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LECTURE 3: Circular accelerators

August 2021

Introduction to Particle Accelerator Science and Technology

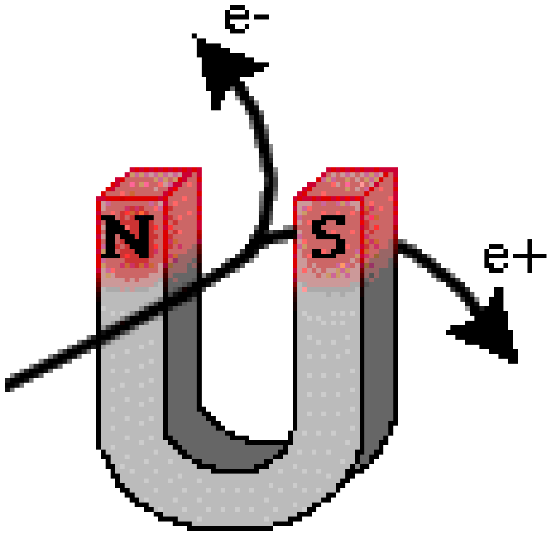
Limitations of linear accelerator technology

Can we use a linac to go to very high energy?

A quick calculation:
With the electric field achievable across drift tubes (limited by breakdown) we can make about 2 MeV per meter.

A linear LHC (7 GeV) would be long 3’500 km !

We need another trick: bending the particles in a magnetic field

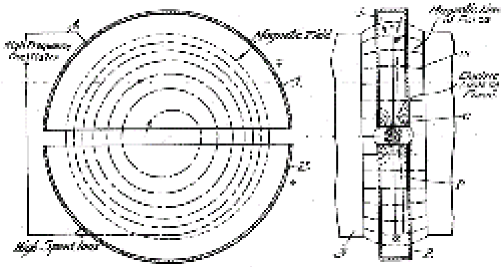


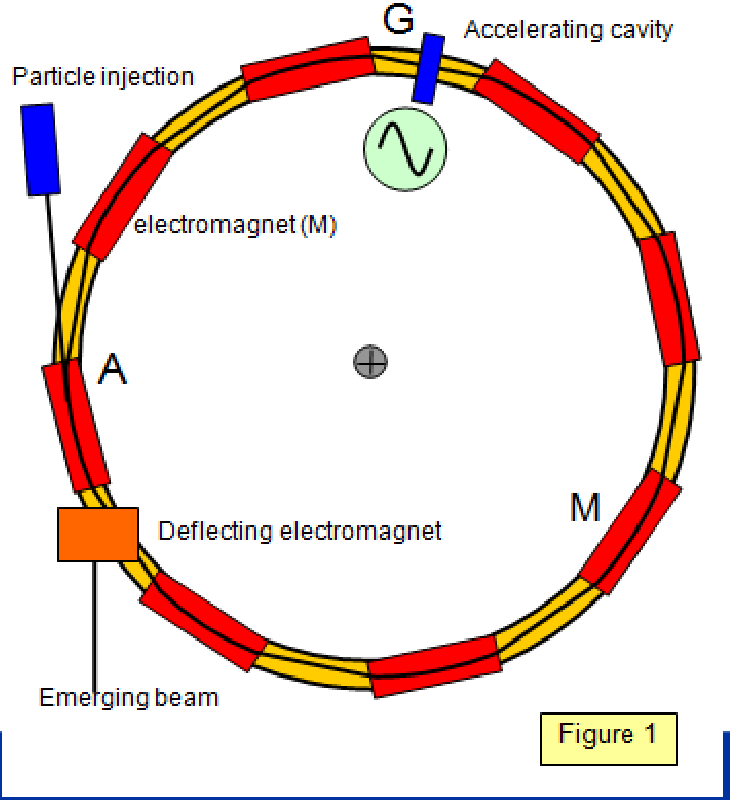
The synchrotron

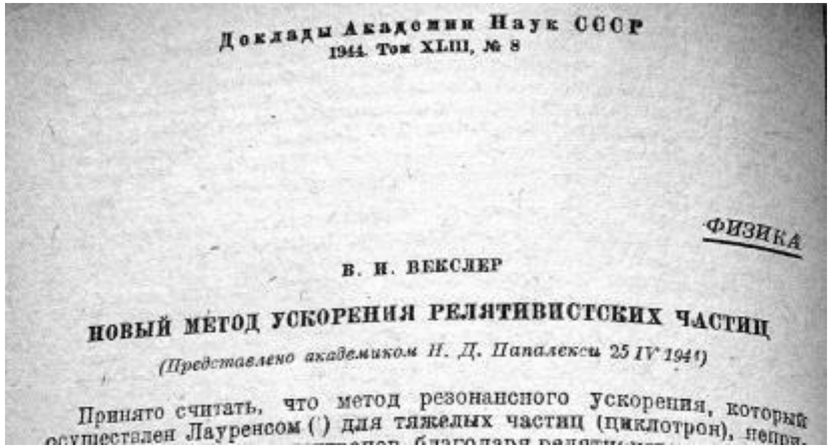
- The cyclotron is limited to particle energies in the «classic» regime ($\beta << 1$) and requires very large magnets.
- Can we imagine an accelerator that minimises the magnetic field region, and that allows going to relativistic energies?

The **synchrotron**: an accelerator where both **accelerating frequency** and **magnetic field** vary to keep synchronicity of the fields with an accelerated particle **moving on a closed orbit**.

Independently proposed by V. Veksler in the URSS (1944) and by M. Oliphant in USA (1943) and codified by E. McMillan at Berkeley (1945). 1st 8 MeV electron synchrotron UK, 1946.







Доклады Академии Наук СССР
1944. Том XLIII, № 8

ФИЗИКА

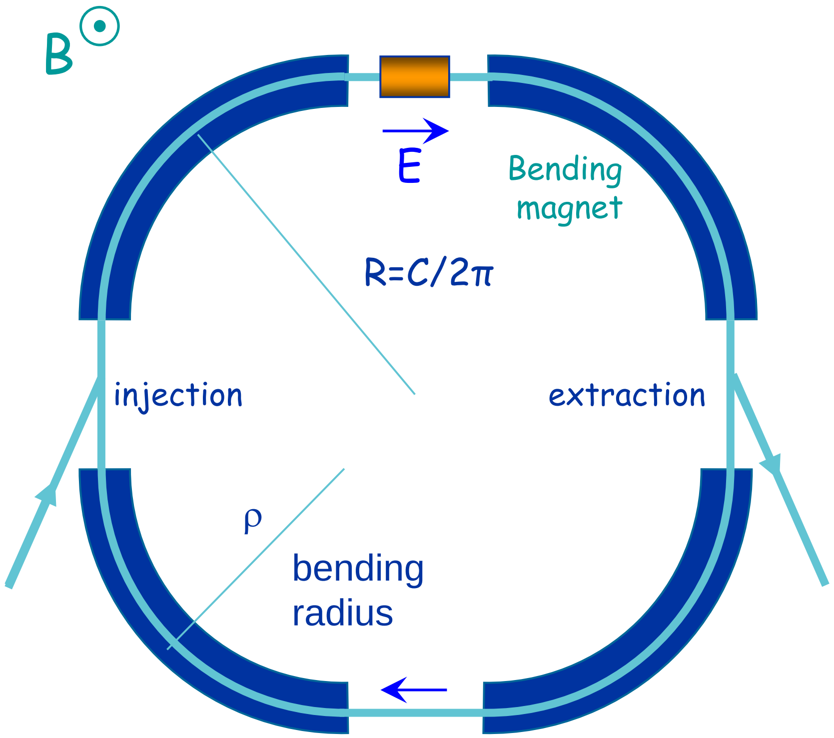
В. И. ВЕКСЛЕР

НОВЫЙ МЕТОД УСКОРЕНИЯ РЕЛЯТИВИСТСКИХ ЧАСТИЦ

(Представлено академиком Н. Д. Папалекси 25 IV 1944)

Принято считать, что метод резонансного ускорения, который осуществлен Лауренсом ( ) для тяжелых частиц (циклотрон), непри-

The synchrotron – some important relations



1. Constant orbit → magnetic field must increase during acceleration

Lorentz force $$F = \frac{mv^2}{\rho} = evB \quad \Rightarrow \quad \boxed{mv = p = eB\rho}$$

$$B\rho\,[\mathrm{Tm}] \approx \frac{p\,[\mathrm{GeV/c}]}{0.3}$$

ρ = bending radius (inside magnets)
p = relativistic momentum, $m=\gamma m_0$

2. Acceleration → RF frequency must be a multiple of revolution frequency and increase with energy

$$\boxed{T = \frac{2\pi R}{v},\, f_r = \frac{v}{2\pi R}} \qquad \boxed{f_{RF} = hf_r = h\frac{v}{2\pi R}}$$

h = harmonic number (integer), number of RF cycles per revolution
It corresponds to the maximum number of bunches in the synchrotron

Example of the LHC:

f_r = 27km/c = 11 kHz
f_{RF} = 400 MHz = 35640 f_r
→ the LHC can accelerate 35460 bunches (but only 2556 "slots" are used)

The synchrotron can reach high energies by passing **millions of times** through the accelerating gaps(s), but because the B field has to be quickly "**ramped**" it can operate only in "**pulsed**" mode (while the cyclotron can run CW!) with a beam intensity per pulse limited by the number of bunches in the ring.

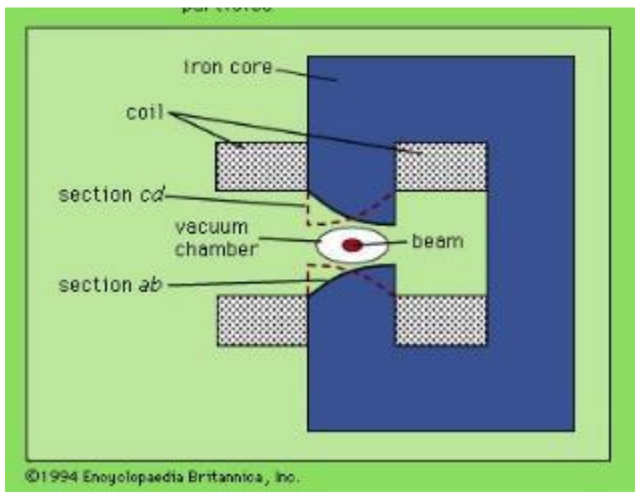
Note: ρ < R , usually dipoles cover only about 2/3 of the ring!

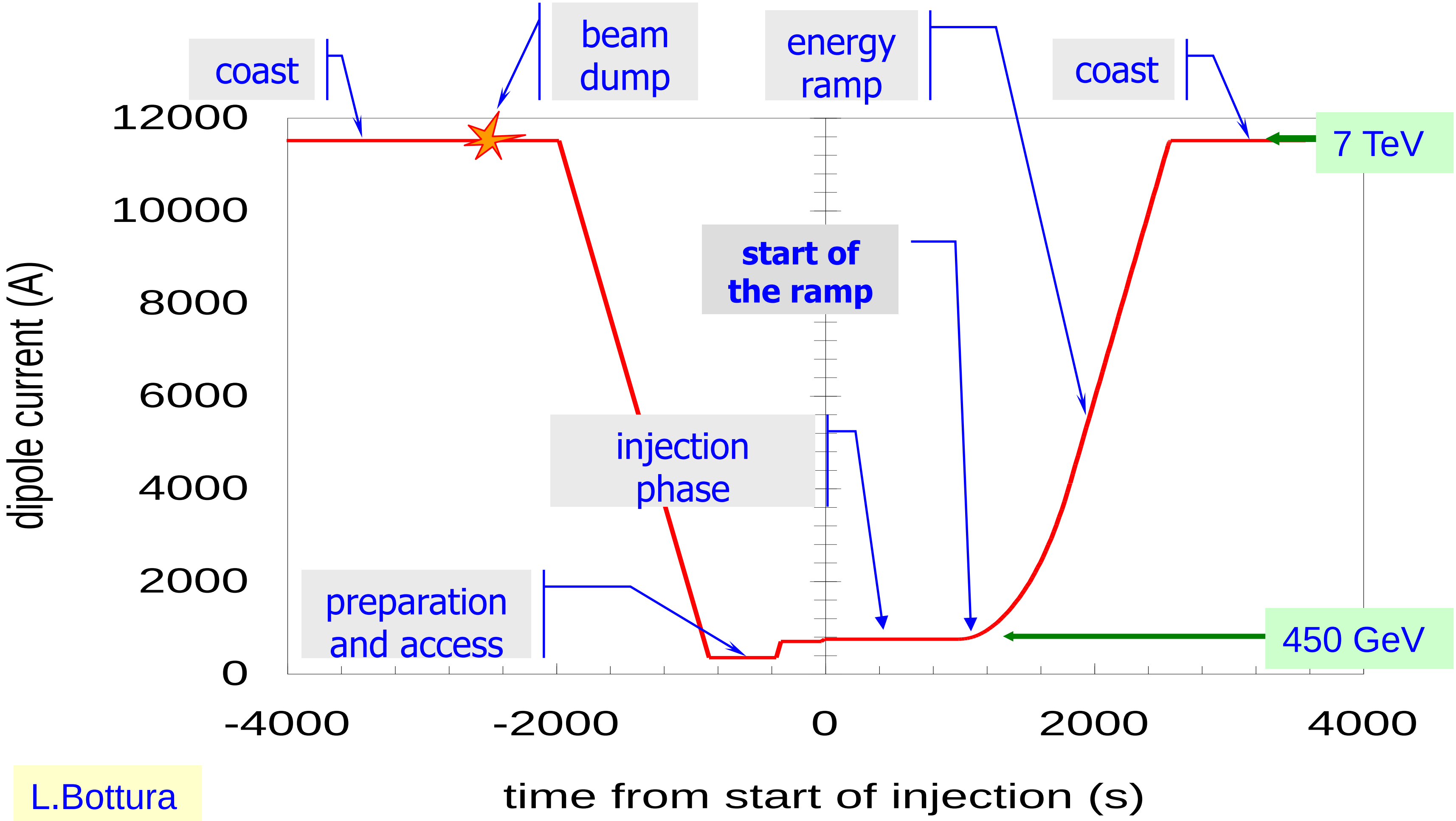
The Synchrotron – LHC Operation Cycle

The magnetic field (dipole current) is increased during the acceleration.

$$\boxed{p = eB\rho}$$

In a conventional dipole magnet (below), $B \propto I$





L.Bottura

The synchrotron – Frequency change

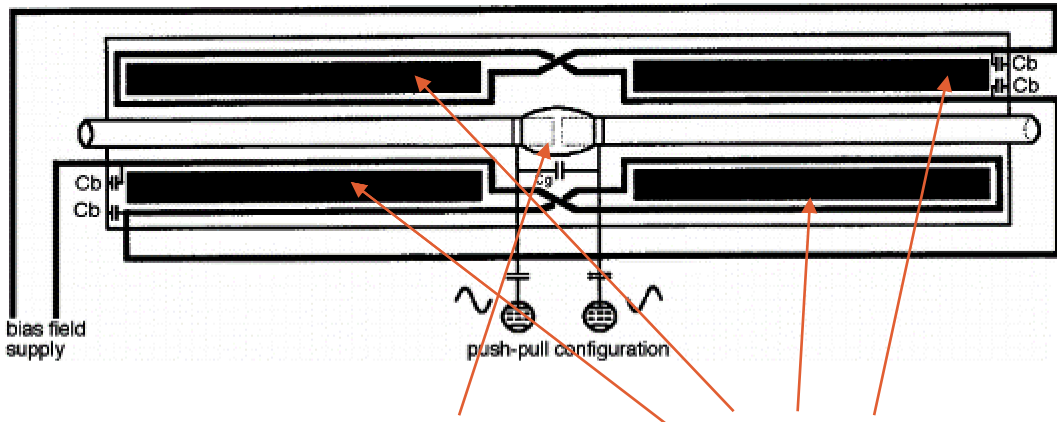
During the energy ramping, the RF frequency increases to follow the increase of the revolution frequency :

$$\frac{f_{RF}(t)}{h} = \frac{v(t)}{2\pi R_s} = \frac{1}{2\pi}\frac{ec^2}{E_s(t)}\frac{\rho}{R_s}B(t)$$

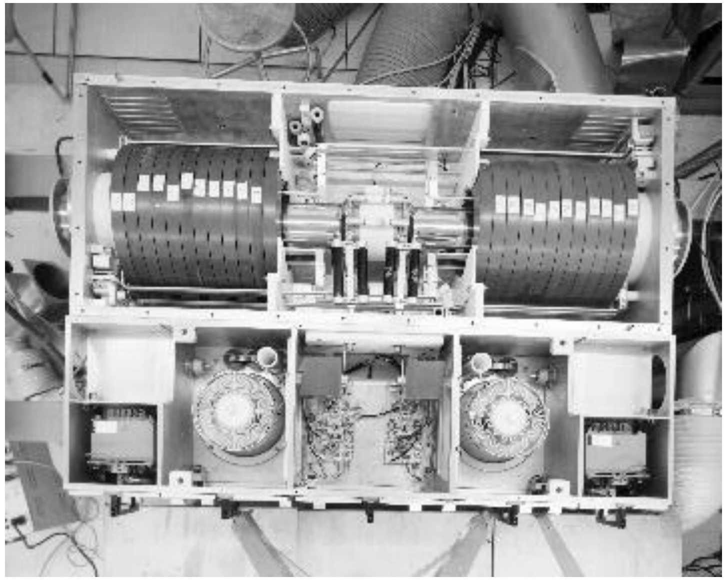
The RF frequency must follow a «frequency programme» during acceleration, defined by B-field

Variation of the frequency is technically challenging for RF cavities and can be achieved only in a small range, that defines the minimum/maximum energy of a synchrotron !

Example: the CERN PS Booster cavity with wide tuning range

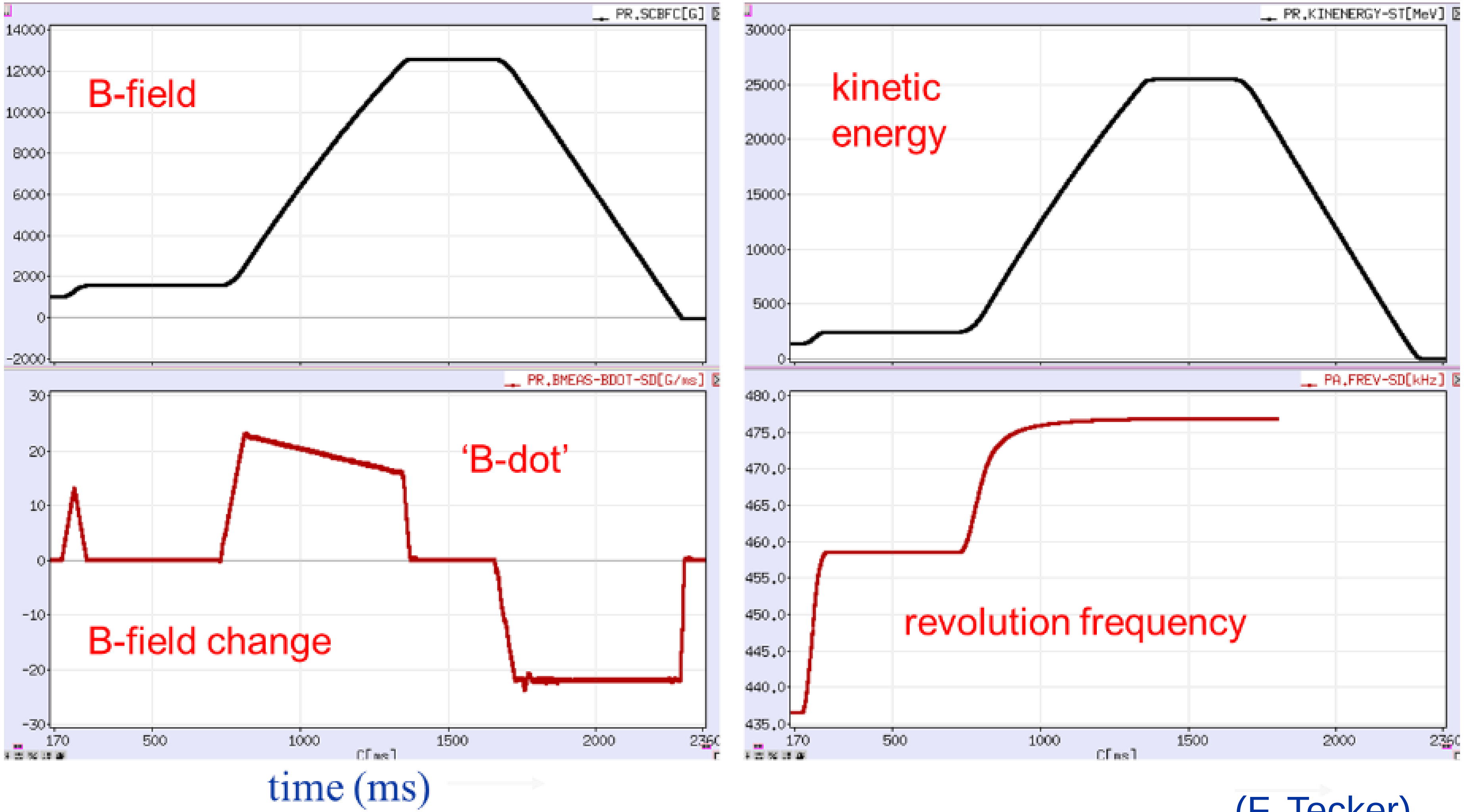


Tuning range:
3 to 8 MHz



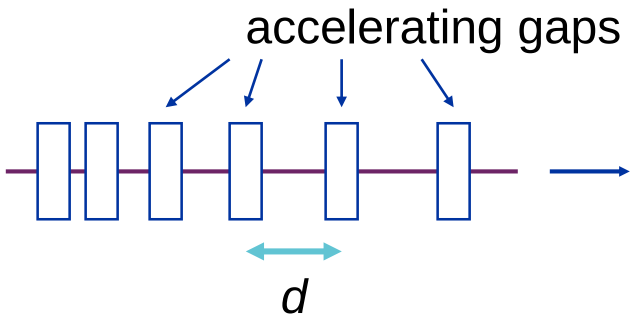
Example: CERN PS Field and Frequency change

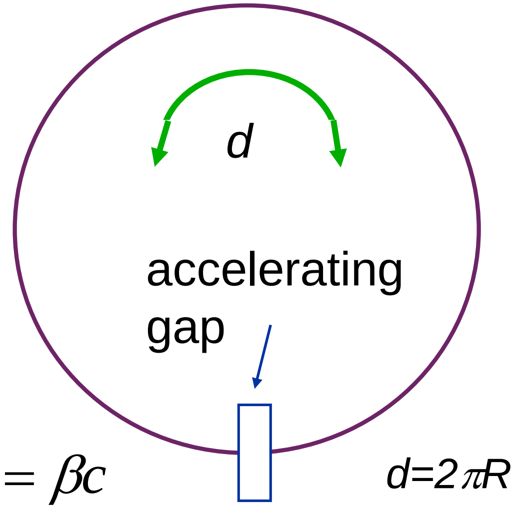
During the energy ramping, the B-field and the revolution frequency increase



(F. Tecker)

Linear and circular accelerators





$d=\beta\lambda/2$=variable

$$d = \frac{\beta c}{2f} = \frac{\beta\lambda}{2}$$

$$2d\,f = \beta c$$

$d=2\pi R$=constant

Linear accelerator:

Particles accelerated by a sequence of gaps (all at the same RF phase).

Distance between gaps increases proportionally to the particle velocity, to keep synchronicity.

Used in the range where β increases. "Newton" machine

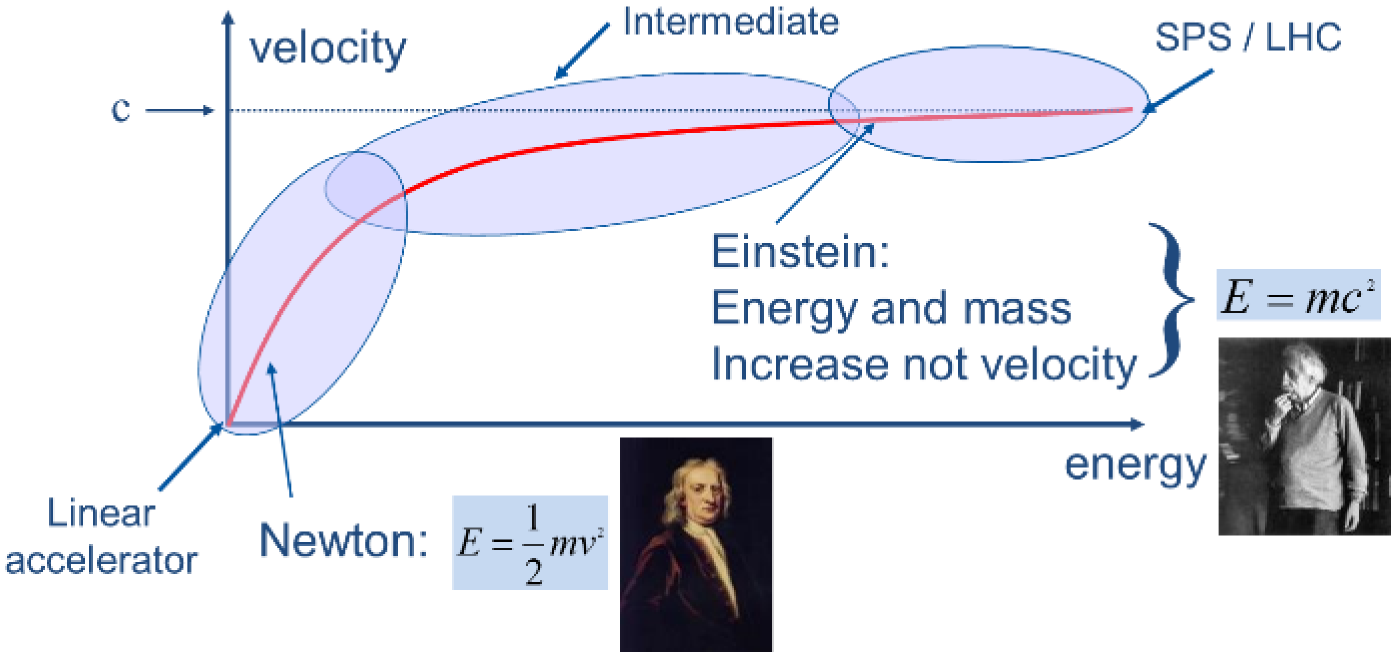
Circular accelerator:

Particles accelerated by one (or more) gaps at given positions in the ring.

Distance between gaps is fixed. Synchronicity only for β~const or varying (in a limited range!) the RF frequency.

Used in the range where β is nearly constant. "Einstein" machine

Newton and Einstein acceleration – what in between?



Linacs
Cyclotrons

Linacs
Small synchrotrons
Synchrocyclotrons

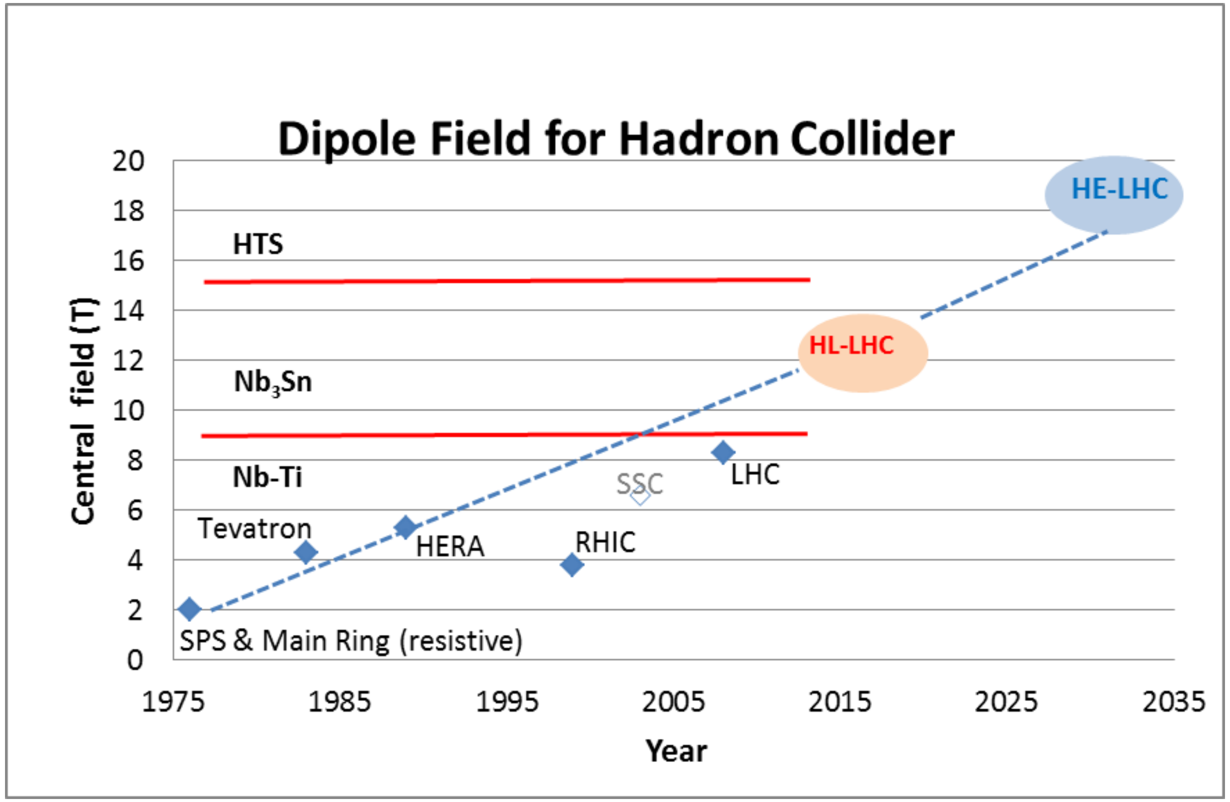
Synchrotrons

In the intermediate region we can use long linear accelerators or small synchrotrons.

Linear accelerators are more expensive but can accelerate a larger number of particles.

Synchrotrons can accept a small variation of velocity by changing the frequency of their RF accelerating system (within some limits).

The magnetic field limitation

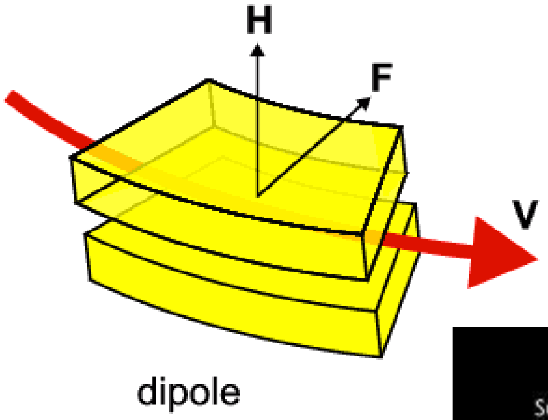


The final limitation to the size and to the energy of a synchrotron comes from the **magnetic field that can be achieved**.

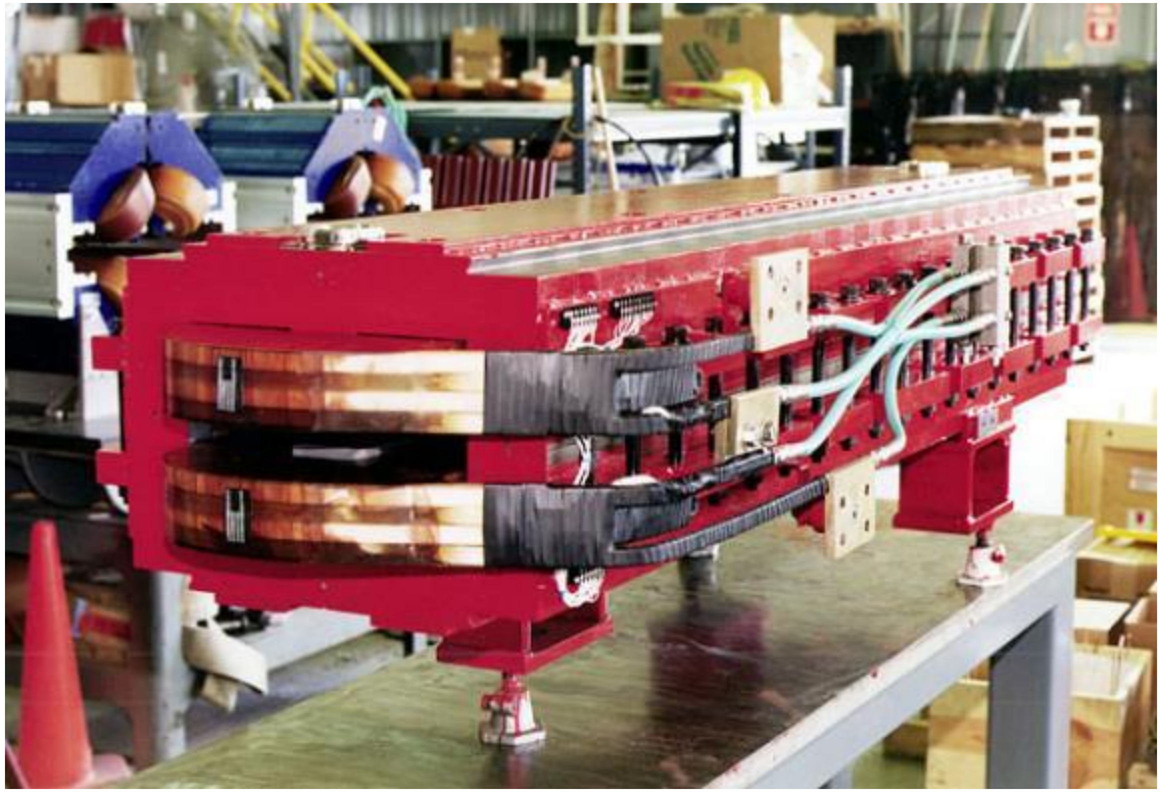
Technological limit:
about 2 T normal conducting magnets
8 T Nb-Ti superconducting magnets
12 T future Nb3Sn superconducting magnets

$$B\rho = \frac{p}{e} \approx \frac{E}{ce} \text{ so } E[\text{GeV}] \approx 0.3\, B[\text{T}]\, \rho[\text{m}] \text{ per unit charge}$$

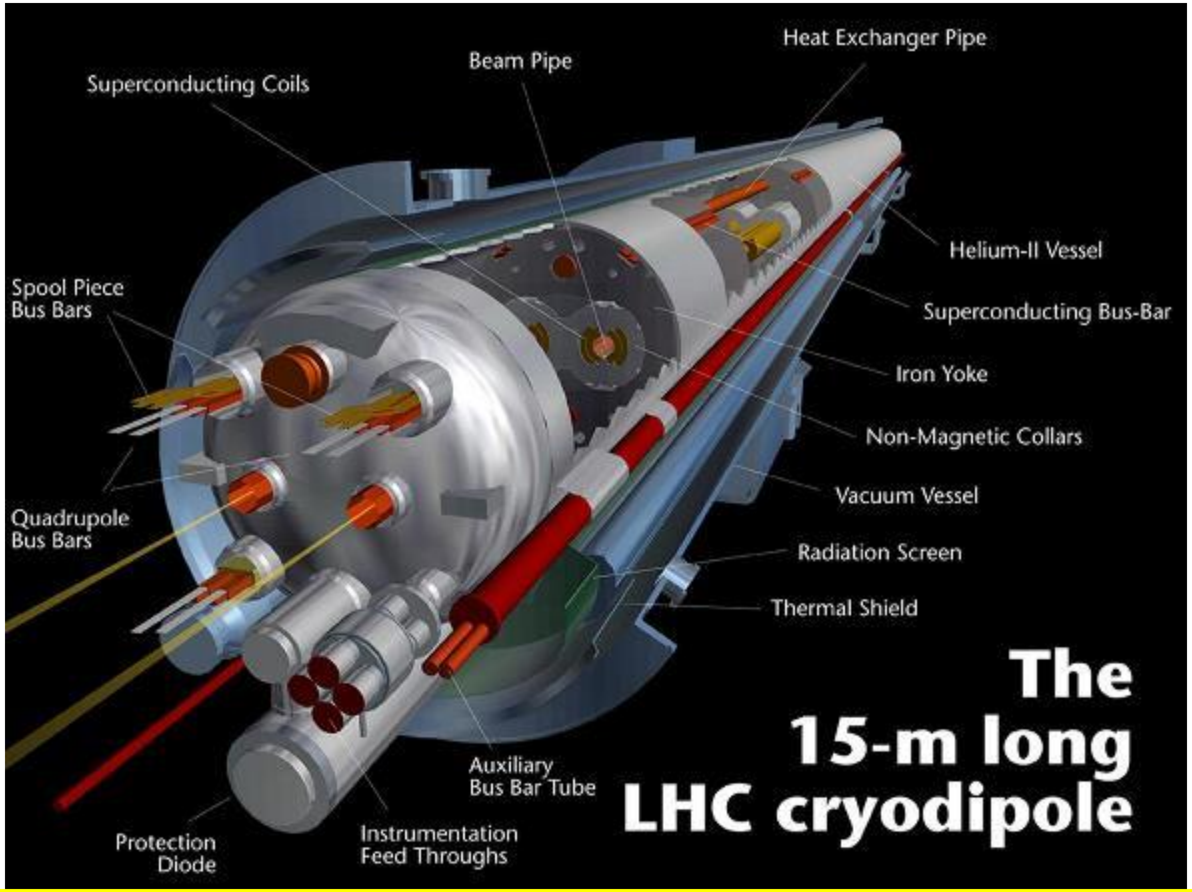
The magnets



Normal conducting

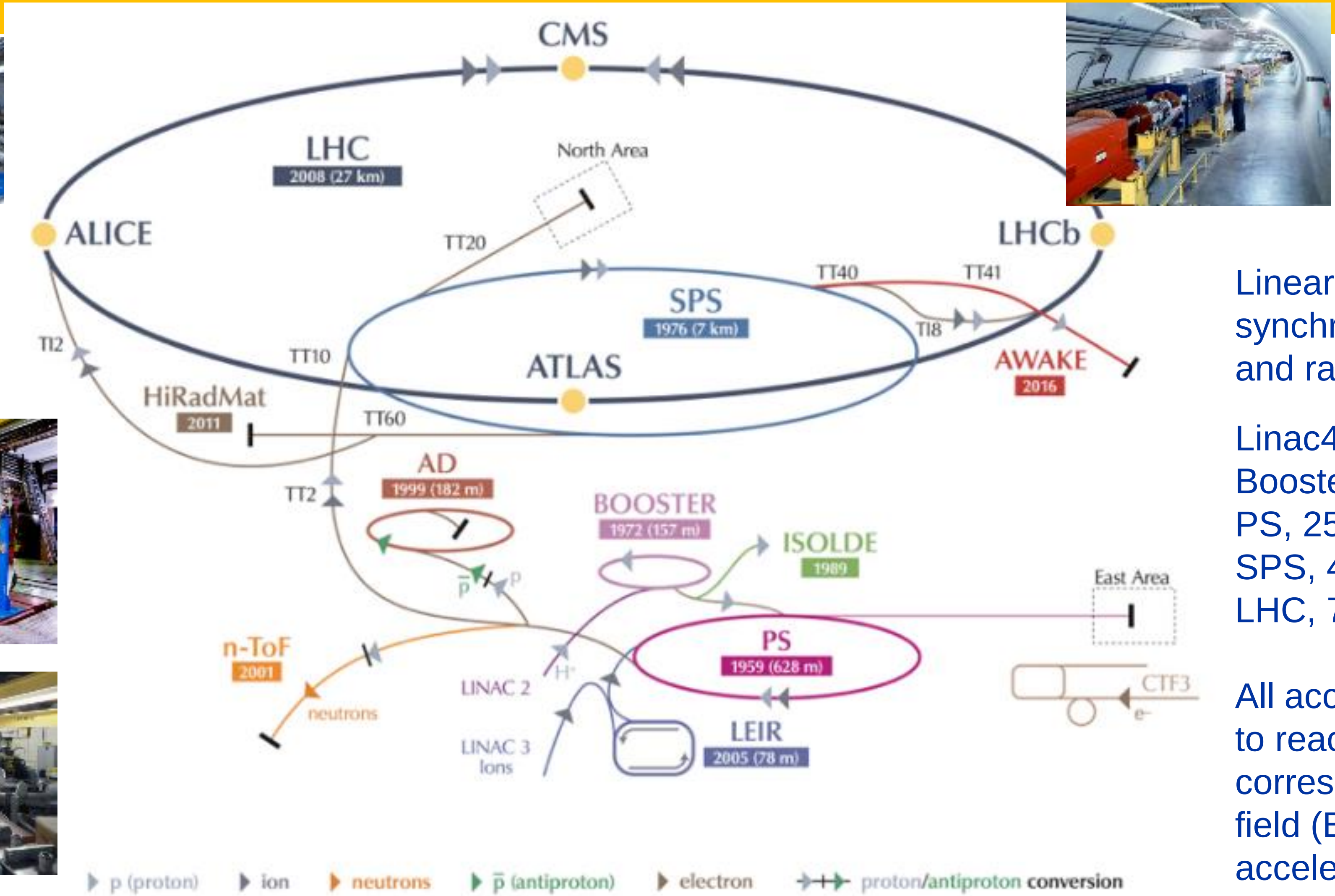


Superconducting



15 metres length, maintained at -269 degrees by a flow of liquid helium. Magnetic field 8 T, 5 times more than conventional magnets and 10’000 times a small house magnet

The CERN chain of accelerators



Linear Accelerator plus a chain of synchrotrons of increasing energy and radius:

Linac4, 160 MeV, β=0.52
Booster, 2 GeV, β=0.948
PS, 25 GeV, β=0.9994
SPS, 450 GeV, β=0.999998
LHC, 7 TeV, β=0.999999991

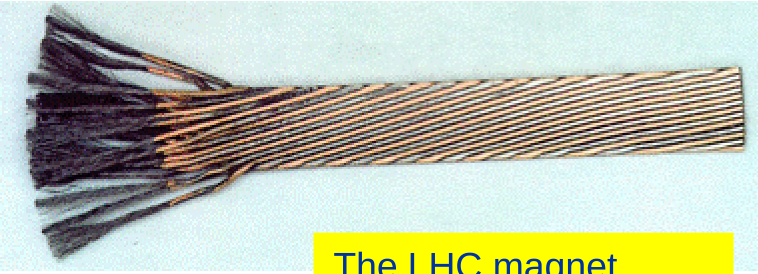
All accelerators ramp the B-field to reach the energy corresponding to their maximum field ($B\rho \sim p$) but high energy accelerators have almost constant RF frequency ($\Delta f \sim \Delta\beta$)

A linear accelerator is always required as injector to limit the frequency swing in the first synchrotron (and to increase the beam intensity!)

Superconductivity and particle accelerators

Some materials present a zero electrical resistance when cooled below a characteristic temperature.
Discovered in 1911, explained in 1958, started to be used for accelerators in the 1970's.
Allows to build magnets that can stand higher electric currents and higher fields (not limited by water cooling) and accelerating RF cavities that do not dissipate power and have higher electrical efficiency.

Materials used in accelerators are
Niobium-Titanium for magnets
Niobium for RF cavities.



The LHC magnet superconducting cable

Clean room assembly of superconducting RF cavities

One of the 8 compressor units of the 4.5 K refrigerator for LHC

BUT: a superconducting accelerator requires a huge cooling system
That keeps all elements at liquid helium temperature

Transverse Beam Dynamics

Particle production – the sources

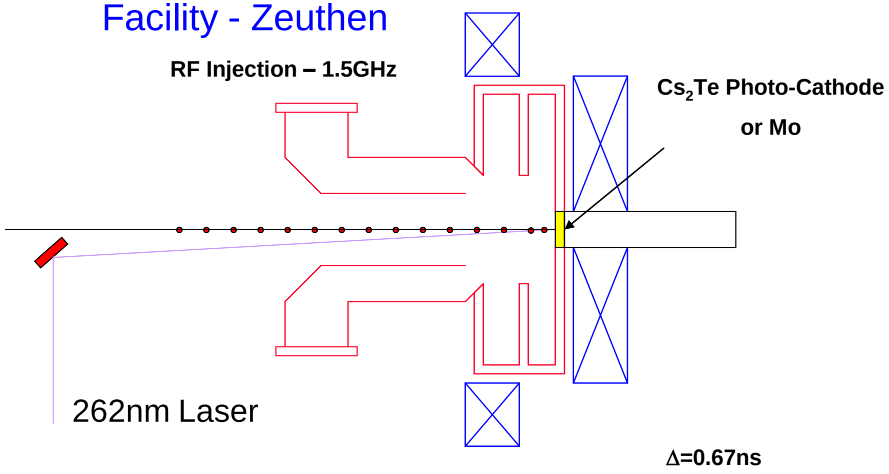
Electron sources:

give energy to the free electrons inside a metal to overcome the potential barrier at the boundary.

Used for electron production:

- thermoionic effect
- laser pulses
- surface plasma

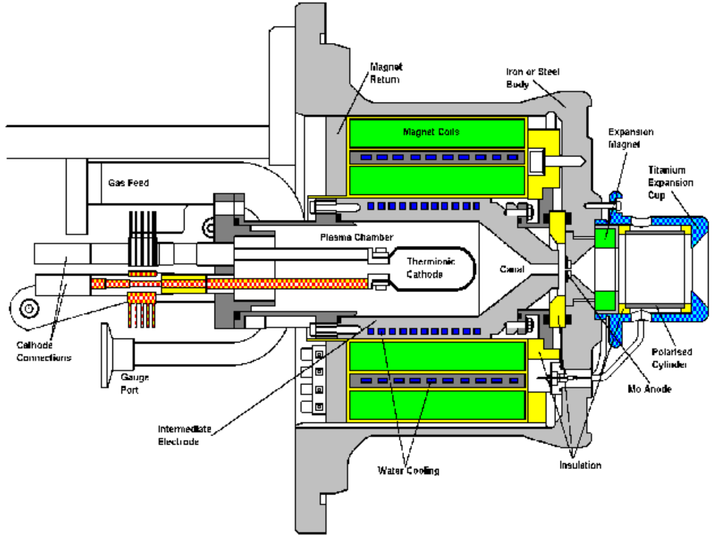
Photo Injector Test
Facility - Zeuthen



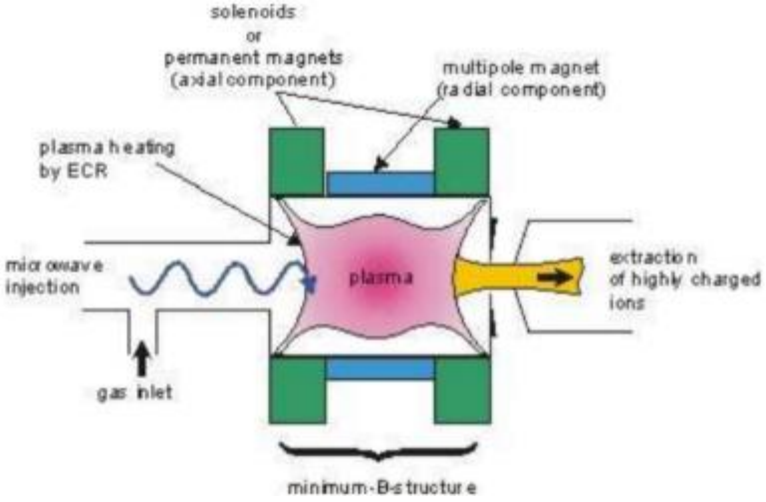
Ion sources:

create a plasma and optimise its conditions (heating, confinement and loss mechanisms) to produce the desired ion type. Remove ions from the plasma via an aperture and a strong electric field.

CERN Duoplasmatron
proton Source



ECR Ion Source



From a hydrogen bottle to the particle beam

Scheme of the ion source delivering protons to all CERN accelerators

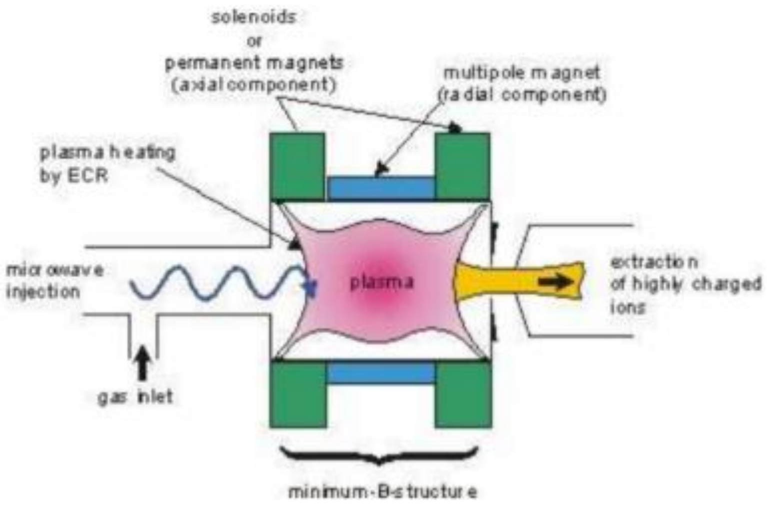
Where do the protons come from?
The ion source is fed by a bottle of industrial hydrogen

A 5 kg bottle of hydrogen contains 3’000’000’000’000’000’000 billions of protons!

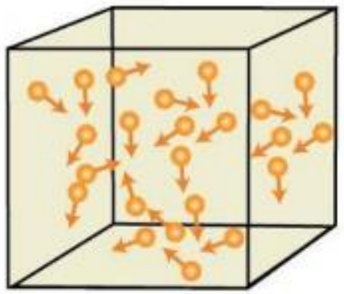
And the LHC at CERN needs only 1’200’000 billions of protons per day.

The ion source at the CNAO hadron therapy facility

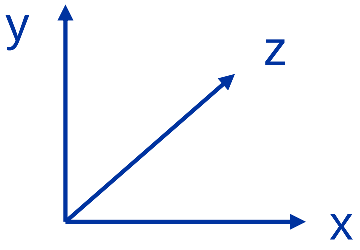
Beam properties out of the ion source



In the ion source, the beam is extracted from a plasma that has its own “**temperature**”:
The extracted particles are in (chaotic) motion, with a given velocity distribution and a given spatial distribution around some “mean” values.



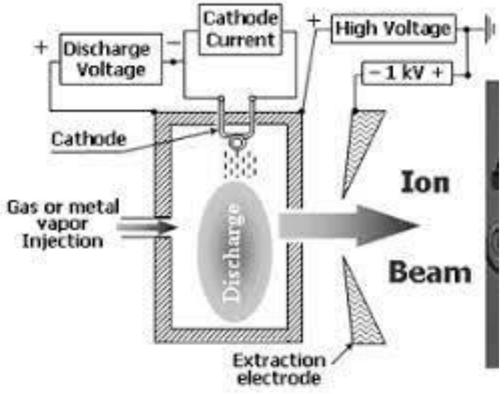
The ion source designer makes a huge effort in controlling and minimising the **velocity and position dispersion**, but some dispersion will always remain, related to the plasma properties and aggravated by the Coulomb repulsion of the particles inside the beam.



Particle accelerator convention:

- *z is always the “acceleration” axis of the beam*
- *(x,y) define the «transverse» plane (to the direction of motion)*

The **initial acceleration** through a high voltage (10 – 50 kV) at ion source extraction overimposes a longitudinal velocity component (along z) to the “chaotic” motion of the particles.



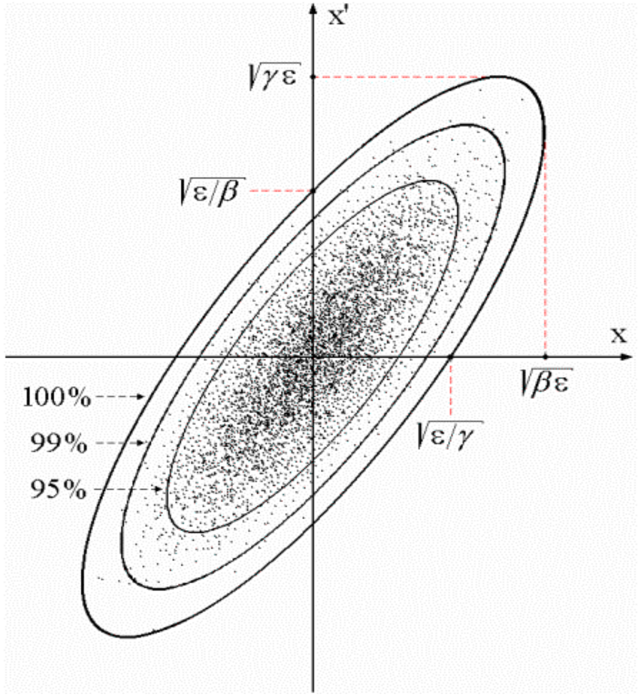
The concept of “beam emittance”

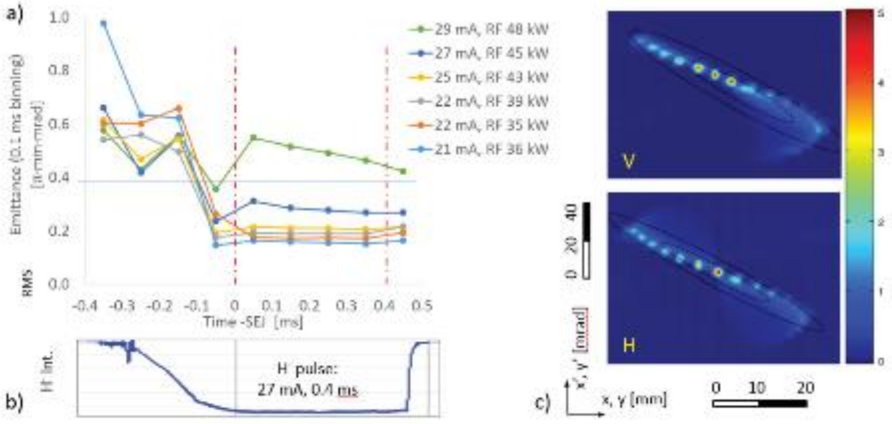
To describe the collective properties of the particles inside a beam is introduced the concept of **beam emittance(s)**

A particle beam at a given position is characterised by three distributions:

- ❑ position and velocity in x: **(x, x’)**
- ❑ position and velocity in y: **(y, y’)**
- ❑ position and velocity in z, referred to the synchronous particle: **(ΔW, Δϕ)**

→ every particle is described by a **6-dimension vector** (x, x’, y, y’, ΔW, Δϕ) and the “ensemble” of all particles is described by a given **volume in the 6D phase space**. The 3 emittances in (x, y z) are the projection of the 6D volume in the 3 planes corresponding to the 3 coordinates.





Transverse beam emittance optimisation and measured emittance for the Linac4 ion source

In most cases the beam distribution is gaussian or can be approximated with a gaussian → beam emittance is an ellipse.

In real cases where the distribution has a special shape, it can be described with an ellipse surrounding the particles.

→ The quality of the beam distribution can be defined by a **number**, ε that is equal to the **area of the elliptical beam distribution** (for a given fraction of the beam)

More on beam emittance

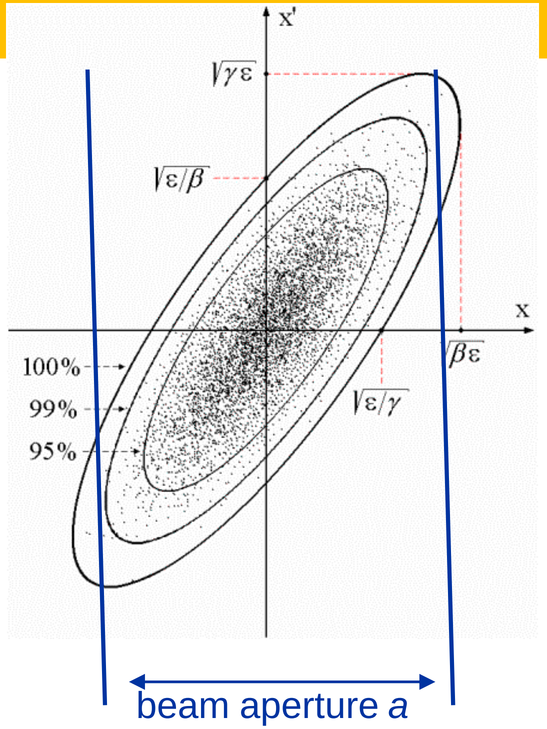
The emittance is important because the **beam brightness**, i.e. the ratio beam current over transverse emittance **I/ε** represents the density of the particle beam and is one of the **main figures of merit** of an accelerator for its key applications. It shows how good it is in concentrating energy for its final application. Our goal is to build accelerators with small emittance.

The emittance is a distribution in the phase space of the particles that is subject to the **Liouville's theorem**: the phase-space distribution function is constant along the trajectories of a conservative Hamiltonian system. Conservative=subject only to linear forces.

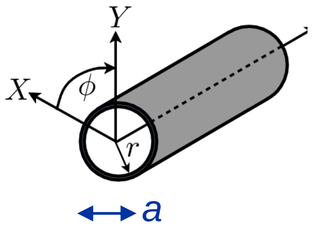
Consequence: the emittance will be **preserved during acceleration** (starting from what is generated in the ion source) unless non-linear forces come to play.

Unfortunately, some of the forces generated by the Coulomb repulsion inside the beam bunch are **non-linear** and lead to emittance increase. Similarly, errors in the synchrotron fields or other effects are non-linear and can increase the emittance.

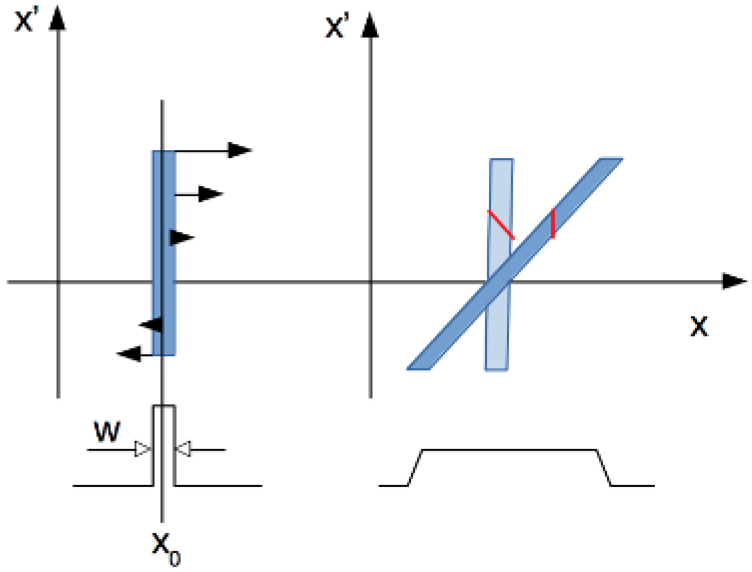
Conclusion: one of the main goals of an accelerator designer is to **minimise emittance growth through all the processes of the accelerator**.



The emittance is also important because it will define the **amount of beam loss** going through an aperture a (e.g. a beam pipe, a gap, …)



Principles of beam transport and focusing

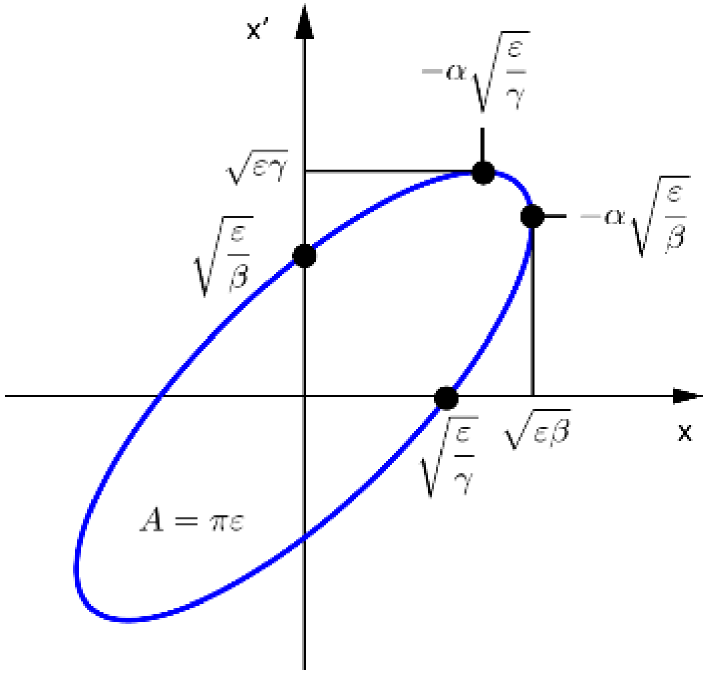


- In early cyclotrons and synchrotrons, some focusing was generated by shaping the magnetic field: “weak focusing”.
- In early linacs, some bars applied to the drift tubes provided some limited focusing.
- In all cases, huge loss of particles!

Evolution in phase-space of a thin vertical rectangle (x,x’) moving in a drift space, without other forces. Emittance is conserved, but transverse dimensions of the beam will constantly increase.

During acceleration we need to apply some external forces to keep under control the transverse dimensions of our beam

Beam focusing

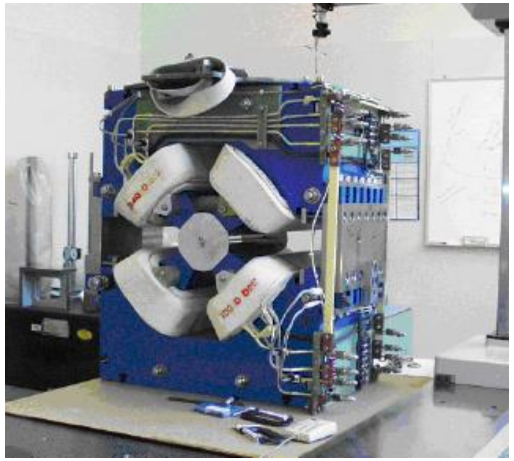


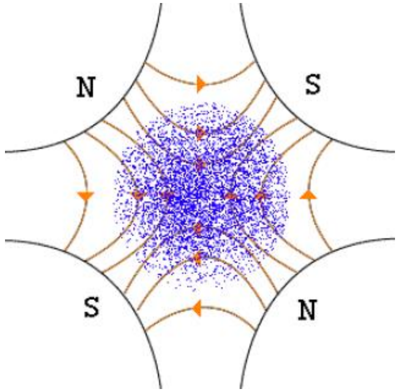
$\alpha, \beta, \gamma, \varepsilon$ are the “Courant-Snyder” parameters defining the beam emittance. ε represents the numerical value of the emittance

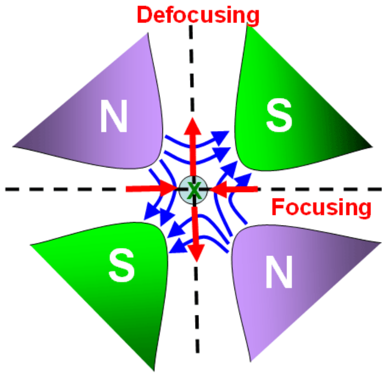
Alternating Gradient (“Strong”) Focusing

Invention (1952) by E. Courant, M. Livingston, H. Snyder, with an earlier enunciation by N. Christofilos

The net effect on a particle beam passing through alternating field gradients is to make the beam converge







www.globalsino.com/EM/

Insert in the beam trajectory electromagnetic quadrupoles of alternate polarity:

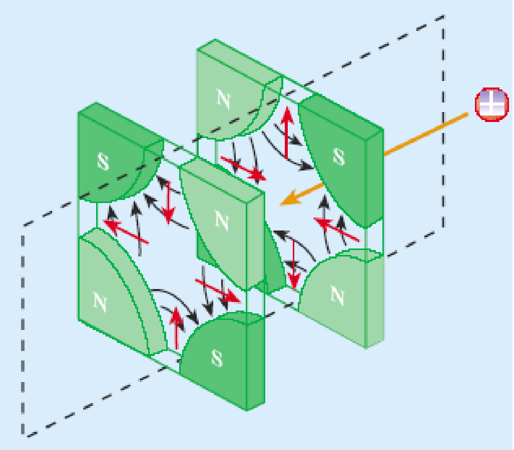
- focusing in x, defocusing in y
- focusing in y, defocusing in x

Quadrupoles:

- have a magnetic field linearly increasing with x or y
- i.e. they have constant “gradient” *g*

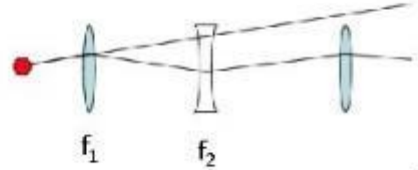
$$B_y = g\,x \qquad g = \frac{dB_y}{dx}$$

«restoring» force $\mathrm{F} = e v_z g\, x$



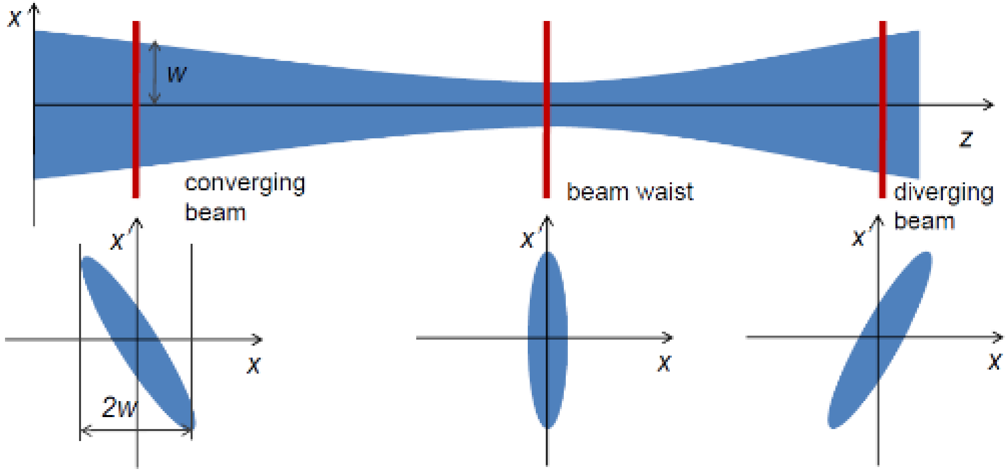
Thin lenses approximation of a strong focusing channel

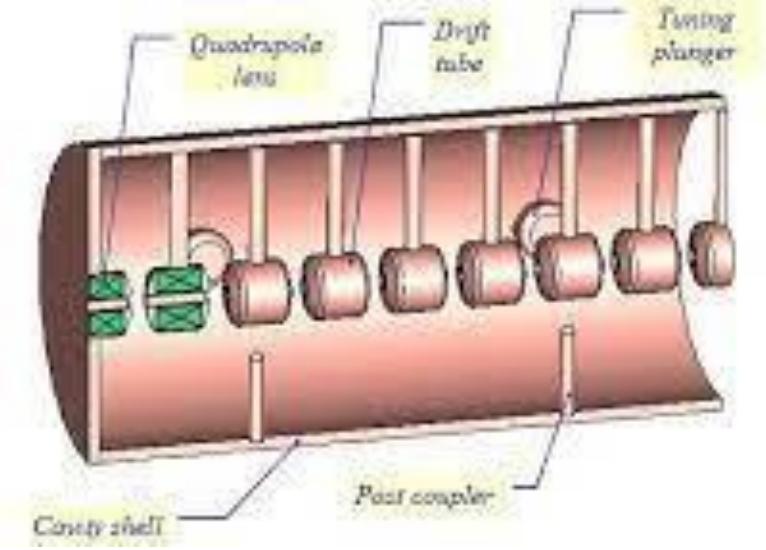
- Analogous to geometrical optics: a series of alternating focusing and defocusing lenses will focus.



$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

Consider $f_1=f$, $f_2 = -f$ → $\boxed{F = d/f^2 > 0}$

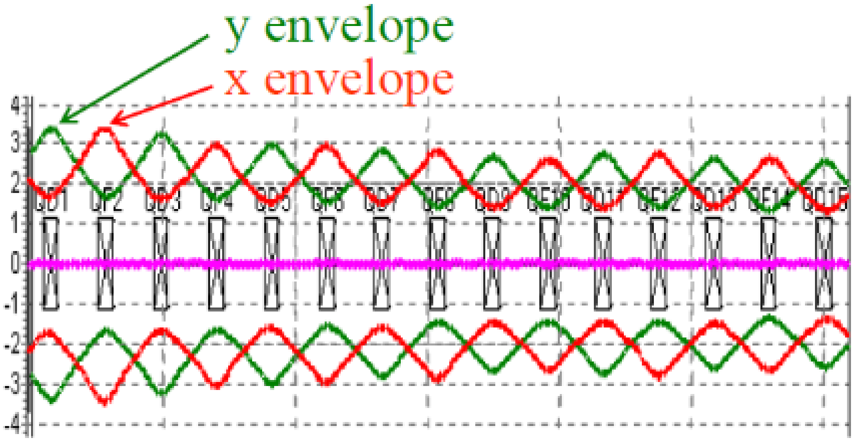




Example: focusing in a Drift Tube Linac with Permanent Magnet Quadrupoles

Transverse («betatron») oscillations

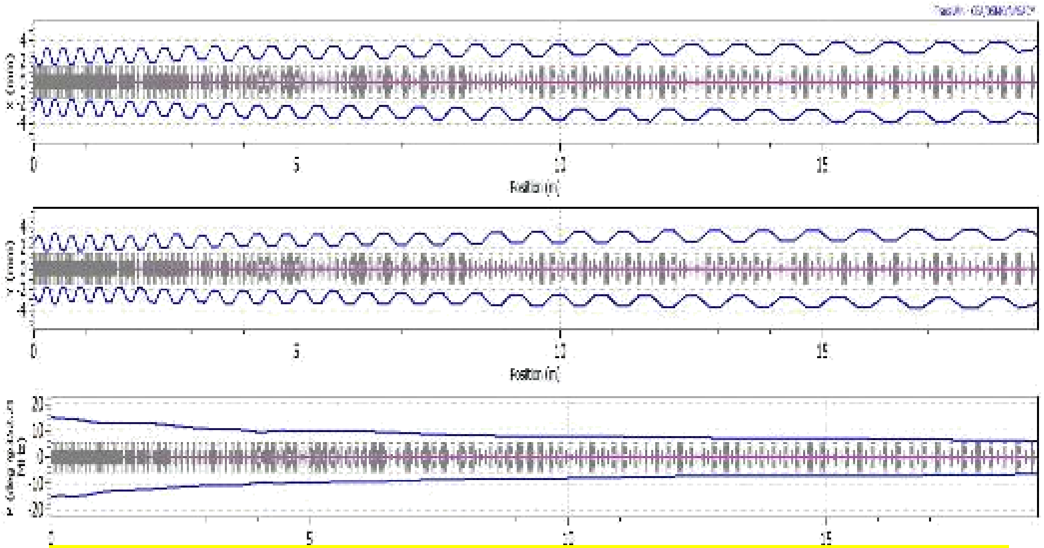
The resulting transverse motion are harmonic oscillations of the beam envelope = betatron oscillations



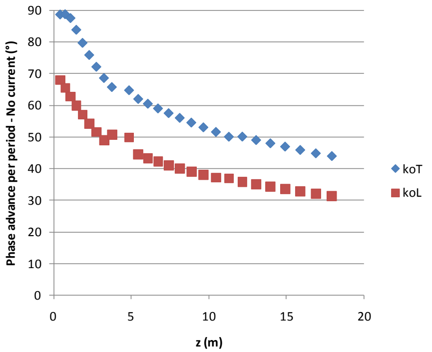
sequence of focusing and defocusing quadrupoles

In synchrotrons the oscillation frequency is relatively stable and we speak of **betatron frequency** (number of oscillations per one revolution of the ring

In linac where space charge forces can be high, we speak of **phase advance per** (focusing) **period**.



Oscillations of the beam envelope (coordinates of the outermost particle) along the Linac 4DTL (x, y, phase)



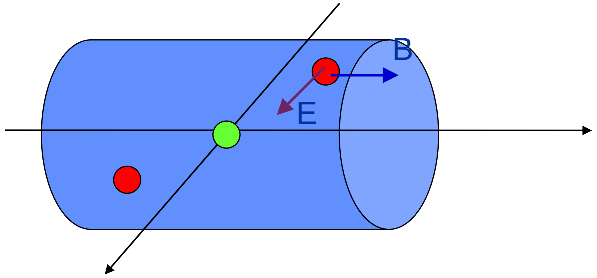
Corresponding phase advance per period

Transverse dynamics - Space charge

→ Large numbers of particles per bunch (~10^{10}).

→ Coulomb repulsion between particles (**space charge**) plays an important role and is the main limitation to the maximum current in linacs and in low energy synchrotrons.

→ But space charge forces ~ $1/\gamma^2$ disappear at relativistic velocity



Force on a particle inside a long bunch with density n(r) traveling at velocity v:

$$E_r = \frac{e}{2\pi\varepsilon r}\int_0^r n(r)\,r\,dr \qquad B_\varphi = \frac{\mu}{2\pi}\frac{e\,v}{r}\int_0^r n(r)\,r\,dr$$

$$F = e(E_r - vB_\varphi) = eE_r\left(1-\frac{v^2}{c^2}\right) = eE_r(1-\beta^2) = \frac{eE_r}{\gamma^2}$$

Transverse beam equilibrium in linacs

The equilibrium between external focusing force and internal defocusing forces defines the frequency of beam oscillations.
Oscillations are characterized in terms of *phase advance per focusing period* σ_t or *phase advance per unit length* k_t.

Ph. advance = Ext. quad focusing - RF defocusing - space charge

$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda}\right)^2 = \left(\frac{q\,G\,l}{2\,mc\,\beta\gamma}\right)^2 - \frac{\pi\,q\,E_0 T\sin(-\varphi)}{mc^2\lambda\,\beta^3\gamma^3} - \frac{3q\,I\,\lambda(1-f)}{8\pi\varepsilon_0\,r_0{}^3 mc^3\beta^2\gamma^3}$$

q=charge
G=quad gradient
l=length foc. element
f=bunch form factor
r_0=bunch radius
λ=wavelength
…

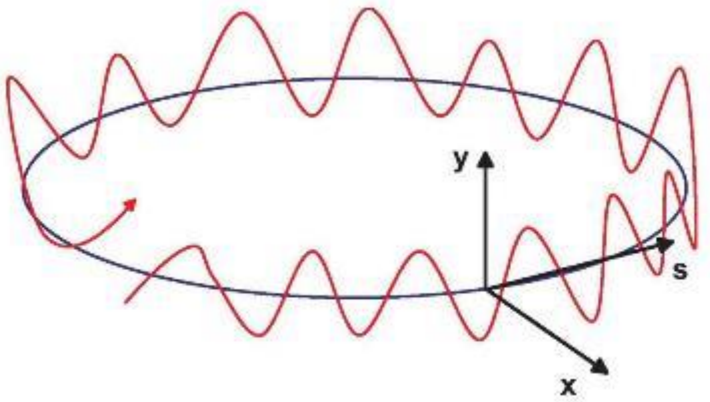
Approximate expression valid for:
FODO lattice, smooth focusing approximation, space charge of a uniform 3D ellipsoidal bunch.

A “low-energy” linac is dominated by space charge and RF defocusing forces !!

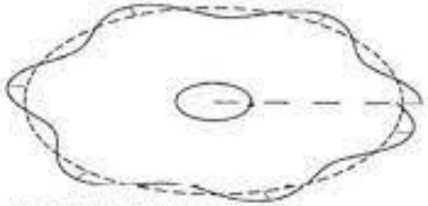
Phase advance per period must stay in reasonable limits (30-80 deg), phase advance per unit length must be continuous (smooth variations) → at low β, we need a strong focusing term to compensate for the defocusing, but the limited space limits the achievable G and l → needs to use short focusing periods N βλ.

Betatron oscillations in a synchrotron

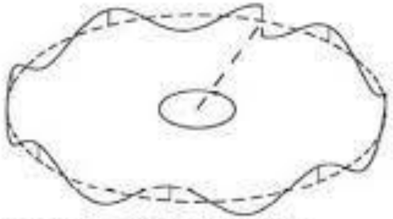
schematic of betatron oscillation around storage ring



tune = number of oscillations per turn

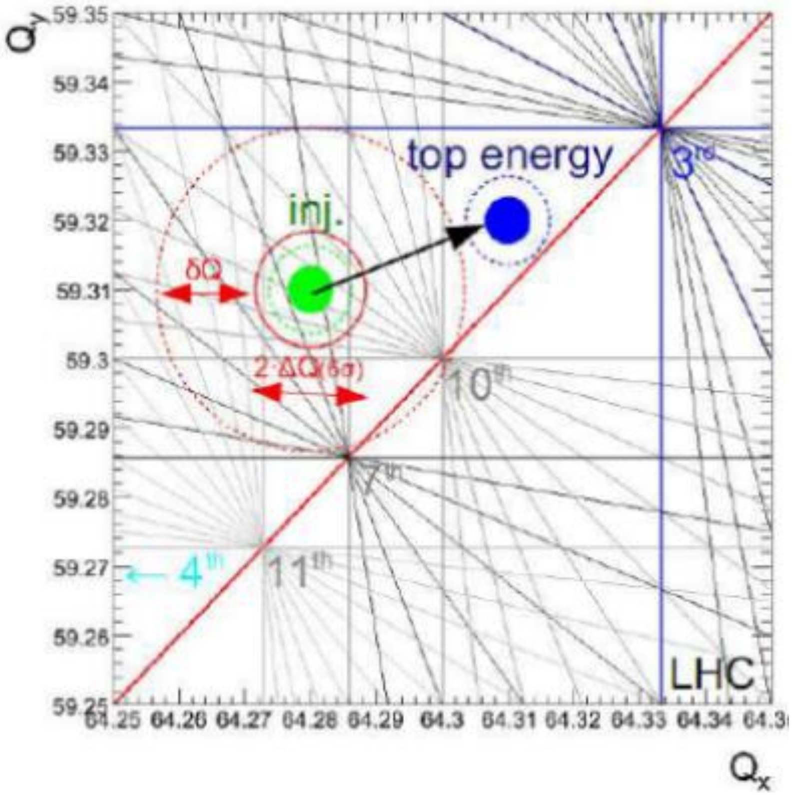


Horizontal Betatron Oscillation
with tune: $Q_h = 6.3$,
i.e., 6.3 oscillations per turn.



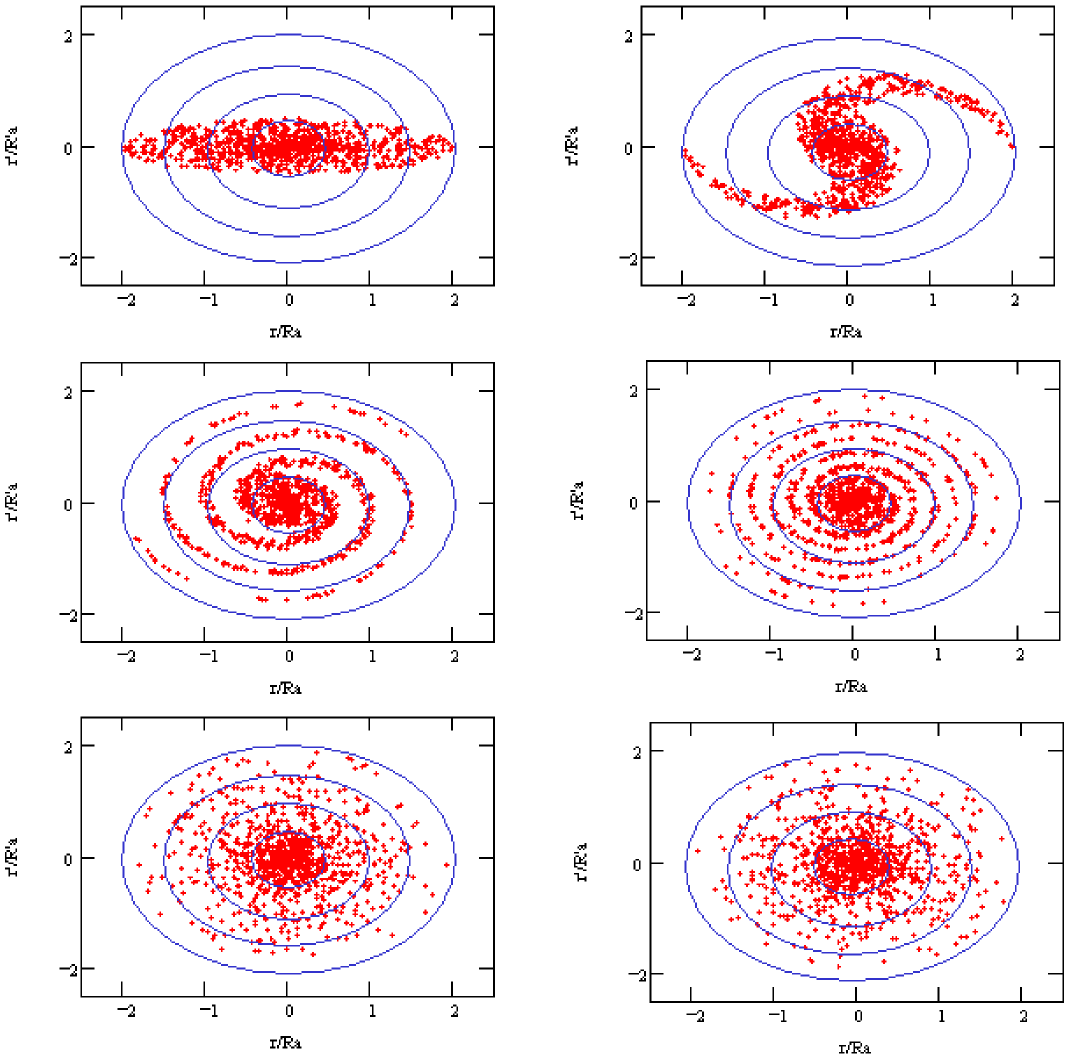
Vertical Betatron Oscillation
with tune: $Q_v = 7.5$,
i.e., 7.5 oscillations per turn.

Compensation of errors in the ring: TUNE DIAGRAM
Goal: avoid resonances that could lead to beam loss



LHC tune diagram (Qx, Qy) with working point for injection (green dot), working point for best collision performance (blue dot). The difficult part is to constrain tune variations far away from strong resonance lines (bold solid lines) in order not to lose the beam partially or totally (courtesy of R. Steinhagen)

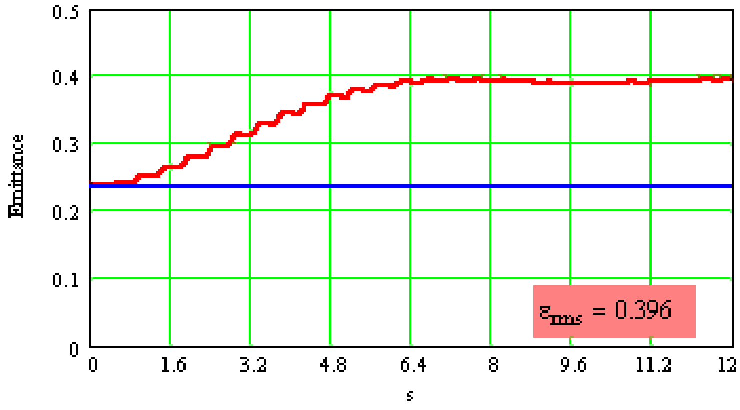
Non linear space charge forces: filamentation

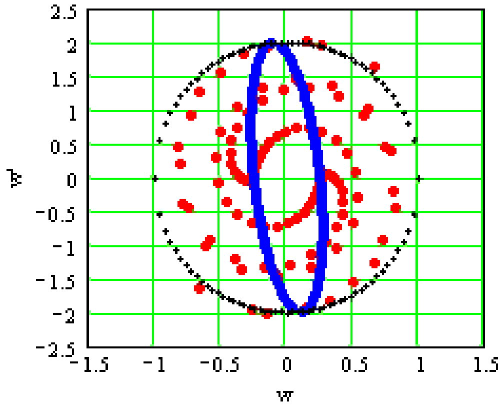


Evolution of the emittance along a linear accelerator under the influence of linear forces only (blue line) or non-linear forces (red line)

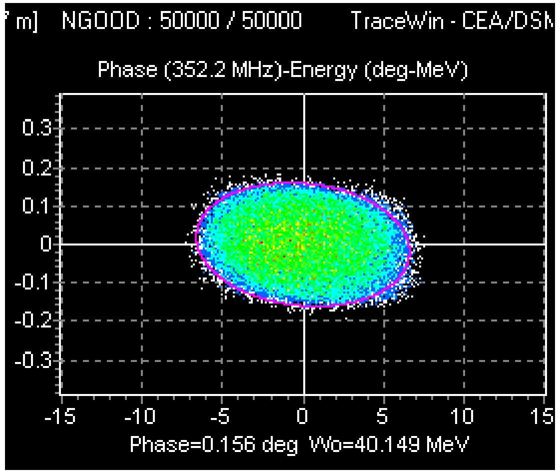
🟦 Linear force

🟥 Non linear force

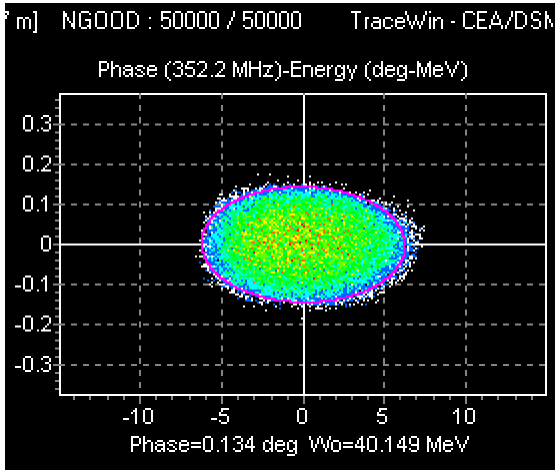




Filamentation in longitudinal plane



(1)



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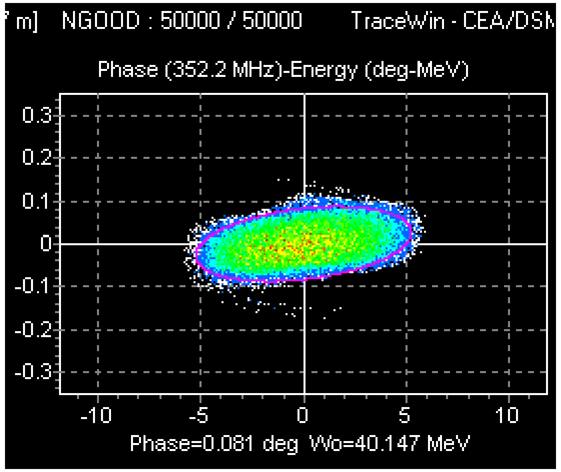

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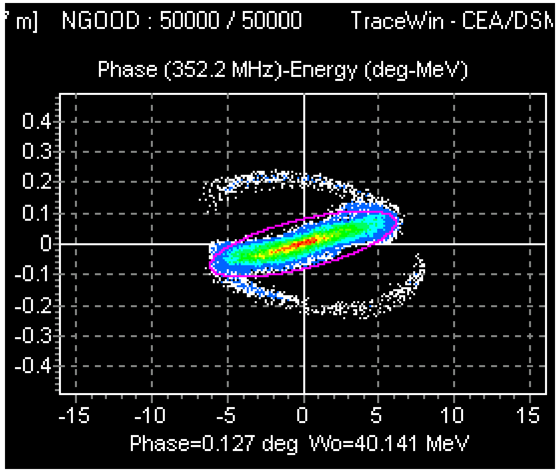


(4)

Longitudinal restoring forces are non-linear!

Non-linear sinusoidal field, linear only in a small region around synchronous phase

(1) el_in=0.21

(2) el_in=0.18

(3) el_in=0.12

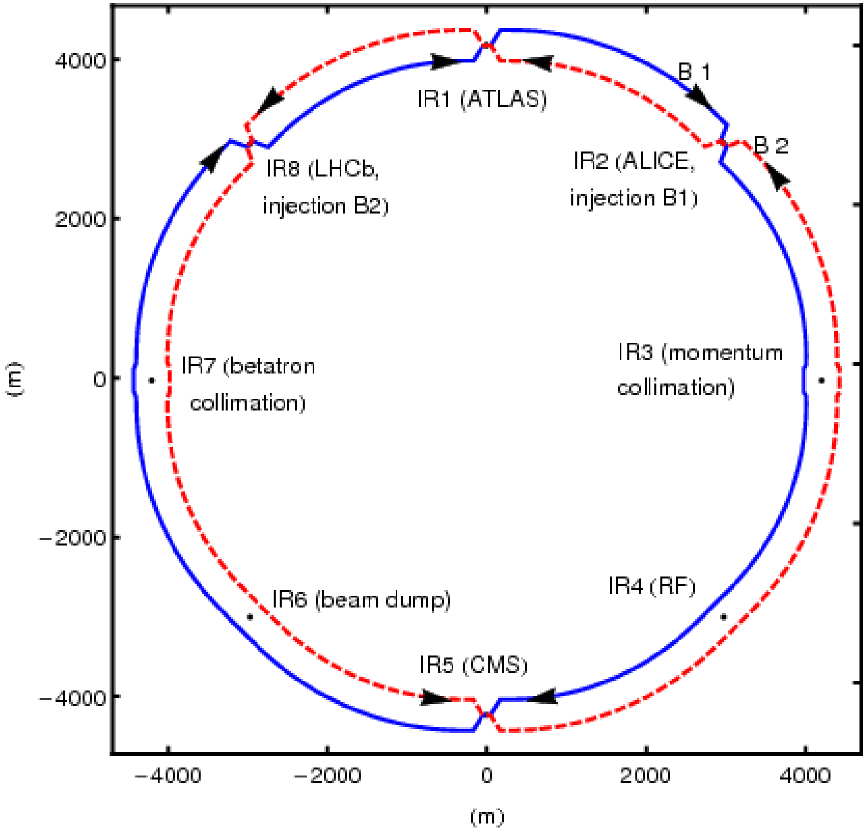
(4) el_in=0.03

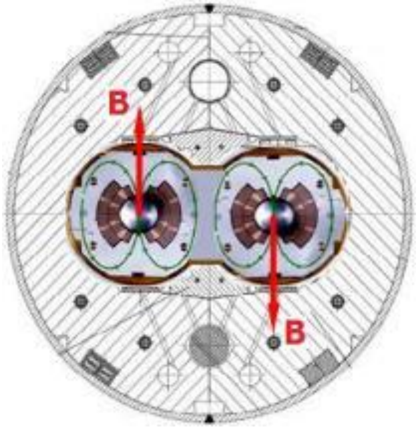
emittance r.m.s. in deg MeV

One more step, the Collider

In a particle collider, 2 flows of particles are accelerated in opposite direction and then collided to increase the energy at collision point compared to collisions on a fixed target

First conceptual design of a collider: Widerøe (1943), first collider (e-p) built end 50’s in Frascati and Princeton





The 1232 LHC magnets contain two pipes, one for each of the counterrotating beams.

Fixed Target Accelerators vs. Colliders

Special relativity: total center of mass energy for collisions of a beam on a fixed target is $E_{\rm cm}^2 = (m_1^2 + m_2^2 + 2m_2 E_{1,\rm lab})$.

Fixed Target

$$E \propto \sqrt{E_{beam}}$$

Much of the energy is lost in the target and only part is used to produce secondary particles

Collider

$$E = E_{beam1} + E_{beam2}$$

All energy will be available for particle production

Example of the LHC (7 TeV):
Collider E_{cm}=14000 GeV
Fixed target E_{cm}=114.6 GeV

Courtesy R. Steerenberg, CERN

Luminosity, the Collider Figure of Merit

ratio of the number of events detected (*dN*) in a certain period of time (*dt*) to the cross-section (σ)

$$LUMINOSITY = \frac{N_{event/\sec}}{\sigma_r} = \frac{N_1 N_2 f_{rev} n_b}{4\pi\sigma_x\sigma_y} F$$

Intensity per bunch

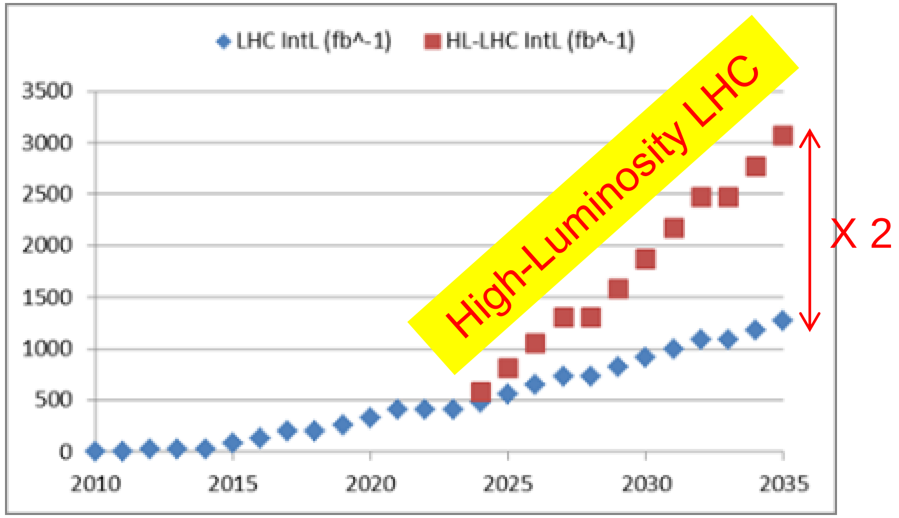
Number of bunches

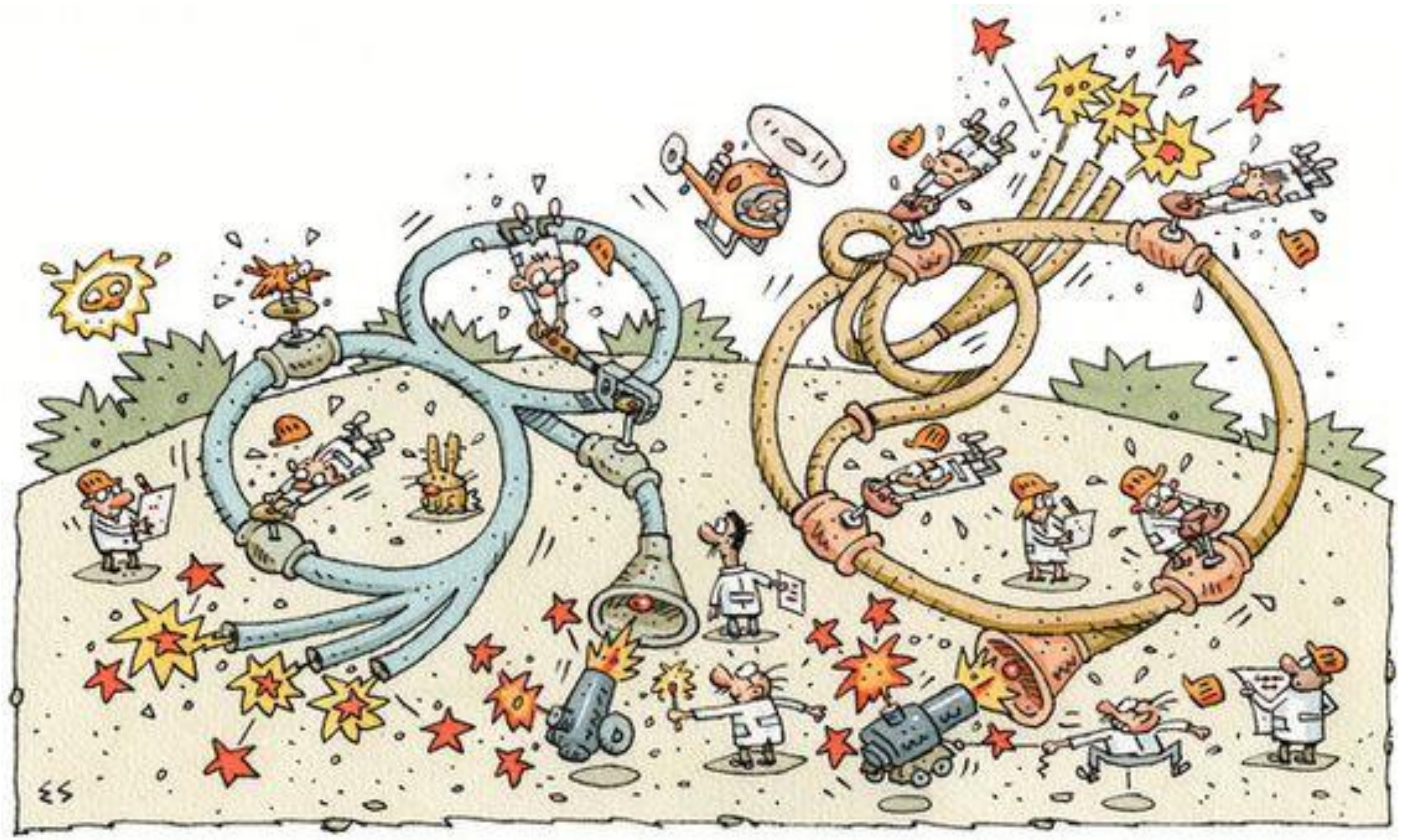
Geometrical Correction factors

Beam dimensions

- More or less fixed:
  - Revolution period
  - Number of bunches

- Parameters to optimise:
  - Number of particles per bunch
  - Beam dimensions
  - Geometrical correction factors

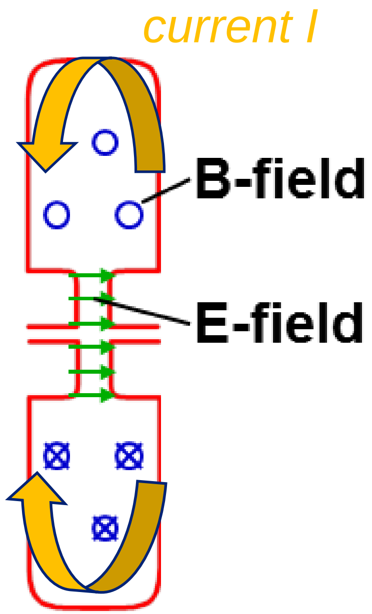




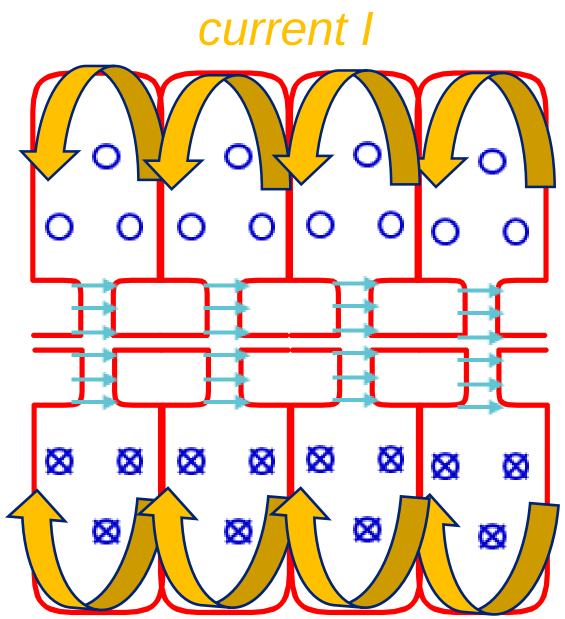
End of Lecture 3

Thank you for your attention!

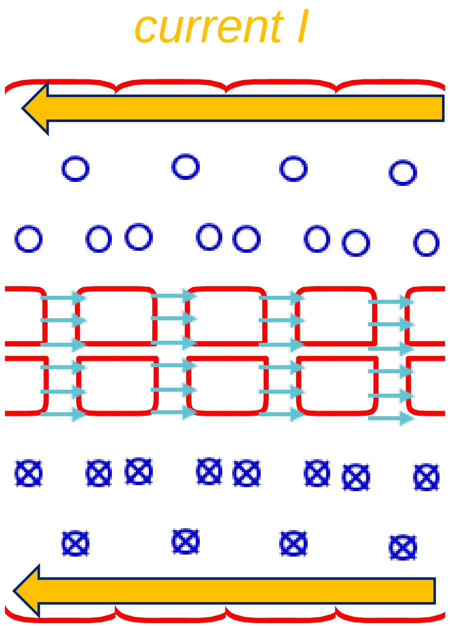
The Alvarez Drift Tube Linac – an efficient 0 mode



Electric current flows on cavity walls, dissipating power



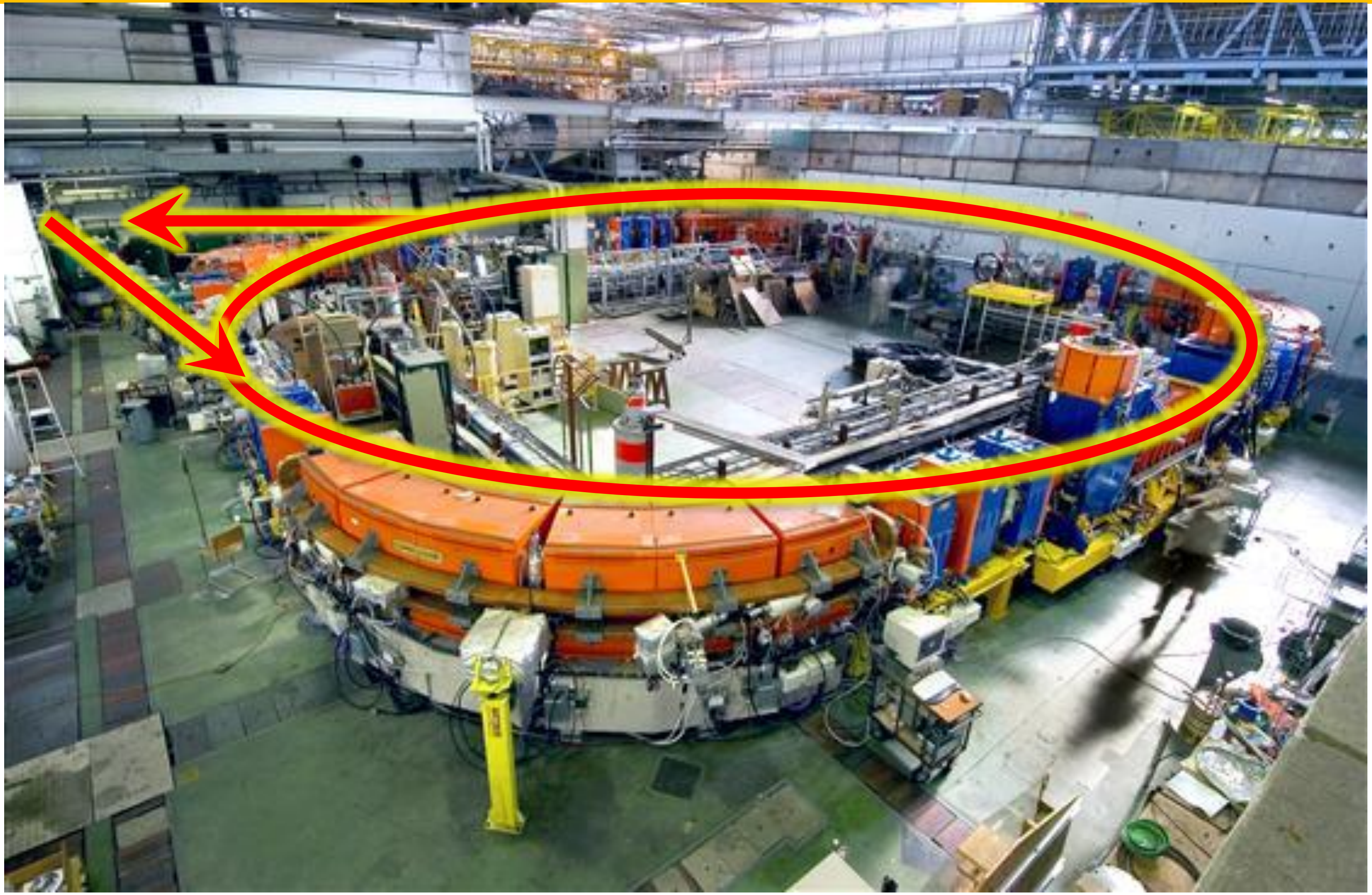
On the walls between cells, the current has opposite sign on adjacent walls → cancels out, we can remove the walls without changing the electromagnetic field



The Alvarez DTL is an open structure with maximum coupling between cells (clean mode spectrum) and minimum power loss on the walls (high shunt impedance corresponding to high efficiency)

Drift Tubes cannot «float» in air → they will be connected to the walls by stems

The smallest CERN synchrotron: LEIR



the Large Hadron Collider