Baltic School of High Energy Physics
and Accelerator Technologies
August 2021, Klapkalnciems, Latvia

QCD, Higher Orders and Jets

Matteo Cacciari
LPTHE Paris and Université de Paris

Lecture 3: jet algorithms and jet substructure

Includes material from
Gavin Salam and Grégory Soyez

Outline

- Jet algorithms
    - How are jets made
- Jet substructure
    - What's inside them

IRC safety

An observable is **infrared and collinear safe** if, in the limit of a **collinear splitting**, or the **emission of an infinitely soft** particle, the observable remains **unchanged:**

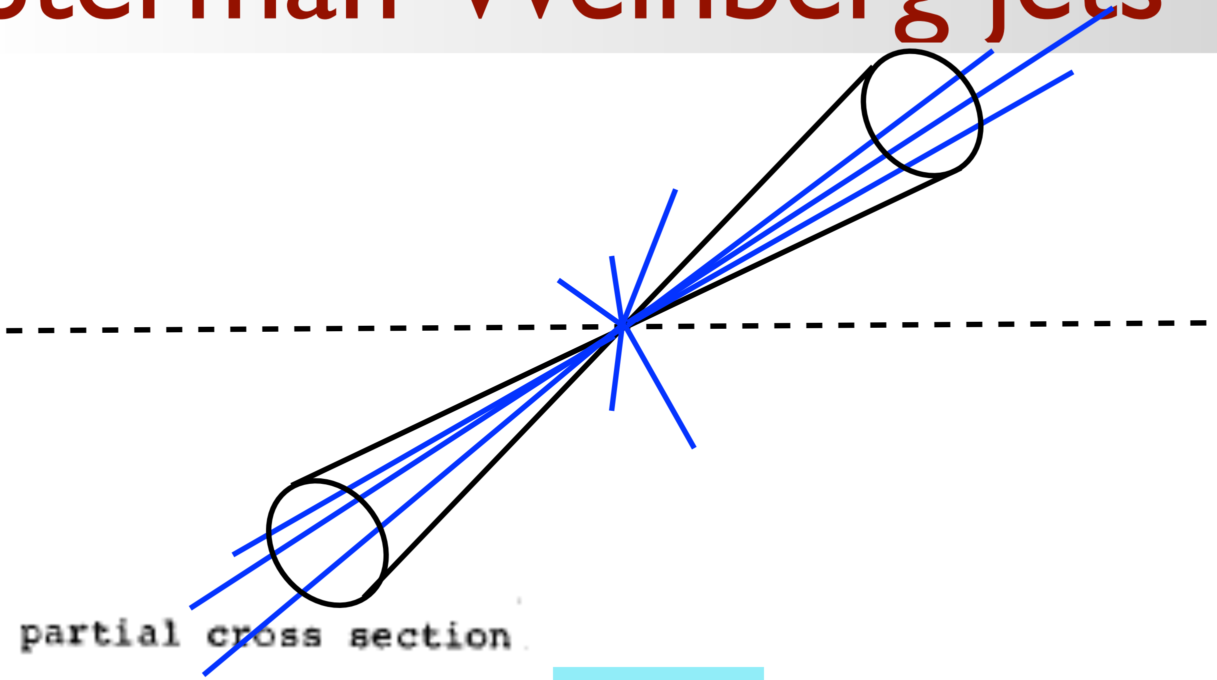
$$O(X;p_1,\ldots,p_n,p_{n+1}\to 0)\to O(X;p_1,\ldots,p_n)$$

$$O(X;p_1,\ldots,p_n\parallel p_{n+1})\to O(X;p_1,\ldots,p_n+p_{n+1})$$

This property ensures cancellation of **real** and **virtual** divergences in higher order calculations

If we wish to be able to calculate a jet rate in perturbative QCD the jet algorithm that we use must be IRC safe:
soft emissions and collinear splittings must not change the hard jets

Sterman-Weinberg jets



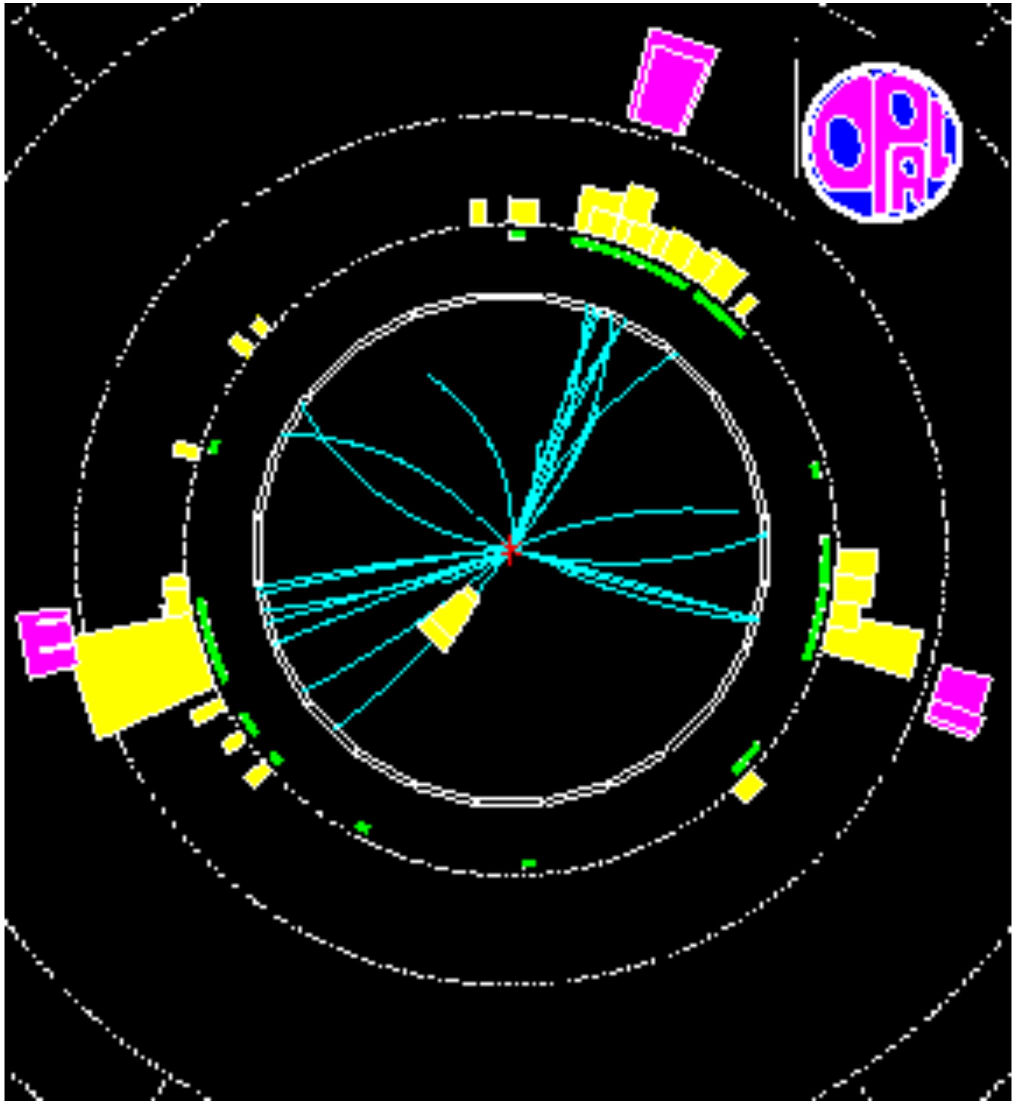
The first rigorous definition of an **infrared and collinear safe** jet in QCD is due to Sterman and Weinberg, Phys. Rev. Lett. **39**, 1436 (1977):

To study jets, we consider the partial cross section $\sigma(E,\theta,\Omega,\epsilon,\delta)$ for e^+e^- hadron production events, in which all but a fraction $\epsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measur-

$$\sigma(E,\theta,\Omega,\epsilon,\delta) = (d\sigma/d\Omega)_0\,\Omega\left[1 - (g_E^2/3\pi^2)\left\{3\ln\delta + 4\ln\delta\,\ln 2\epsilon + \frac{\pi^3}{3} - \frac{5}{2}\right\}\right]$$

Calculable in pQCD (here is the result) but notice the soft and collinear large logs

Why jets

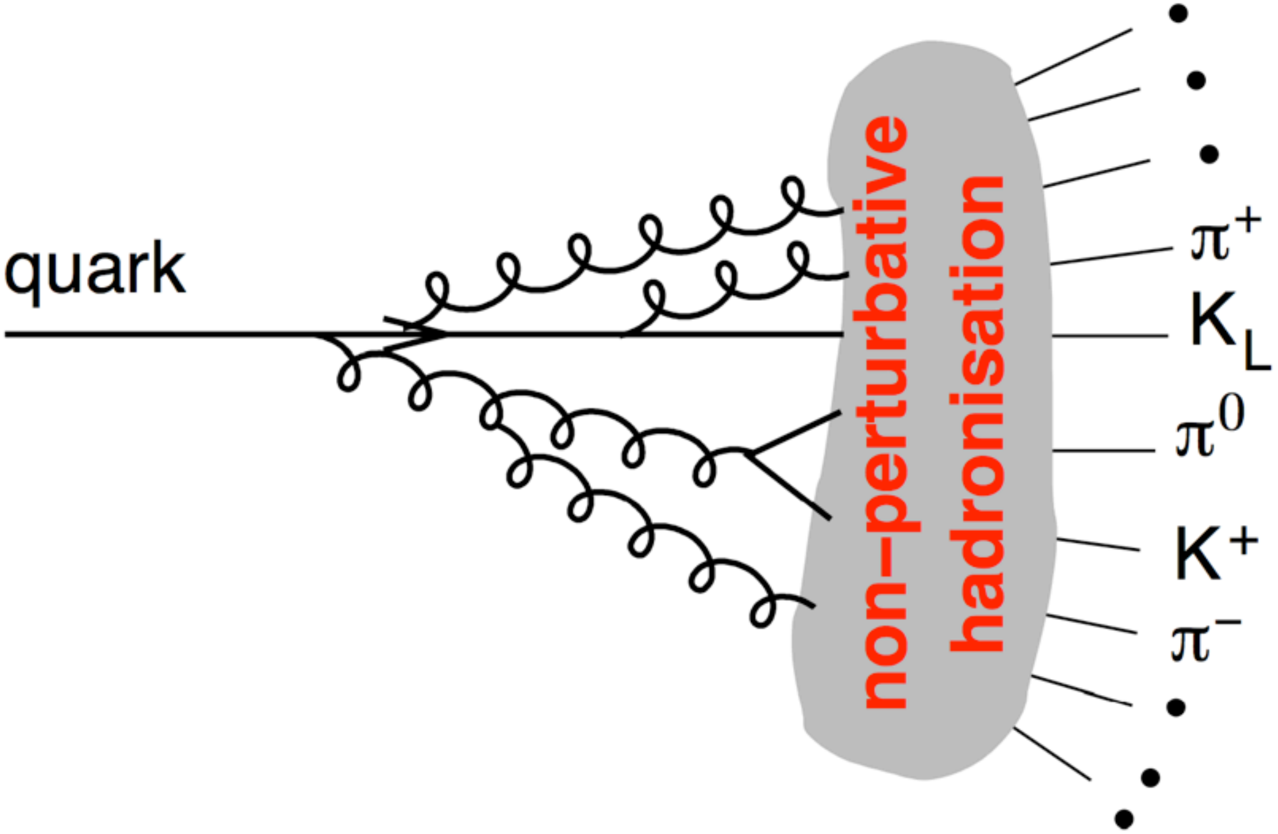


A **jet** is something that happens in high energy events:
a collimated bunch of hadrons flying roughly in the same direction

We could eyeball the collimated bunches, but it becomes impractical with millions of events

The classification of particles into jets is best done using a **clustering algorithm**

Why do jets happen?



Gluon emission

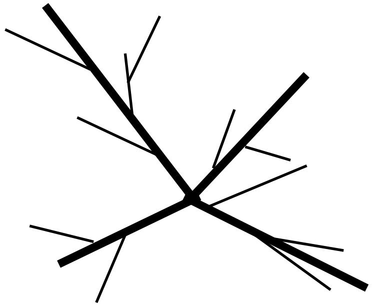
$$\int \alpha_s \frac{dE}{E} \frac{d\theta}{\theta} \gg 1$$

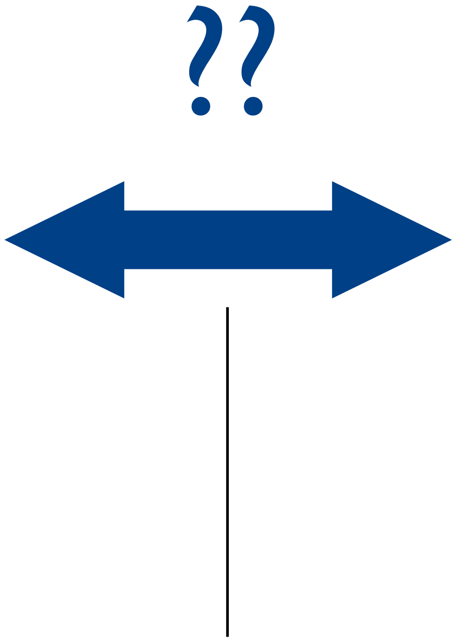
Non-perturbative physics

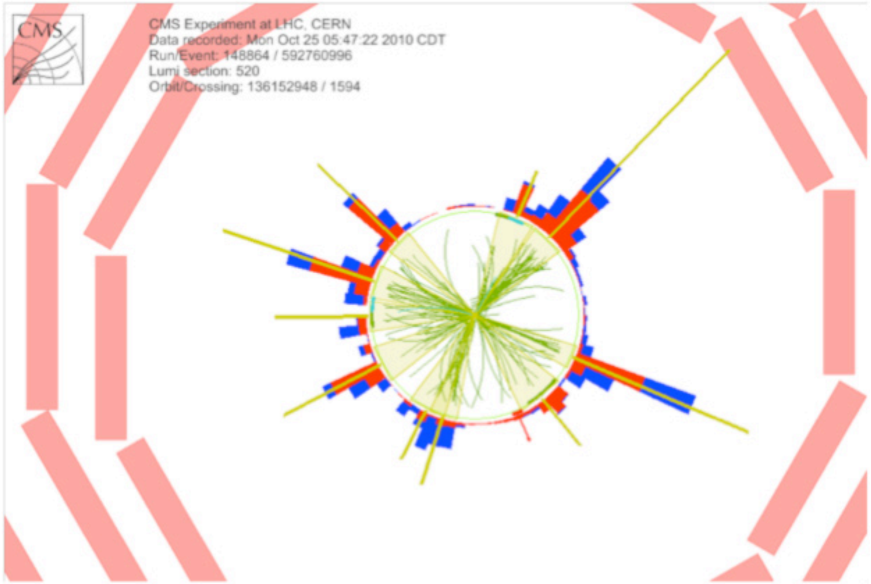
$$\alpha_s \sim 1$$

Taming reality

Multileg + PS







QCD predictions

Real data

Jets

One purpose of a 'jet clustering' algorithm is to **reduce the complexity** of the final state, simplifying many hadrons to **simpler objects** that one can hope to **calculate**

Jets can serve two purposes

- They can be ***observables***, that one can measure and calculate
- They can be ***tools***, that one can employ to extract specific properties of the final state

Different clustering algorithms have different properties and characteristics that can make them more or less appropriate for each of these tasks

Jet clustering algorithm

A **jet algorithm** maps the momenta of the final state particles into the momenta of a certain number of jets:

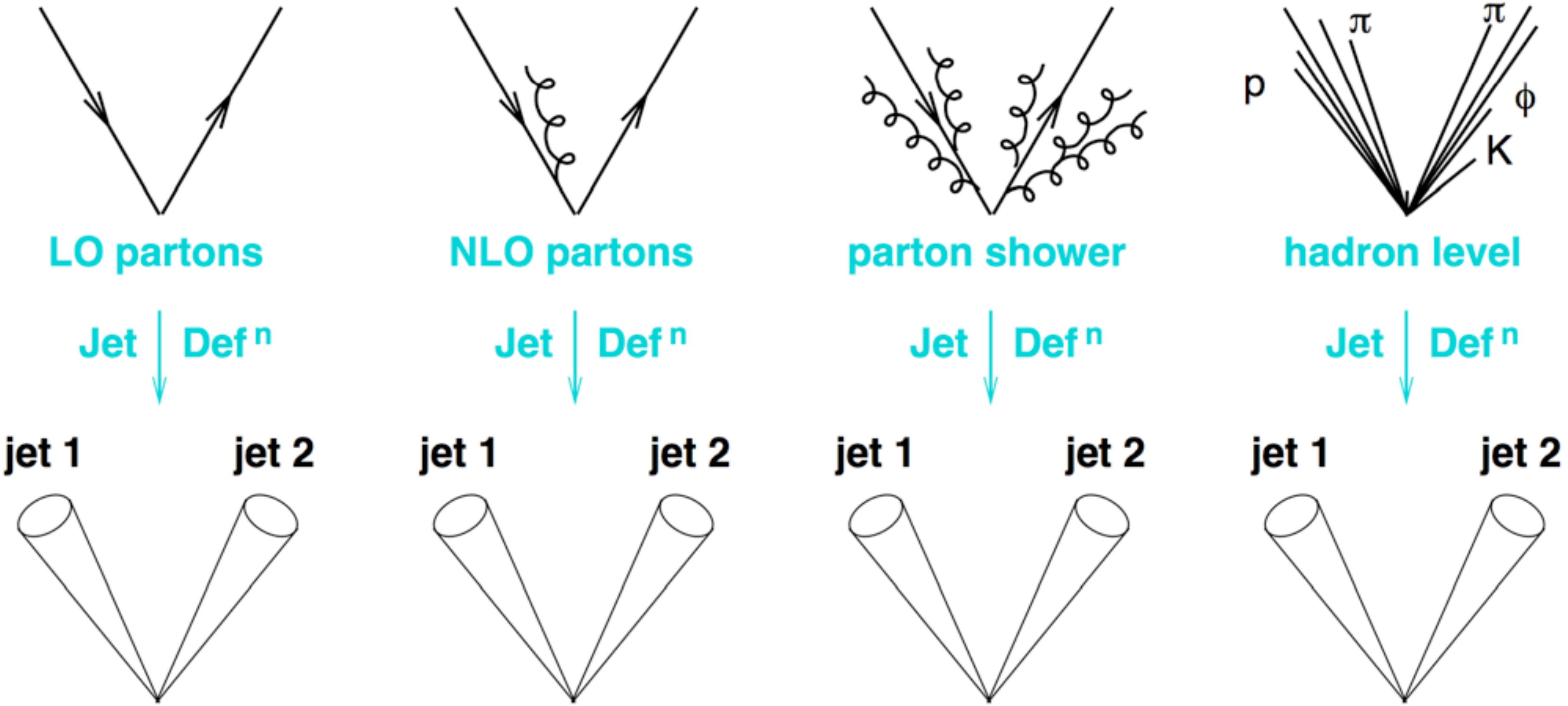
$$\{p_i\} \xrightarrow{\text{jet algorithm}} \{j_k\}$$

particles,
4-momenta,
calorimeter towers, ...

jets

Most algorithms contain a resolution parameter, **R**, which controls the extension of the jet

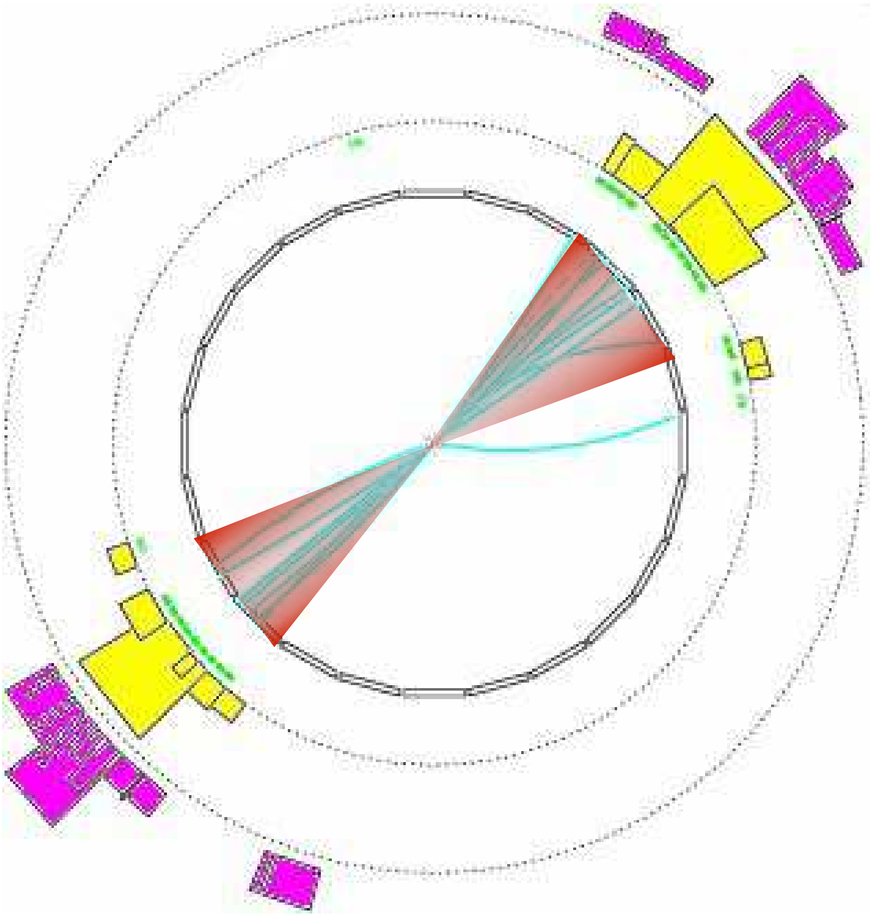
Jet definitions as projections



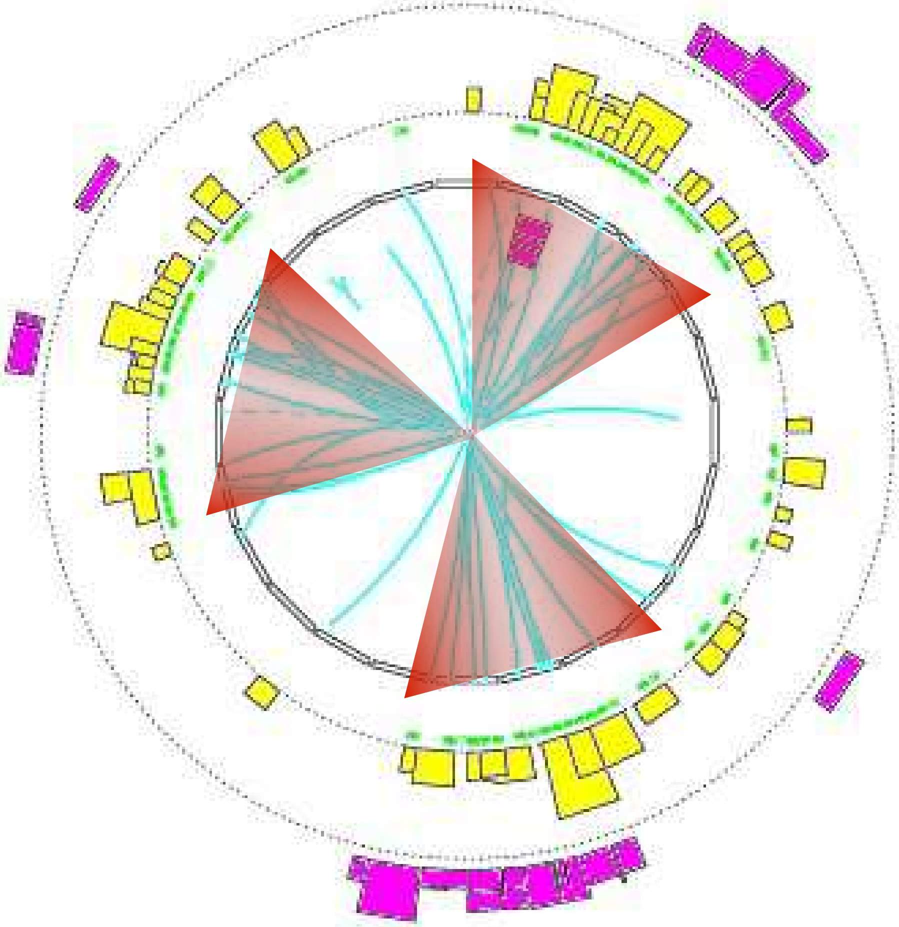
Projection to jets should be resilient to QCD effects

NB: projections are NOT unique:
a jet is NOT EQUIVALENT to a parton

Reconstructing jets is an ambiguous task

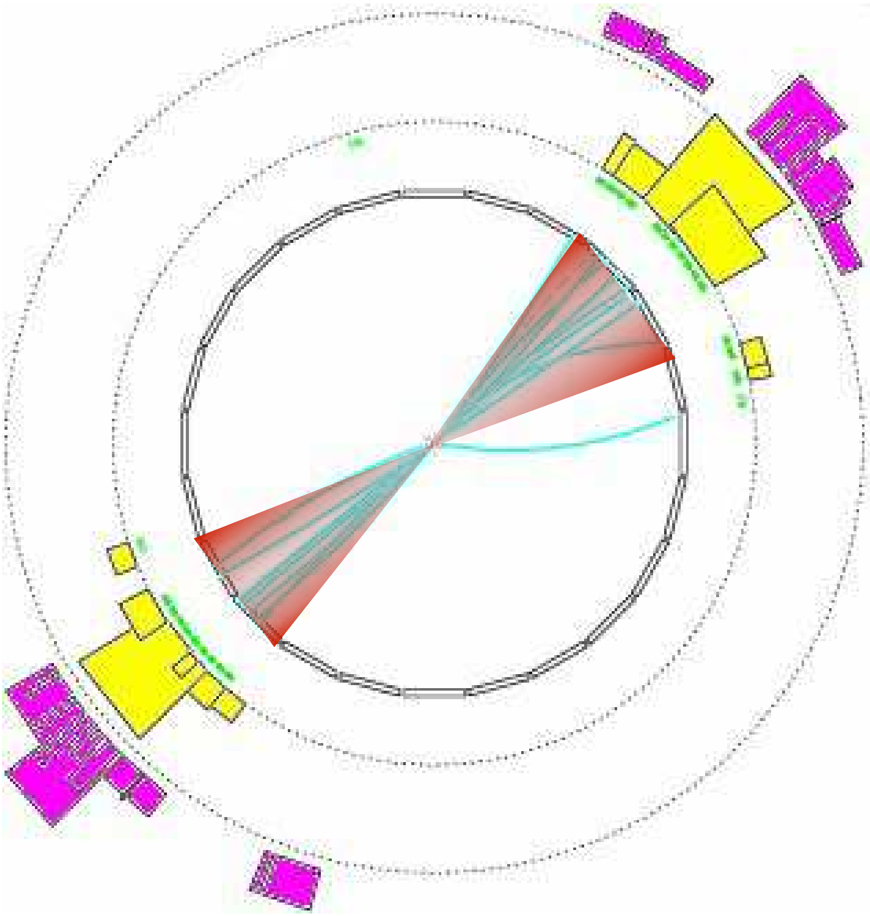


2 clear jets

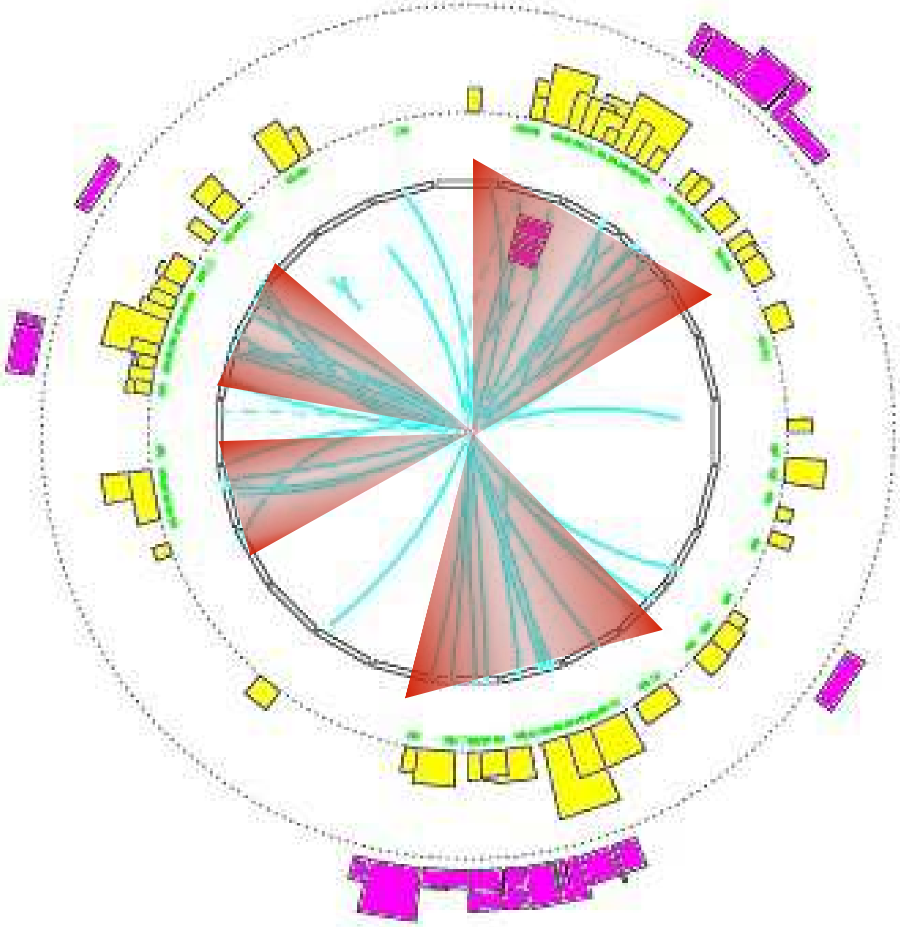


3 jets?

Reconstructing jets is an ambiguous task



2 clear jets

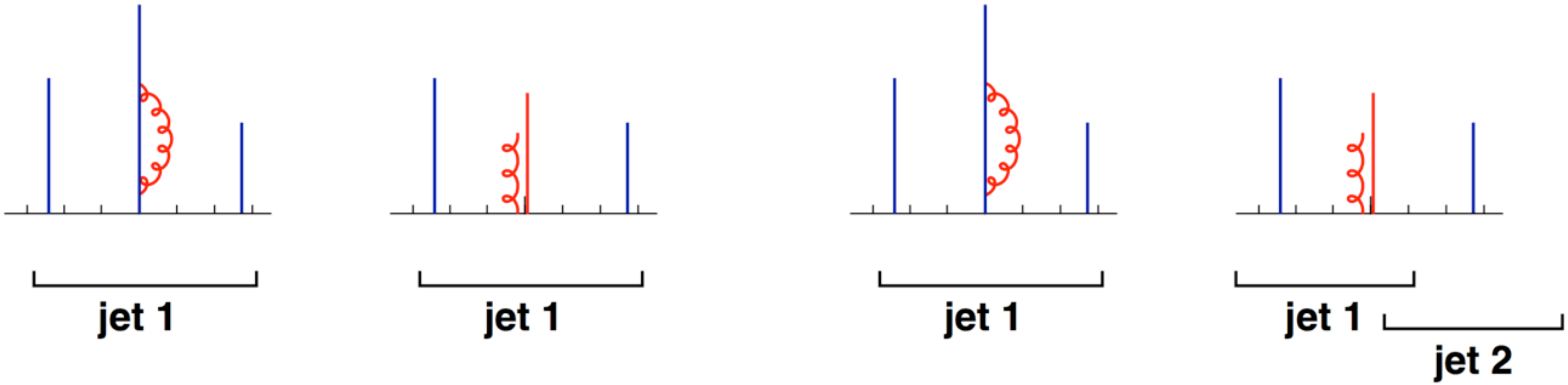


3 jets?

or 4 jets?

Reconstructing jets must respect rules

Collinear Safe **Collinear Unsafe**



$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$ $\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

Infinities cancel **Infinities do not cancel**

Perturbative calculations of jet observable will only be possible with collinear (and infrared) safe jet definitions

Two main classes of jet algorithms

- **Sequential recombination algorithms**

Bottom-up approach: combine particles starting from **closest ones**

How? Choose a **distance measure**, iterate recombination until few objects left, call them jets

Works because of mapping closeness ⇔ QCD divergence

Examples: Jade, k_t, Cambridge/Aachen, anti-k_t, …..

Usually trivially made IRC safe, but their algorithmic complexity scales like N^3

- **Cone algorithms**

Top-down approach: find coarse regions of energy flow.

How? Find **stable cones** (i.e. their axis coincides with sum of momenta of particles in it)

Works because QCD only modifies energy flow on small scales

Examples: JetClu, MidPoint, ATLAS cone, CMS cone, SISCone…...

Can be programmed to be fairly fast, at the price of being complex and IRC unsafe

A little history

- Cone-type jets were introduced first in QCD in the 1970s (Sterman-Weinberg ’77)

- In the 1980s cone-type jets were adapted for use in hadron colliders (SppS, Tevatron...) ➙ iterative cone algorithms

- LEP was a golden era for jets: new algorithms and many relevant calculations during the 1990s
  - Introduction of the ‘theory-friendly’ k_t algorithm
    - sequential recombination type algorithm, IRC safe
    - it allows for all order resummation of jet rates
  - Several accurate calculations in perturbative QCD of jet properties: rates, jet mass, thrust, ...

e^+e^- k_t (Durham) algorithm

[Catani, Dokshitzer, Olsson, Turnock, Webber '91]

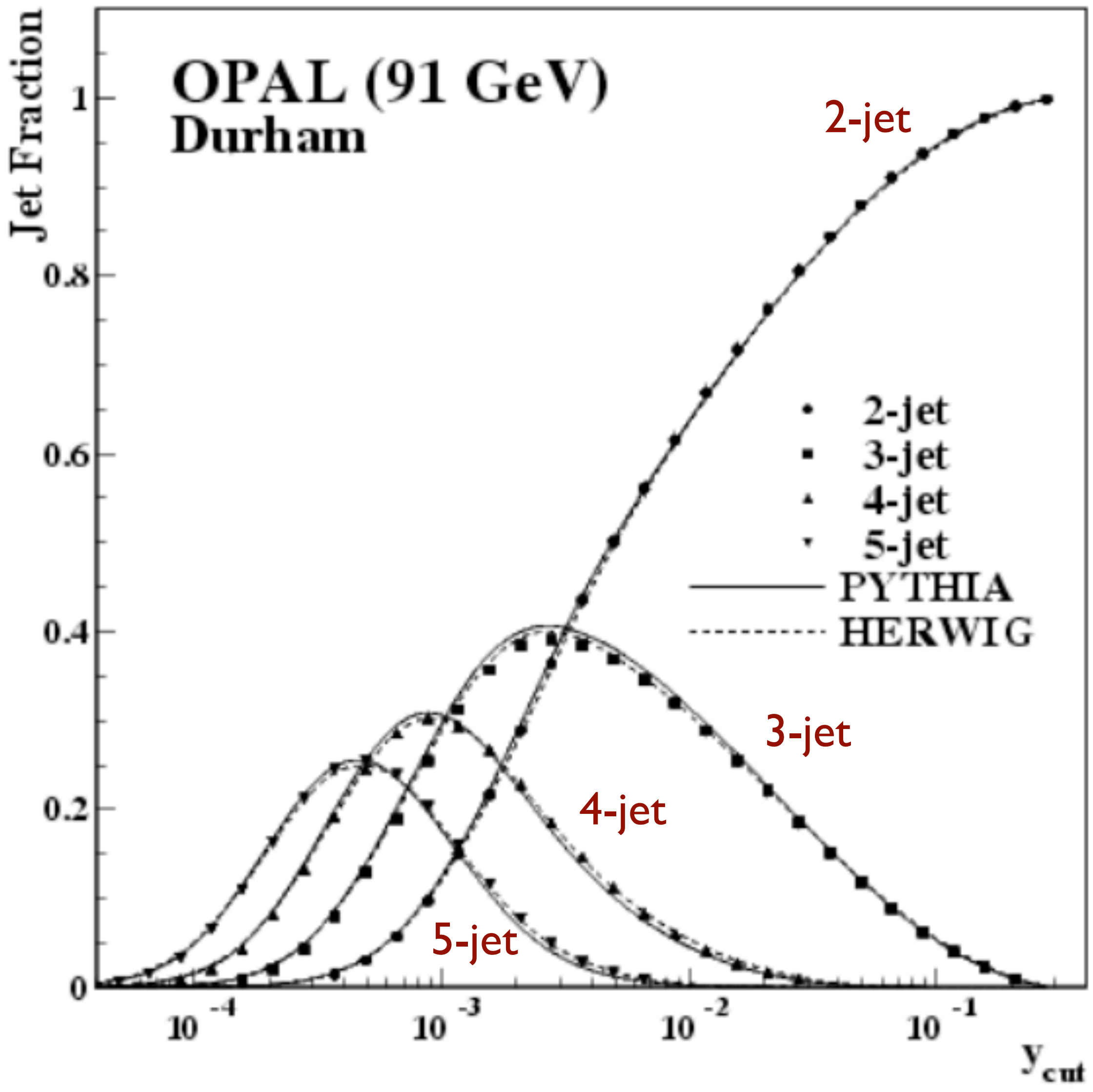
Distance:

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{Q^2}$$

In the collinear limit, the numerator reduces to the **relative transverse momentum** (squared) of the two particles, hence the name of the algorithm

- Find the minimum y_{min} of all y_{ij}
- If y_{min} is below some jet resolution threshold y_{cut}, recombine i and j into a single new particle ('pseudojet'), and repeat
- If no $y_{min} < y_{cut}$ are left, all remaining particles are jets

e^+e^- k_t (Durham) algorithm in action



Characterise events
in terms of number of jets
(as a function of y_{cut})

Resummed calculations for distributions of y_{cut} doable with the k_t algorithm

e^+e^- k_t (Durham) algorithm v. QCD

k_t is a sequential recombination type algorithm

One key feature of the k_t algorithm is its relation to the structure of QCD divergences:

$$\frac{dP_{k\to ij}}{dE_i d\theta_{ij}} \sim \frac{\alpha_s}{\min(E_i, E_j)\theta_{ij}}$$

The y_{ij} distance is the inverse of the emission probability

- The k_t algorithm roughly inverts the QCD branching sequence (the pair which is recombined first is the one with the largest probability to have branched)
- The history of successive clusterings has physical meaning

hadron-collider k_t algorithm

Two parameters, *R* and $p_{t,min}$

(These are the two parameters in essentially every widely used hadron-collider jet algorithm)

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{R^2}, \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

Sequential recombination algorithm

1. Find smallest of d_{ij}, d_{iB}
2. If *ij*, recombine them
3. If *iB*, call i a jet and remove from list of particles
4. repeat from step 1 until no particles left

Only use jets with $p_t > p_{t,min}$

Inclusive k_t algorithm

S.D. Ellis & Soper, 1993

Catani, Dokshitzer, Seymour & Webber, 1993

The k_t algorithm and its siblings

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta y^2 + \Delta\phi^2}{R^2} \qquad d_{iB} = p_{ti}^{2p}$$

p = 1 k_t algorithm

S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187
S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

p = 0 Cambridge/Aachen algorithm

Y. Dokshitzer, G. Leder, S.Moretti and B. Webber, JHEP 08 (1997) 001
M.Wobisch and T.Wengler, hep-ph/9907280

p = -1 **anti-k_t algorithm**

MC, G. Salam and G. Soyez, arXiv:0802.1189

NB: in anti-kt pairs with a **hard** particle will cluster first: if no other hard particles are close by, the algorithm will give **perfect cones**

Quite ironically, a sequential recombination algorithm is the 'perfect' cone algorithm

IRC safety of generalised-k_t algorithms

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta y^2 + \Delta\phi^2}{R^2} \qquad d_{iB} = p_{ti}^{2p}$$

p > 0

New **soft** particle ($p_t \to 0$) means that d → 0 ⇒ clustered first, no effect on jets

New **collinear** particle ($\Delta y^2 + \Delta\Phi^2 \to 0$) means that d → 0 ⇒ clustered first, no effect on jets

p = 0

New **soft** particle ($p_t \to 0$) can be new jet of zero momentum ⇒ no effect on hard jets

New **collinear** particle ($\Delta y^2 + \Delta\Phi^2 \to 0$) means that d → 0 ⇒ clustered first, no effect on jets

p < 0

New **soft** particle ($p_t \to 0$) means d → ∞ ⇒ clustered last or new zero-jet, no effect on hard jets

New **collinear** particle ($\Delta y^2 + \Delta\Phi^2 \to 0$) means that d → 0 ⇒ clustered first, no effect on jets

IRC safe algorithms

k_t	SR $d_{ij} = \min(p_{ti}^2, p_{tj}^2)\Delta R_{ij}^2/R^2$ hierarchical in rel p_t	Catani et al '91 Ellis, Soper '93	NlnN
Cambridge/ Aachen	SR $d_{ij} = \Delta R_{ij}^2/R^2$ hierarchical in angle	Dokshitzer et al '97 Wengler, Wobish '98	NlnN
anti-k_t	SR $d_{ij} = \min(p_{ti}^{-2}, p_{tj}^{-2})\Delta R_{ij}^2/R^2$ gives perfectly conical hard jets	MC, Salam, Soyez '08 (Delsart, Loch)	$N^{3/2}$
SISCone	Seedless iterative cone with split-merge gives 'economical' jets	Salam, Soyez '07	$N^2 \ln N$

'second-generation' algorithms

All are available in FastJet, http://fastjet.fr

(As well as many IRC unsafe ones)

Anti-k$_t$ in action

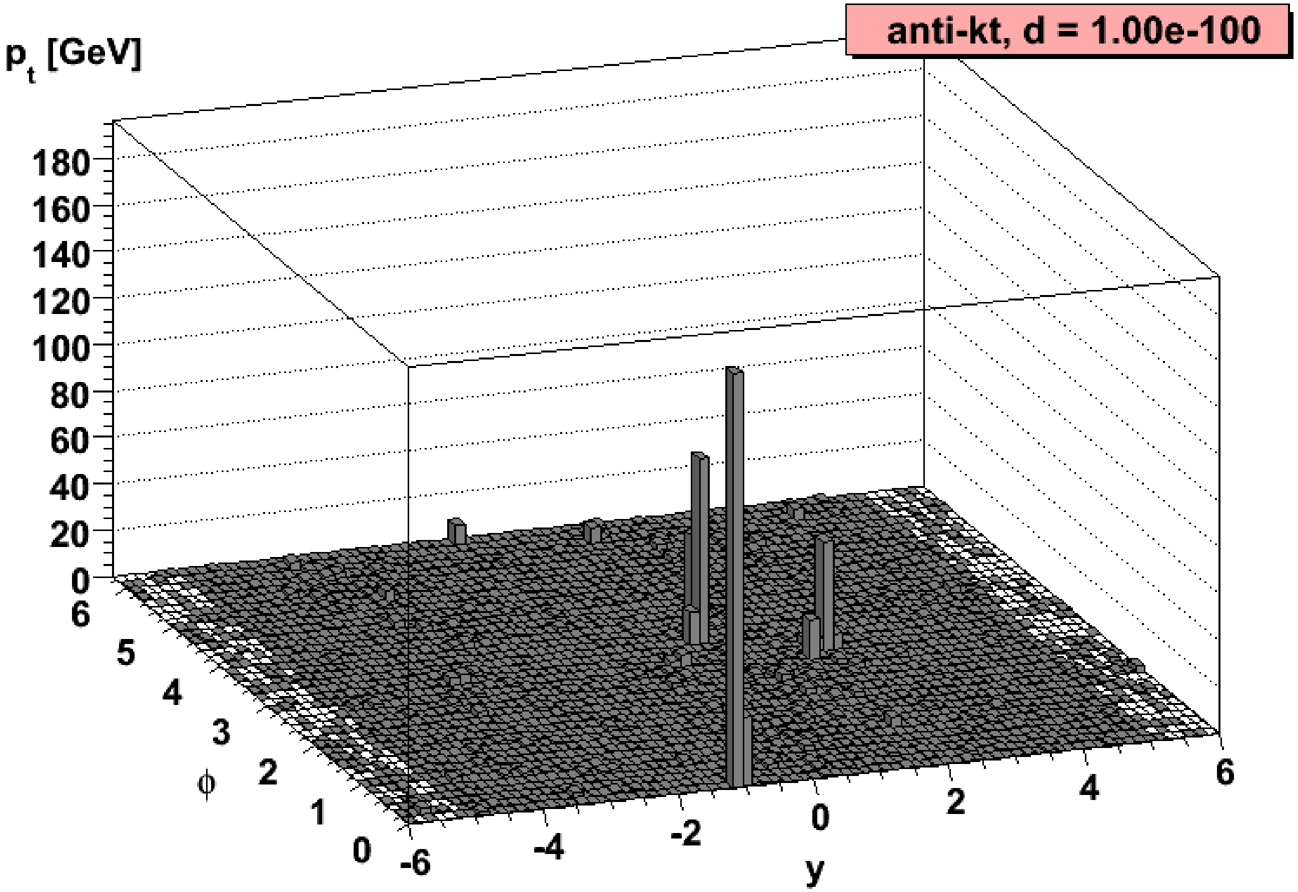
Clustering grows
around hard cores

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$

Anti-k_t in action

Clustering grows around hard cores

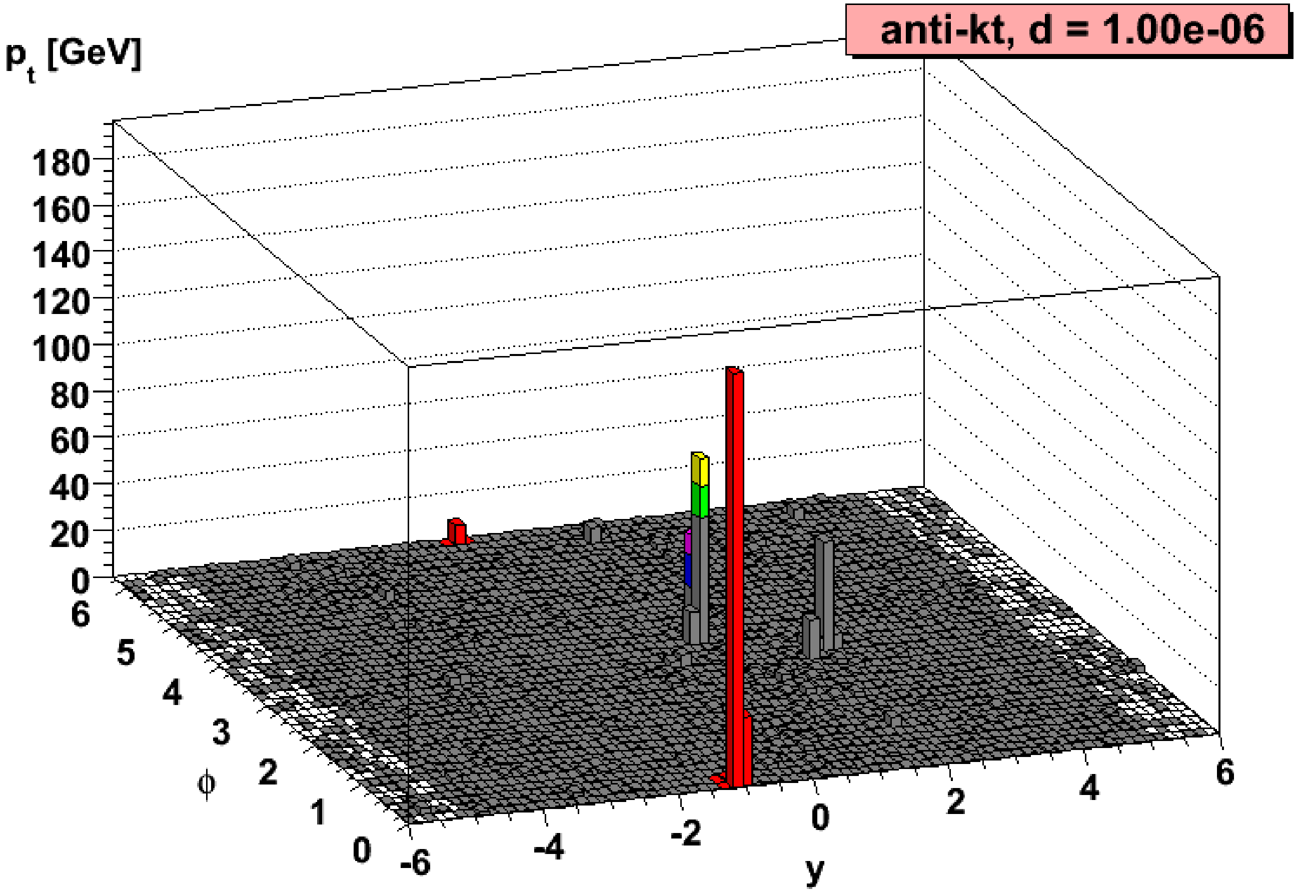
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows
around hard cores

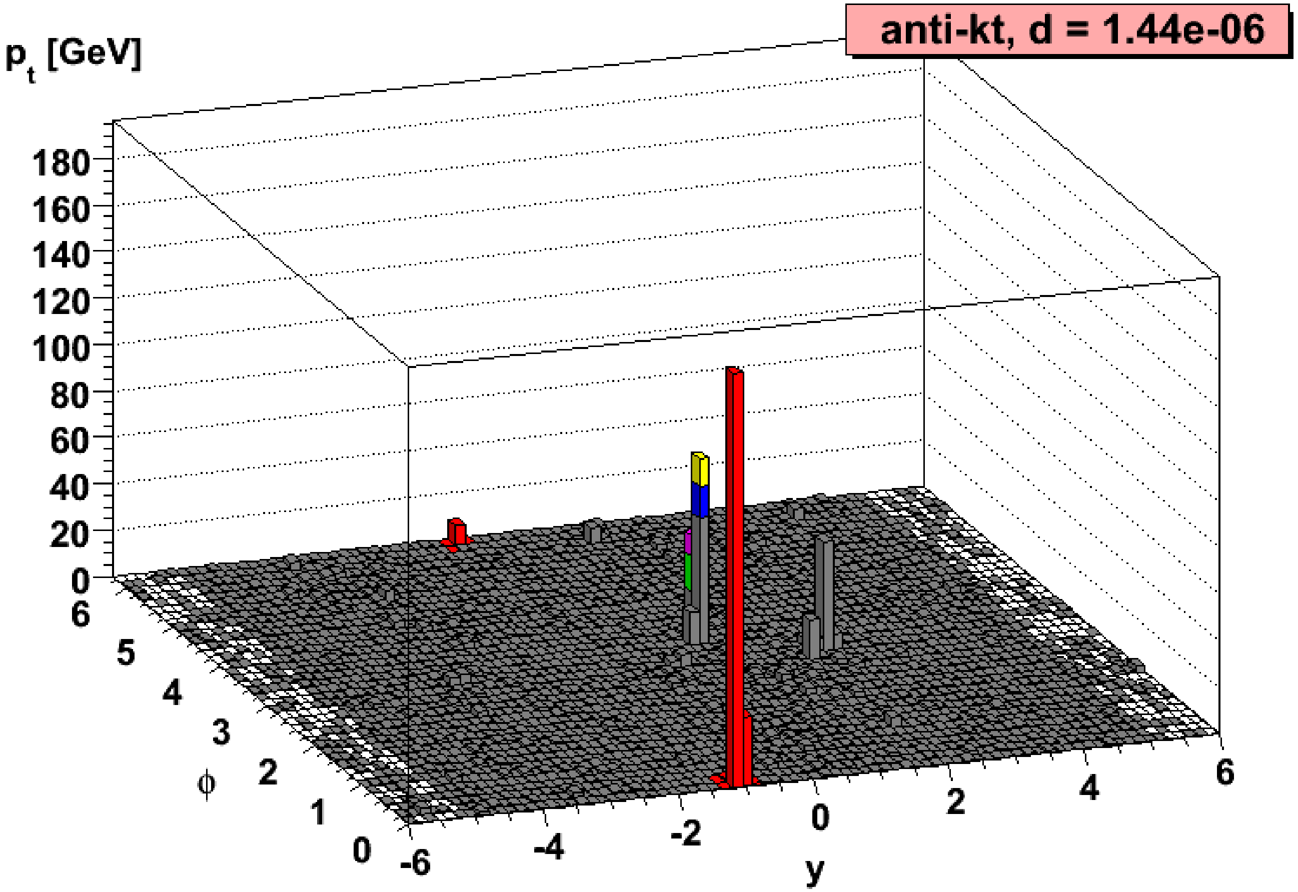
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows
around hard cores

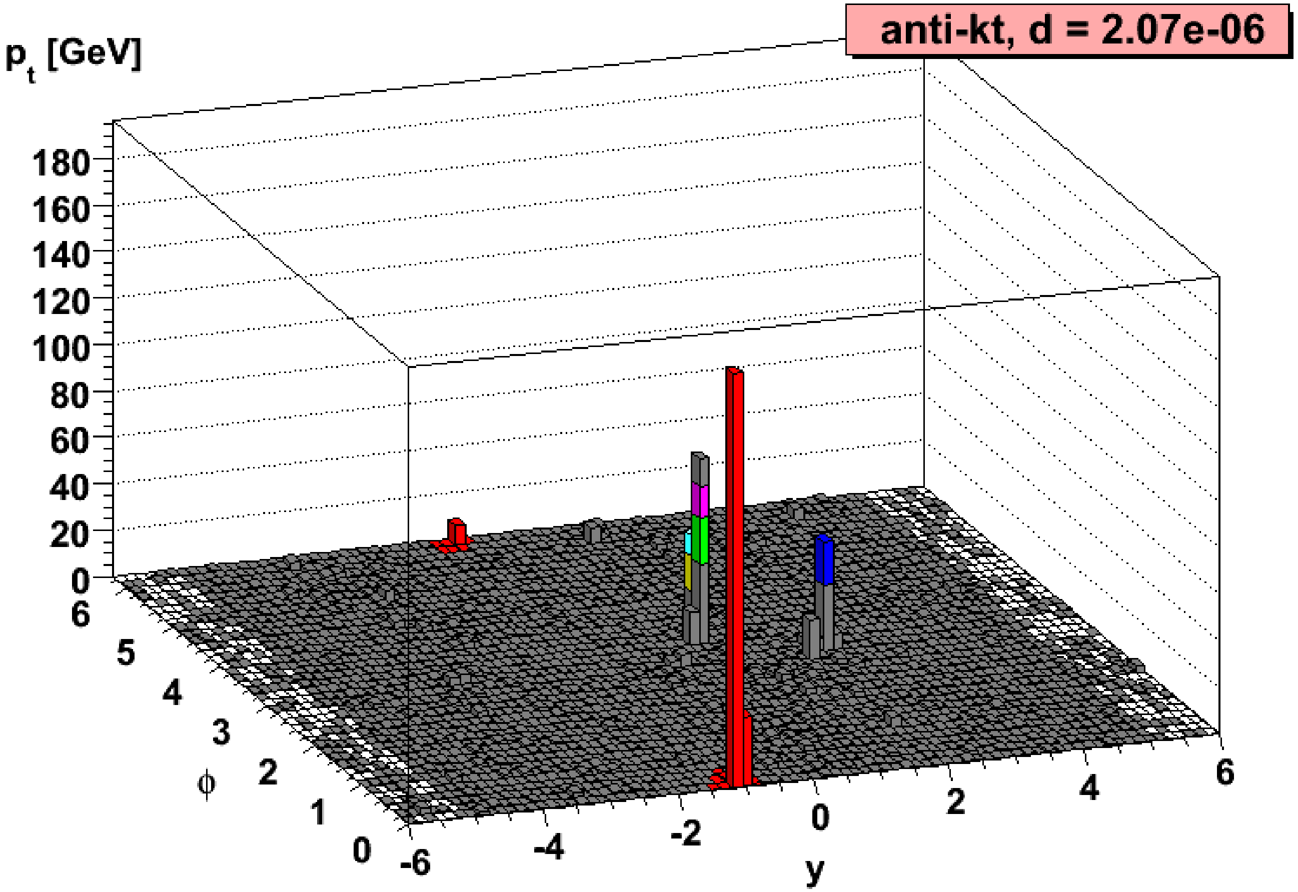
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows around hard cores

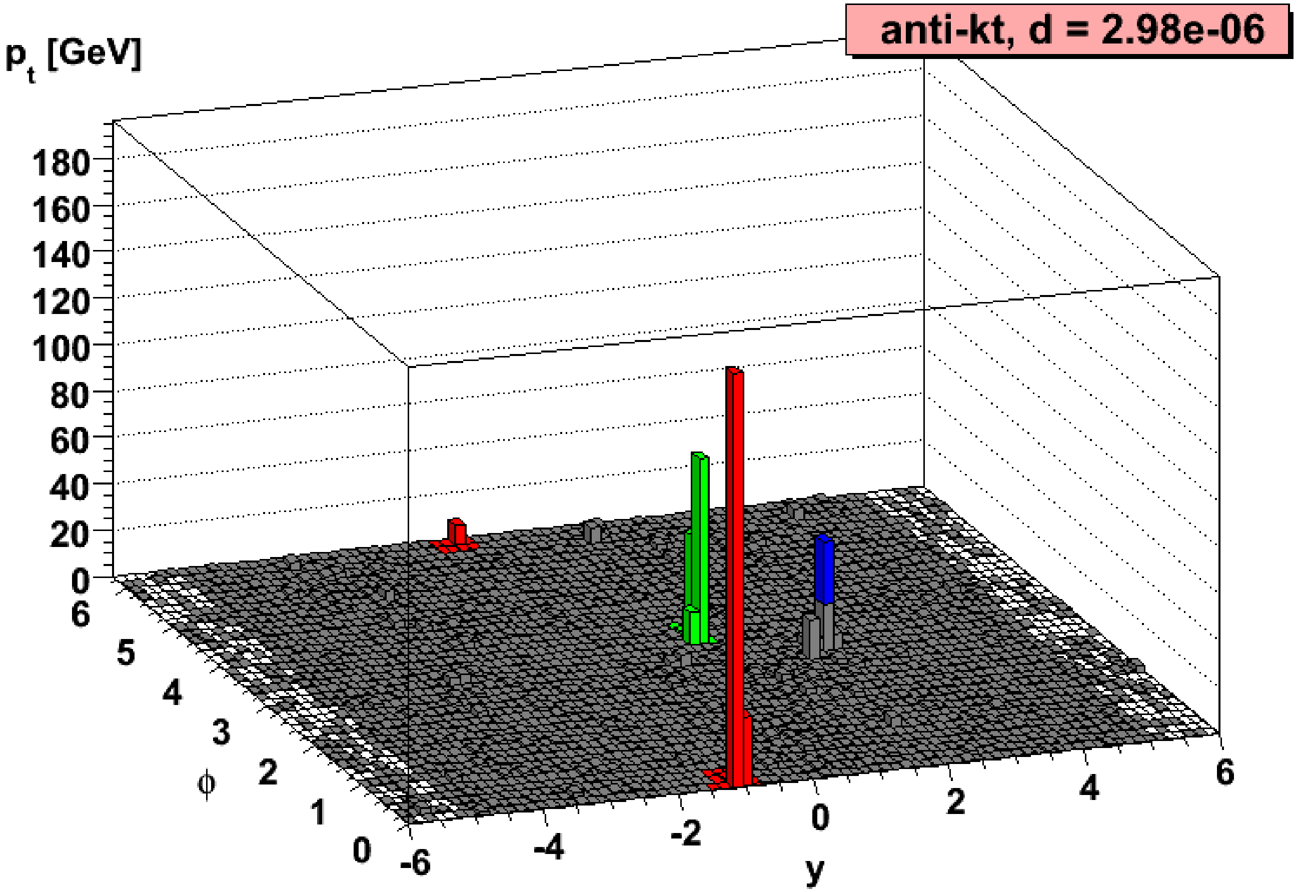
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows
around hard cores

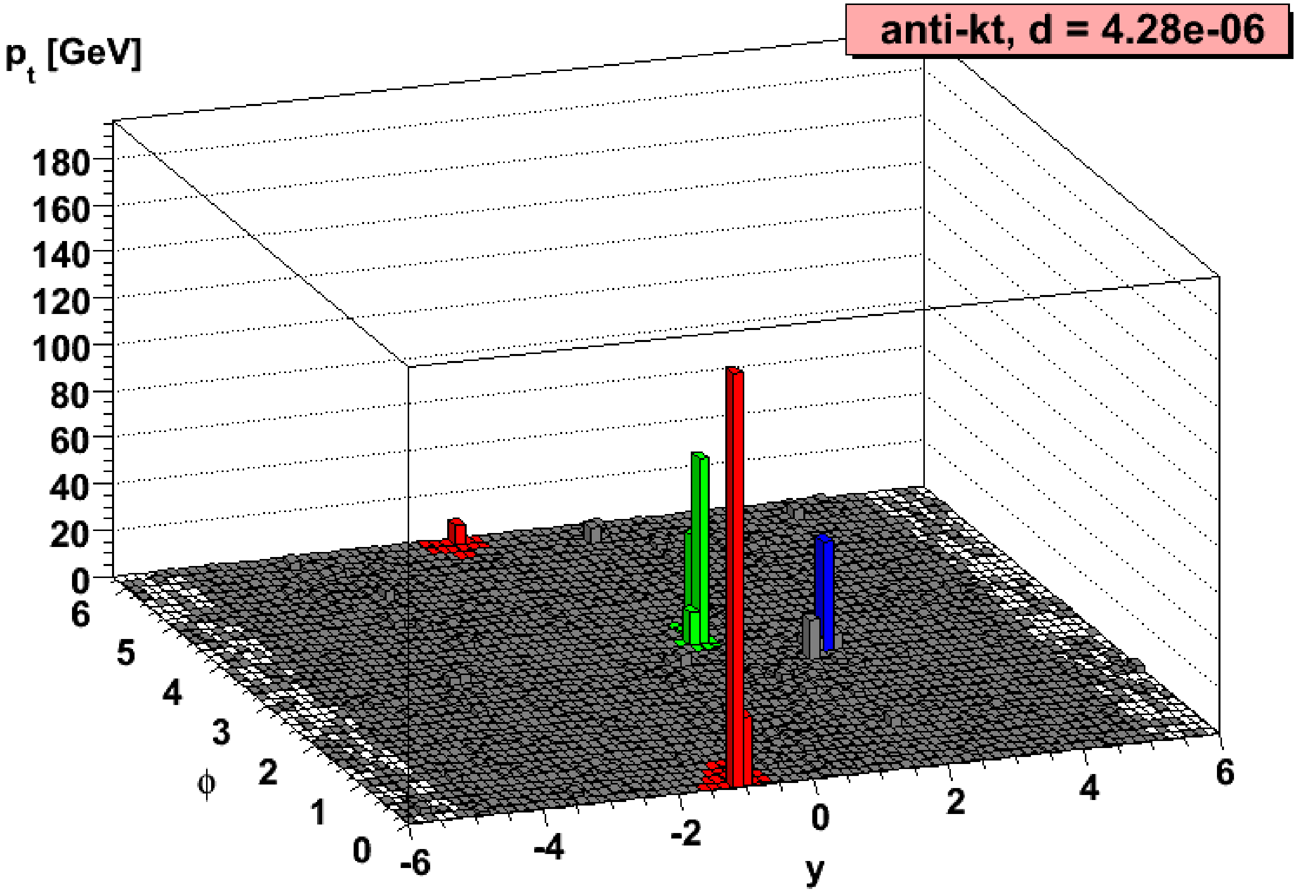
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows around hard cores

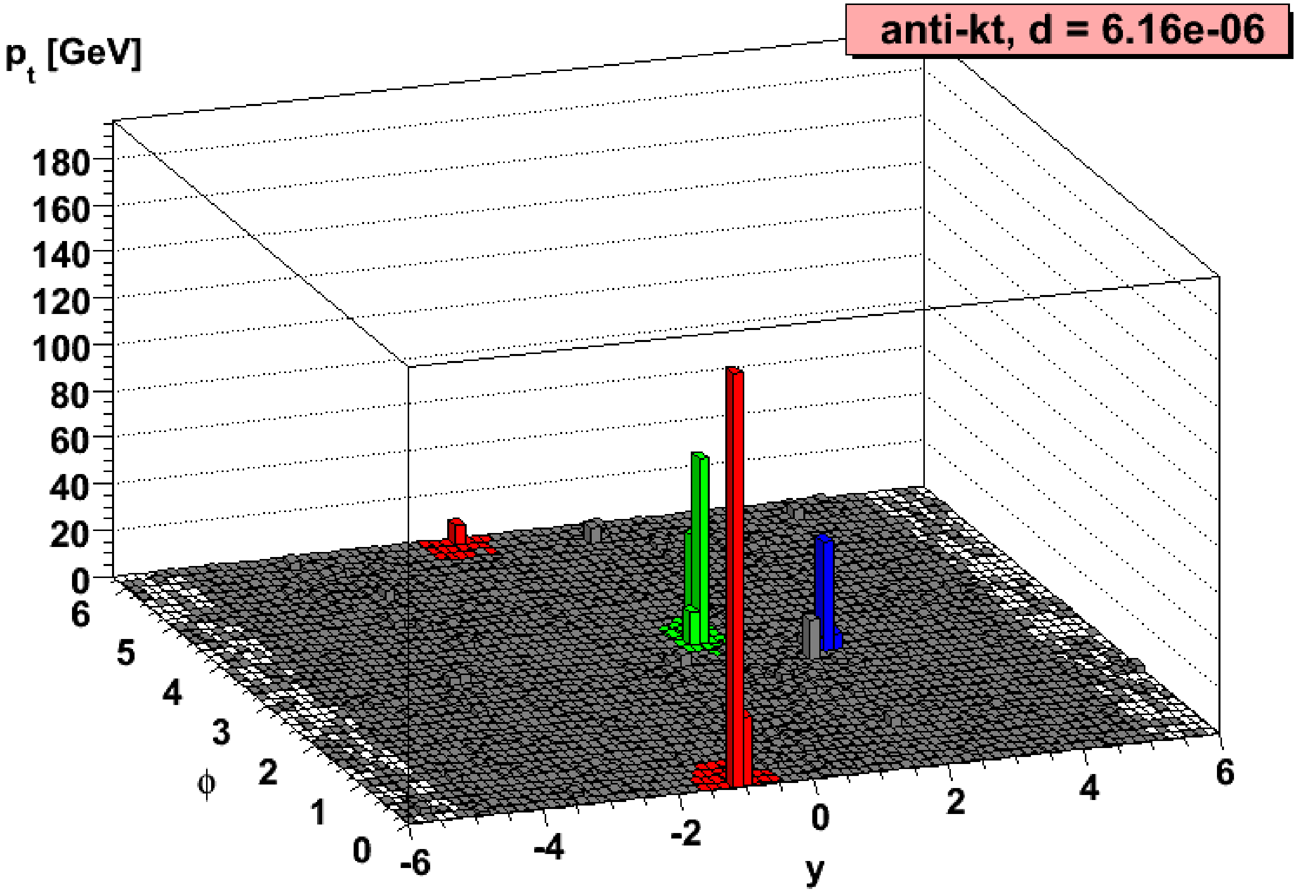
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows around hard cores

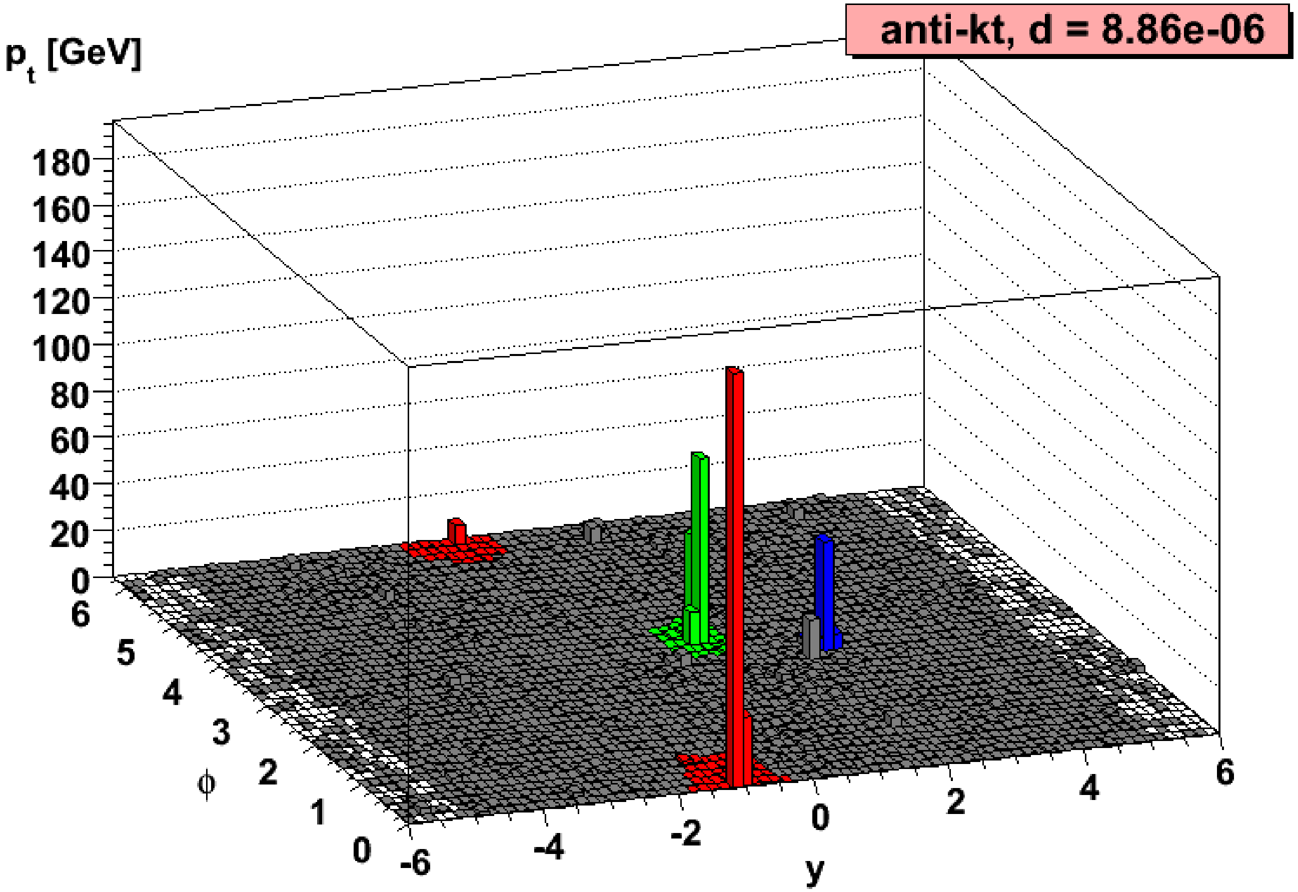
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows around hard cores

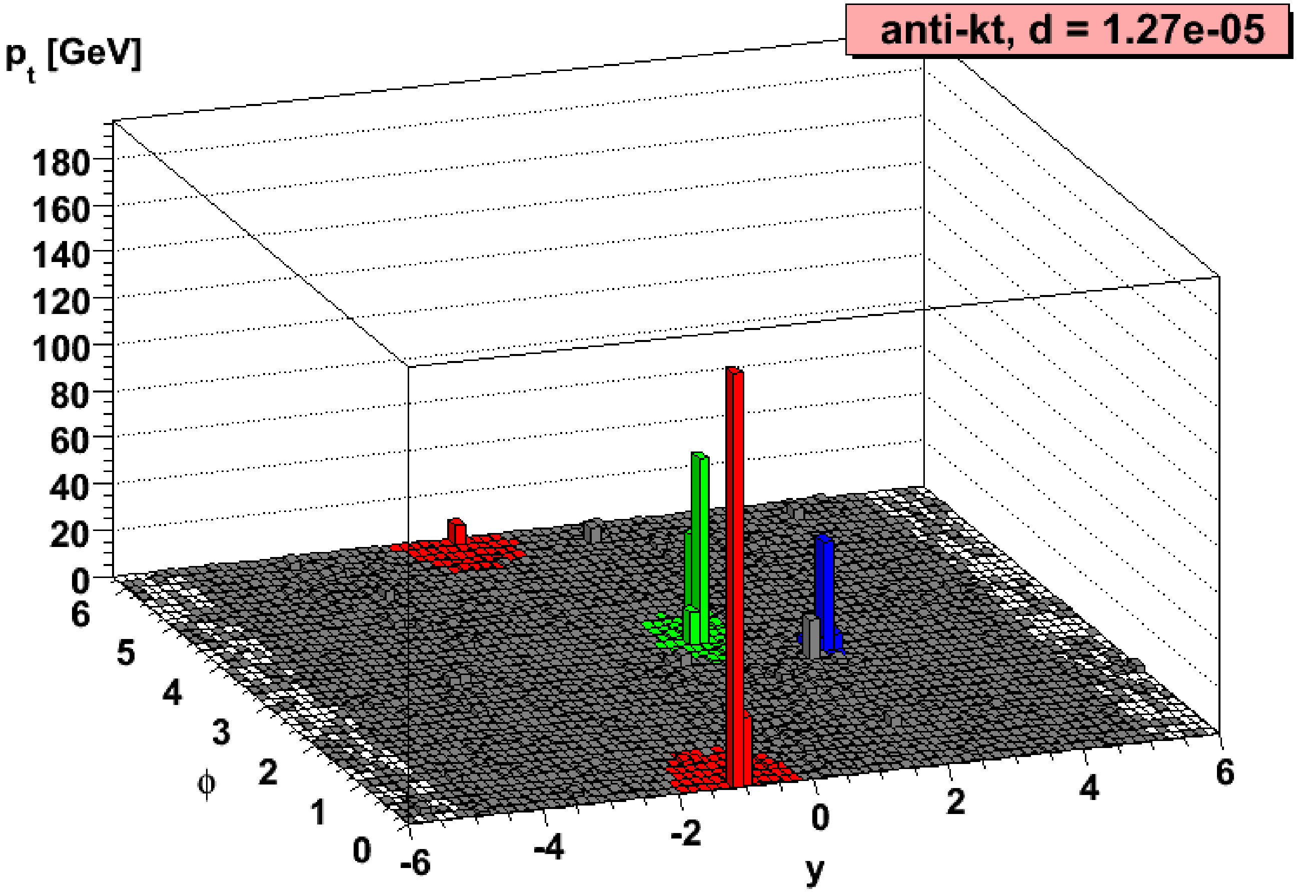
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows around hard cores

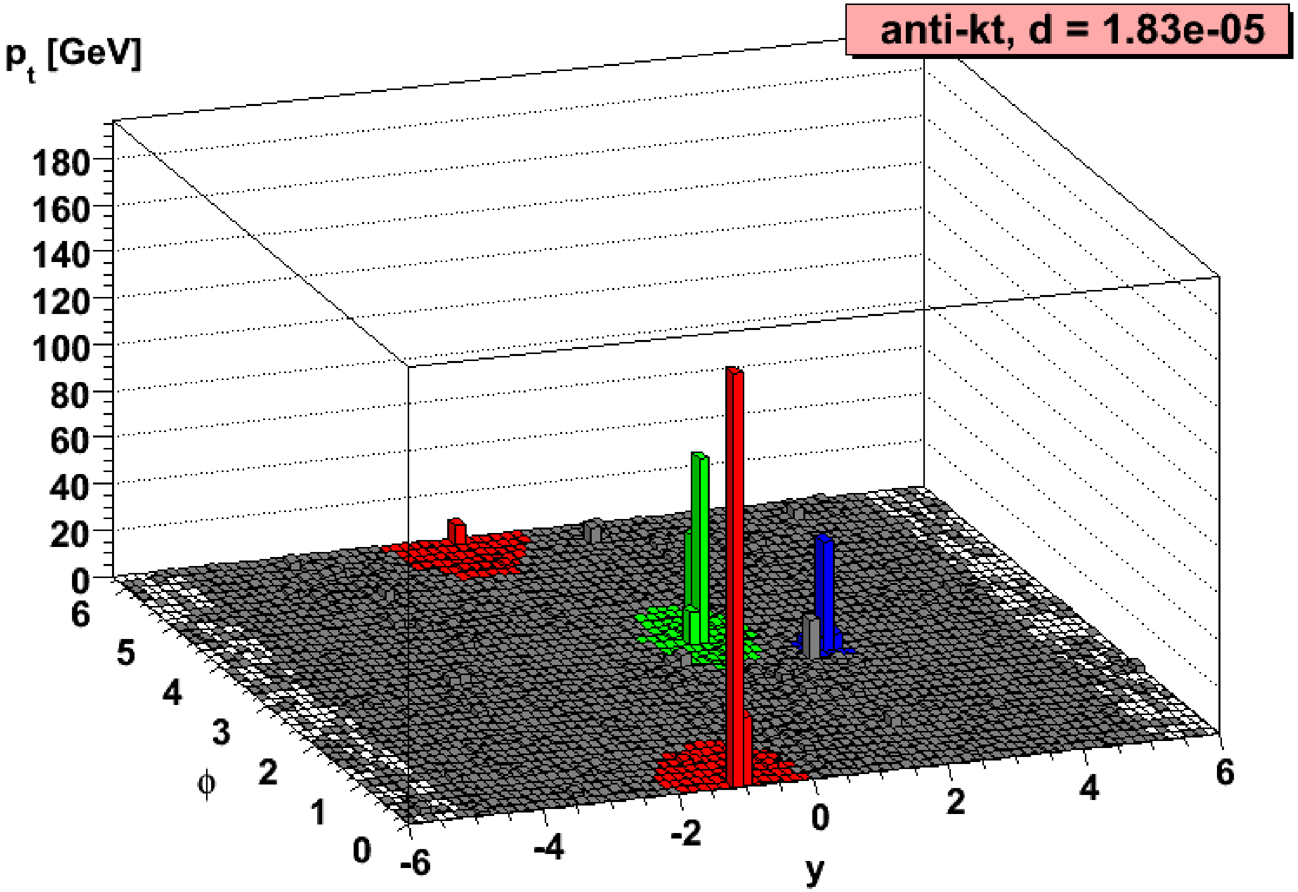
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows
around hard cores

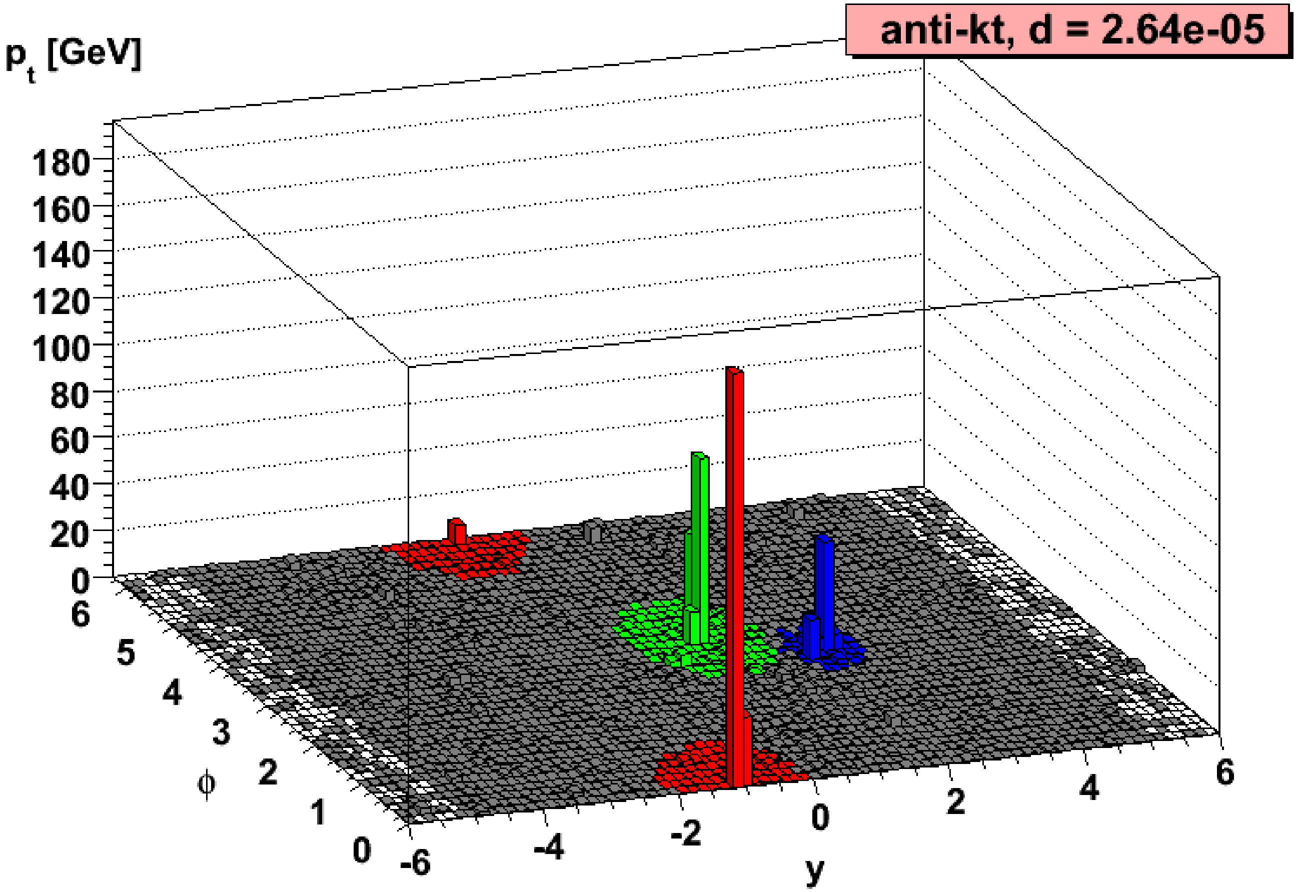
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows
around hard cores

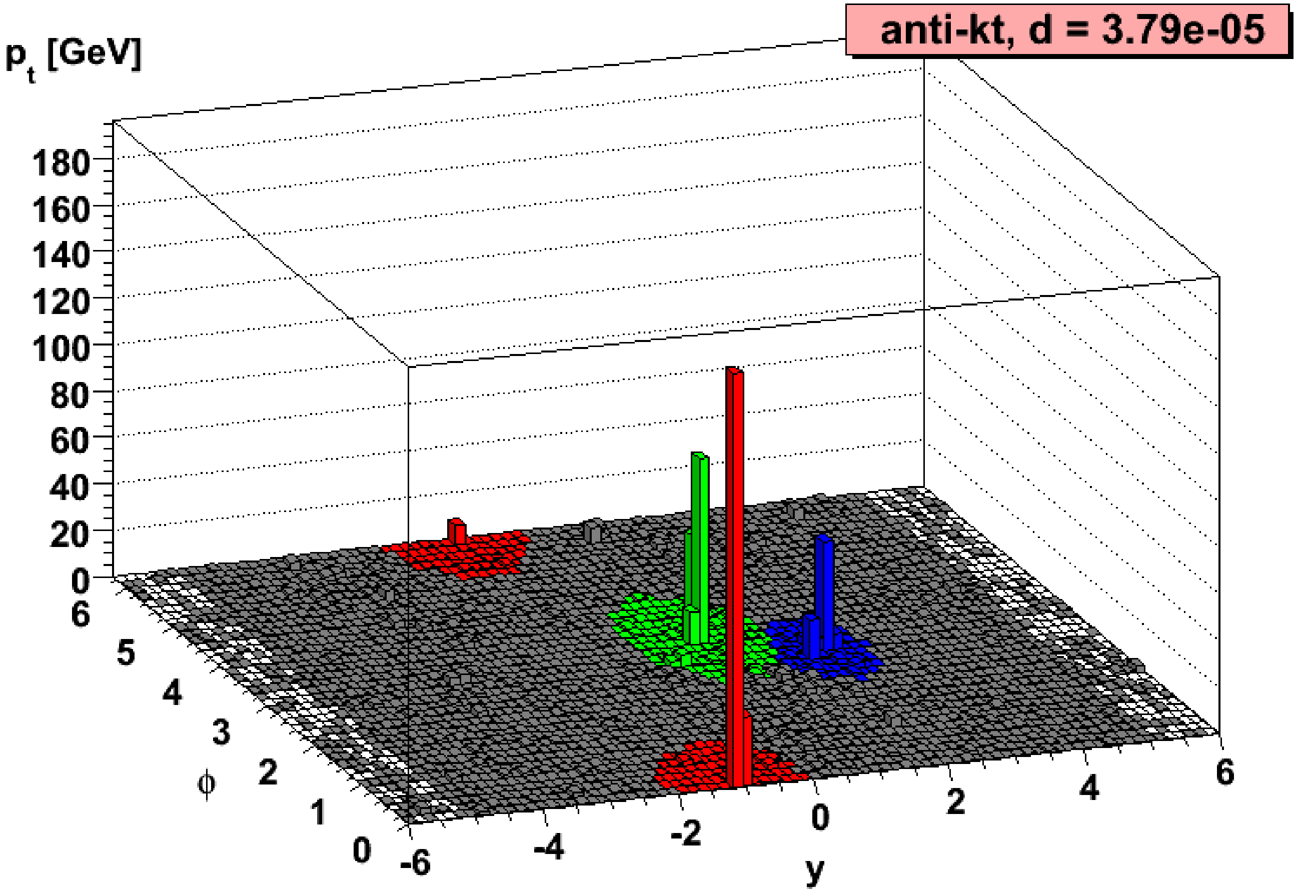
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows around hard cores

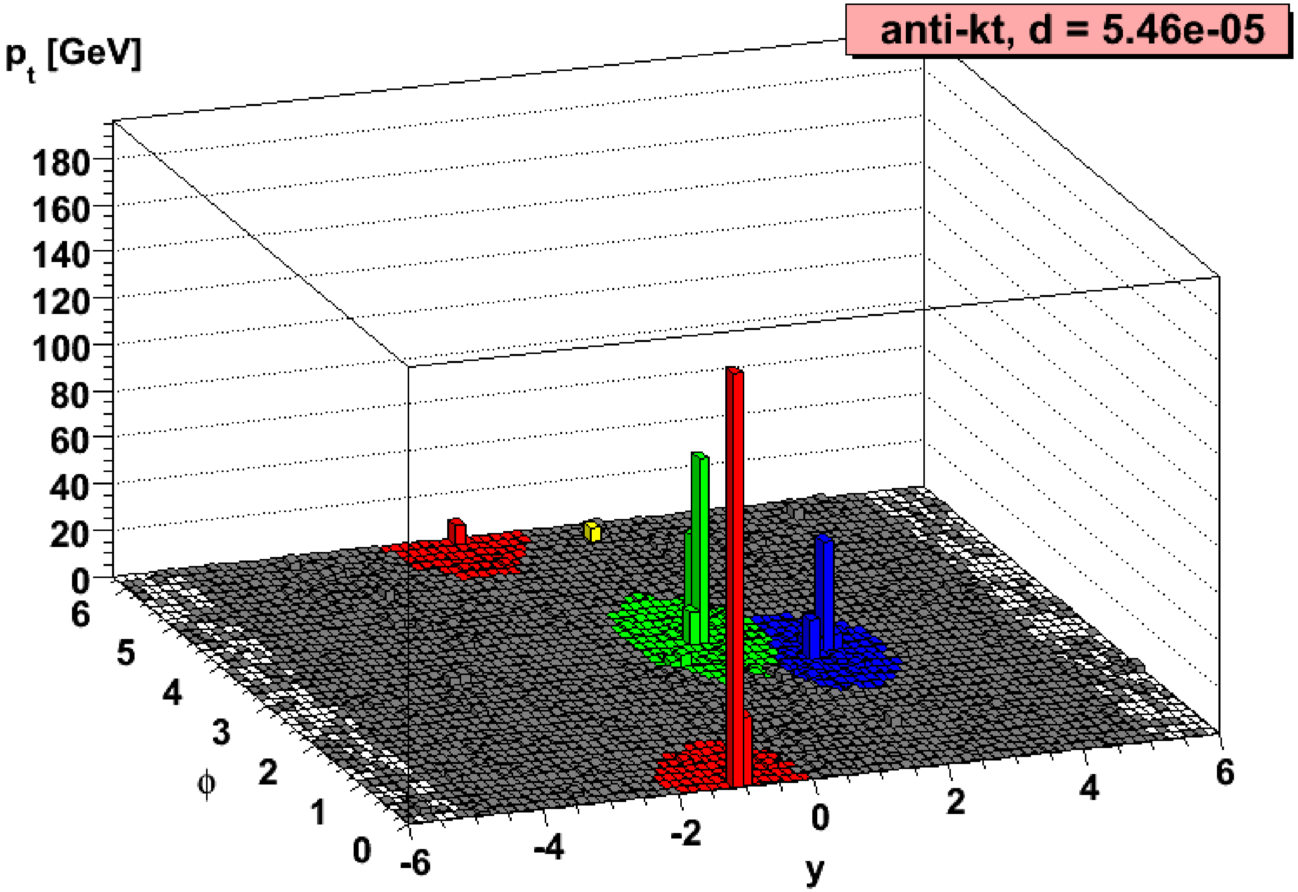
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows
around hard cores

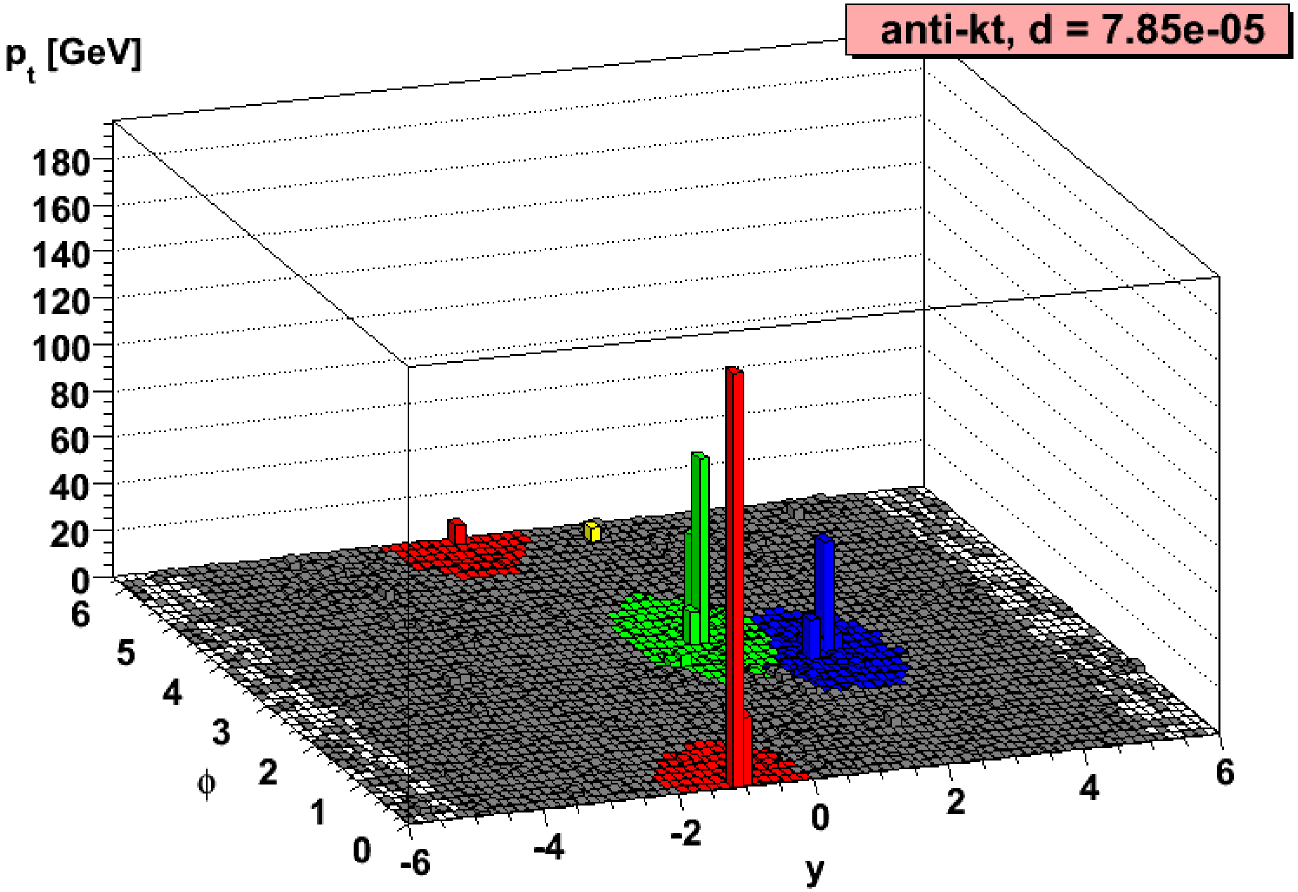
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



Anti-k_t in action

Clustering grows around hard cores

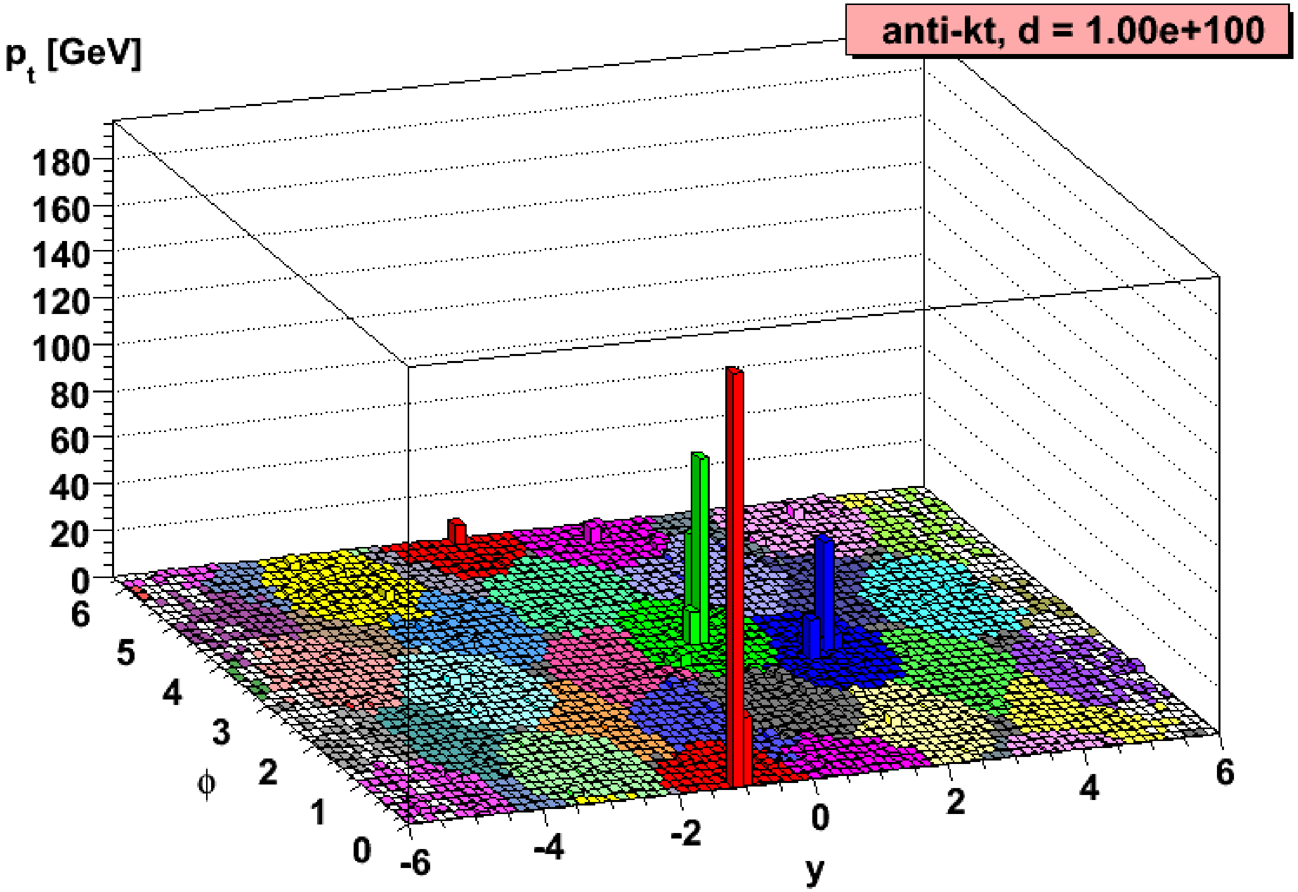
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = \frac{1}{p_{ti}^2}$$



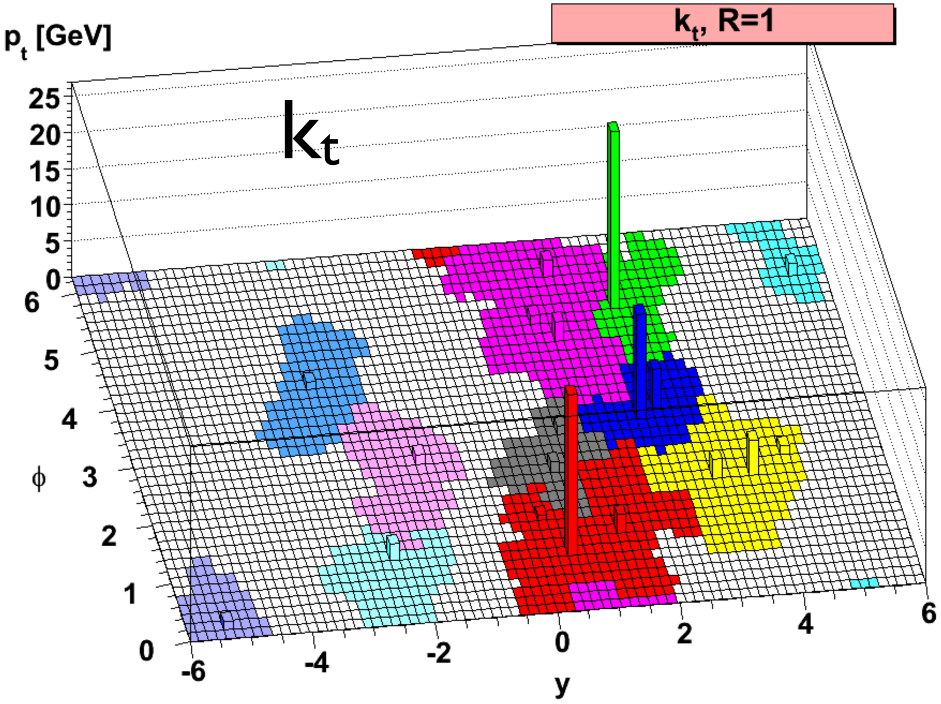
Anti-k_t in action

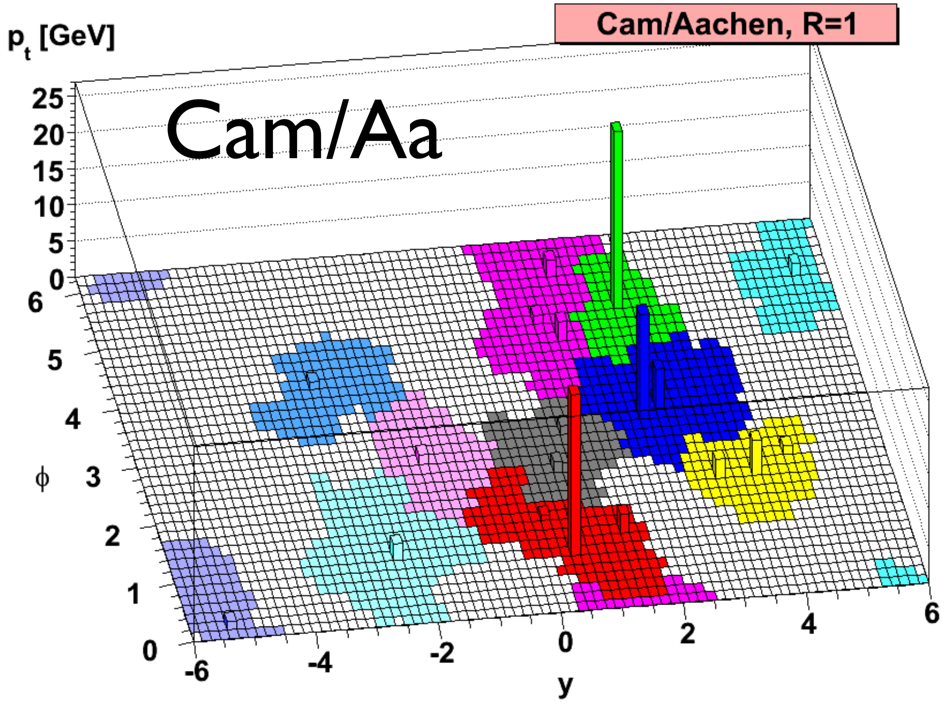
Clustering grows around hard cores

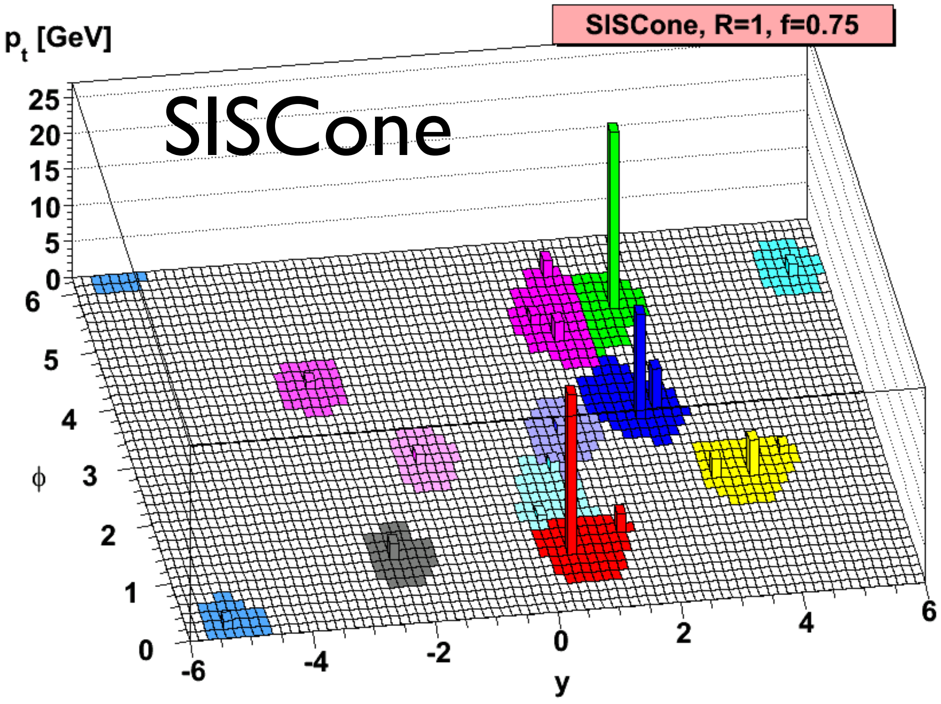
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$

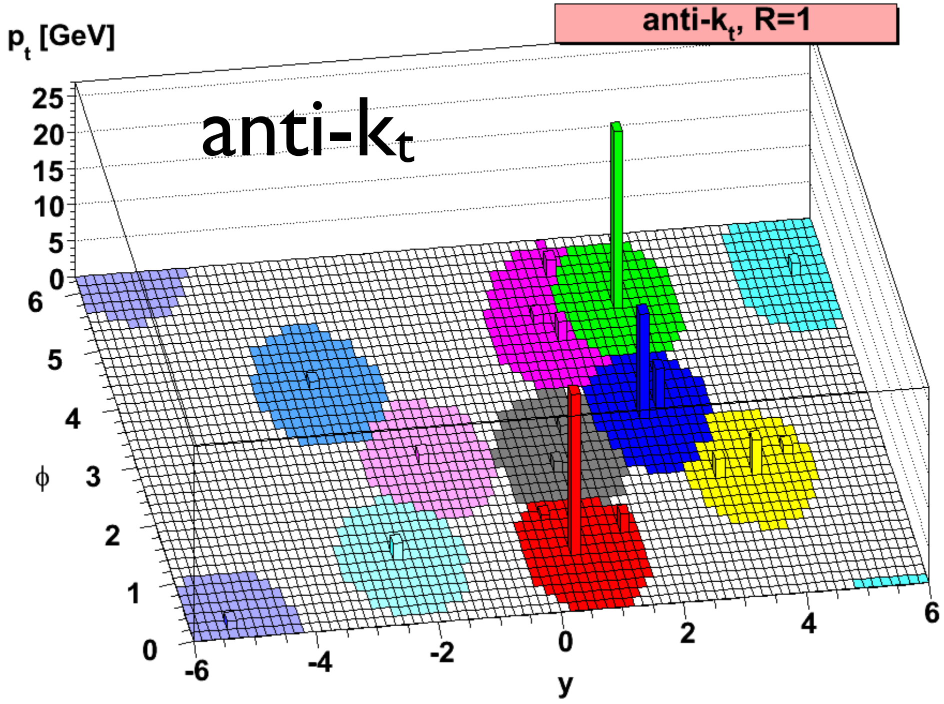


Anti-k_t gives circular jets ("cone-like") in a way that's infrared safe

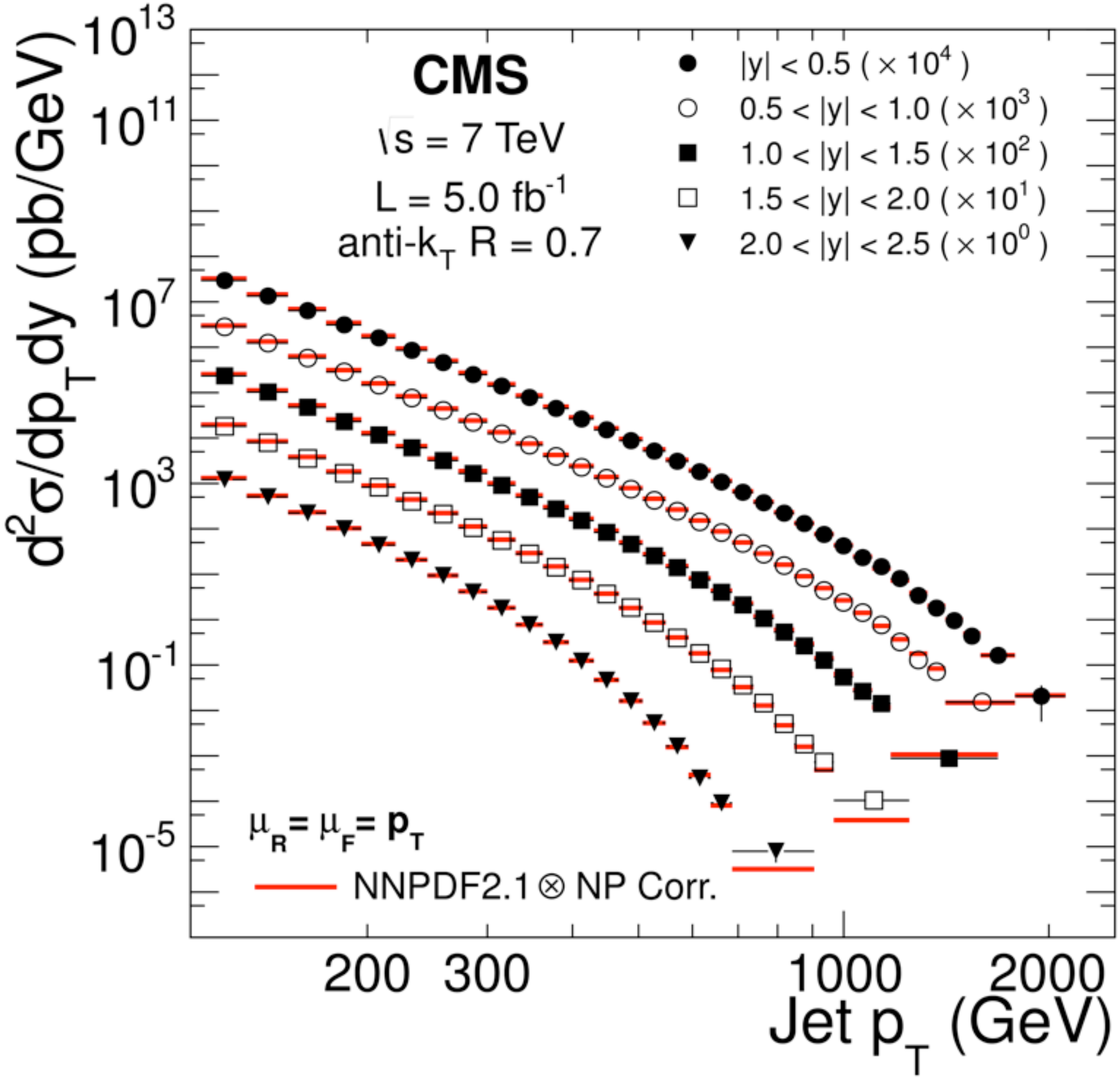

k_t


Cam/Aa


SISCone


anti-k_t

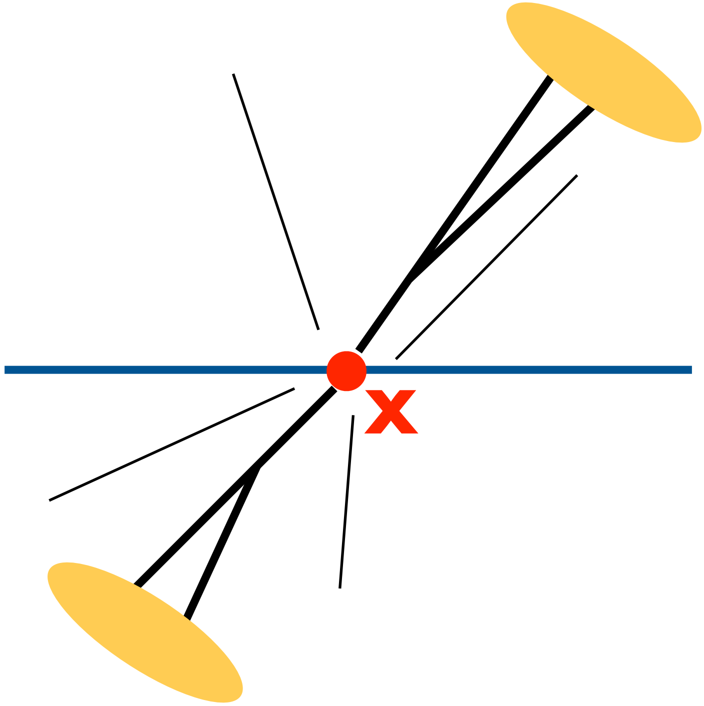
Example of jet observable



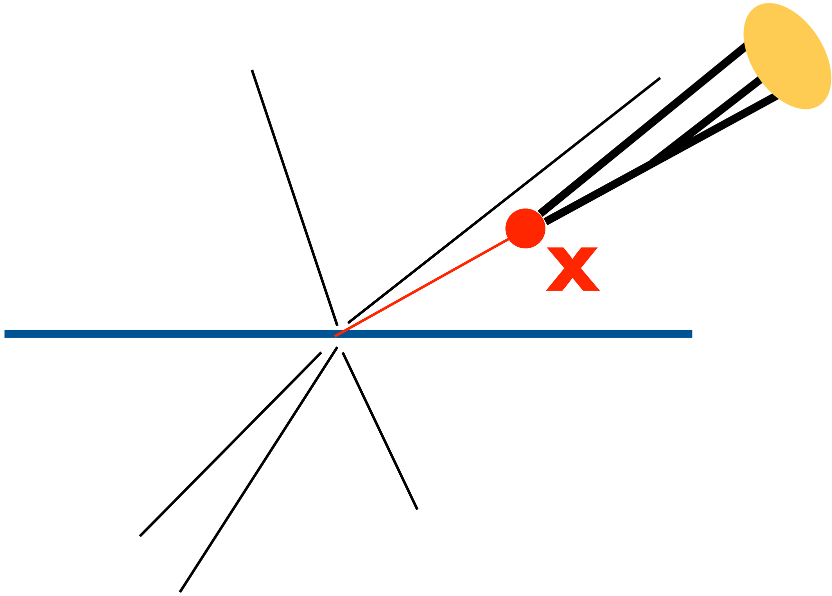
Inclusive jet cross section

Excellent theory-data agreement over many orders of magnitude

Why boosted objects



Heavy particle X at **rest**



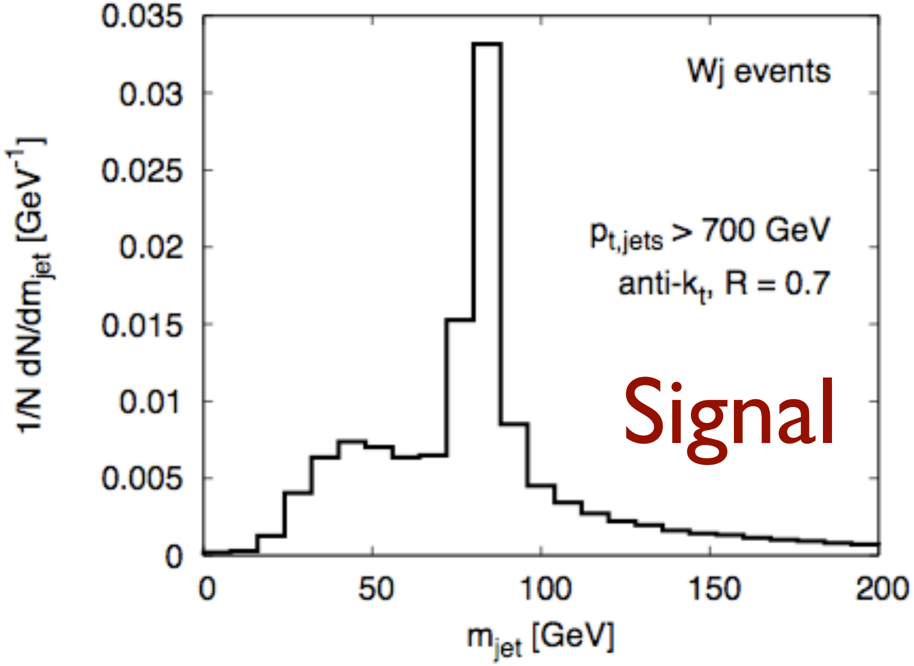
Boosted heavy particle X

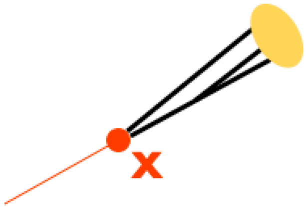
Easy to resolve jets and calculate invariant mass, but signal very likely swamped by background (eg H→bb v. tt →WbWb)

Cross section very much reduced, but acceptance better and some backgrounds smaller/ reducible

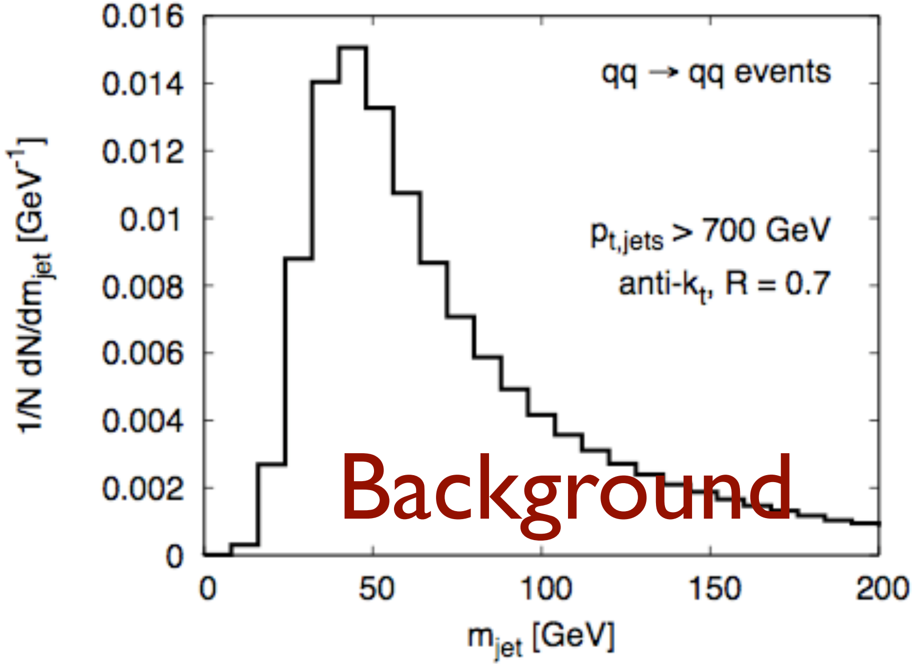
Mass of a single jet

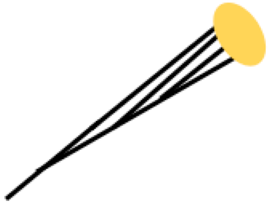
G. Salam





A heavy object decaying into a single jet naturally gives it a mass...



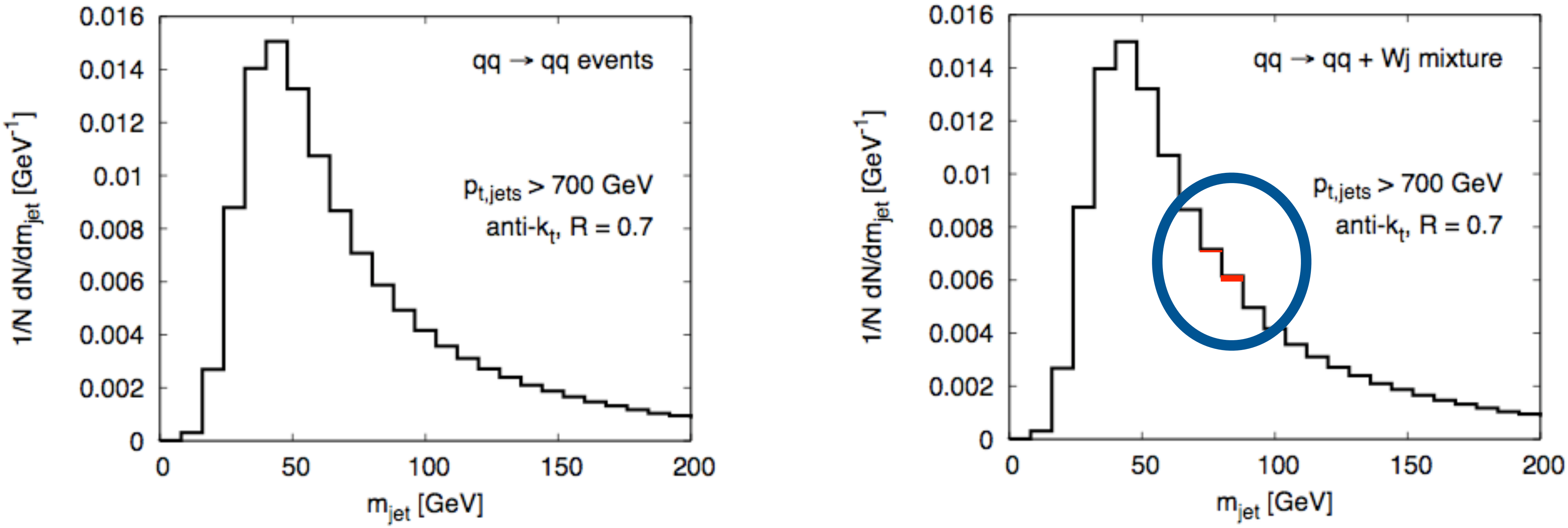


... but pure QCD jets can be massive too:

$$\frac{dN}{d\ln m} \sim \alpha_s \ln \frac{p_t R}{m} \times \mathrm{Sudakov}$$

Mass of a single jet

Summing 'signal' and 'background' (with appropriate cross sections) shows how much the background dominates

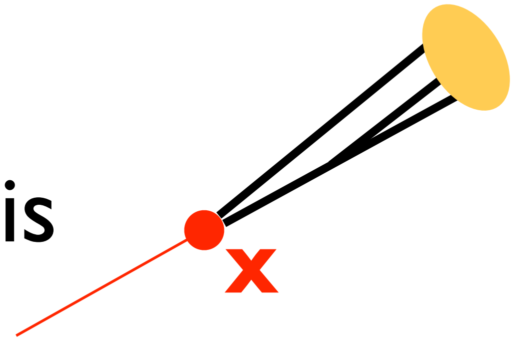
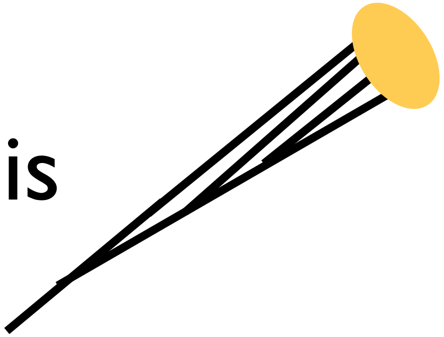


Background only **Signal + background**

Practically identical

This means that one can't rely on the invariant mass only.
An appropriate strategy must be found to reduce the background and enhance the signal

Tagging

How to tell this  from this  ?

Decay of a heavy (boosted) object

Light parton fragmentation

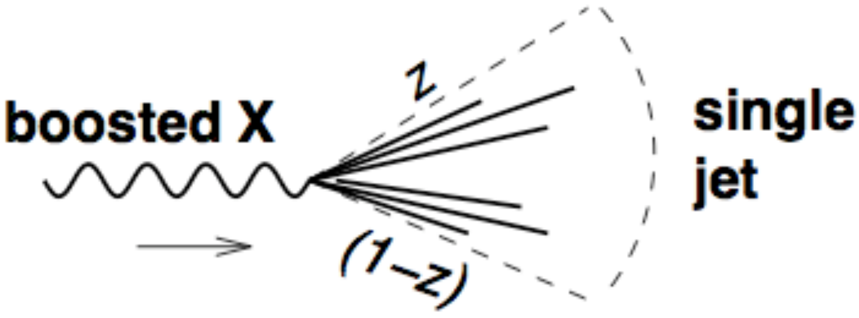
Tagging and Grooming

- The substructure of a jet can be exploited to
  - ***tag*** *a particular structure inside the jet, i.e. a massive particle*
    - *First examples: Higgs (2-prong decay), top (3-prong decay)*
  - *remove background contamination from the jet or its components, while keeping the bulk of the perturbative radiation (often generically denoted as* ***grooming****)*
    - *First examples: filtering, trimming, pruning*

Why substructure

Scales: m ~ 100 GeV, p_t ~ 500 GeV

(e.g. electroweak particle from decay of ~ 1TeV BSM particle)



$$R \gtrsim \frac{m}{p_t} \frac{1}{\sqrt{z(1-z)}}$$

- need small R (< $2m/p_t$ ~ 0.4) to resolve two prongs
- need large R (>~ $3m/p_t$ ~ 0.6) to cluster into a single jet

Possible strategies

- Use large R, get a single jet : background large
- Use small R, resolve the jets : what is the right scale?
  - Also: small jets lead to huge combinatorial issues

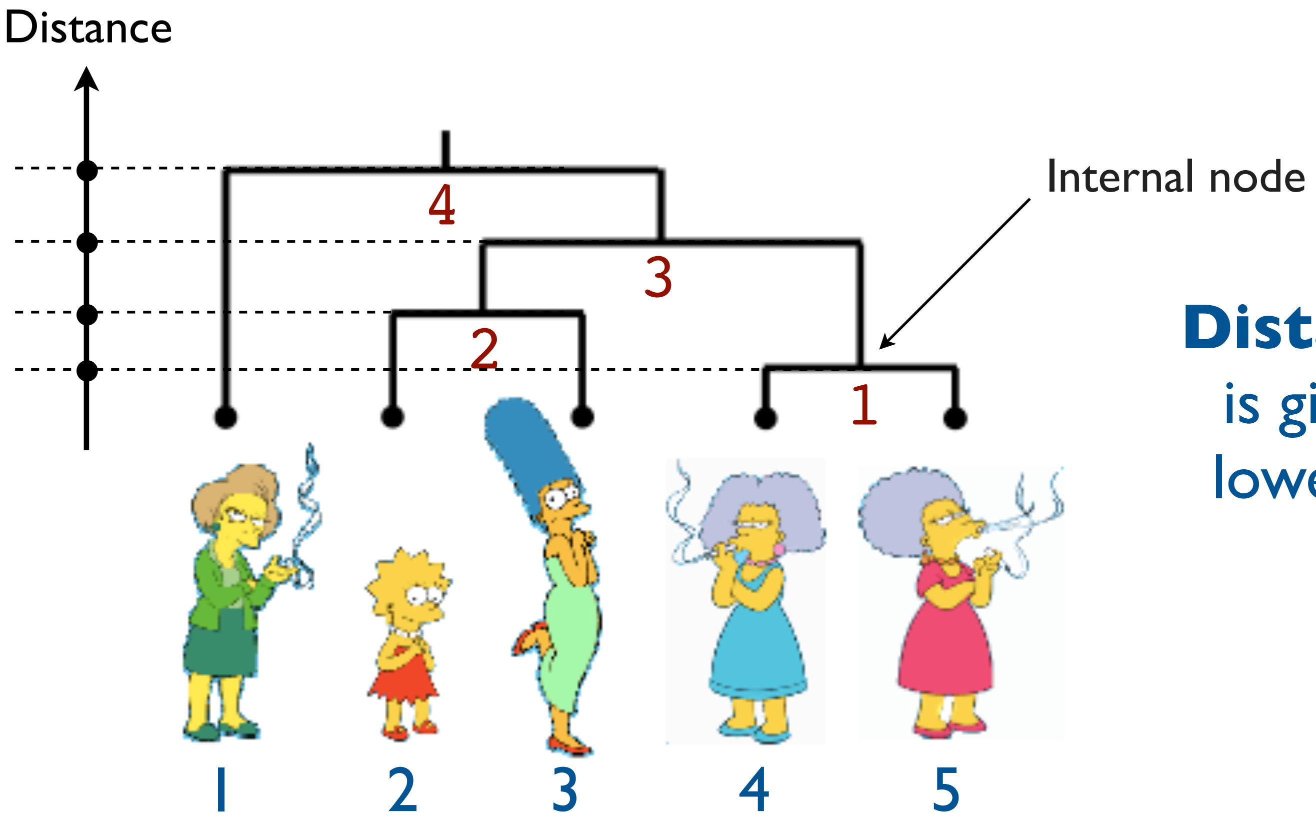
Let an algorithm find the 'right' substructure

What jets to use for substructure?

Different jet algorithms will give different 'pictures' of what's inside a jet

Dendrogram

Used to represent graphically the sequence of clustering steps in a sequential recombination algorithm



Distance between two objects is given by the **height** of the lowest internal node that they share.

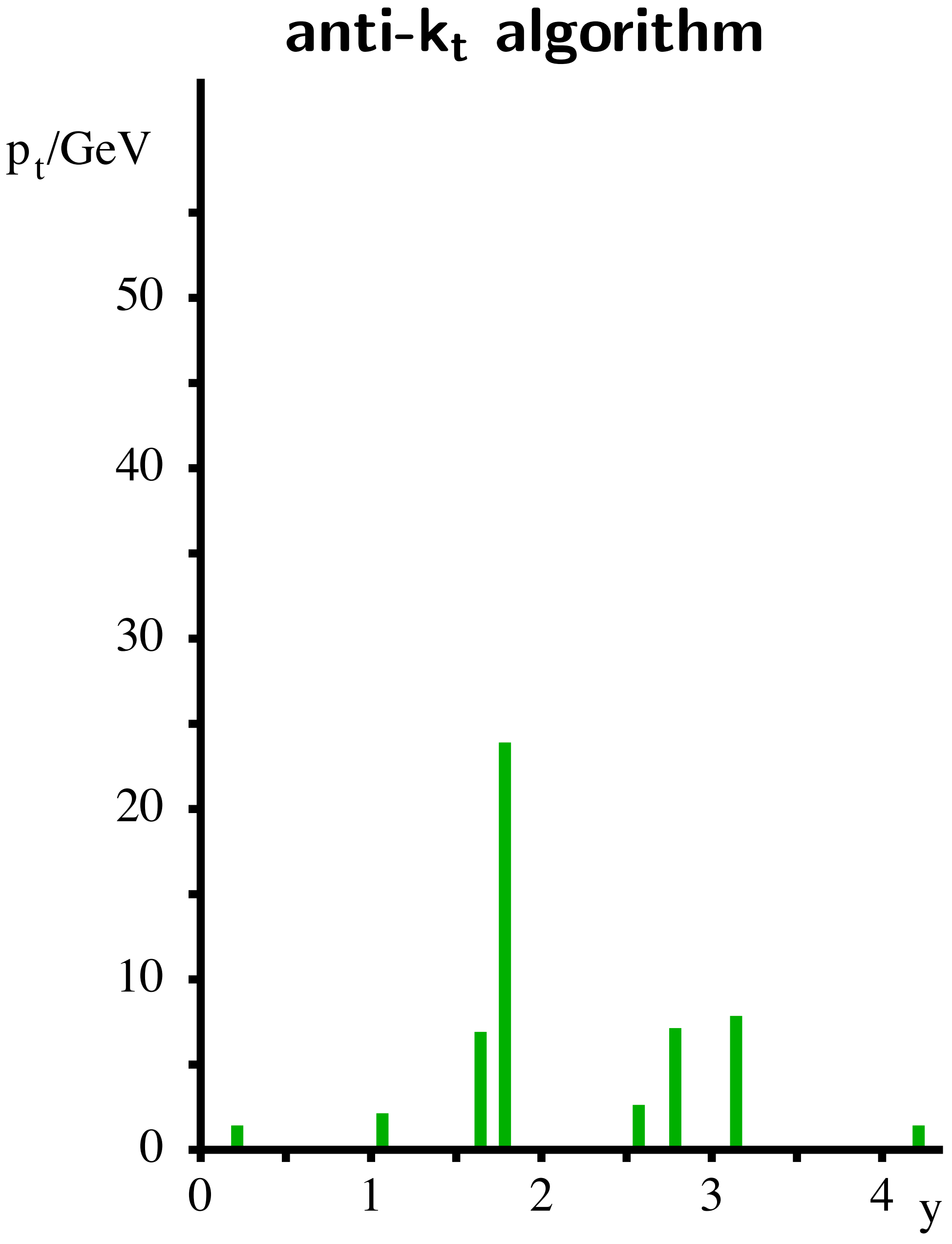
Order of clustering here is 1,2,3,4

The **clustering sequence** is 4-5 (1), 2-3 (2), 23-45 (3), 1-2345 (4)

First try

anti-kt

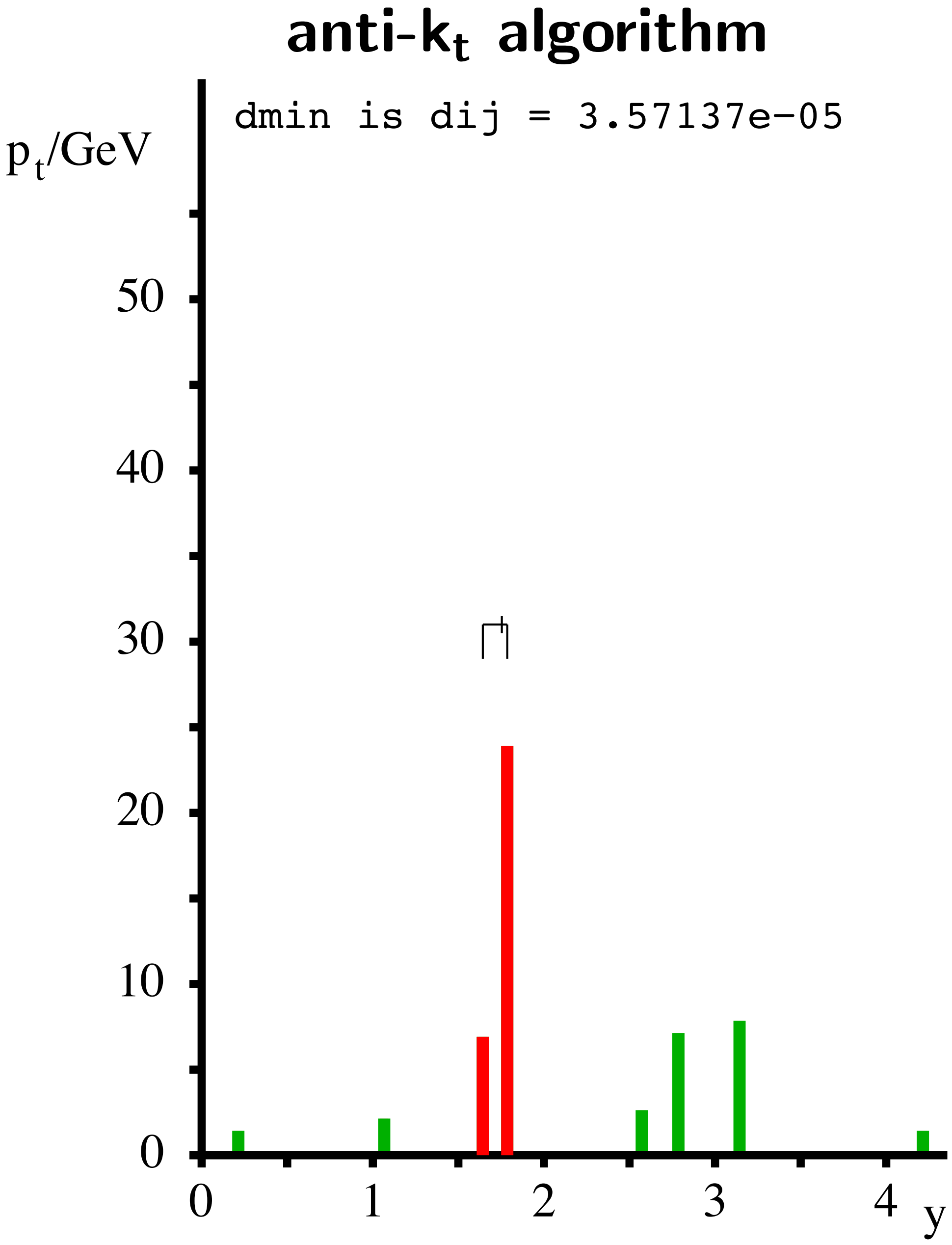
Identifying jet substructure: try out anti-k_t



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

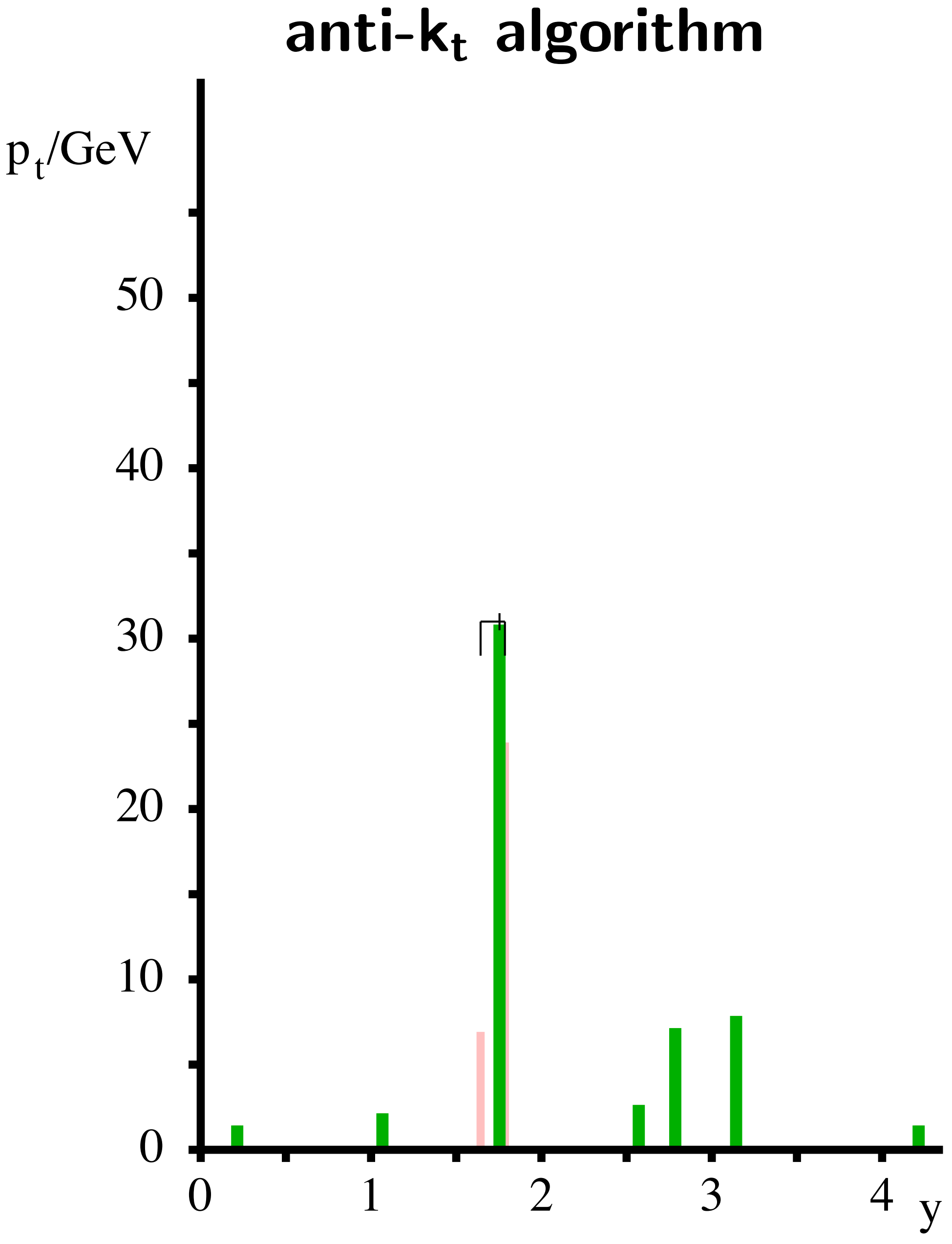
Identifying jet substructure: try out anti-k_t



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

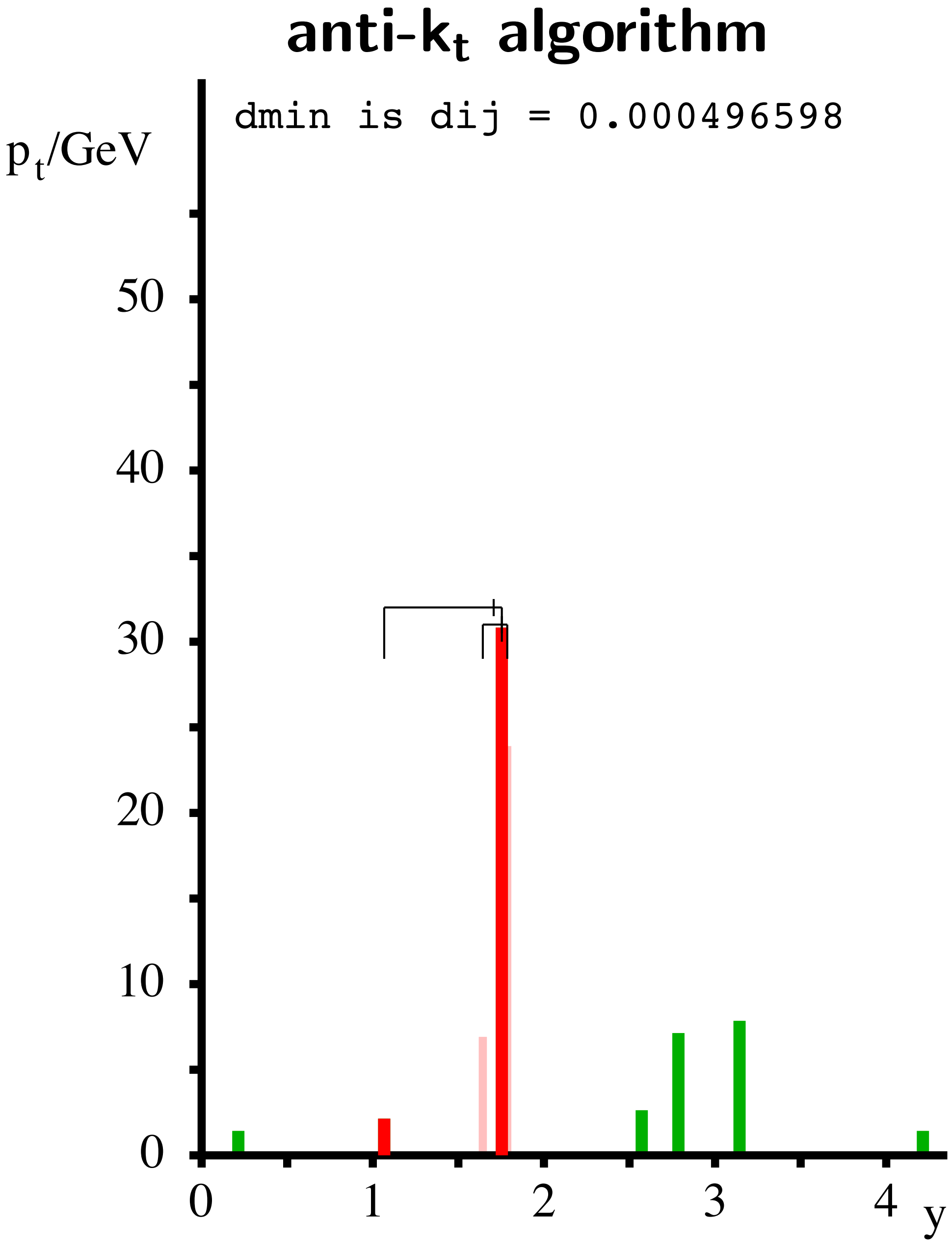
Identifying jet substructure: try out anti-k_t



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

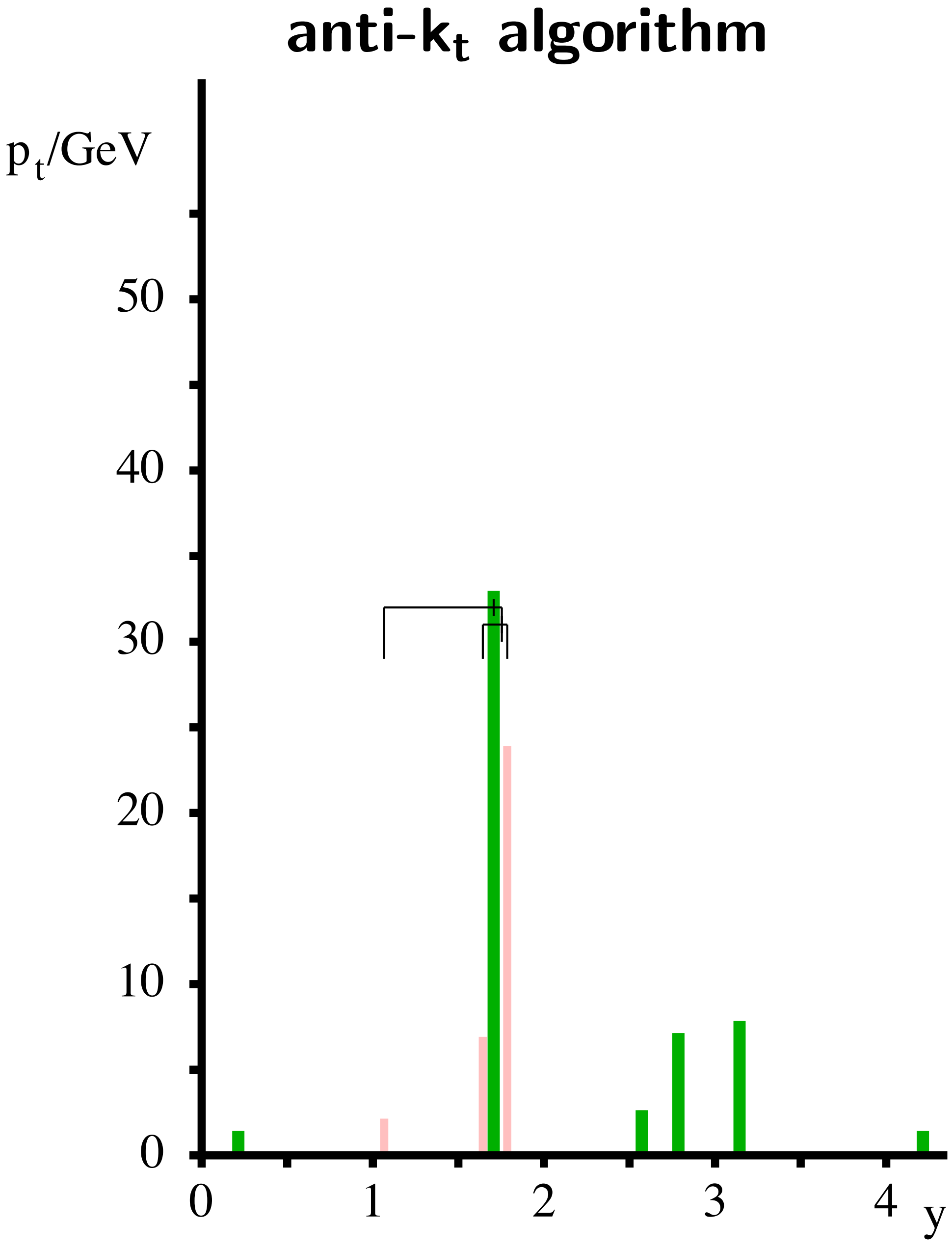
Identifying jet substructure: try out anti-k_t



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

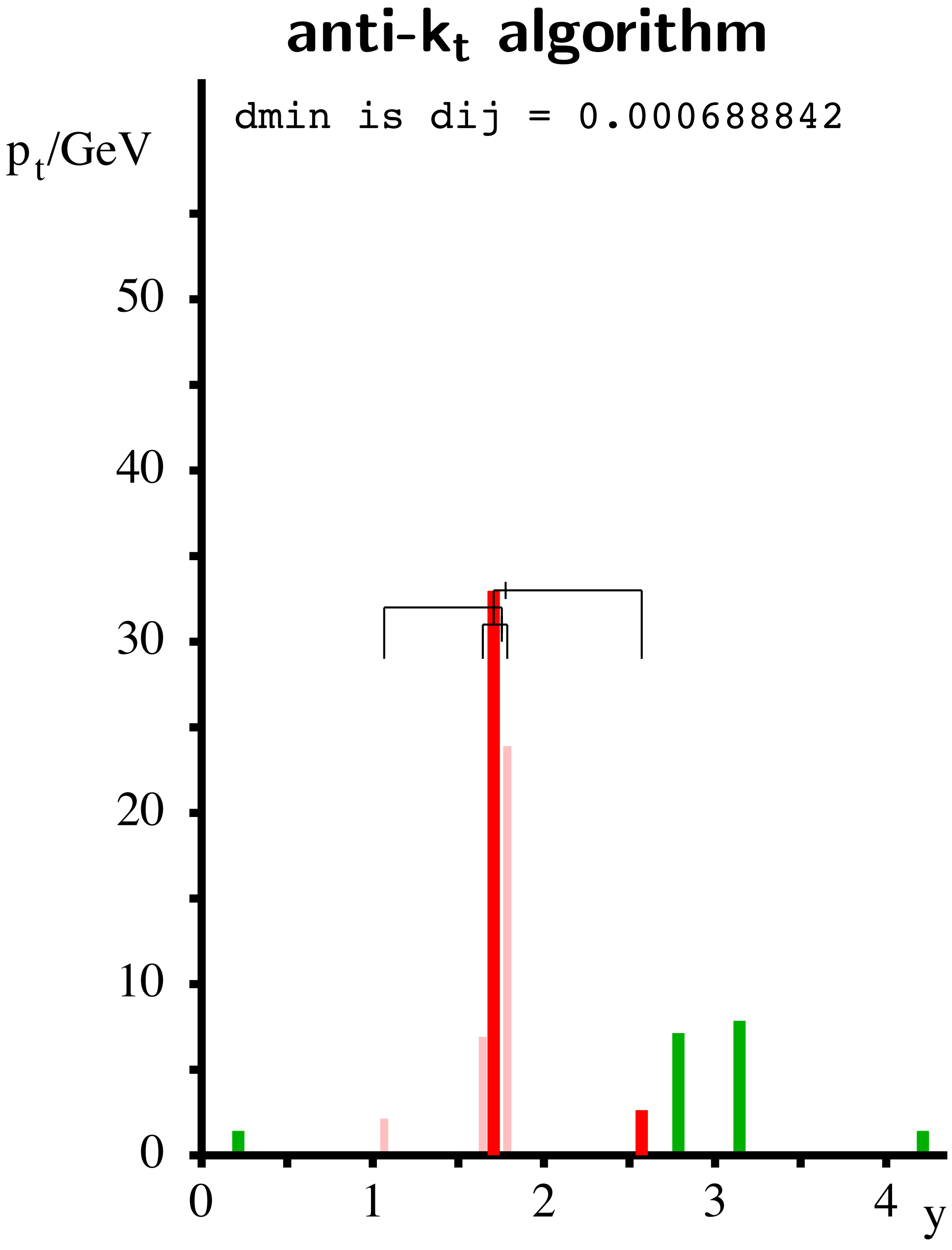
Identifying jet substructure: try out anti-k_t



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

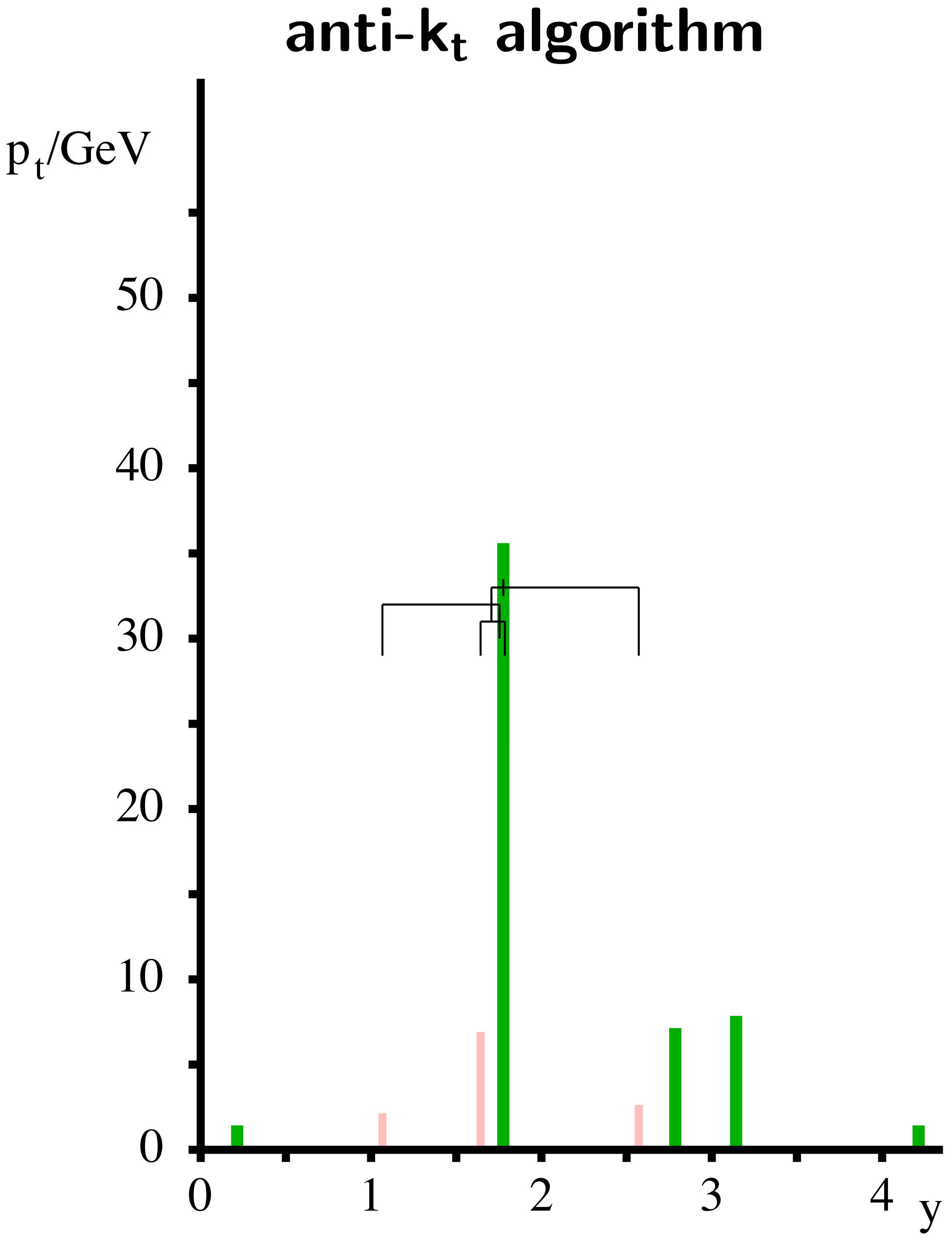
Identifying jet substructure: try out anti-k_t



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

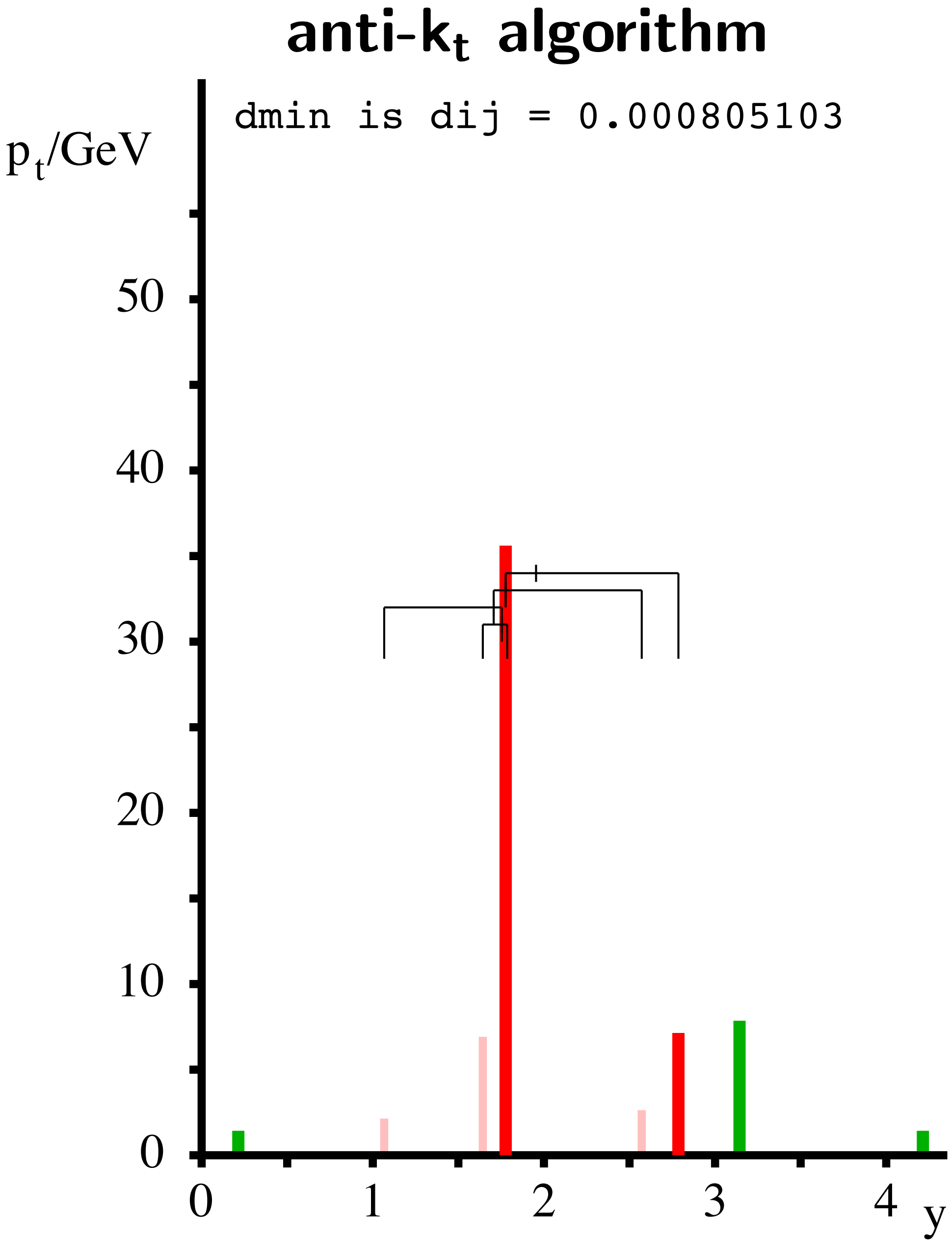
Identifying jet substructure: try out anti-k_t



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Identifying jet substructure: try out anti-k_t

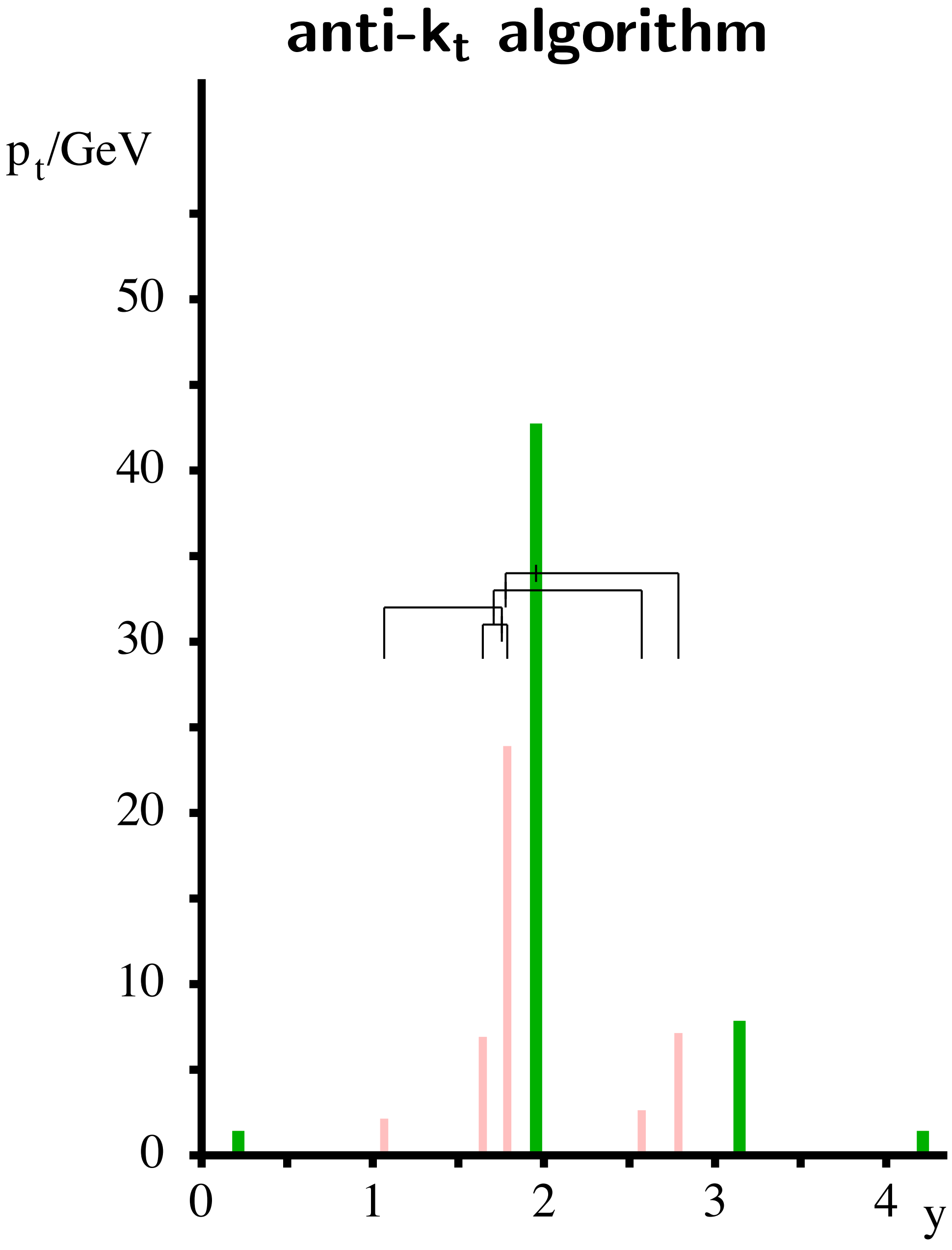


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t

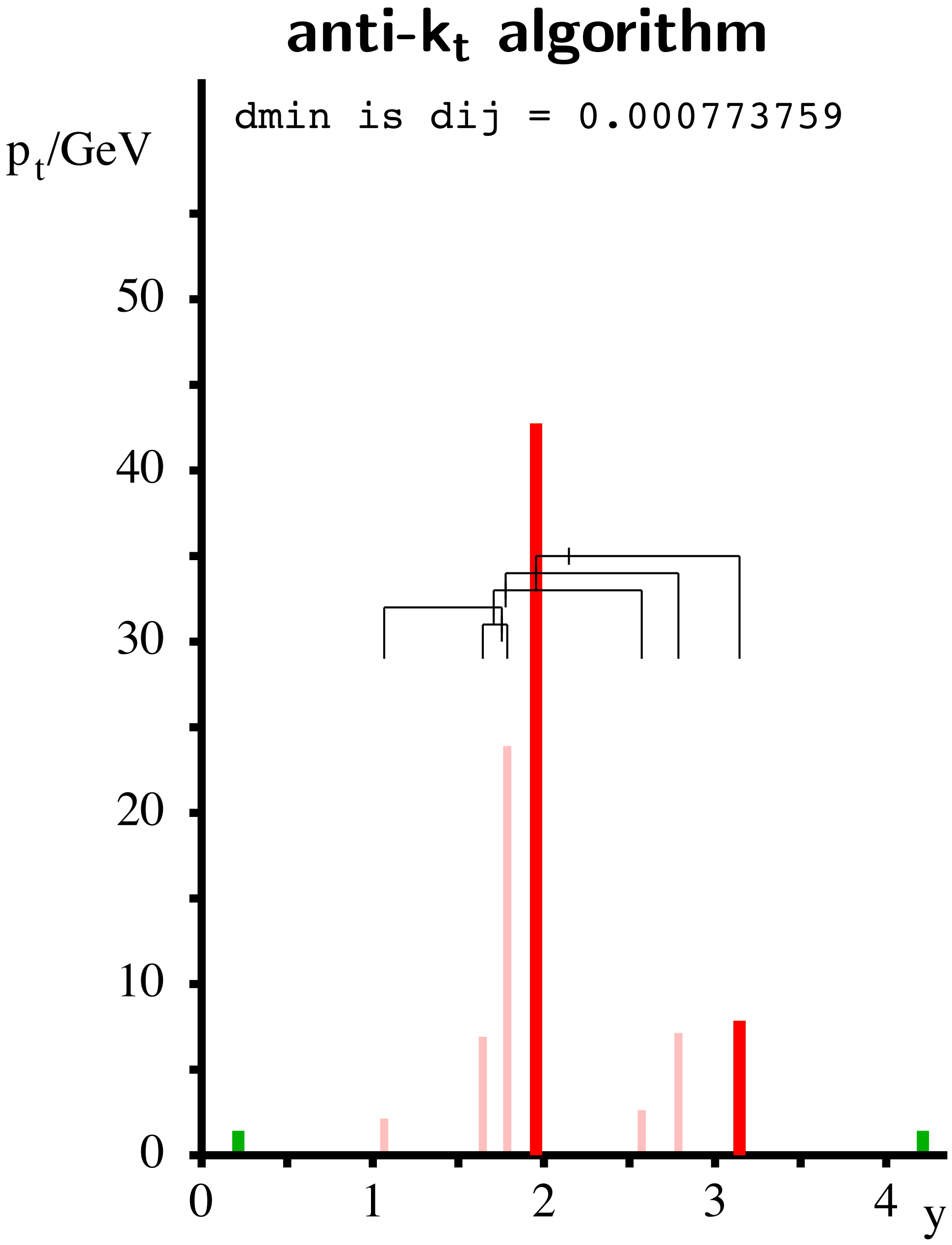


How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t

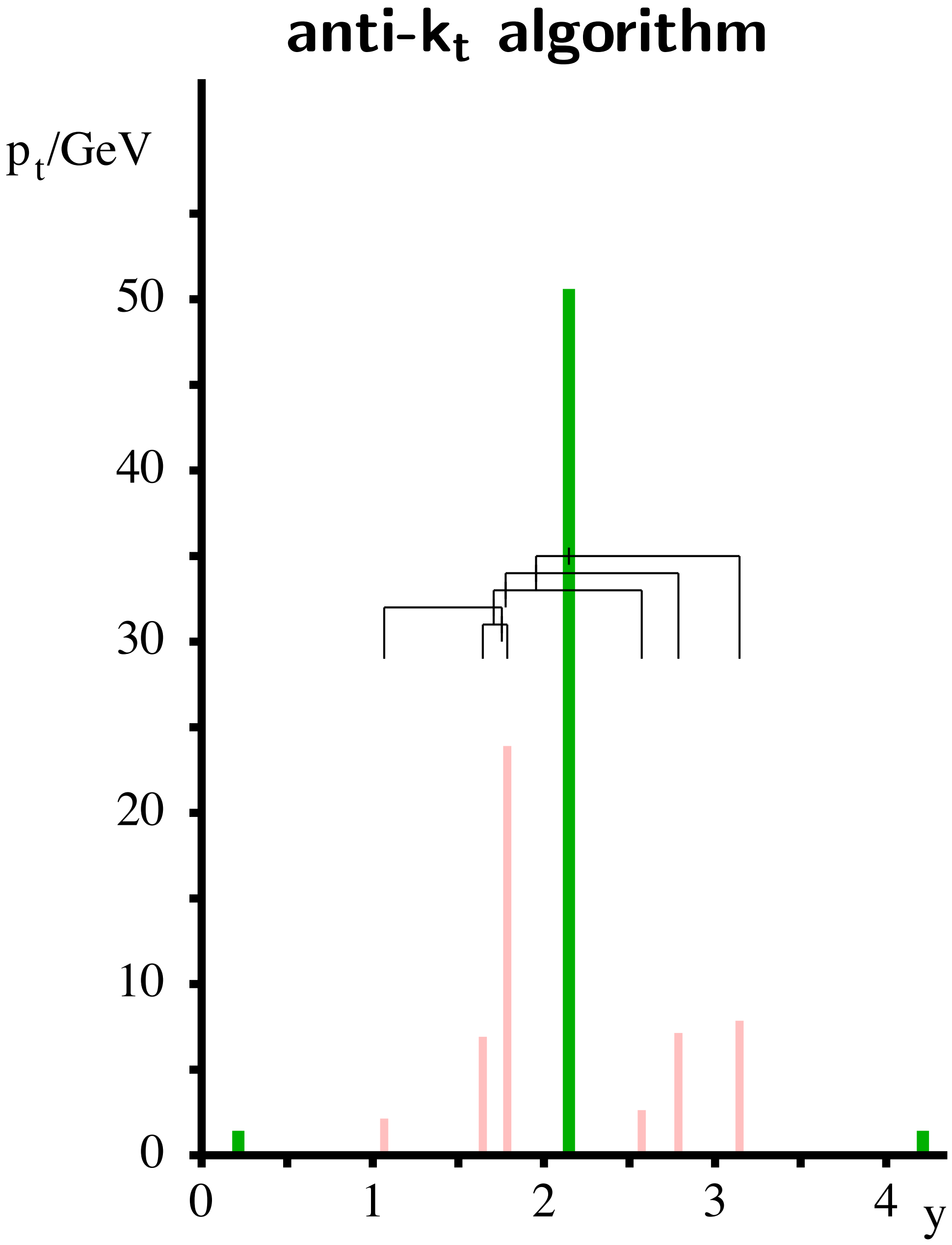


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t

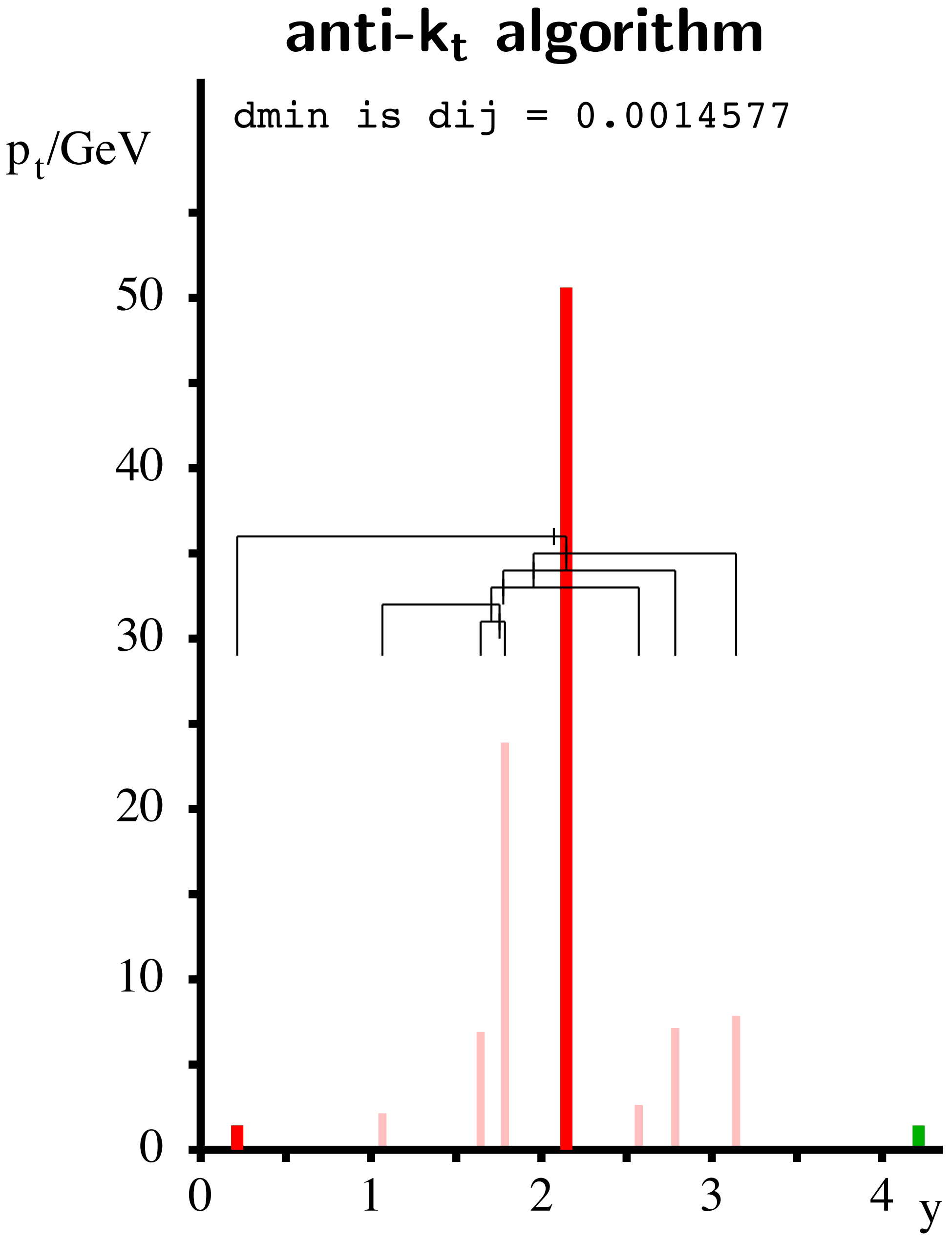


How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t

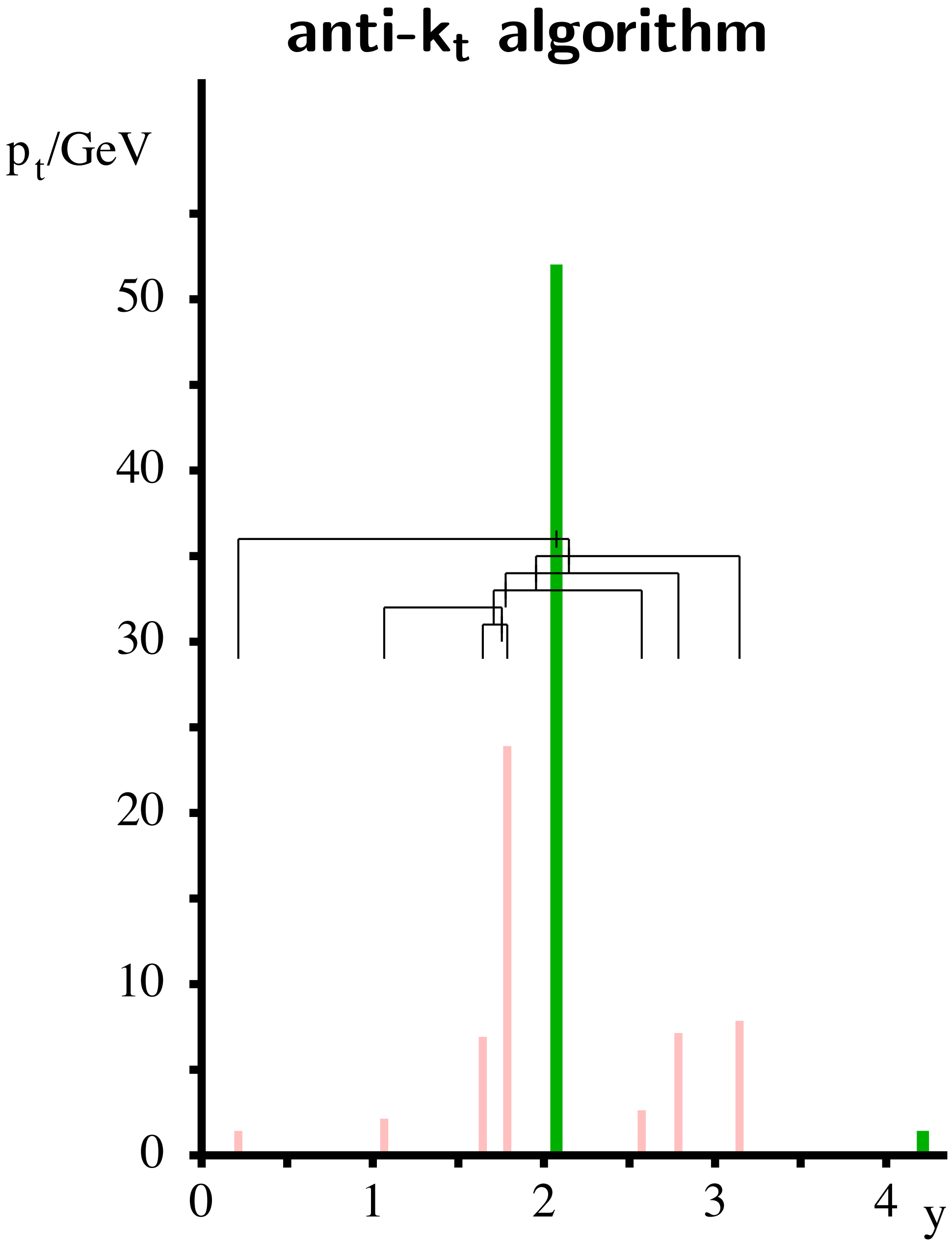


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t

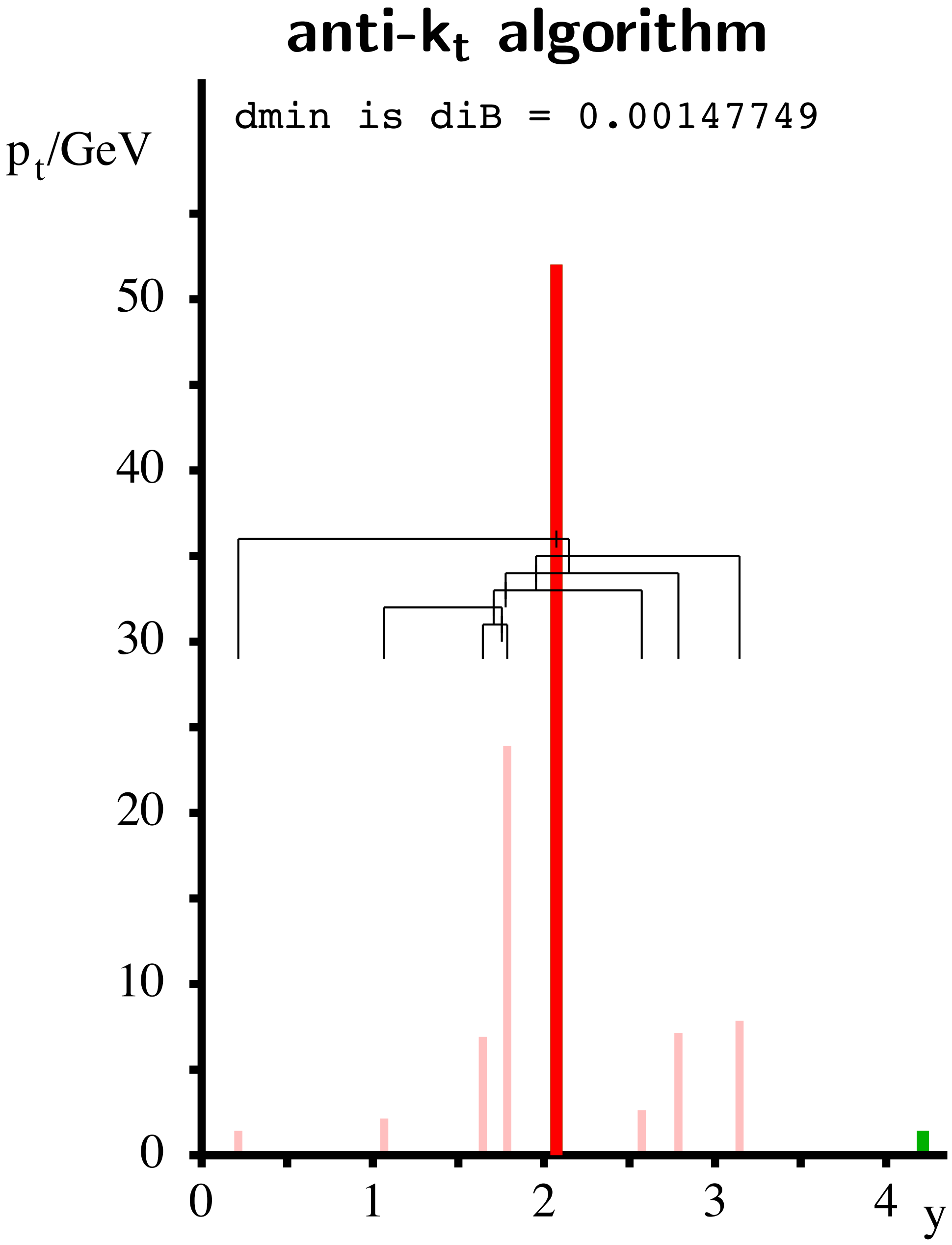


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t

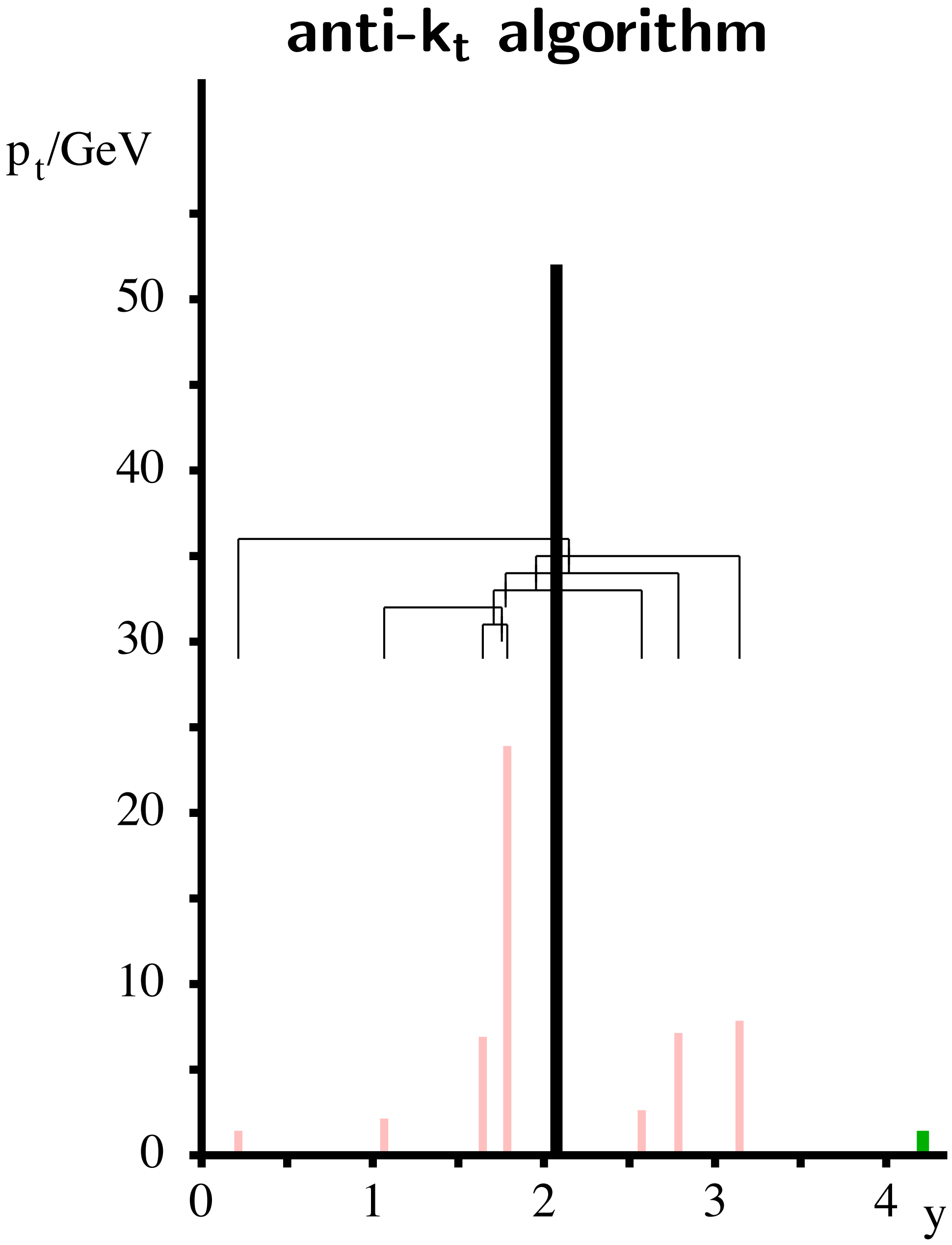


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t

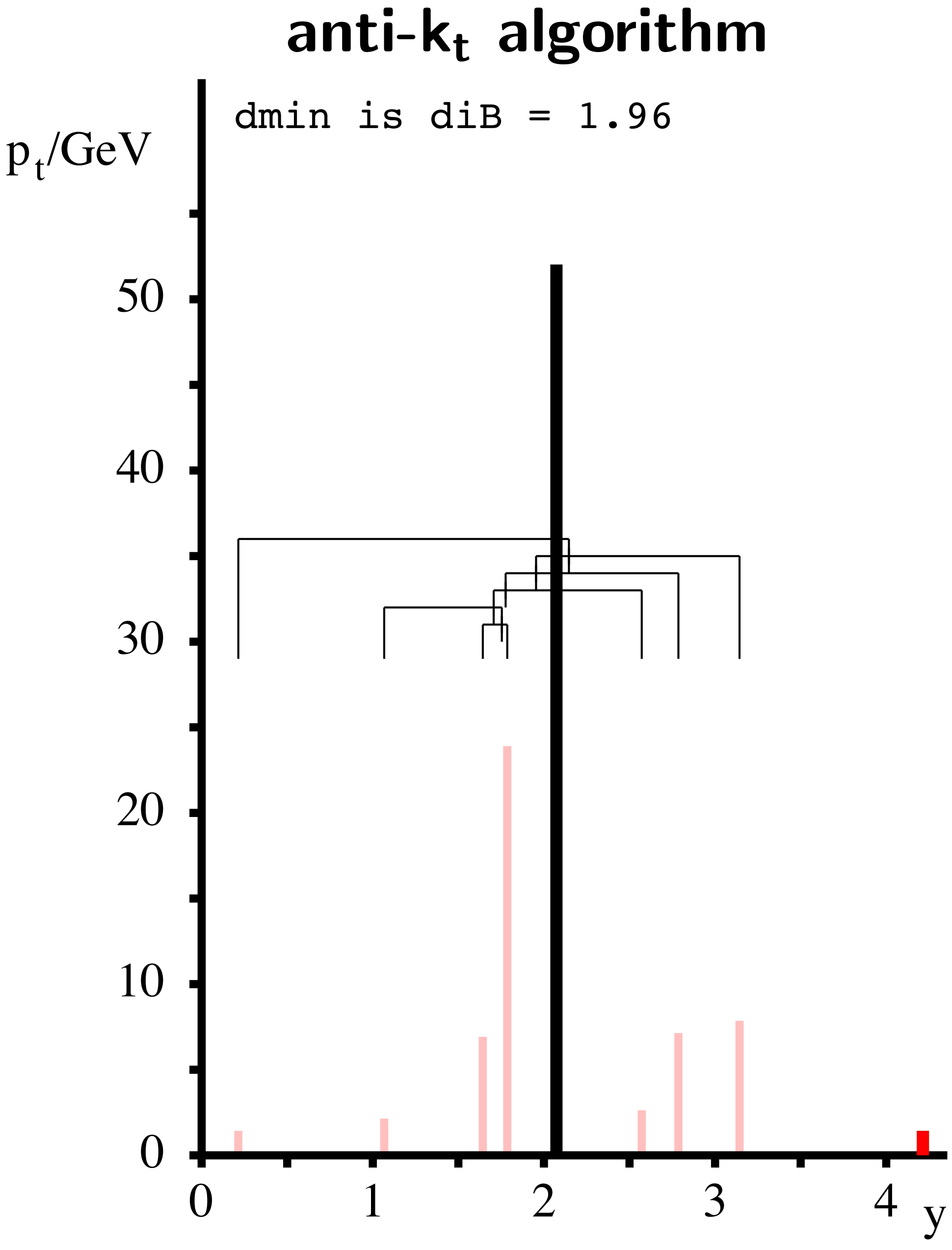


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t

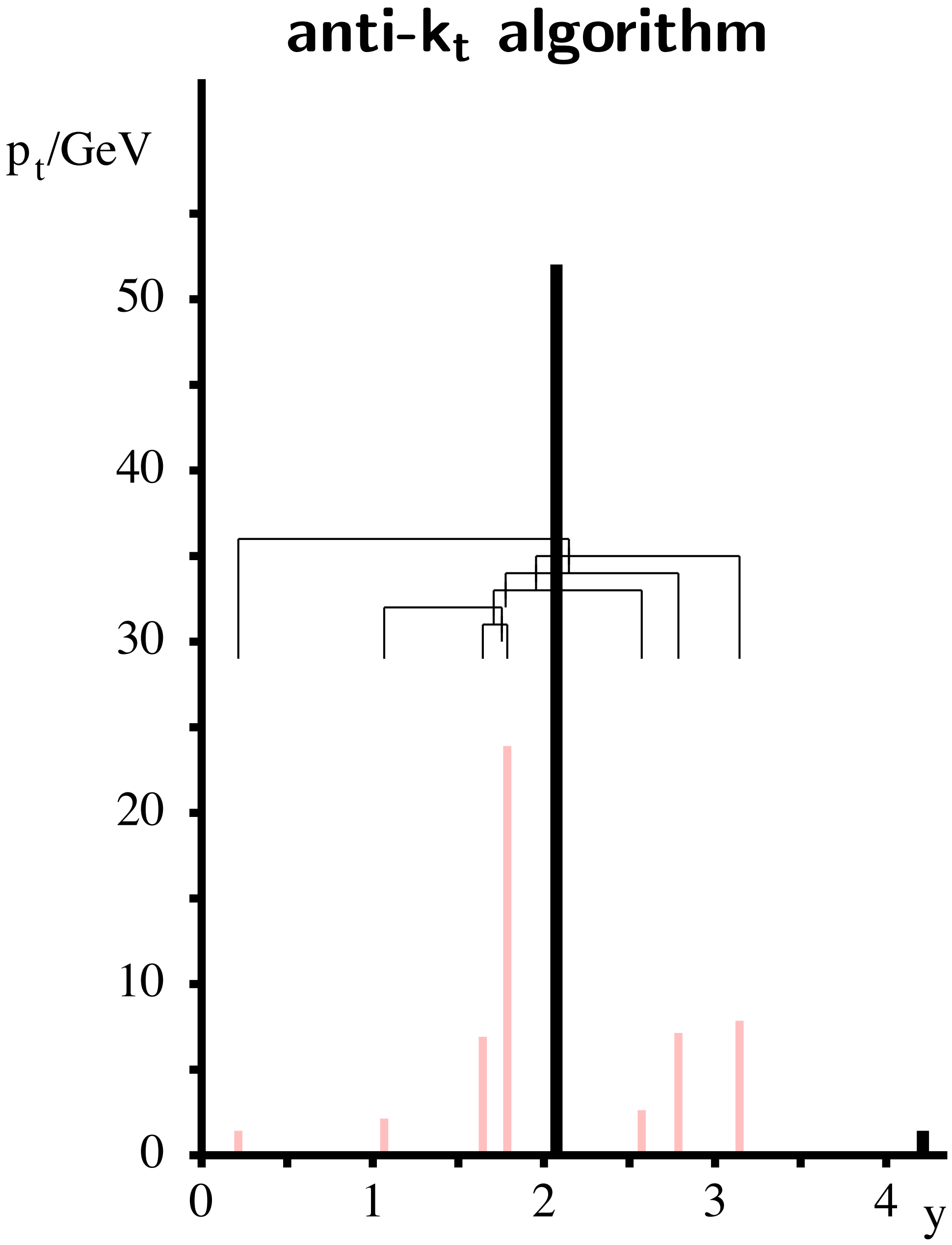


How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Identifying jet substructure: try out anti-k_t



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

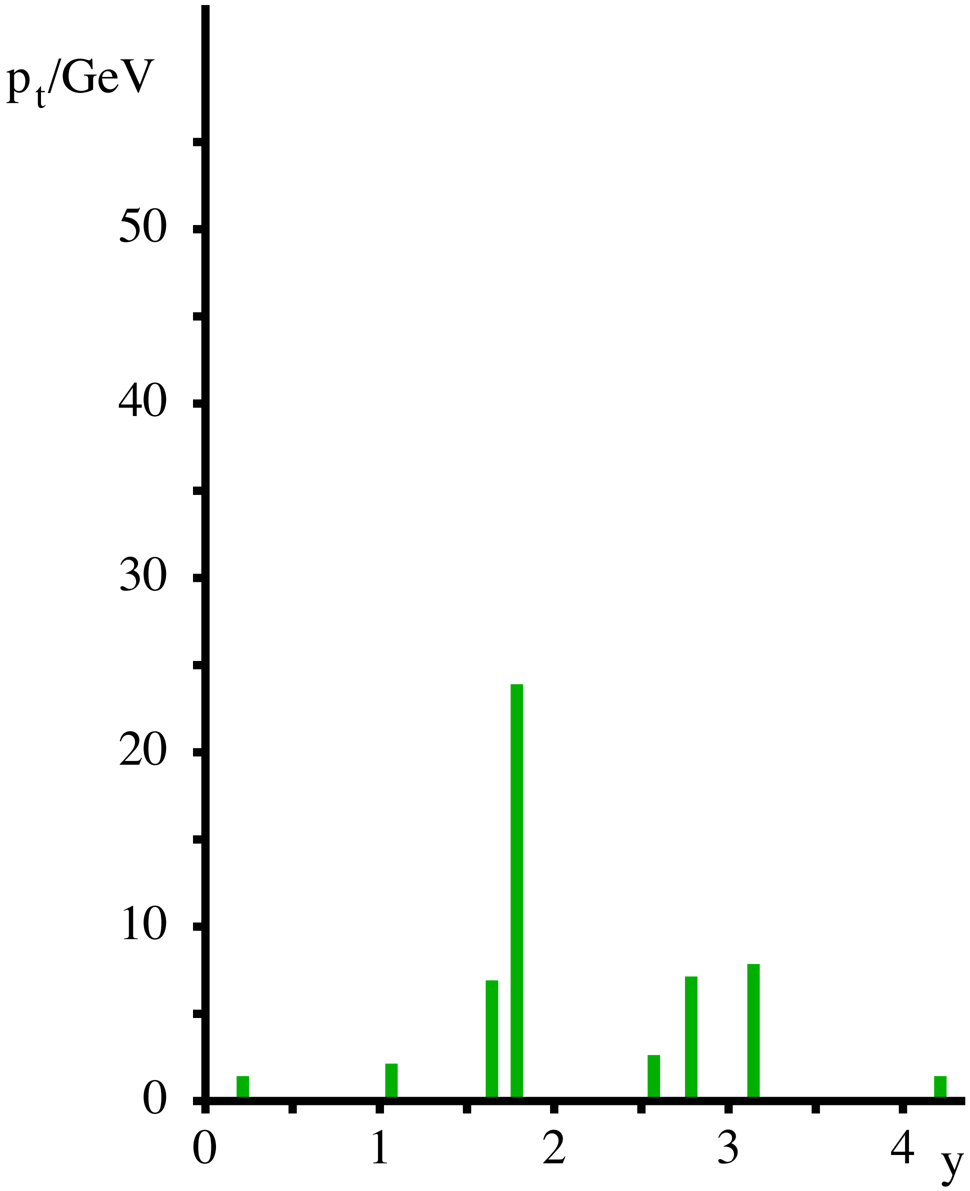
Anti-k_t gradually makes its way through the secondary blob → no clear identification of substructure associated with 2nd parton.

Second try

kt

Identifying jet substructure: try out k_t

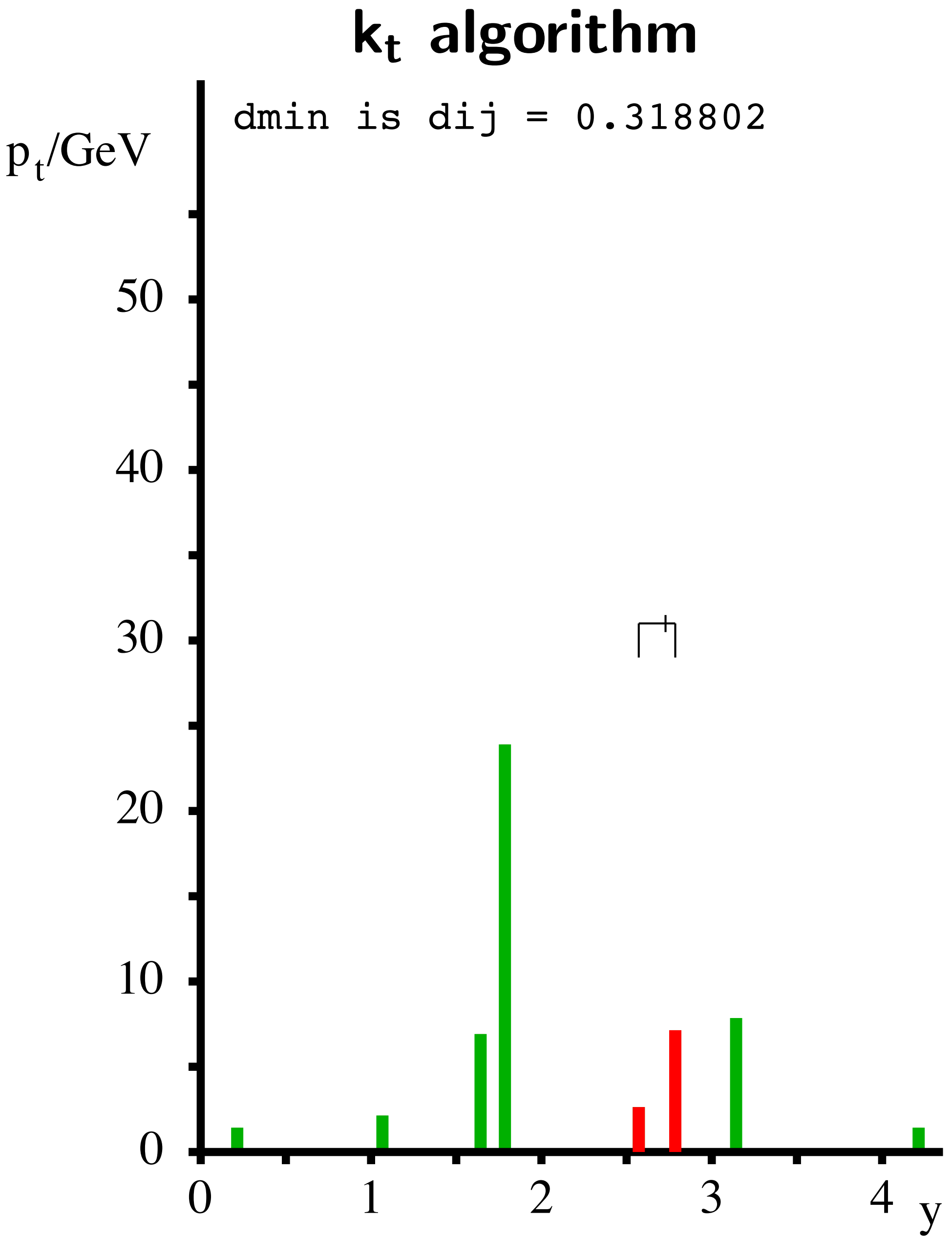
$\mathbf{k_t}$ algorithm



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

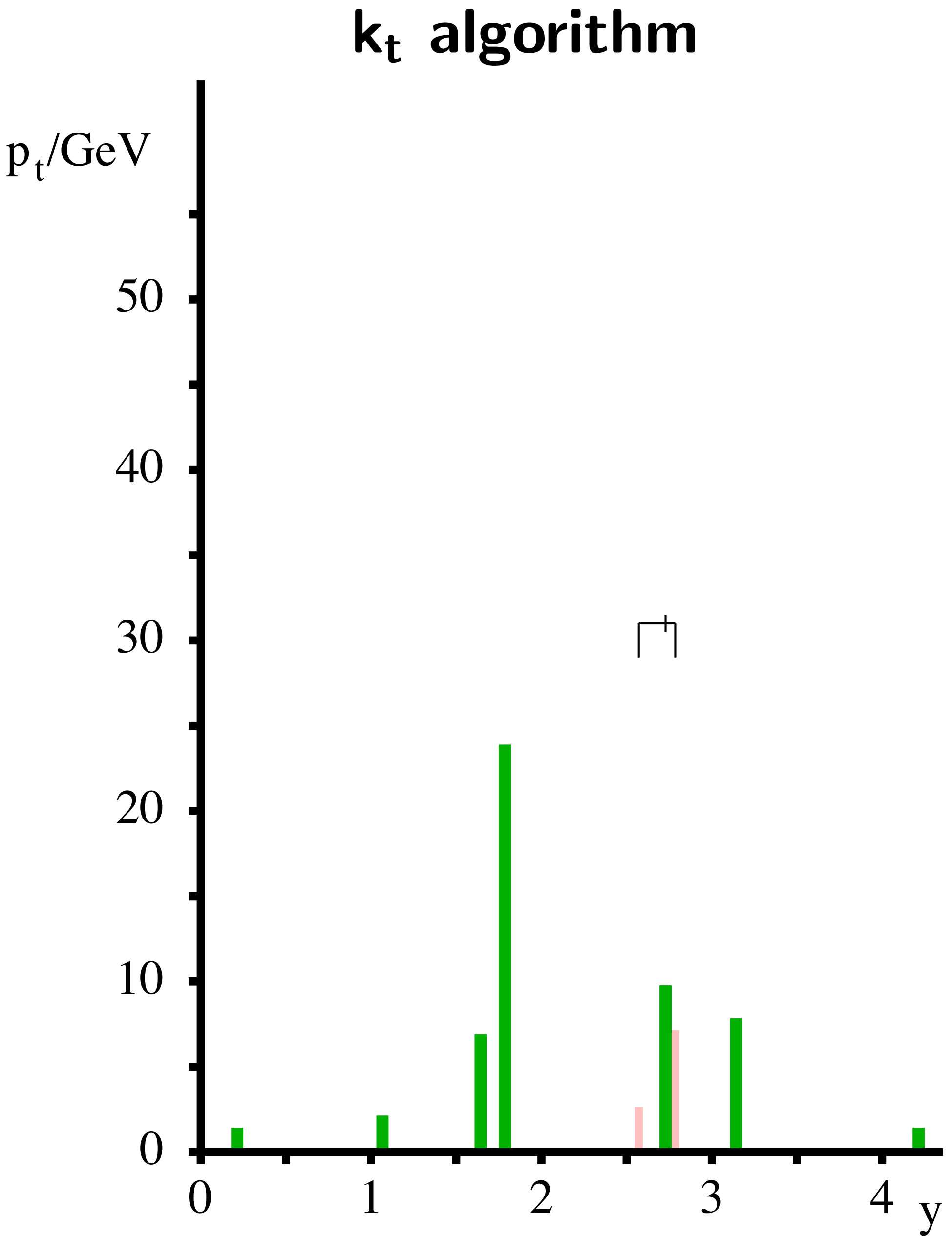
Identifying jet substructure: try out k_t



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

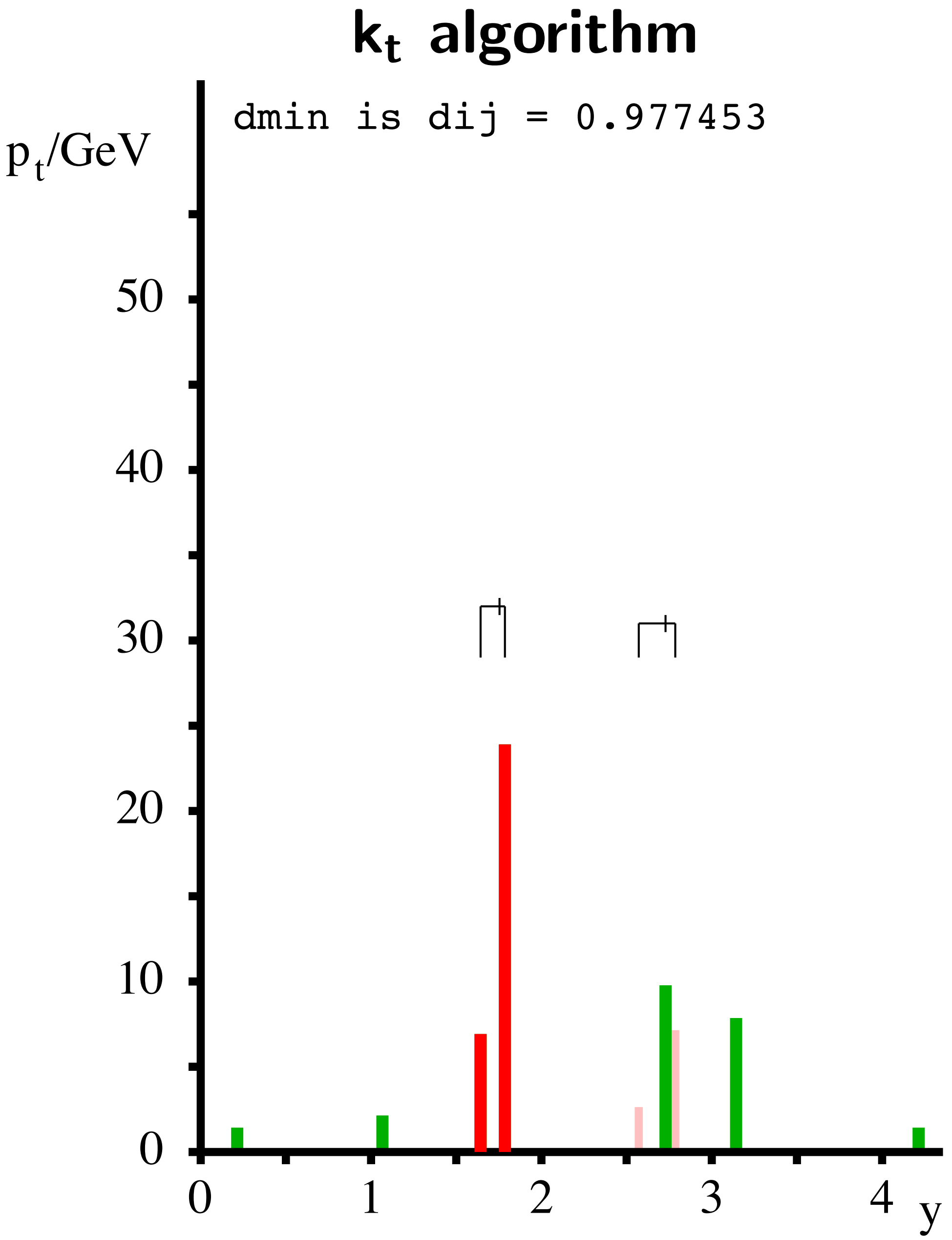
Identifying jet substructure: try out k_t



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

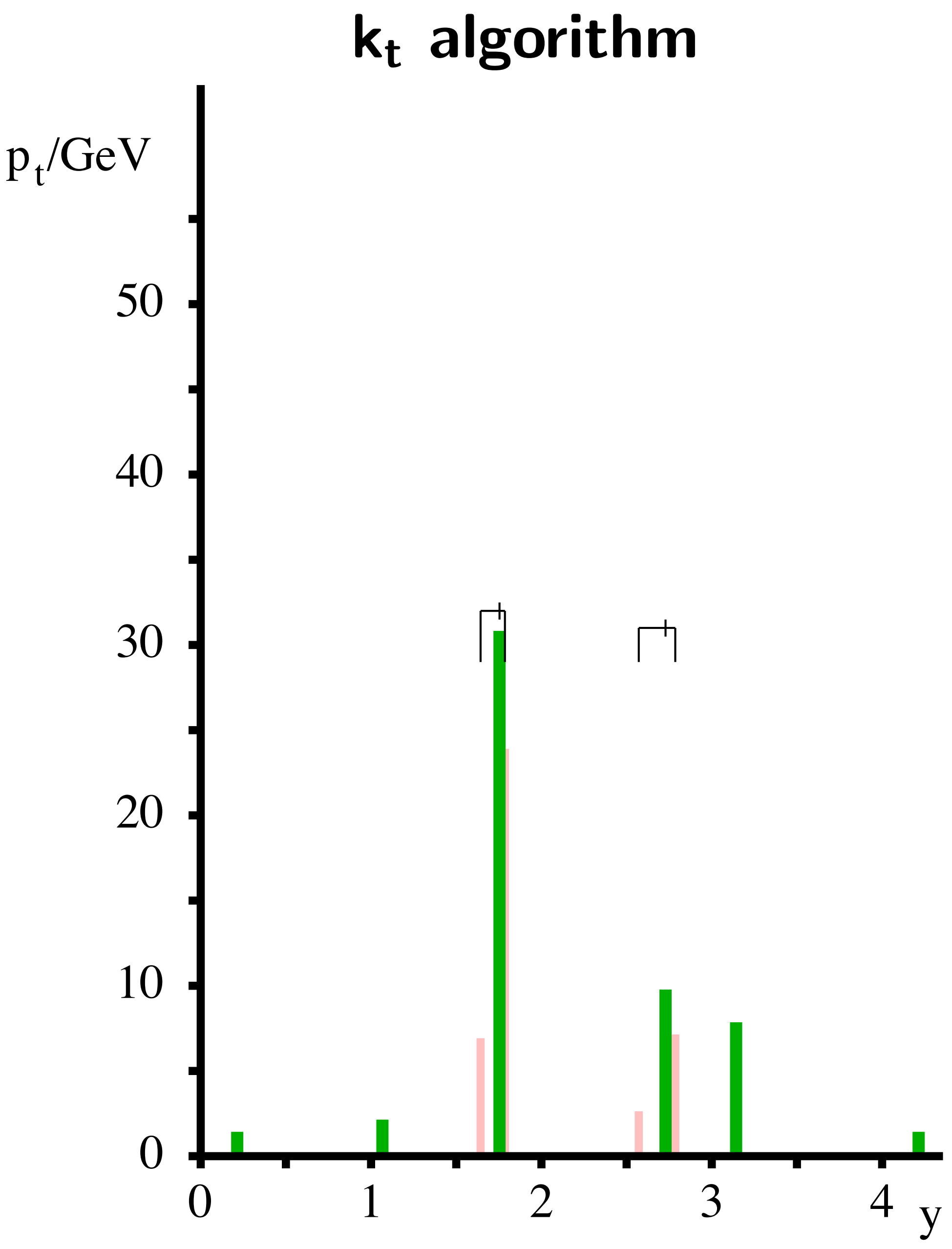
Identifying jet substructure: try out k_t



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

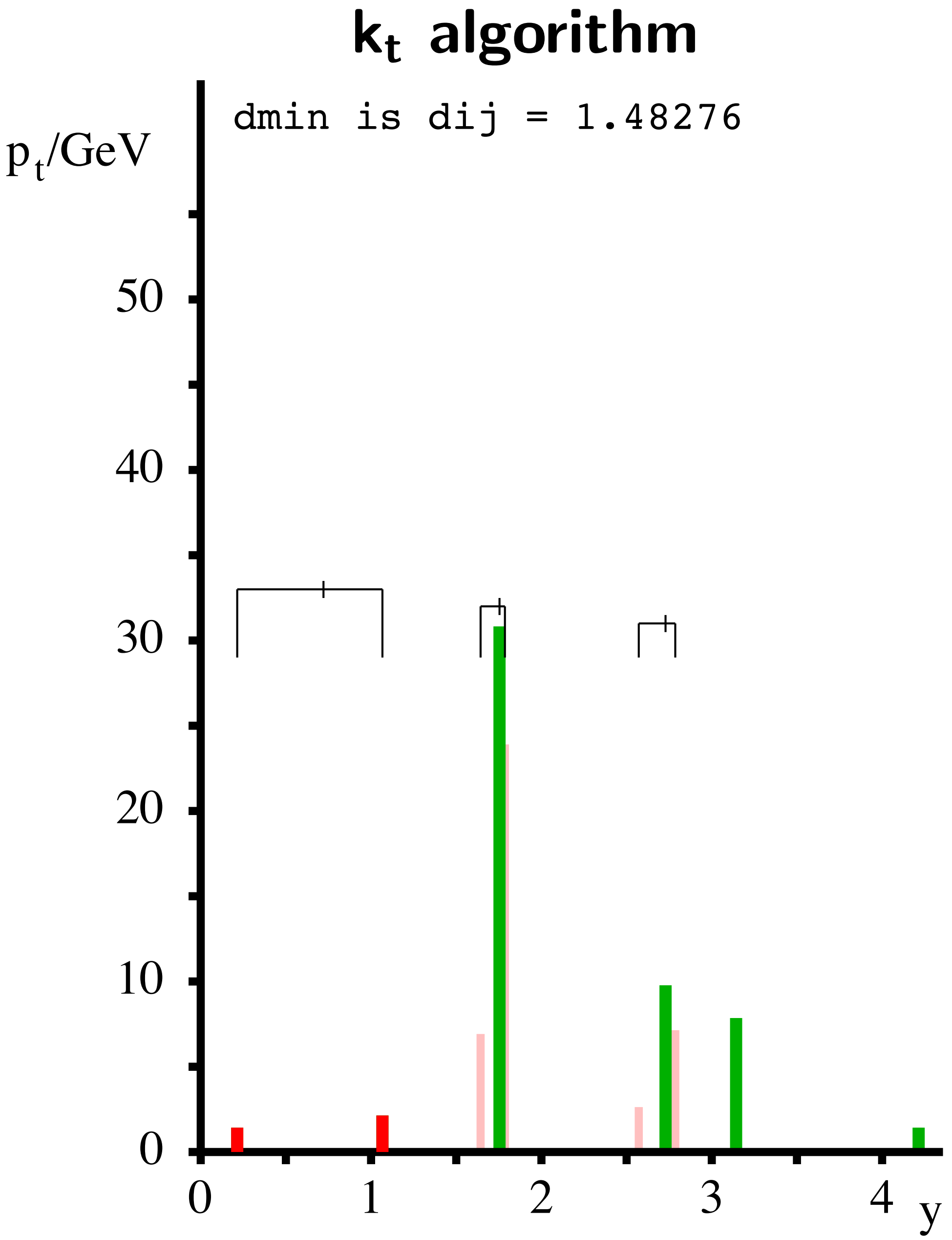
Identifying jet substructure: try out k_t



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

Identifying jet substructure: try out k_t

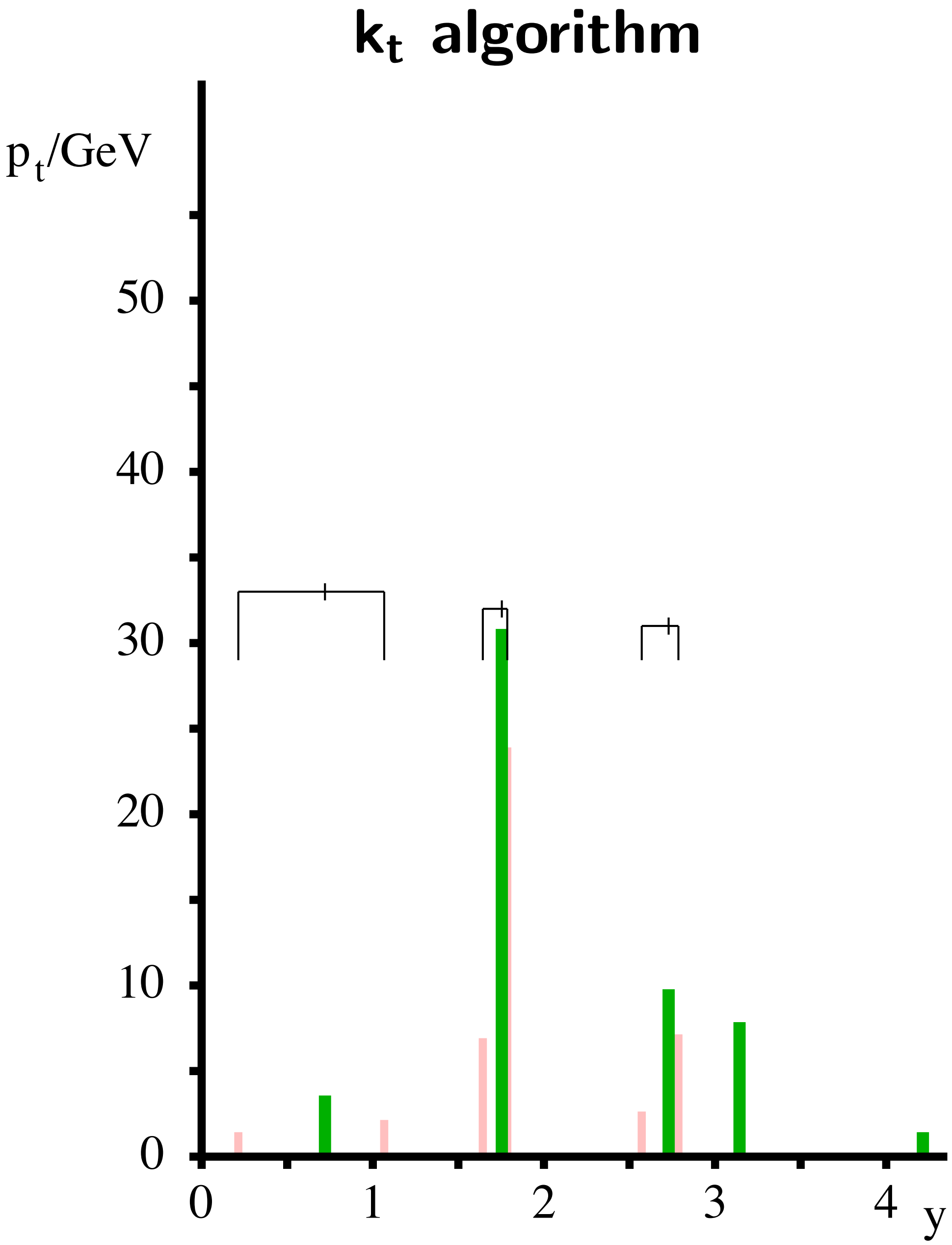


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Identifying jet substructure: try out k_t

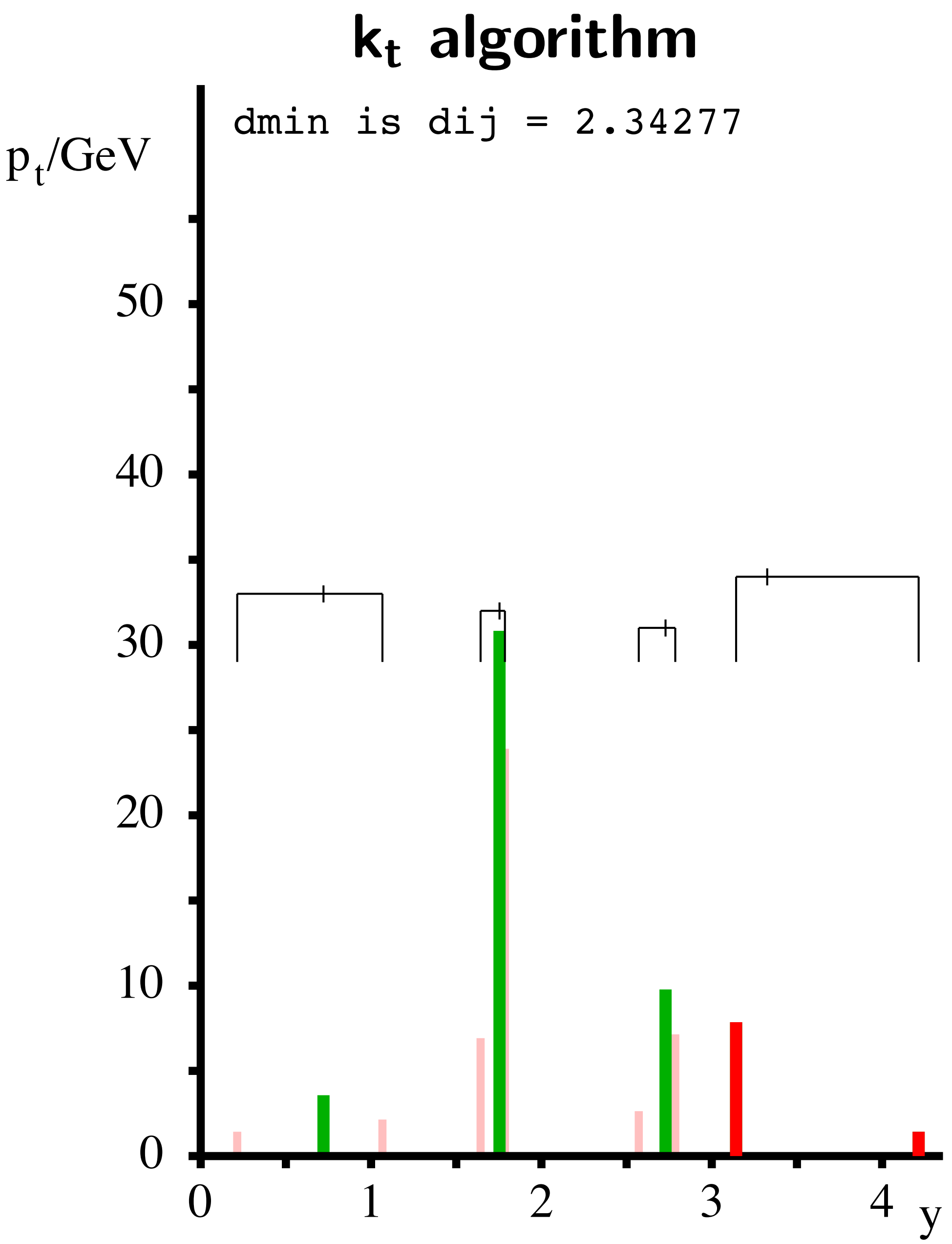


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Identifying jet substructure: try out k_t

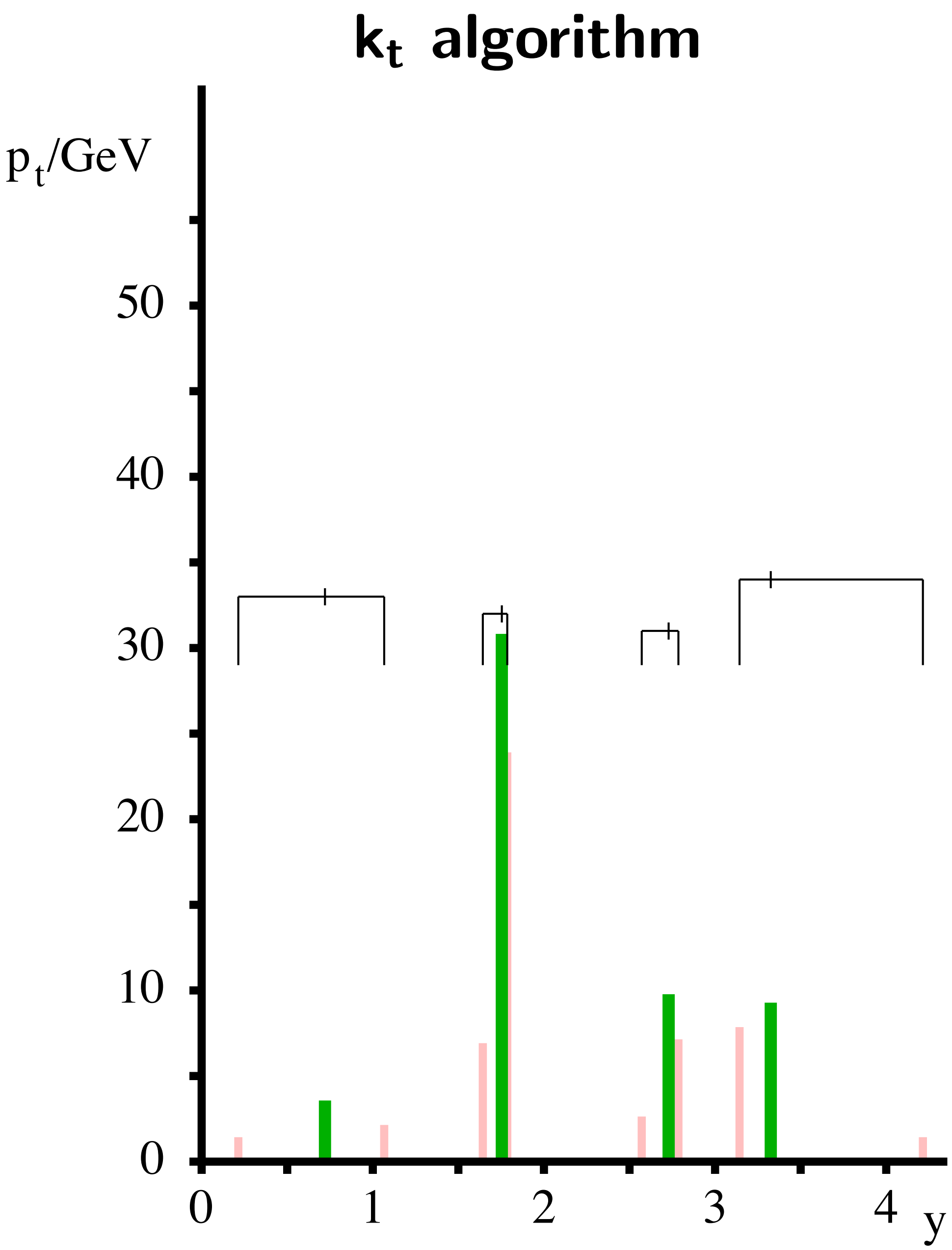


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Identifying jet substructure: try out k_t

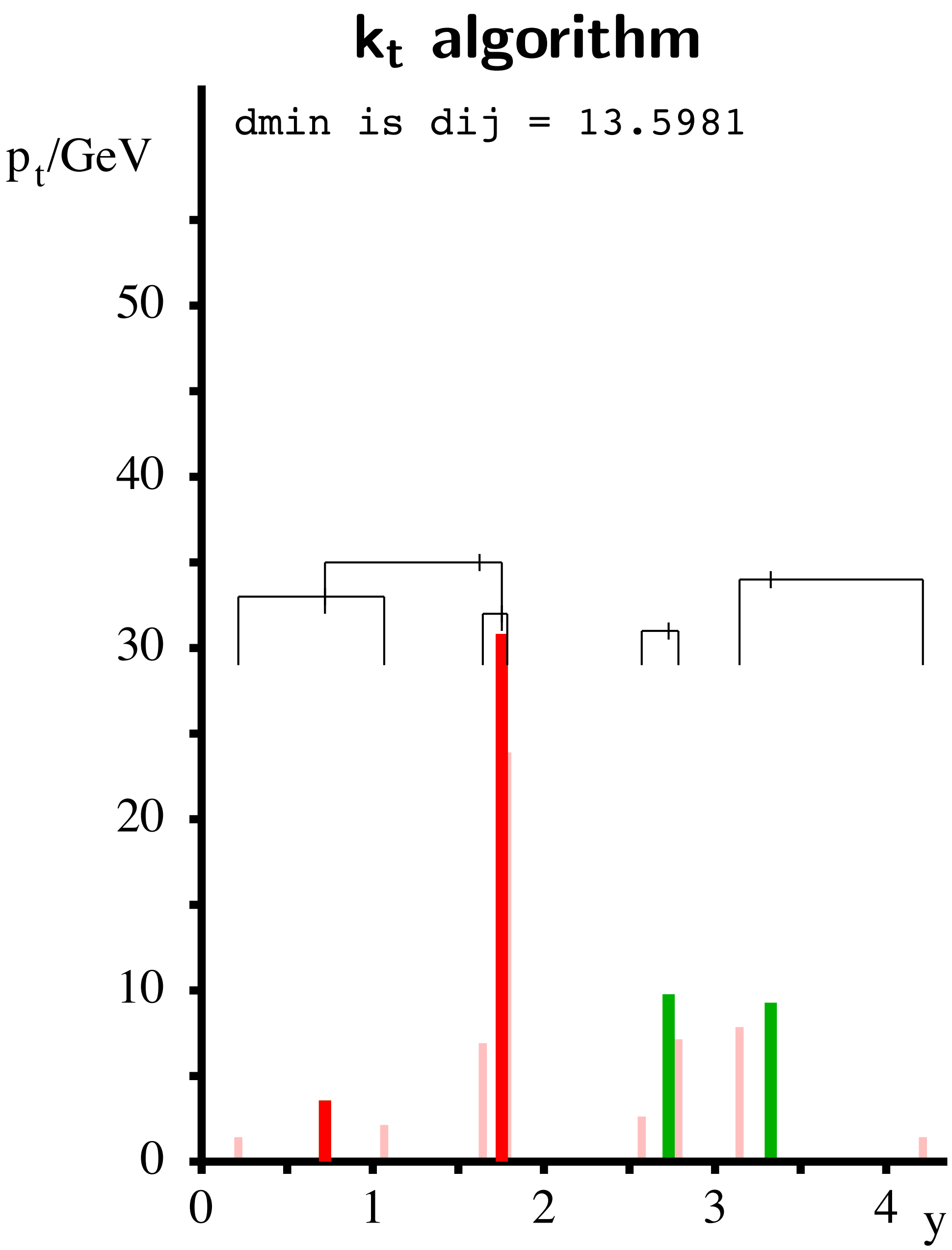


How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft “junk” early on in the clustering

Identifying jet substructure: try out k_t

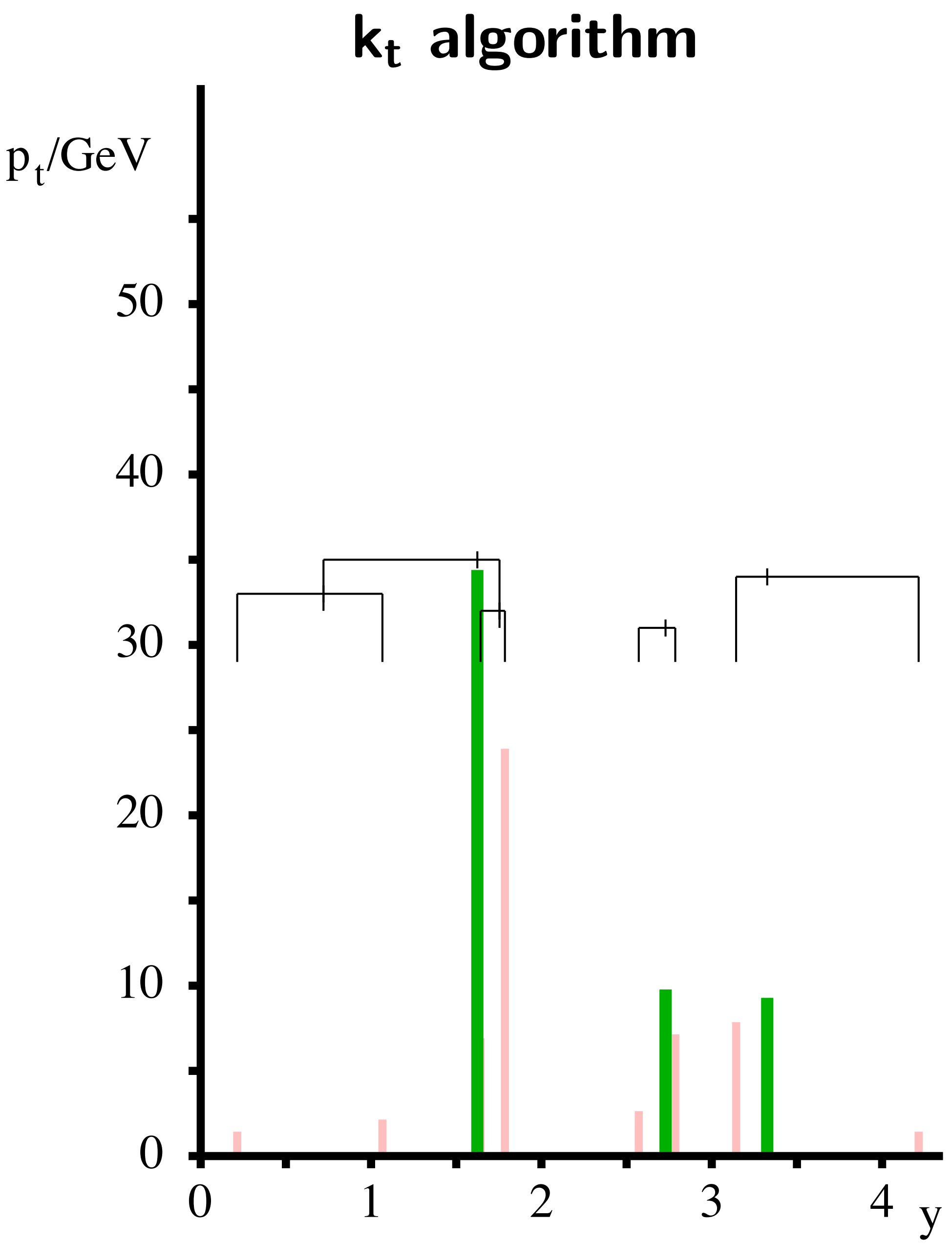


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Identifying jet substructure: try out k_t

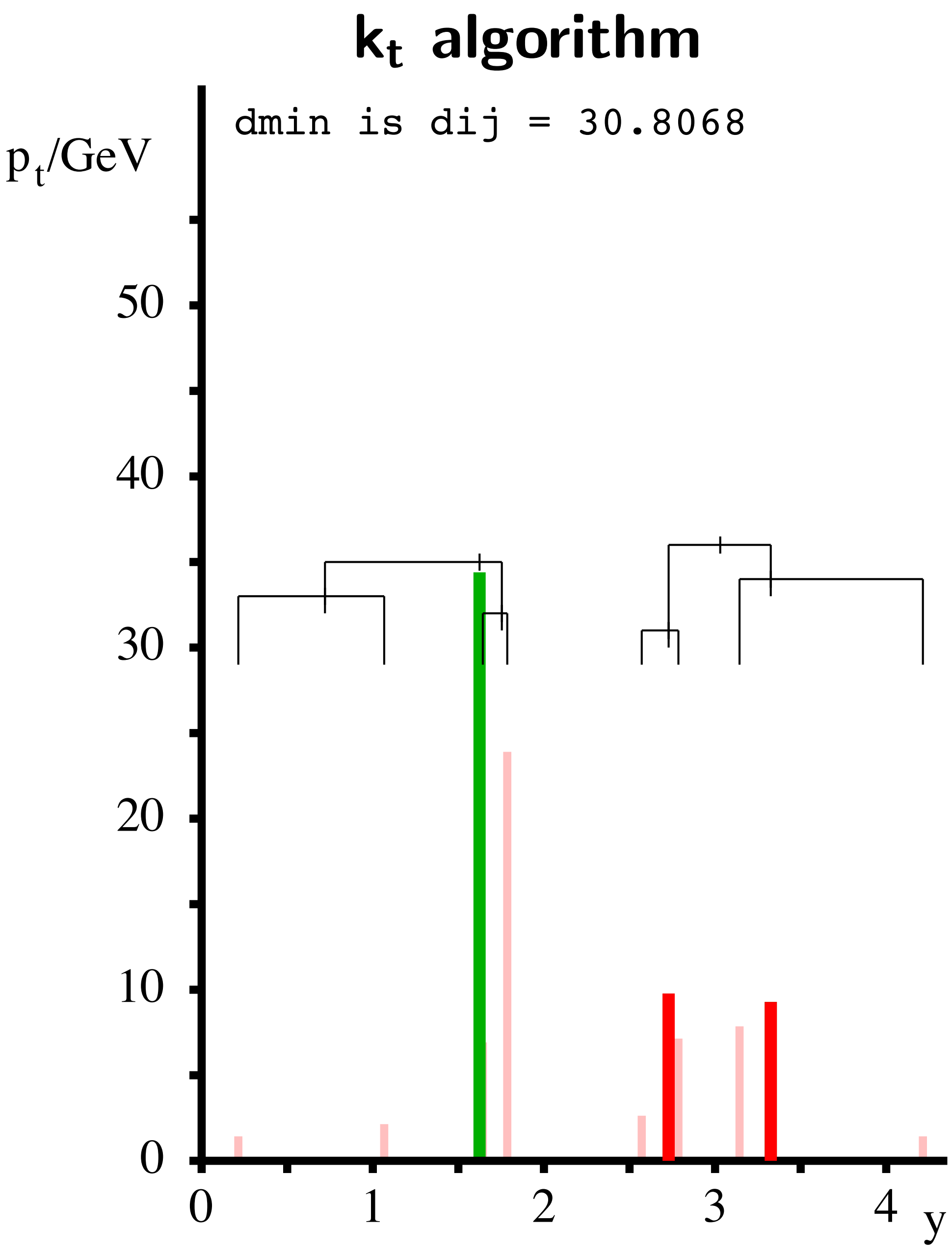


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Identifying jet substructure: try out k_t

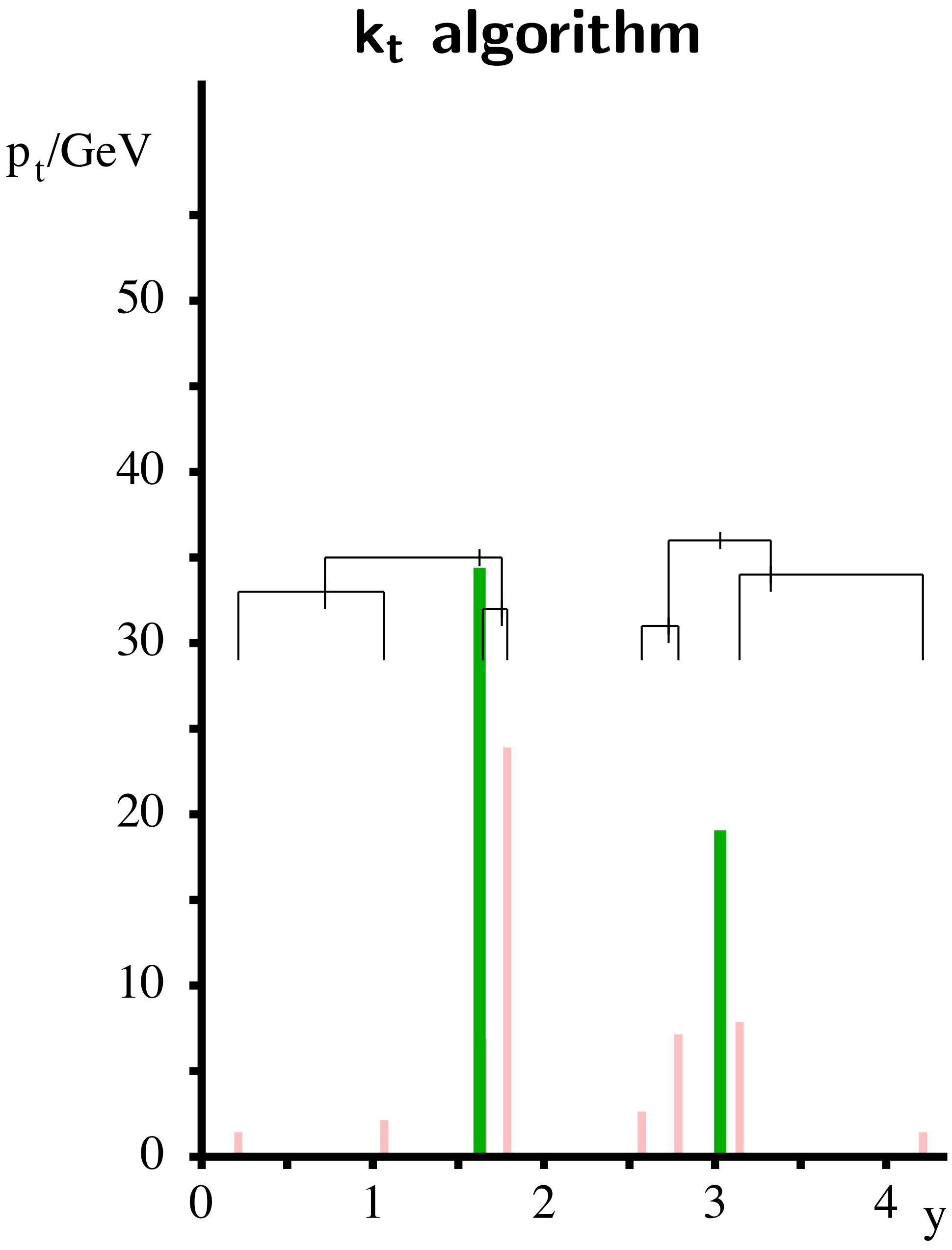


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Identifying jet substructure: try out k_t

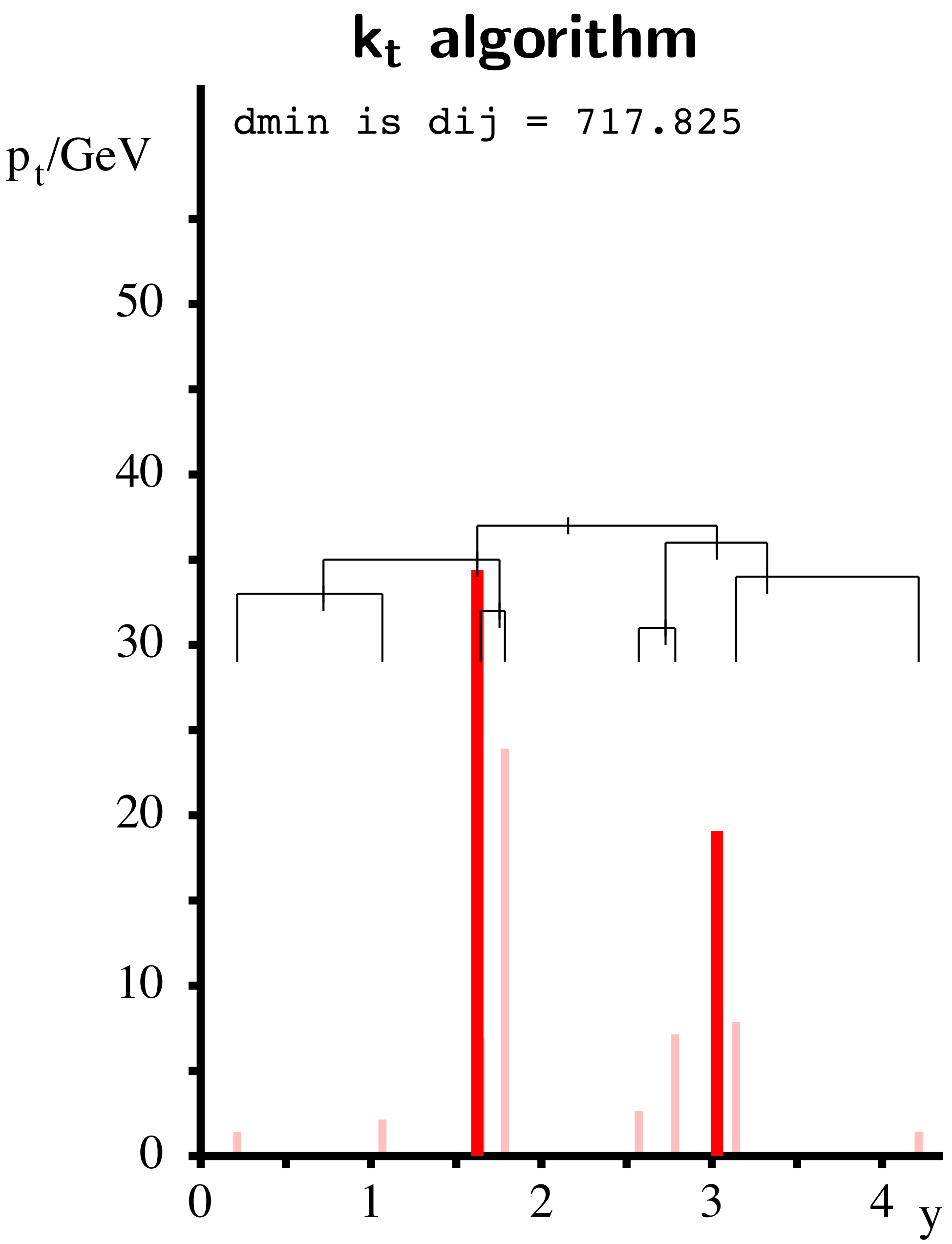


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Identifying jet substructure: try out k_t



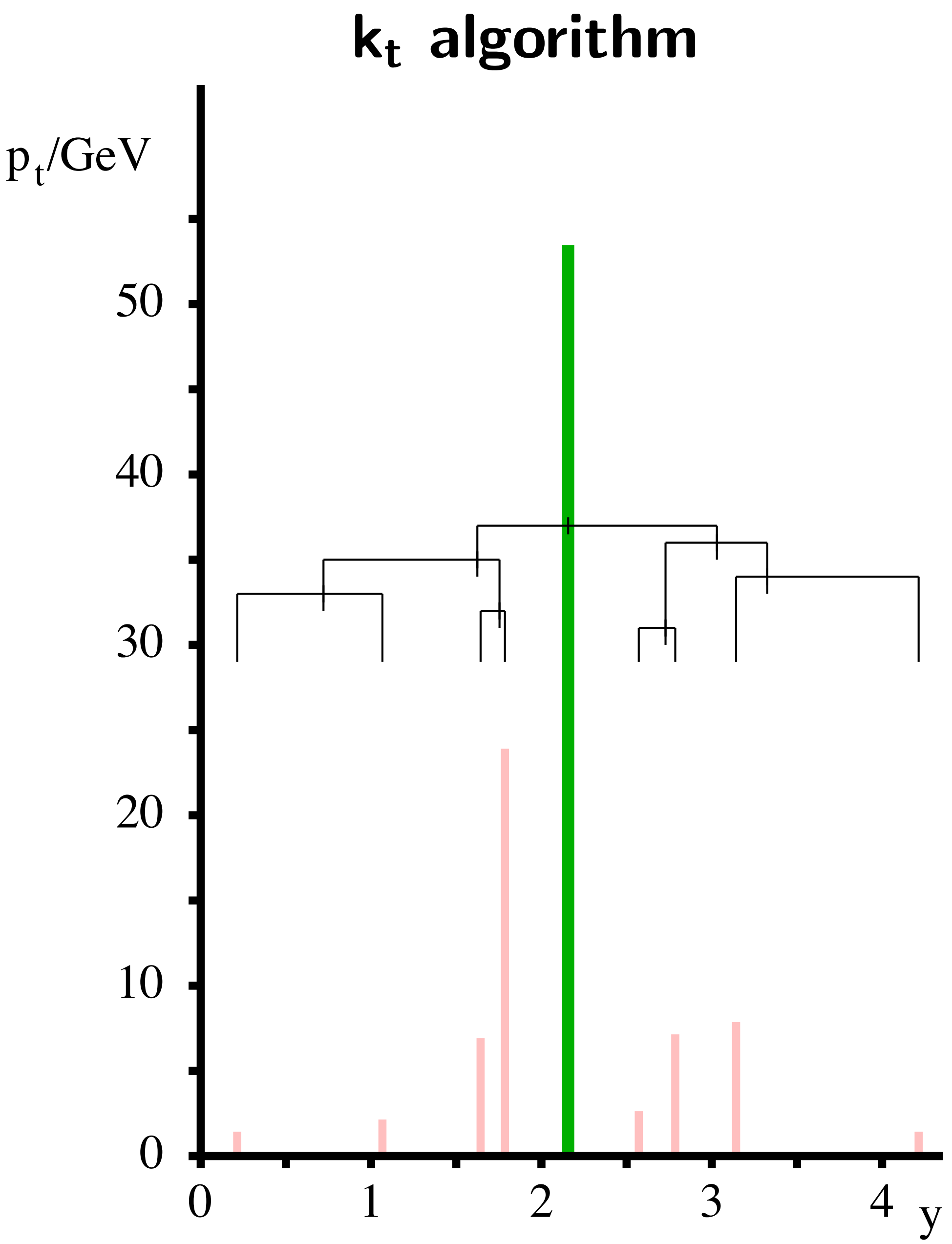
How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Its last step is to merge two hard pieces. Easily undone to identify underlying kinematics

Identifying jet substructure: try out k_t



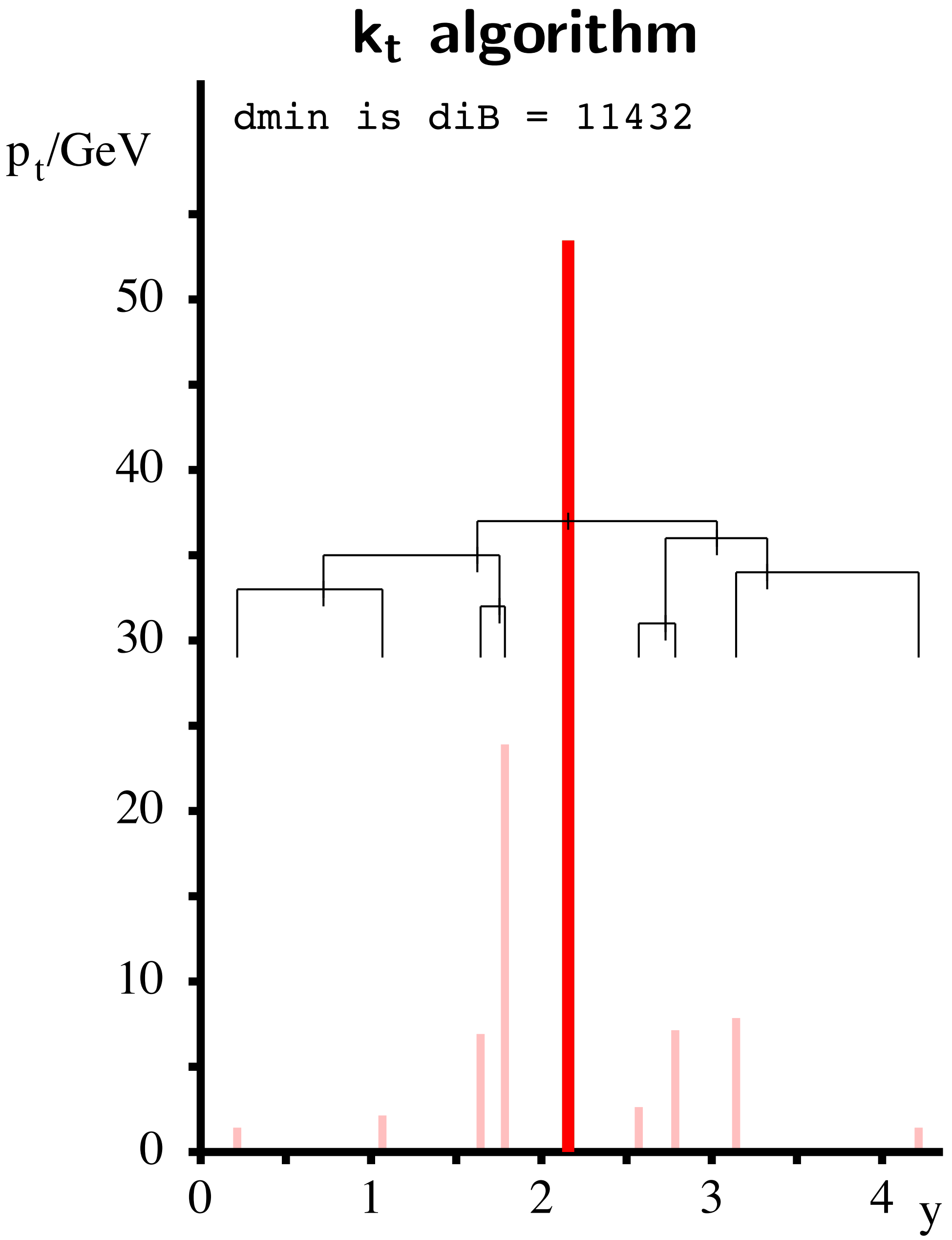
How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Its last step is to merge two hard pieces. Easily undone to identify underlying kinematics

Identifying jet substructure: try out k_t



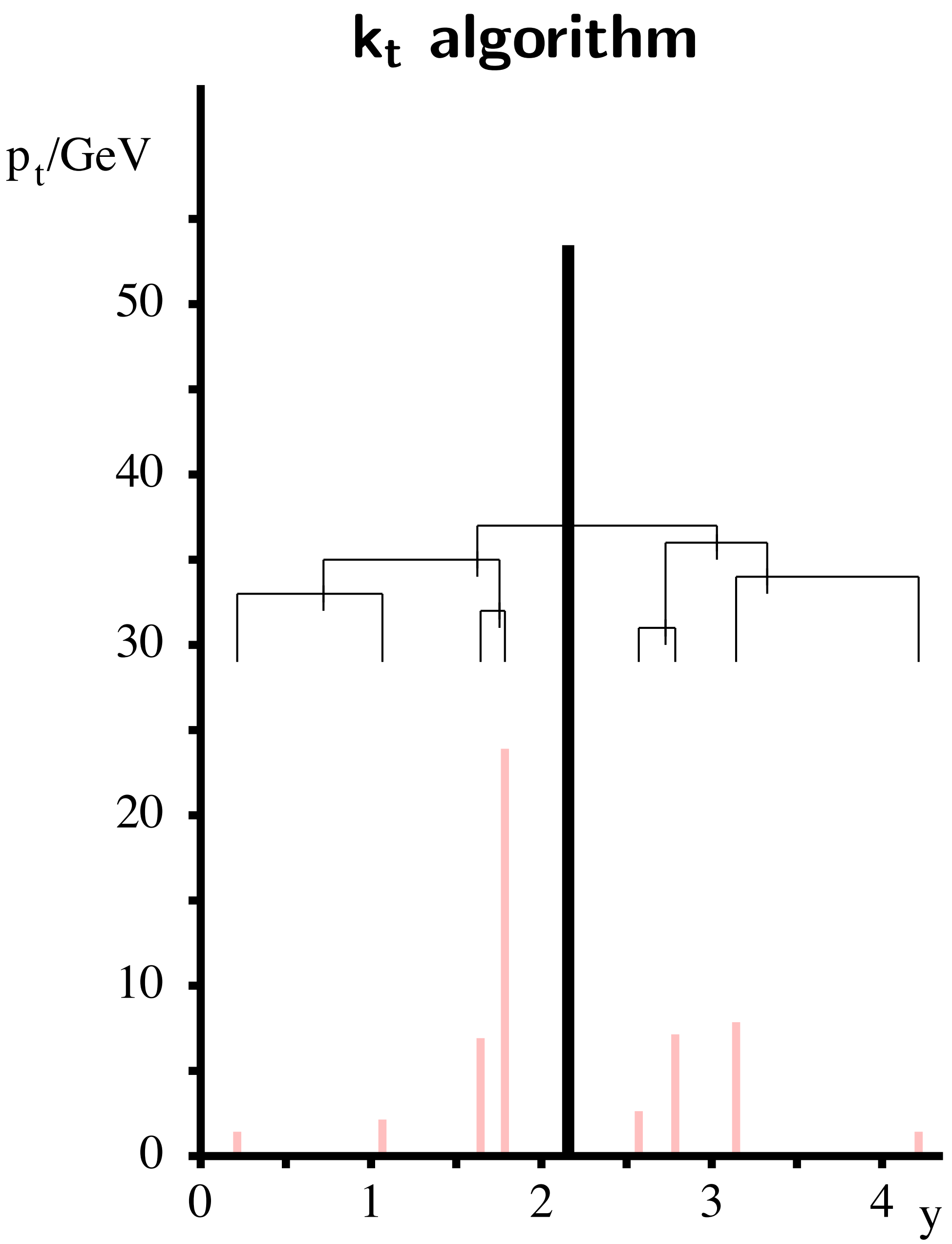
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft “junk” early on in the clustering

Its last step is to merge two hard pieces. Easily undone to identify underlying kinematics

Identifying jet substructure: try out k_t



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g. z).

k_t clusters soft "junk" early on in the clustering

Its last step is to merge two hard pieces. Easily undone to identify underlying kinematics

This meant it was the first algorithm to be used for jet substructure.

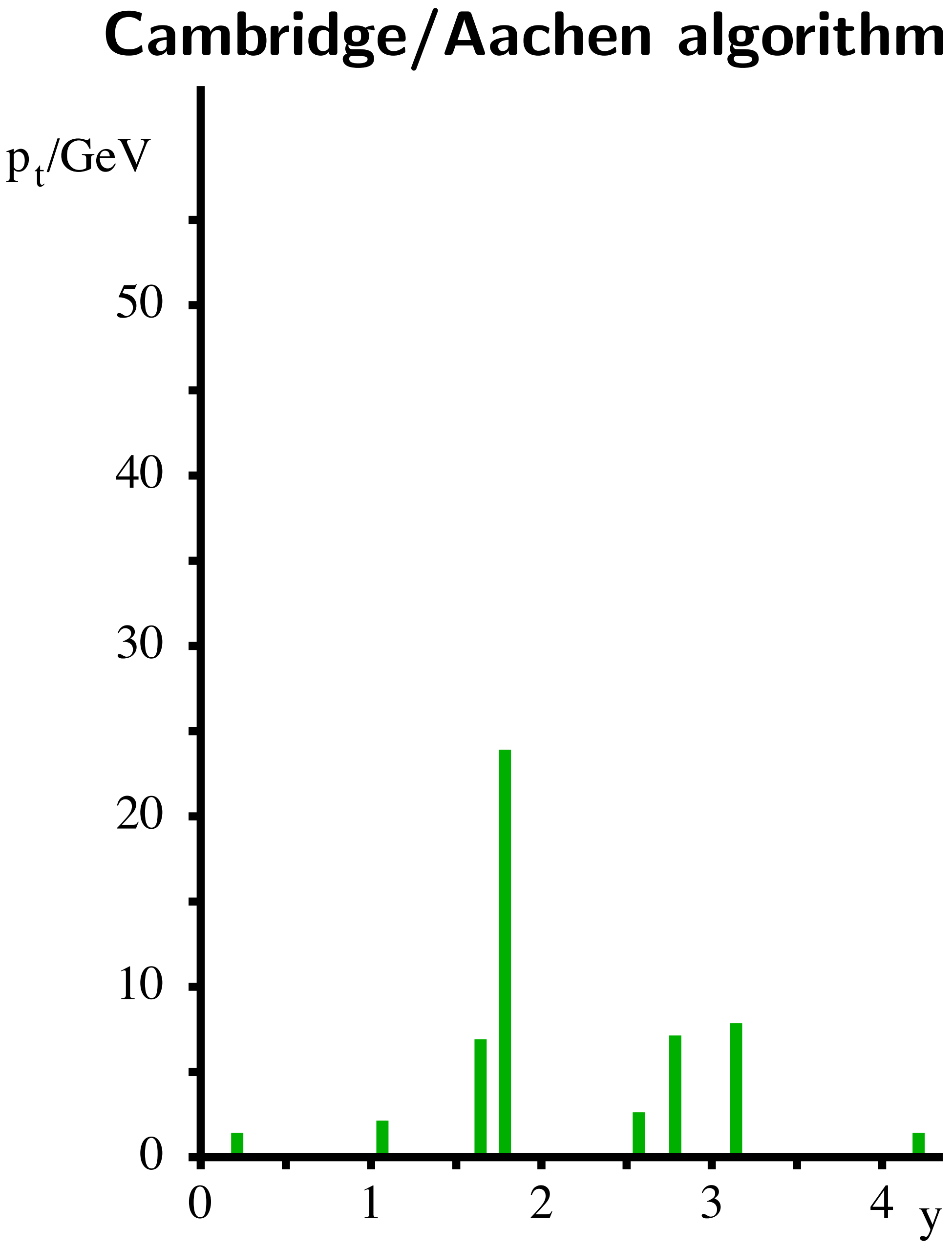
Seymour '93

Butterworth, Cox & Forshaw '02

Third try

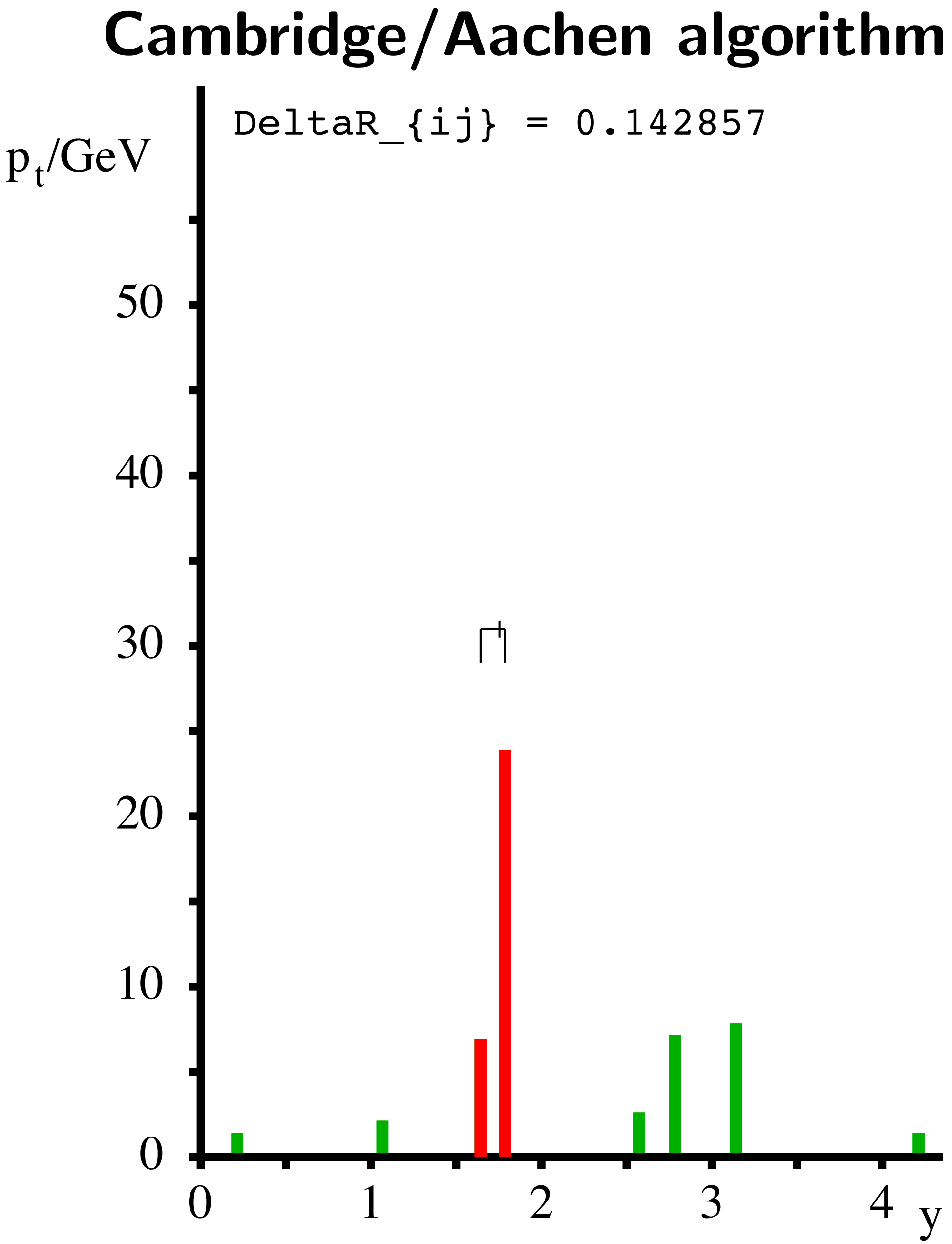
Cambridge/Aachen

Identifying jet substructure: Cam/Aachen



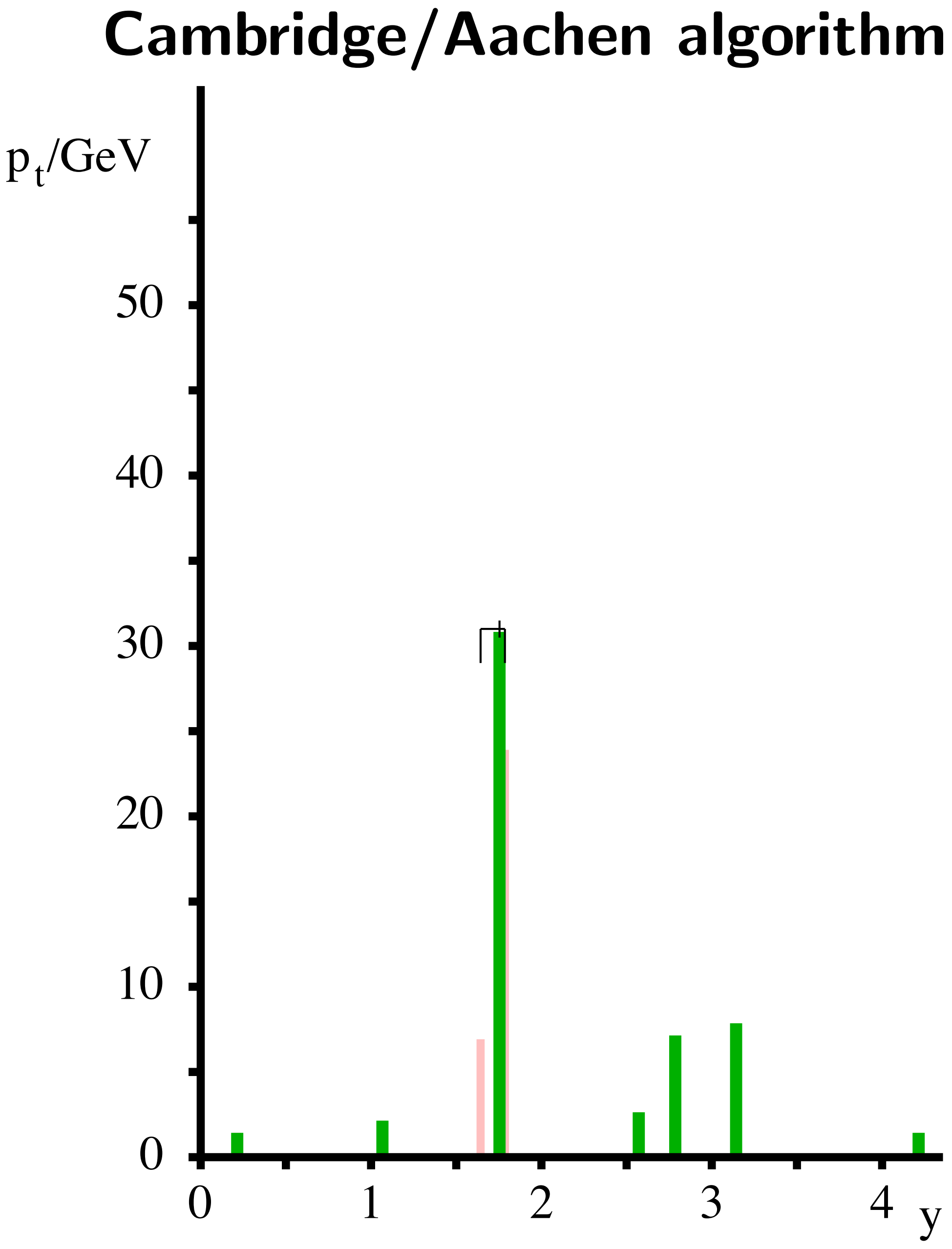
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

Identifying jet substructure: Cam/Aachen



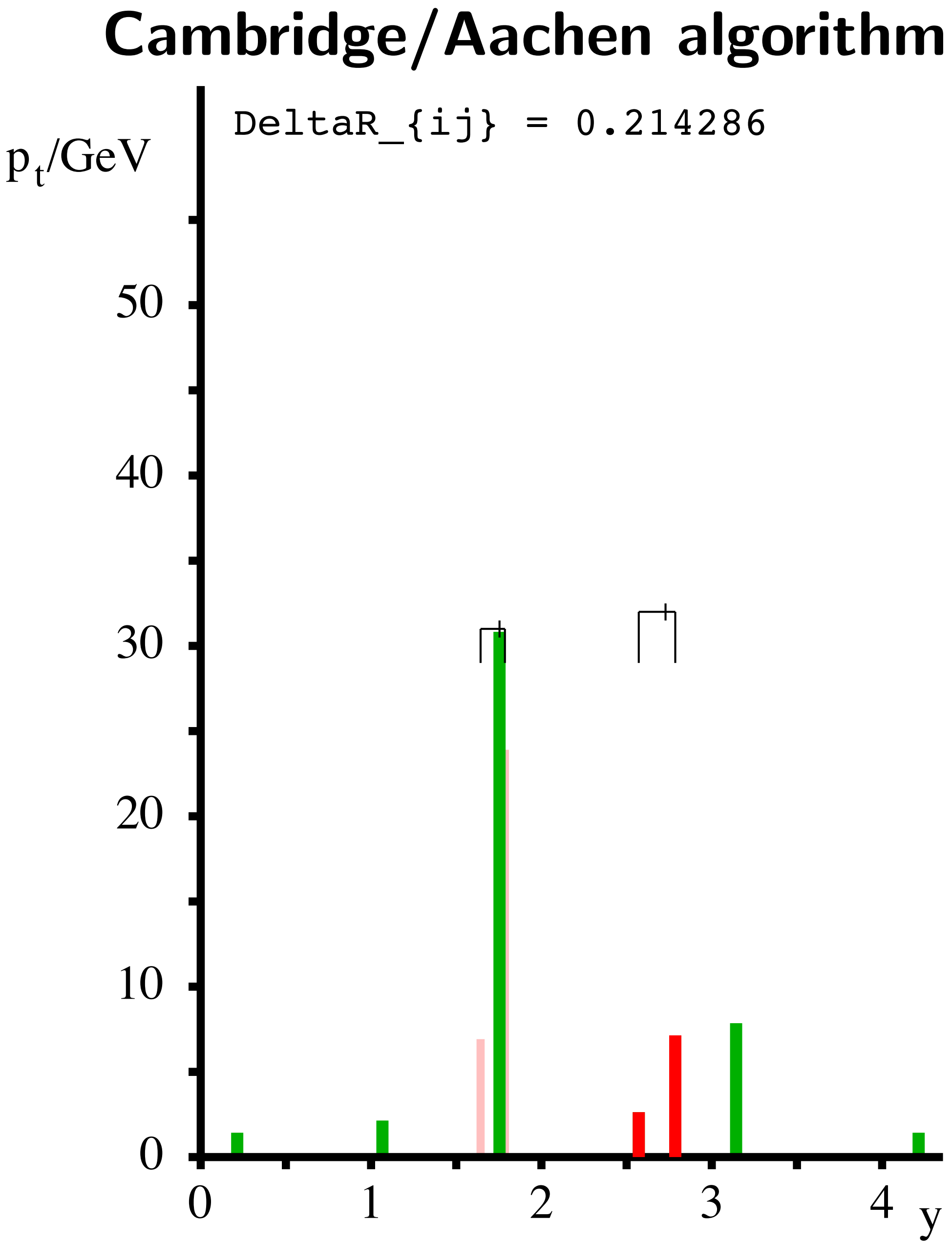
How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

Identifying jet substructure: Cam/Aachen



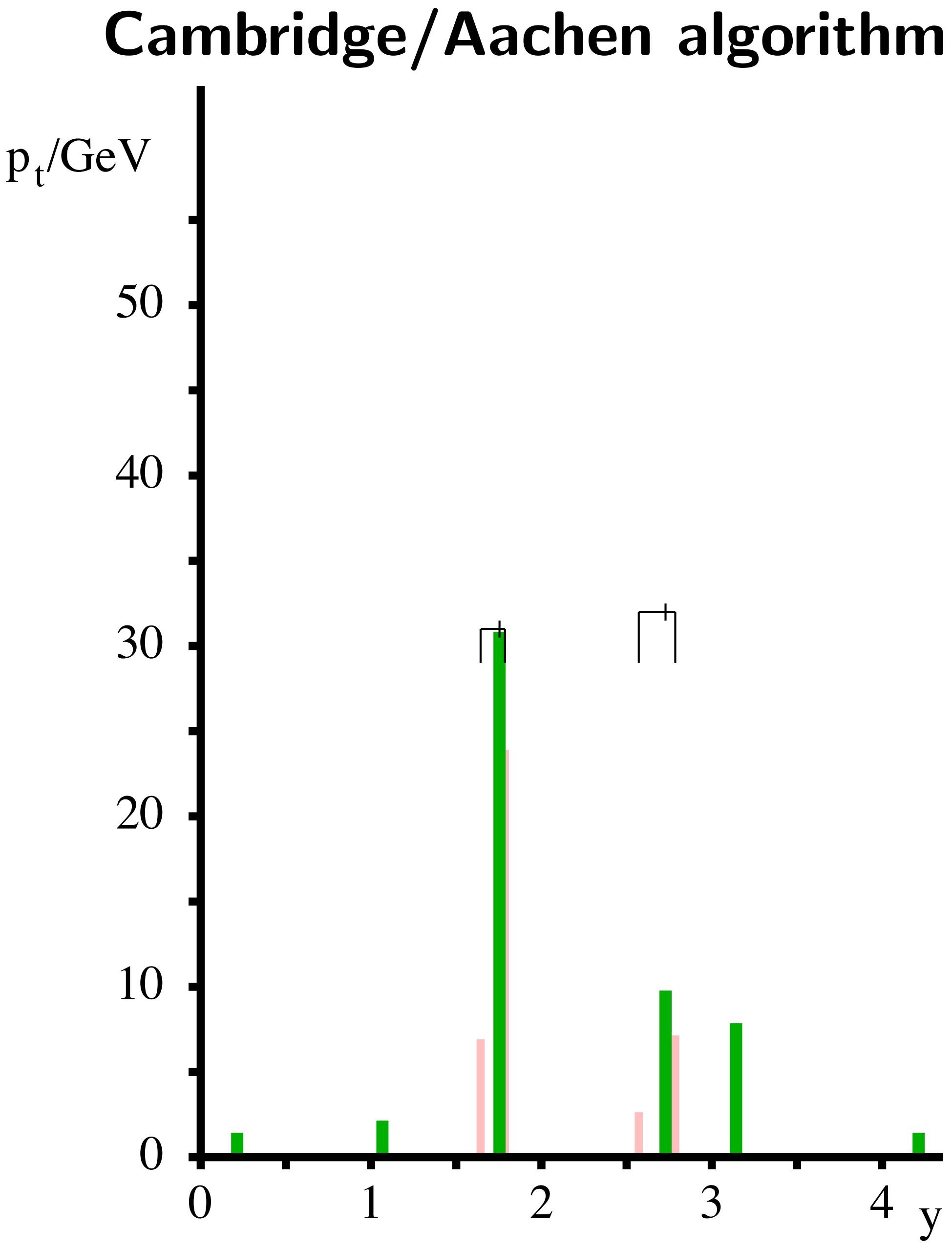
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

Identifying jet substructure: Cam/Aachen



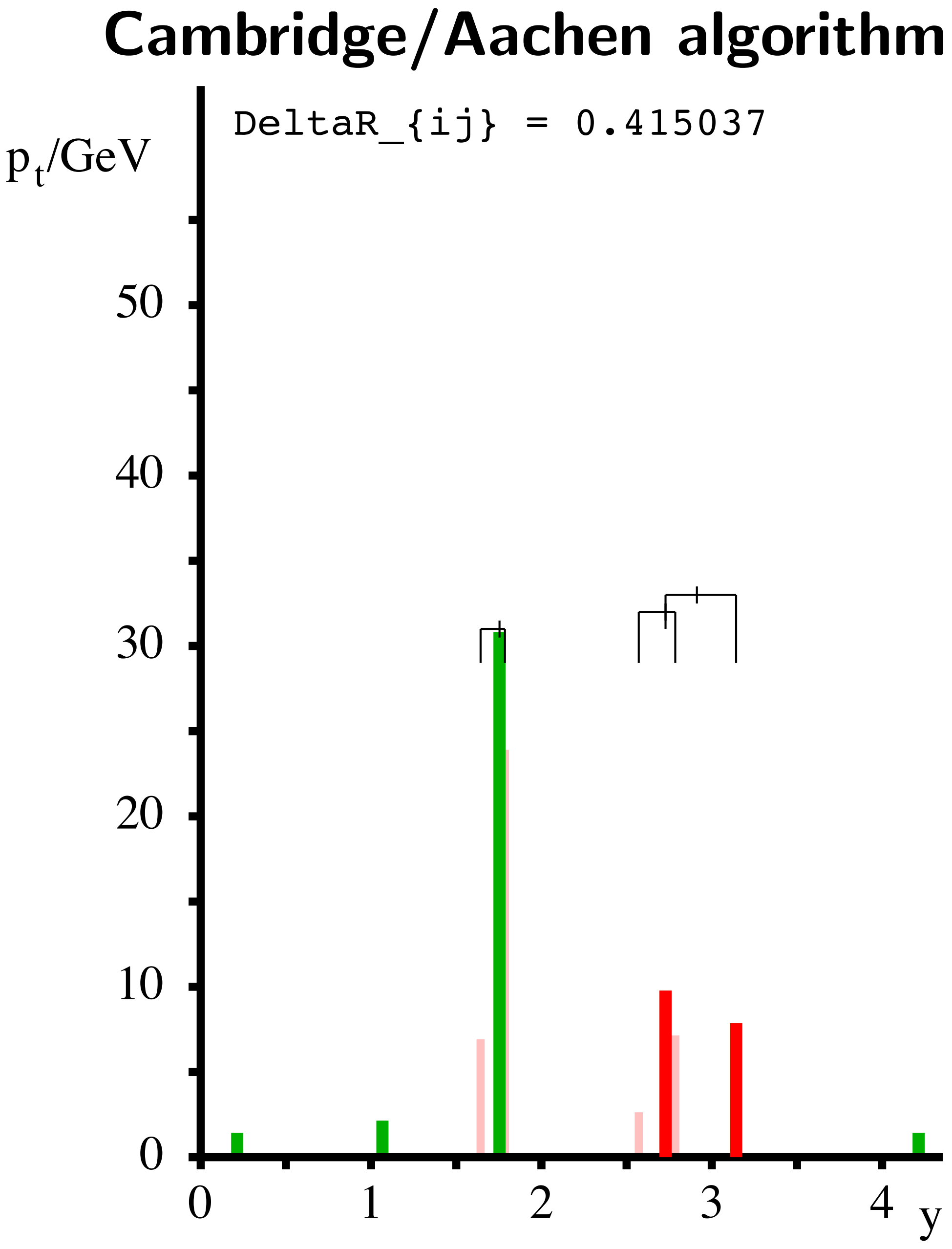
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

Identifying jet substructure: Cam/Aachen



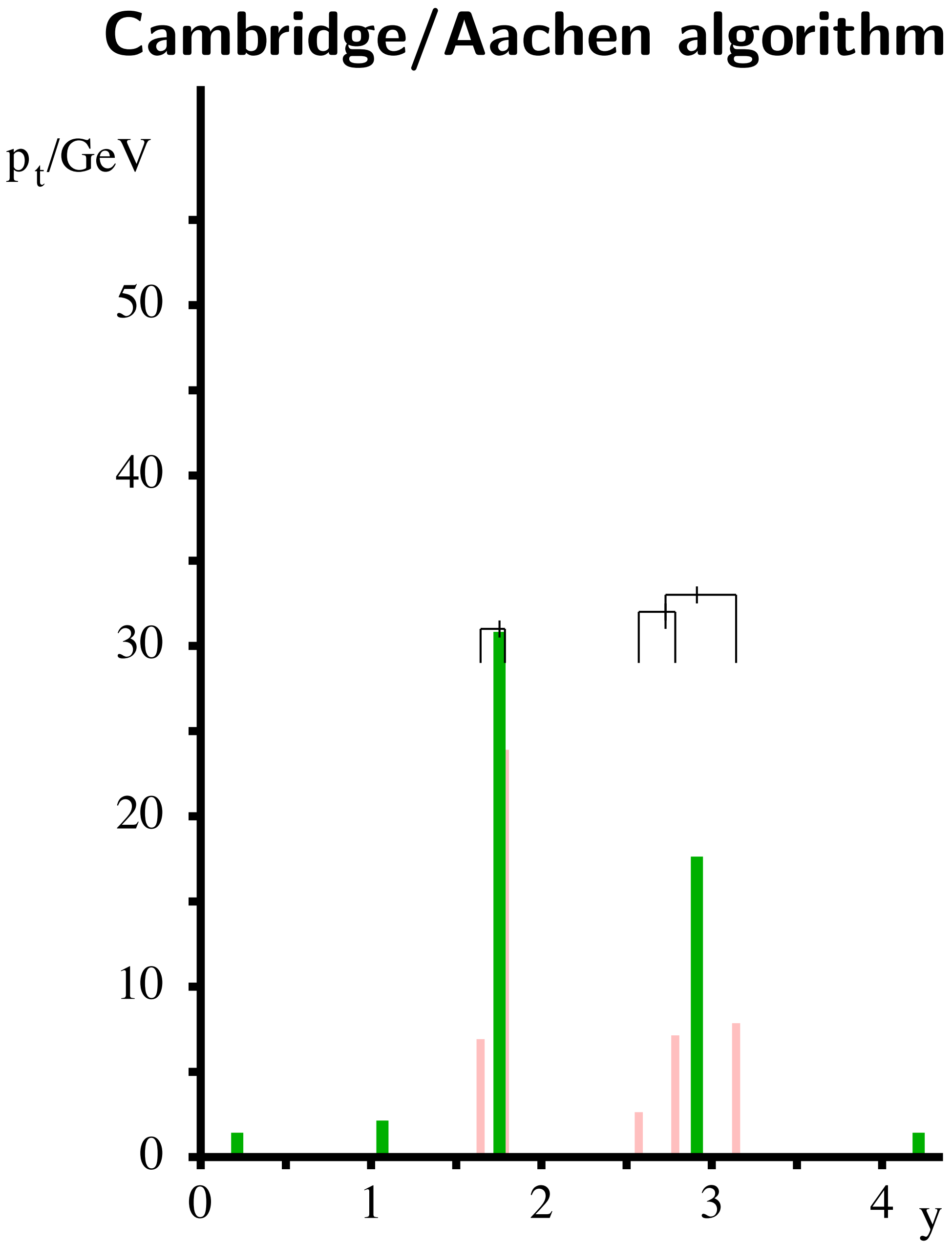
How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

Identifying jet substructure: Cam/Aachen



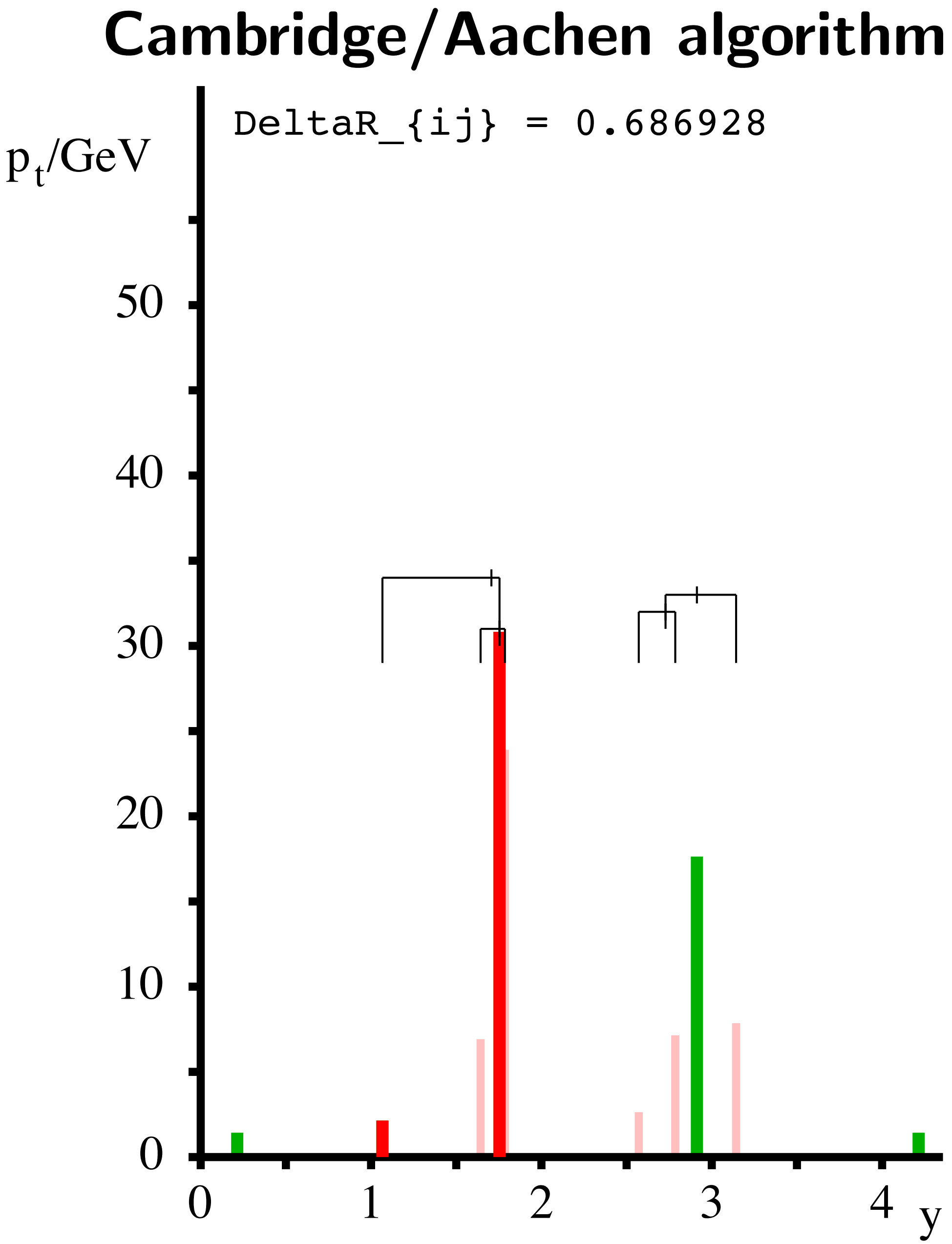
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

Identifying jet substructure: Cam/Aachen



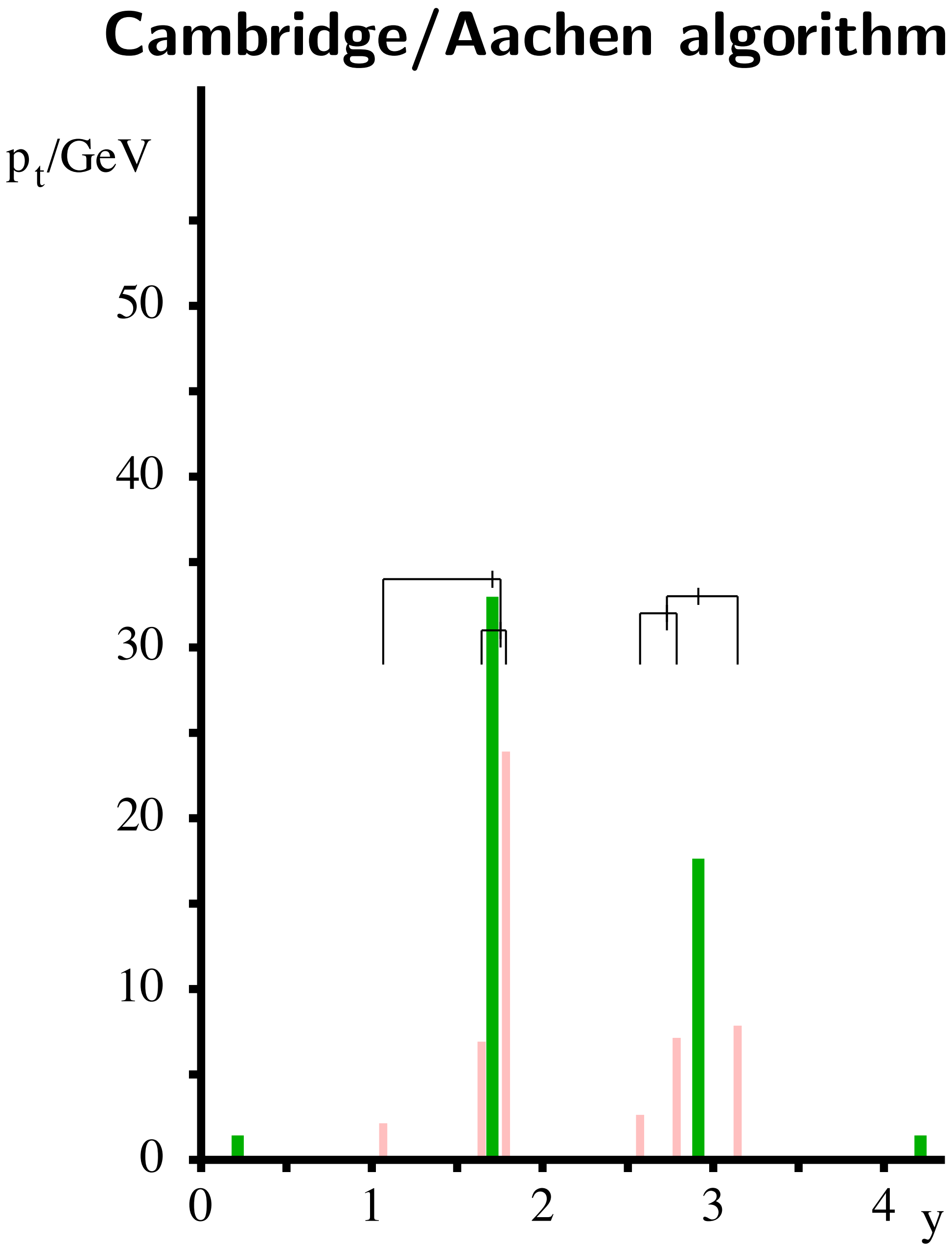
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

Identifying jet substructure: Cam/Aachen



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

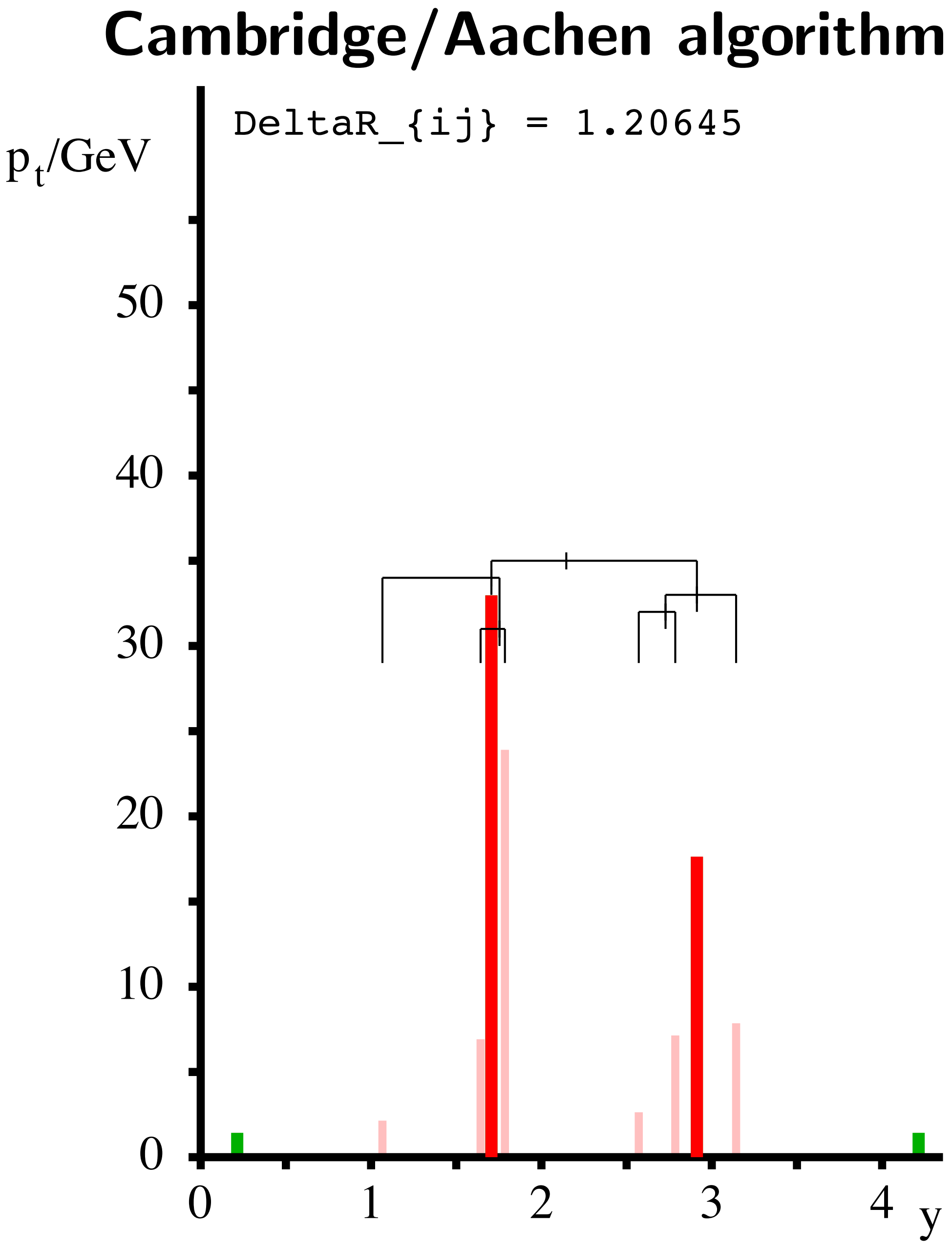
Identifying jet substructure: Cam/Aachen



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination

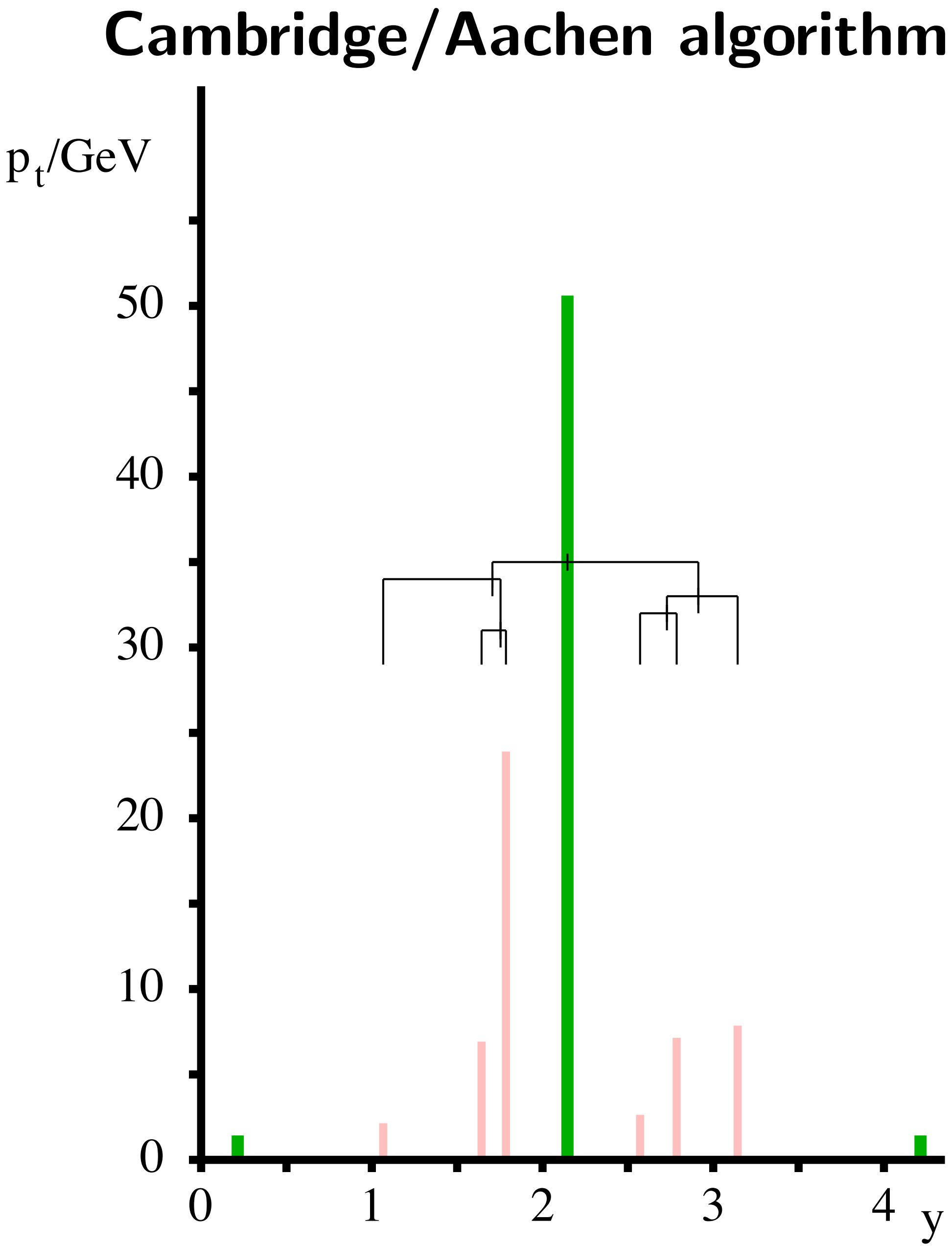
Identifying jet substructure: Cam/Aachen



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them

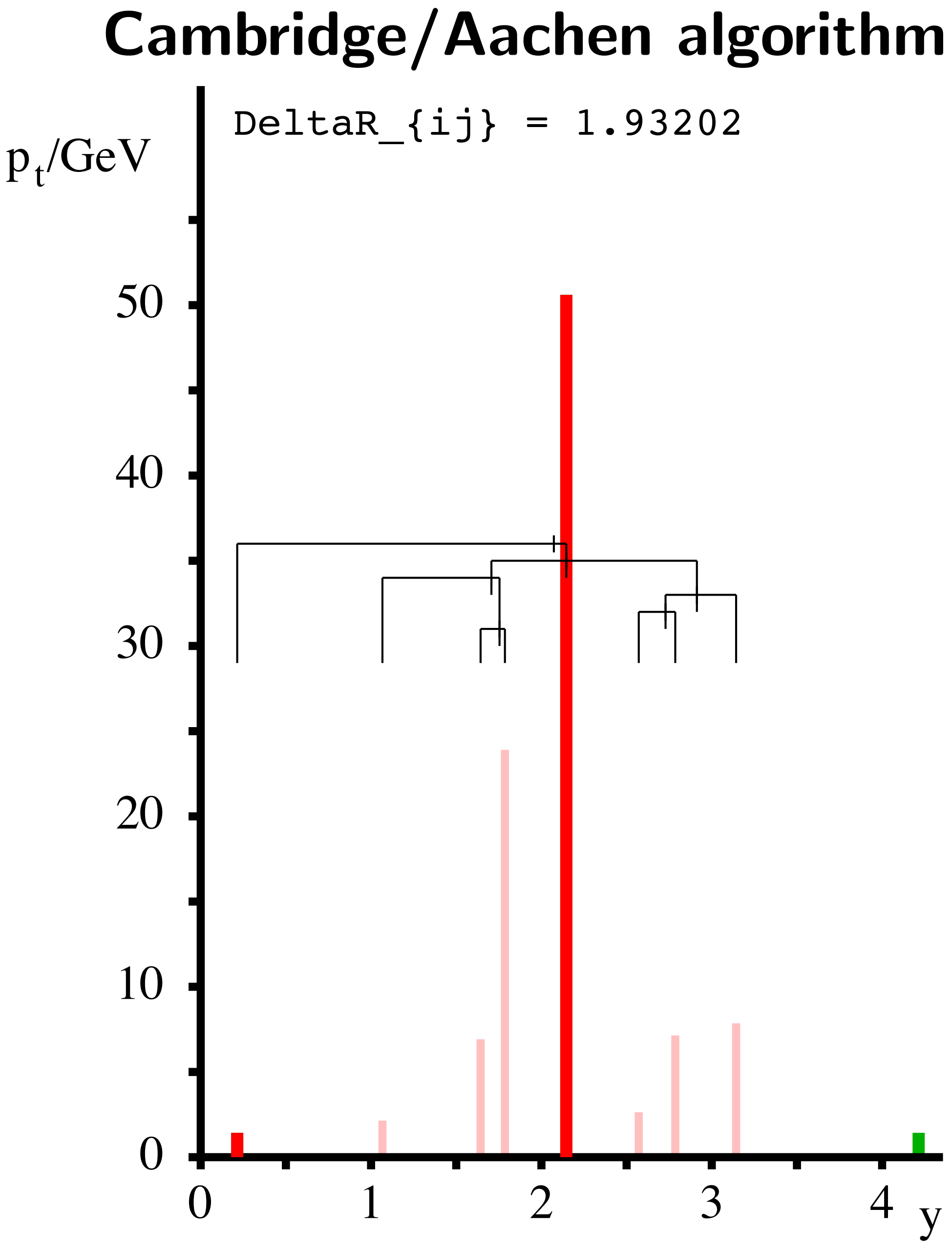
Identifying jet substructure: Cam/Aachen



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them

Identifying jet substructure: Cam/Aachen

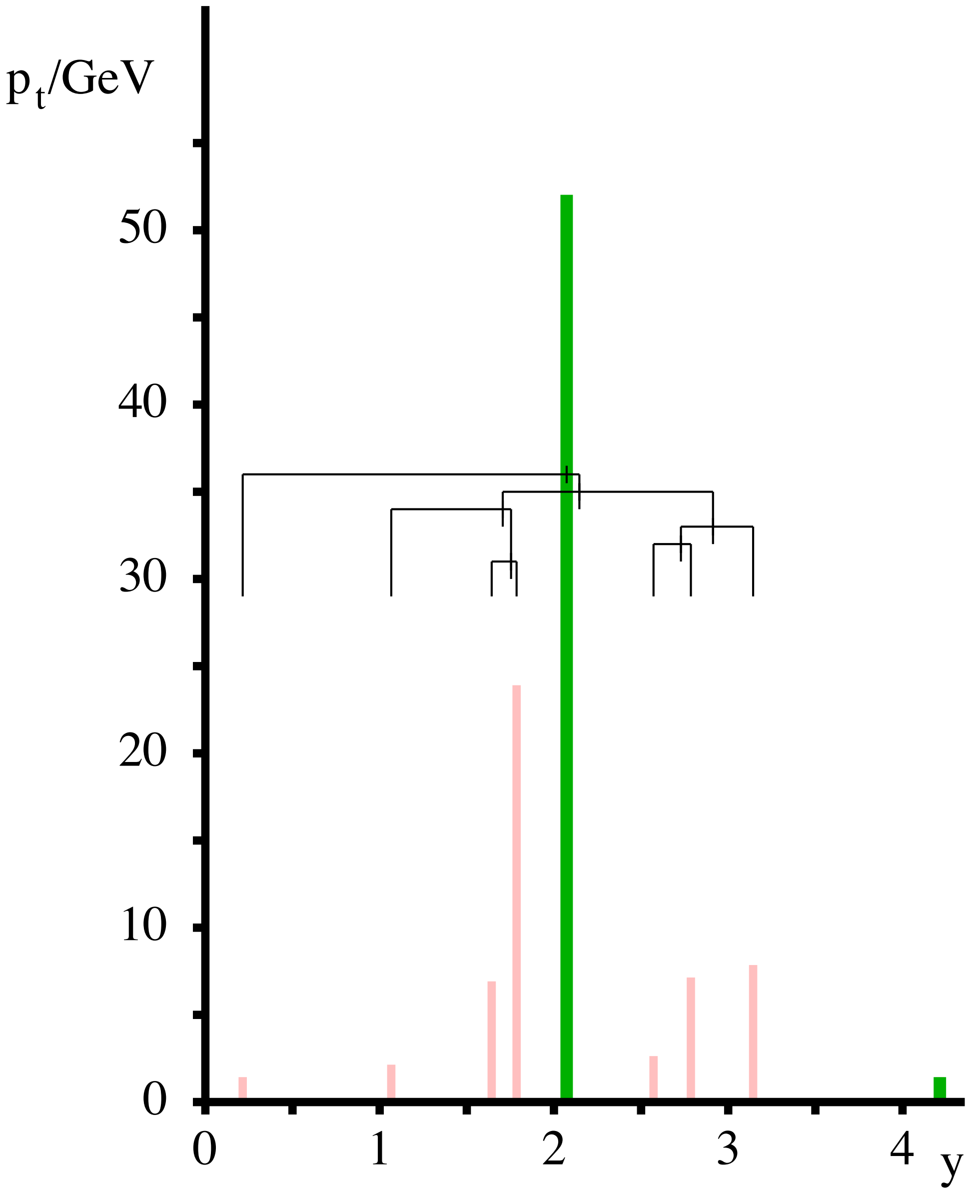


How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them, and then adds in remaining soft junk

Identifying jet substructure: Cam/Aachen

Cambridge/Aachen algorithm

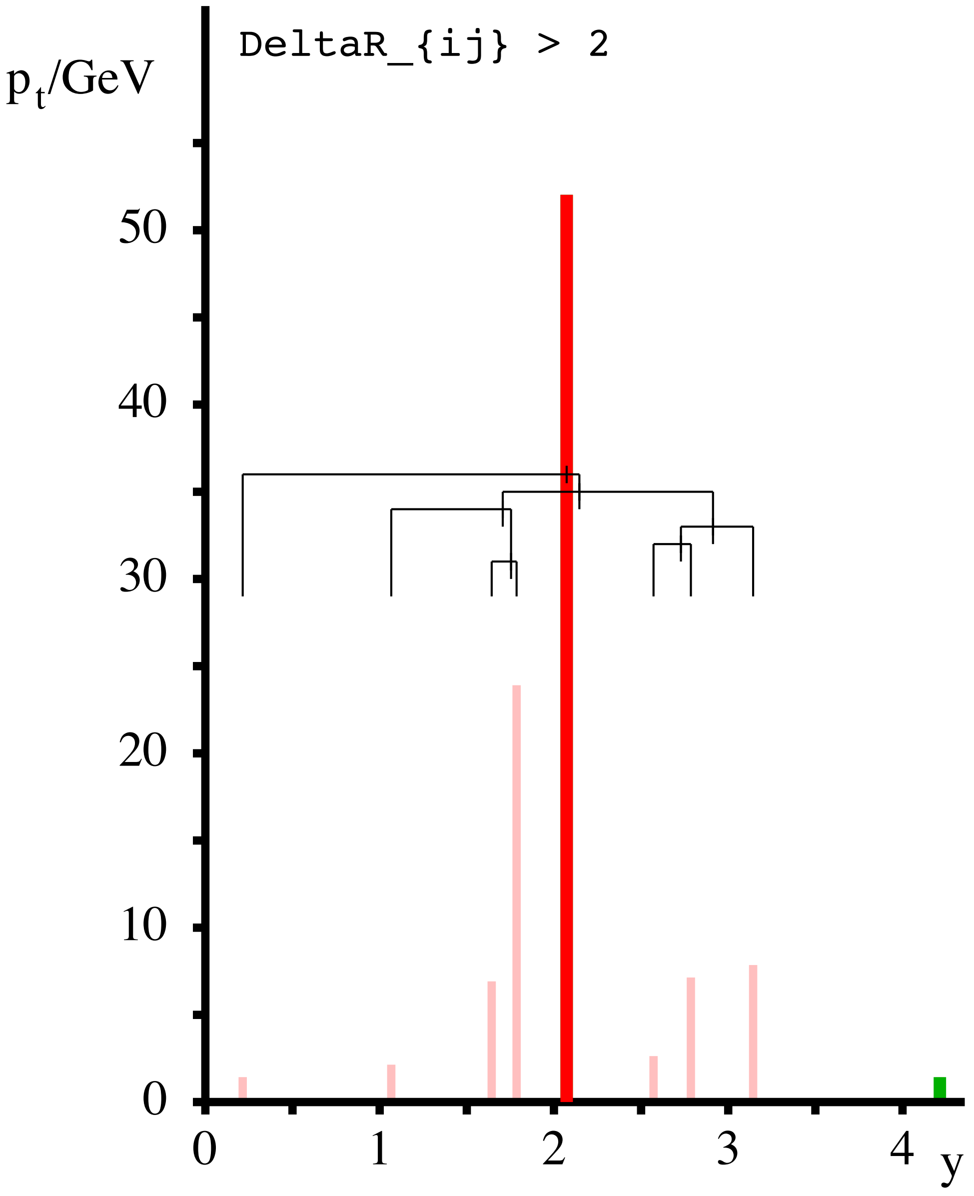


How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them, and then adds in remaining soft junk

Identifying jet substructure: Cam/Aachen

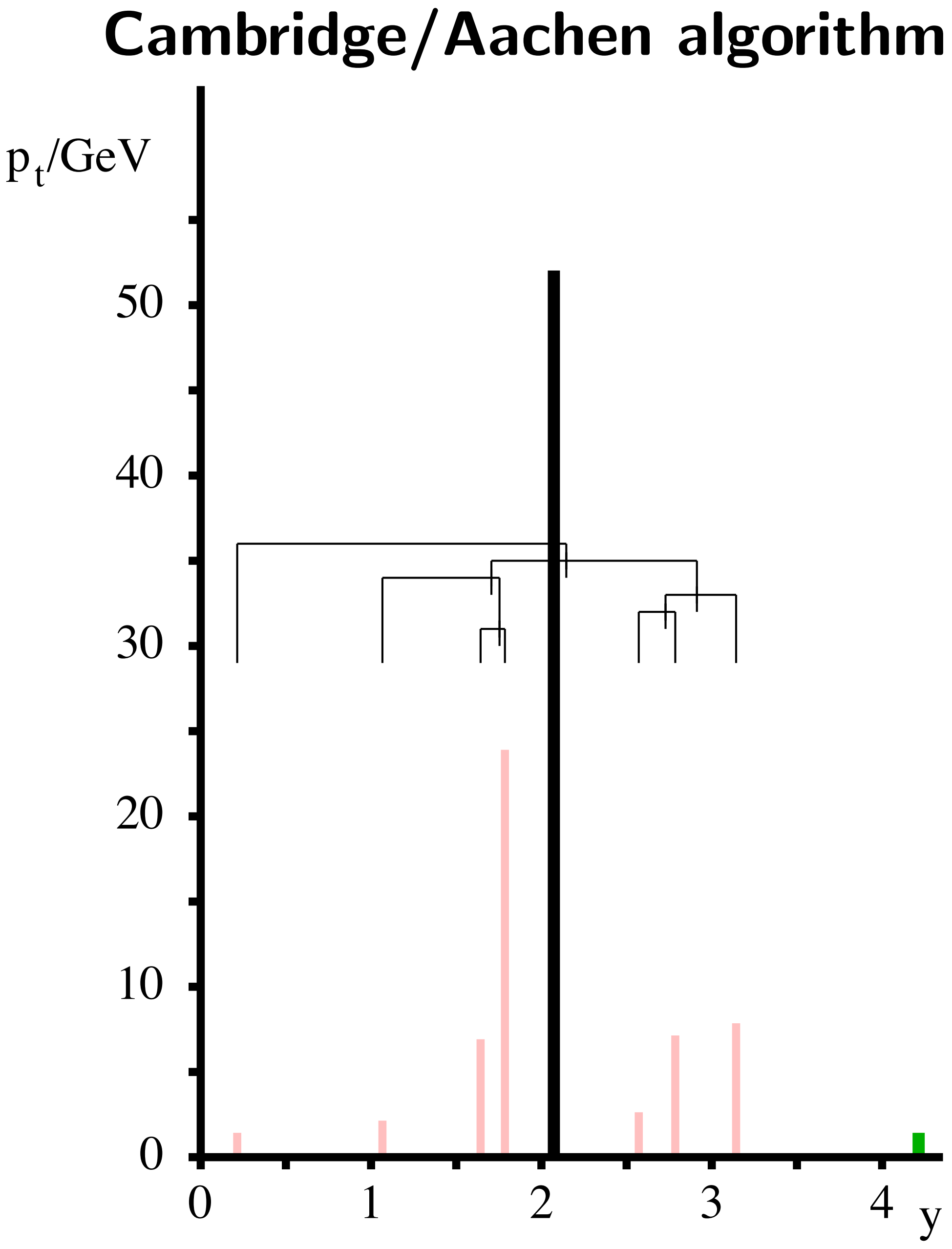
Cambridge/Aachen algorithm



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them, and then adds in remaining soft junk

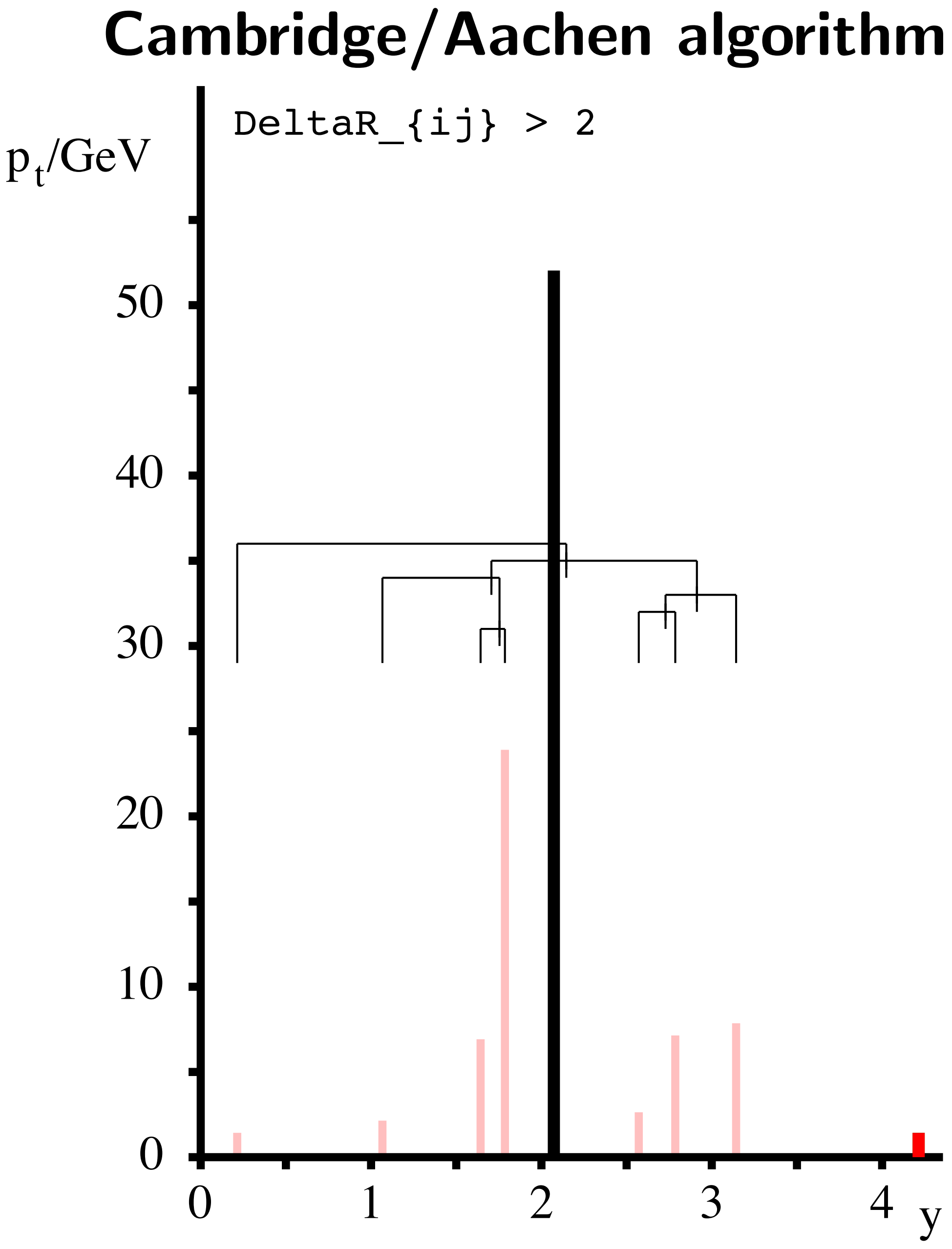
Identifying jet substructure: Cam/Aachen



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them, and then adds in remaining soft junk

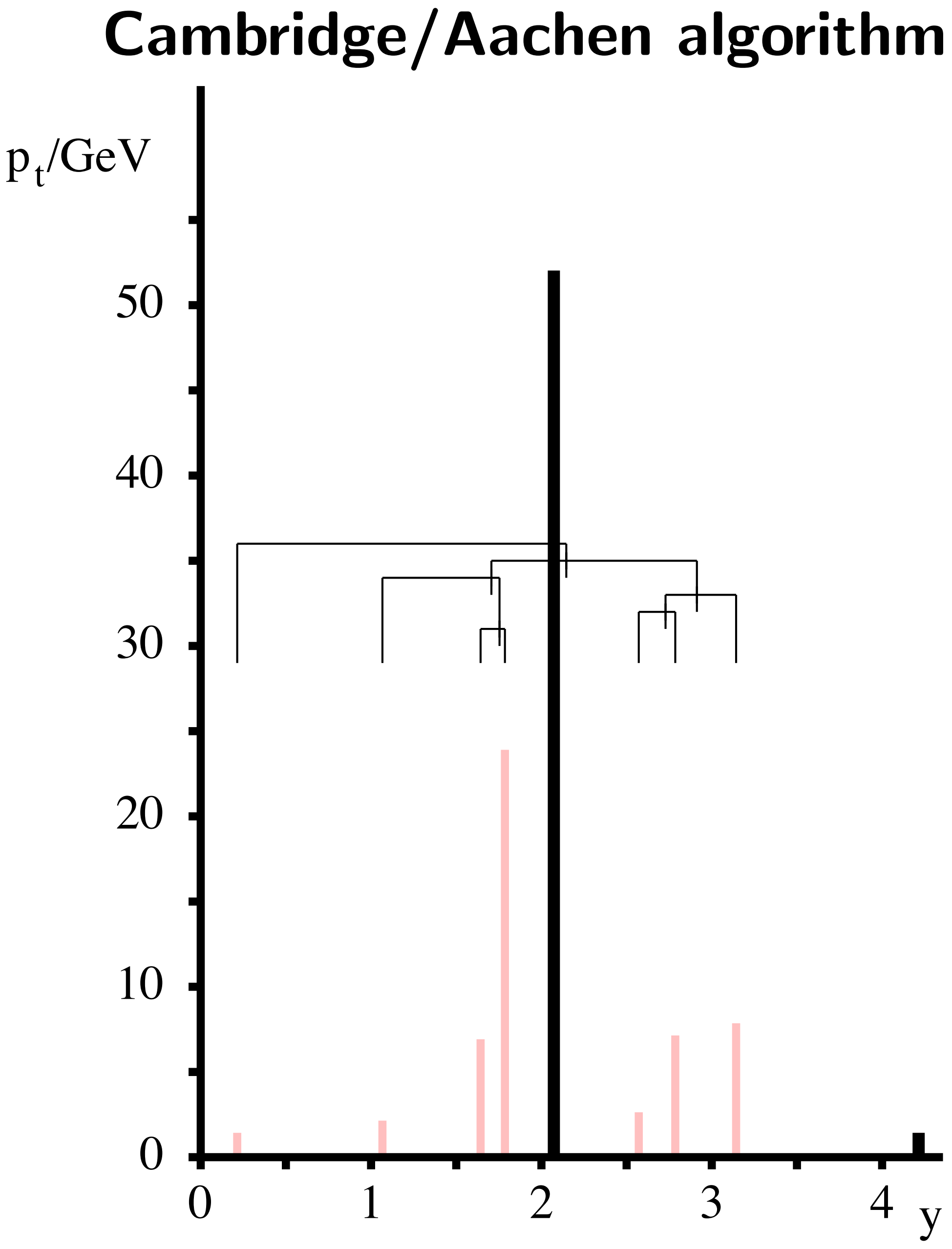
Identifying jet substructure: Cam/Aachen



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them, and then adds in remaining soft junk

Identifying jet substructure: Cam/Aachen



How well can an algorithm identify the "blobs" of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them, and then adds in remaining soft junk

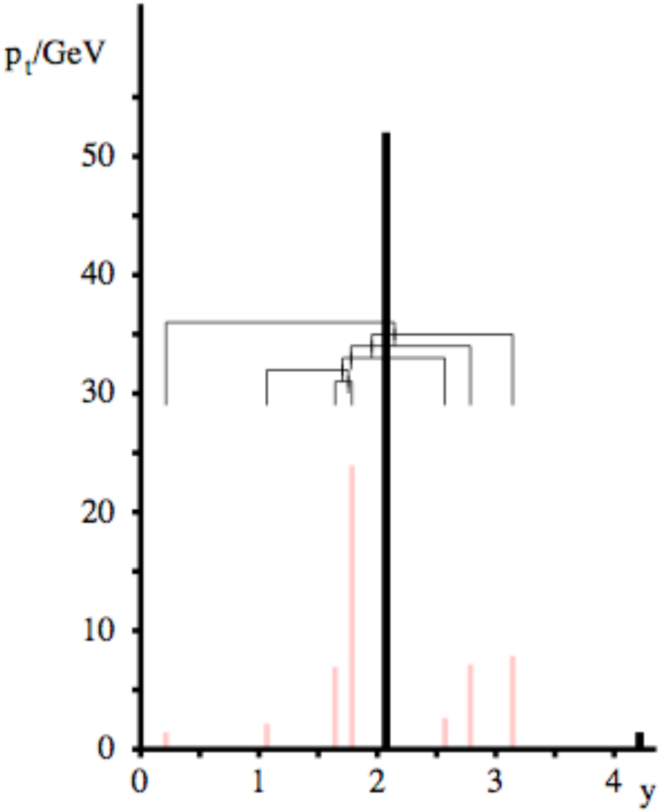
The interesting substructure is buried inside the clustering sequence — it's less contaminated by soft junk, but needs to be pulled out with special techniques

Butterworth, Davison, Rubin & GPS '08
Kaplan, Schwartz, Reherman & Tweedie '08
Butterworth, Ellis, Rubin & GPS '09
Ellis, Vermilion & Walsh '09

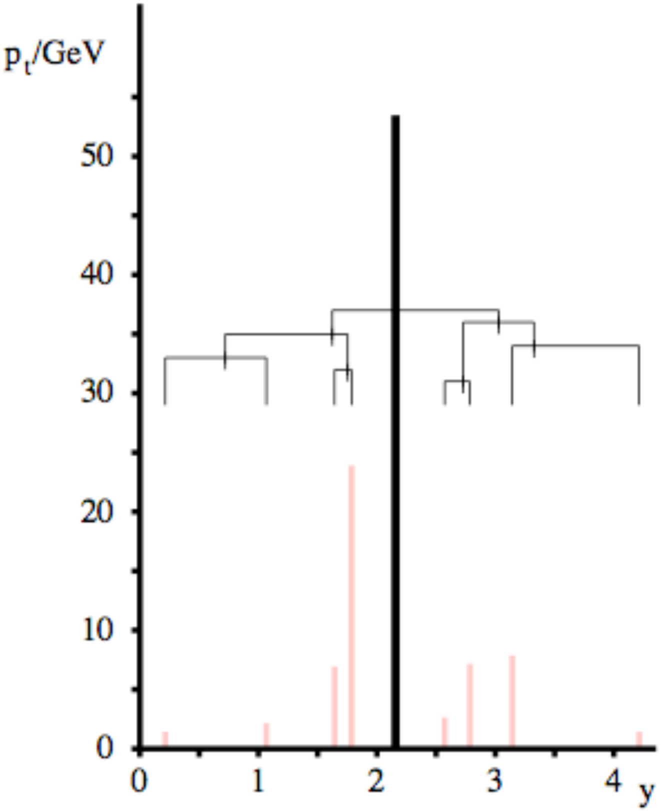
Slide by
Gavin Salam

Hierarchical substructure

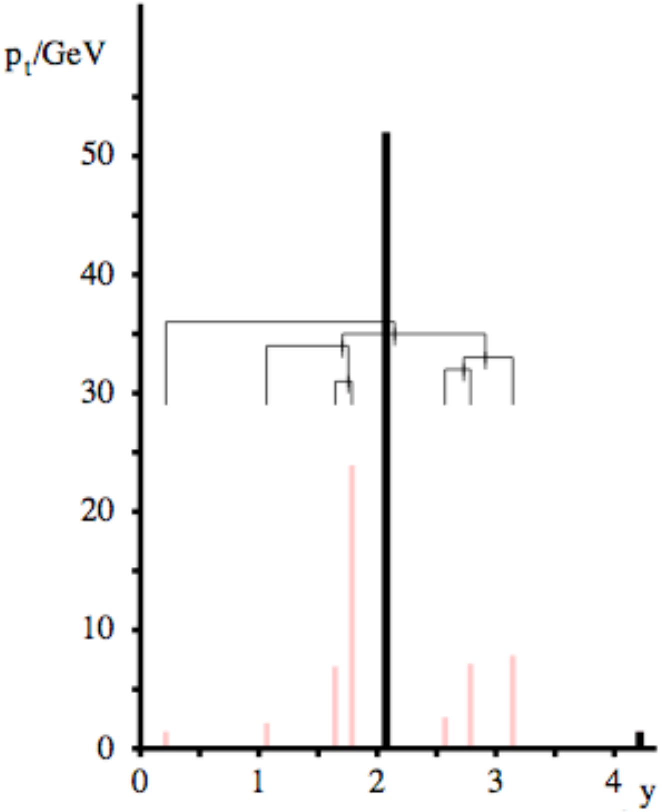
anti-k_t algorithm



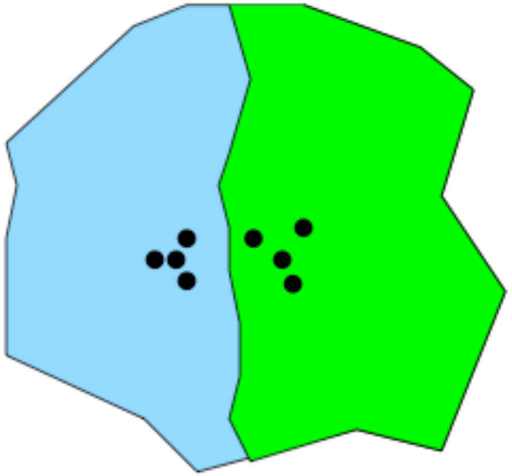
k_t algorithm

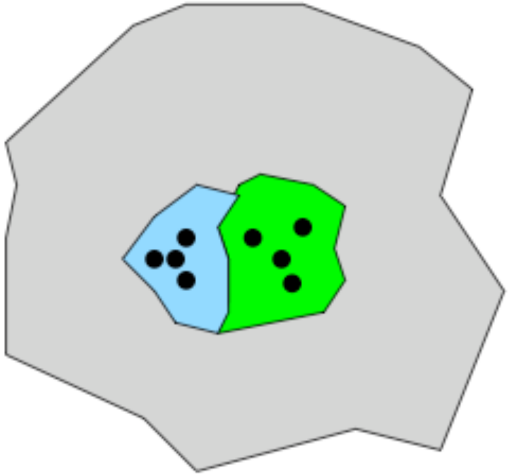


Cambridge/Aachen



Undo the last clustering step(s) →





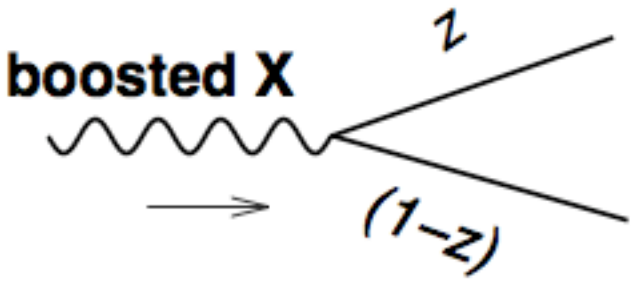
The IRC safe algorithms

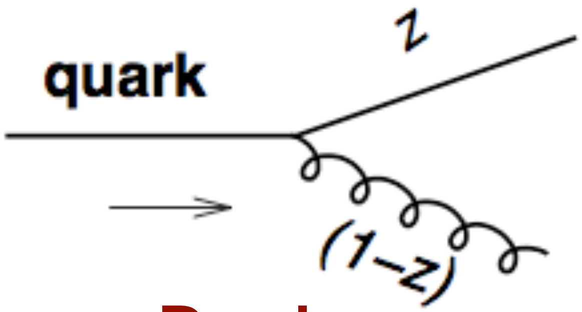
	Speed	Regularity	UE contamination	Backreaction	Hierarchical substructure
k_t	☺☺☺	☂	☂☂	☁☁	☺☺
Cambridge /Aachen	☺☺☺	☂	☂	☁☁	☺☺☺
anti-k_t	☺☺☺	☺☺	☁/☺	☺☺	✗
SISCone	☺	☁	☺☺	☁	✗

Array of tools with different characteristics.
Pick the right one for the job

QCD v. heavy decay

A possible approach for reducing the QCD background is to identify the two prongs of the heavy particle decay, and put a cut on their momentum fraction





Signal:

$$P(z) \sim 1$$

Will split mainly
symmetrically

Background:

$$P(z) \sim \frac{1+z^2}{1-z} \qquad P(z) \sim \frac{1+(1-z)^2}{z}$$

Will split mainly
asymmetrically

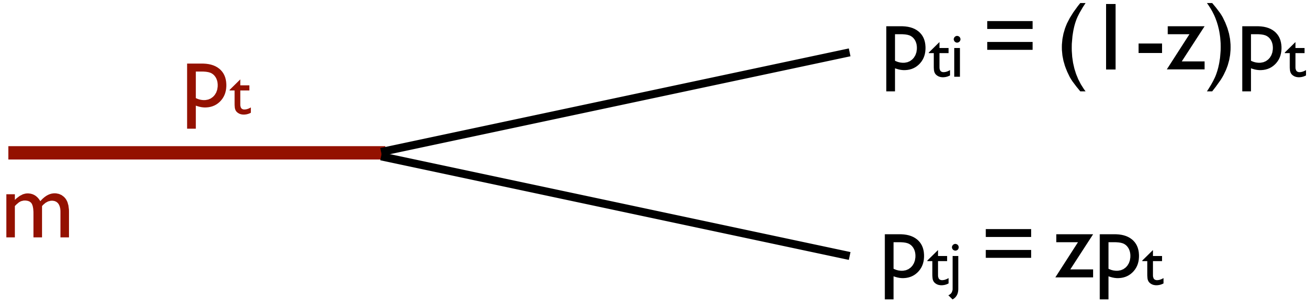
Potential tagger: asymmetric splitting

Possibly implemented via a cut on

$$y = min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{m^2} \simeq \frac{min(p_{ti}, p_{tj})}{max(p_{ti}, p_{tj})}$$

Splittings and distances

Quasi-collinear splitting ($p_{tj} < p_{ti}$)



p_t

m

$p_{ti} = (1-z)p_t$

$p_{tj} = zp_t$

Invariant mass:

$$m^2 \simeq p_{ti}p_{tj}\Delta R_{ij}^2 = (1-z)zp_t^2\Delta R_{ij}^2$$

k_t distance:

$$d_{ij} \overset{(p_{tj} < p_{ti})}{=} z^2 p_t^2 \Delta R_{ij}^2 \simeq \frac{z}{1-z} m^2$$

For a given mass, the **background** will have smaller distance d_{ij} than the signal, i.e. it will tend to **cluster earlier** in the k_t algorithm

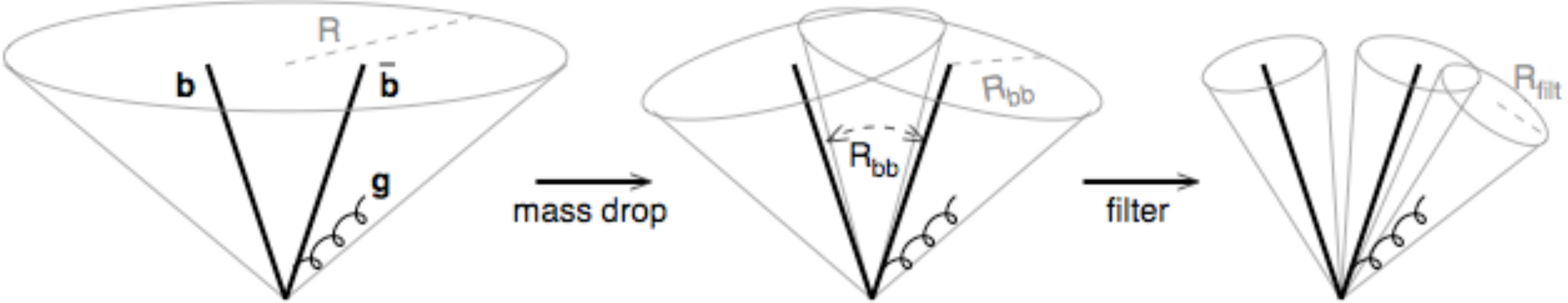
Potential tagger: last clustering in k_t algorithm

This is where the hierarchy of the k_t algorithm becomes relevant.
QCD radiation is clustered first, and only at the end the symmetric, large-angle splittings due to decays are reclustered

$pp \to ZH \to \nu\bar{\nu}b\bar{b}$

The BDRS tagger/groomer

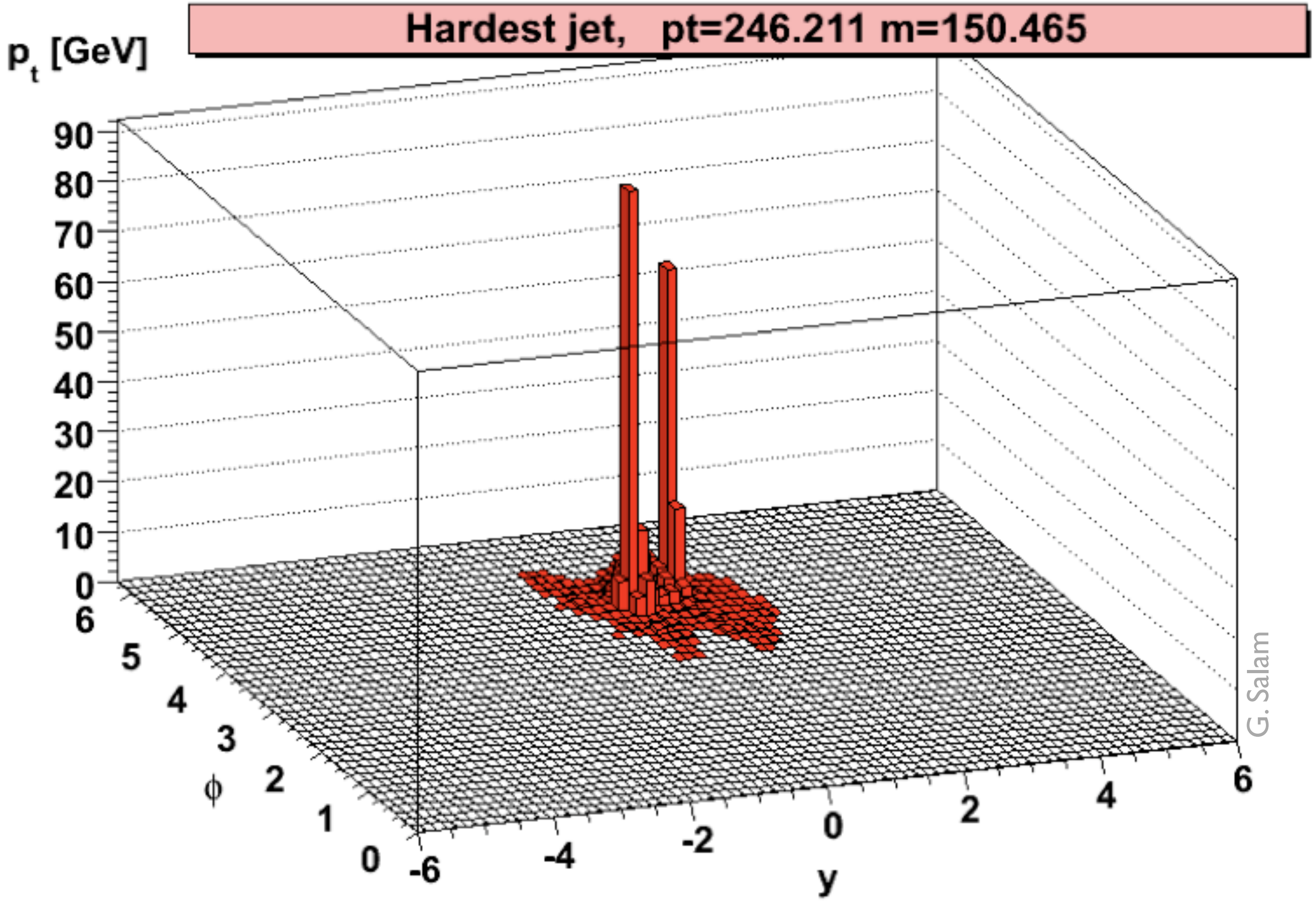
Butterworth, Davison, Rubin, Salam, 2008



- A two-prong tagger/groomer for boosted Higgs, which
  - Uses the **Cambridge/Aachen** algorithm (because it's 'physical')
  - Employs a **Mass-Drop** condition, as well as an **asymmetry cut** to find the **relevant splitting** (i.e. '**tag**' the heavy particle)
  - Includes a post-processing step, using 'filtering' (introduced in the same paper) to clean as much as possible the resulting jets of UE contamination ('**grooming**')

BDRS: tagging

$pp \to ZH \to \nu\bar{\nu}b\bar{b}$



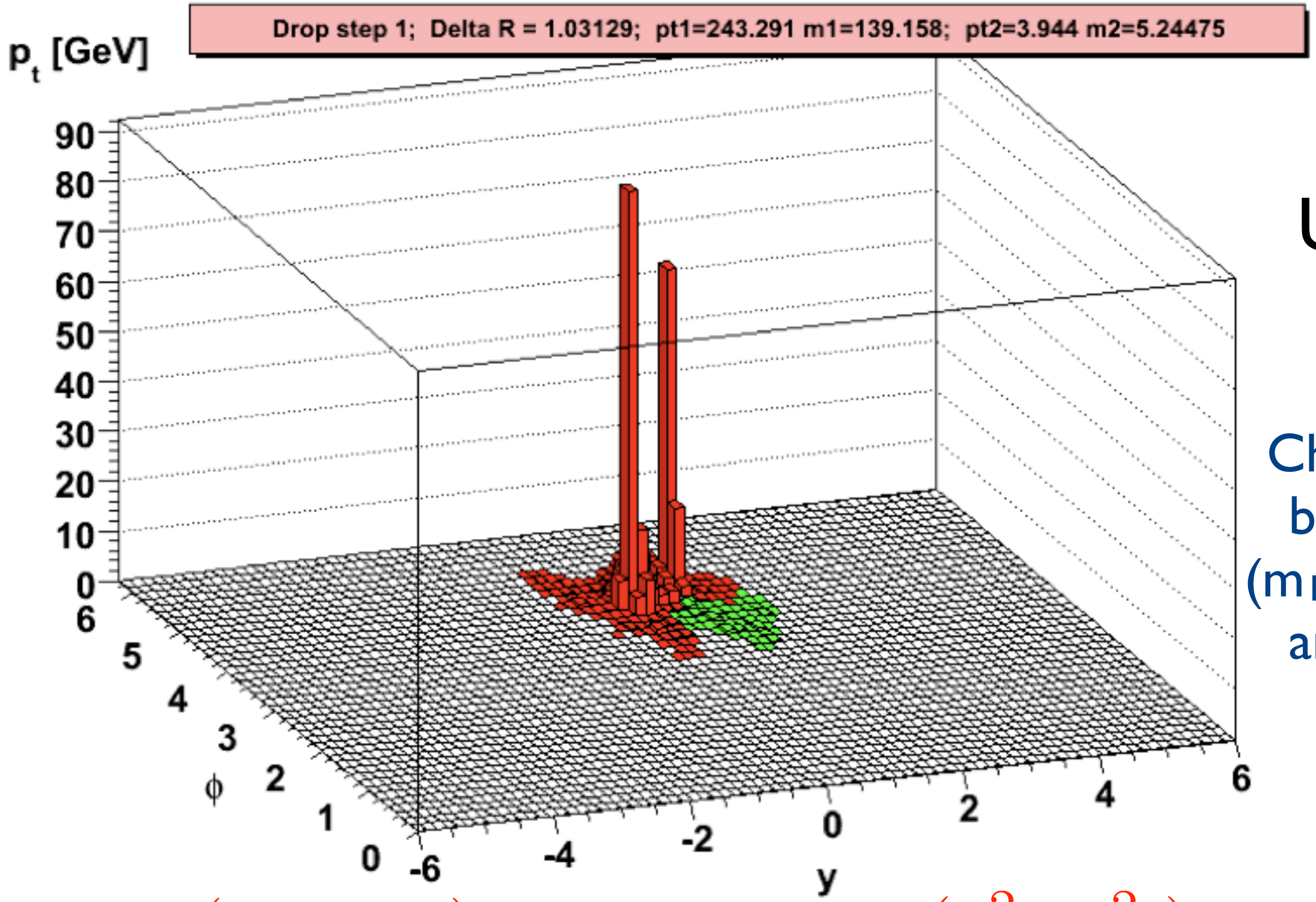
Start with the hardest jet

Use C/A with large R=1.2

m_j = 150 GeV

BDRS: tagging

pp → ZH → ννbb



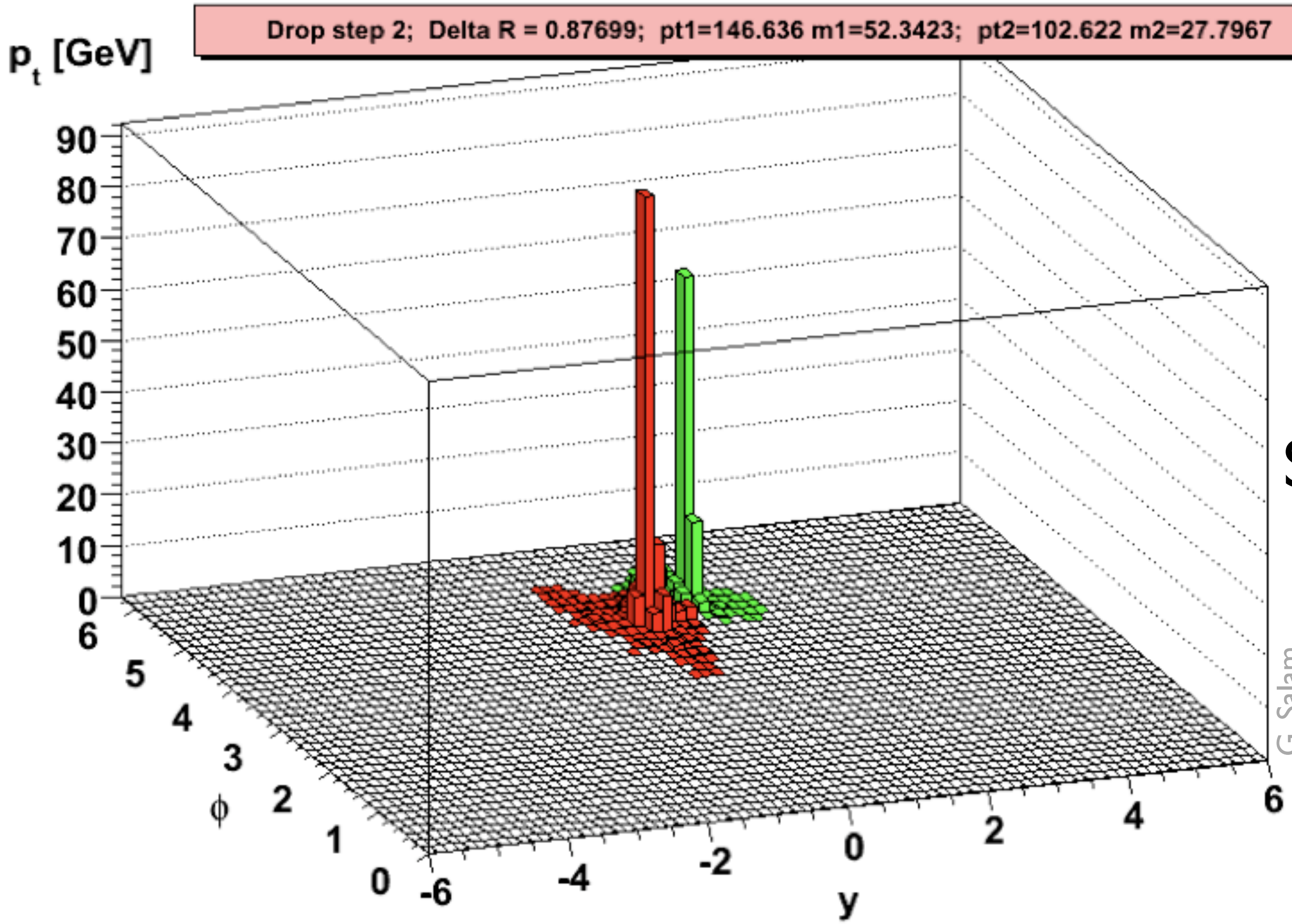
Undo last step of clustering

Check how the mass splits between the two subjets (m_1 = 139 GeV, m_2 = 5 GeV) and how asymmetric the splitting is

If $\dfrac{max(m_1, m_2)}{m_j} > \mu$ or $\dfrac{min(p_{t1}^2, p_{t2}^2)}{m_j^2}\Delta R_{12}^2 < y_{cut}$ **repeat**

BDRS: tagging

$pp \to ZH \to \nu\nu bb$



G. Salam

$m_1 = 52$ GeV, $m_2 = 28$ GeV

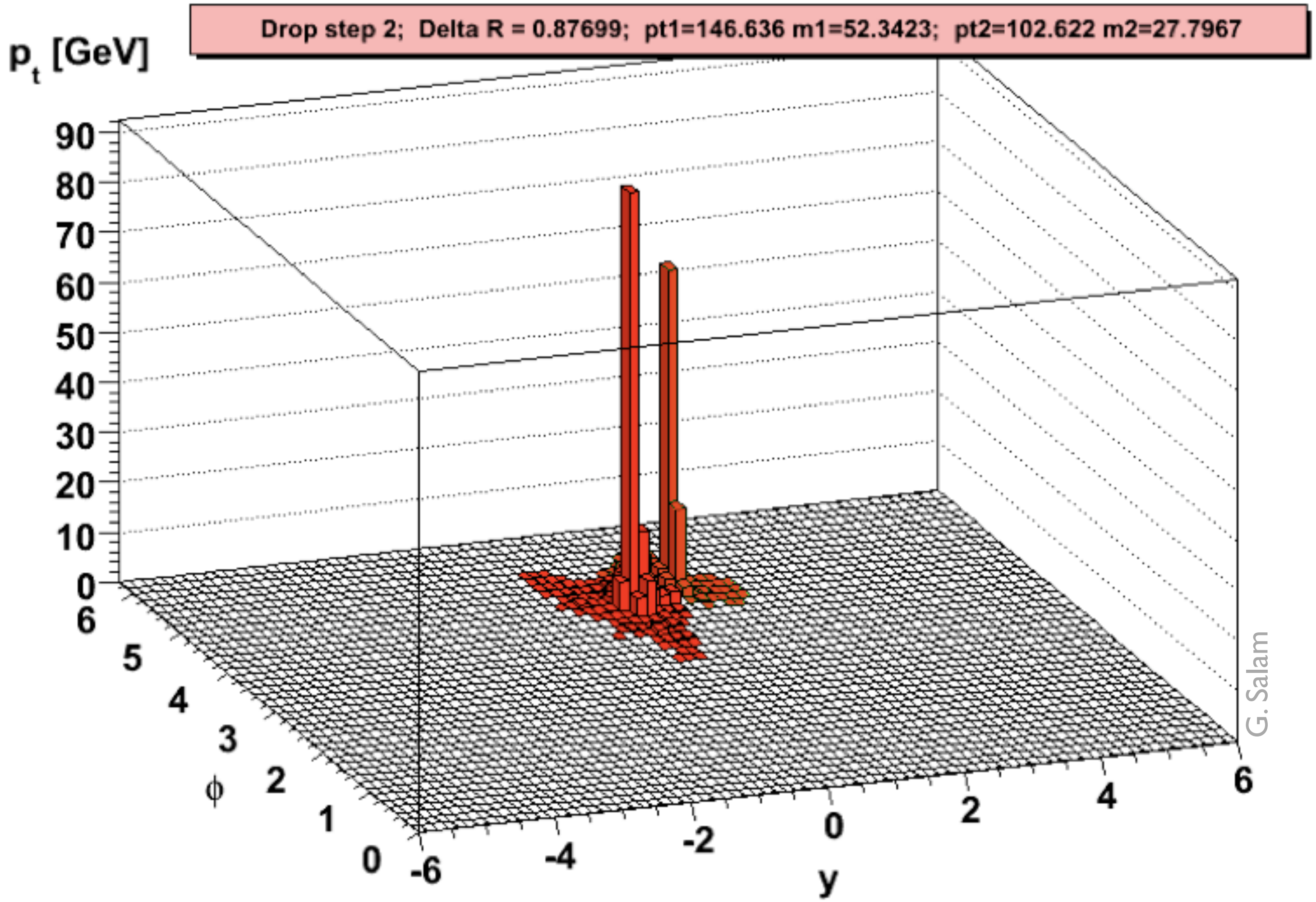
Stop when a large mass drop is observed

(and **recombine** these two jets)

[NB. Parameters used $\mu = 0.67$ and $y_{cut} = 0.09$]

BDRS: filtering

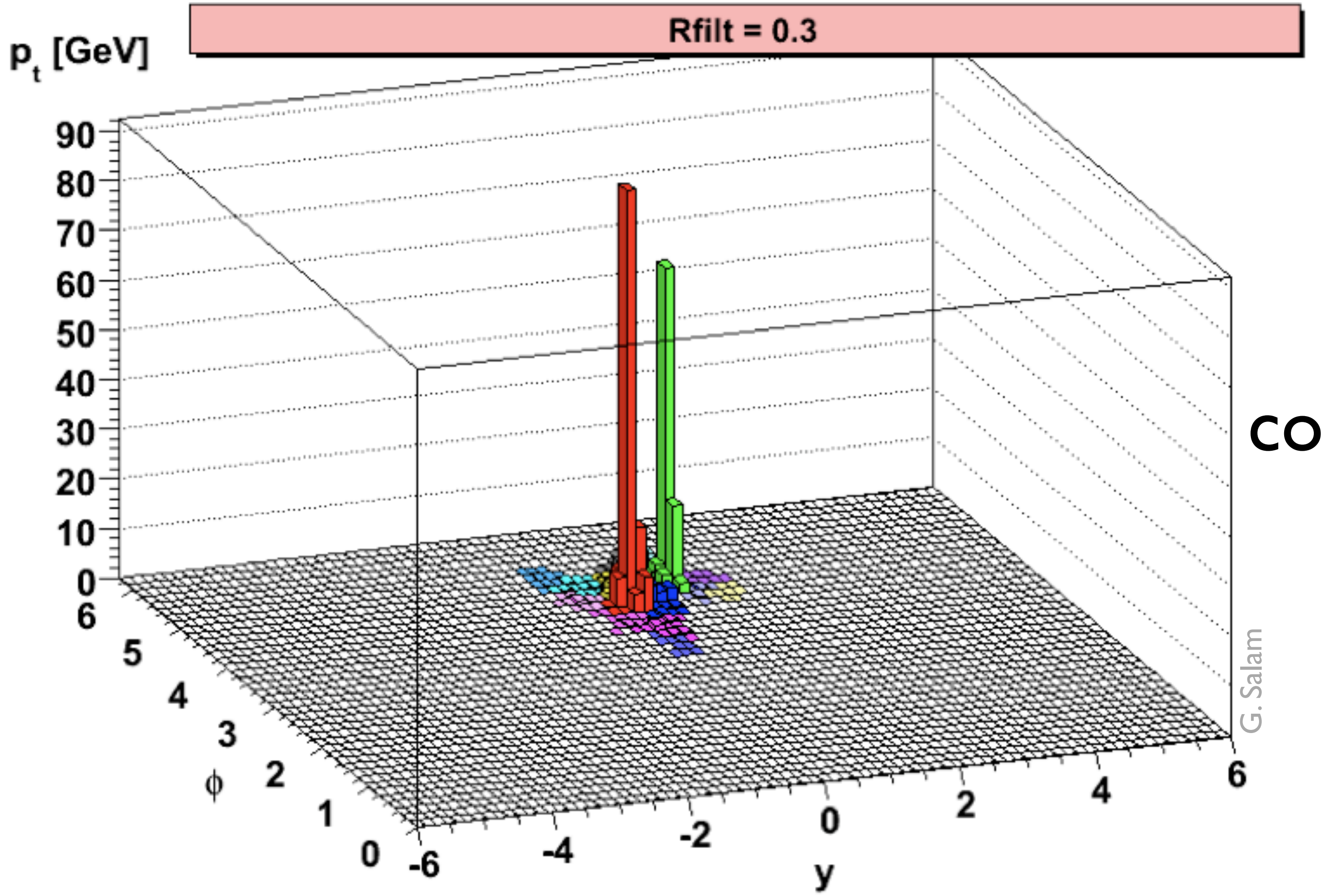
pp $\to$ ZH $\to$ $\nu\nu bb$



Start with the recombined jet

BDRS: filtering

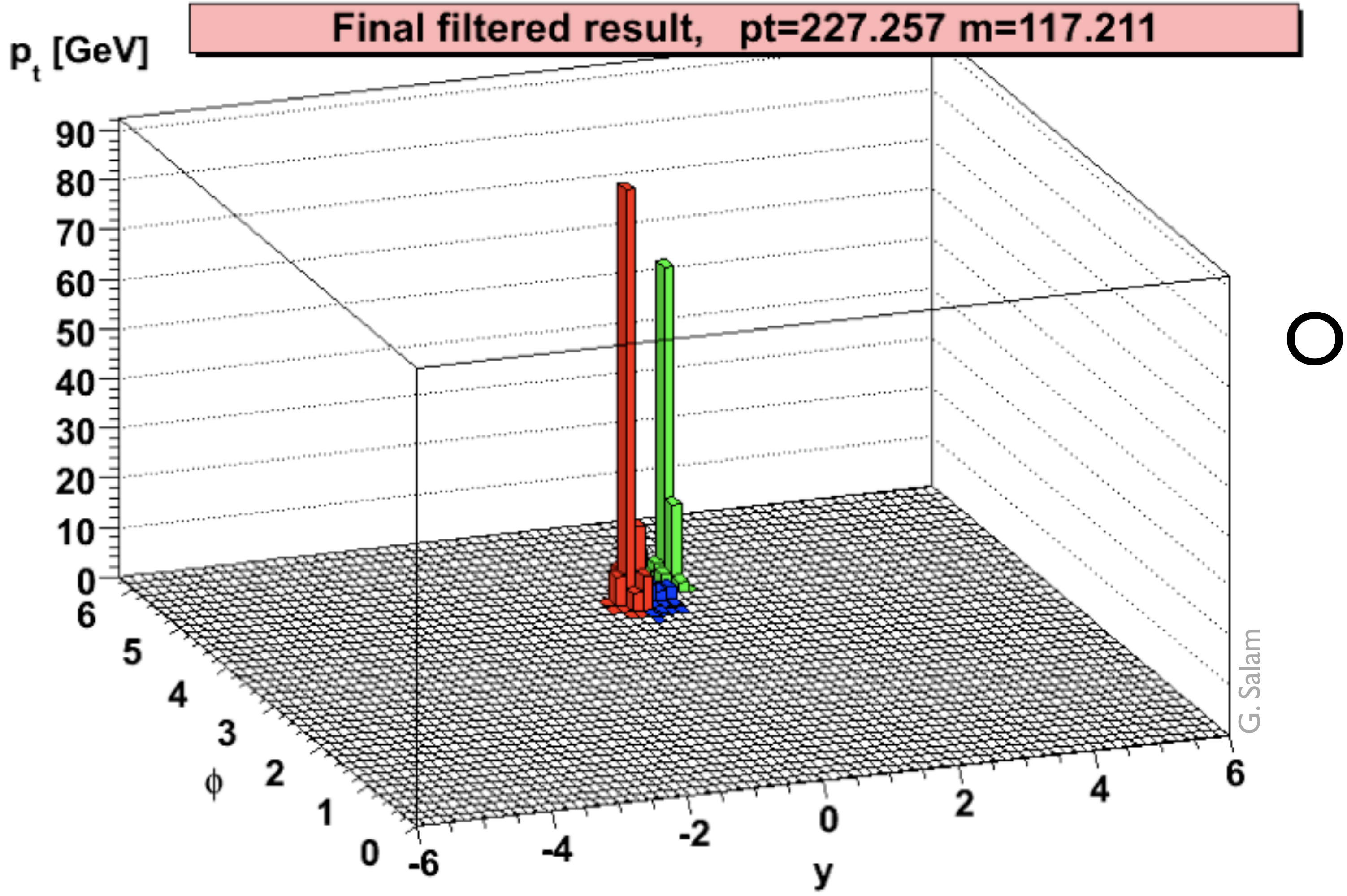
$pp \to ZH \to \nu\nu bb$



Recluster the contituents with R_{filt}

BDRS: filtering

pp →ZH → ννbb



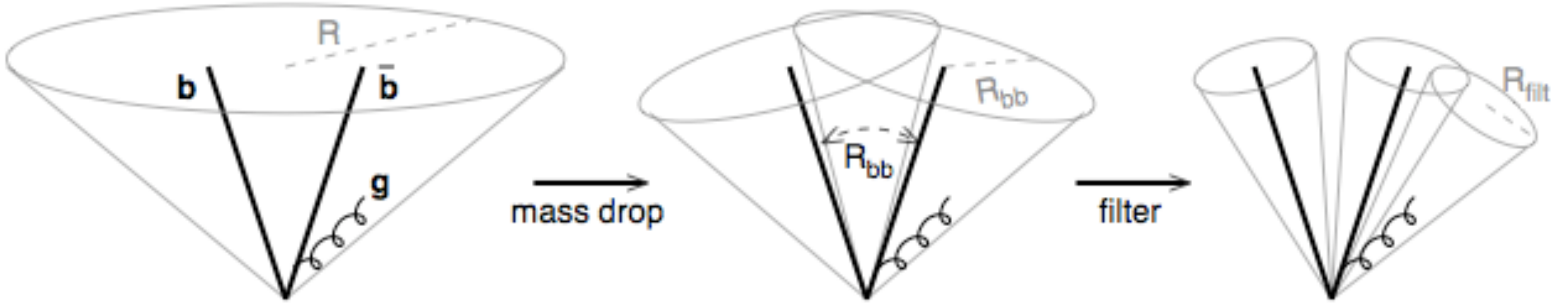
Only keep the n_{filt} hardest jets

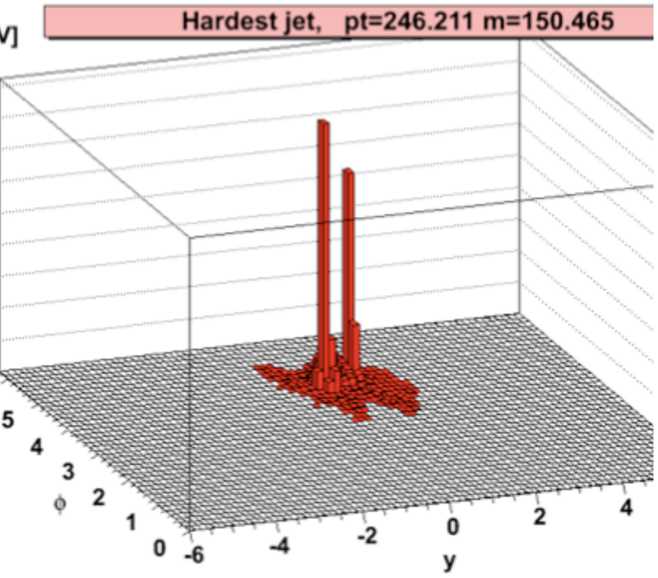
The low-momentum stuff surrounding the hard particles has been removed

$pp \to ZH \to \nu\bar{\nu}b\bar{b}$

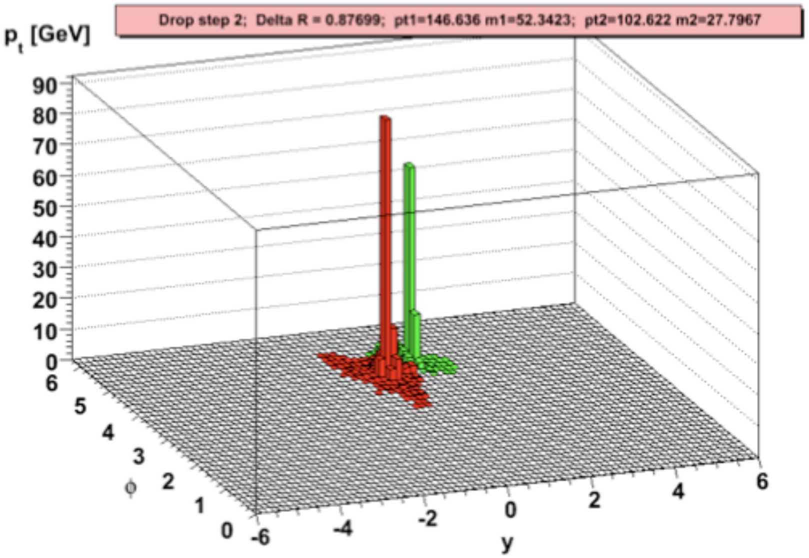
Visualisation of BDRS

Butterworth, Davison, Rubin, Salam, 2008

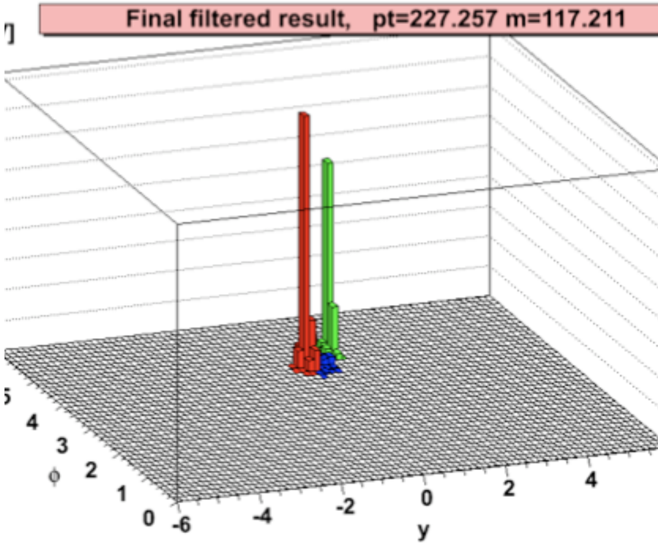




Cluster with a large R



Undo the clustering into subjets, until a large asymmetry/mass drop is observed: tagging step



Re-cluster with smaller R, and keep only 3 hardest jets: grooming step

Take-home messages

- A number of different IRC-safe jet algorithms exist
    - They all try to be good proxies for hard partons, but they have different characteristics, especially with respect to soft particles

- Jets from all algorithms inevitably suffer from pileup contamination
    - Techniques exist (though not covered today) to subtract it, either at jet-level, or at particle-level

- The past ~10 years have seen the emergence of jet-substructure analysis techniques, aimed at helping classifying a jet according to its origin (quark or gluon, heavy particle, etc)
    - These techniques have been successfully validated in Standard Model analyses and are being employed in BSM searches and even in heavy ion analyses.