



**Università
di Genova**

Jet Angularities in Z + jet production at the LHC

Simone Caletti

LHC EW Working Group on Jets and Bosons

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Outline:

- Jets from a 'pheno' perspective
- Jets from a 'theoretical' perspective

 Introduction

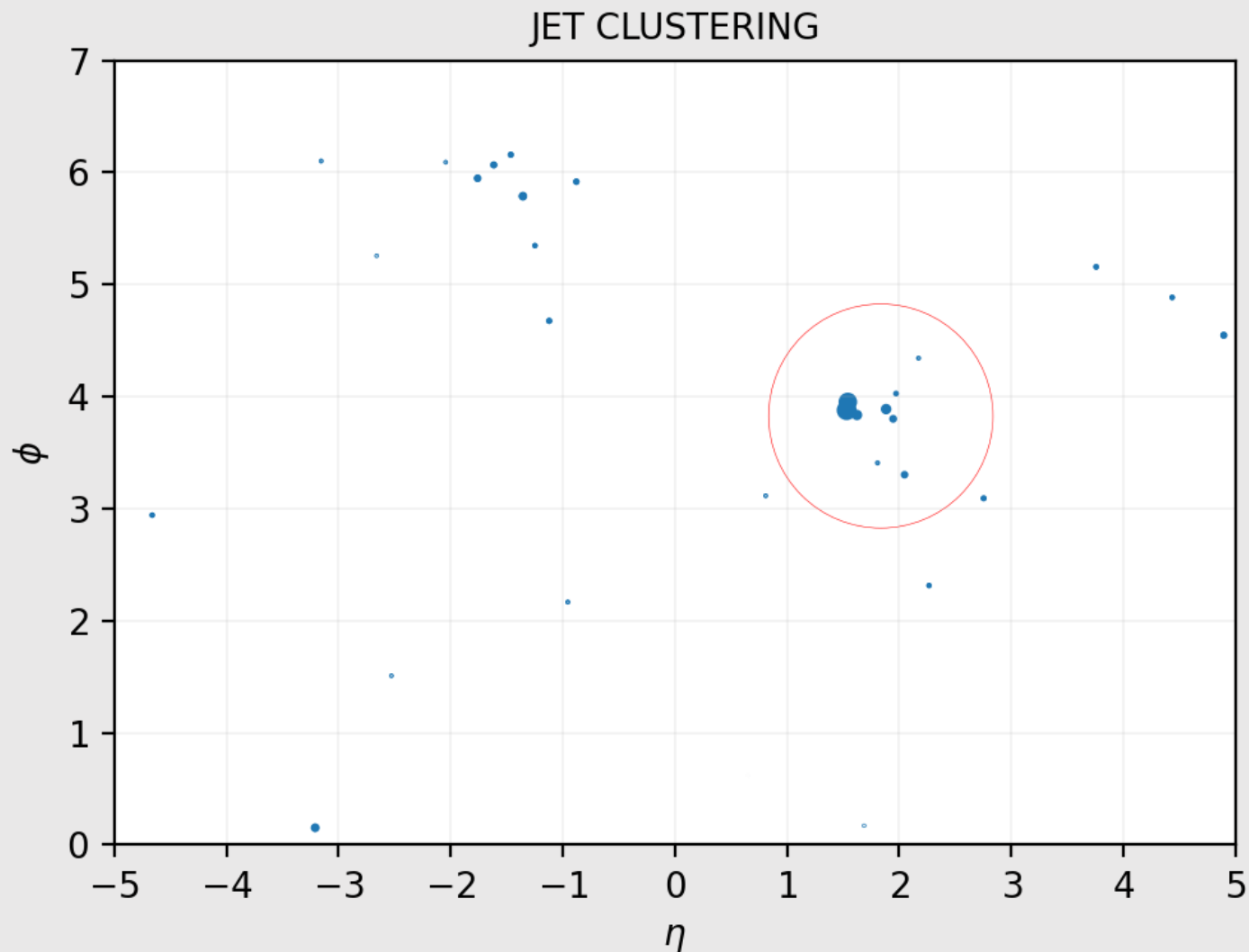
- NLO + NLL' resummed results
- Plots and comparison with MC

 The reason why I am here

- Outlook and conclusions

PART 1

Pheno perspective:



- Take the particles in the events as our initial list of objects.
- From this list build the *inter-particle distance* as

$$d_{ij} = \min \left(p_{T,i}^{2p}, p_{T,j}^{2p} \right) \Delta_{ij}^2$$

and the *beam distance* as

$$d_{B,i} = p_{T,i}^{2p} R^2$$

with R the jet radius.

- Iteratively find the smallest among all the two distances:

If $d_{ij} < d_{B,i}$ then remove i and j and recombine them into a new object k which is added to the list.

If $d_{B,i} < d_{ij}$ then i is called a *jet* and removed from the list.

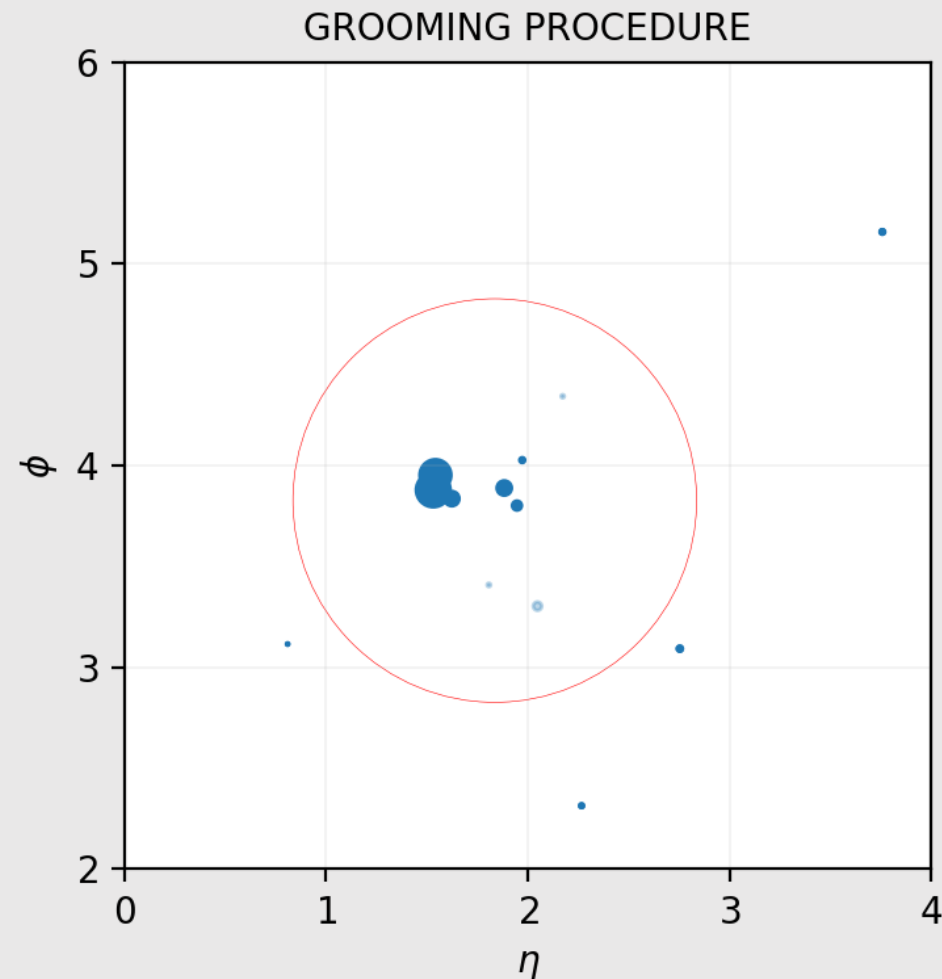
while(! list is empty)

Pheno perspective:

Here we consider the SoftDrop declustering

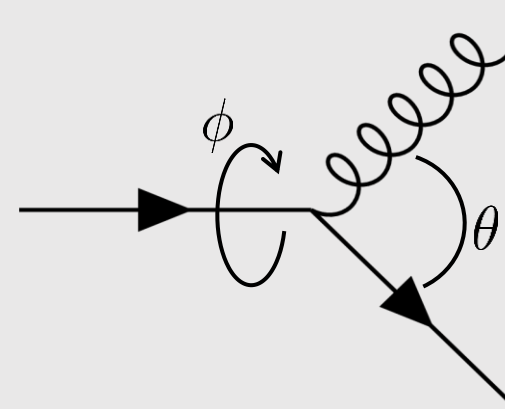
$$\frac{\min(p_{T,i}, p_{T,j})}{p_{T,i} + p_{T,j}} > z_{\text{cut}} \left(\frac{\Delta_{ij}}{R} \right)^\beta$$

1. Break the jet j into two subjects by undoing the last stage of C/A clustering and label them as j_1 and j_2 .
2. If j_1 and j_2 pass the SD condition then deem j to be the final soft-drop jet.
3. Else: redefine $j = \max_{p_T}[j_1, j_2]$ while(! SD)



Theory perspective:

1. At high energies QCD is an approximately scale-invariant quantum field theory. Hence, we can consider α_S to be a constant at least in the first approximation.
2. In addition α_S is also small ($\alpha_S \ll 1$) at high energies.



The gluon is massless thus $E_g = |\vec{p}|$

$$p_g = (E_g, p_x, p_y, p_z) = E_g(1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

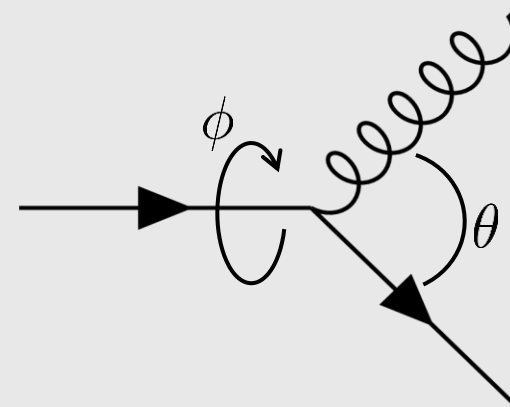
Calling E the energy of the incoming parton we can write $E_g = zE$, with $z \in [0, 1]$ the energy fraction taken by the gluon.

$$p_g = zE(1, \sin \theta, 0, \cos \theta)$$

$$p_q = E(1 - z)(1, 0, 0, 1)$$

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The relevant degrees of freedom for the gluon are θ and z . Hence, the splitting probability is

$$P_{qg \leftarrow q} = p(z, \theta) dz d\theta$$

α_S controls the strength with which quarks and gluons interact, thus

$$p(z, \theta) \propto \frac{2\alpha_S}{\pi} C_F$$

But what about the z, θ dependence of $p(z, \theta)$?

Theory perspective:

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$$p(z, \theta) = ?$$

Given our quark-gluon system, the only Lorentz invariant we can construct is the dot product of their momenta:

$$p_q \cdot p_g = E^2 z(1-z)(1-\cos\theta)$$

1. $\Rightarrow p_q \cdot p_g \rightarrow \lambda p_q \cdot p_g$ leads to the same physical phenomena for any $\lambda > 0$

In general this is not simply implemented in term of z and θ but there is a limit in which is simple

$$\begin{cases} z(1-z) \xrightarrow{z \ll 1} z \\ (1-\cos\theta) \xrightarrow{\theta \ll 1} \frac{\theta^2}{2} \end{cases}$$

Theory perspective:

1. At high energies QCD is an approximately scale-invariant quantum field theory. Hence, we can consider α_S to be a constant at least in the first approximation.
2. In addition α_S is also small ($\alpha_S \ll 1$) at high energies.

$$p(z, \theta) = ?$$

$$p_q \cdot p_g \xrightarrow{z \ll 1, \theta \ll 1} z\theta^2 \frac{E^2}{2}$$

The scaling of $p_q \cdot p_g$ can therefore be accomplished by scaling of either z or θ (or both).

1. \Rightarrow QCD is invariant under $z \rightarrow \lambda z$ or $\theta^2 \rightarrow \lambda\theta^2$ for $\lambda > 0$

Oss. There is a coordinate scale transformation under which the product $z\theta^2$ is unchanged. This is

$$z \rightarrow \lambda z \quad \theta^2 \rightarrow \frac{\theta^2}{\lambda}$$

Theory perspective:

1. At high energies QCD is an approximately scale-invariant quantum field theory. Hence, we can consider α_S to be a constant at least in the first approximation.
2. In addition α_S is also small ($\alpha_S \ll 1$) at high energies.

$$p(z, \theta) = ?$$

Now, consider to perform two independent scale transformations by independent scalings $z \rightarrow \lambda_1 z$ and $\theta^2 \rightarrow \lambda_2 \theta^2$

$$P_{qg \leftarrow q} = p(z, \theta) dz d\theta = \frac{2\alpha_S}{\pi} C_F \frac{dz}{z} \frac{d\theta}{\theta}$$

This is a pretty simple functional form. But keep in mind we are working in the soft and collinear limit!

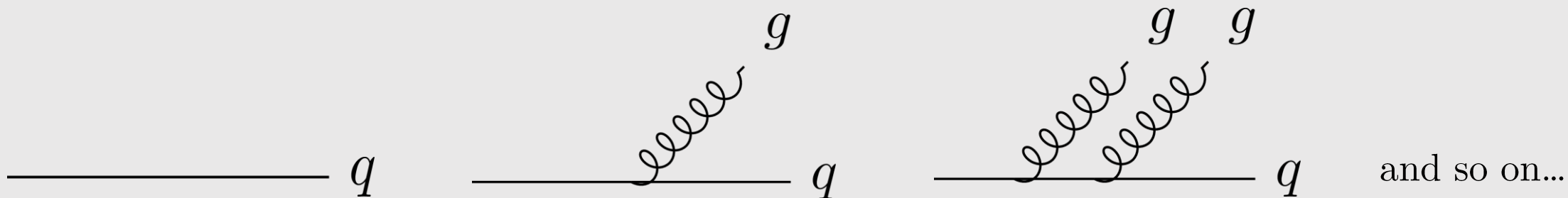
This probability distribution diverges in the $z, \theta \rightarrow 0$ limits.

Theory perspective:

1. At high energies QCD is an approximately scale-invariant quantum field theory. Hence, we can consider α_S to be a constant at least in the first approximation.
2. In addition α_S is also small ($\alpha_S \ll 1$) at high energies.

$$P_{qg \leftarrow q} = \frac{2\alpha_S}{\pi} C_F \frac{dz}{z} \frac{d\theta}{\theta}$$

There is no measurement we can perform to distinguish between:



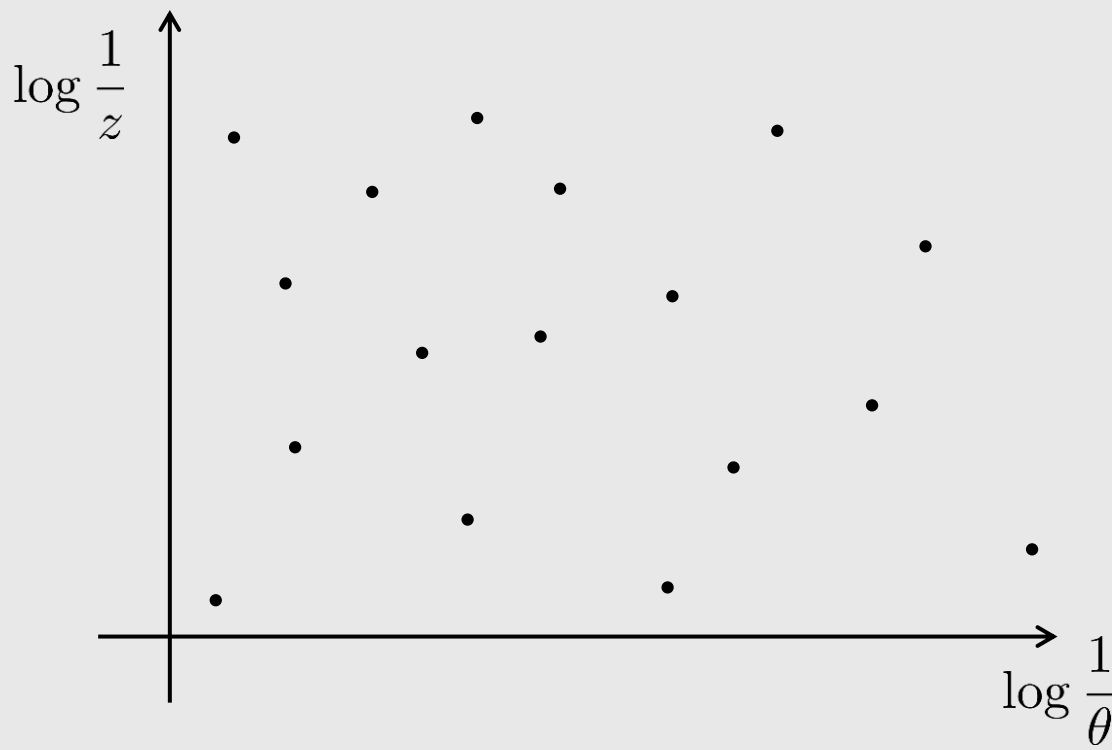
Every one these configurations of soft and/or collinear gluons and a quark have a divergent probability. So how do we proceed?

KLN th.

Theory perspective:

1. At high energies QCD is an approximately scale-invariant quantum field theory. Hence, we can consider α_S to be a constant at least in the first approximation.
2. In addition α_S is also small ($\alpha_S \ll 1$) at high energies.

$$P_{qg \leftarrow q} = \frac{2\alpha_S}{\pi} C_F \frac{dz}{z} \frac{d\theta}{\theta}$$



$$P_{qg \leftarrow q} = \frac{2\alpha_S}{\pi} C_F d \left(\log \frac{1}{z} \right) d \left(\log \frac{1}{\theta} \right)$$

Theory perspective:

Now we introduce the angularity

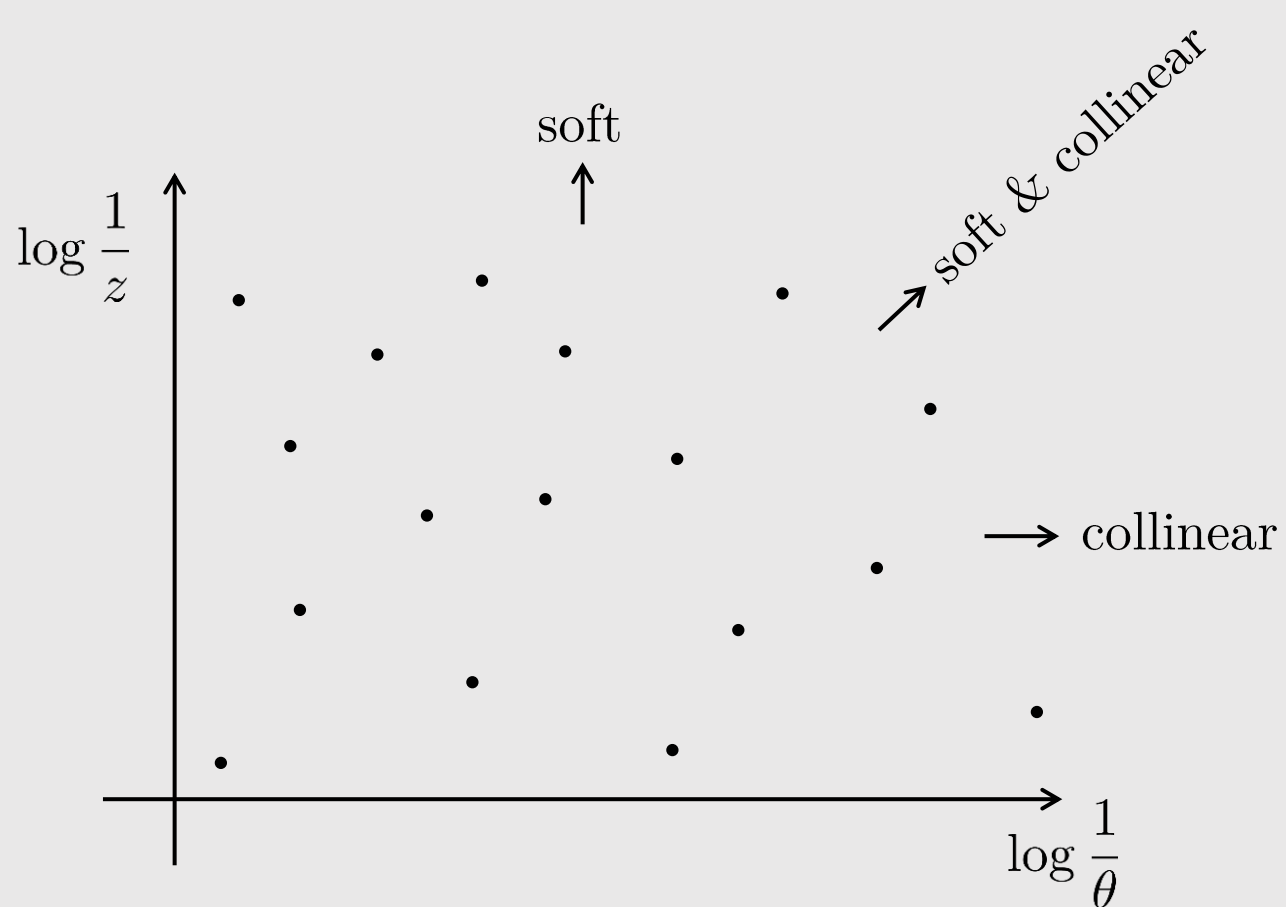
$$\lambda_\alpha^\kappa = \sum_{j \in \text{Jet}} \left(\frac{p_{T,j}}{\sum_{j \in \text{Jet}} p_{T,j}} \right)^\kappa \left(\frac{\Delta_j}{R} \right)^\alpha$$
$$\simeq \sum_{j \in \text{Jet}} z_j^\kappa \theta_j^\alpha$$

with $\Delta_j = \sqrt{(y_i - y_{\text{Jet}})^2 + (\phi_i - \phi_{\text{Jet}})^2}$

IRC safety poses the restrictions $\kappa = 1$ and $\alpha > 0$.

Uniformly distributed emissions in the Lund plane means they are exponentially far apart in the real space.

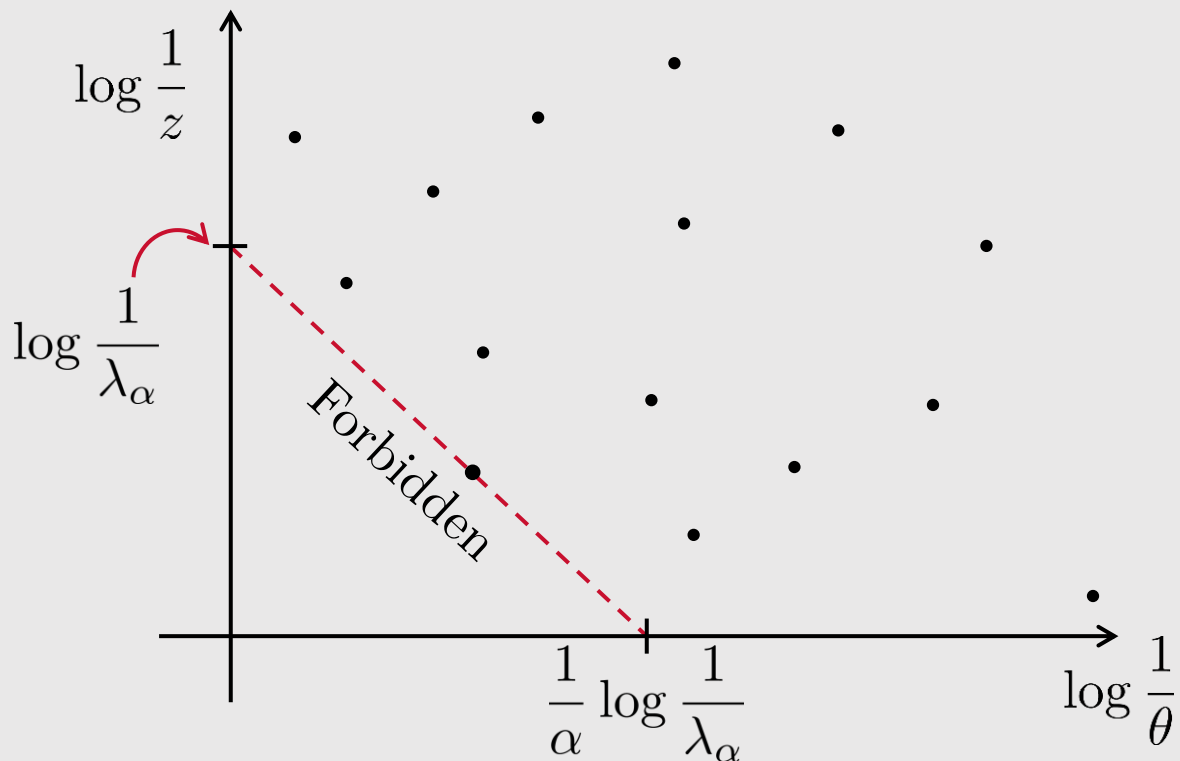
So one emission will dominate the others: $\lambda_\alpha = z\theta^\alpha$



$$\log \frac{1}{\lambda_\alpha} = \log \frac{1}{z} + \alpha \log \frac{1}{\theta}$$

which is a straight line in the Lund plane.

Theory perspective:



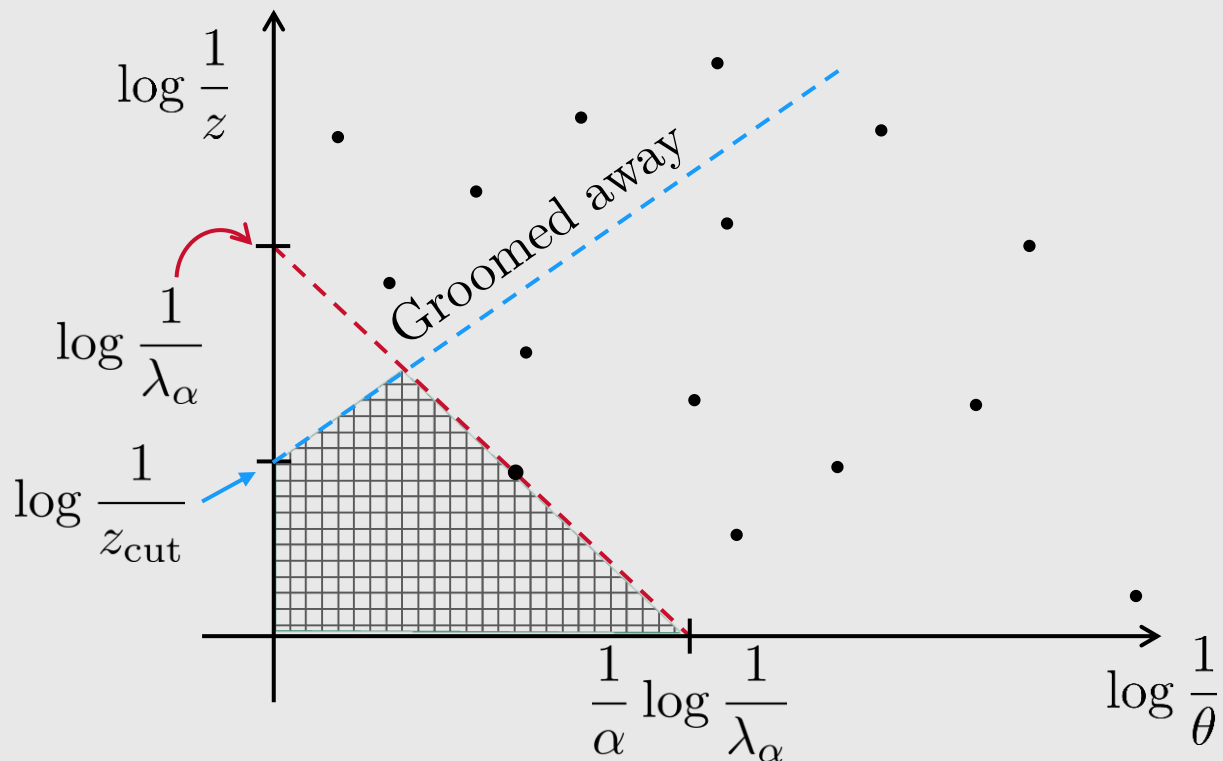
$$\log \frac{1}{\lambda_\alpha} = \log \frac{1}{z} + \alpha \log \frac{1}{\theta}$$

Probability for emission = $\frac{2\alpha_S}{\pi} C_F \frac{\triangle}{N}$
 where $\triangle = \frac{1}{2\alpha} \log^2 \lambda_\alpha$

Probability for no emission = $1 - \frac{2\alpha_S}{\pi} C_F \frac{\triangle}{N}$

$$P(\lambda_\alpha < \text{measured value}) = \lim_{N \rightarrow \infty} \left(1 - \frac{2\alpha_S}{\pi} C_F \frac{\triangle}{N} \right)^N = e^{-\frac{2\alpha_S}{\pi} C_F \triangle}$$

Theory perspective:



If we consider also the Soft Drop condition

$$z > z_{\text{cut}} \theta^\beta$$

$$\log \frac{1}{z} < \log \frac{1}{z_{\text{cut}}} + \beta \log \frac{1}{\theta}$$

$$P(\lambda_\alpha < \text{measured value}) = e^{-\frac{2\alpha_S}{\pi} C_F} \int_0^{\lambda_\alpha} \frac{d\lambda}{\lambda} \int_0^{\lambda} \frac{d\lambda'}{\lambda'} \int_0^{\lambda'} \frac{d\lambda''}{\lambda''} \dots$$

What we have neglected in this naive picture?

- We achieved just the LL accuracy. We can do better!
- Non-Global Logs (NGL)
- Strong coupling runs, it is not a constant!
- etc.

PART 2

NLL Resummation:

CAESAR resummation formalism
 arXiv:hep-ph/0407286
 (A. Banfi, G. Salam, G. Zanderighi)

$$\Sigma_{\text{res}}(v) = \sum_{\delta} \Sigma_{\text{res}}^{\delta}(v)$$

$$\Sigma_{\text{res}}^{\delta}(v) = \int d\mathcal{B}_{\delta} \frac{d\sigma_{\delta}}{d\mathcal{B}_{\delta}} \exp \left[- \sum_{l \in \delta} R_l^{\mathcal{B}_{\delta}}(L) \right] \mathcal{P}^{\mathcal{B}_{\delta}}(L) \mathcal{S}^{\mathcal{B}_{\delta}}(L) \mathcal{F}^{\mathcal{B}_{\delta}}(L) \mathcal{H}^{\delta}(\mathcal{B}_{\delta})$$

RADIATOR
soft function
kinematic cuts

pdf ratio
multiple emission

The collinear radiators R_l for the hard legs l were computed in for a general observable with these $k_t^{(l)}$ and $\eta^{(l)}$ scalings

$$V(k) = \left(\frac{k_t^{(l)}}{\mu_Q} \right)^a e^{-b_l \eta^{(l)}} d_l(\mu_Q) g_l(\phi)$$

With this choice of the parameters $\begin{cases} a = 1 \\ b_3 = \alpha - 1 \\ g_3 d_3(\mu_Q) = \left(2 \frac{\cosh(y_3)}{R_0} \right)^{\alpha-1} \frac{\mu_Q}{p_{T, \text{Jet}} R_0} \end{cases}$ we obtain angularities.

Matching to NLO:

$$\Sigma_{\text{match, mult}}(\lambda_\alpha) = \sum_{\delta} \Sigma_{\text{match, mult}}^{\delta}(\lambda_\alpha)$$

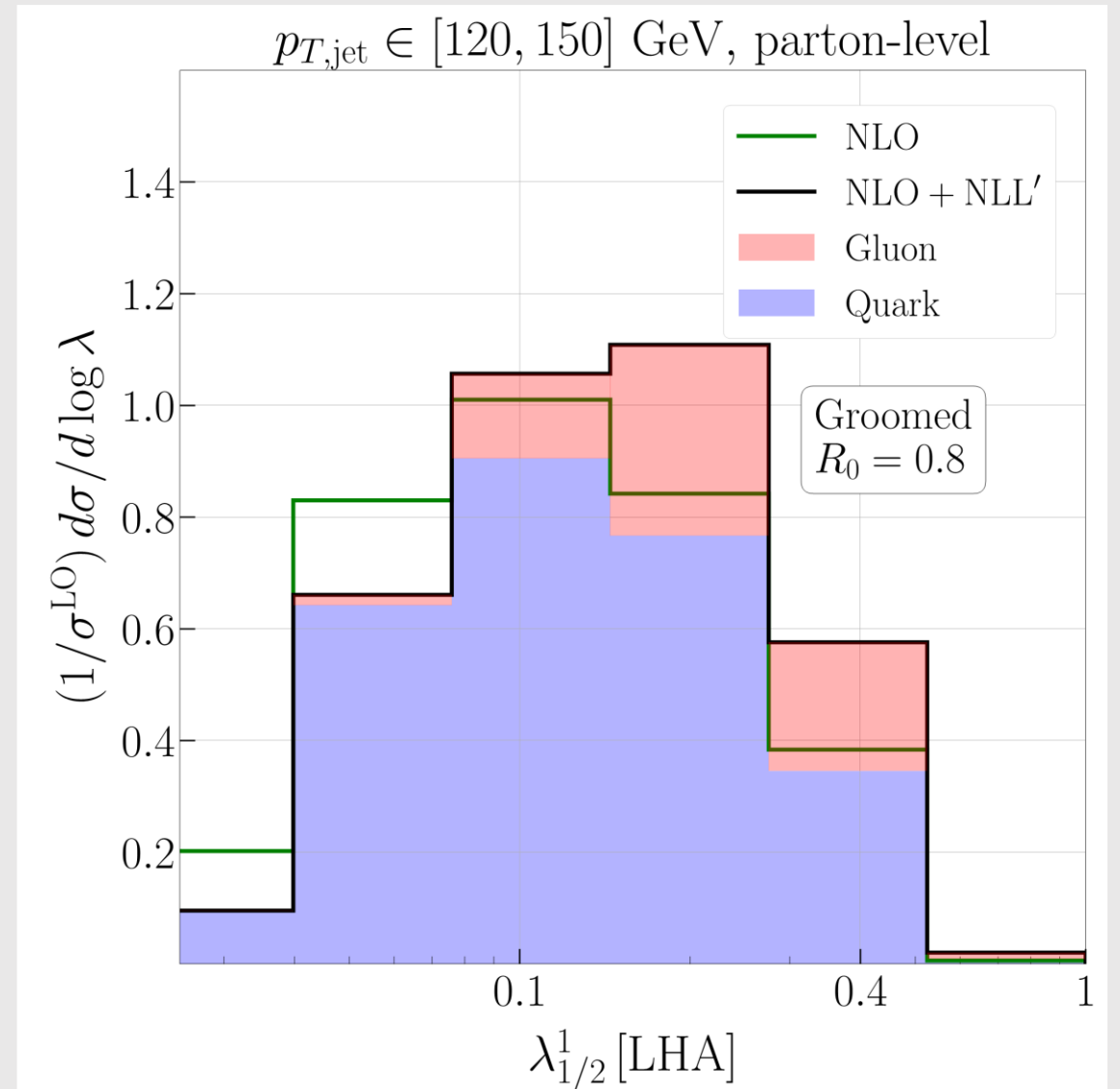
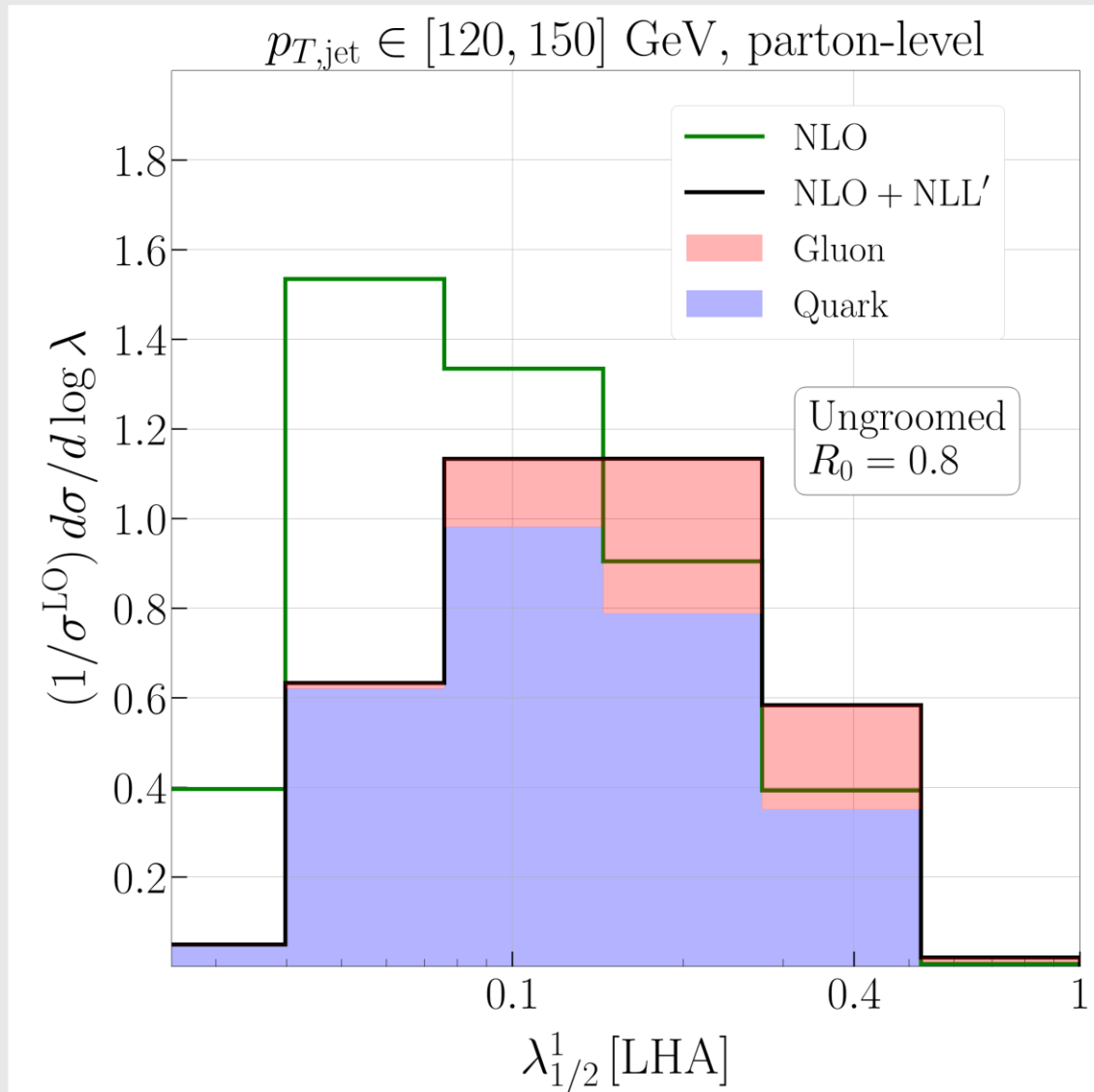
1. The matched result should be correct up to NLL terms in the exponent and the expanded matched result should be correct up to and including $O(\alpha_S^n L^{2n-2})$ terms.
2. The expanded matched result should coincide with the fixed order result up to and including the NLO terms.

$$\Sigma_{\text{match, mult}}^{\delta}(\lambda_\alpha) = \Sigma_{\text{res}}^{\delta}(\lambda_\alpha) \left[1 + \frac{\Sigma_{\text{fo}}^{\delta,(1)}(\lambda_\alpha) - \Sigma_{\text{res}}^{\delta,(1)}(\lambda_\alpha)}{\sigma^{\delta,(0)}} + \frac{1}{\sigma^{\delta,(0)}} \left(-\bar{\Sigma}_{\text{fo}}^{\delta,(2)}(\lambda_\alpha) - \Sigma_{\text{res}}^{\delta,(2)}(\lambda_\alpha) - \Sigma_{\text{res}}^{\delta,(1)}(\lambda_\alpha) \frac{\Sigma_{\text{fo}}^{\delta,(1)}(\lambda_\alpha) - \Sigma_{\text{res}}^{\delta,(1)}(\lambda_\alpha)}{\sigma^{\delta,(0)}} \right) \right]$$

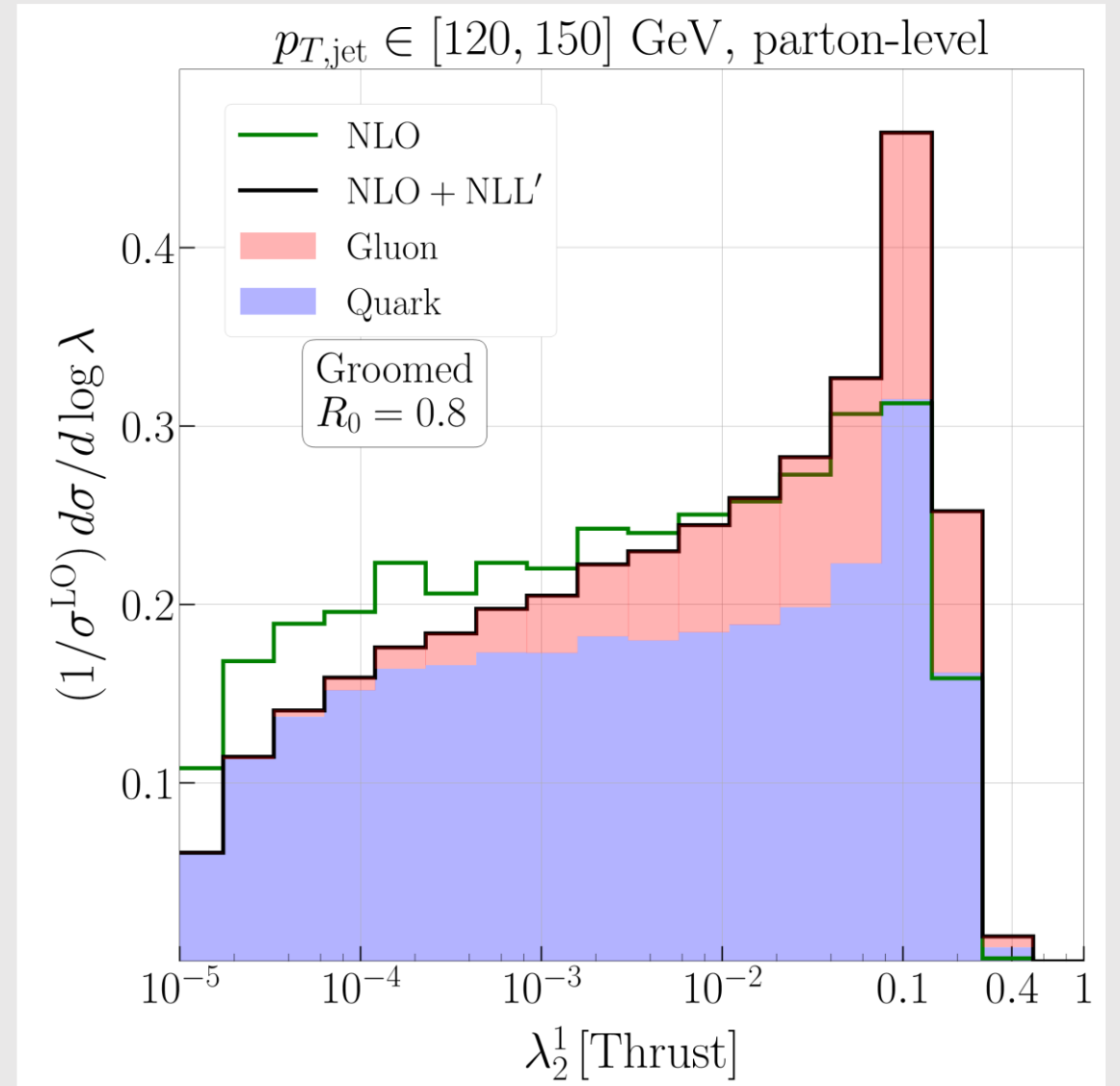
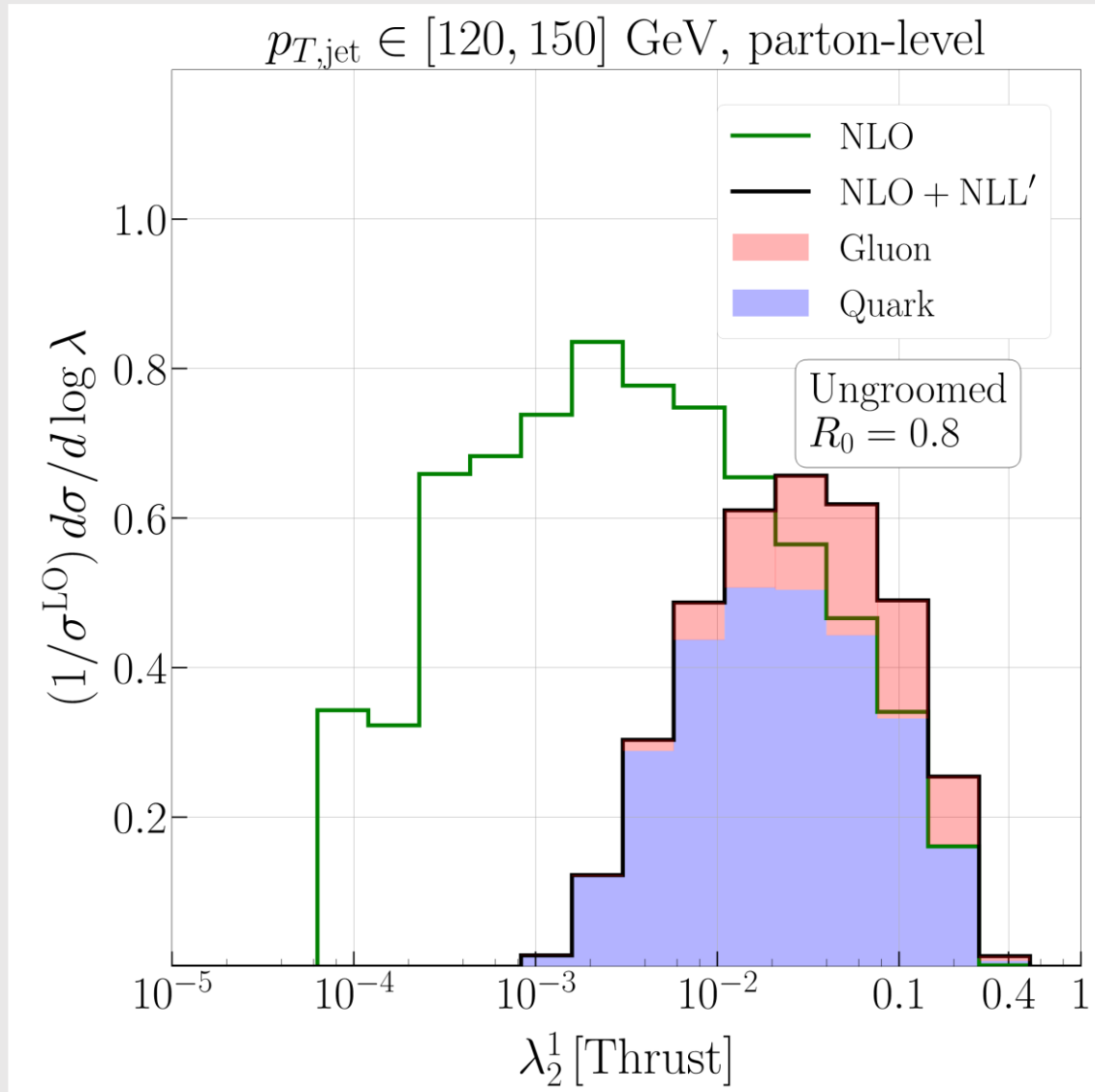
$$\left\{ \begin{array}{l} \sigma = \Sigma(1) \\ \Sigma^{(k)} \propto \alpha_{\text{EW}}^2 \alpha_S^{1+k} \\ \bar{\Sigma} = \sigma - \Sigma \end{array} \right.$$

$$\frac{\alpha_S}{2\pi} C^{\delta,(1)} \equiv \lim_{\lambda \rightarrow 0} \frac{\Sigma_{\text{fo}}^{\delta,(1)}(\lambda_\alpha) - \Sigma_{\text{res}}^{\delta,(1)}(\lambda_\alpha)}{\sigma^{\delta,(0)}}$$

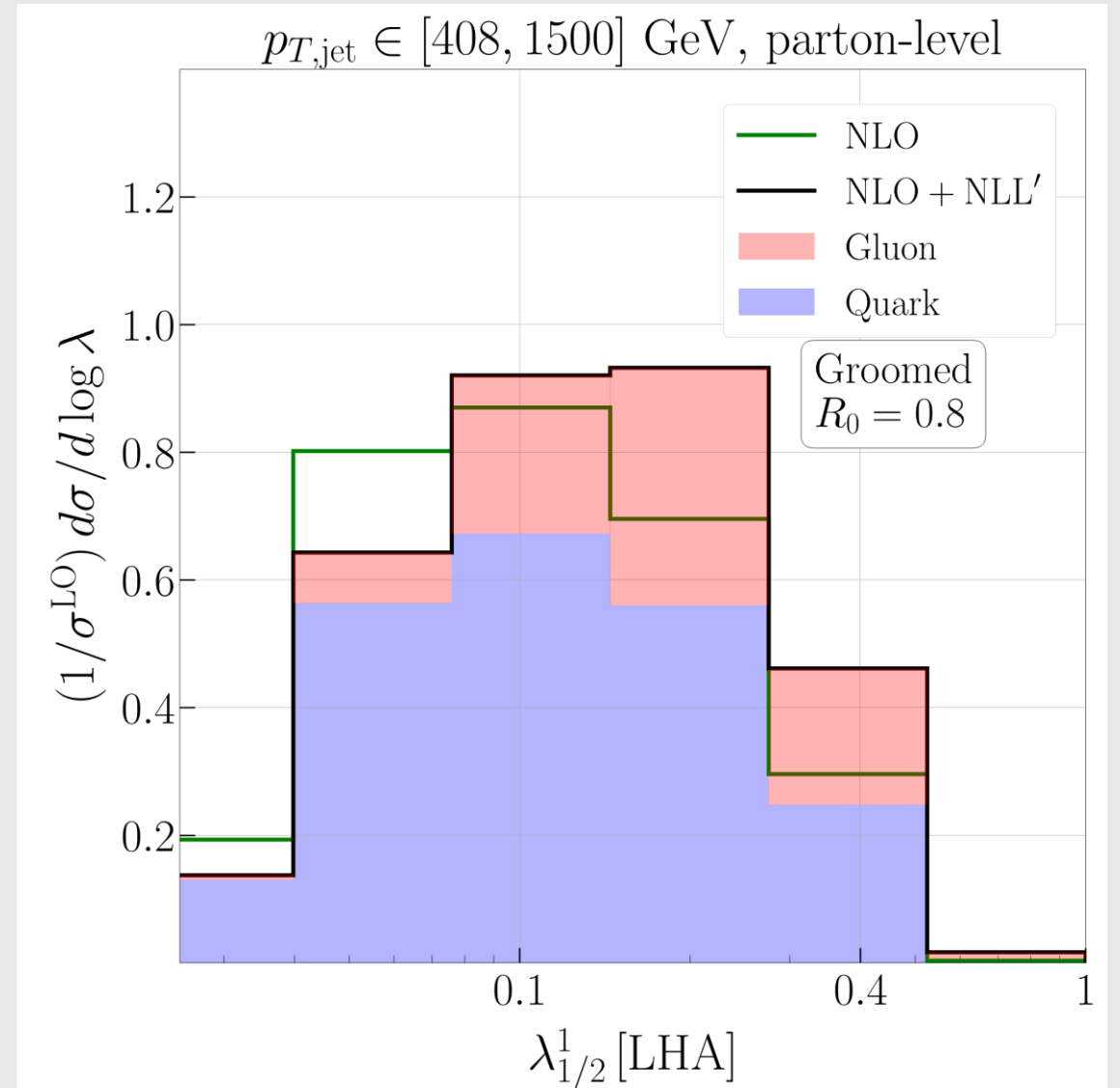
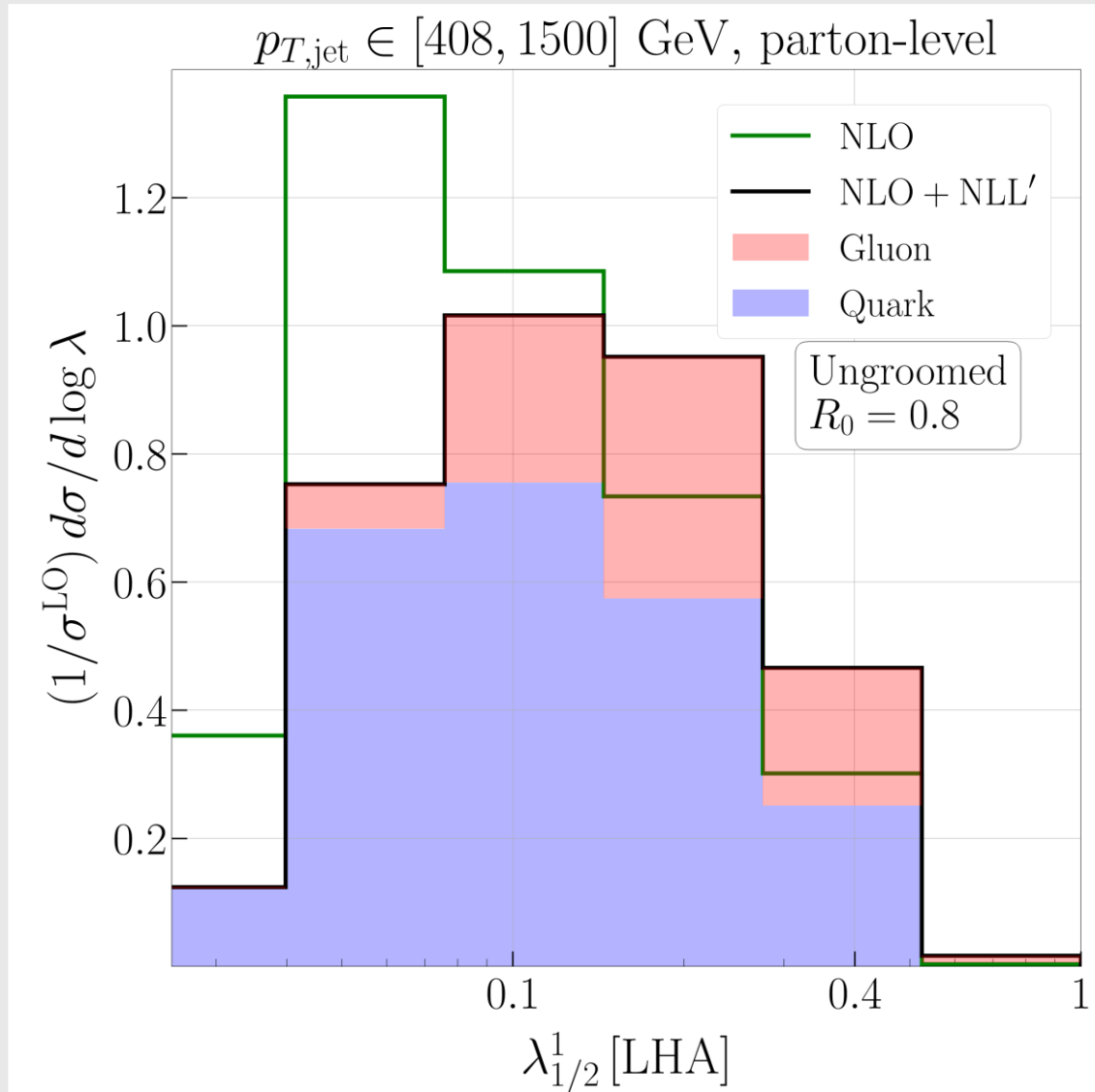
Fixed-order vs. matched:



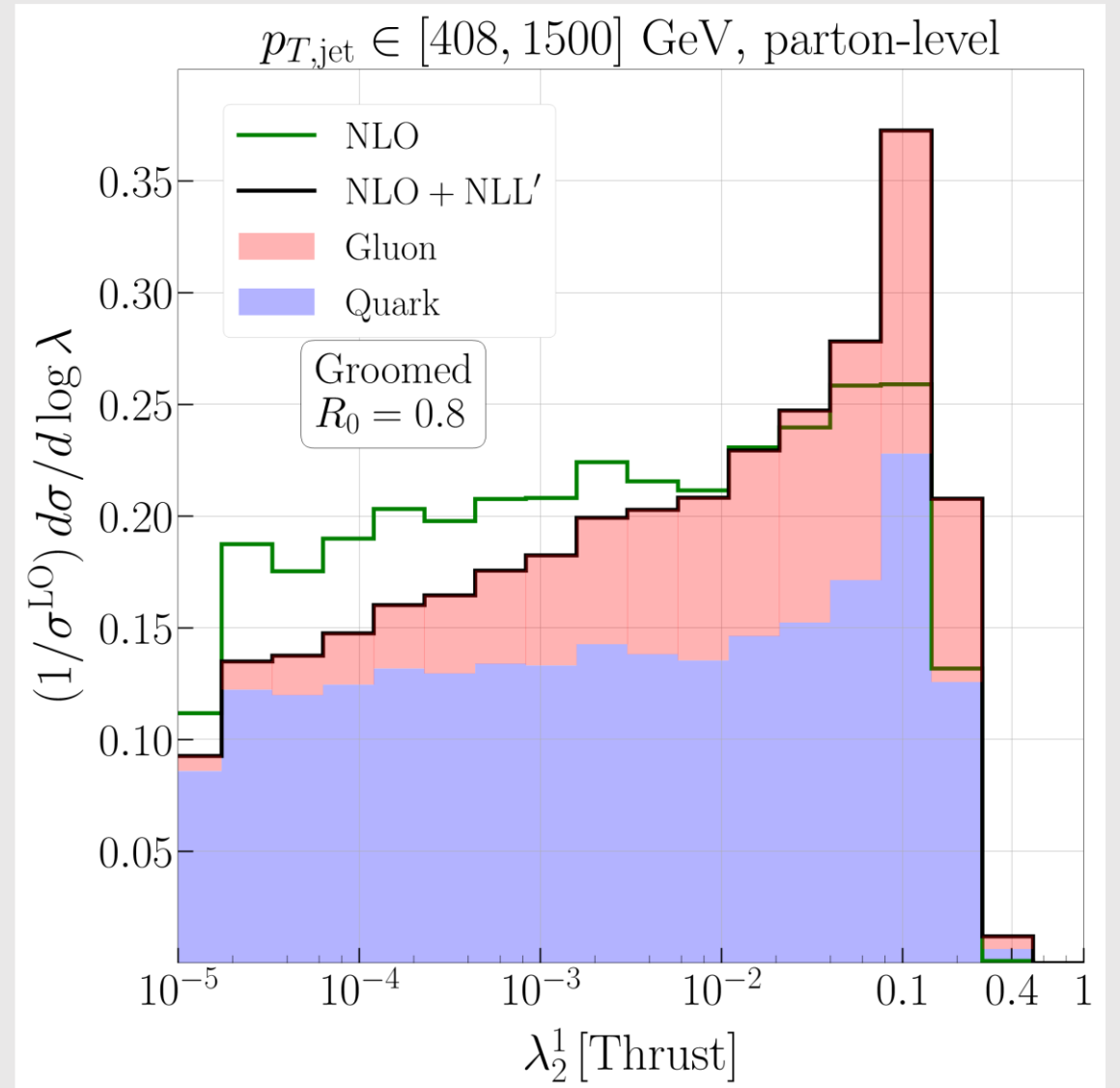
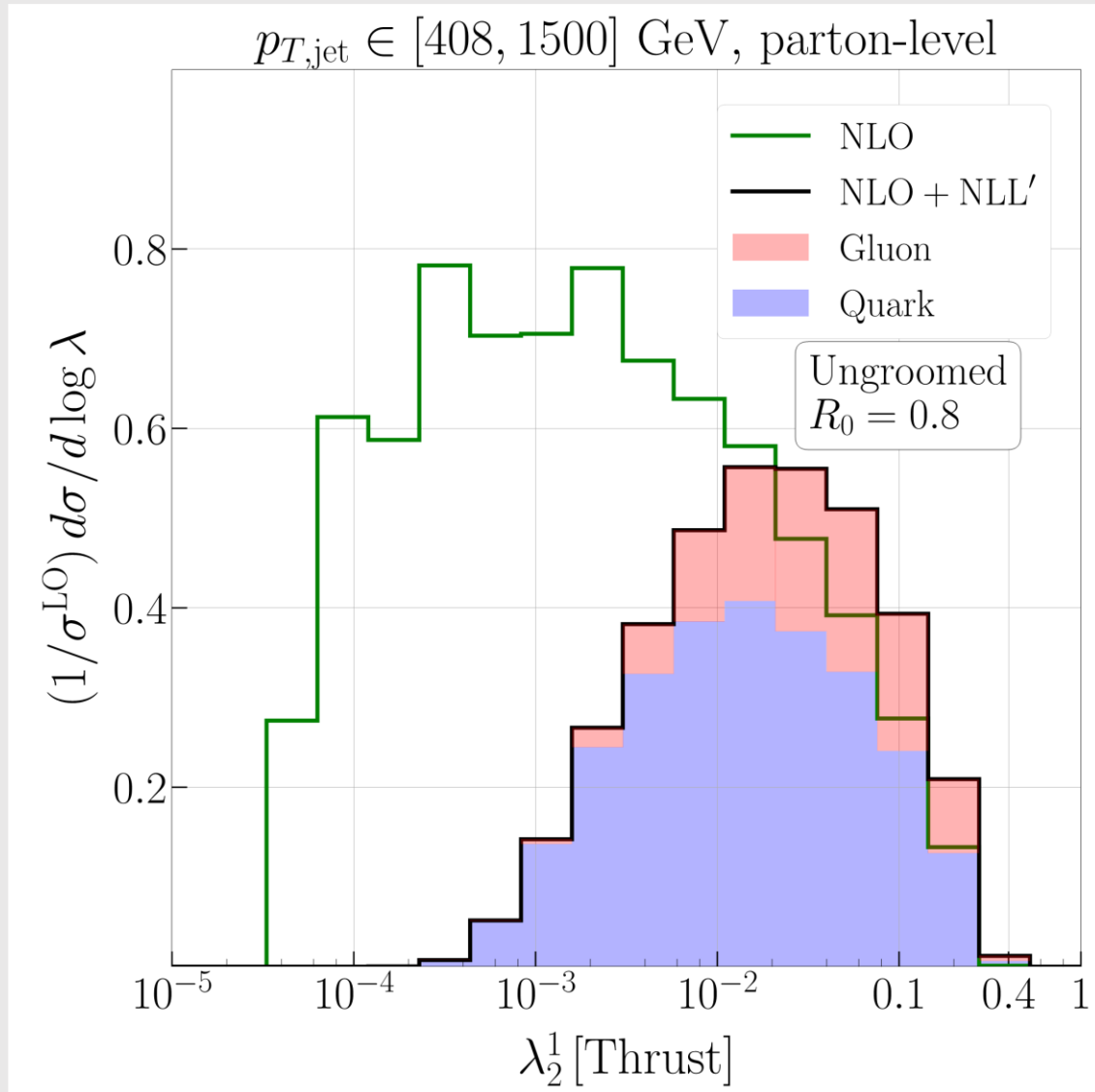
Fixed-order vs. matched:



Fixed-order vs. matched:

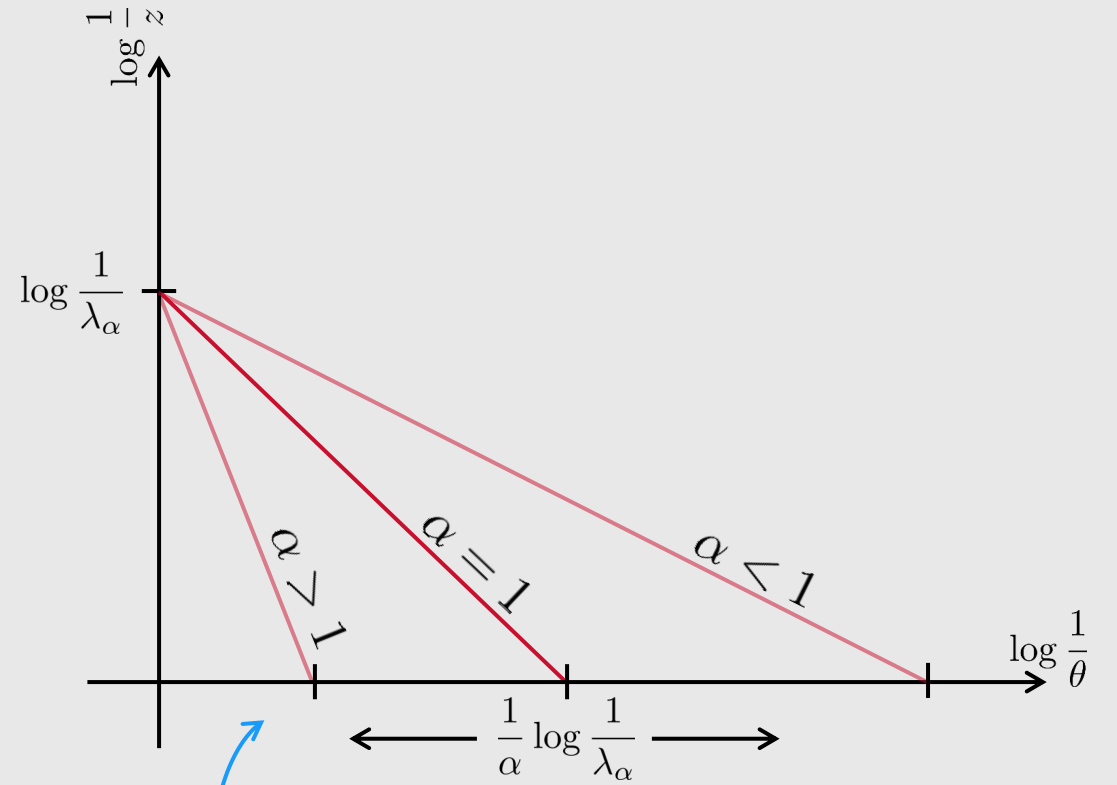
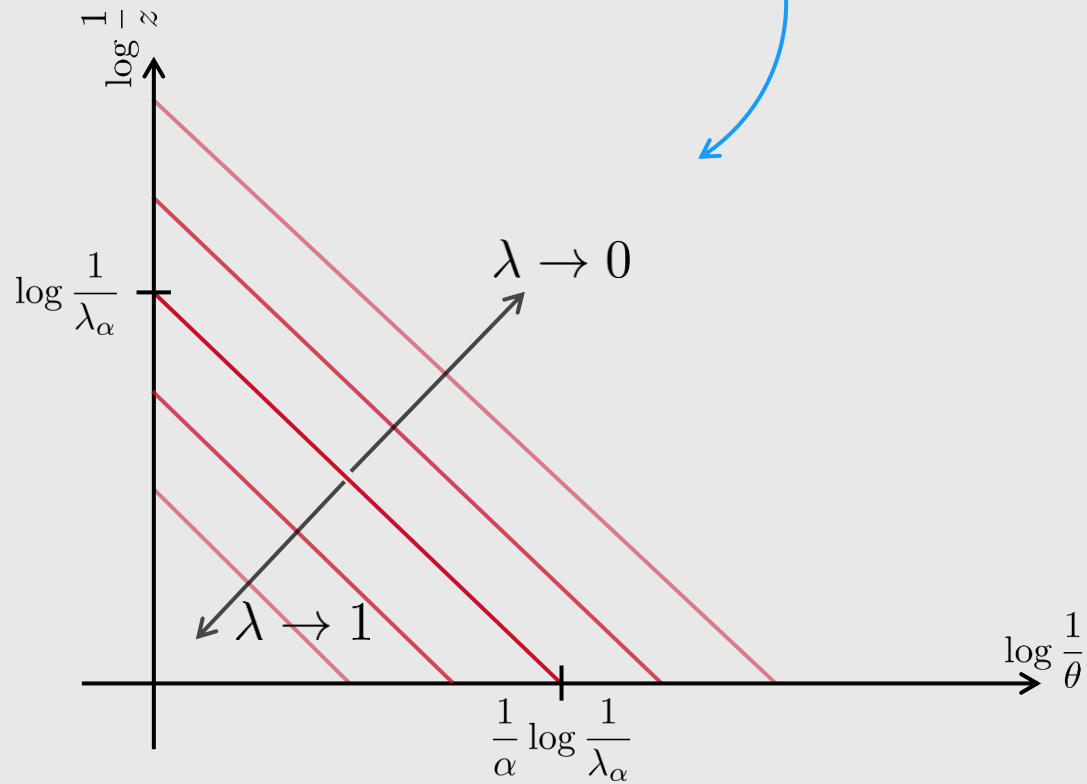


Fixed-order vs. matched:



Lund plane geography :

Varying λ corresponds to...



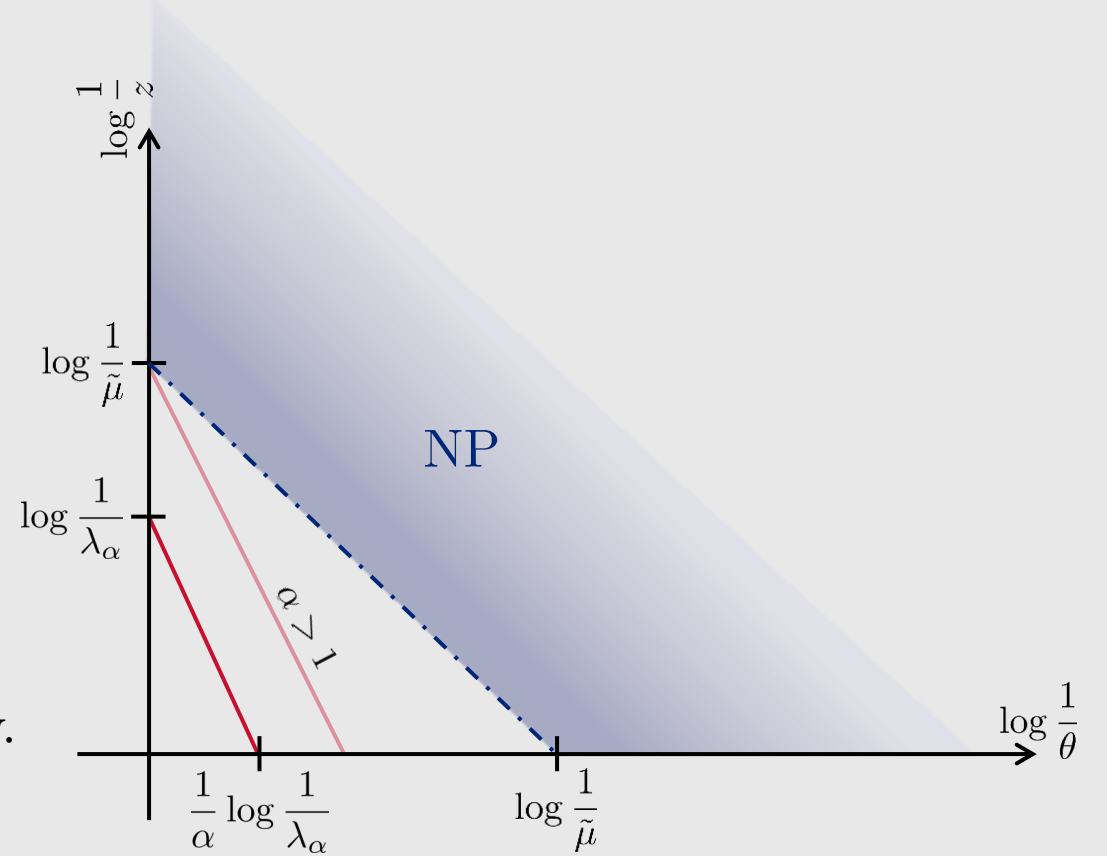
Varying α corresponds to...

Lund plane geography:

$$z\theta \gtrsim \tilde{\mu} \equiv \frac{1 \text{ GeV}}{p_{T,\text{Jet}} R} \quad \text{Perturbative region}$$

$$\text{Which corresponds to } \log \frac{1}{z} \lesssim \log \frac{1}{\tilde{\mu}} - \log \frac{1}{\theta}$$

We can push $\lambda \rightarrow 0$ until we run into the NP boundary.



In the ungroomed case which end-point of the angularity line do that for first depends on α . For $\alpha > 1$:

$$\log \frac{1}{\lambda_\alpha} = \log \frac{1}{\tilde{\mu}} \quad \Rightarrow \quad \lambda_\alpha \simeq \tilde{\mu}$$

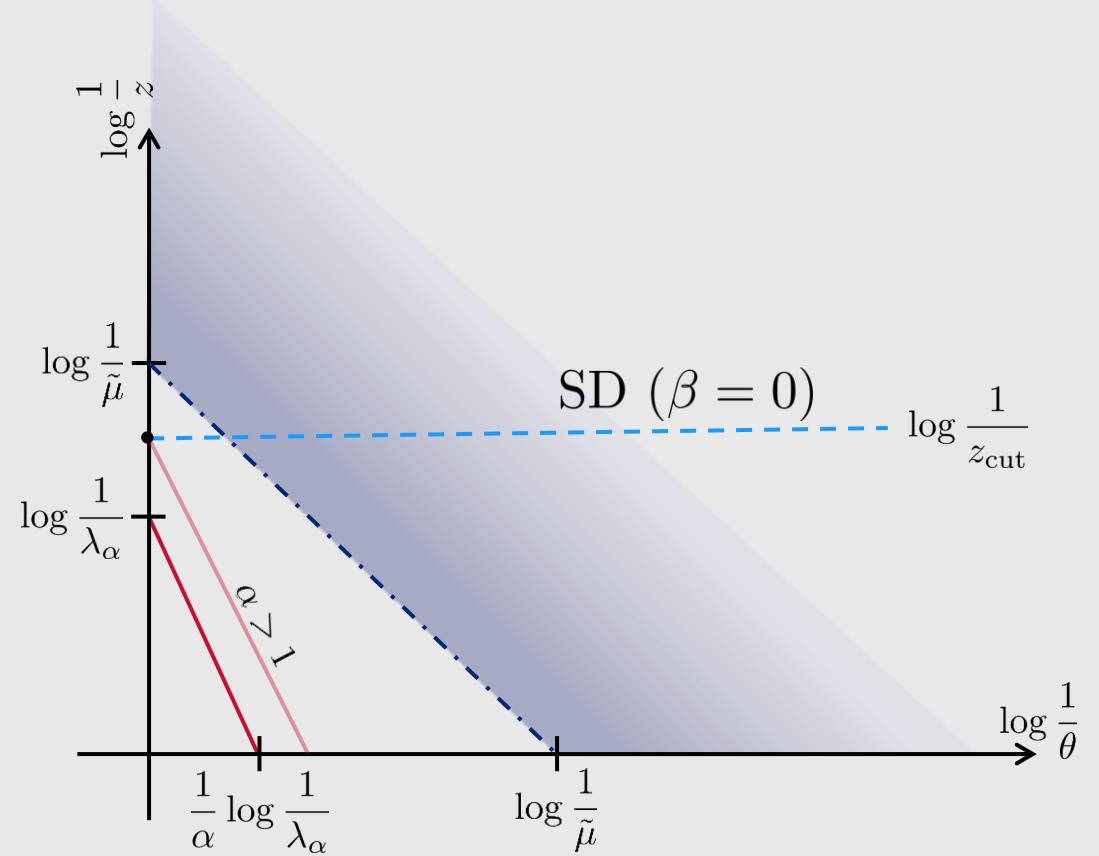
Instead for $\alpha < 1$:

$$\frac{1}{\alpha} \log \frac{1}{\lambda_\alpha} \simeq \log \frac{1}{\tilde{\mu}} \quad \Rightarrow \quad \lambda_\alpha \simeq \tilde{\mu}^\alpha$$

Lund plane geografy:

$$\log \frac{1}{z} < \log \frac{1}{z_{\text{cut}}} + \beta \log \frac{1}{\theta} \quad \text{SoftDrop condition}$$

$$\log \frac{1}{z} < \log \frac{1}{\lambda_\alpha} - \alpha \log \frac{1}{\theta} \quad \text{Angulariry veto}$$



If we add the SoftDrop condition we have a transition point before the angulariry line run into the NP boundary.

$$\log \frac{1}{z_{\text{cut}}} = \log \frac{1}{\lambda_\alpha} \quad \Rightarrow \quad \lambda_\alpha = z_{\text{cut}}$$

Lund plane geography:

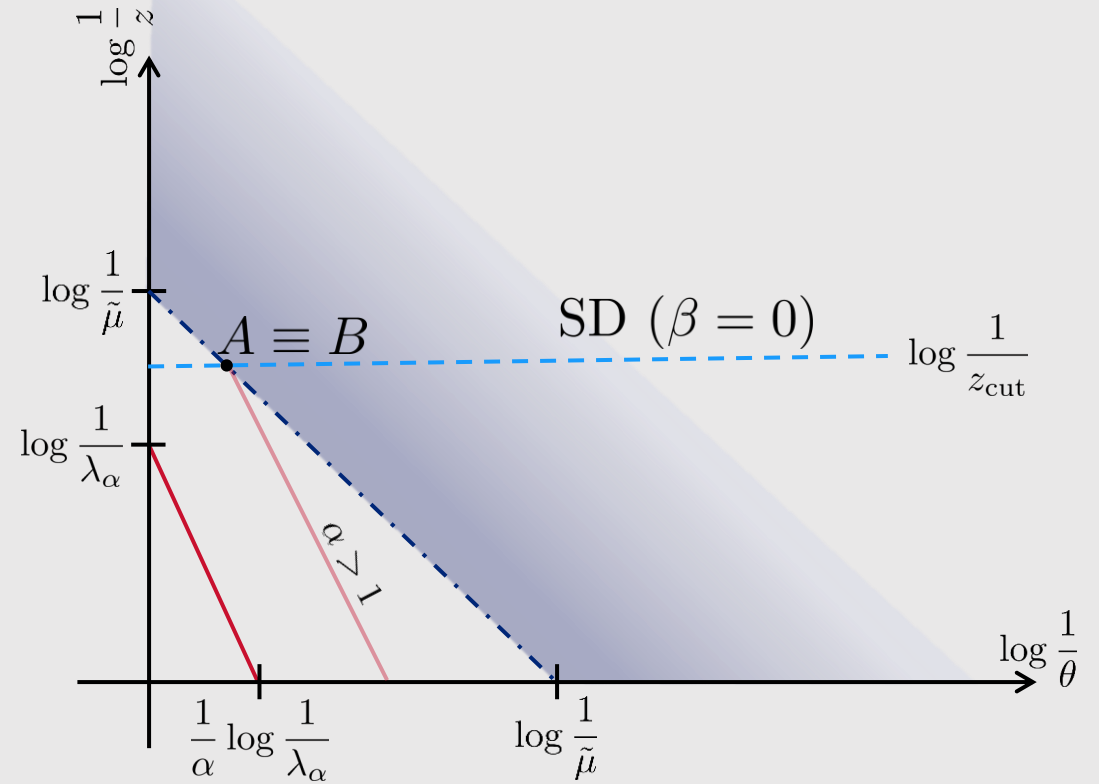
$$\log \frac{1}{z} < \log \frac{1}{z_{\text{cut}}} + \beta \log \frac{1}{\theta} \quad \text{SoftDrop condition}$$

$$\log \frac{1}{z} \lesssim \log \frac{1}{\tilde{\mu}} - \log \frac{1}{\theta} \quad \text{Perturbative region}$$

$$\log \frac{1}{z} < \log \frac{1}{\lambda_\alpha} - \alpha \log \frac{1}{\theta} \quad \text{Angularity veto}$$

$$x_A = y_A = \frac{1}{\beta + 1} \log \frac{z_{\text{cut}}}{\tilde{\mu}}$$

$$x_B = y_B = \frac{1}{\alpha - 1} \log \frac{\tilde{\mu}}{\lambda_\alpha}$$

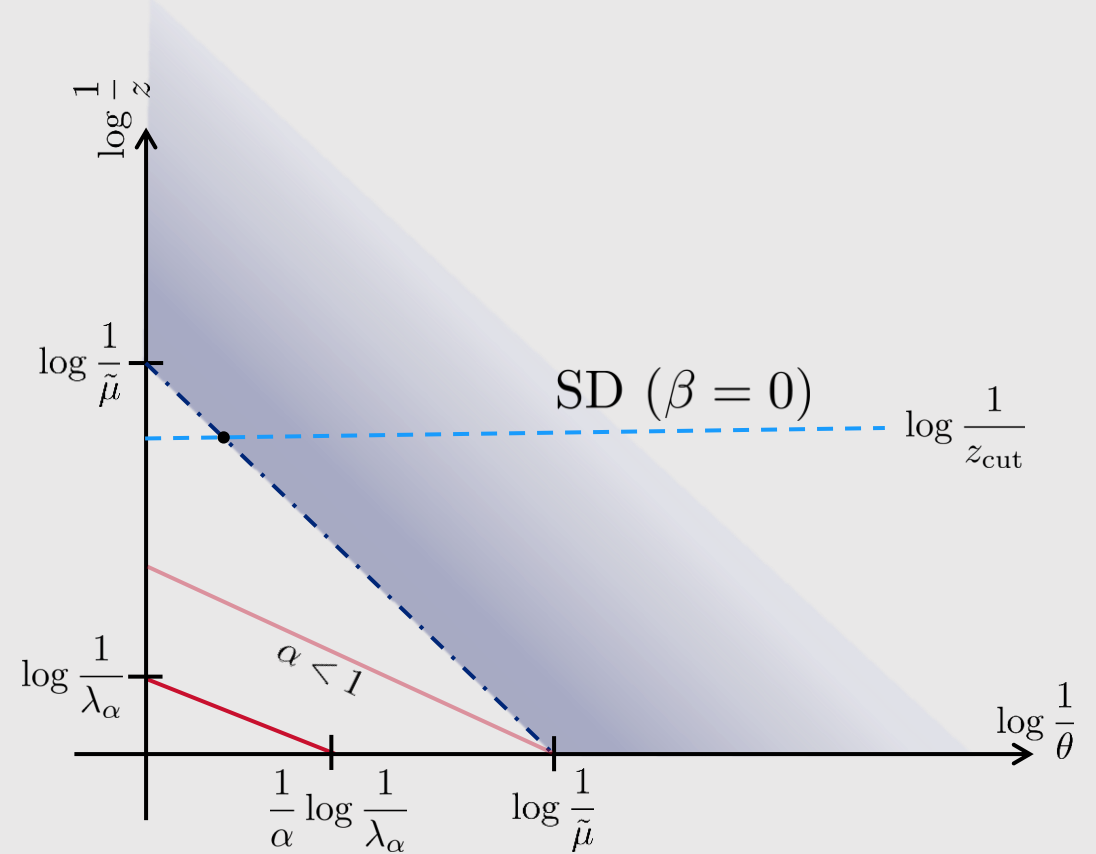


Now, considering also the SoftDrop condition the $\alpha < 1$ and the $\alpha > 1$ situations become different. For $\alpha > 1$ we have:

$$\lambda_\alpha \gtrsim \tilde{\mu} \left(\frac{\tilde{\mu}}{z_{\text{cut}}} \right)^{\frac{\alpha-1}{\beta+1}}$$

Lund plane geography:

For the $\alpha < 1$ case the SoftDrop condition doesn't affect the intersection point with the NP boundary.



Summarizing, we expect (resummed) perturbative to provide a good description of the physical process for

$$\text{Ungroomed jets or SD jets with } \alpha \leq 1: \quad \lambda_\alpha \gtrsim \tilde{\mu}^{\min[\alpha, 1]}$$

$$\text{SD jets with } \alpha > 1: \quad \lambda_\alpha \gtrsim \tilde{\mu} \left(\frac{\tilde{\mu}}{z_{\text{cut}}} \right)^{\frac{\alpha-1}{\beta+1}}$$

Lund plane geografy:

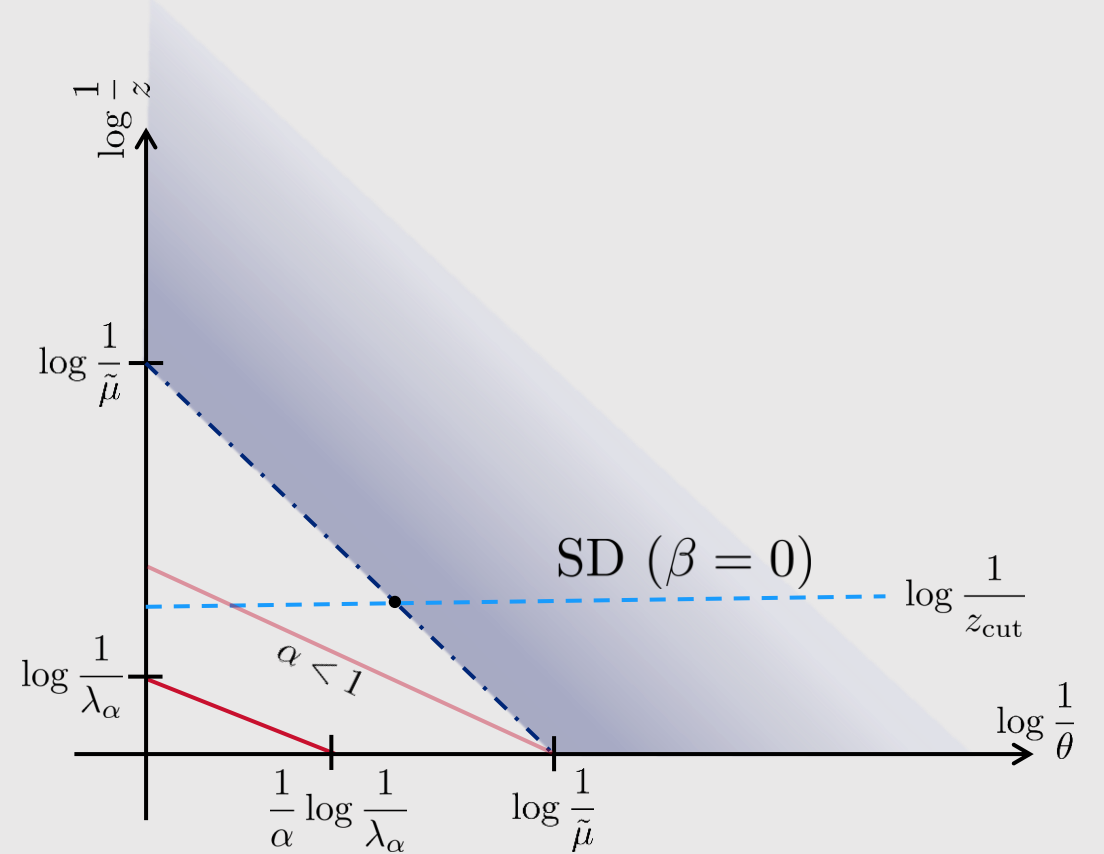


However it is possible that for big enough z_{cut} we hit SD transition point before reaching the NP boundary also for the $\alpha < 1$ case.

$$\log \frac{1}{z_{\text{cut}}} \lesssim \left[\log \frac{1}{\lambda_\alpha} - \alpha \log \frac{1}{\theta} \right]_{\text{at the NP boundary}}$$

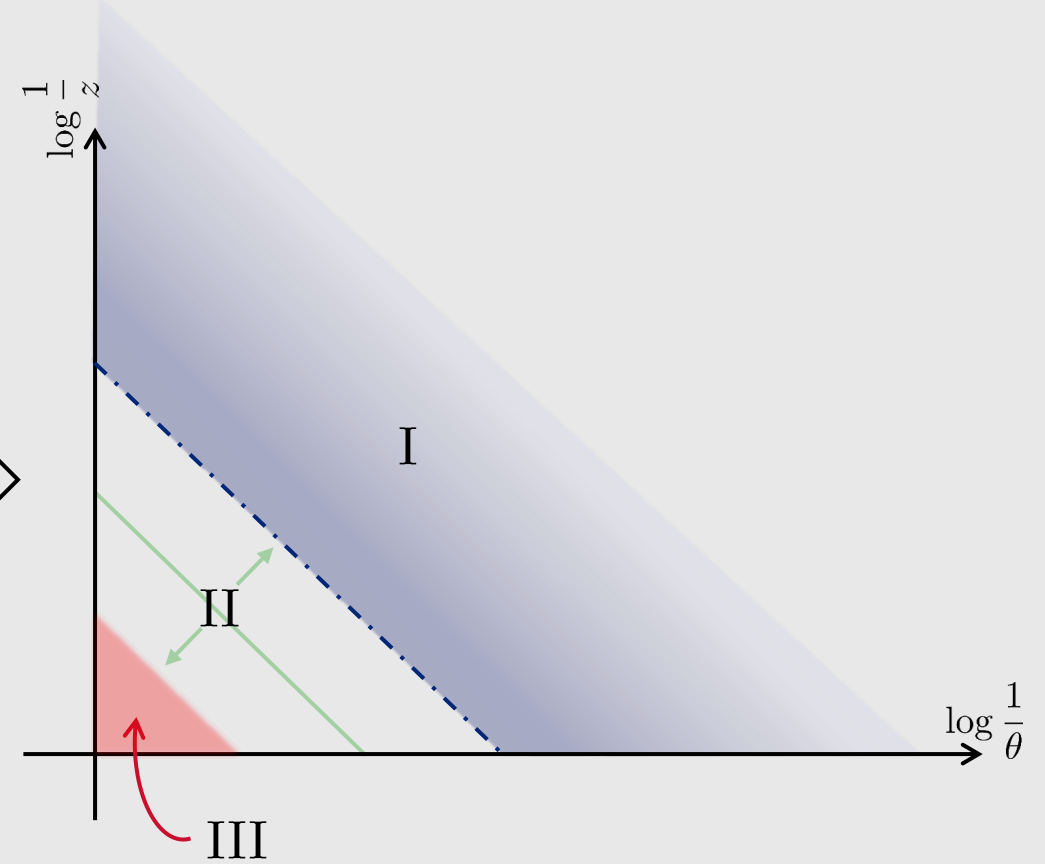
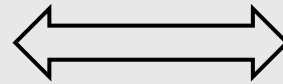
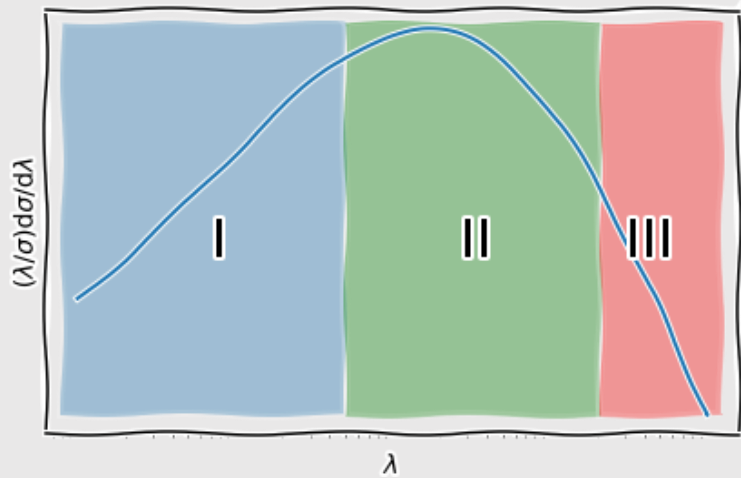
$$\lesssim \alpha \log \frac{1}{\tilde{\mu}} \Rightarrow z_{\text{cut}} \gtrsim \tilde{\mu}^\alpha$$

$$z_{\text{cut}} \gtrsim \left(\frac{1\text{GeV}}{100\text{GeV} \cdot 0.8} \right)^{1/2} \simeq 0.11$$



In this case we hit the SD transition point before the NP region but this does not affect the intersection point with the NP boundary.

Lund plane geography:

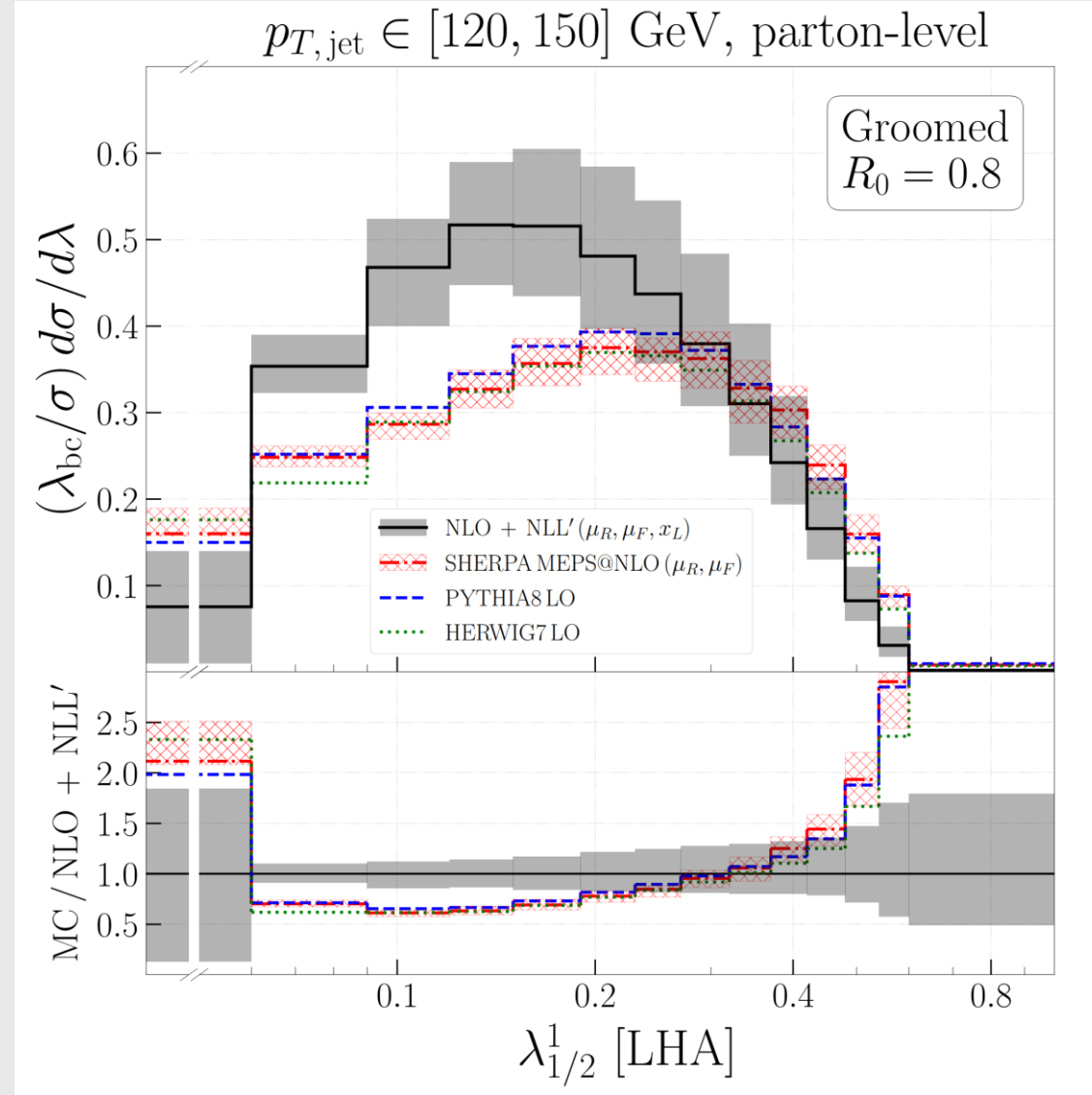
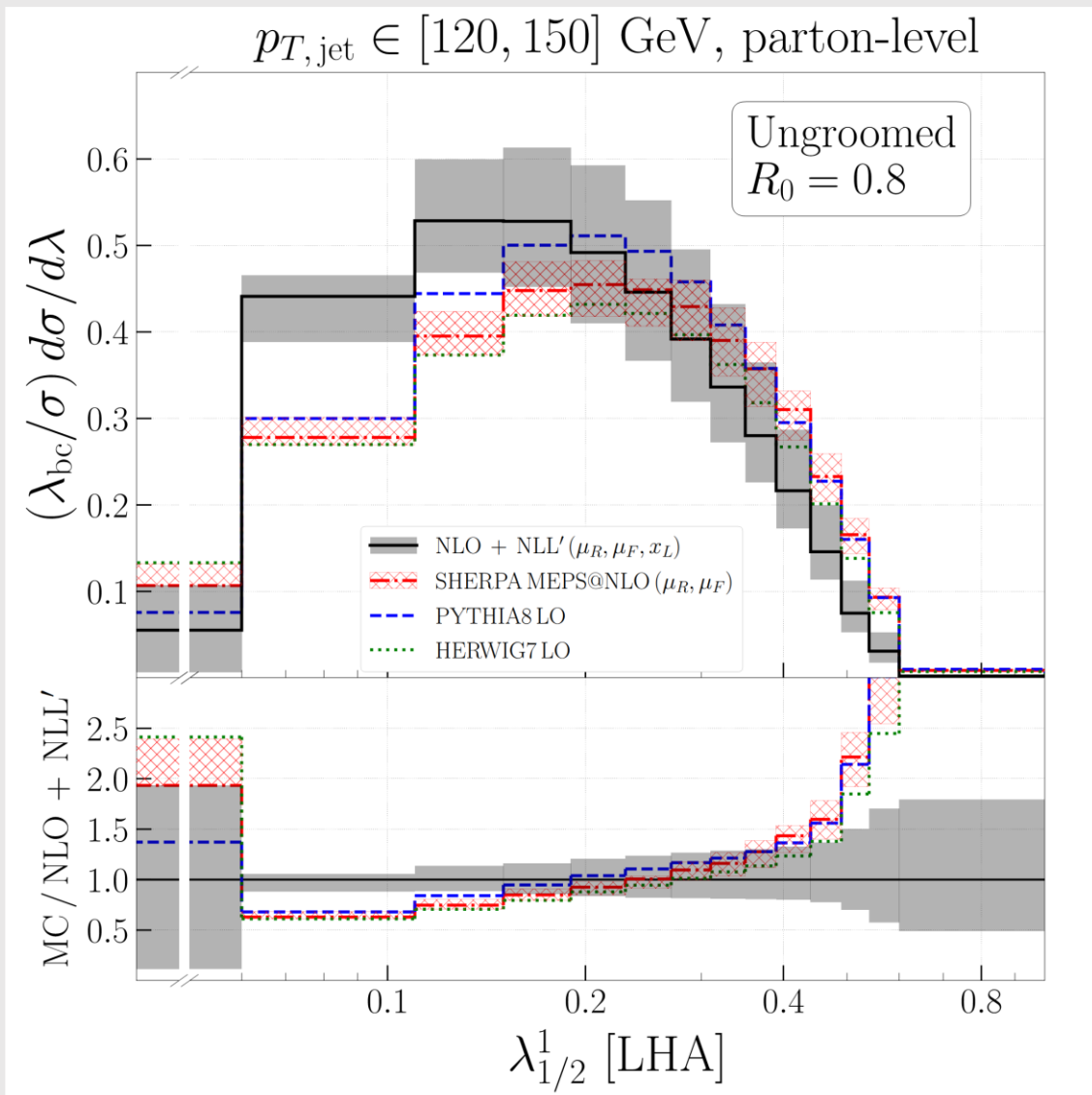


In addition from our LL picture we have that:

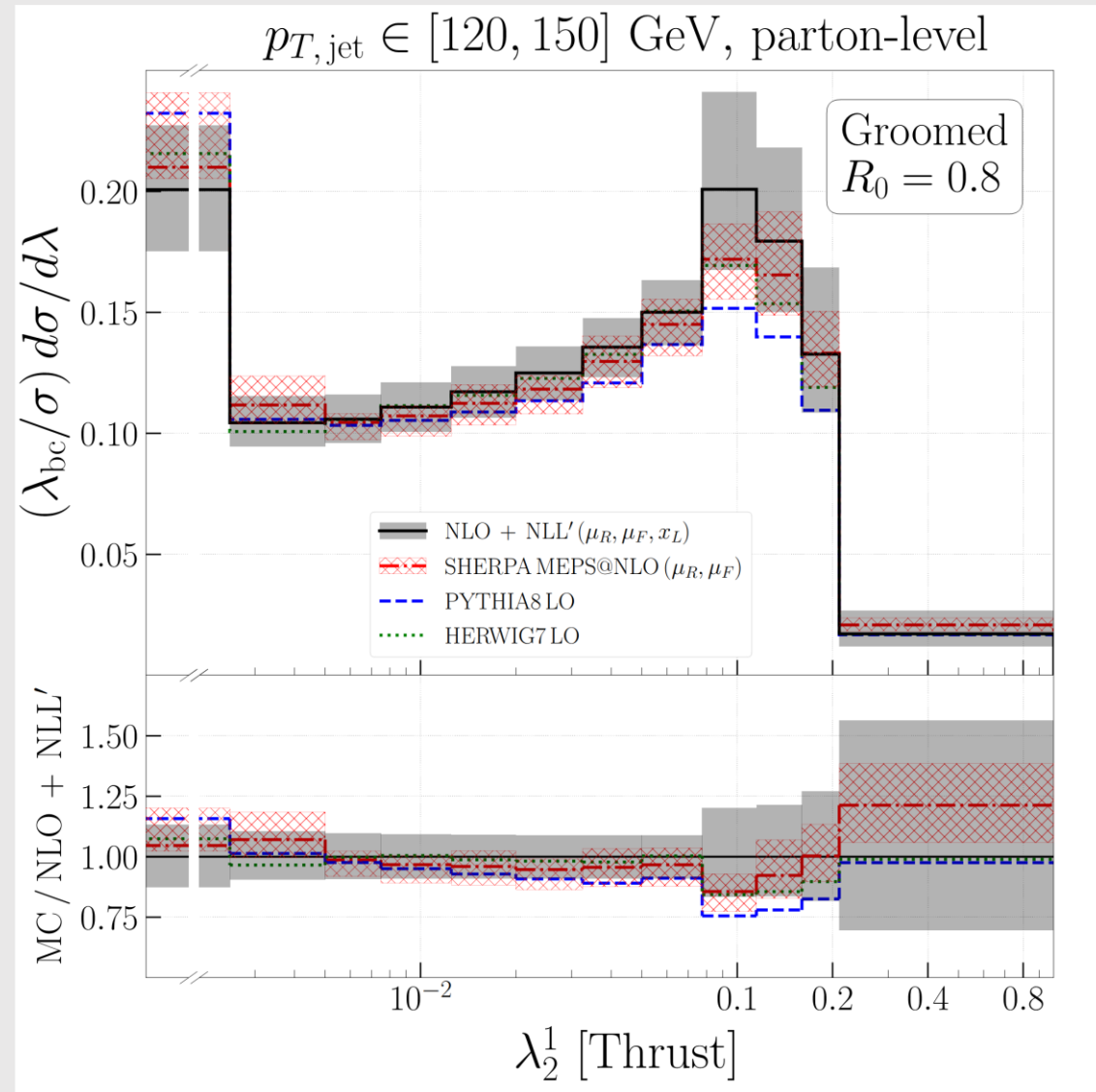
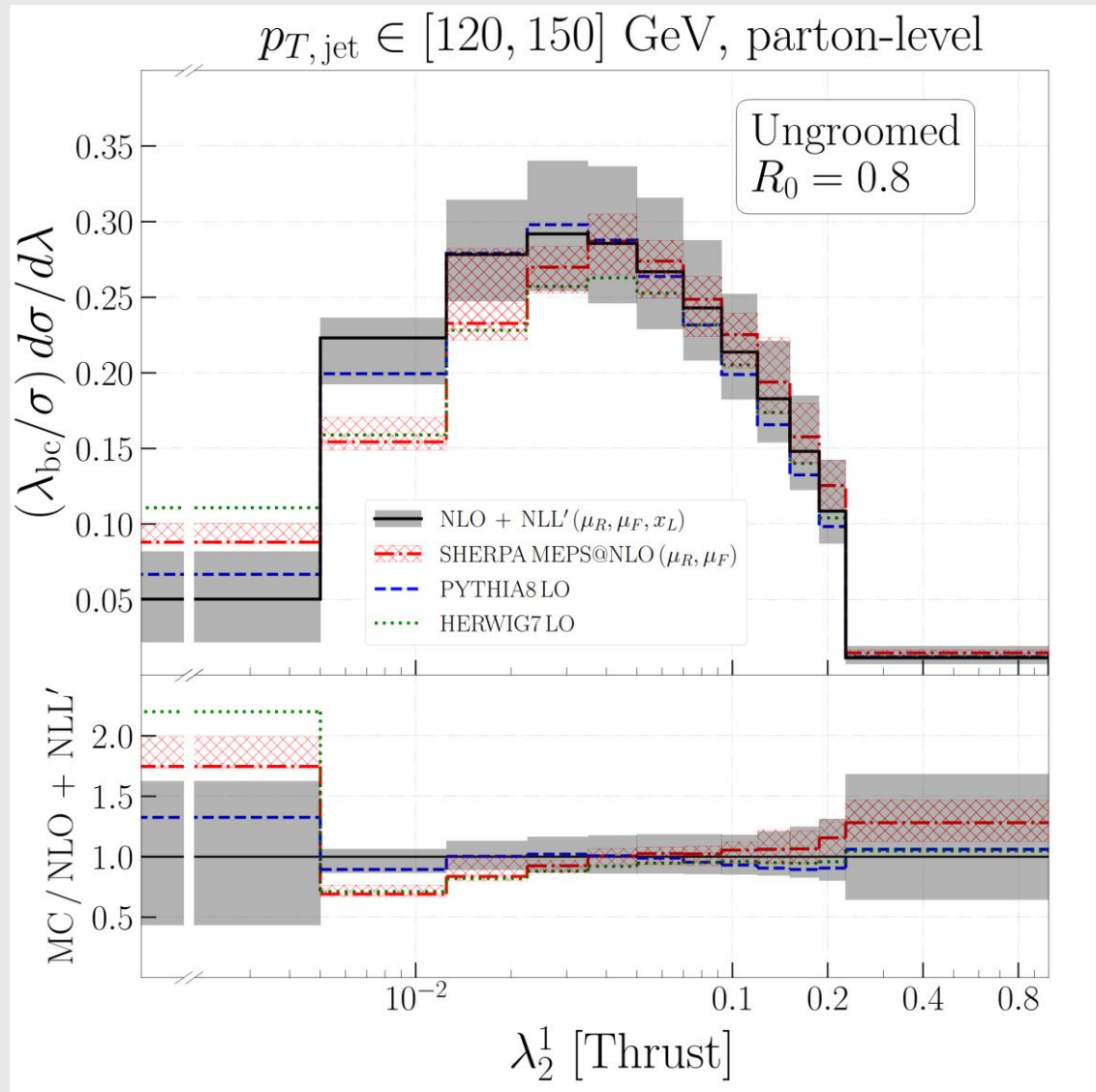
$$p(\lambda_\alpha) \simeq \frac{d}{d \log \lambda_\alpha} e^{-\frac{2 \cdot 0.118}{\pi} C_i \frac{\log^2 \lambda_\alpha}{2\alpha}} = -\frac{0.075}{\alpha} C_i \log \lambda_\alpha \cdot e^{-0.075 \cdot C_i \frac{\log^2 \lambda_\alpha}{2\alpha}}$$

α	$\lambda_\alpha^{\max, \text{quark}}$	$\lambda_\alpha^{\max, \text{gluon}}$
0.5	0.107	0.225
1	0.042	0.122
2	0.011	0.051

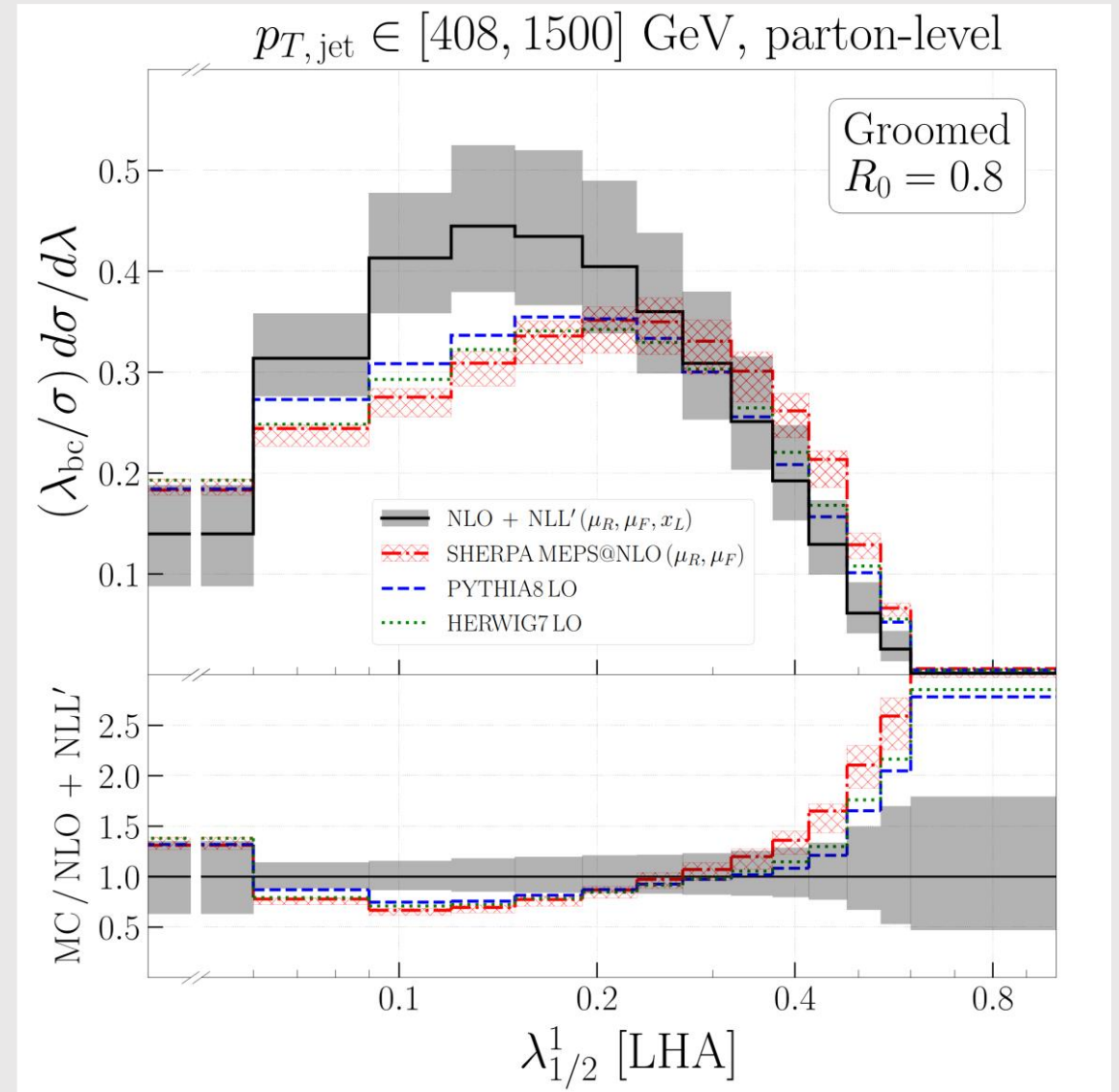
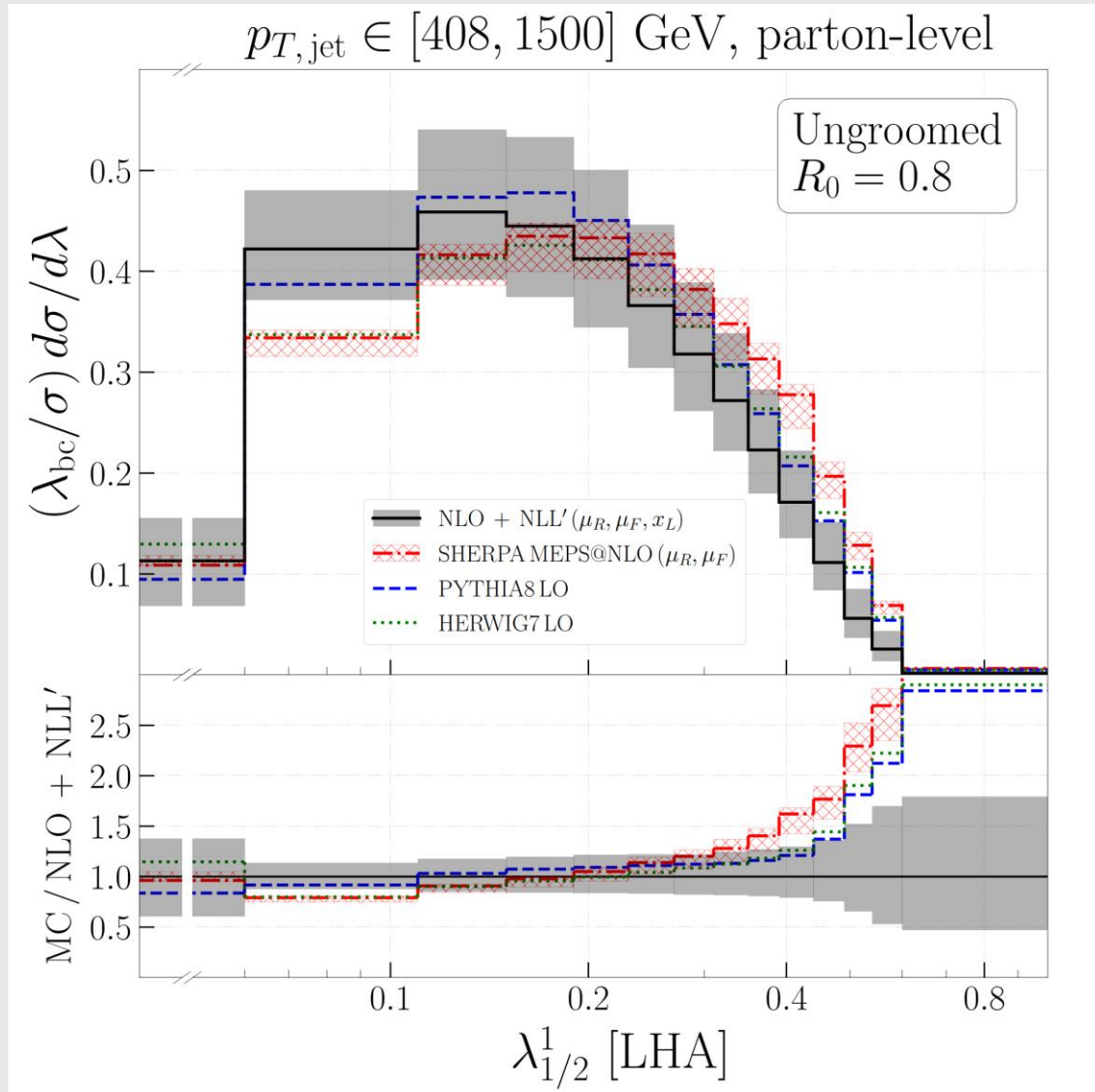
NLO + NLL':



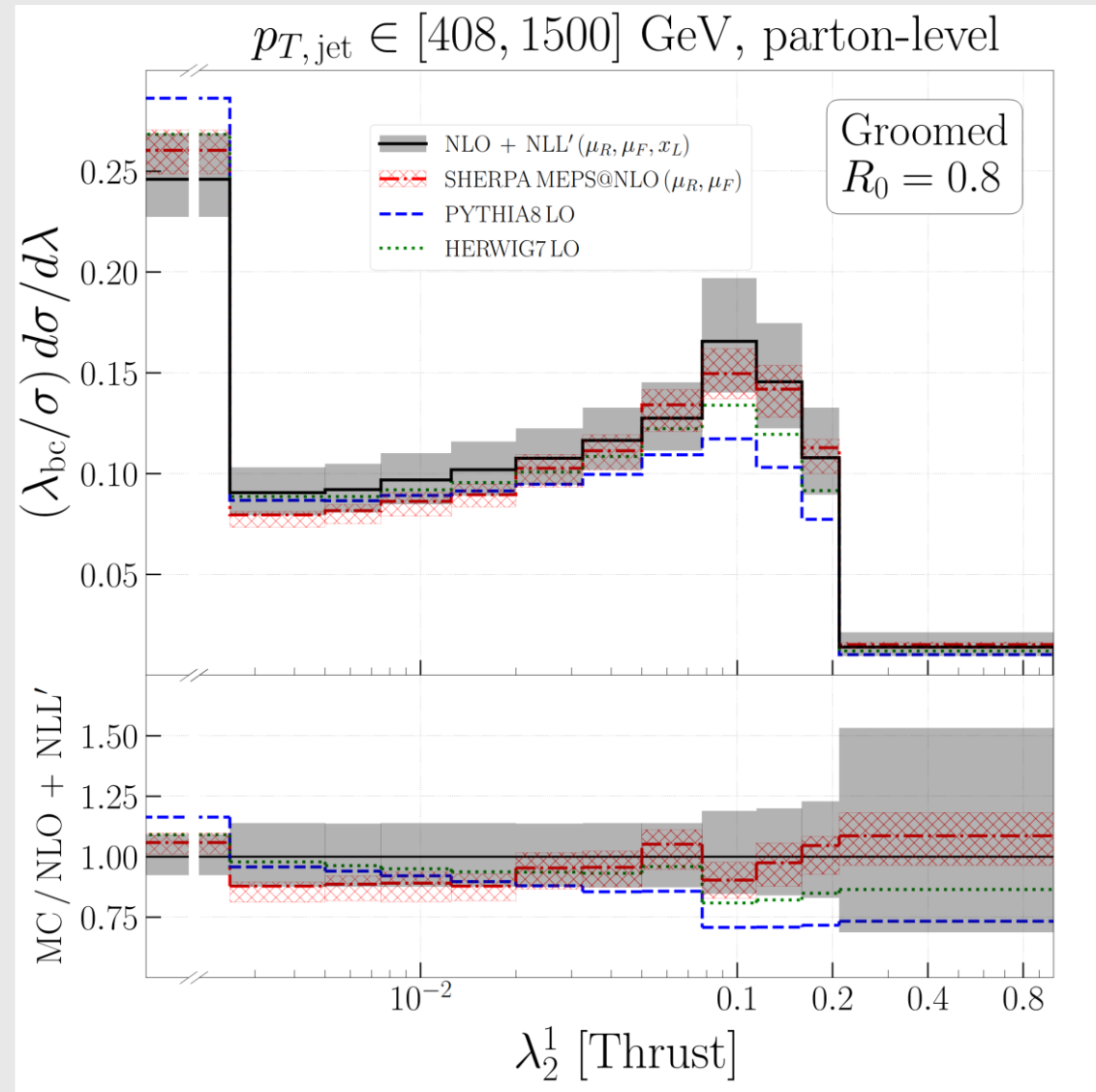
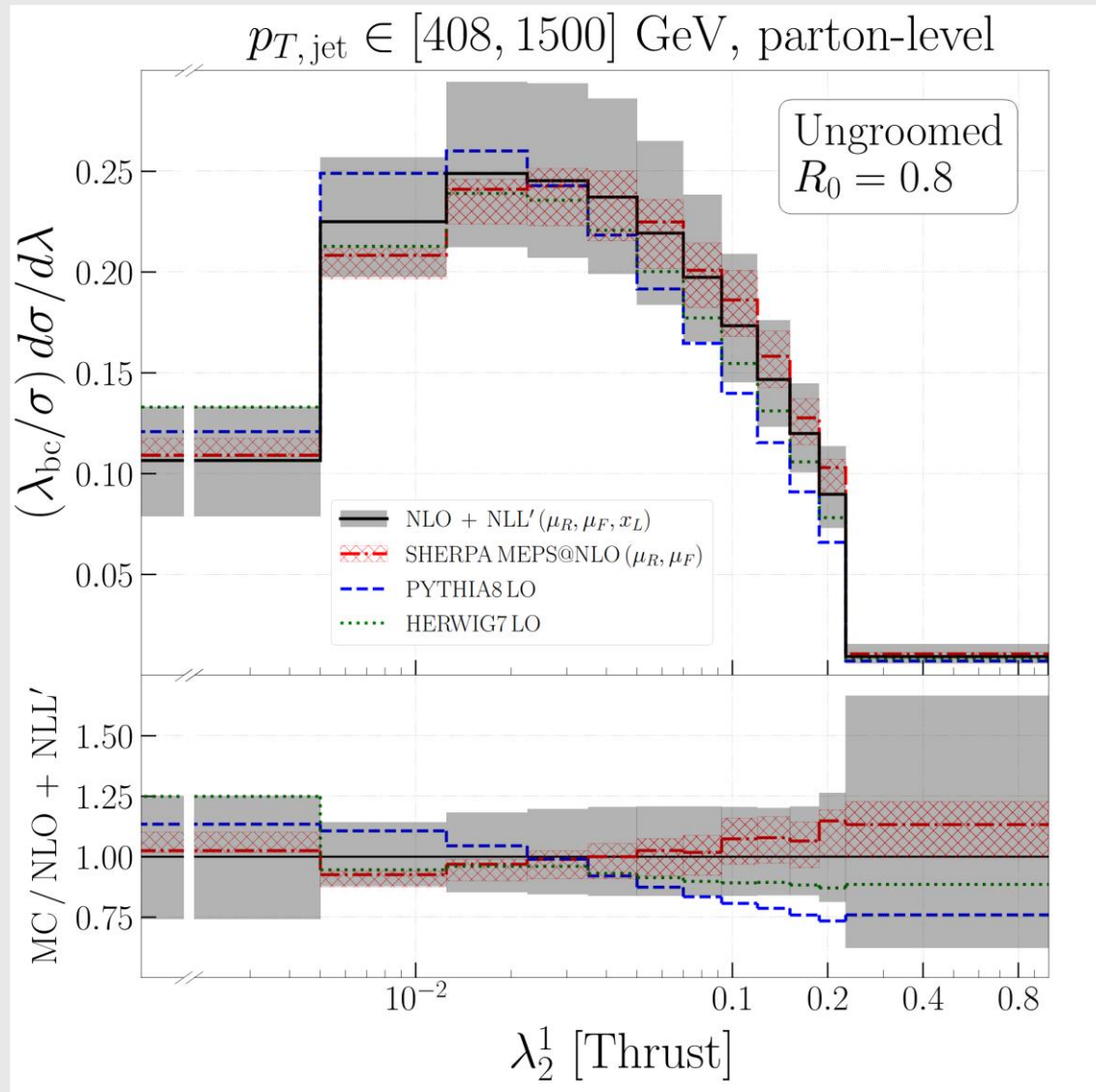
NLO + NLL':



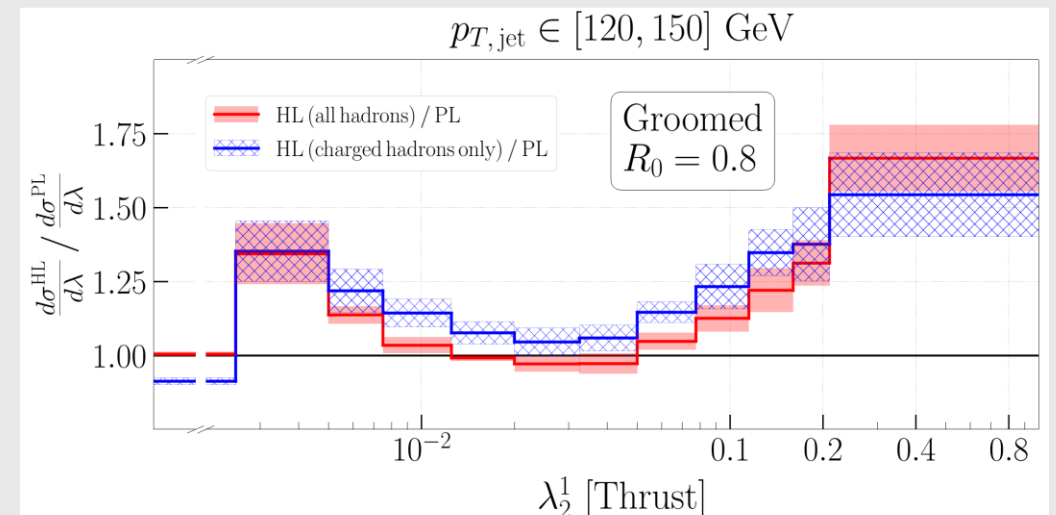
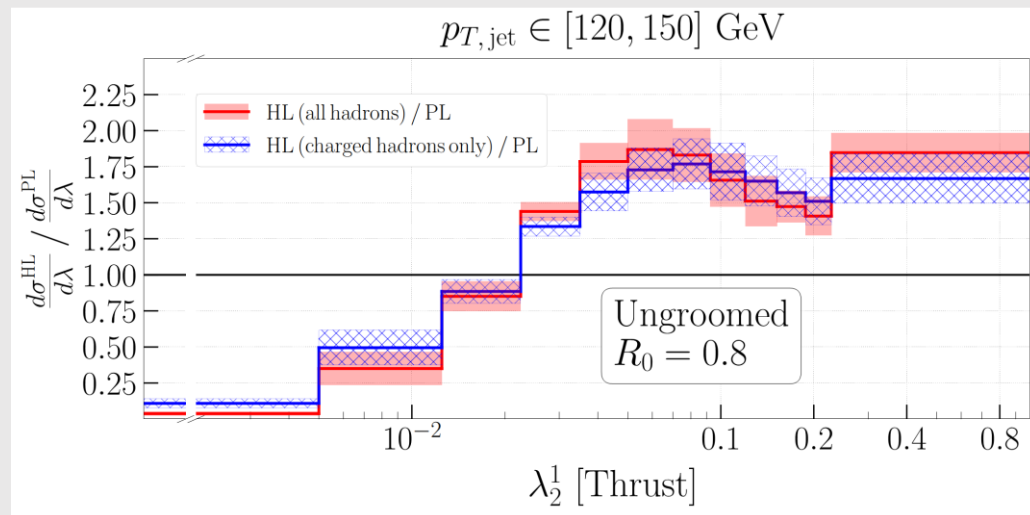
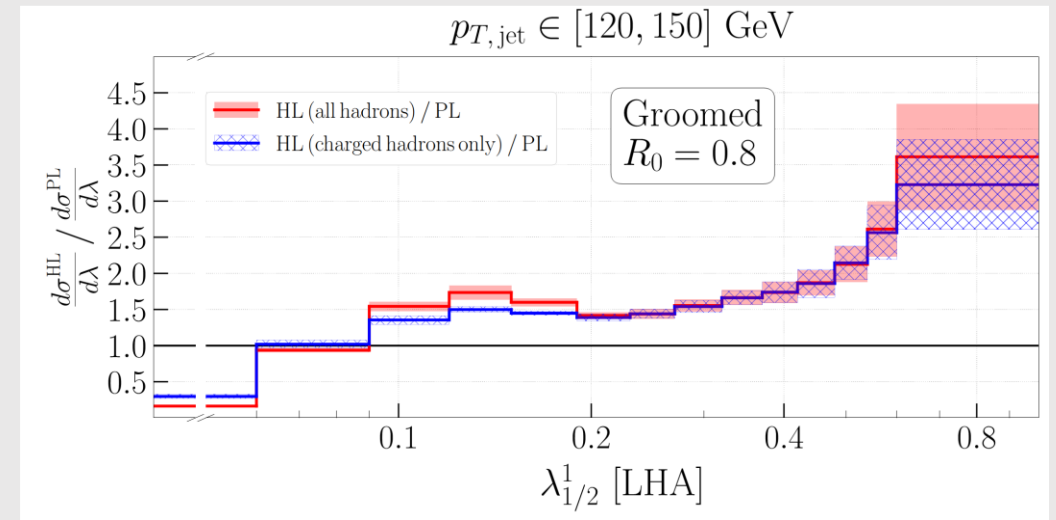
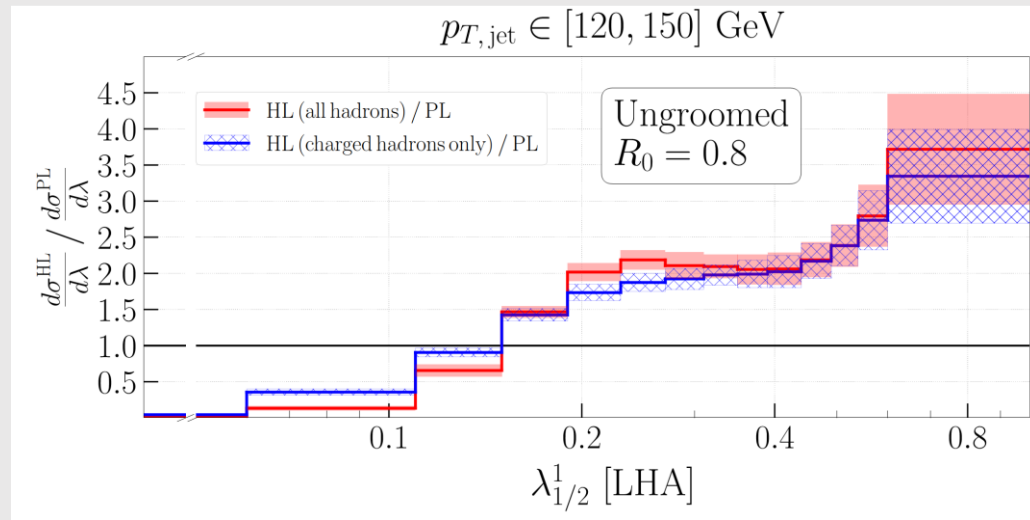
NLO + NLL'



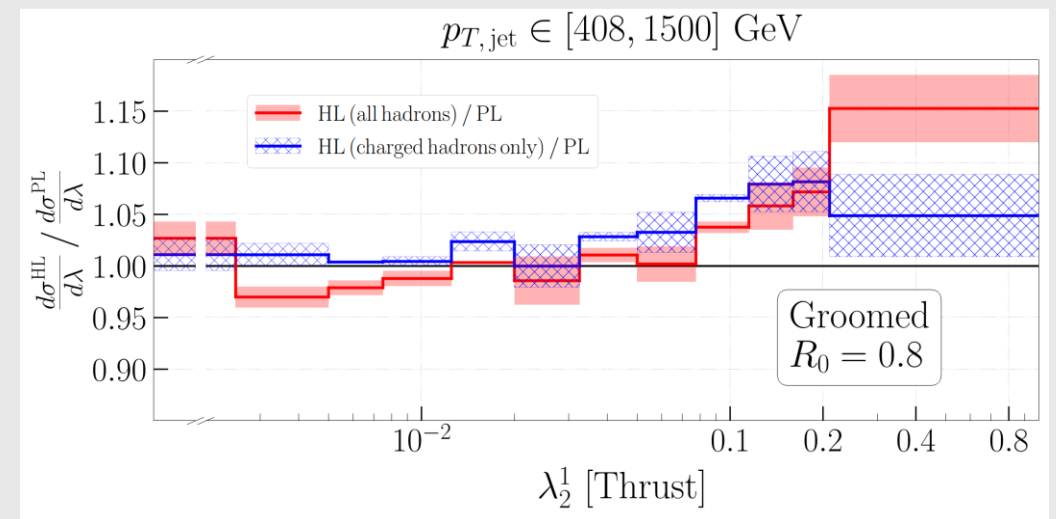
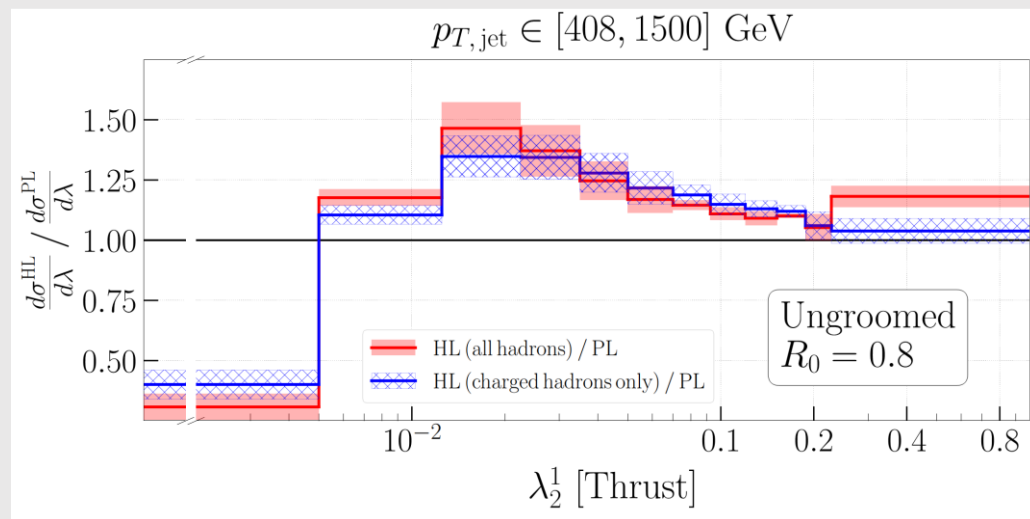
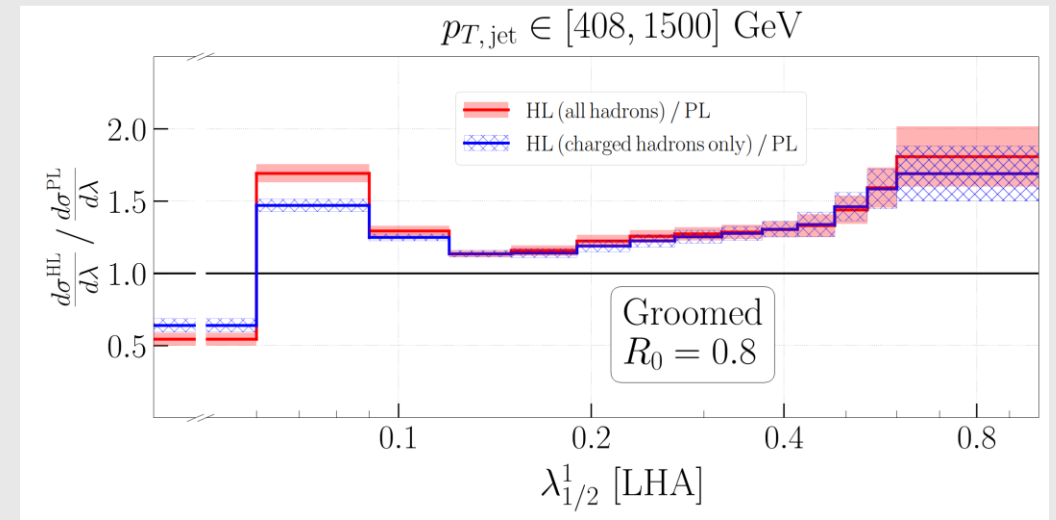
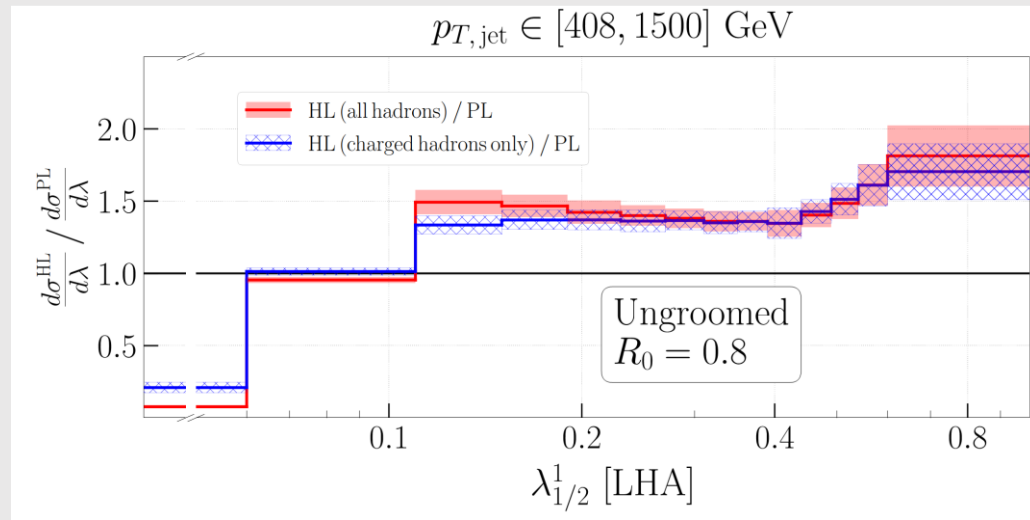
NLO + NLL'



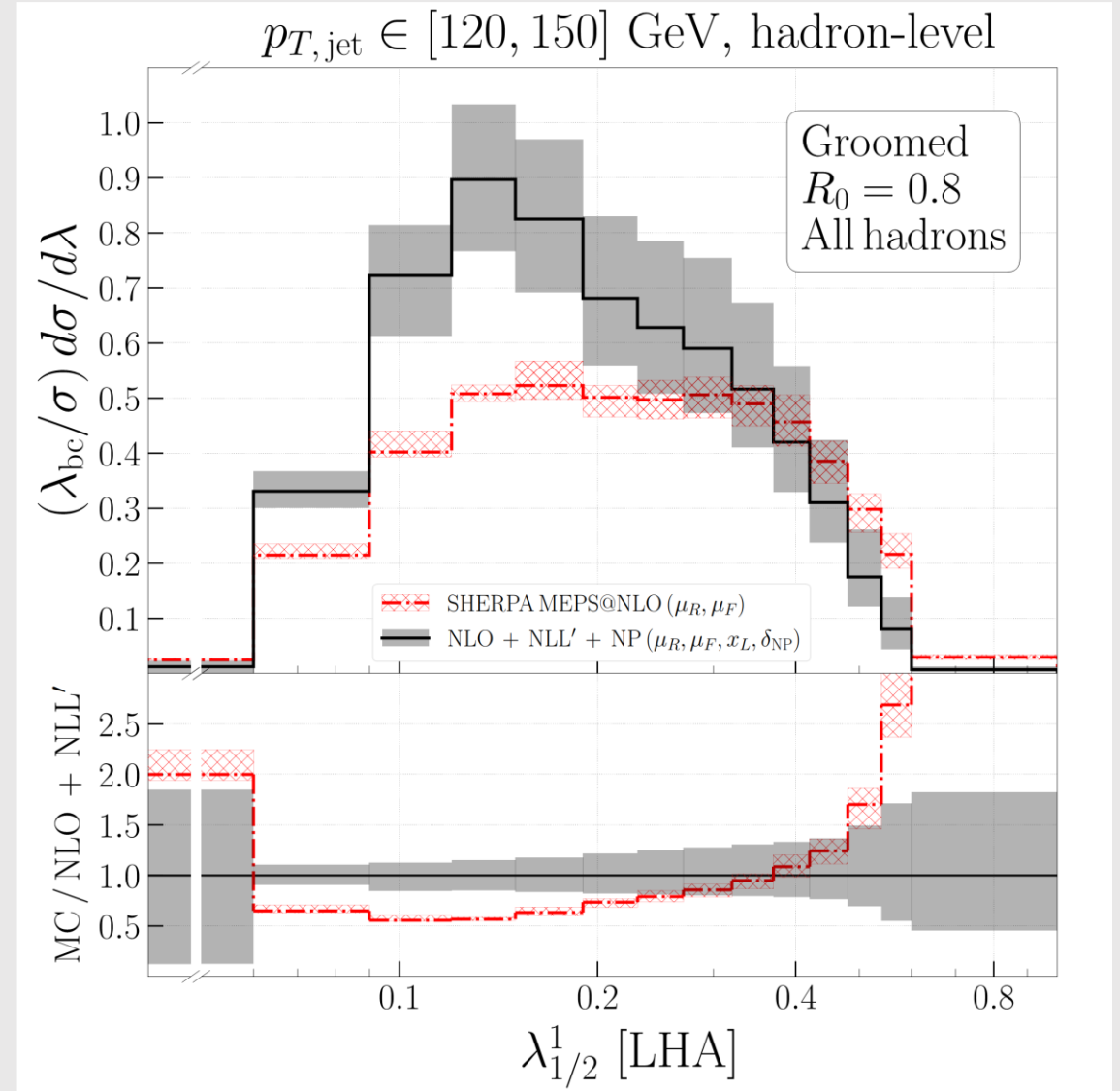
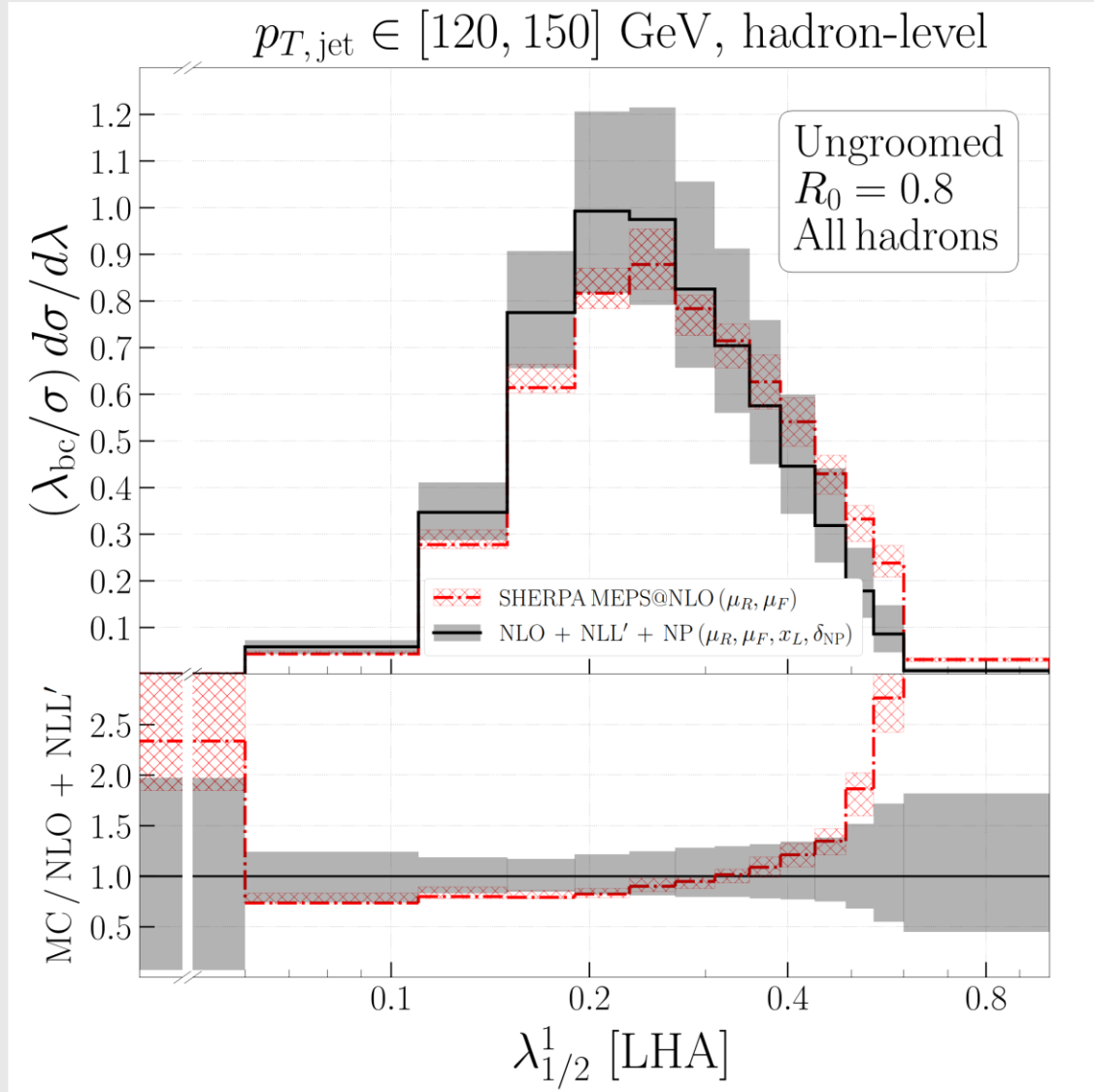
NP corrections from MC:



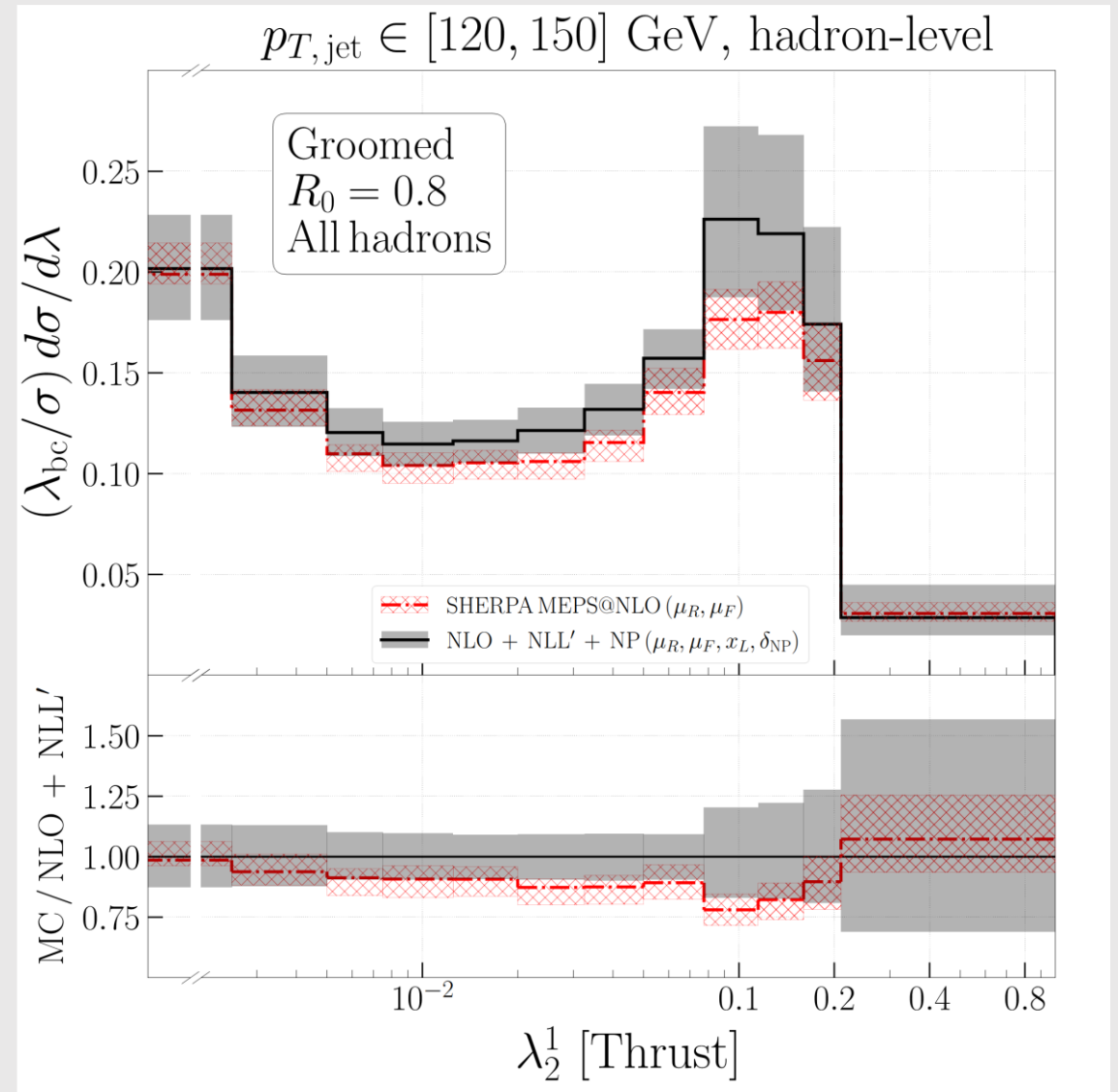
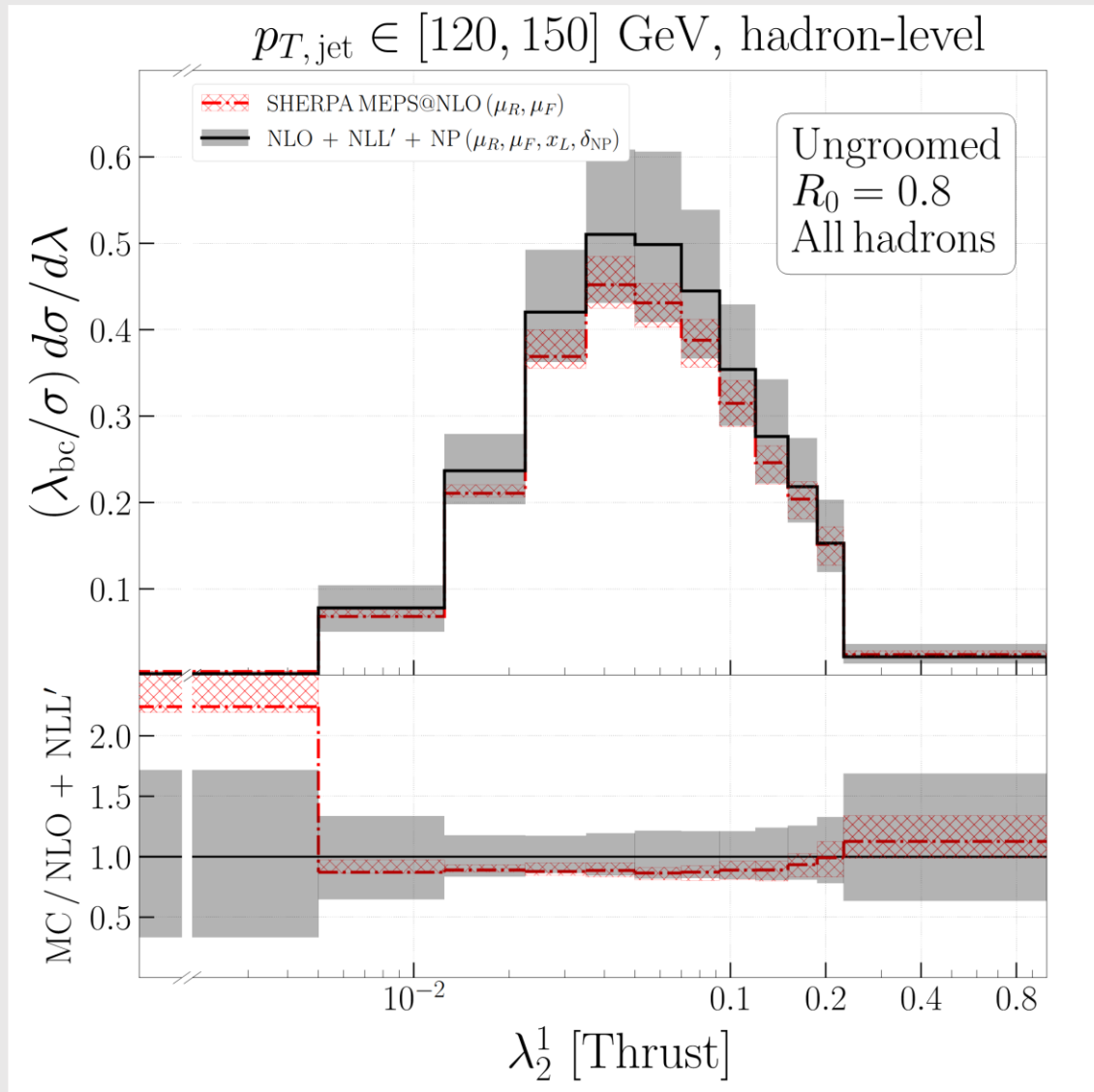
NP corrections from MC:



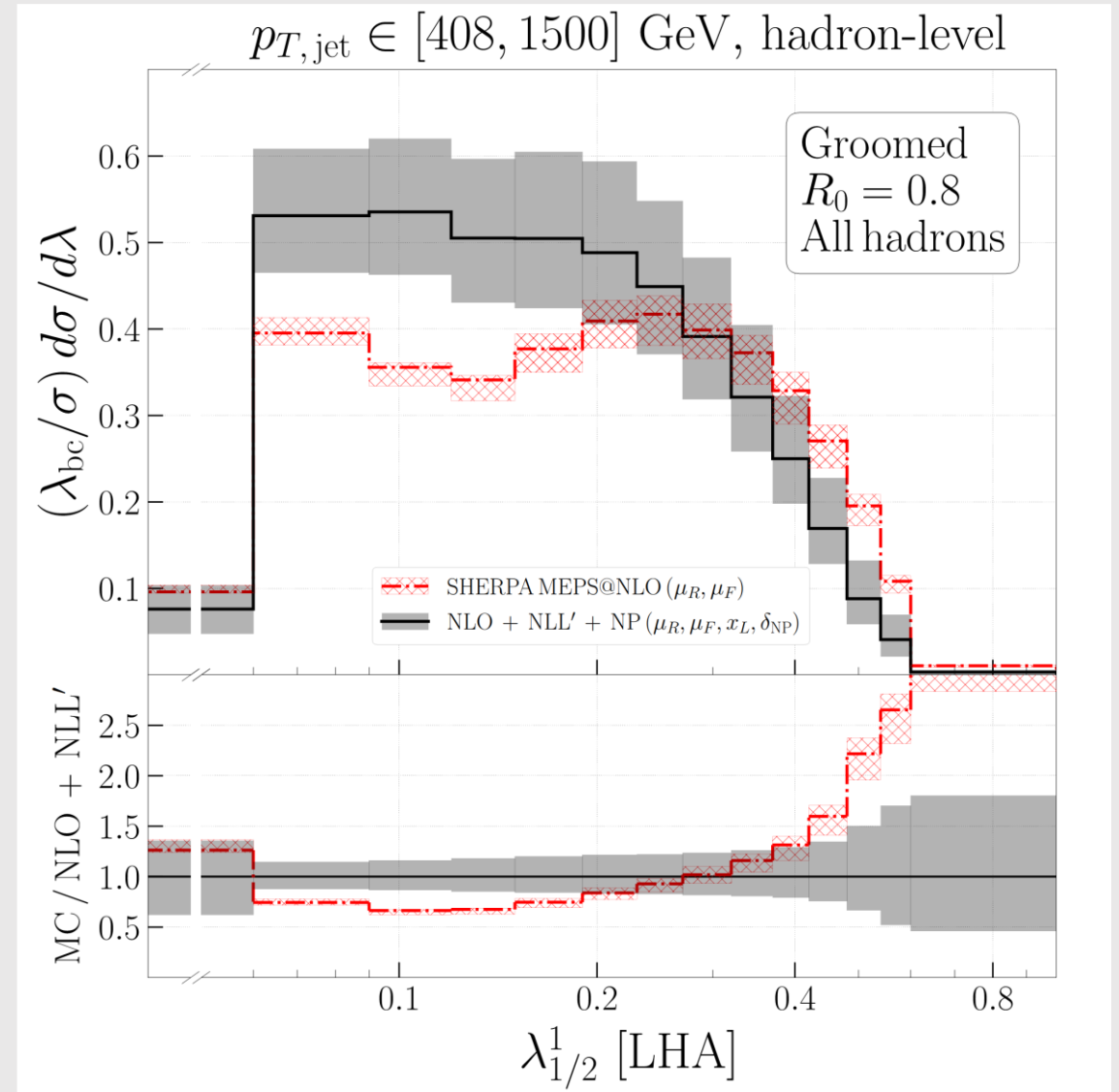
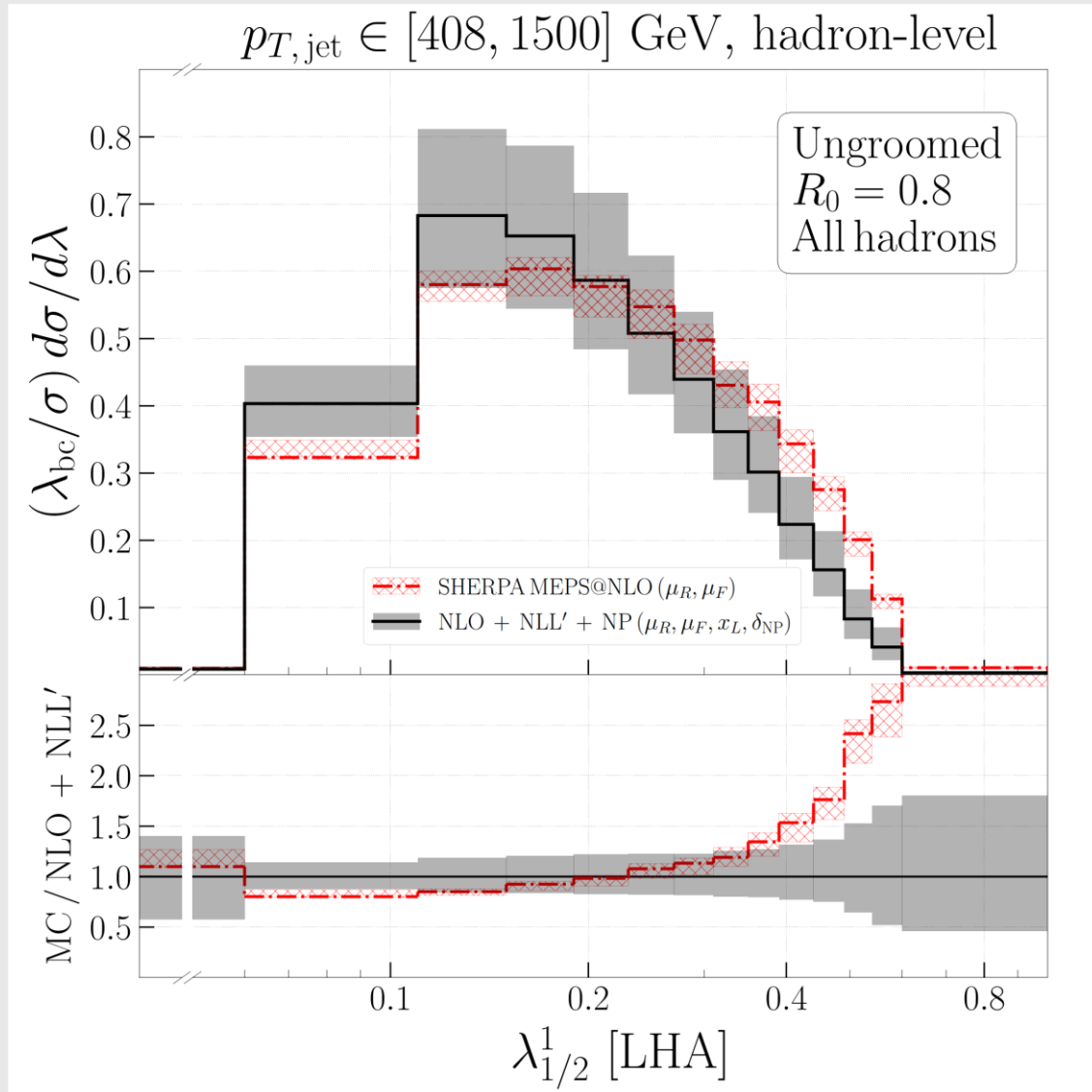
NLO + NLL' + NP:



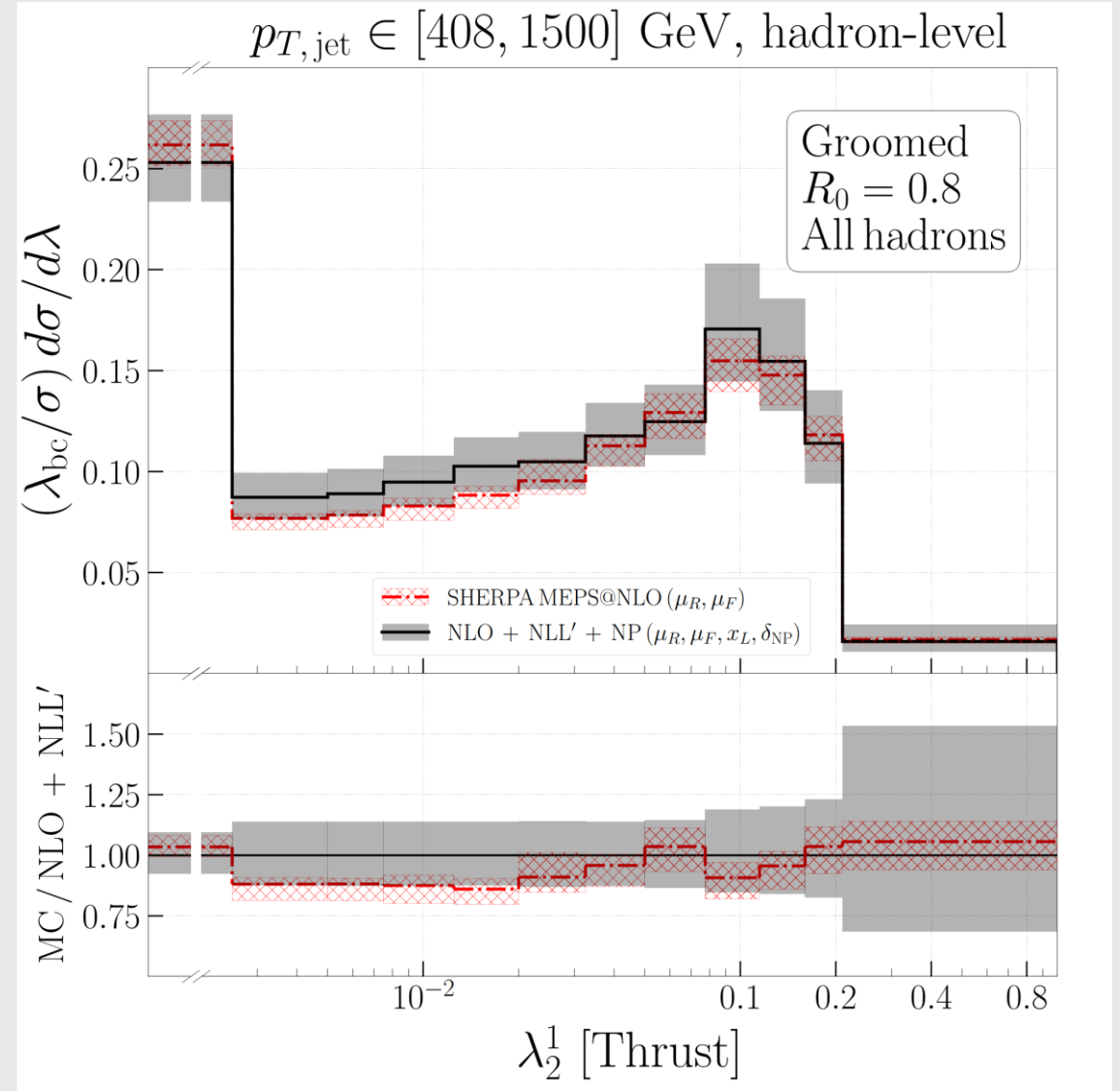
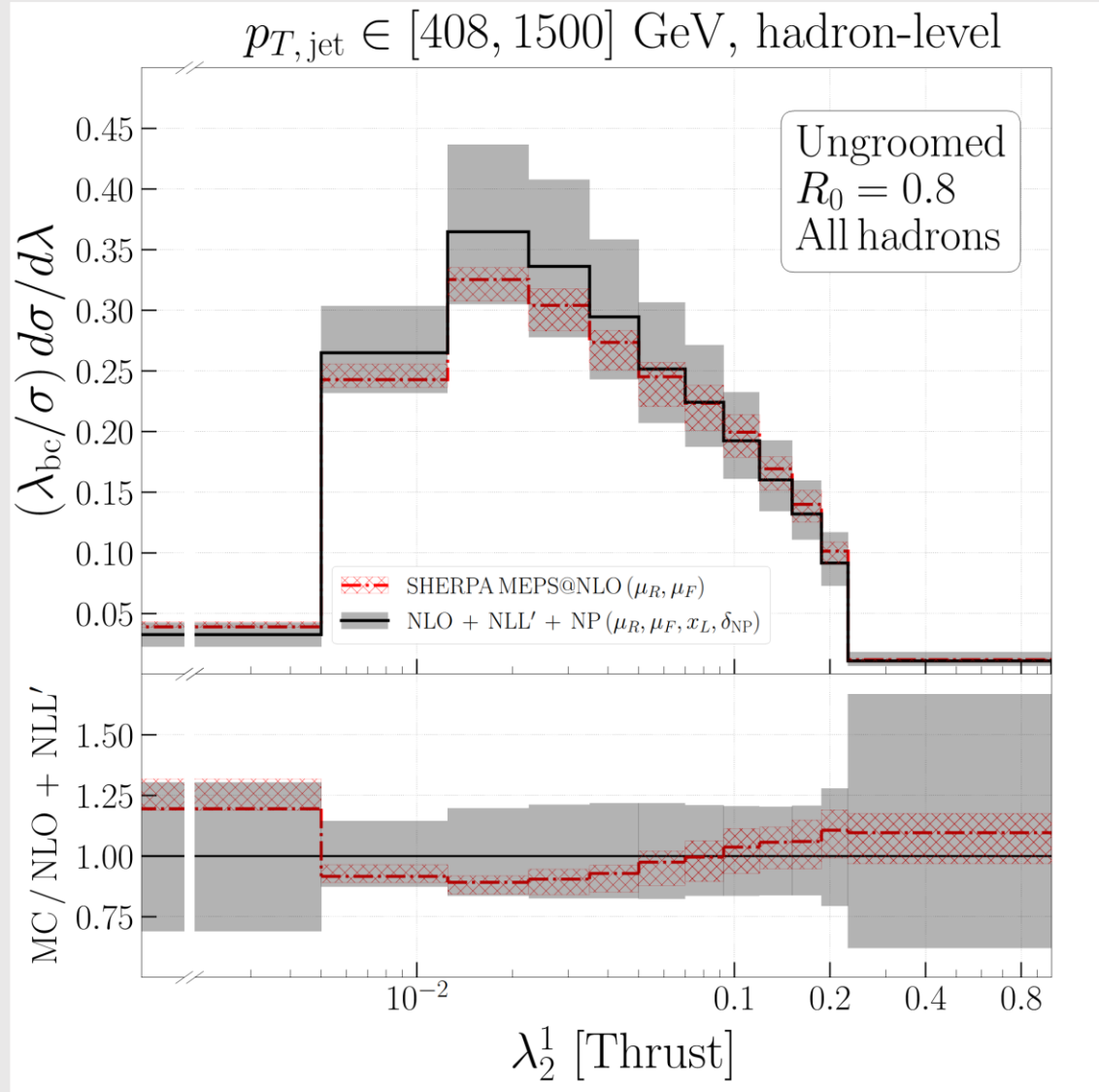
NLO + NLL' + NP:



NLO + NLL' + NP:



NLO + NLL' + NP:

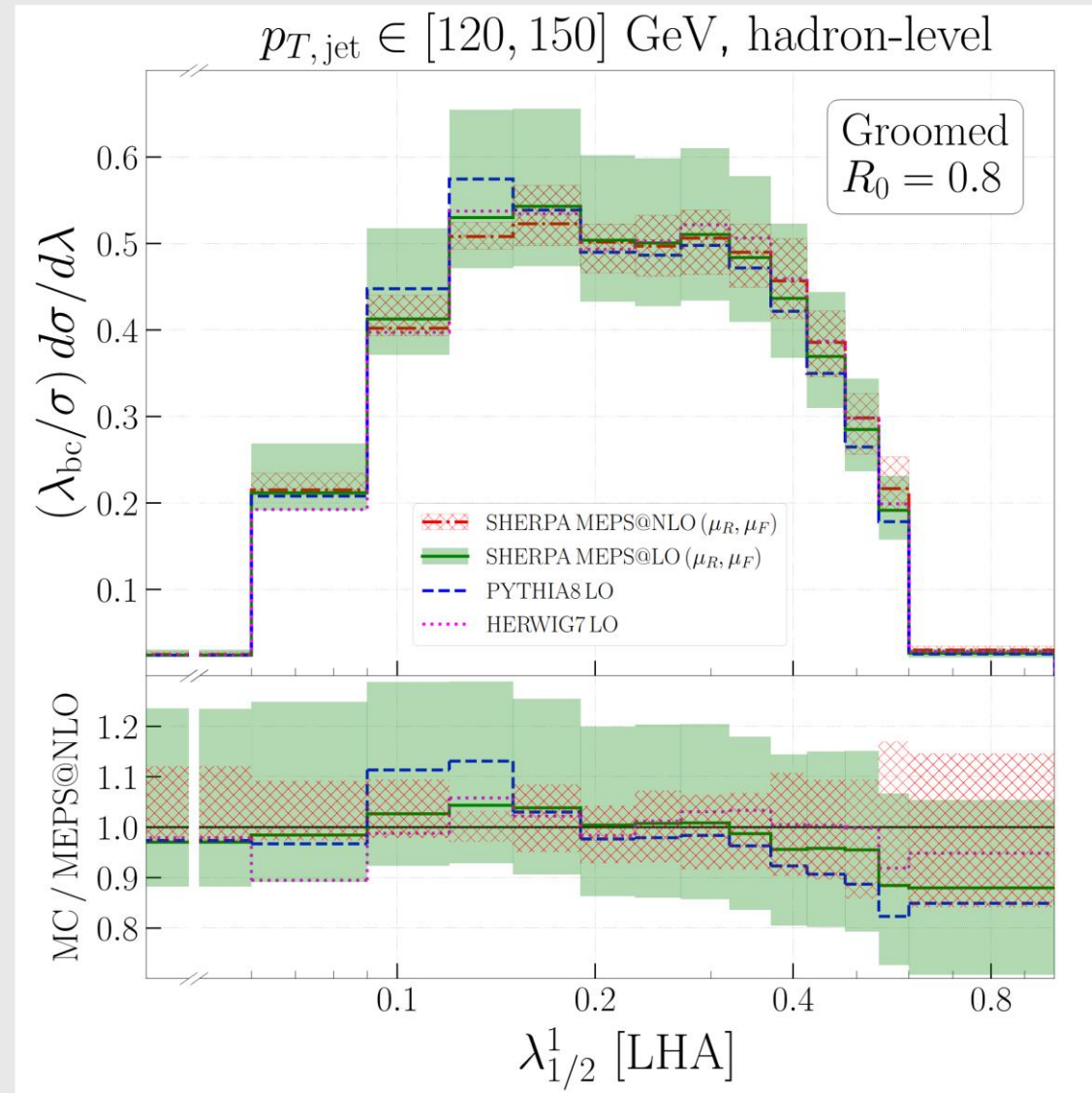
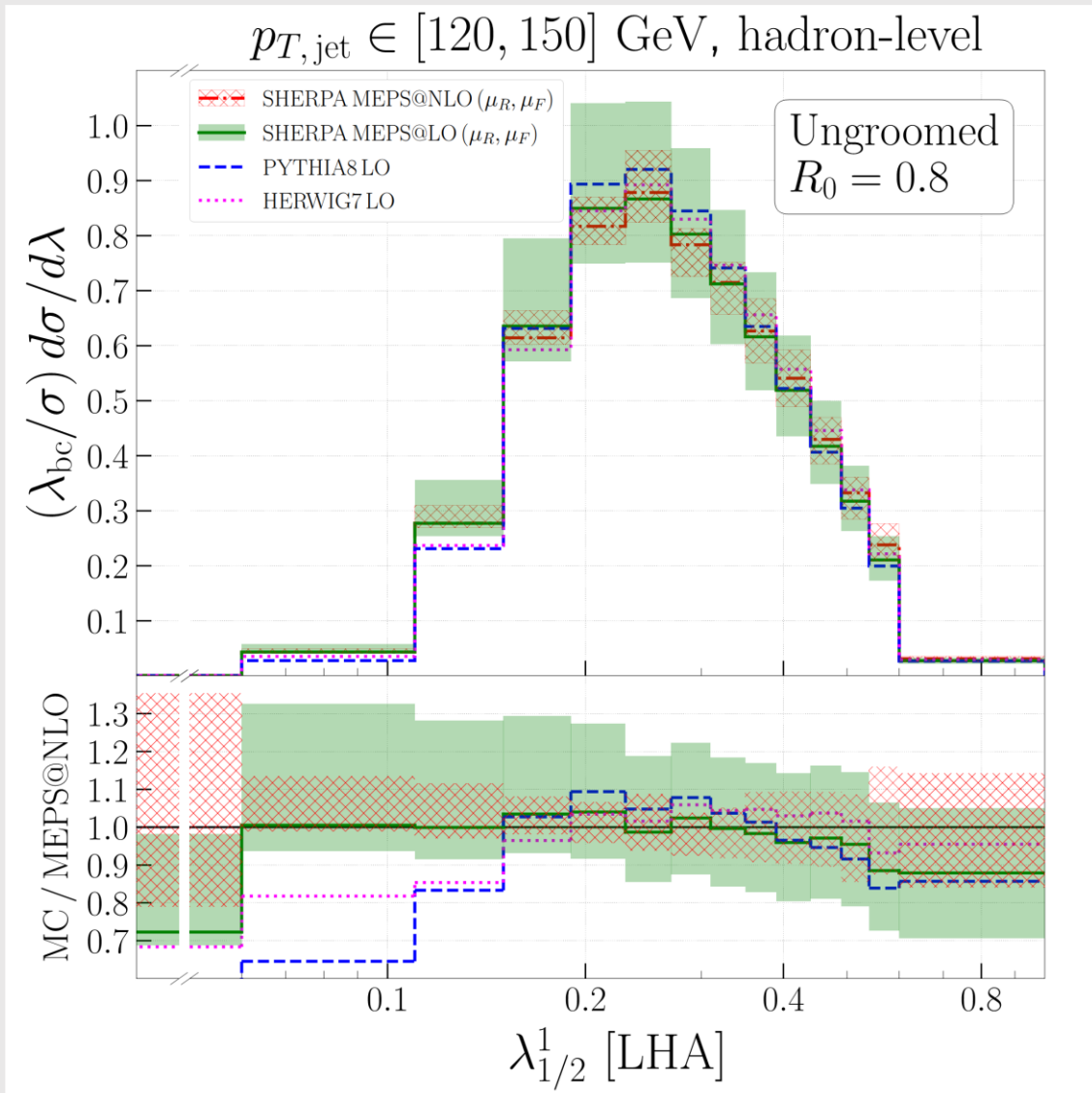


Conclusions and Outlook:

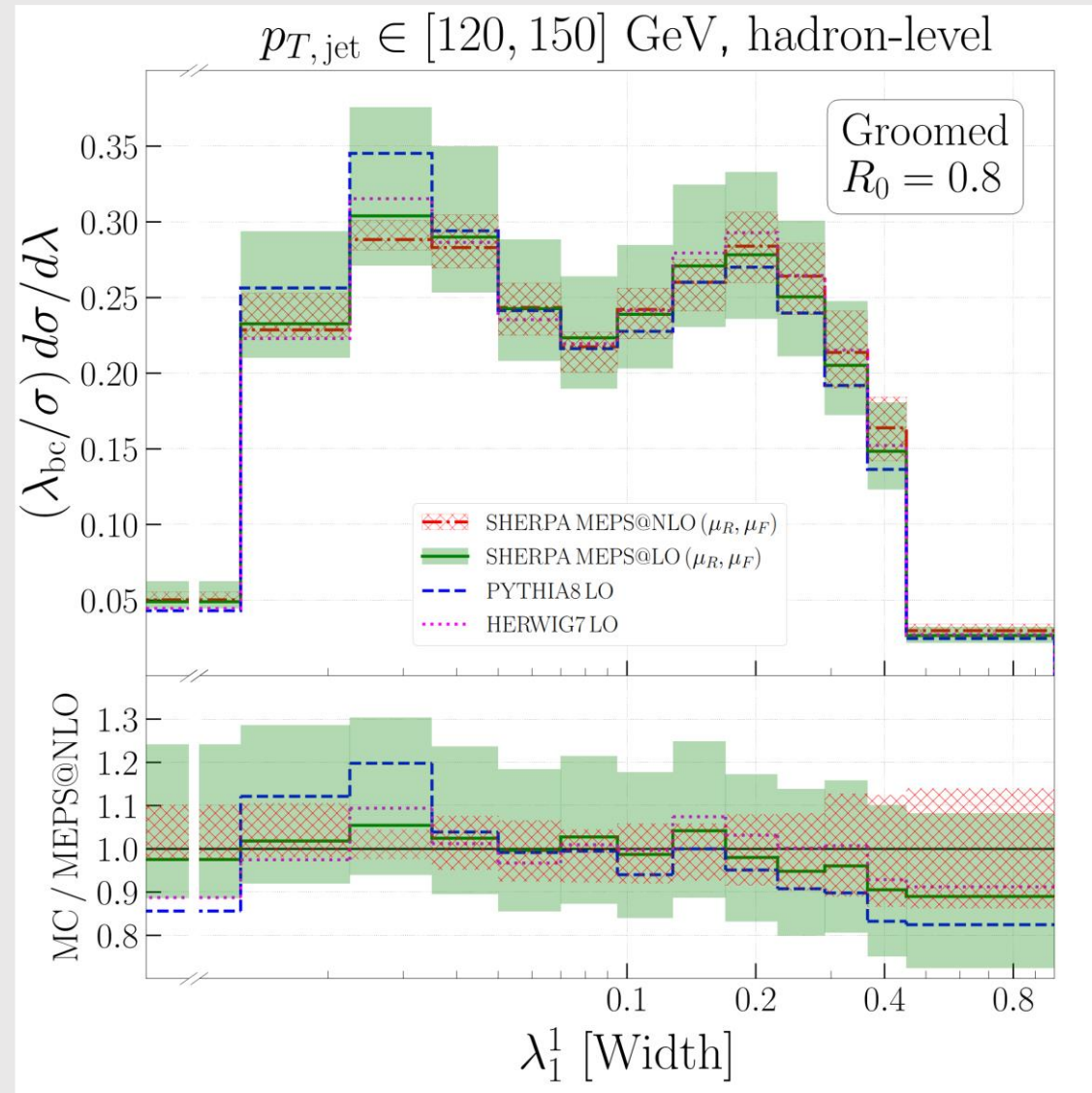
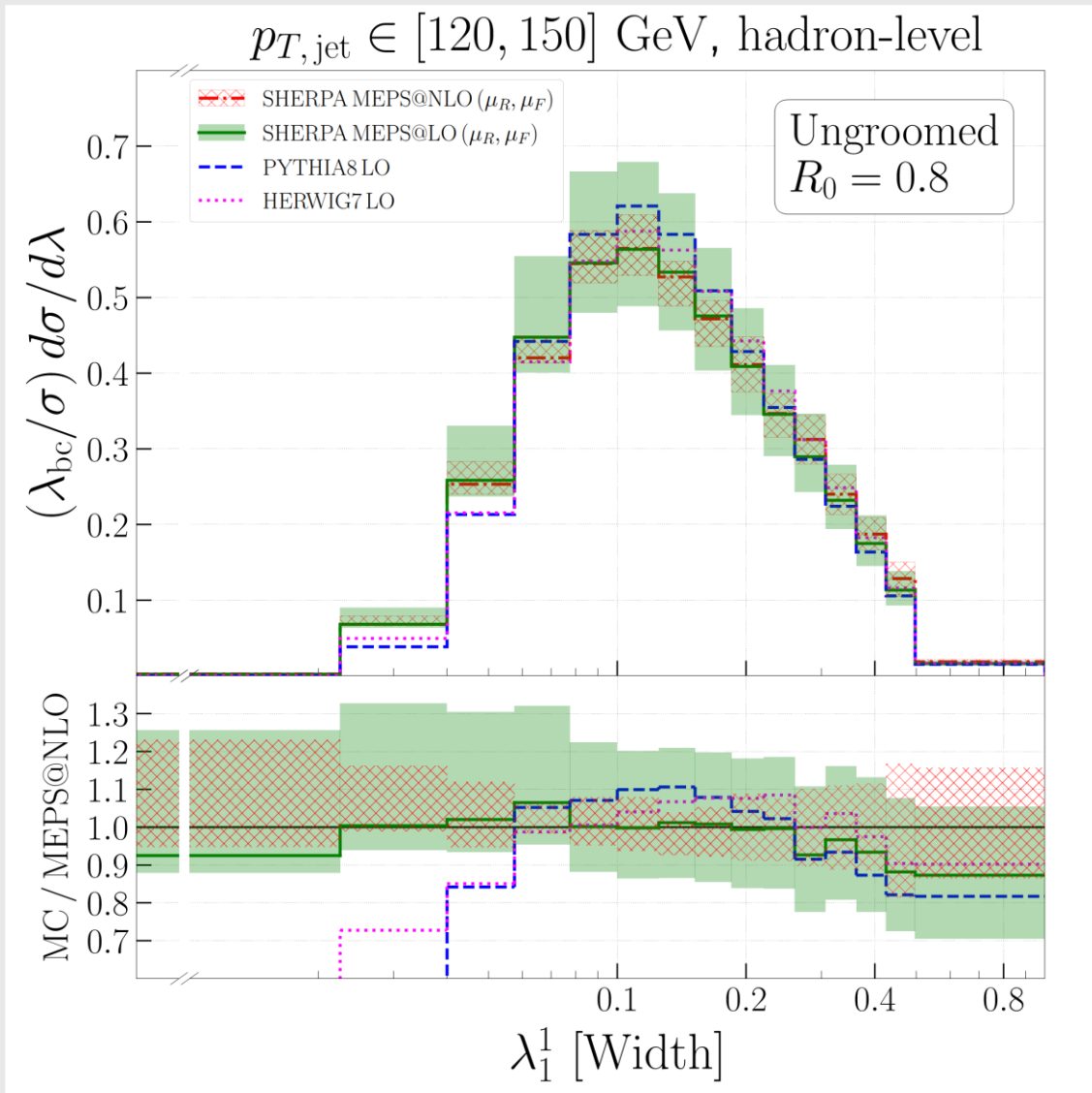
- We see a good agreement between the two predictions for the jet Thrust (and Width). Hence, we expect that a comparison to upcoming experimental data would find agreement with them.
- For LHA event-generator prediction and resummed calculation are not in agreement, especially in the groomed case. In this case upcoming LHC data may help us to shed light on the discrepancy. The data may favour the MC prediction, indicating that modelling used in the resummed calculation is not appropriate (e.g. recoil treatment?). On the other hand, the data may favour the resummation, indicating the need for a better understanding of the logarithmic structure that is achieved by the PS models.
- We have seen that there is a more significant contribution from gluon jets for larger values of the angularities, while the low tails are entirely dominated by quark jets. This is an evidence that a cut on the jet angularity can serve as a theoretically well-defined and IRC safe quark-gluon discriminant.

Backup Slides

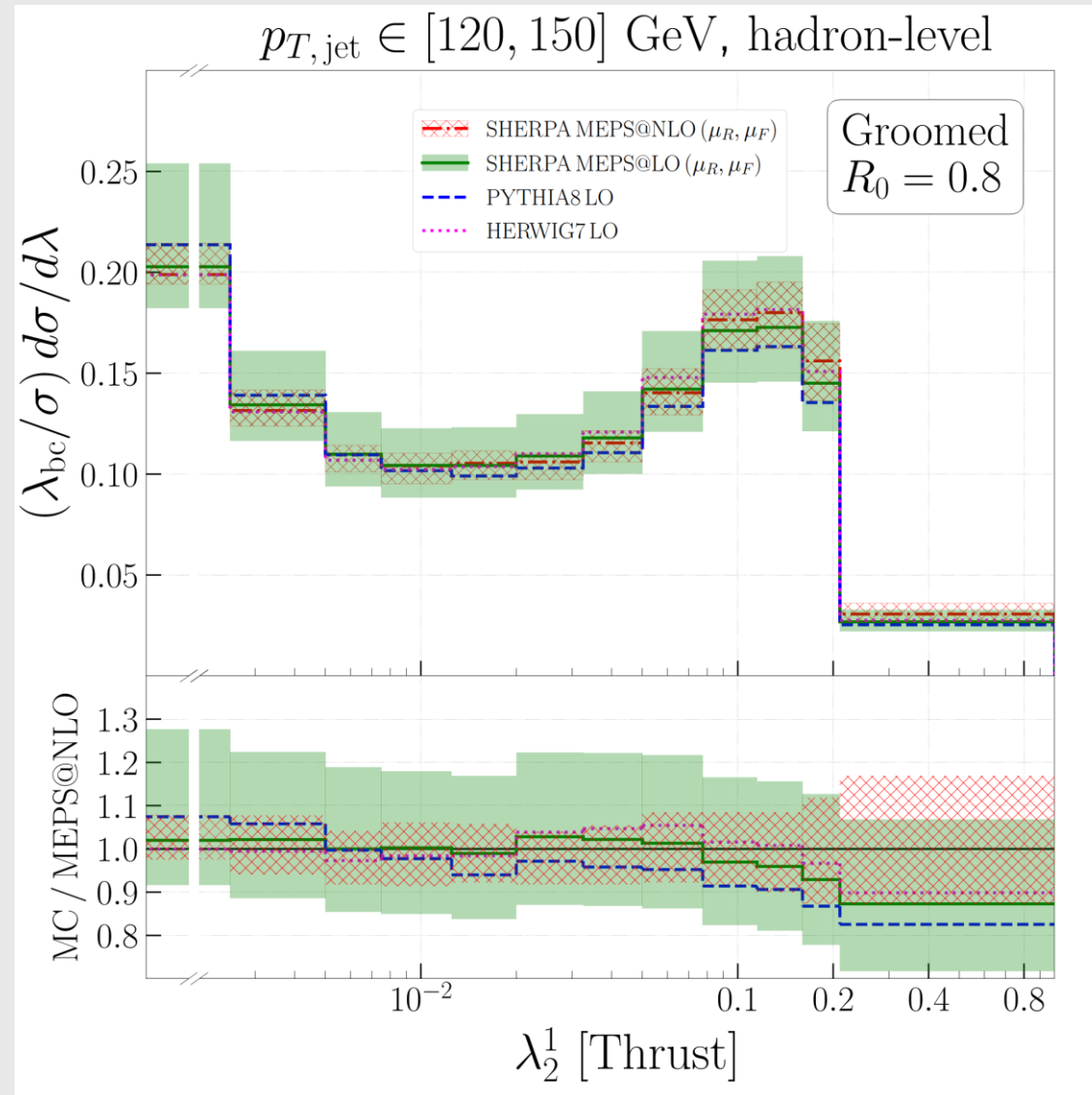
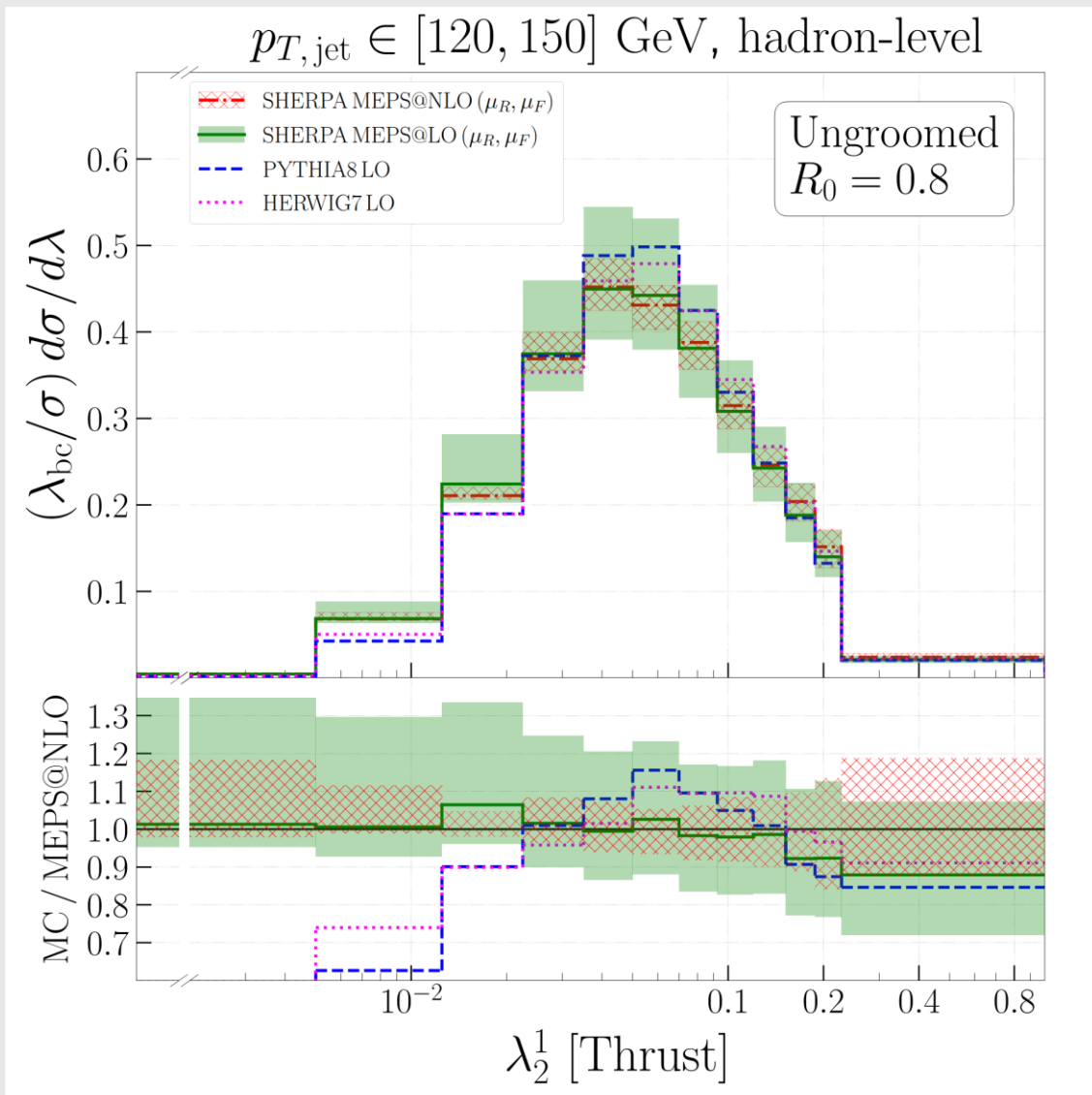
All MCs (low-pT):



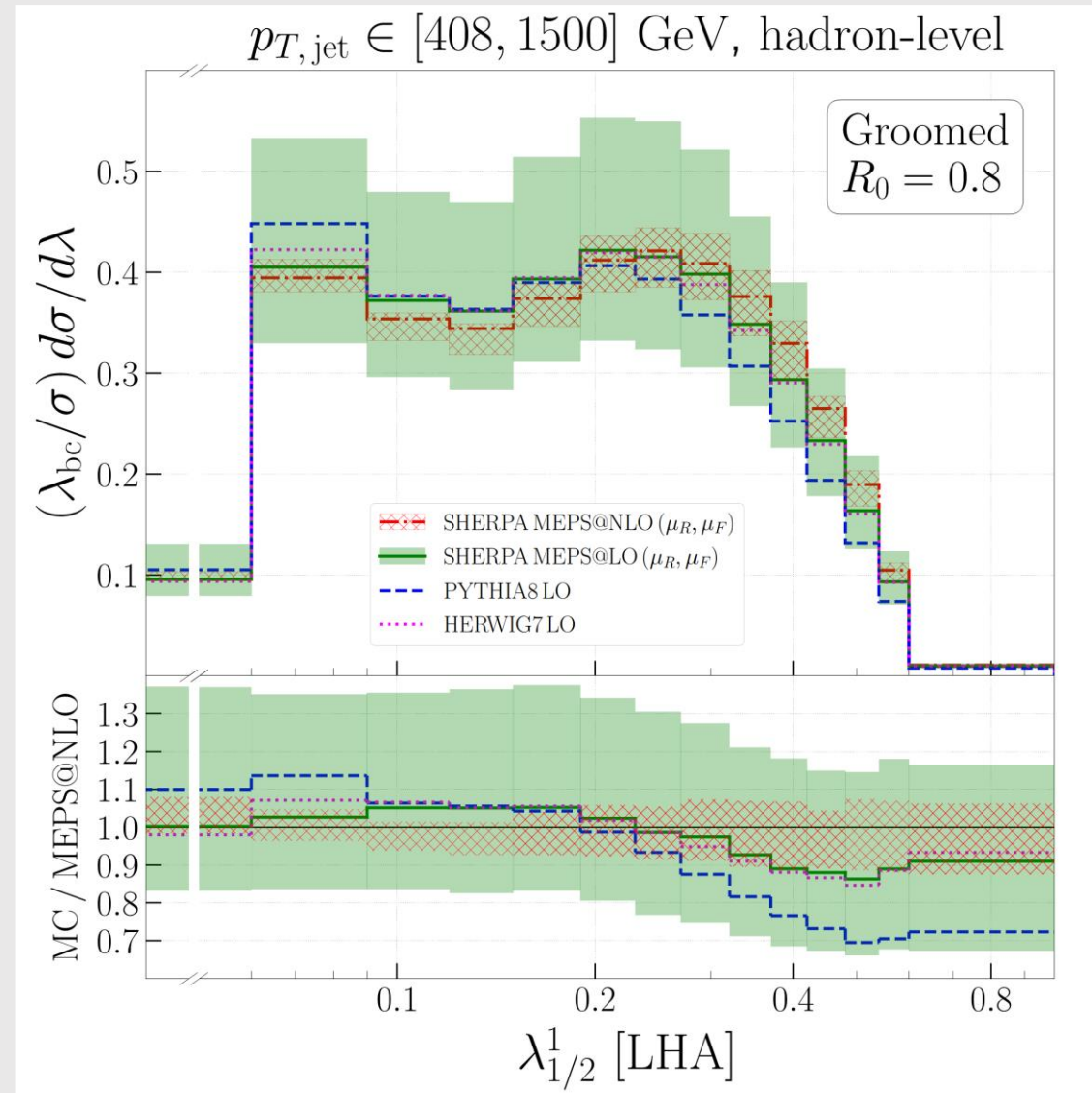
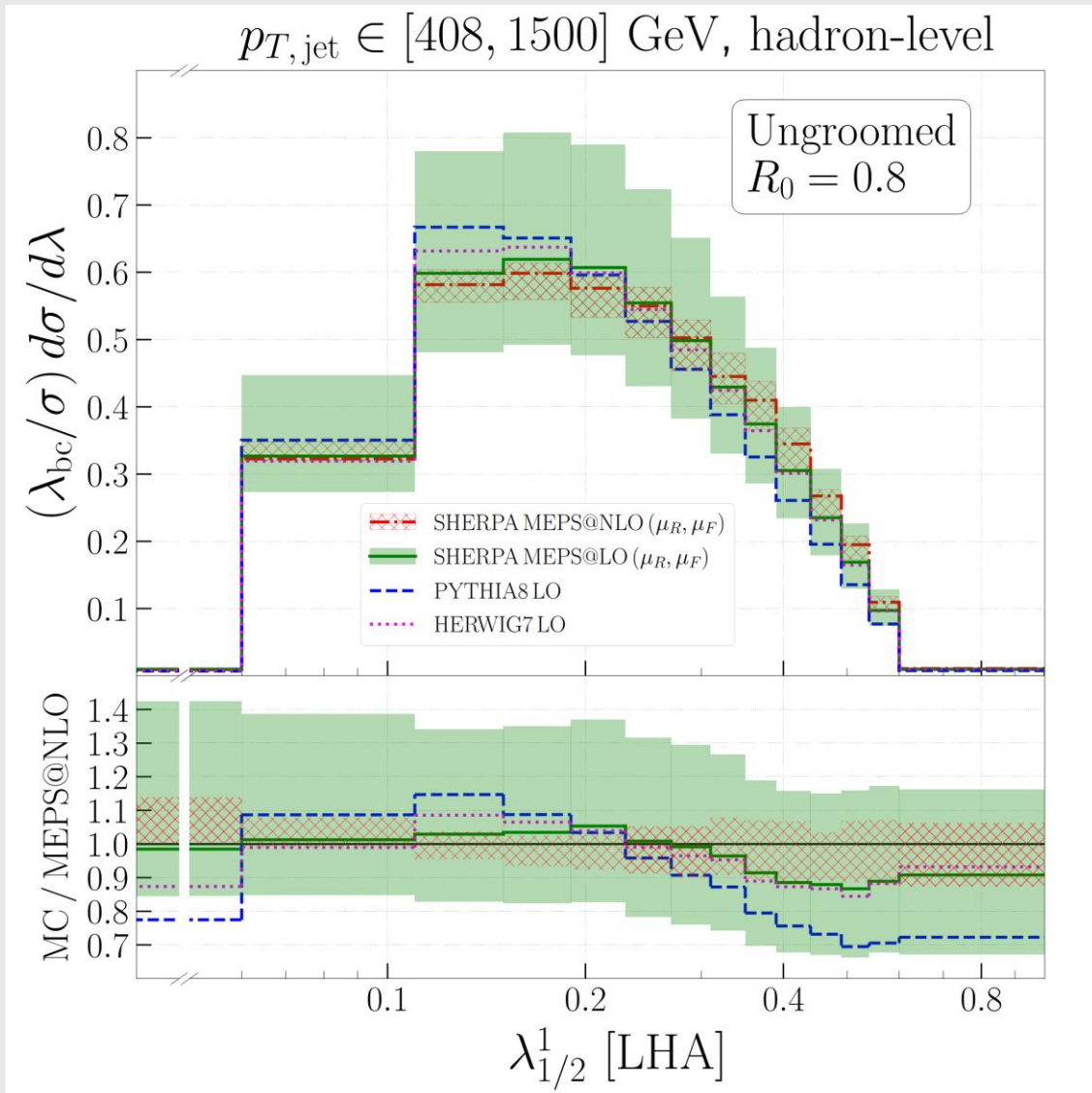
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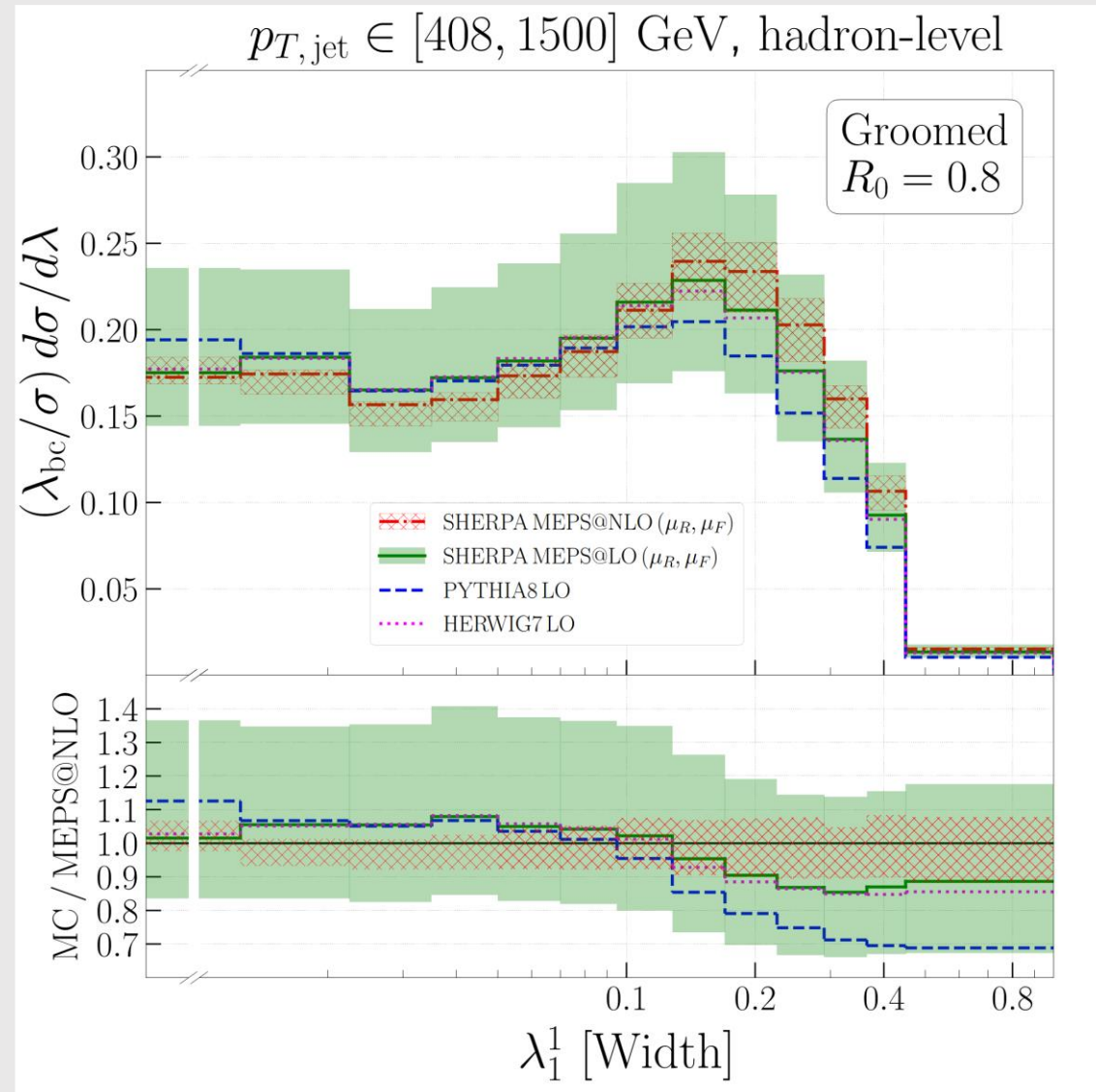
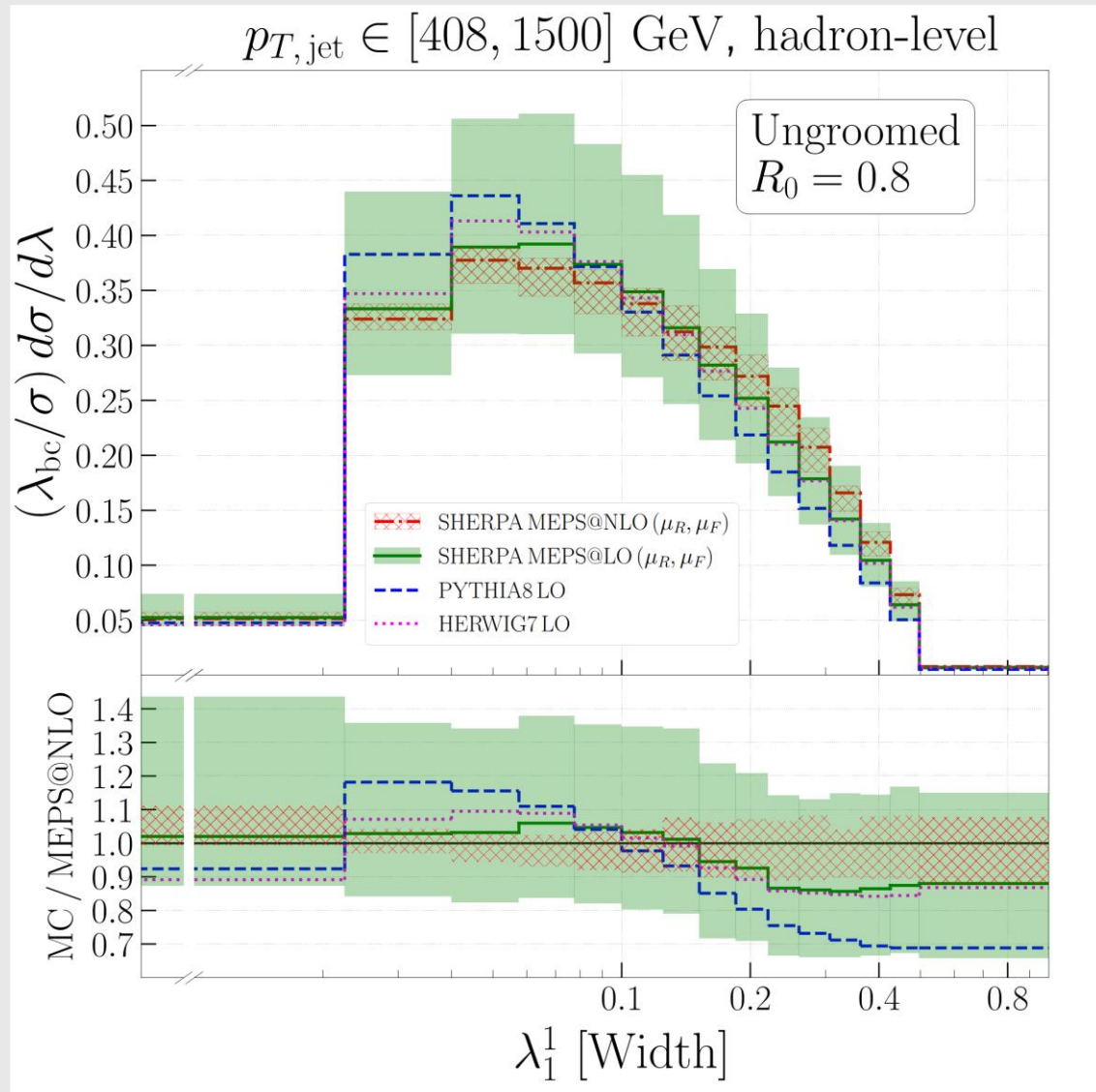
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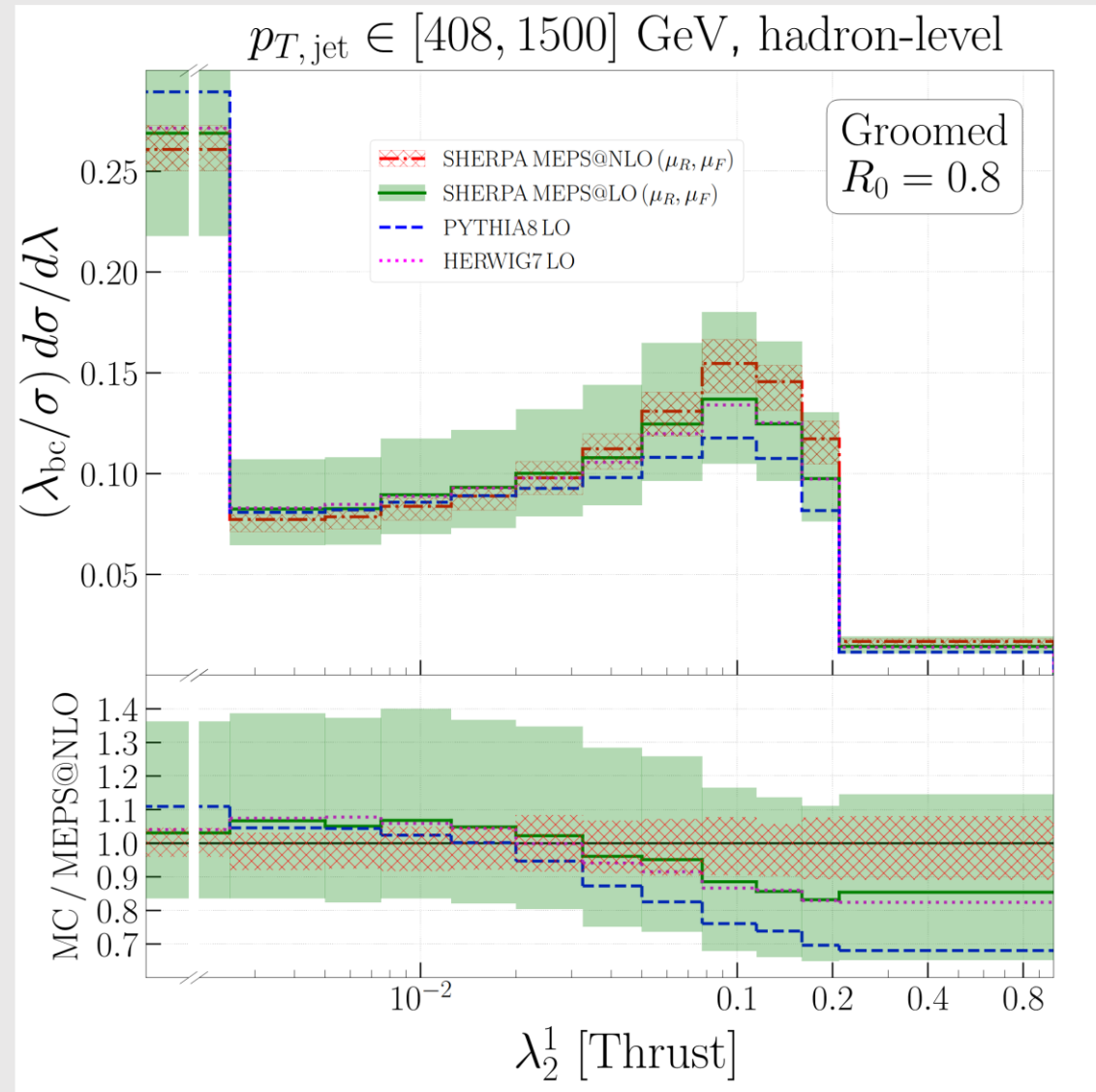
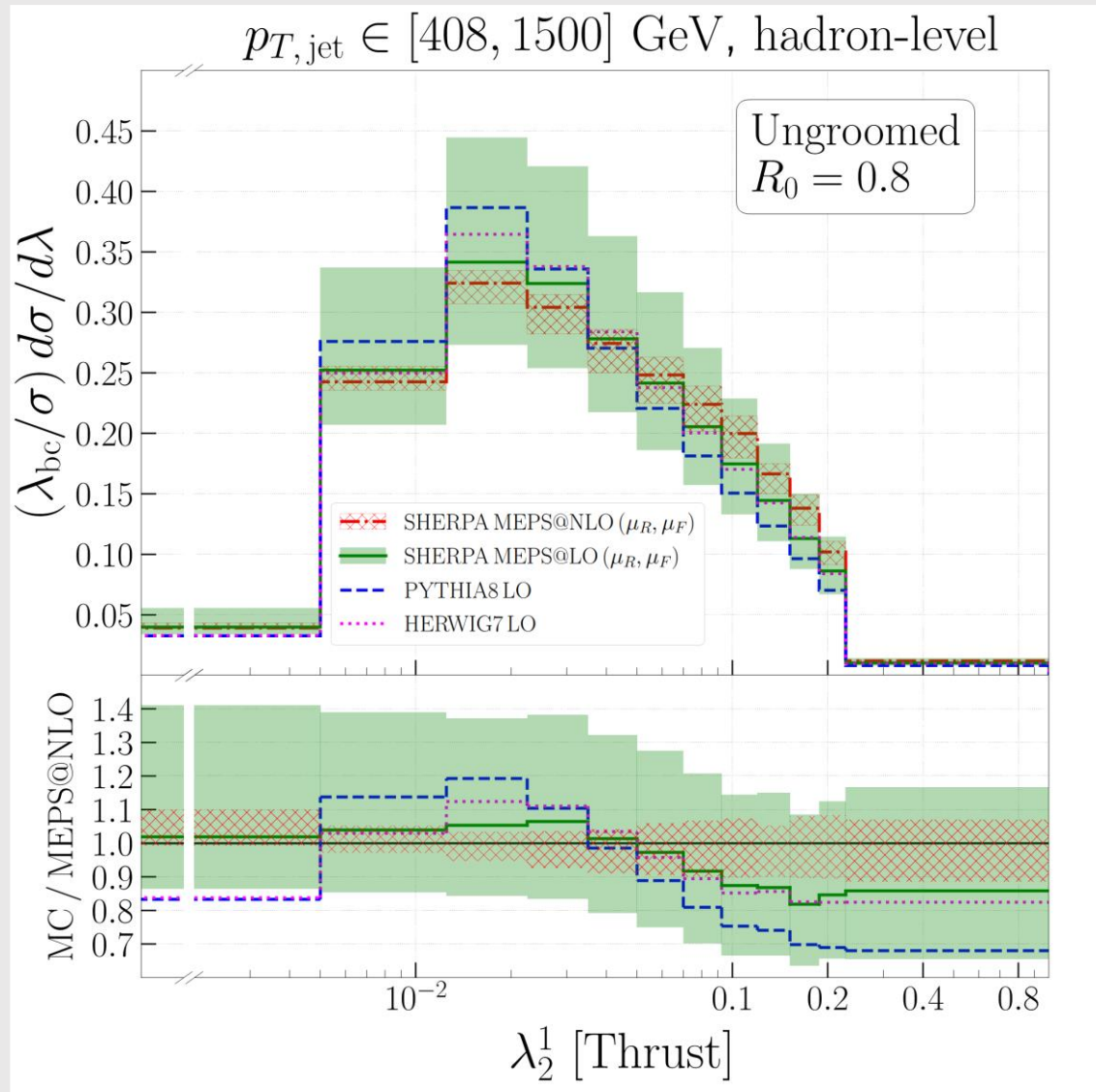
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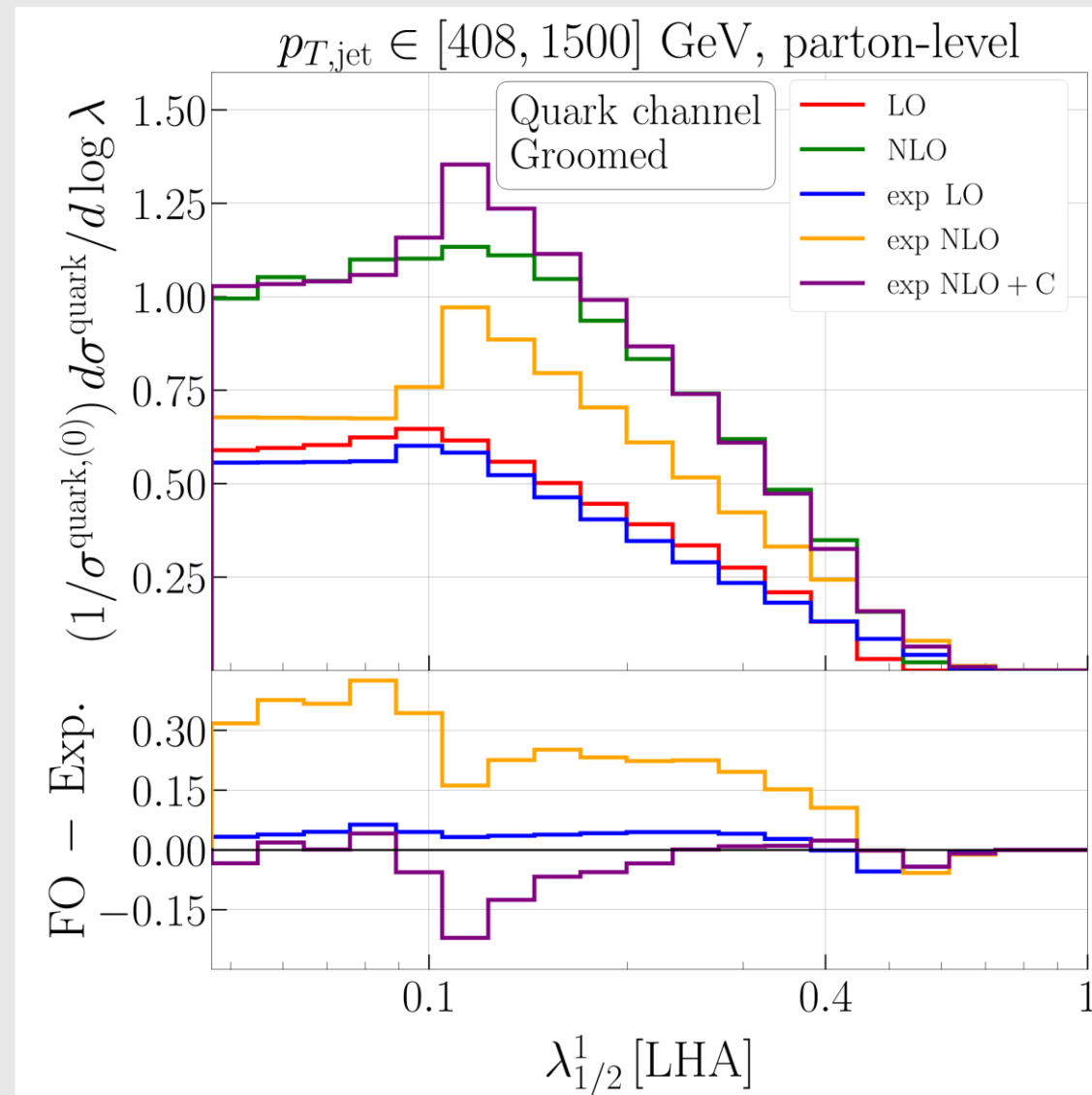
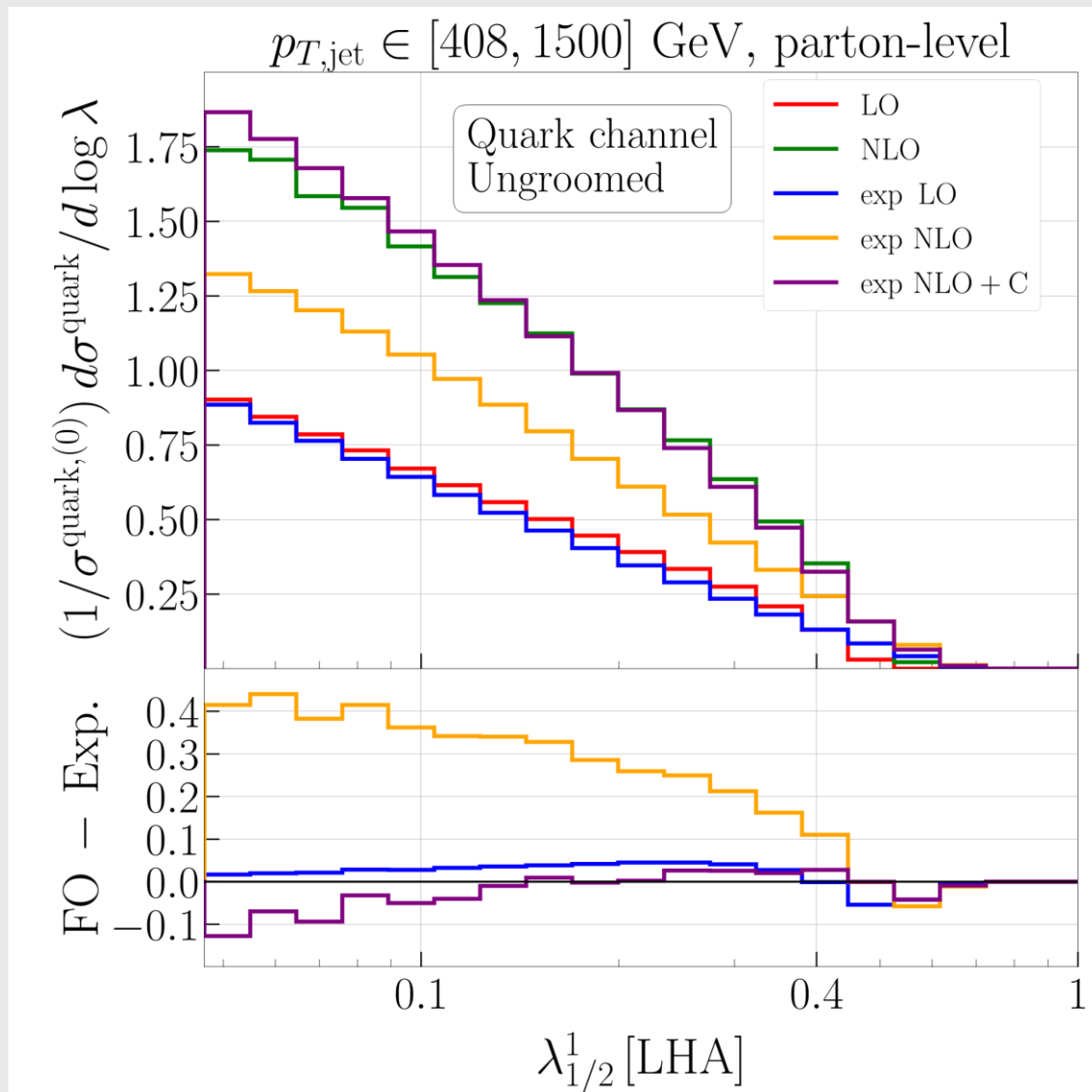
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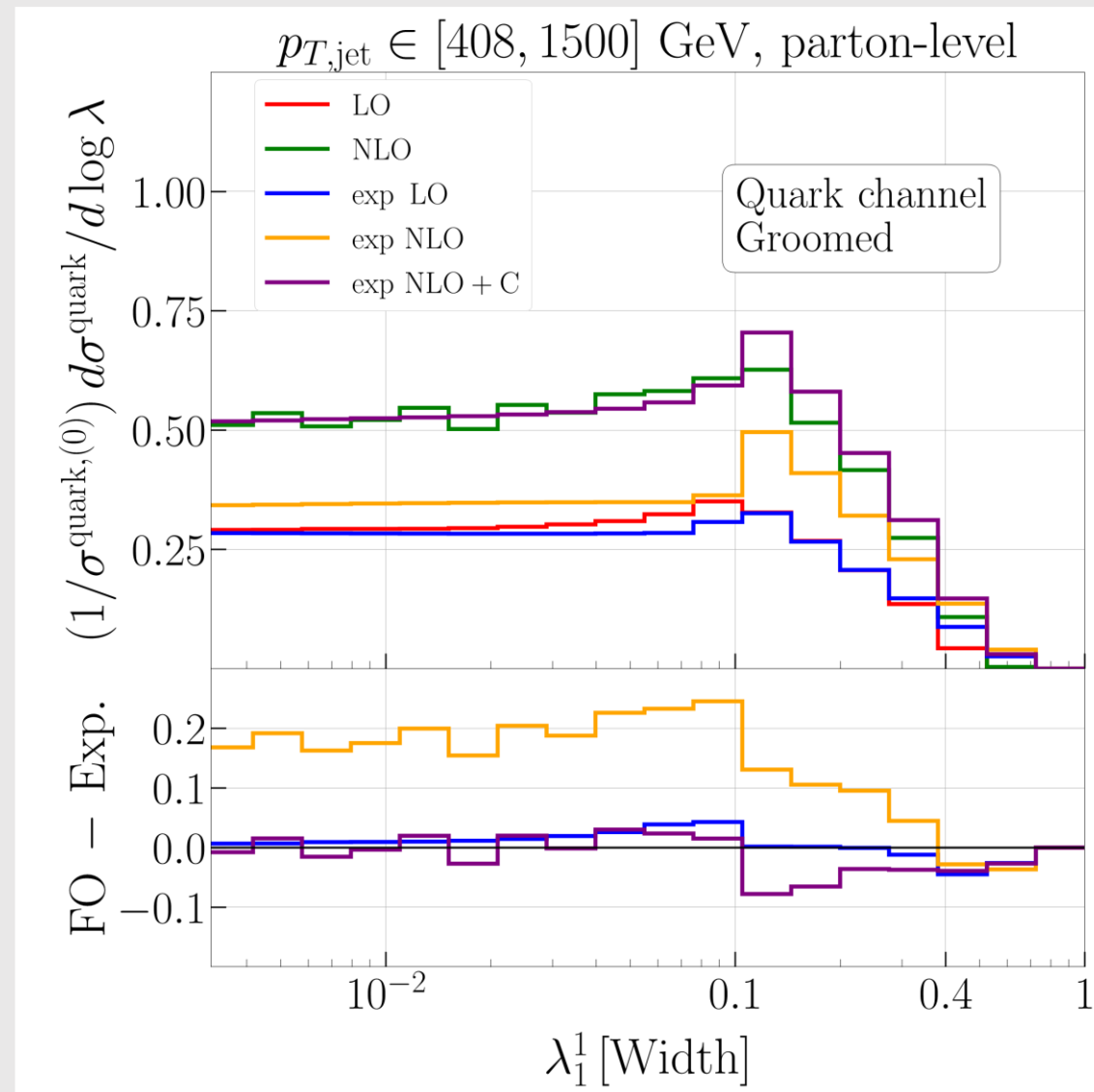
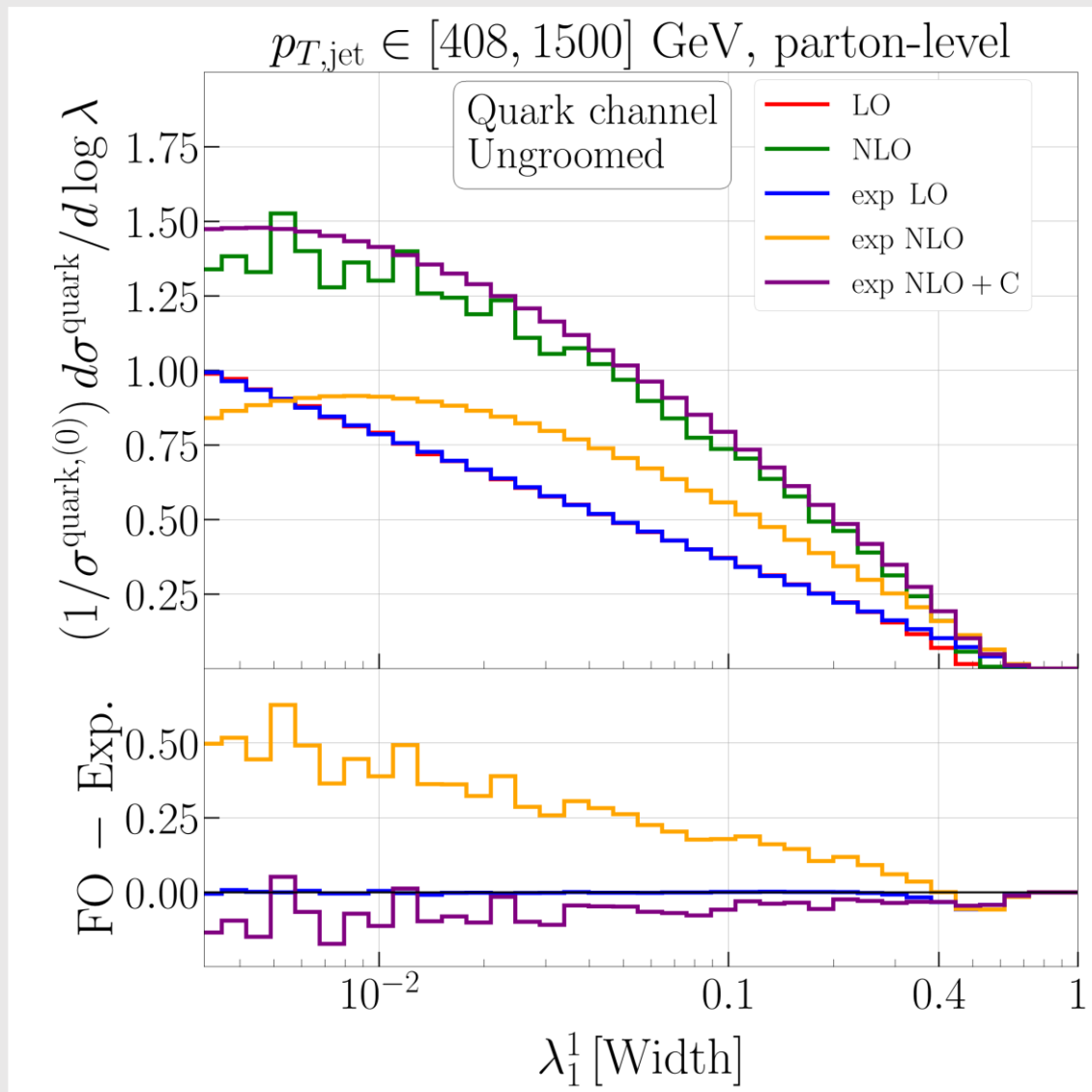
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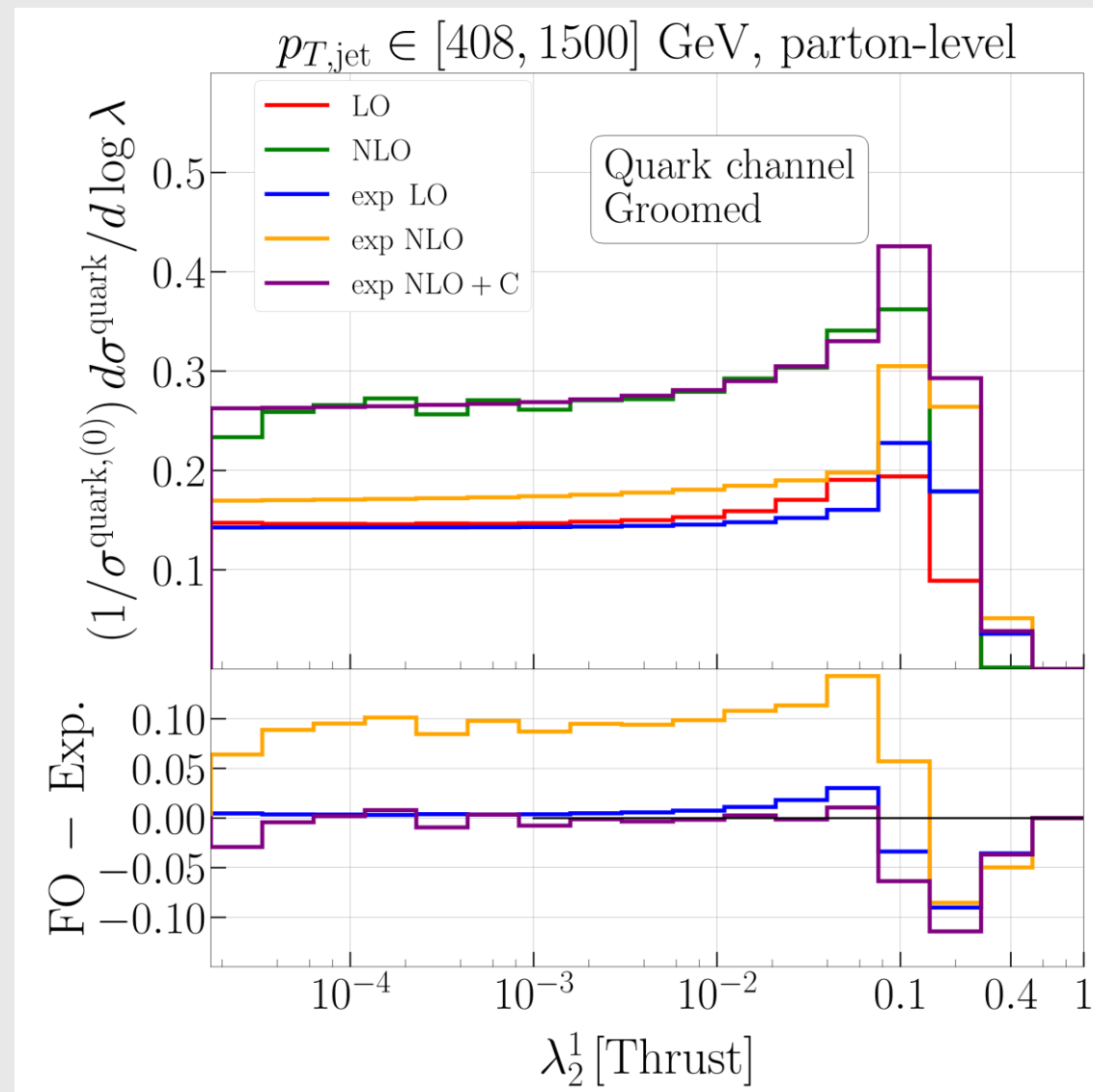
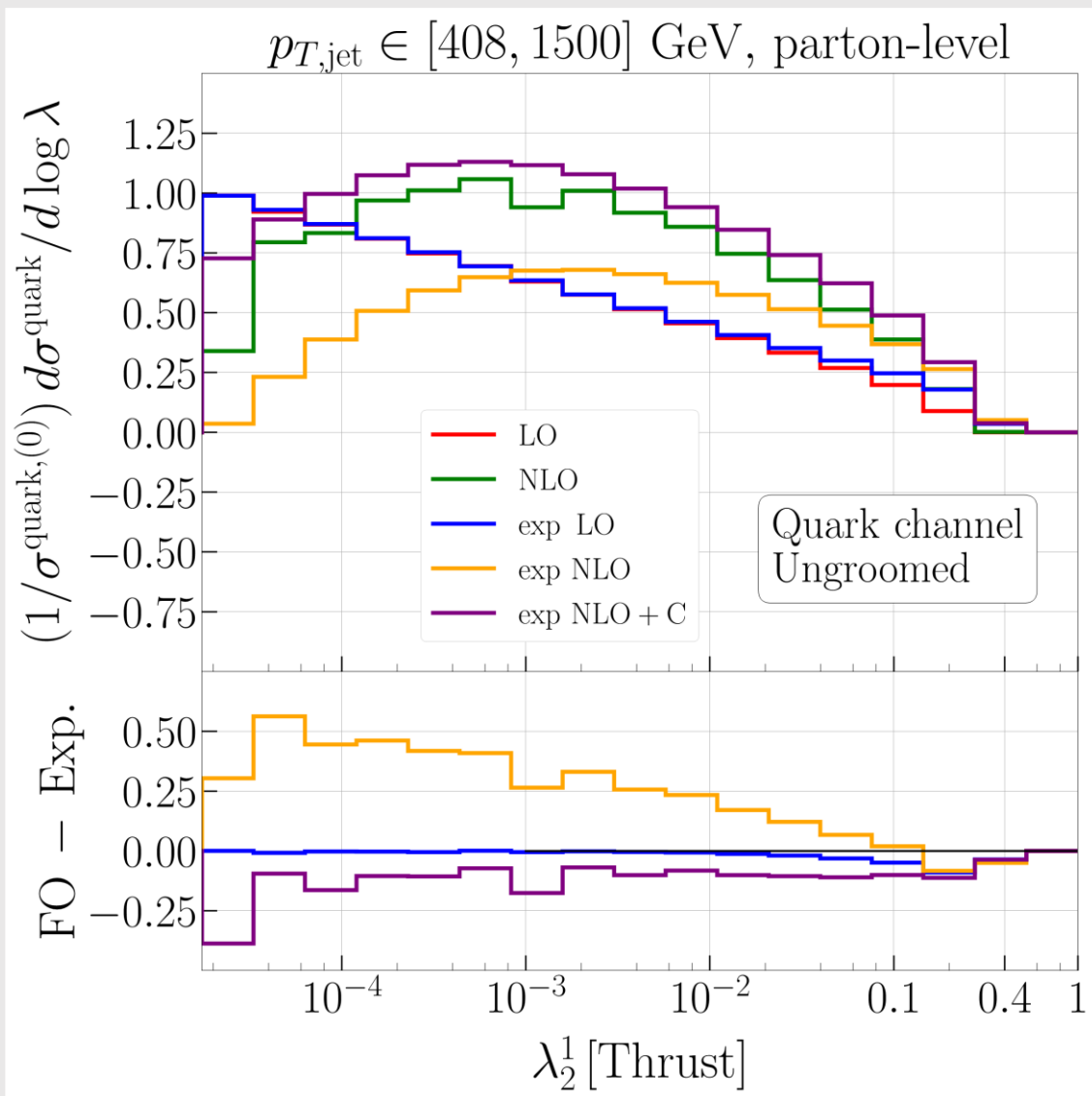
Validation test (quark channel):



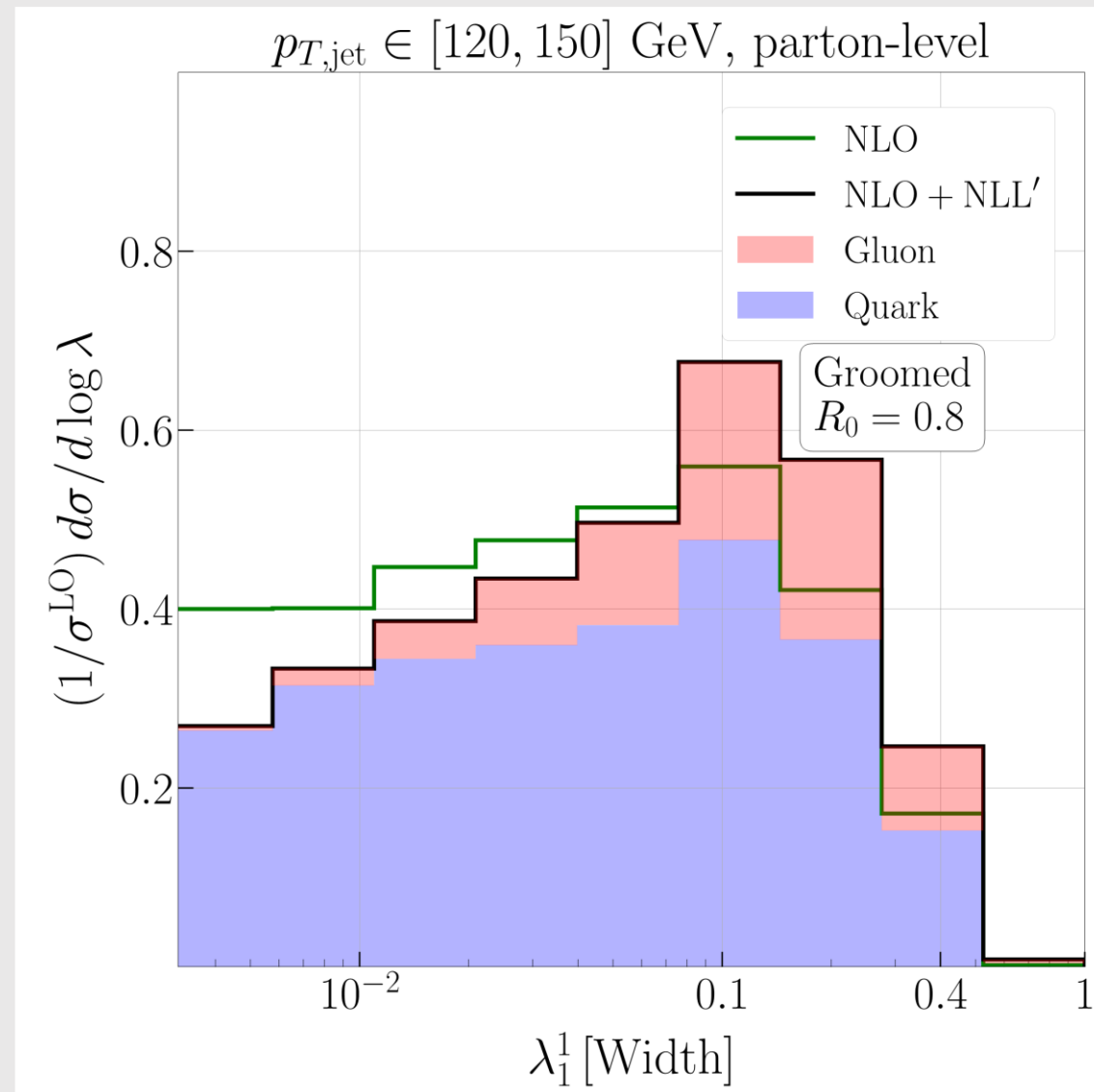
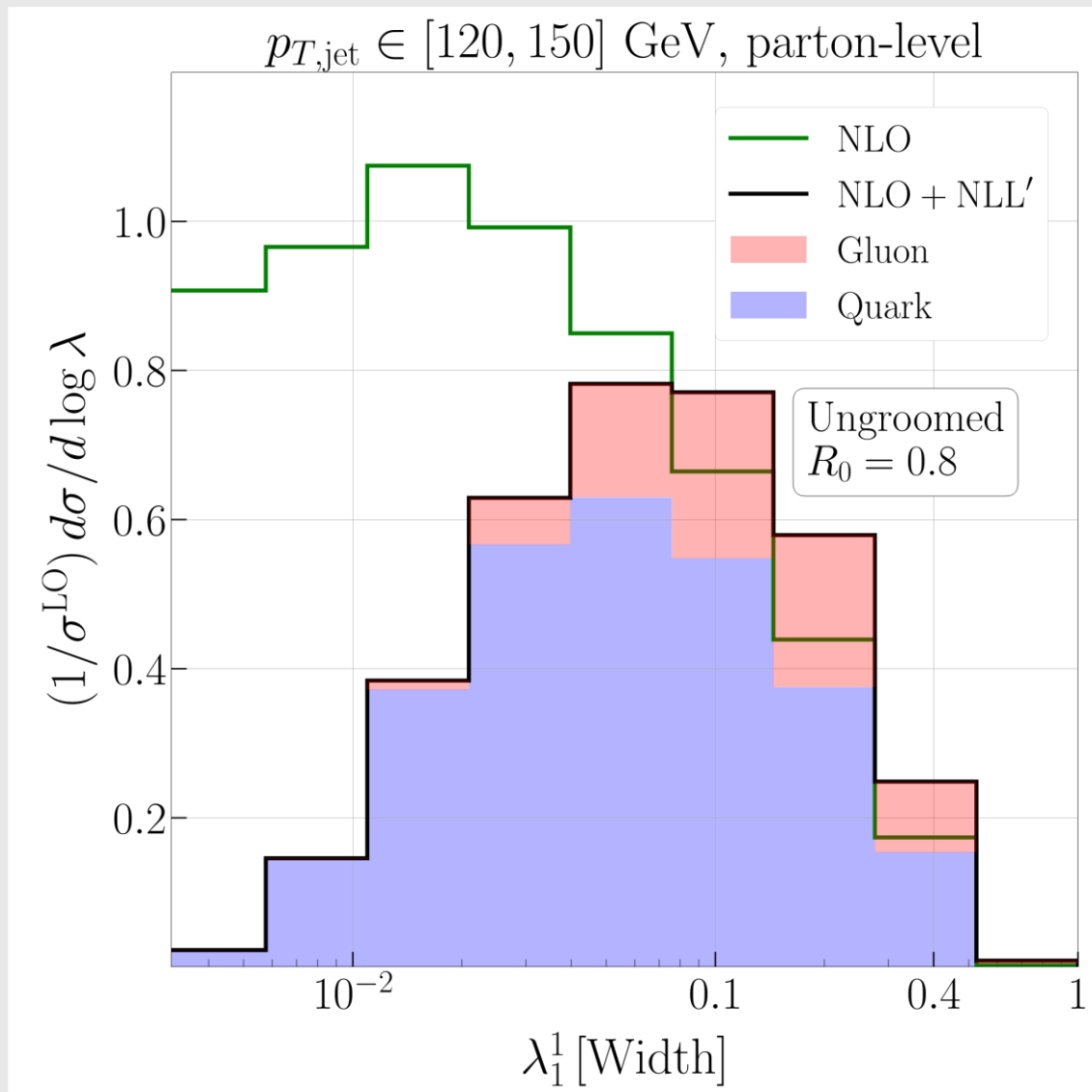
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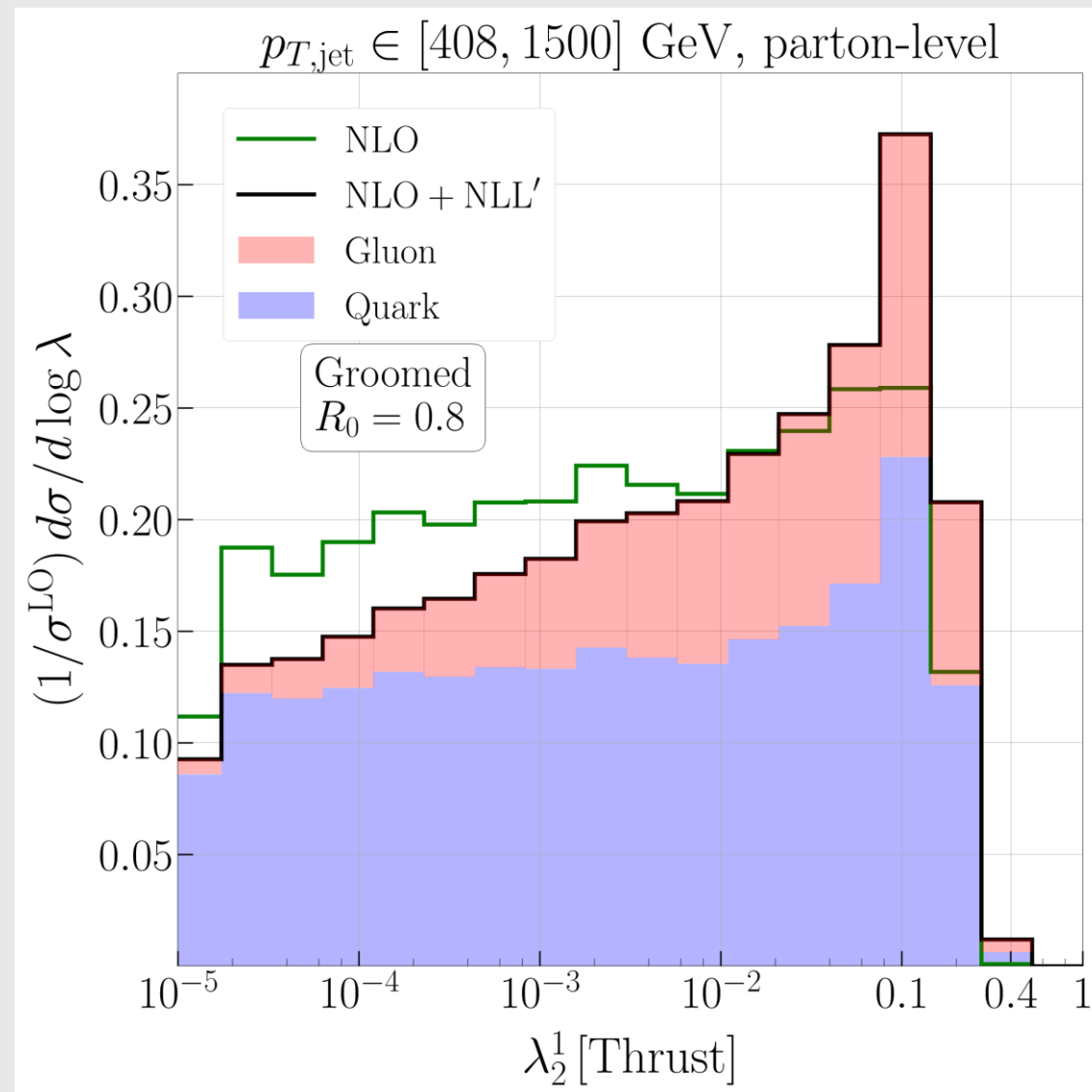
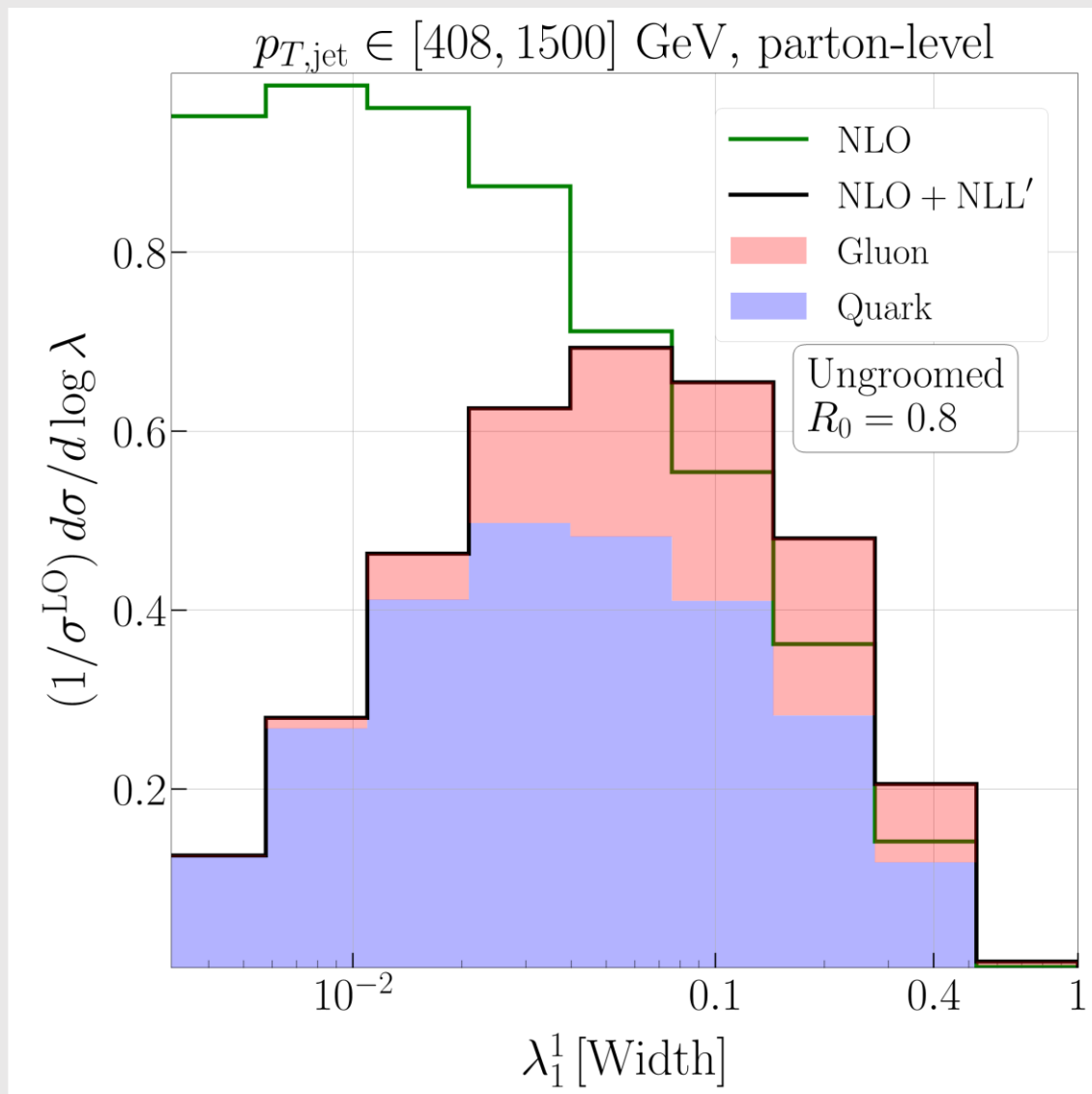
Validation test (quark channel):



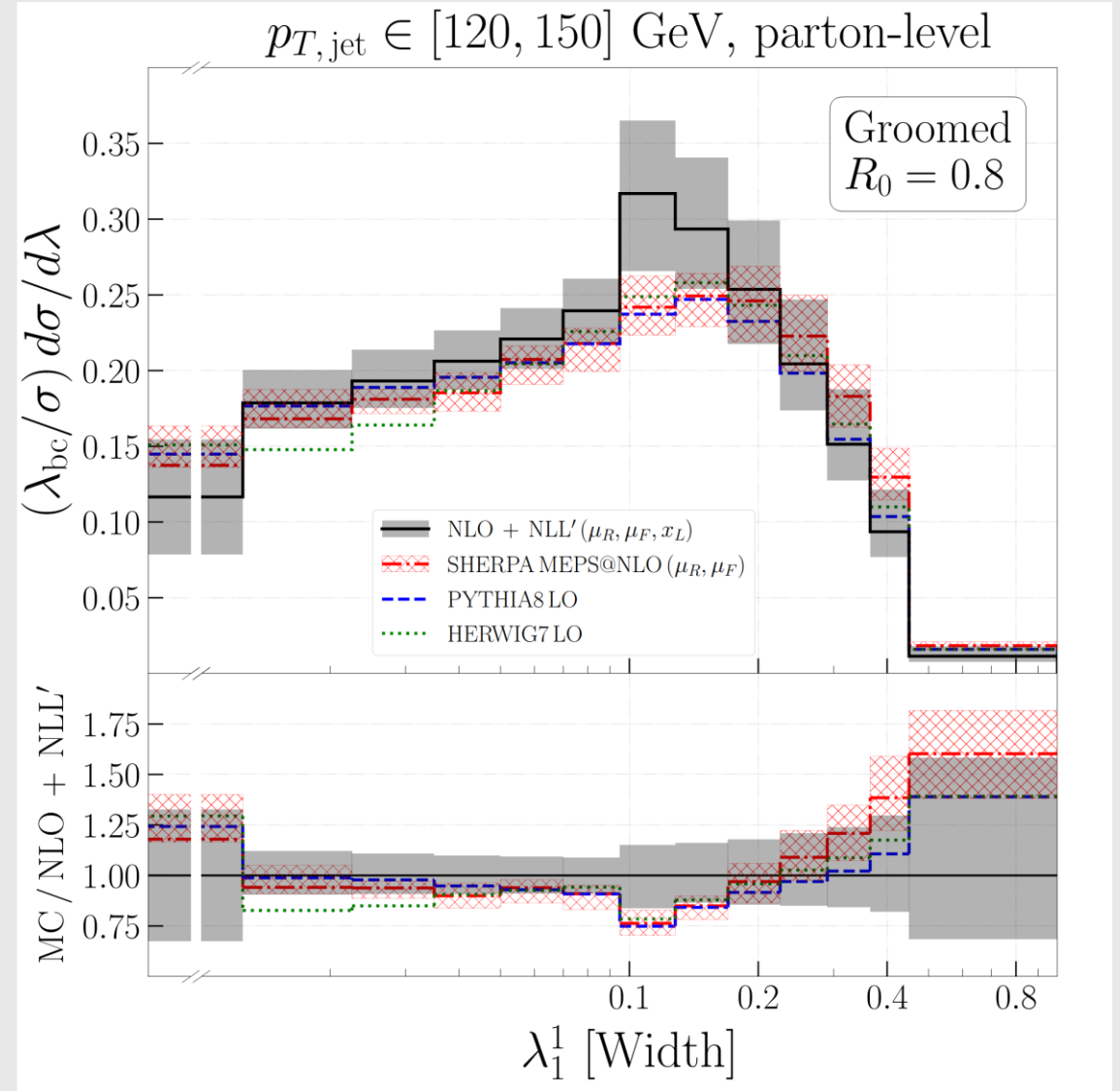
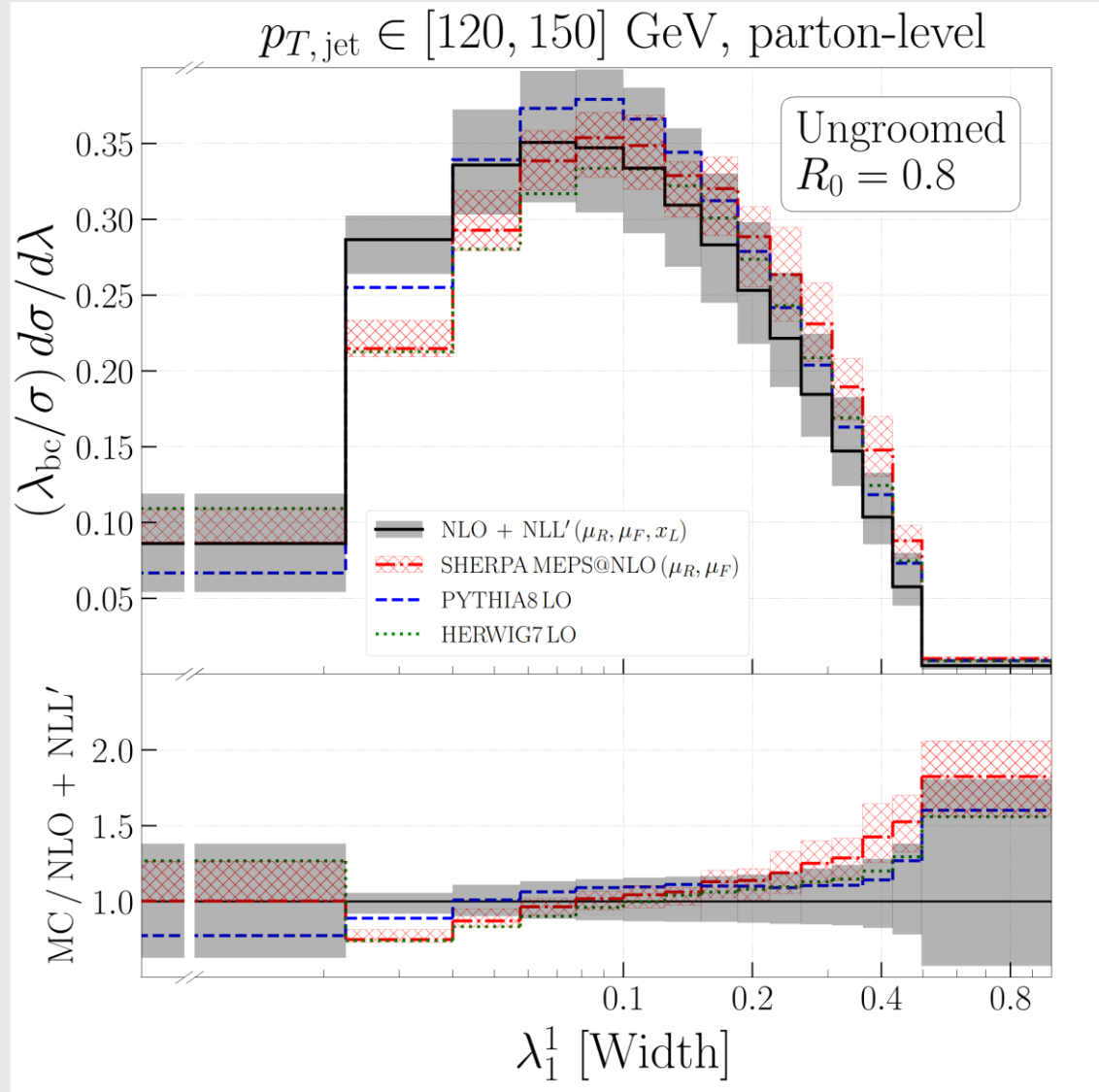
Fixed-order vs. matched(width):



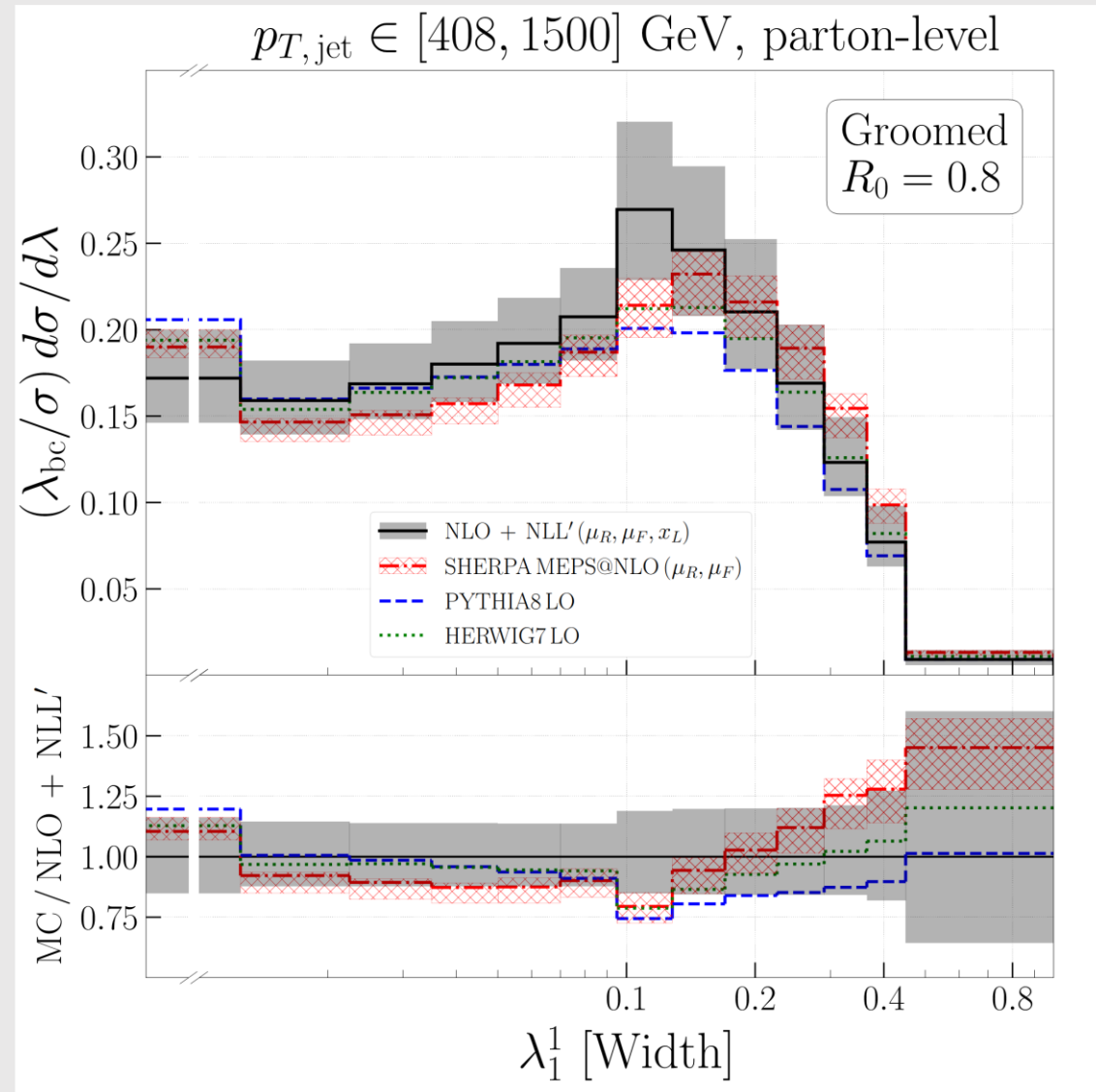
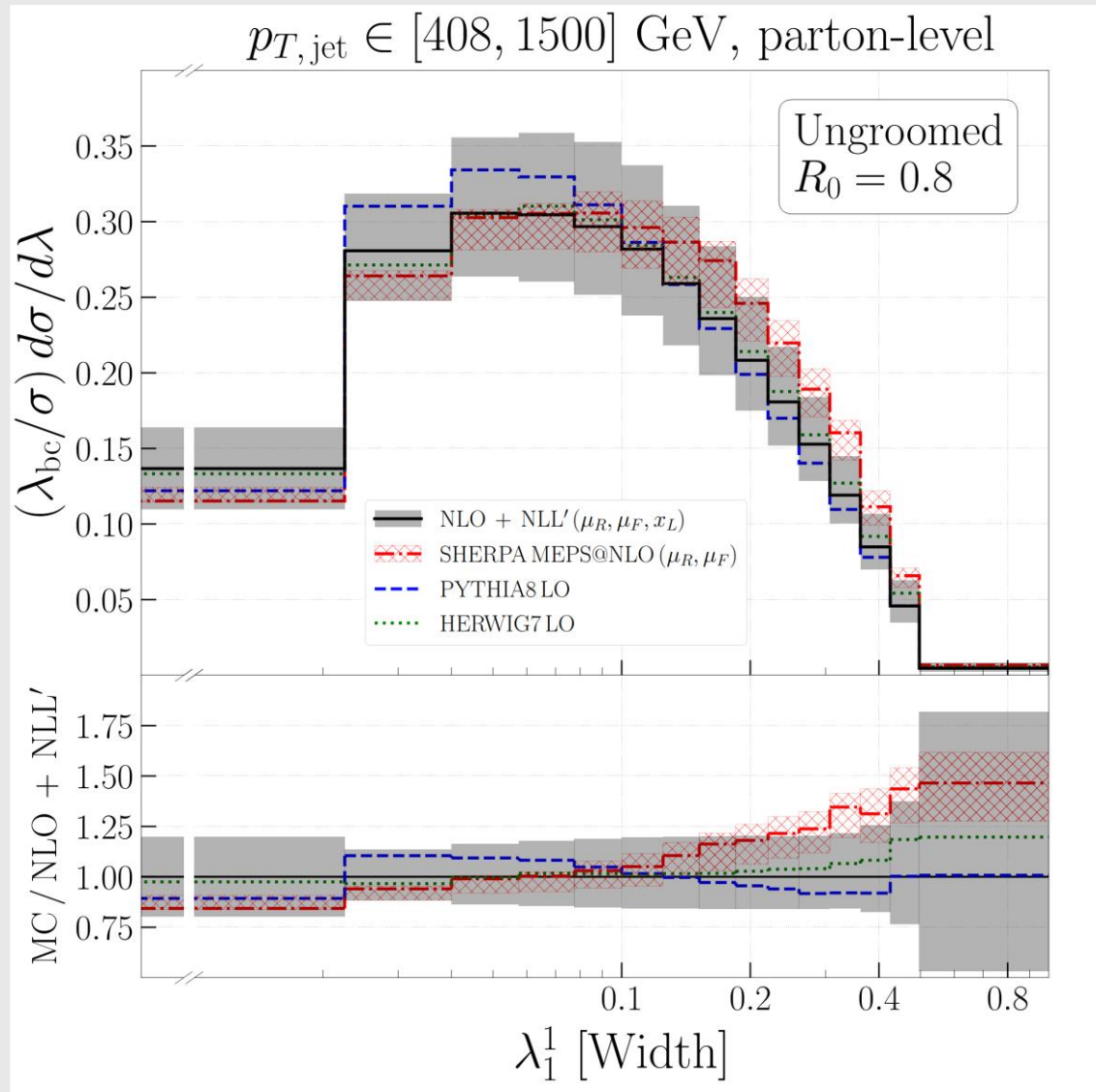
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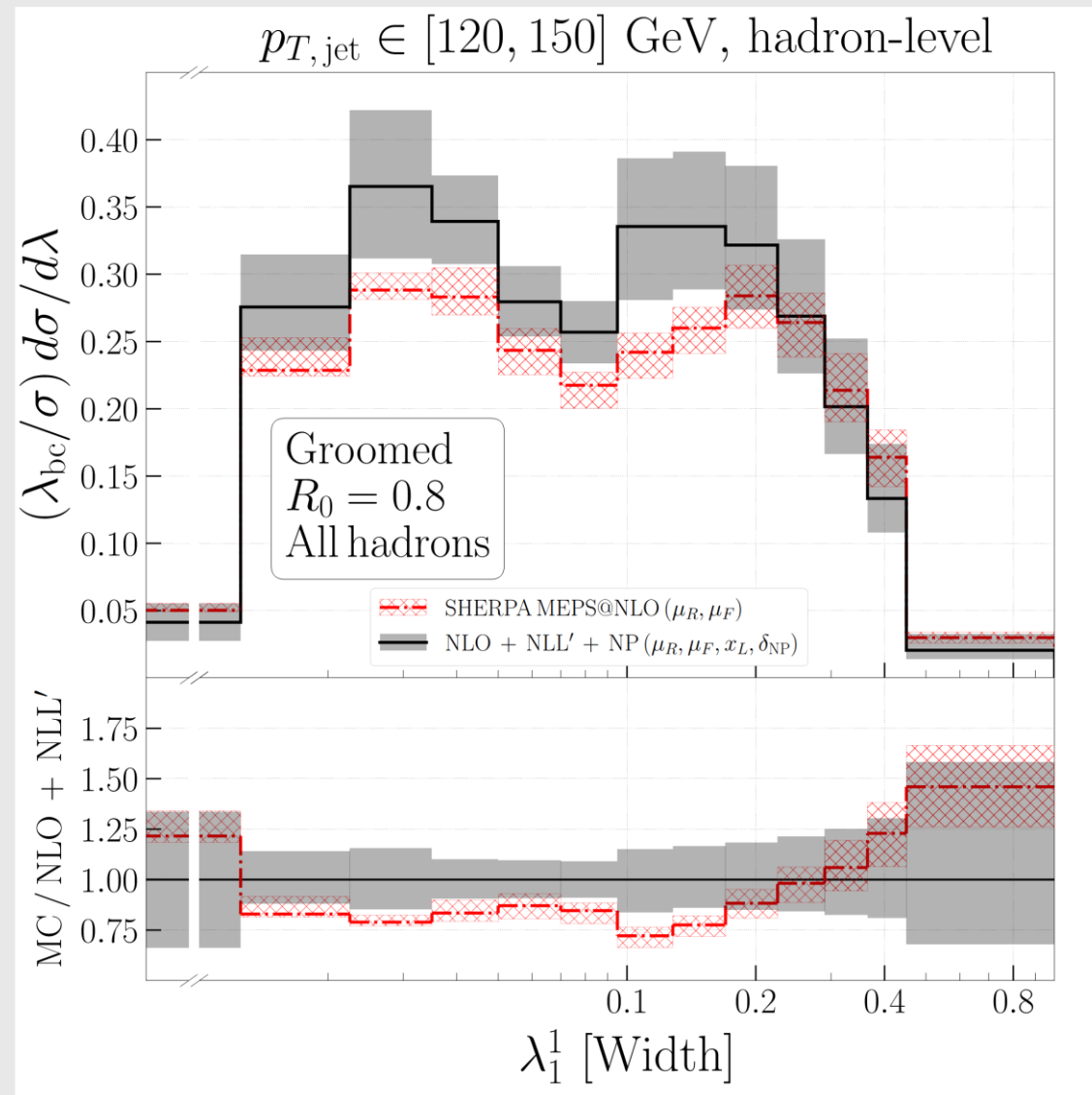
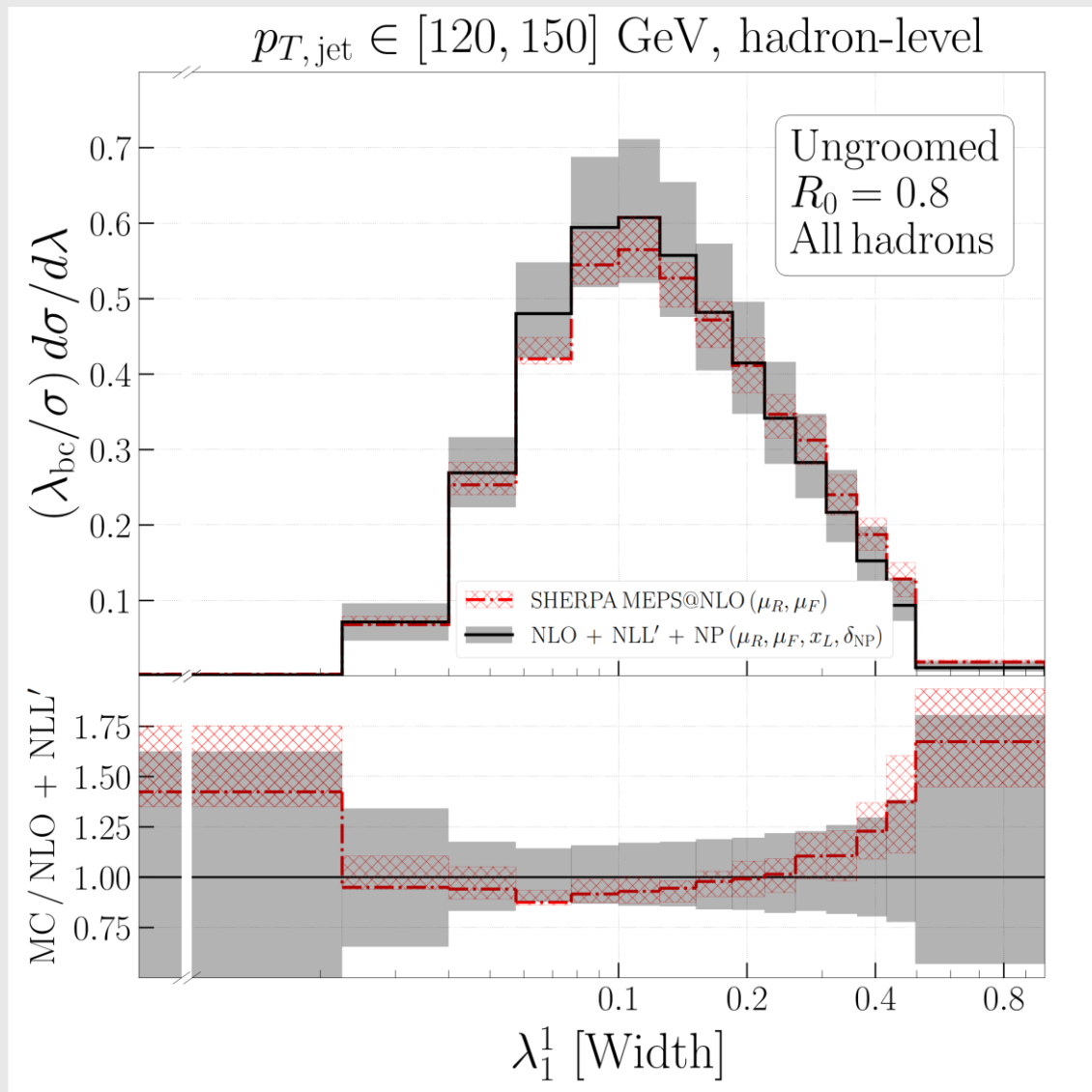
NLO + NLL' (width):



NLO + NLL' (width):



NLO + NLL' + NP (width):



NLO + NLL' + NP (width):

