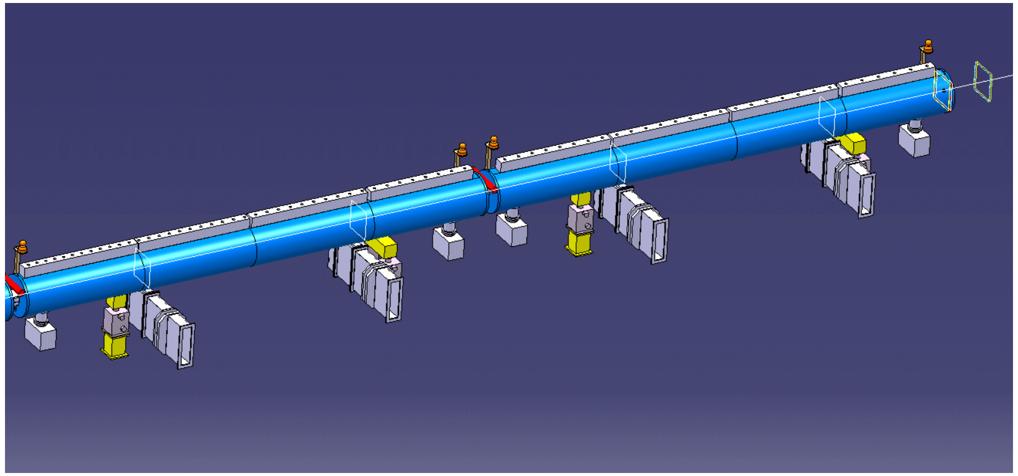
50 MeV,
β = 0.31

d=βλ

RF Frequency:
352.2 MHz

Tank 2 and 3 of the new Linac4 at CERN:

57 coupled accelerating gaps
Frequency 352.2 MHz, λ = 85 cm
Cell length (d=βλ) from 12.3 cm to 26.4 cm
(factor 2 !).

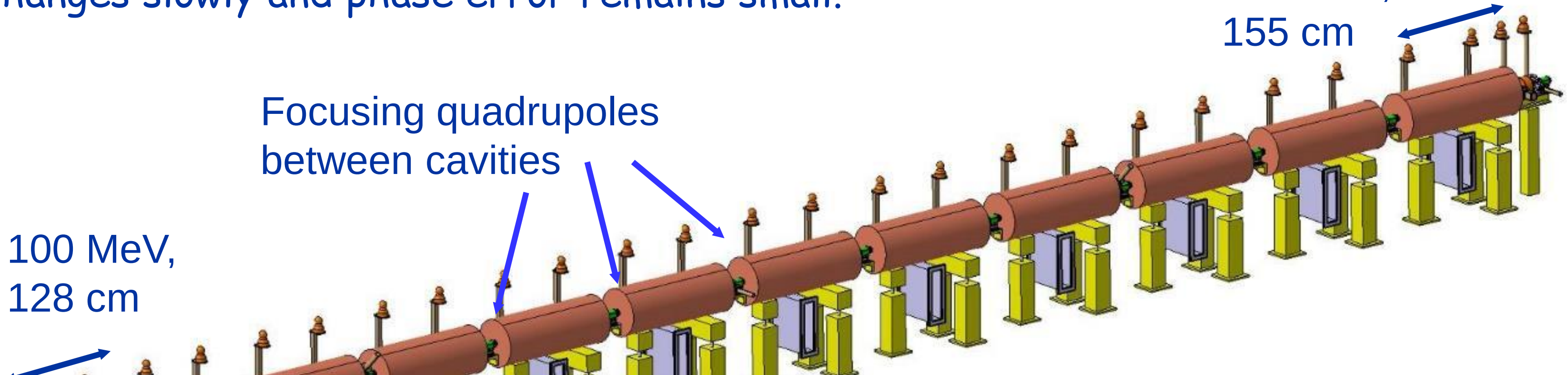


The Drift Tube Linac of the CERN Linac4

Case 3 (intermediate): the Linac4 PIMS structure

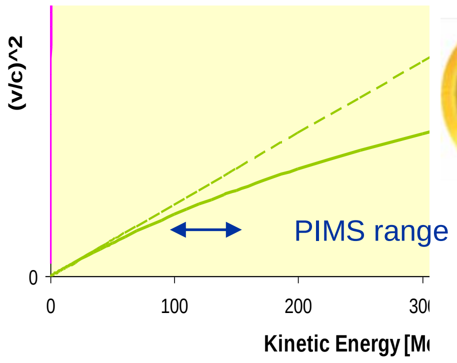
Between the 2 basic cases there are “intermediate” cases: a) single-gap cavities are expensive and b) structures with each cell matched to the beta profile are mechanically complicated → as soon as the increase of beta with energy becomes smaller we can accept a small phase error and allow short sequences of identical cells.

PIMS: cells have same length inside a cavity (7 cells) but increase from one cavity to the next. At high energy (>100 MeV) beta changes slowly and phase error remains small.

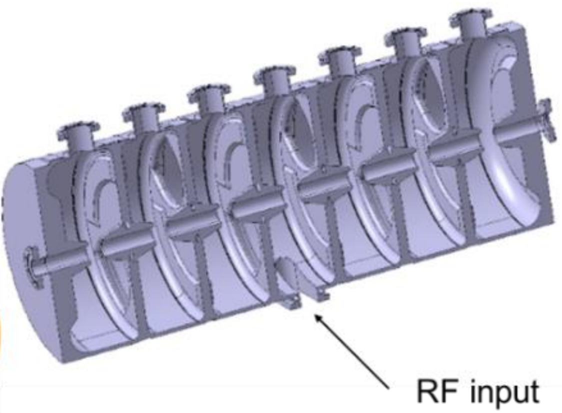

160 MeV,
155 cm

Focusing quadrupoles
between cavities

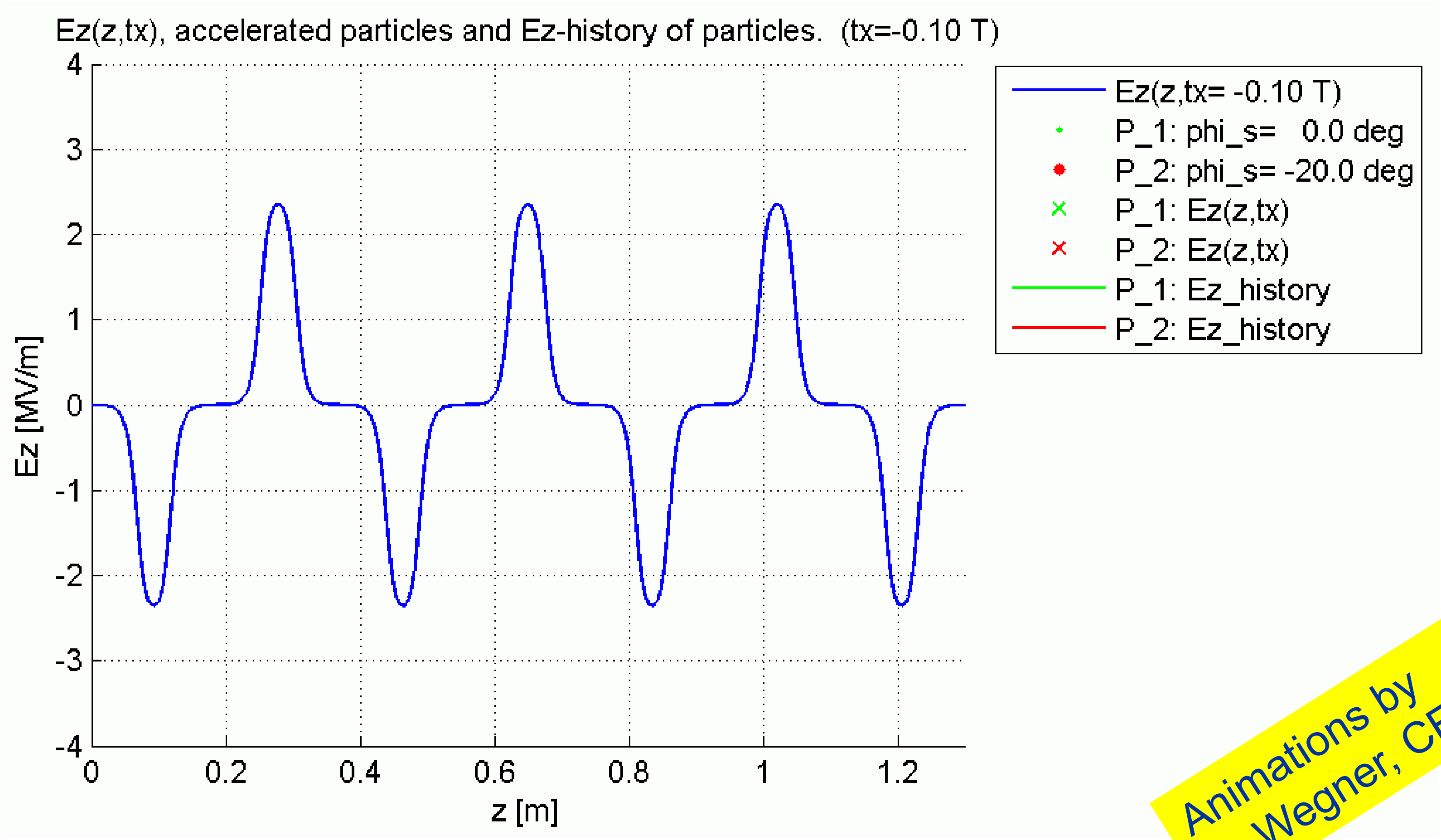
100 MeV,
128 cm



PIMS range



Acceleration in the PIMS



Animations by R. Wegner, CERN

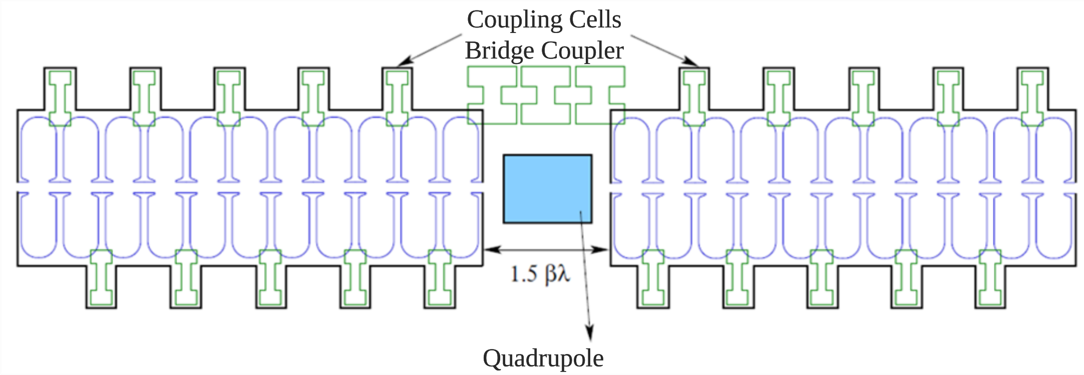
Please note that this animation presents the effect on 2 particles:

One is exactly riding on the crest of the wave (green one)

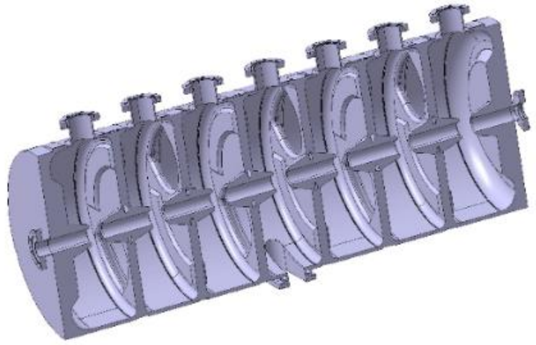
Another is slightly in advance on the wave (red one)

Multi-gap coupled-cell cavities (for protons and ions)

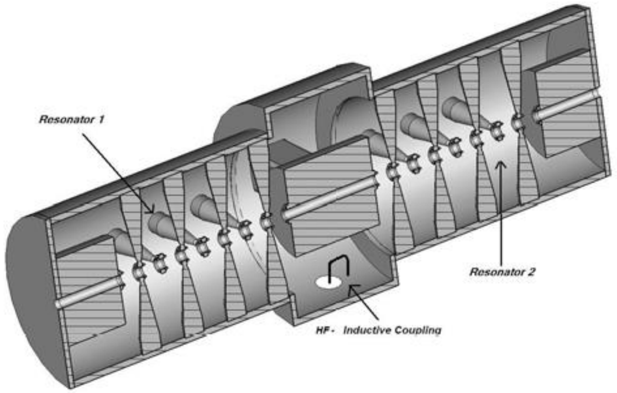
Between the 2 extreme cases (array of independently phased single-gap cavities / single long chain of coupled cells with lengths matching the particle beta) there can be a large number of variations (number of gaps per cavity, length of the cavity, type of coupling) each optimized for a certain range of energy and type of particle.



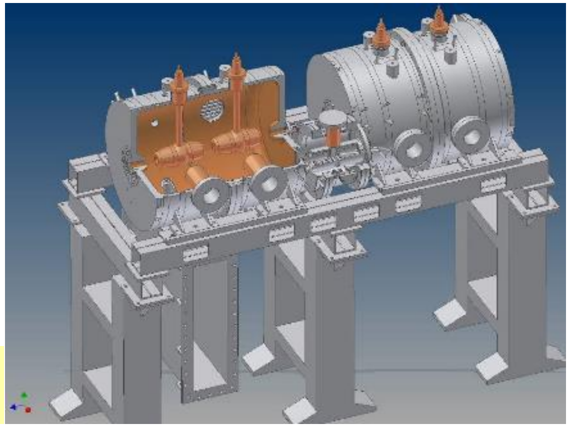
SCL: side coupled linac, operating in π/2 mode for higher stability



PIMS: Pi-Mode Structure

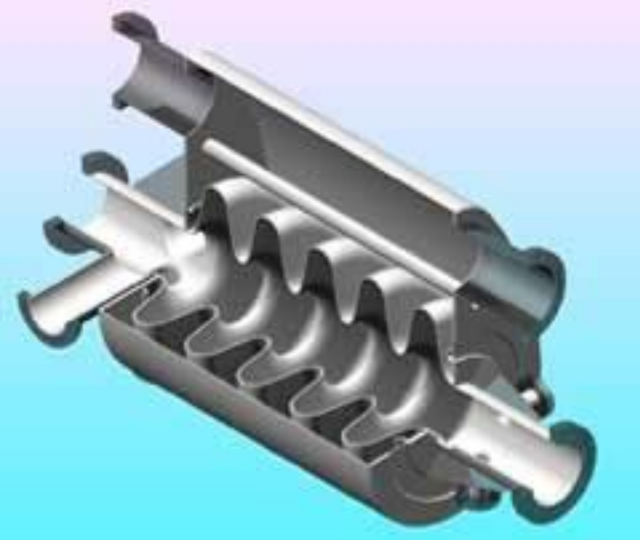


CH: interdigital Cross-H



CCDTL: Cell-Coupled Drift Tube Linac

Multi-gap superconducting linac structures (elliptical)

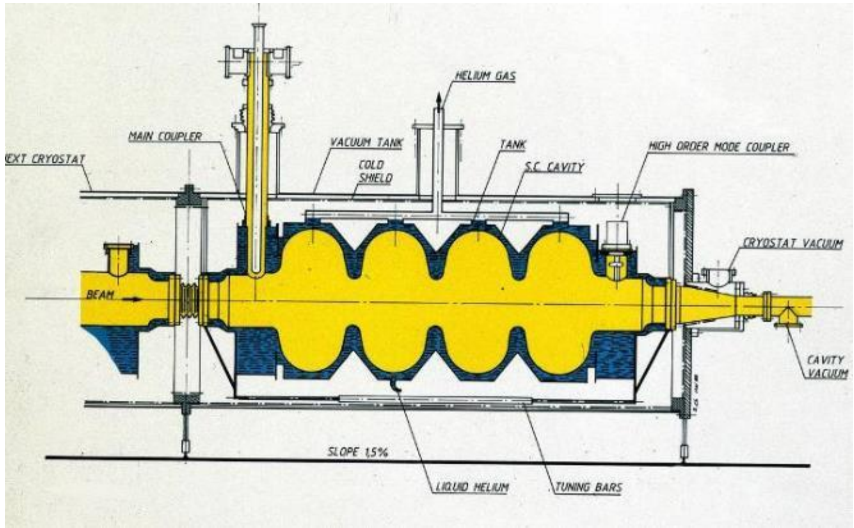


Standing wave structures for particles at β>0.5-0.7, widely used for protons (SNS, etc.) and electrons (ILC, etc.) f=350-700 MHz (protons), f=350 MHz – 3 GHz (electrons)

Chain of cells electrically coupled, large apertures Operating in π-mode, cell length $\beta\lambda/2$

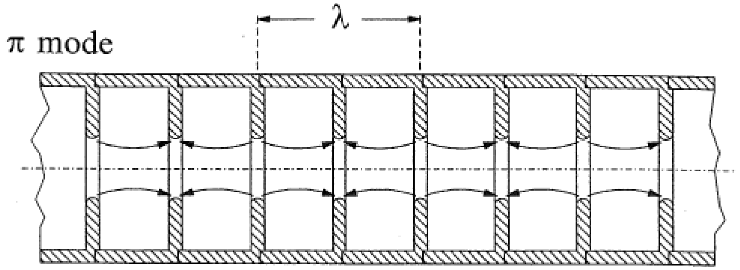
Input coupler placed at one end.

Commonly used in the accelerating section of synchrotrons (e.g. LHC)

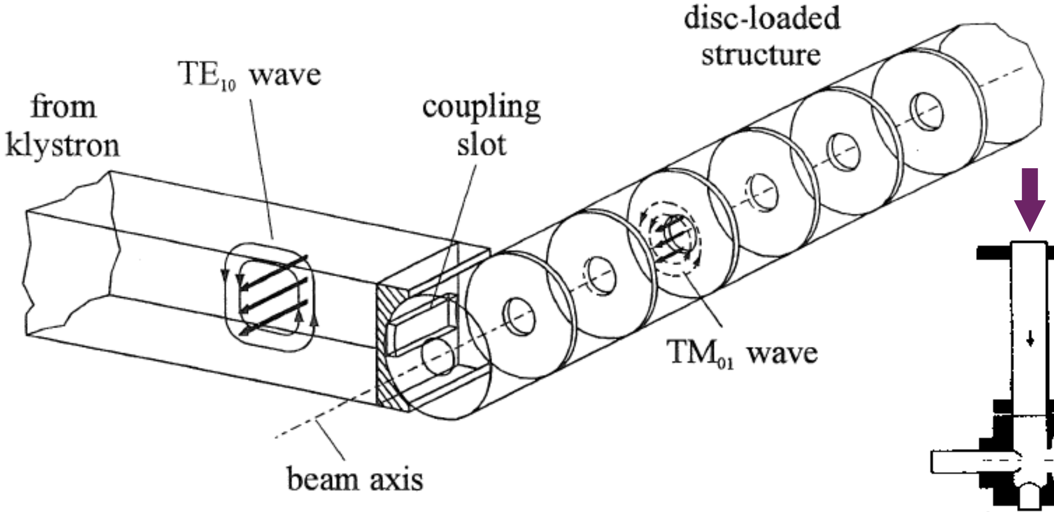


Used in linacs and for the accelerating section of synchrotrons

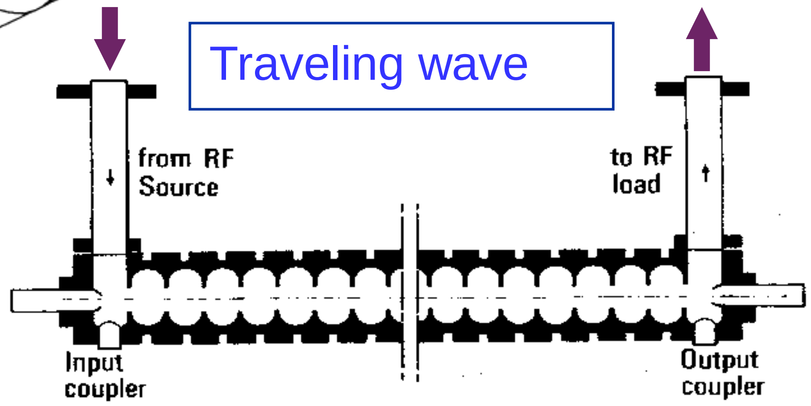
Electron linacs



1. In an electron linac velocity is ~ constant. To use the fundamental accelerating mode cell length must be d = βλ / 2.
2. the linac structure will be made of a sequence of identical cells. Because of the limits of the RF source, the cells will be grouped in cavities operating in travelling wave mode.



Pictures from K. Wille, The Physics of Particle Accelerators


Traveling wave

A 3 GHz accelerating structure, 4.5 meters long, energy gain 45 MeV.

Why Linear Accelerators

Linear Accelerators are used for:

1. Low-Energy acceleration (injectors to synchrotrons or stand-alone): for protons and ions, linacs can be synchronous with the RF fields in the range where velocity increases with energy. (both "Newton" and "Einstein" regions!).

When velocity is ~constant, synchrotrons are more efficient (multiple crossings instead of single crossing).

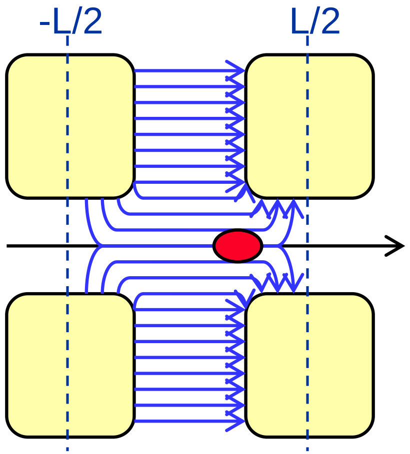
Protons : $\beta = v/c = 0.51$ at 150 MeV, 0.95 at 2 GeV.

2. High-Energy acceleration in the case of:

- Production of high-intensity proton beams, in comparison with synchrotrons, linacs can go to higher repetition rate, are less affected by resonances and instabilities and have more distributed beam losses. Higher injection energy from linacs to synchrotrons leads to lower space charge effects in the synchrotron and allows increasing the beam intensity.
- High energy linear colliders for leptons, where the main advantage is the absence of synchrotron radiation.

Accelerating structure parameters and beam dynamics

Energy gain in an accelerating gap



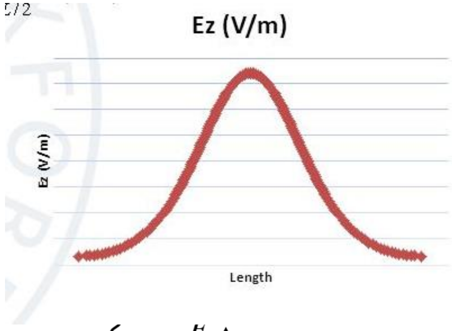
Particle crossing an RF gap in a linac cell of length L:

E-field is $E_z(z,t) = E(z)\cos(\omega t + \varphi)$

Shape of E_z depends on gap length and aperture radius

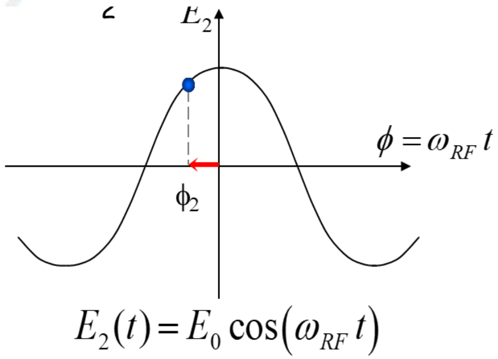
Voltage is $V_0 = \int_{-L/2}^{L/2} E(z)dz = E_0 L$

E_0 mean field



Energy gain by a particle crossing the gap at phase ϕ is:

$$\Delta W = e\int_{-L/2}^{L/2} E(z)\cos[\omega t(z) + \varphi)]dz = e\int_{-L/2}^{L/2} E(z)[\cos\omega t\cos\varphi - \sin\omega t\sin\varphi]dz = eV_0 T\cos\varphi$$



with a Transit Time Factor defined as:

$$T = \frac{\int_{-L/2}^{L/2} E(z)\cos\omega t(z)dz}{\int_{-L/2}^{L/2} E(z)dz} - \tan\varphi\frac{\int_{-L/2}^{L/2} E(z)\sin\omega t(z)dz}{\int_{-L/2}^{L/2} E(z)dz} \underset{\uparrow}{=} \frac{\int_{-L/2}^{L/2} E(z)\cos\omega t(z)dz}{\int_{-L/2}^{L/2} E(z)dz} = \frac{1}{V_0}\int_{-L/2}^{L/2} E(z)\cos\omega t(z)dz$$

For E(z) even function of z

In linacs, phase is counted from the crest of the wave.

Note that in synchrotrons it is counted from the zero crossing (-90^0 for linacs) !

$$\boxed{\Delta W = eE_0 TL\cos\varphi}$$

« Panofsky equation » very simple, but the physics is in the transit time factor!

Transit time factor

$$\Delta W = eE_0 TL\cos\varphi$$

design parameter

from frequency, gap geometry and particle velocity

cell length, βλ or βλ/2

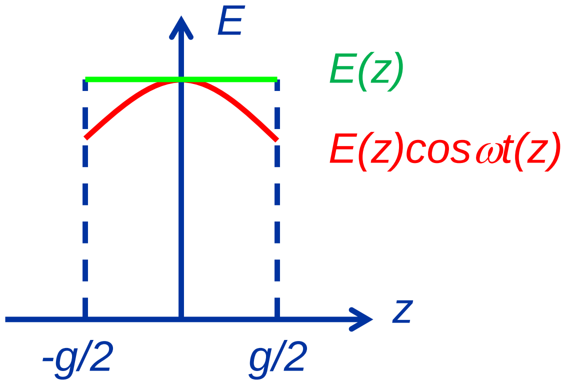
synchronous phase of particle

$$T = \frac{\int_{-L/2}^{L/2} E(z)\cos\omega t(z)dz}{\int_{-L/2}^{L/2} E(z)dz}$$

energy gain by a particle in the middle of the gap at t=0

energy gain by a particle crossing the gap with infinite velocity at t=0

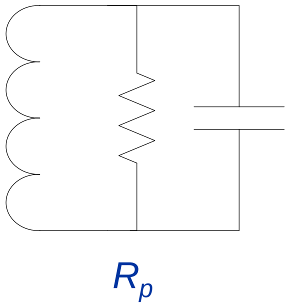
T is a characteristic of your design; can be easily calculated by the design codes



The Transit Time Factor T tells us how much of the E-field that we have provided on the gap has been really seen by a particle moving at velocity $v=\beta c$. Is the ratio between 2 integrals ($0 \leq T \leq 1$)

Power efficiency and Shunt impedance

Power lost on the walls of an accelerating cavity:
$$P_c = \frac{V_0^2}{Z} = \frac{(E_0 L)^2}{Z} = \frac{(\Delta W)^2/\cos^2\varphi}{ZT^2}$$



$$R_p = \frac{1}{2} Z$$

We use the “shunt impedance” Z that (apart a factor 2!) corresponds to the shunt resistance in the cavity equivalent circuit – is measured in ohm/meter

$$efficiency = \frac{P_b}{(P_b + P_c)} \cong \frac{P_b}{P_c} = \frac{I\,\Delta W\, ZT^2}{\Delta W^2/\cos^2\varphi} = \frac{I\, ZT^2 \cos^2\varphi}{\Delta W/e} = \frac{I\, ZT^2 \cos\varphi}{(E_0 T)L} = \frac{I}{E_0 T}\frac{ZT^2}{L}\cos\varphi$$

$$ZT^2/L \quad [\Omega/m]$$

$$efficiency = \frac{I}{E_0 T}\frac{ZT^2}{L}\cos\varphi$$

Main design parameters: beam current and effective gradient

Shunt impedance per unit length

synchronous phase

For a normal linac with low beam loading, the RF power efficiency (proportional to the wall plug efficiency) is:

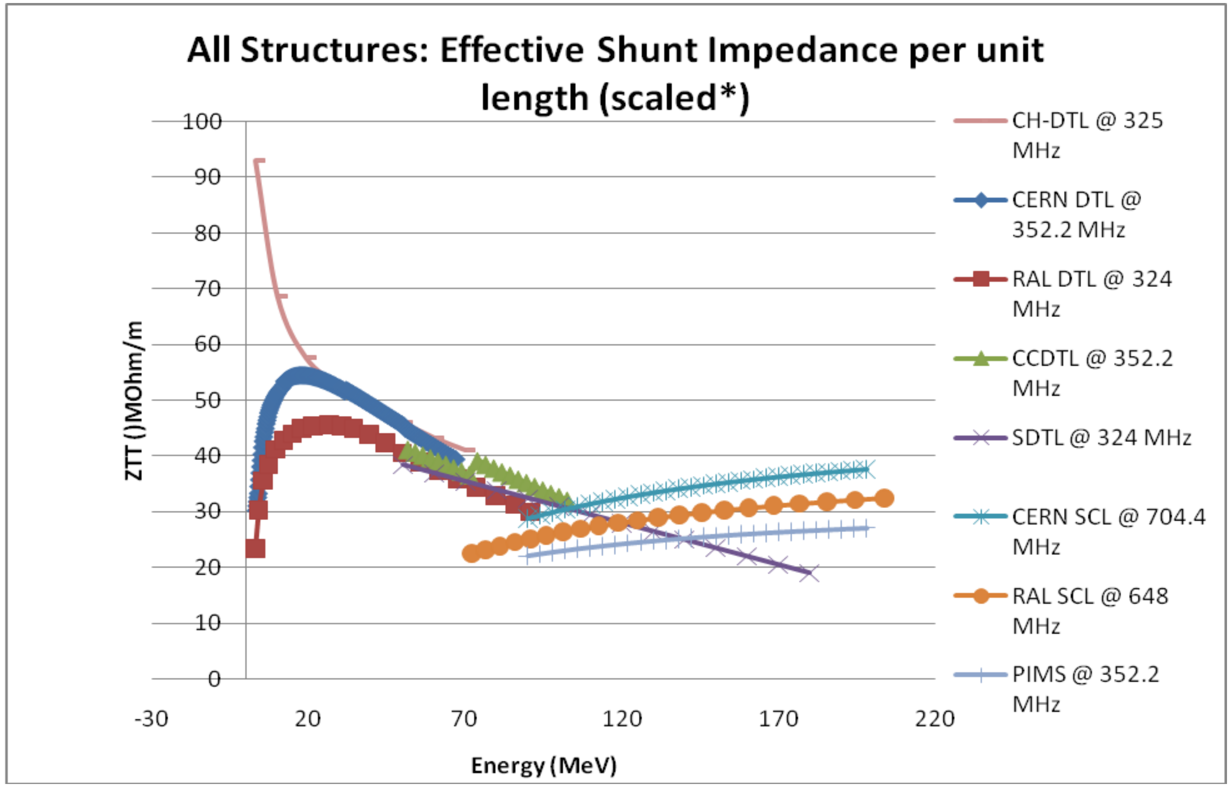
- proportional to peak beam current, shunt impedance, and cosine of the synchronous phase.
- inversely proportional to the effective accelerating gradient (long linacs with low gradient have a higher efficiency, but higher construction cost…).

Selection of linac structures on shunt impedance

How to choose the best linac structure?
Main figure of merit is the power efficiency = shunt impedance
Ratio between energy gain (square) and power dissipation, is a measure of the energy efficiency of a structure.
Depends on beta, energy and mode of operation.

$$Z = \frac{V_0^2}{P_c}$$

But the choice of the best accelerating structure for a certain energy range depends as well on beam dynamics and on construction cost.



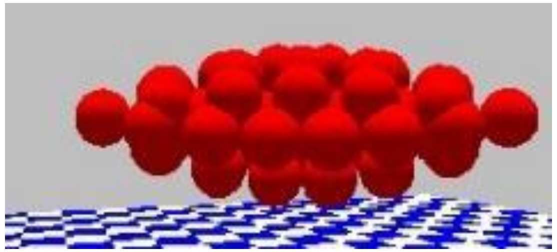
Comparison of computed shunt-impedances for different low-beta structures done in 2005-08 by the “HIPPI” EU-funded Activity.

In general terms, a DTL structure is preferred at low-energy, and π-mode structures at high-energy.
CH is excellent at very low energies (ions).

For Linac4 at CERN, were selected:
DTL up to 50 MeV
CCDTL 50 – 100 MeV
PIMMS 100 – 160 MeV

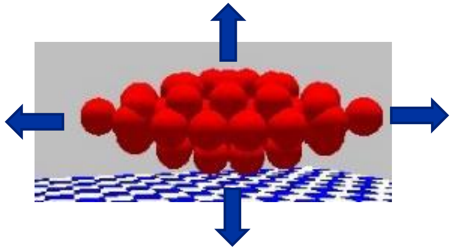
Why beam dynamics is so important

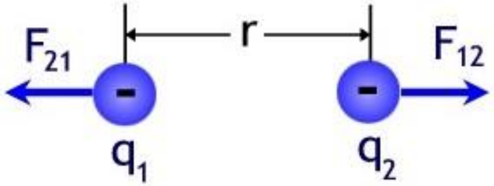
Up to now we have been considering only one particle accelerated on the on the crest of the wave and traveling exactly on the axis of the accelerator.



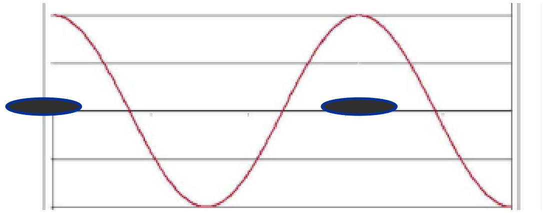
In reality, in a bunch we can have up to **10^{10}-10^{13}** particles in a volume of few mm^3 , which will be distributed around the synchronous particle, and having all the same sign will repel each other (Coulomb force)!

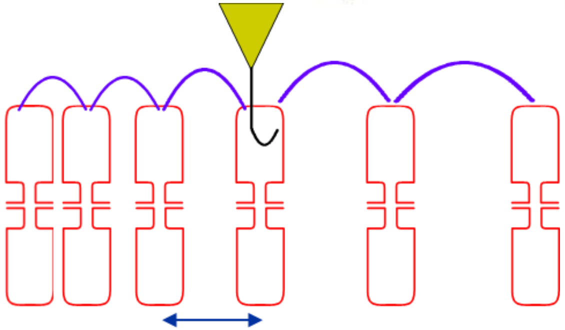
If we do nothing, our beam will end up going in every direction and will be lost on the walls of the vacuum chamber!





In particular, the bunch will have a length in the longitudinal direction (Δz, or $\Delta\phi$) and **not all particles will be synchronous with the wave !**



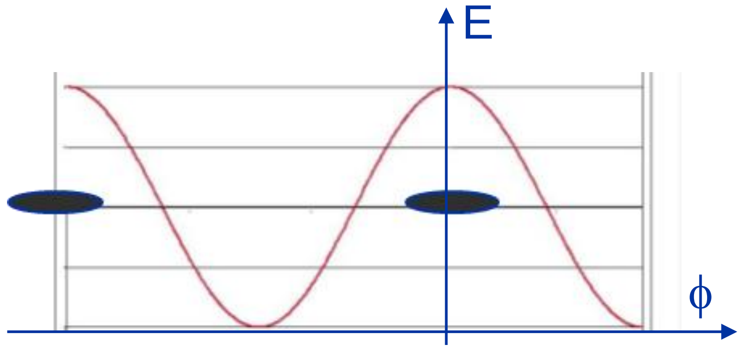


d = $\beta\lambda/2$, but if the energy gain is lower or higher than the design length, the particle will not be synchronous on the next gap!

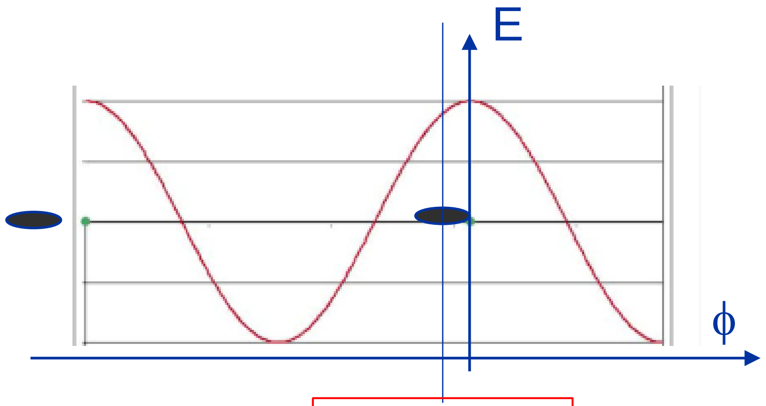
The concept of stable (or synchronous) phase

PHASE STABILITY PRINCIPLE

Independently introduced by V. Veksler (Moscow) and E. McMillan (Berkeley), 1945



If we accelerate around the crest, all particles around the particle at $\phi=0$ will receive less energy and will arrive late at the following gap, receiving even less energy → the beam will be rapidly "debunched" and most of it will go to the "decelerating" phases.



$$\phi = -\phi_s$$

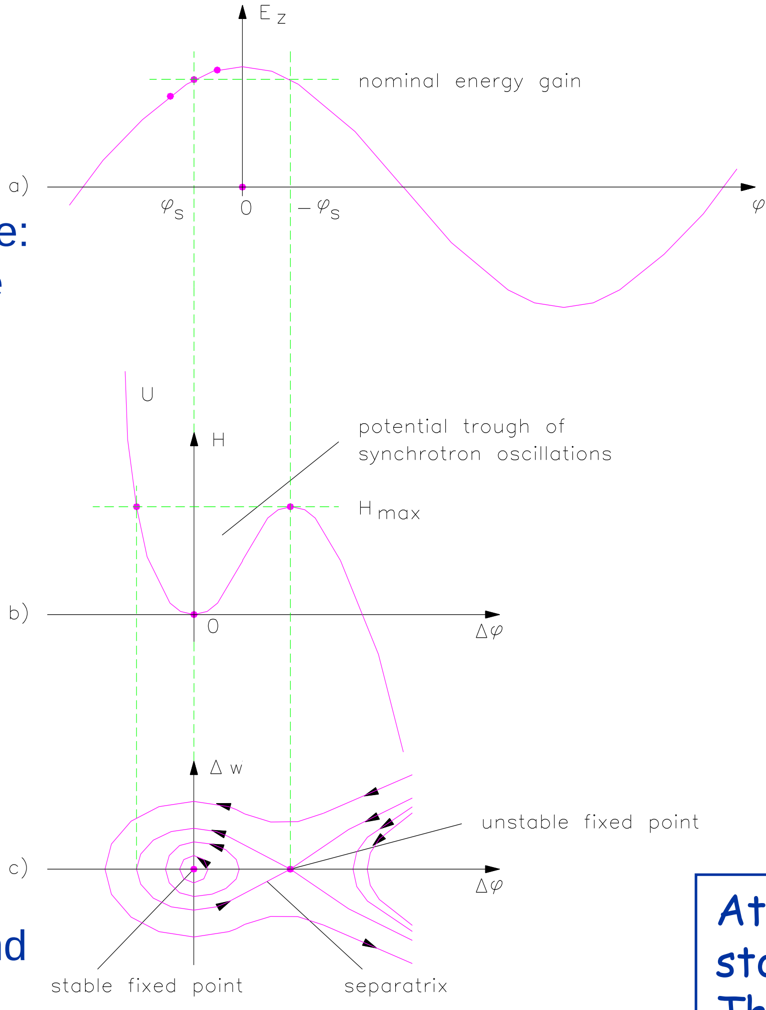
If instead we **accelerate around a negative phase – ϕ_s** , the "synchronous phase", usually between -30^0 and -20^0, we lose some acceleration but the beam will be "longitudinally stable" → particles too fast will receive less energy, and particle too slow will receive more energy. The result is a stable oscillation of all particles around the synchronous one.

Longitudinal beam dynamics (linacs and rings)

Electric field as a function of particle phase with respect to the reference (synchronous) particle: ϕ=0 when the particle is on the crest.

Potential energy of particles oscillating around the synchronous particle.

Longitudinal phase plane energy/phase (ΔE , Δϕ)
Particles rotate around a central stable point



→ Ions are accelerated around a (negative = linac definition) synchronous phase.

→ Particles around the synchronous one perform oscillations in the longitudinal phase space.

→ Frequency of small oscillations:

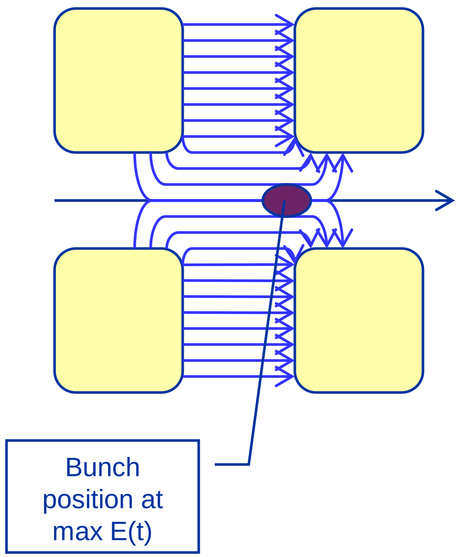
$$\omega_l^2 = \omega_0^2 \frac{qE_0 T \sin(-\varphi)\lambda}{2\pi mc^2 \beta\gamma^3}$$

- → Tends to zero for relativistic particles γ>>1.
- → Note phase damping of oscillations:

$$\Delta\varphi = \frac{const}{(\beta\gamma)^{3/4}} \qquad \Delta W = const \times (\beta\gamma)^{3/4}$$

At relativistic velocities (e.g. electron linac) phase oscillations stop, and the beam is compressed in phase around the initial phase. The crest of the wave can be used for acceleration.

First transverse dynamics problem - RF defocusing



→ RF defocusing experienced by particles crossing a gap on a longitudinally stable phase. Increasing field means that the defocusing effect going out of the gap is stronger than the focusing effect going in.

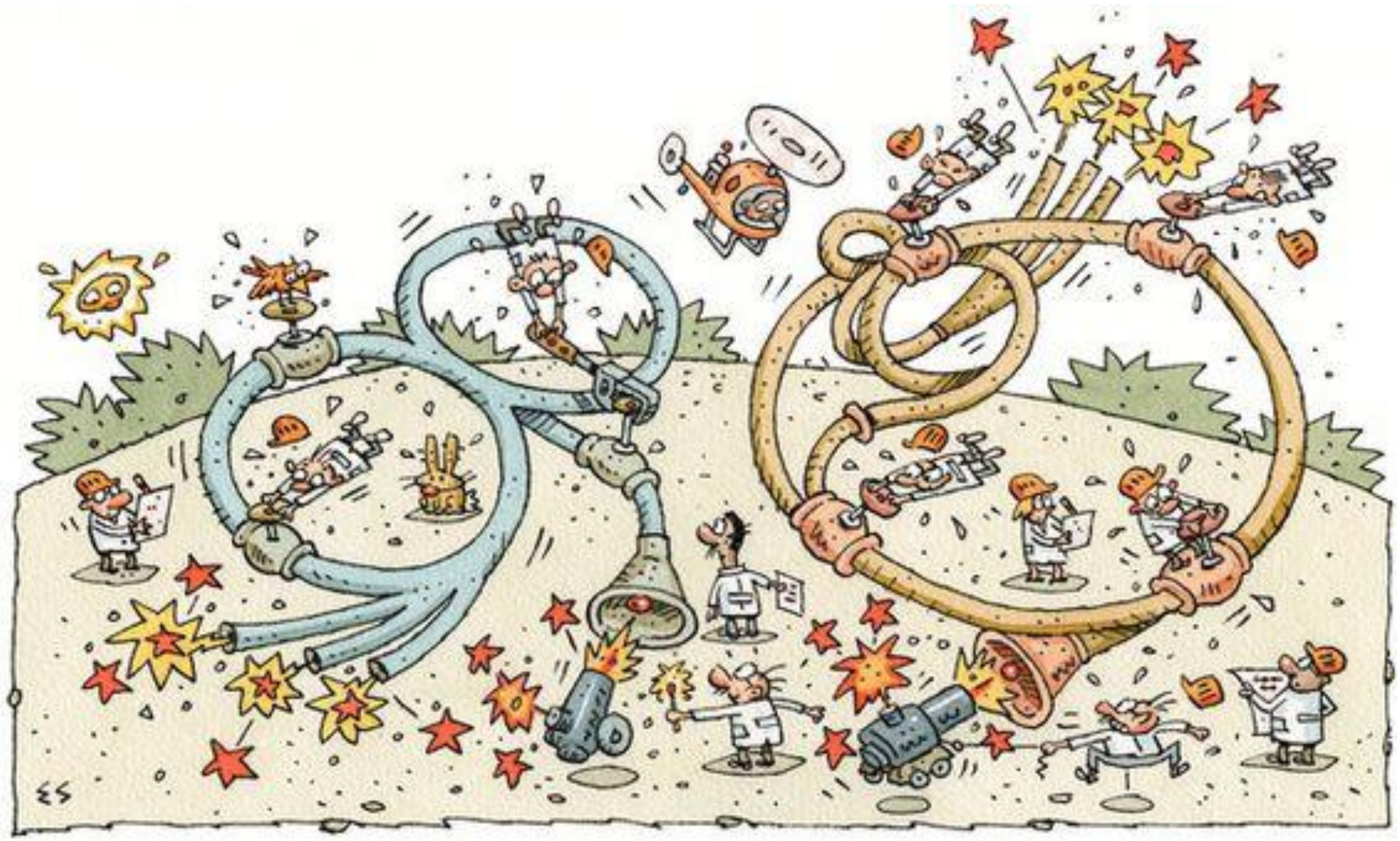
→ In the rest frame of the particle, only electrostatic forces → no stable points (maximum or minimum) → radial defocusing.

→ Lorentz transformation and calculation of radial momentum impulse per period (from electric and magnetic field contribution in the laboratory frame):

$$\Delta p_r = -\frac{\pi\, e\, E_0\, T\, L\, r \sin\varphi}{c\, \beta^2\, \gamma^2\, \lambda}$$

→ Transverse defocusing ~ $1/\gamma^2$ disappears at relativistic velocity

Longitudinal stability is not compatible with transverse stability: you can't win!
To counteract transverse defocusing effects, we have to enter the domain of transverse beam dynamics and transverse focusing.



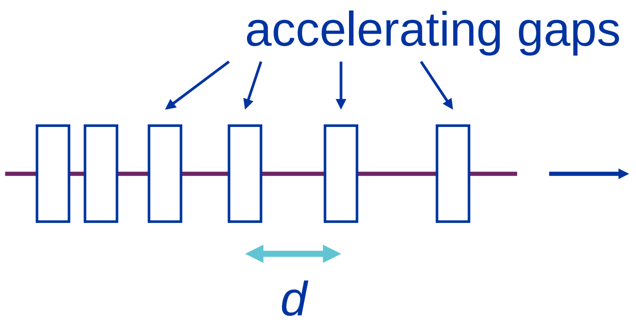
End of Lecture 2

Thank you for your attention!

Linear and circular accelerators

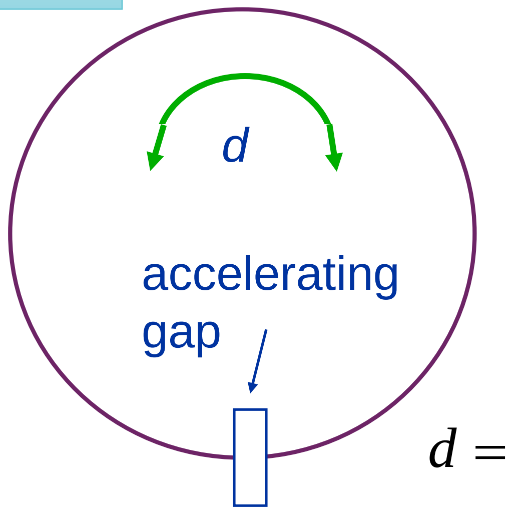
Synchronism conditions

d=βλ/2=variable



$$\frac{d}{v}=T \Rightarrow \frac{d}{\beta c}=\frac{1}{f} \Rightarrow d=\frac{\beta c}{f}=\beta\lambda$$

d=2πR=constant



$$d=\frac{\beta c}{f}=\beta\lambda$$

Linear accelerator:

Particles accelerated by a sequence of gaps inside cavity resonators (all at the same RF phase).

Distance between gaps increases proportionally to the particle velocity, to keep synchronicity.

Used in the range where β increases.
"Newton" machine

Circular accelerator:

Particles accelerated by one (or more) gaps inside cavity resonators at given positions in the ring.

Distance between gaps is fixed. Synchronicity only for β~const, or varying (in a limited range!) the RF frequency.

Used in the range where β is nearly constant.
"Einstein" machine