

The background of the slide is a complex, abstract composition of various geometric shapes, lines, and colors. It includes circles, squares, triangles, and irregular polygons in shades of purple, blue, yellow, and grey, all set against a light, textured background. The overall style is reminiscent of mid-century modern or abstract art.

Silicon Detectors: Principles and Technology

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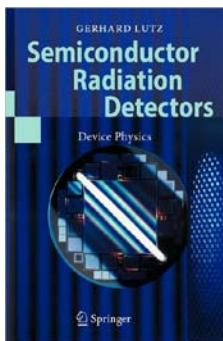
Introduction

- The basics of semiconductor sensors and readout have been addressed, in depth, in a number of recent books and multi-day lecture series (see next slide).
- We will provide a brief summary of these topics.
- The rest of the lecture will focus on a variety of technical matters and recent developments.
- The hope is to provide a useful reference point.

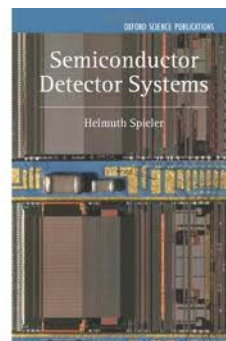
Suggestions for further reading

- H. Spieler, Semiconductor Detector Systems, Oxford Science Publications, 2005
See also: <http://www-physics.lbl.gov/~spieler/>
- G. Lutz, Semiconductor Radiation Detectors: Device Physics , Springer (July 11, 2007)
- G. Knoll, Radiation Detection and Measurement Wiley; 4 edition (August 16, 2010)
- A.S. Grove, Physics and Technology of Semiconductor Devices, (1967) John Wiley & Sons; ISBN: 0471329983
- S.Sze, Physics of Semiconductor Devices, J.Wiley, 1981
- T. Ferbel, Experimental Techniques in High Energy Nuclear and Particle Physics, World Scientific, 1992
- [K. Nakamura *et al.*](#) (Particle Data Group), J. Phys. G **37**, 075021 (2010)
<http://pdg.lbl.gov/2009/reviews/rpp2009-rev-particle-detectors-accel.pdf>

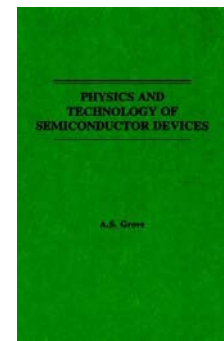
...and references therein



June 10, 2011



Silicon Detectors TIPP 2011



Carl Haber LBNL

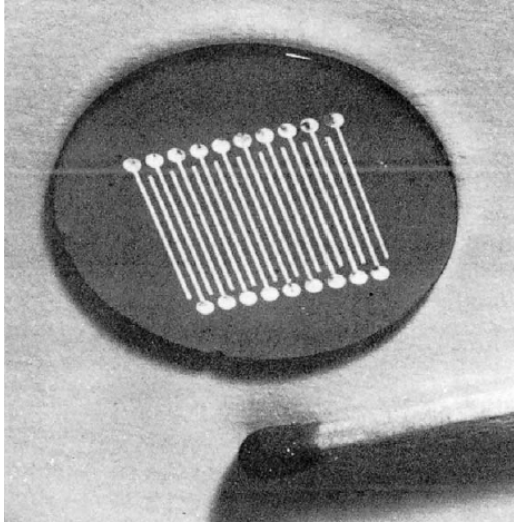
Outline

- Historical Perspective
- Basics
- Readout architectures
- Electronics Technology and Systems
- Mechanical and Metrological Aspects
- Radiation issues
- Future directions

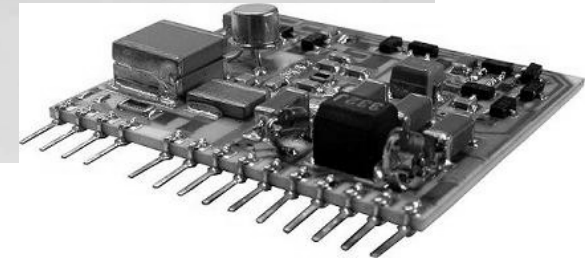
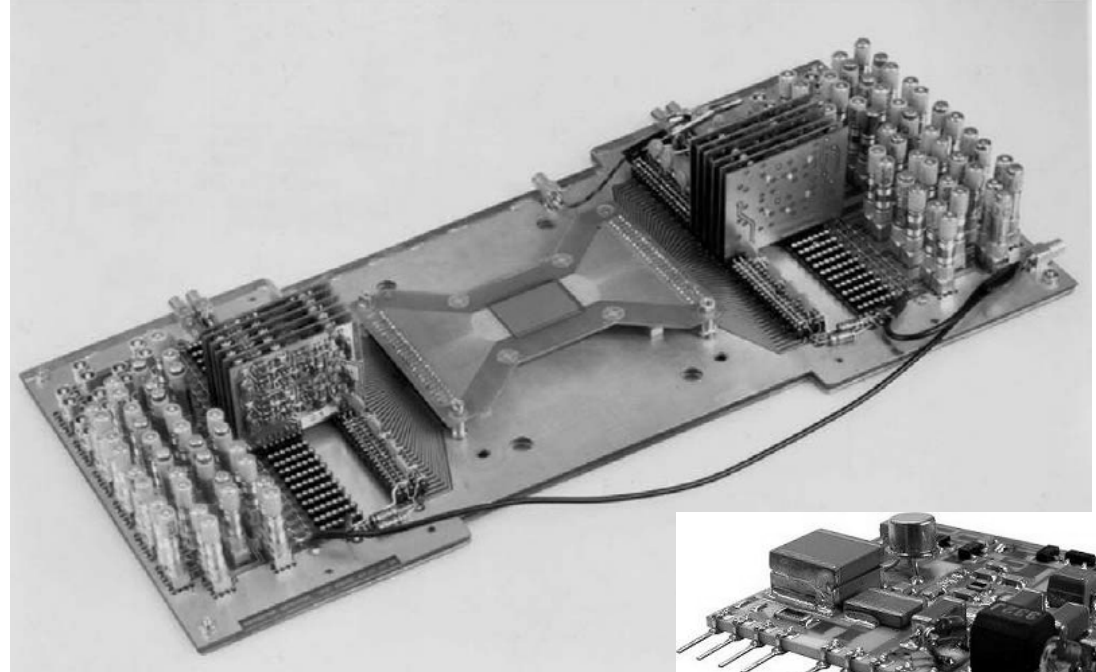
Why Silicon?

- Microelectronics and lithography allow for the precise patterning of sensor and readout elements on the $\sim 1 \mu\text{m}$ scale
- There exists a huge industrial and academic base which supports this technology, ever improving
- The natural scale maps well onto the experimental requirements for interesting physics measurements
 - Momentum determination at high p_T
 - Tracking in dense environments
 - Heavy flavor decay processes

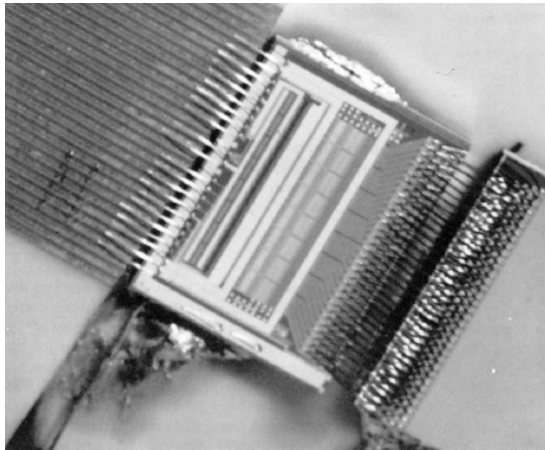
Looking for charm in fixed target hadronic interactions.....b's at lepton colliders



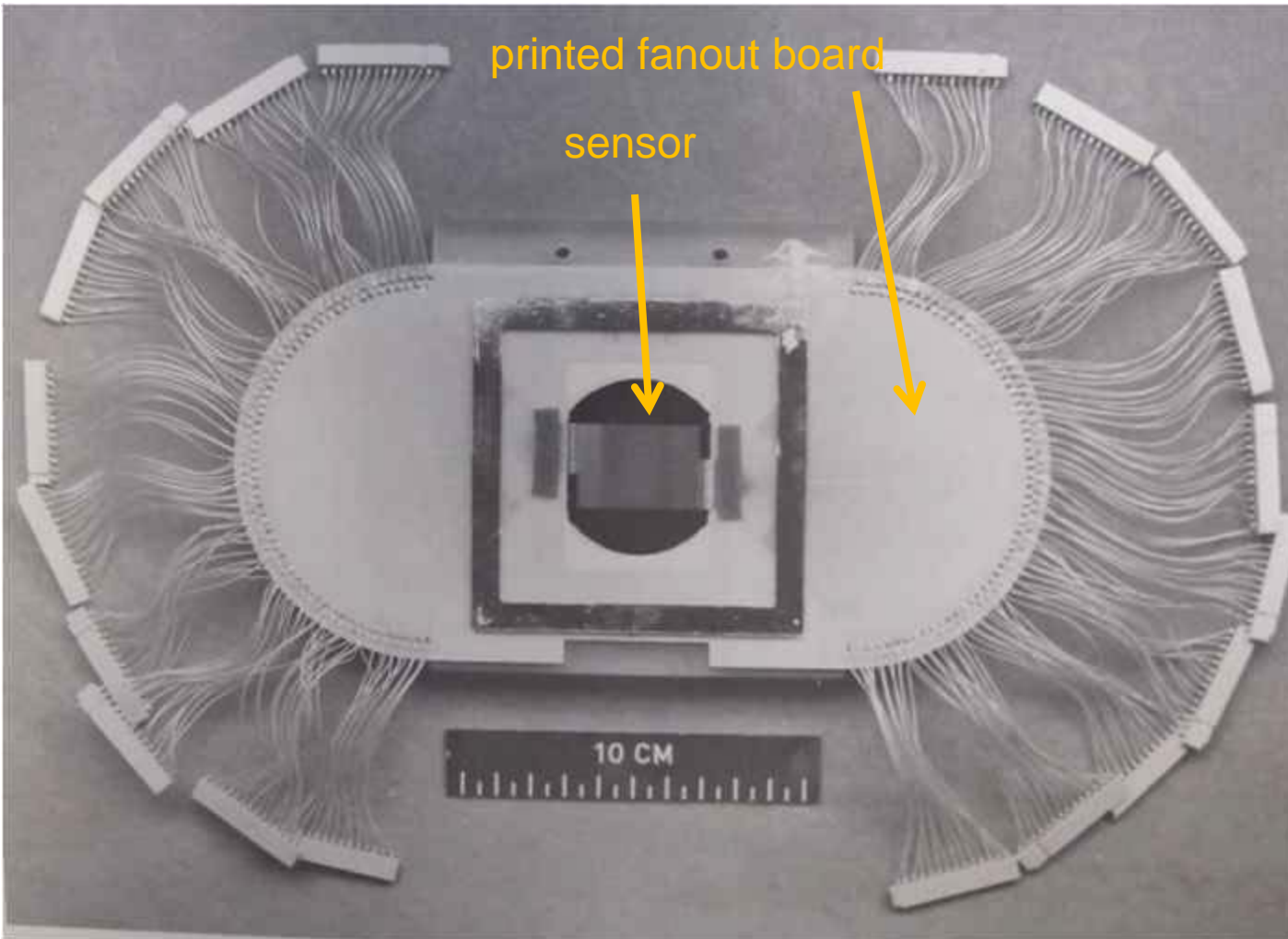
Late 1970's surface barrier strip detector (Pisa)



~1980, 128 discrete channels, ~14 mW/channel (CERN)

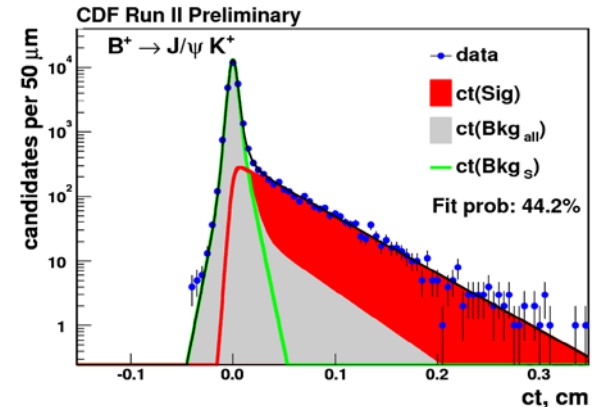
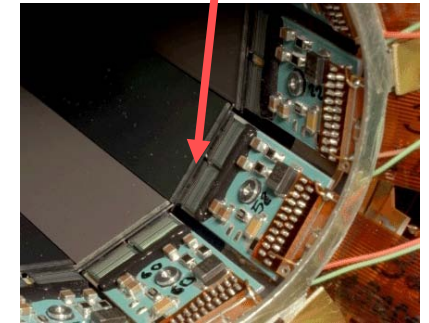
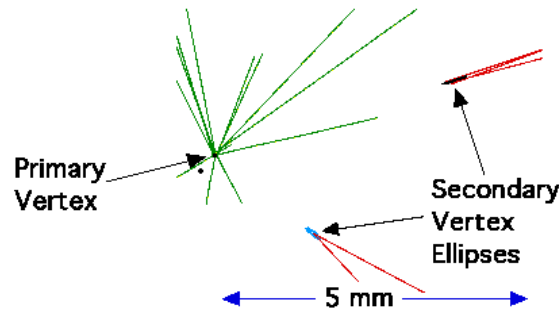
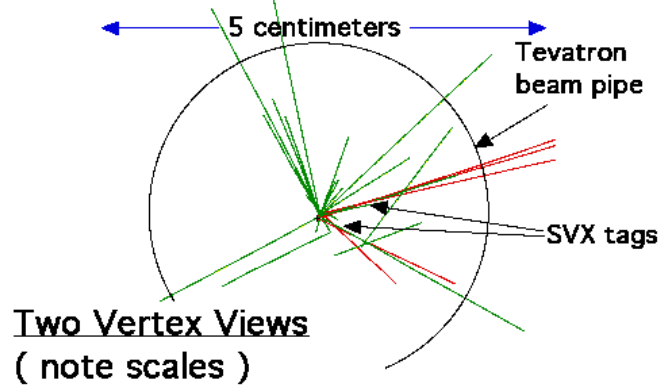
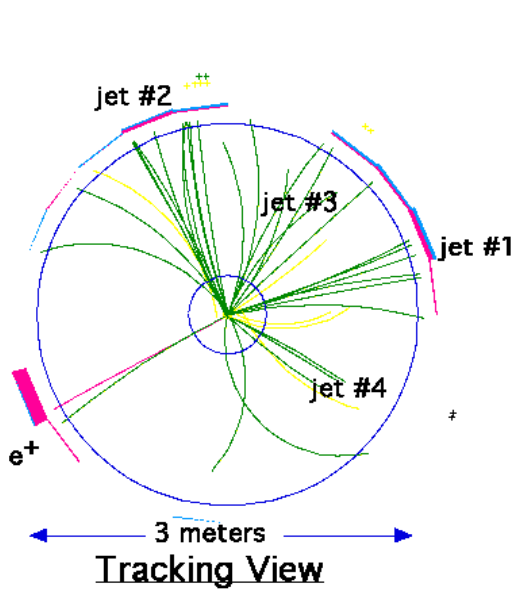
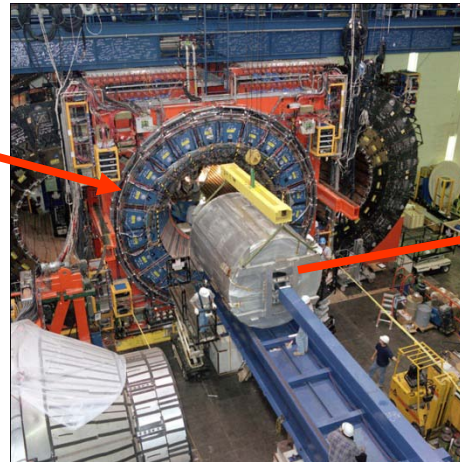
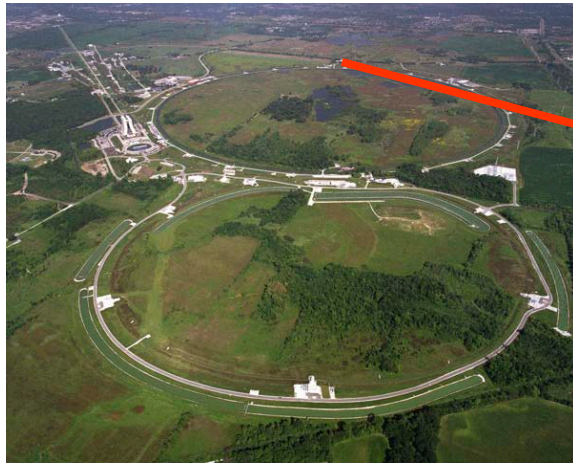


~1985, "Microplex", 1st 128 readout ASIC, 3 mW/chan (Parker, Hyams, Walker)



Circa 1980, state of the art, 256 channel strip detector for use in a fixed target experiment, (NA11 charm production) all strips are fanned out to a rack of discrete amplifiers and line drivers

Finding top and bottom at the Tevatron ~1995.....

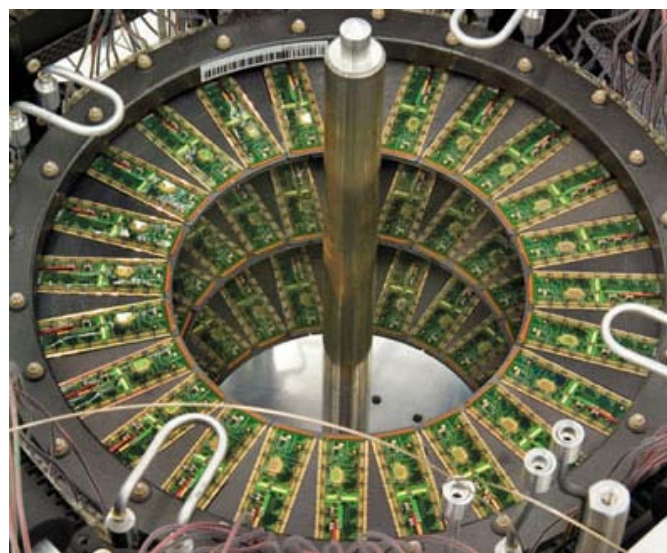
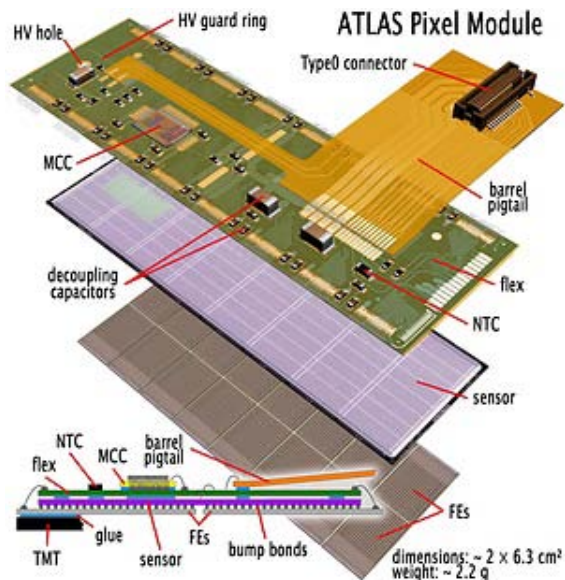
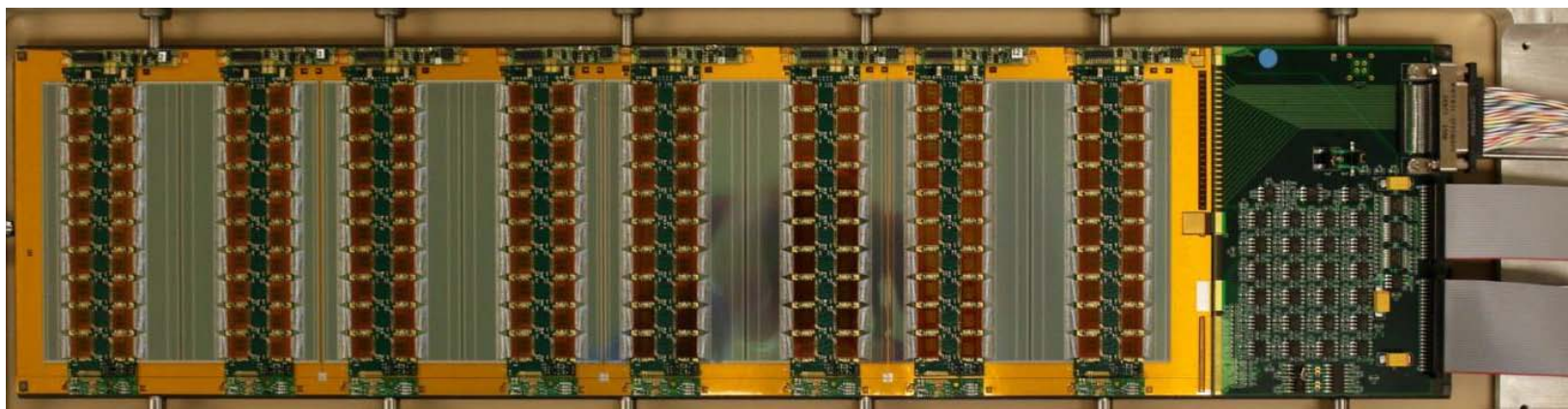


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Silicon Detectors TIPP 2011

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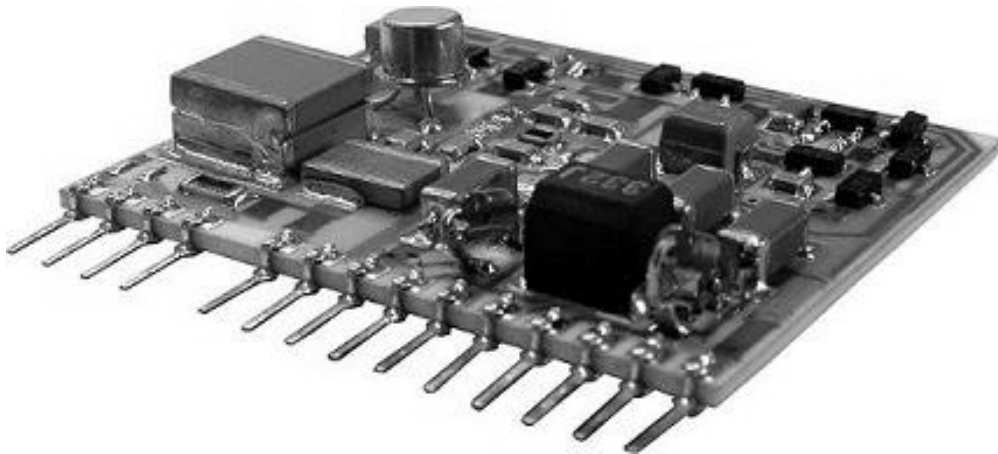
Prototype multi-modular silicon strip stave for use at the High Luminosity LHC



Present generation ATLAS pixel module in use today at the LHC...Higgs decays?

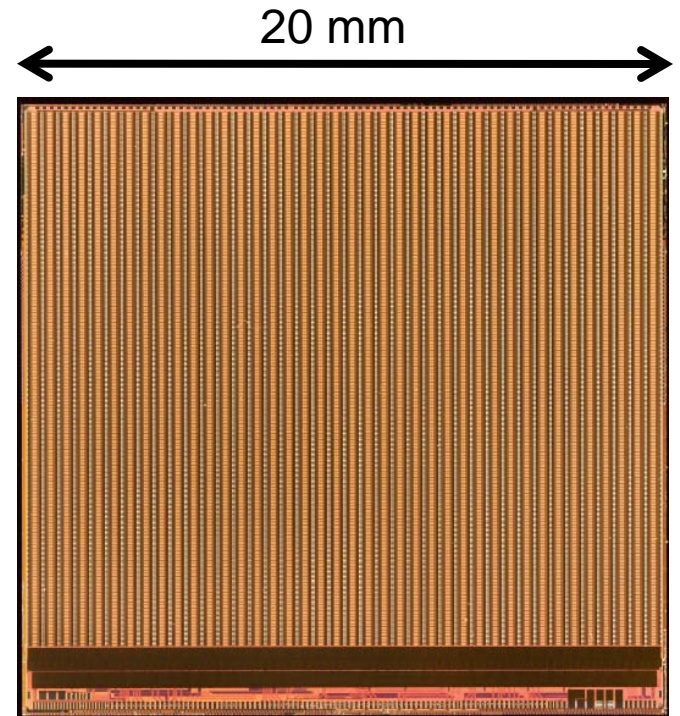
Evolution of Electronics

~1980 single channel of discrete hybrid pre-amp, a few transistors, 14 mW/channel

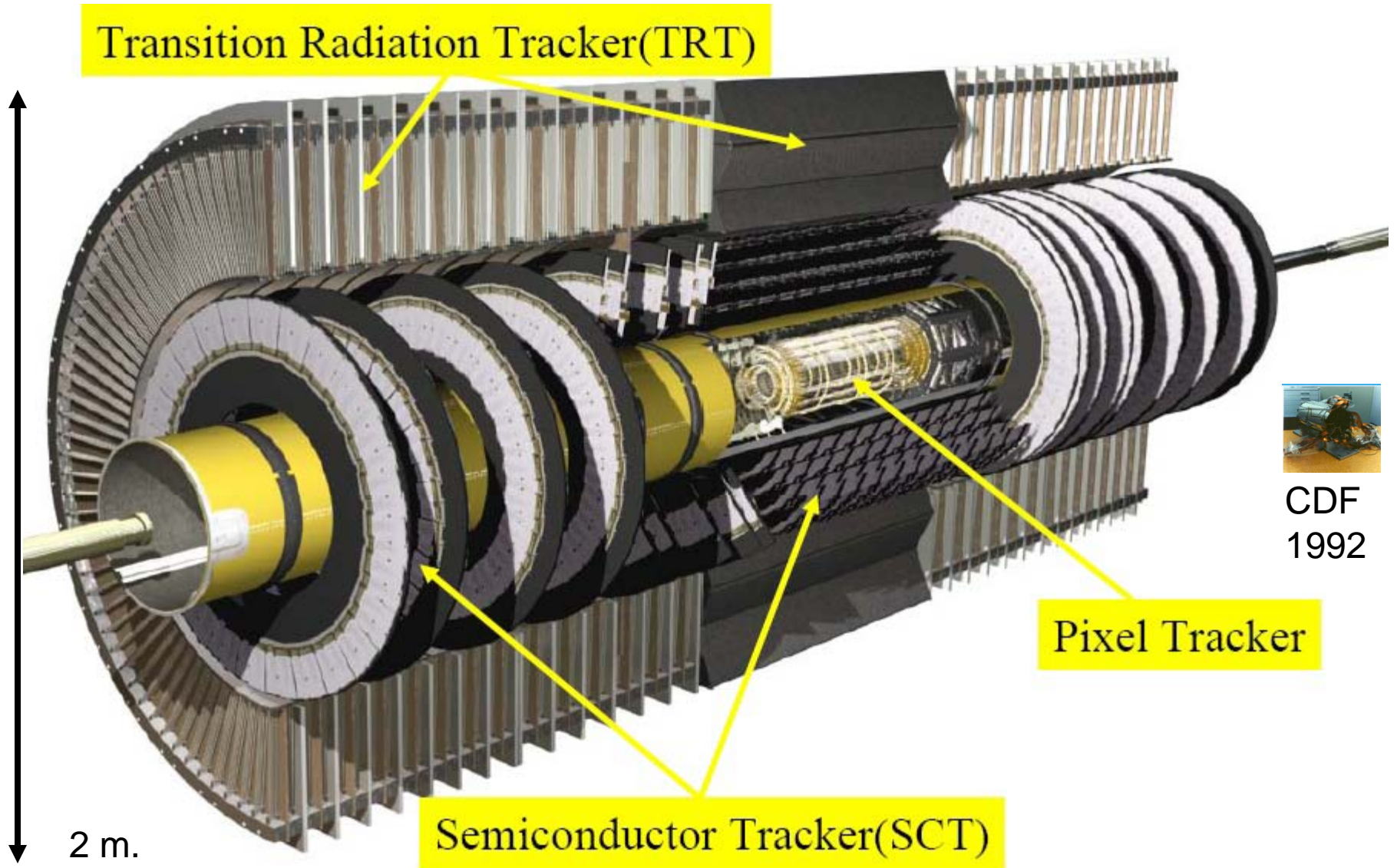


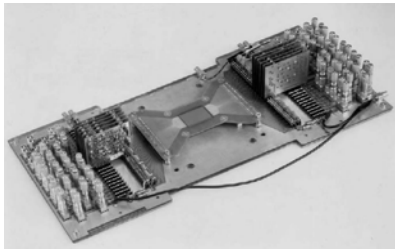
2011 ATLAS FEI4 Chip

26880 pixels, $30 \mu\text{W}/\text{pixel}$
3000 transistors/pixel

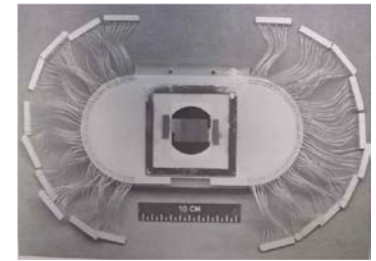


CERN ATLAS tracker (4th generation, beam in 2008)





Development



generation	year	luminosity	ΔT	chan/area	dose	readout
1 CDF SVX	1990	10^{29}	3.5 μ s	50K/ 0.68 m ²	25 Krad	3 μ m CMOS
2 CDF SVX*	1995	10^{30}	3.5 μ s	50K	100 Krad	1.2 μ m RHCMOS
3 Run 2	2000	10^{32}	128 ns	600K / 5 m ²	1 Mrad, $10^{13}/\text{cm}^2$	0.8 μ m RHCMOS
4 LHC	2009	10^{34}	25 ns	5×10^6 / 68 m ² 10^8 pixels	10 Mrad 10^{15}	0.25 μ m CMOS RH Bi-CMOS
5 HL-LHC	2020	10^{35}	25 ns	10^8 / 200m ² 10^9 pixels	100 Mrad 10^{16}	65 – 130 nm CMOS SiGe, Commercial

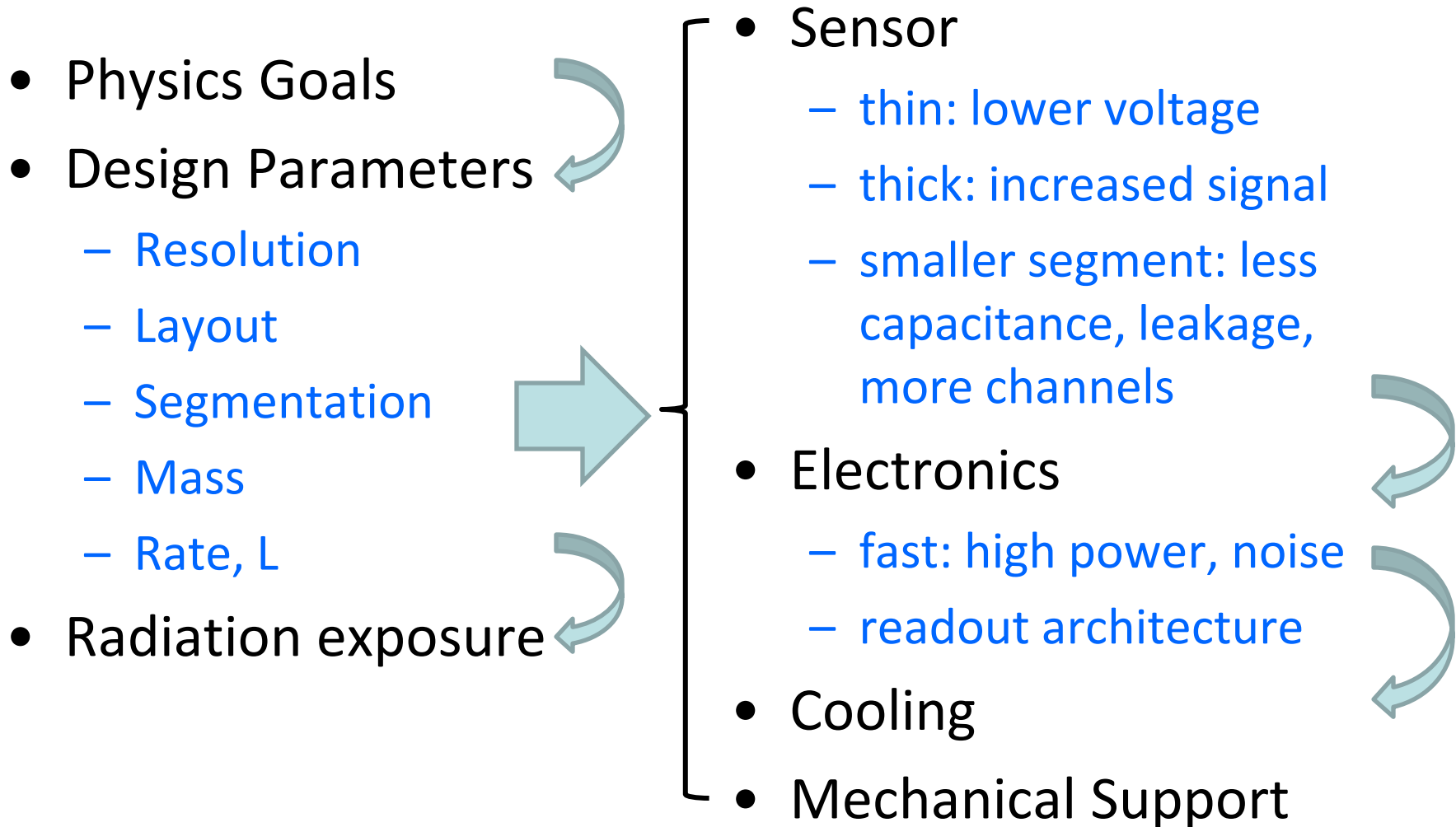
Technology

- The basic principles and structures have remained the same yet semiconductor detectors continue to function over a range of “ $\sim 10^6$ ”
- Application specific integrated circuits
- Digital design and simulation tools
- Wafer size 2”,.....,10”; feature size, circuit performance
- Interconnections, wire and bump bonding
- High density electronic packaging
- Advanced power management
- Composite mechanics
- Advanced thermal/mechanical materials
- Precision optical metrology
- Highly parallel DAQ with embedded processing (FPGA’s)

What drives the present and future developments?

- Today's silicon trackers are large systems typically in use at colliders for momentum vertex measurement
 - Physical size ~ 1 m radius
 - Channel counts $\sim 10^7$
- High rate of interactions and track density: 40 MHz
- High radiation levels: 10^{15} - $10^{16}/\text{cm}^2$
- Inaccessible: few years
- Mass ruins the response of other systems
- **As the field progresses all of these aspects increase!**

Specs and Optimizations



Silicon Detectors

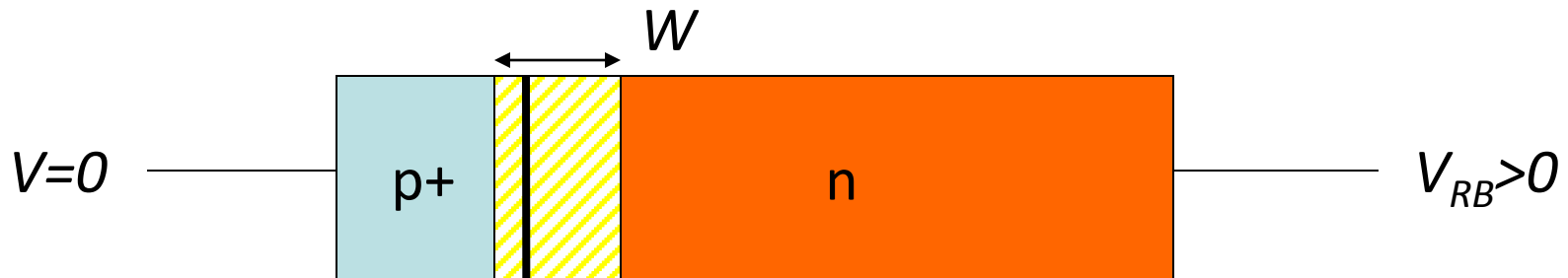
- Semiconductor band structure ' energy gap
- Asymmetric diode junction: example p(+) into contact with n ($N_A \gg N_D$)
- Space charge region formed by diffusion of free charges, can be increased with "reverse bias"

$$\text{junction width : } W = \sqrt{2\mu\rho\varepsilon(V_{BI} + V_{RB})} = 0.5\mu m\sqrt{\rho(V_{BI} + V_{RB})}$$

μ = electron mobility, $\varepsilon = 11.9\varepsilon_0$

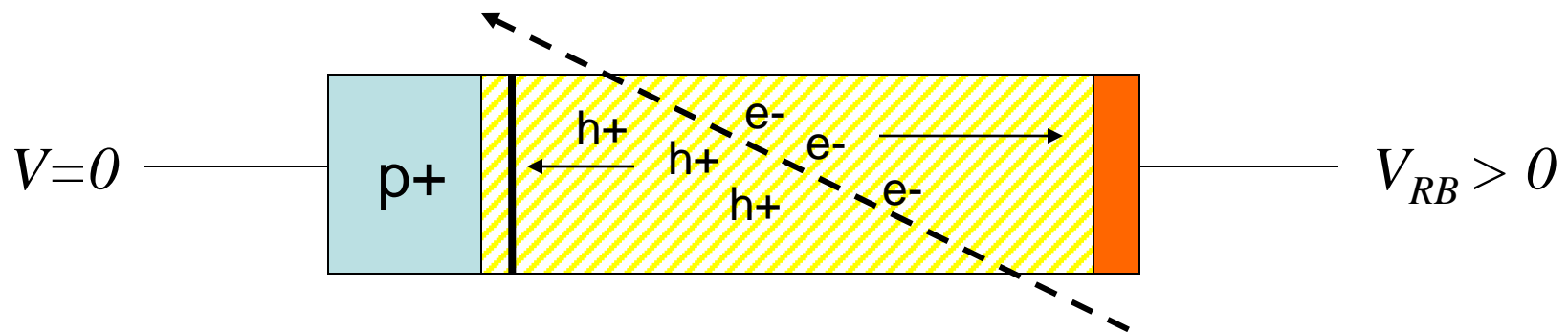
$$\rho = \text{resistivity of n type material} = \frac{1}{e\mu N_D} \approx 1 - 10k\Omega \text{ cm}$$

V_{BI} = built in potential ($\sim 0.8 \text{ V}$) V_{RB} = applied reverse bias



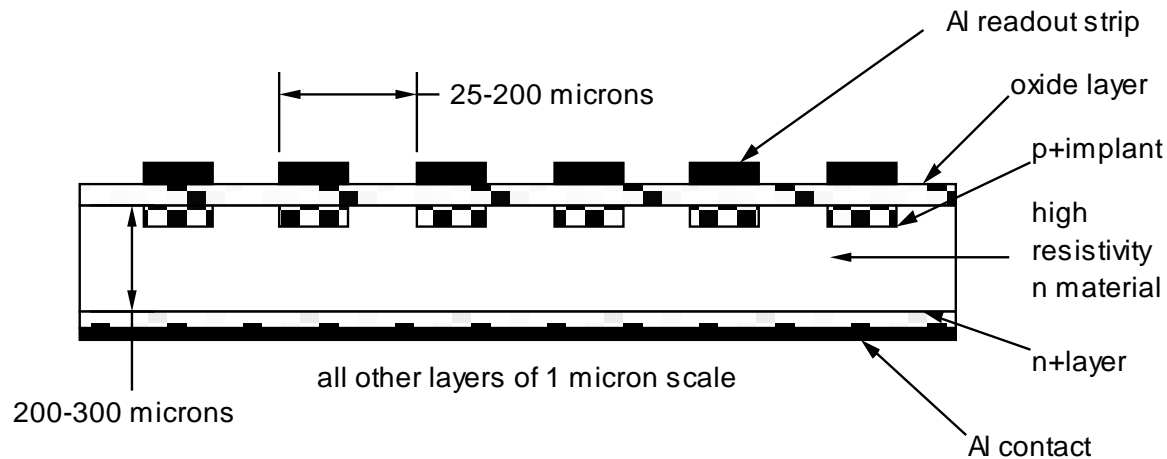
Response to Ionization

- Electron-hole pairs formed in the depletion zone drift under the influence of the electric field
- Signal depends on width of depletion zone
- Drift time determined by mobility and field
 - ~ 7 ns to cross 300 microns
- Drifting charge is a current which can be measured

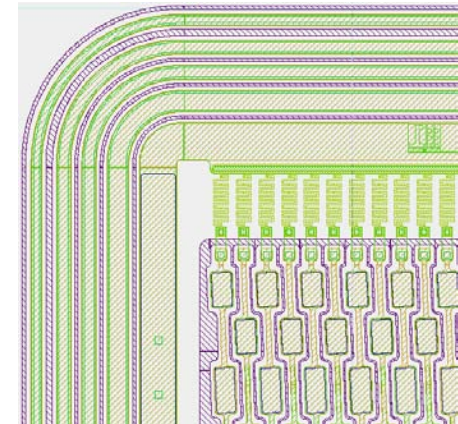
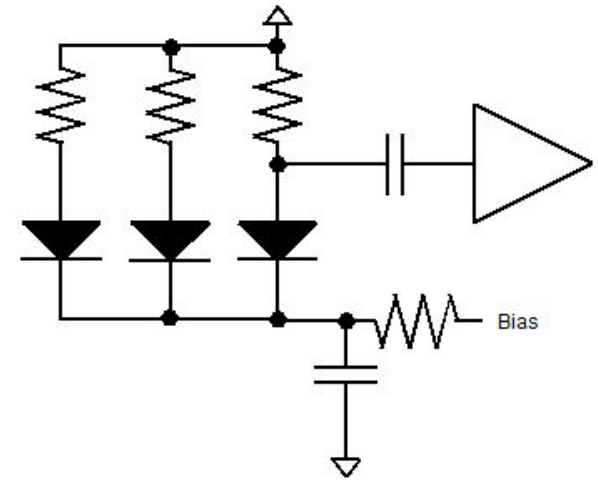
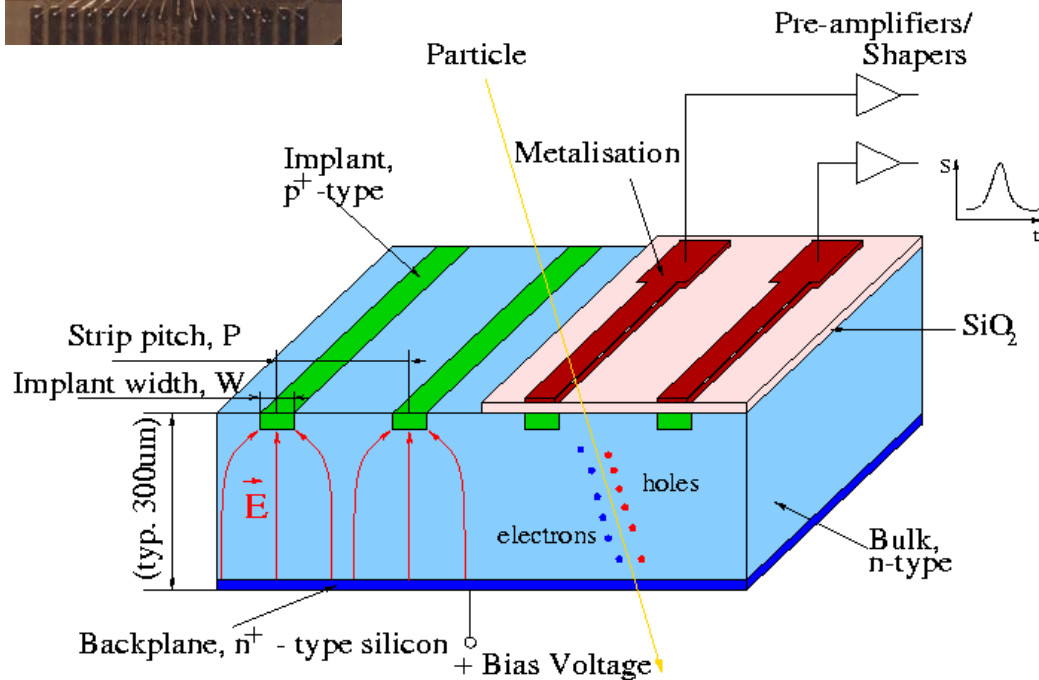
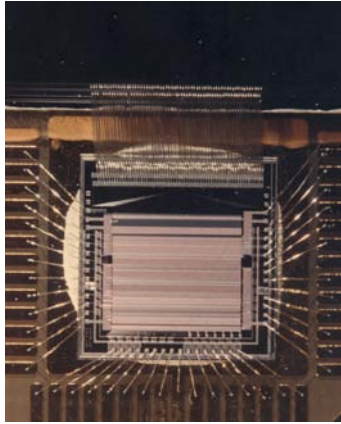


Planar Processing

- Using micro-lithographic techniques arrays of diode structures can be patterned on silicon wafers.
- The “Silicon Microstrip” detector was introduced in the late 1970’s and is the basis of all precision types in use today

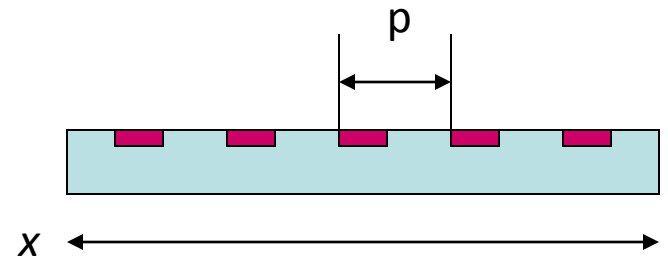


Sensor Details

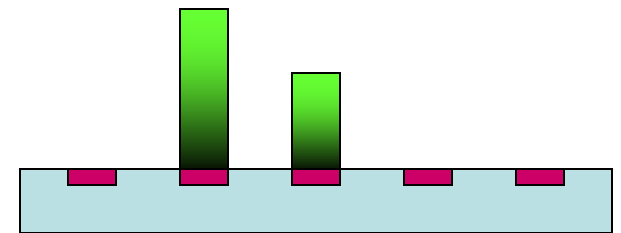


Point Resolution: Segmentation

- Discrete sensing elements (binary response, hit or no hit), on a pitch p , measuring a coordinate x
- Discrete sensing elements (analog response with signal to noise ratio S/N) on a pitch p , where f is a factor depending on pitch, threshold, cluster width



$$\sigma_x = \frac{p}{\sqrt{12}}$$



$$\sigma_x \sim fp\left(\frac{N}{S}\right) < \frac{p}{\sqrt{12}}$$

2D Pixel Structure

Single sided configuration
Pixel readout

Charge sensitive
Preamp + shaping,
signal processing,
pipelines, digitization

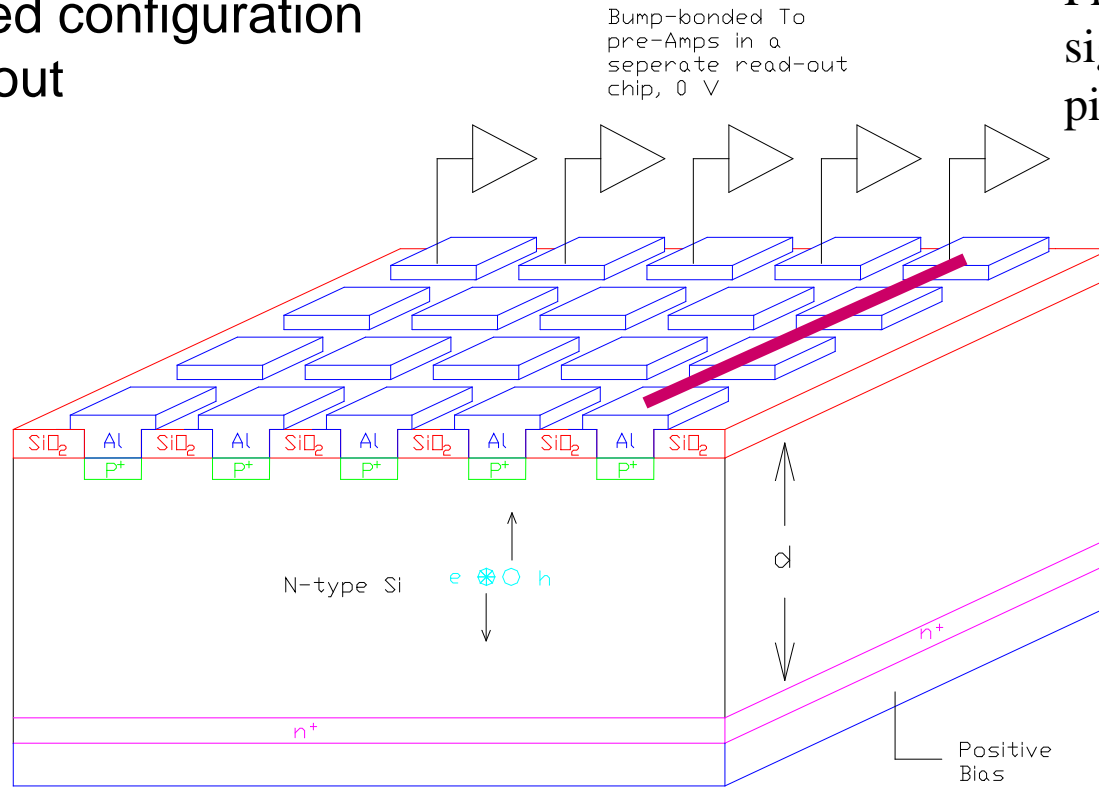
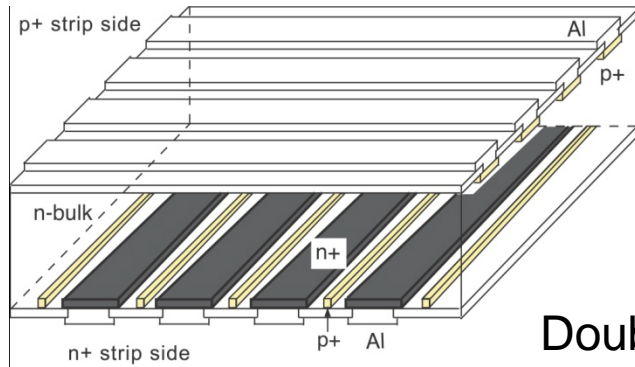


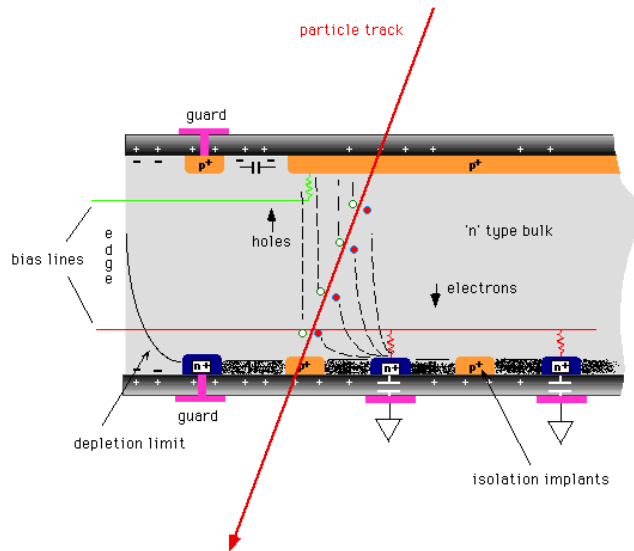
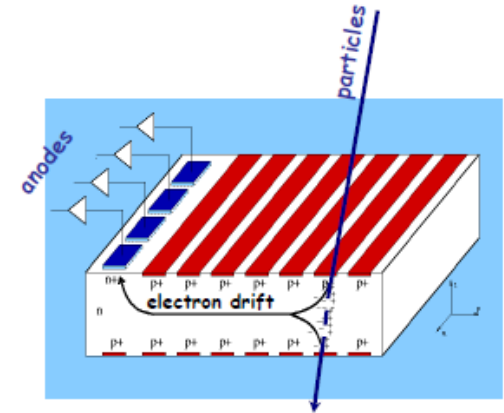
Diagram courtesy of Z.Li and V.Radeka

Further Variants

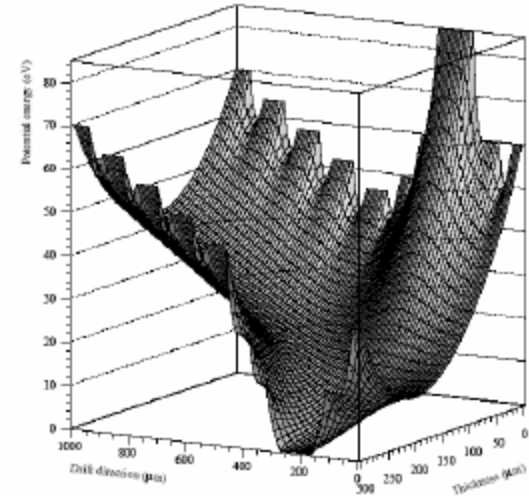
Silicon drift detector



Double sided detector



p+ in n
n+ in p
n+ in n



Pixel Detector Types

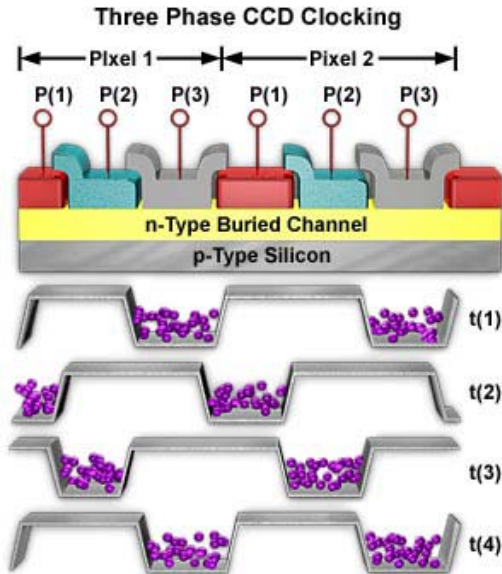
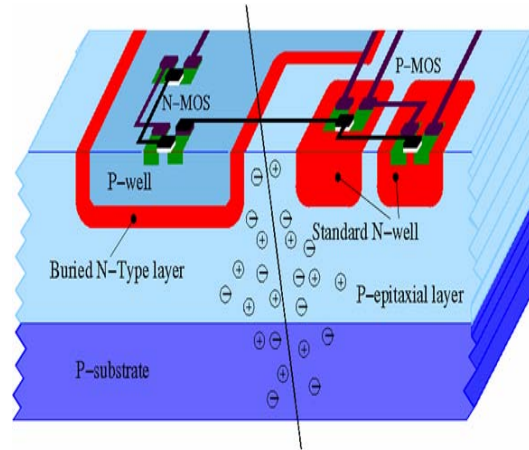
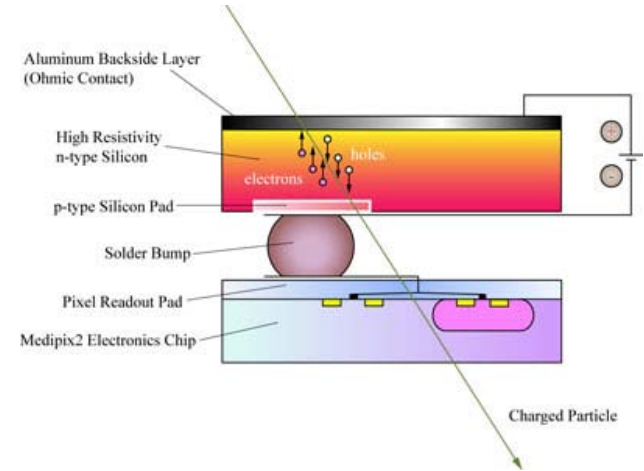


Figure 1

CCD
Charge Coupled Device
Relatively slow
Low noise, Low mass



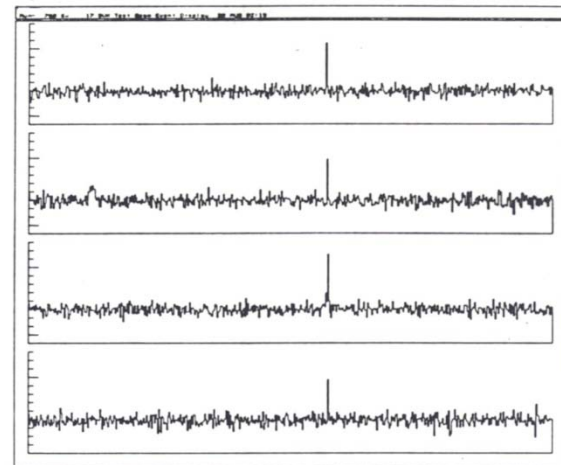
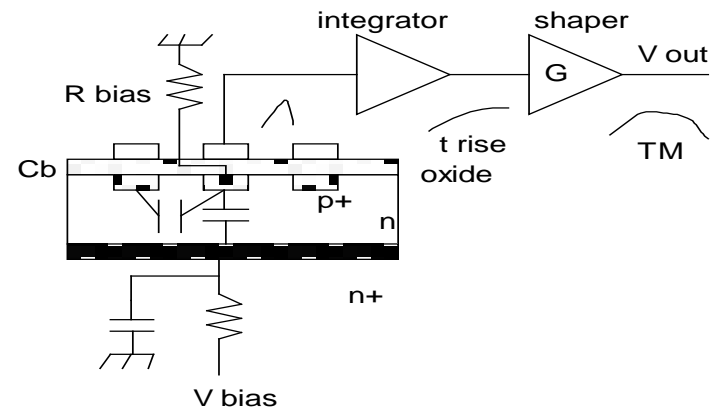
Monolithic Active Pixel
electronics and charge
formation/collection in a
thin epitaxial layer, diffusion
Moderate speed
Low noise, Low mass



Hybrid pixel
FE IC and sensor joined
by a bump bond
Fast
Radiation Hard

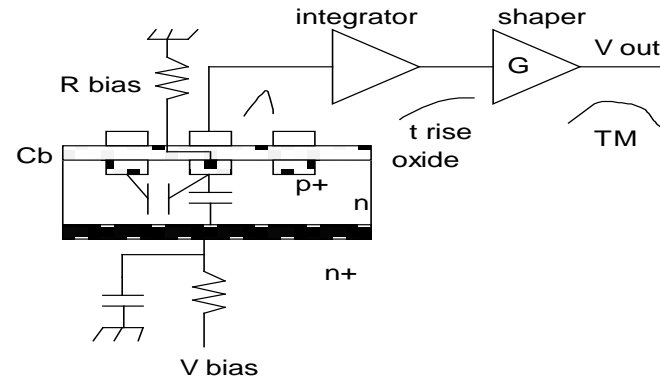
Signal Processing Issues

- Signal: expressed as input charge, typically 25,000 electron-hole pairs (4 fC)
- Gain: determined by feedback and capacitance
- Noise: various sources, must be small compared to signal ($S/N > \sim 10$)



Leakage Current

- Leakage current
 - DC component blocked by oxide capacitor
 - If DC coupled then must be compensated by filter, feedback, or injection
 - Before (after) radiation damage ~ 1 nA (1 ma)
 - AC component is seen by pre-amp: noise source



$$I_L = \frac{en_i(\sigma v_{thermal} N_T)WA}{2}$$

n_i = intrinsic carrier concentration

σ = recombination cross section

$v_{thermal}$ = carrier thermal velocity

N_T = trap density

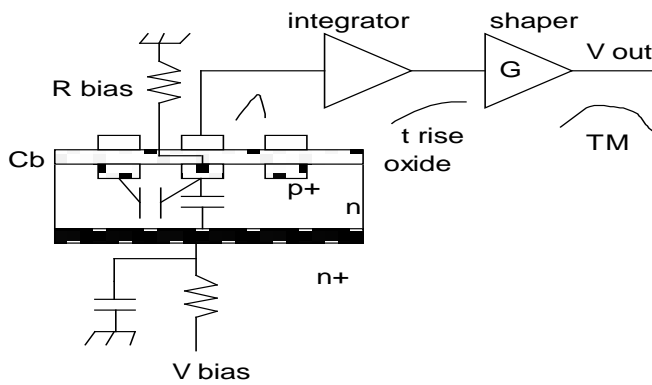
A = junction area

$$I_L(T) \propto T^2 e^{-E_a/2kT}$$

Noise

- Fluctuations \sim Gaussian σ_N
 - Leakage Current
 - Preamp “input noise charge”, white noise, decreases with pre-amp current, increases with faster risetime, a, b are constants and C_D is the detector capacitance
 - Bias resistor: source of thermal noise
 - Radiation activated

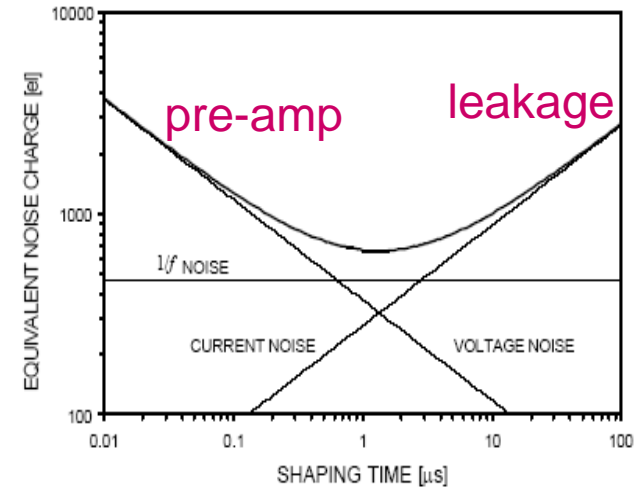
- Extraneous Noise



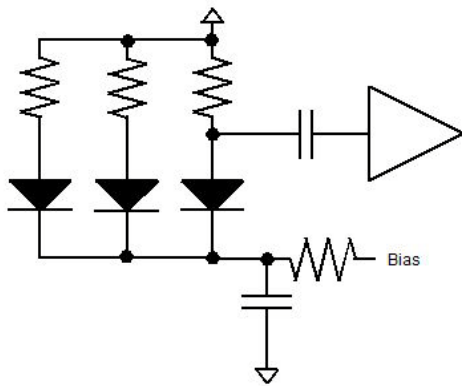
$$\sigma_N \propto \sqrt{I_{LEAK} T_M}$$

$$\sigma_N \propto a + bC_D$$

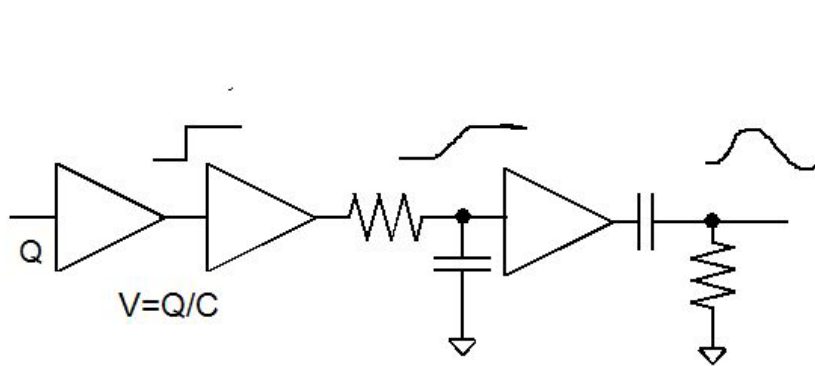
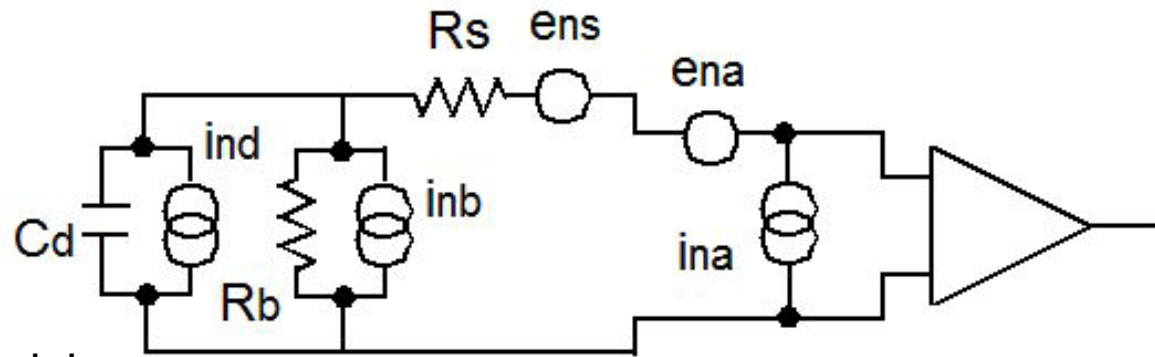
$$\sigma_N \propto \frac{1}{R_{BIAS}}$$



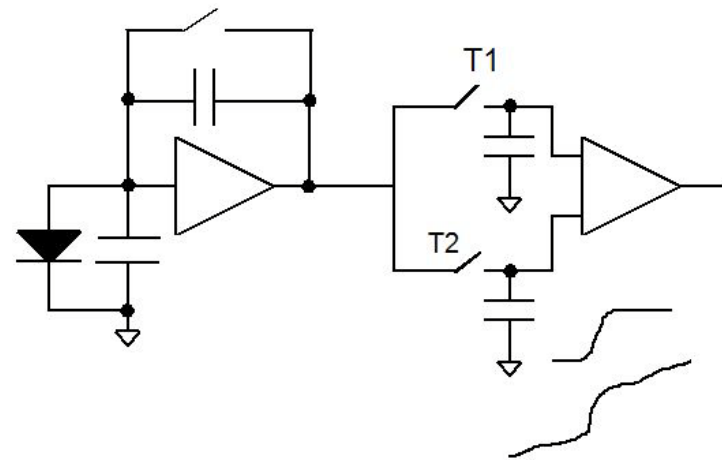
Full Evaluation of S/N



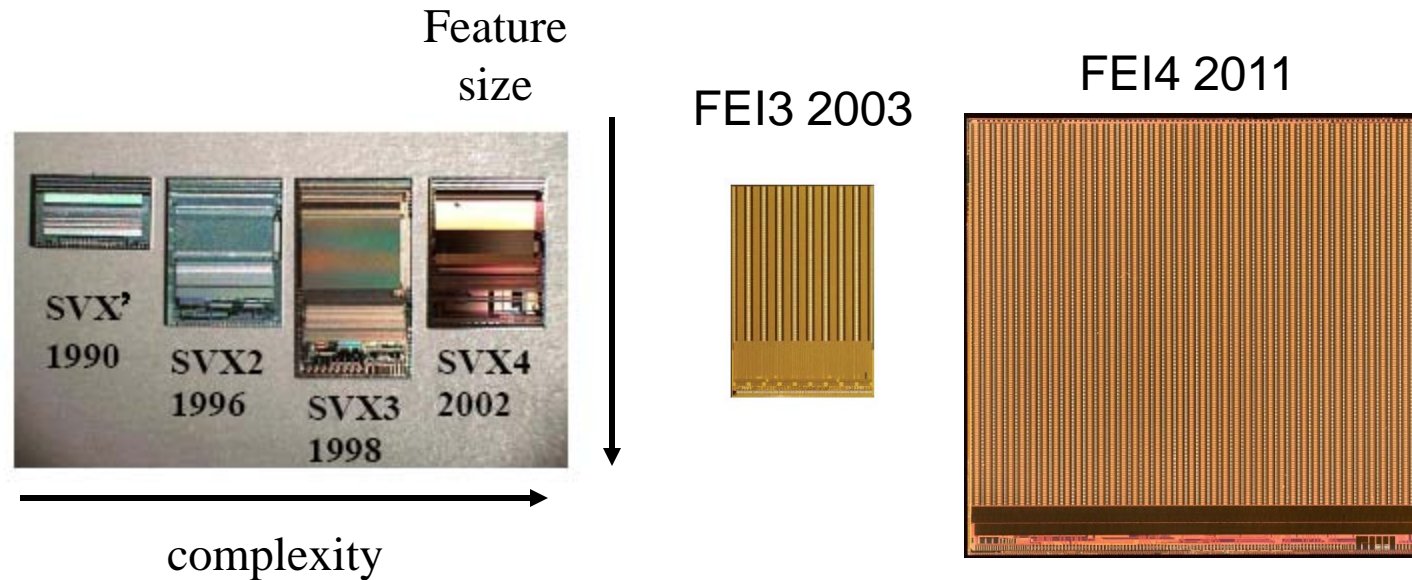
Noise model



Shaping models



Readout Electronics

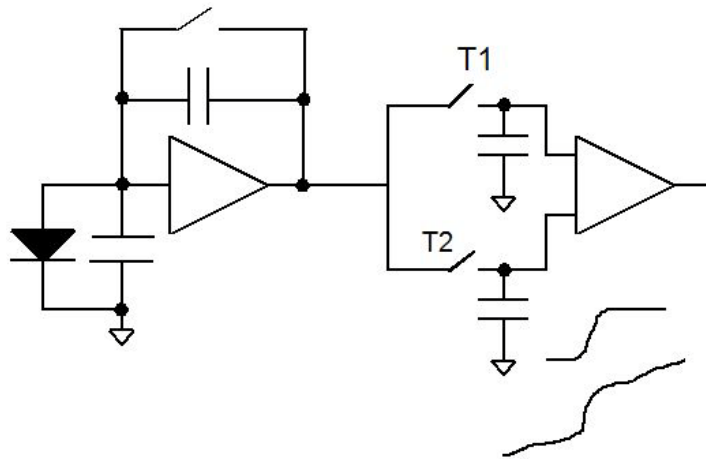


- Large channel count and complexity require custom readout chips (ASICs)
- On-chip complexity increases with process evolution
- Impact of powerful design and simulation tools
- Mixed analog-digital signals on the same chip
- Speed and noise performance have kept up with requirements but S/N often remains an issue, particularly with longer strips and irradiation

Readout Architectures

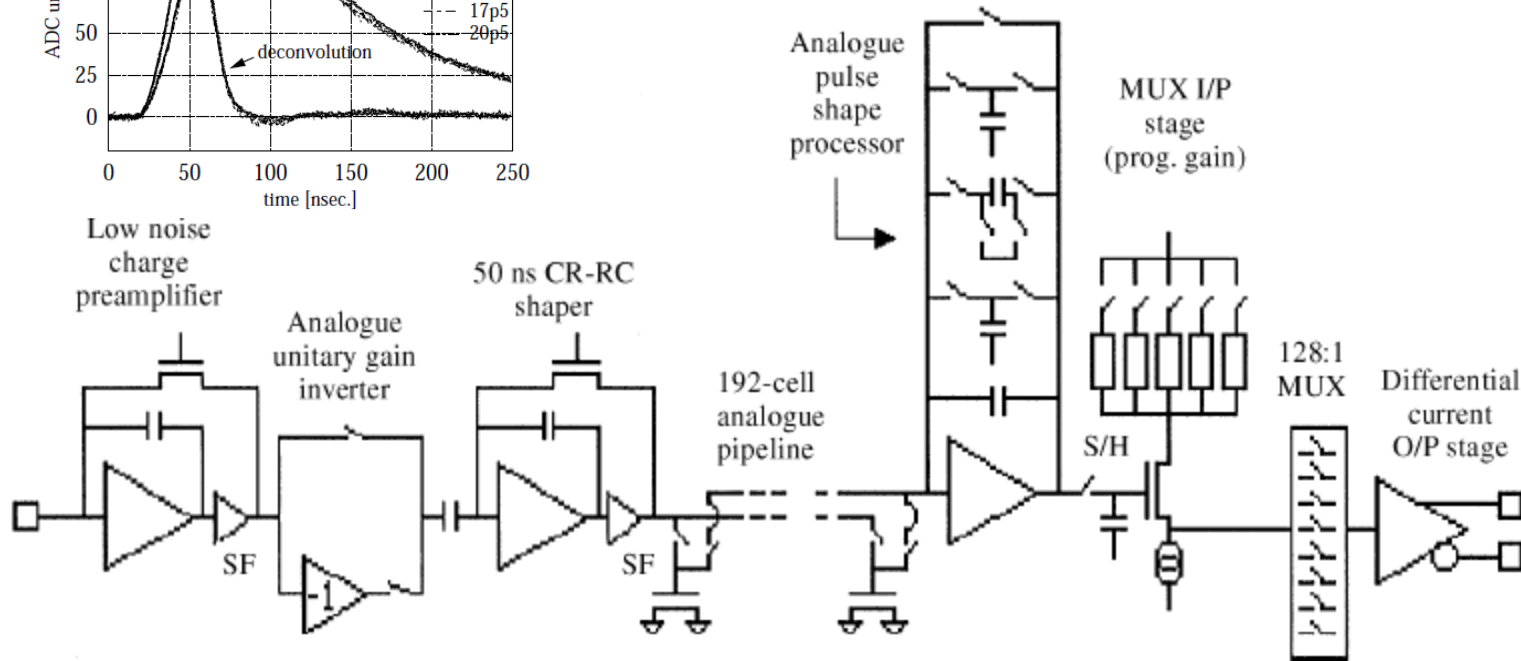
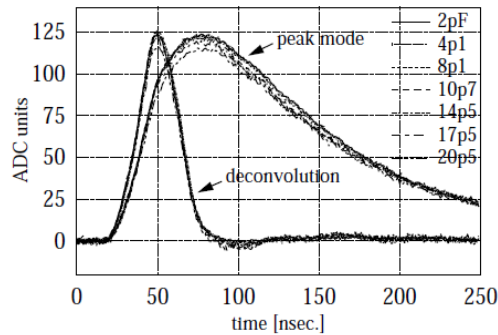
- Experimental conditions, have, to some extent driven the development of a variety of readout architectures
- Accommodate properties and limitations of available IC processes, a moving target
- Subjective aspects have entered as well
- Analog: process analog pulse heights off detector, full resolution, diagnostics
- Digital: digitize on detector, full resolution
- Binary: on detector threshold, simplify readout

Double Correlated Sample and Hold



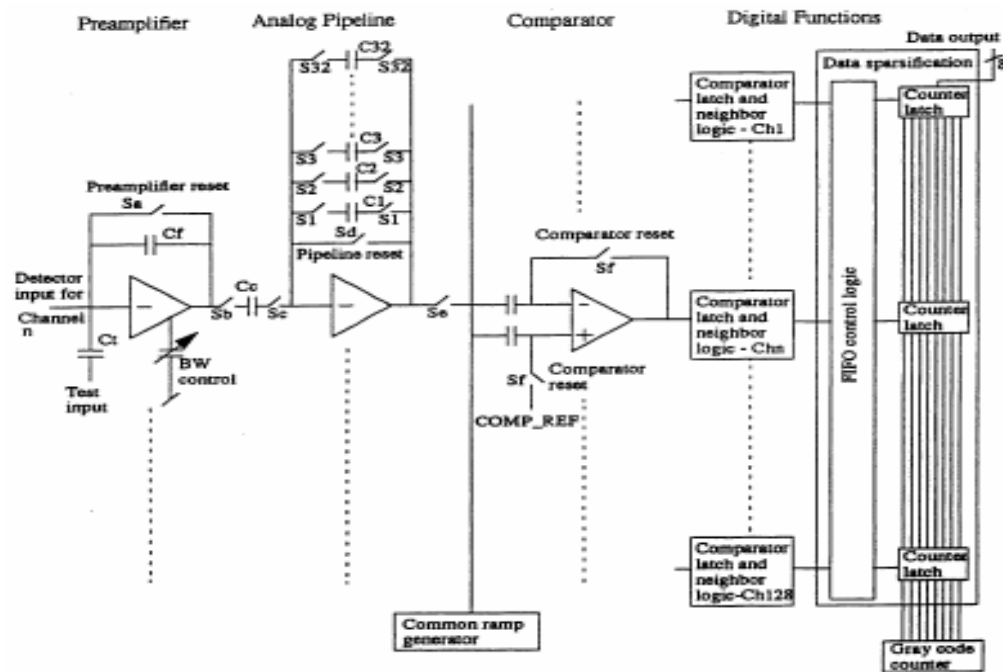
- Original MOS monolithic architecture, no resistors
- High frequency bandwidth limited by pre-amp
- Low frequency limited by $\Delta T = T2 - T1$
- Can be generalized to N samples but incurs a noise penalty factor of $\sqrt{2}$ for each pair

Analog



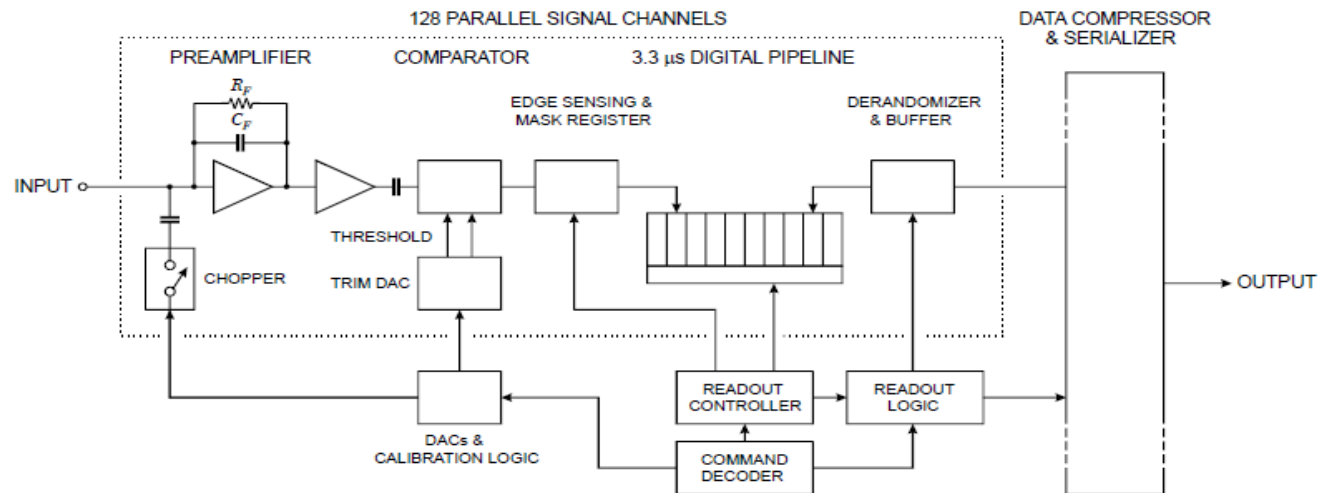
- Example is the APV25 chip developed for CMS
- Readout all analog pulse heights, no sparsification
- Dual function: fast time mode, slow low noise mode
- Utilize analog signal processing on-chip to measure pulse time

Digital



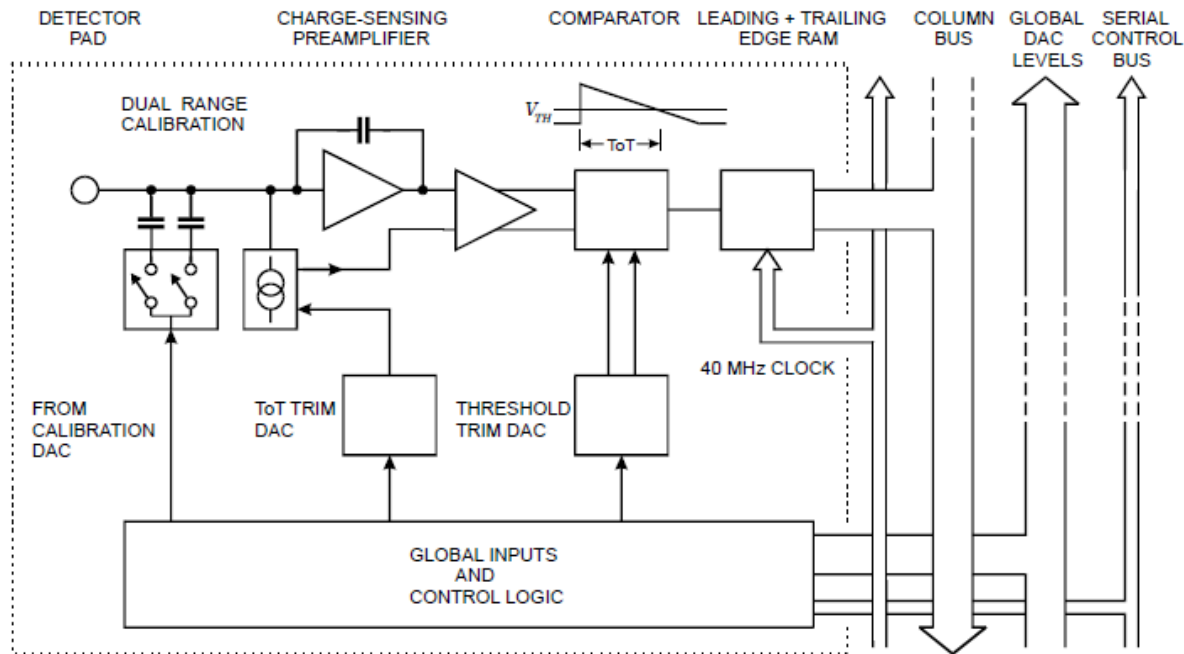
- Example is the SVX4 chip developed for CDF
- Switched capacitor analog pipeline
- Combined analog threshold + 8 bit digitization
- Sparse readout

Binary



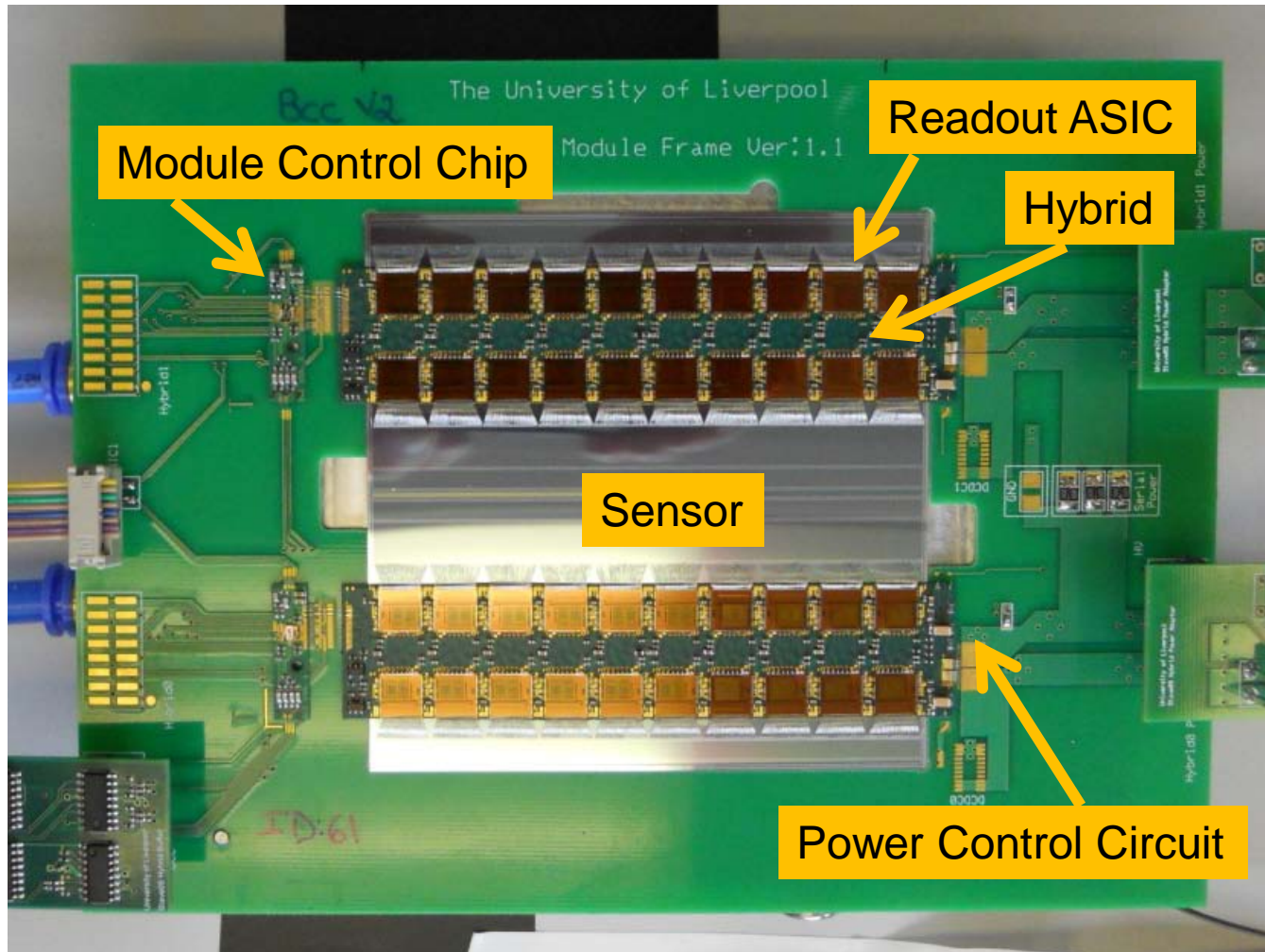
- Example is the ABCD chip developed for ATLAS, BiCMOS
- AC coupled pre-amplifier shaper for 25 ns collisions
- Comparator + trim DAC per channel
- 1 bit pipeline clocked at 40 MHz, L1 buffer
- Data compression
- Control and configuration protocol
- DSM CMOS version exists as well, 130 nm underway

Pixel



- Example is ATLAS FE13 pixel cell
- DC couple preamp with leakage current compensation
- Pulse height is measured by “Time over Threshold”
- Additional architecture organizes the data in columns for readout

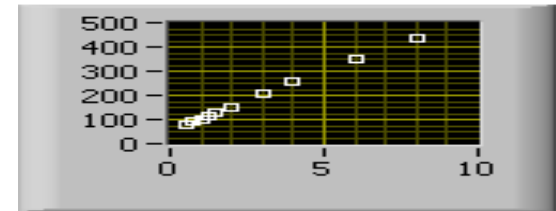
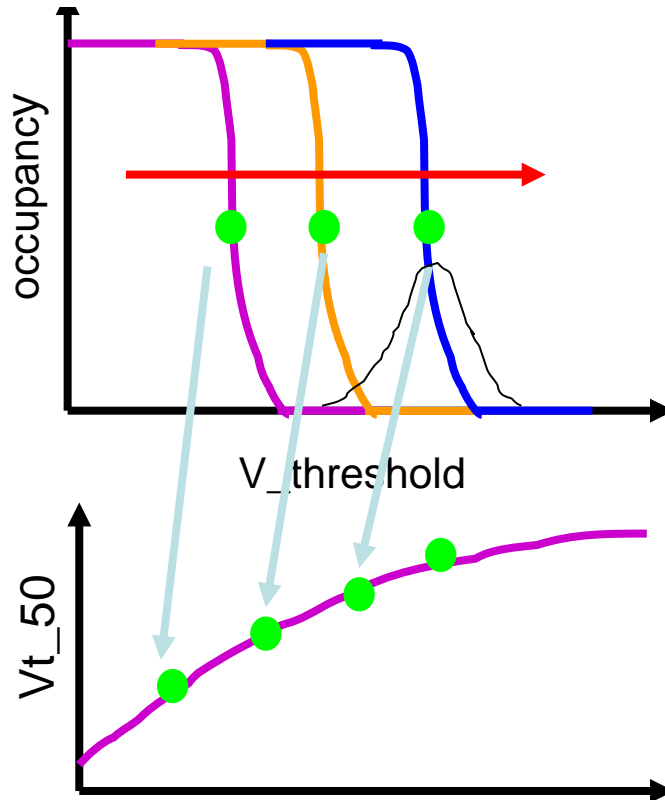
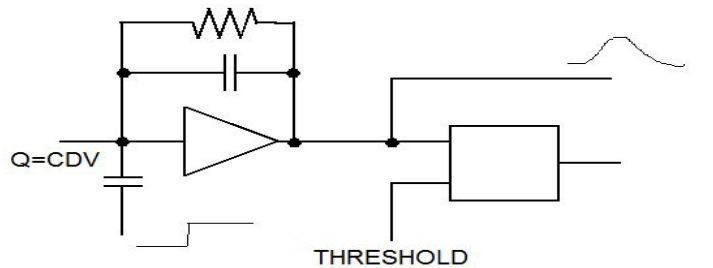
Going to the next level



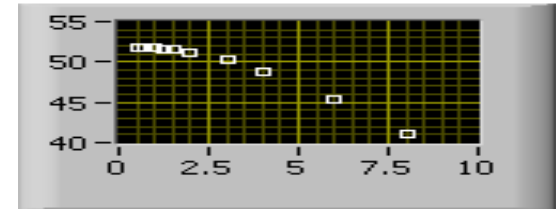
Diagnostics

- Single channels can be studied in detail “on the bench”
- Channels are integrated in chips, chips into modules, etc.....
- Large detector systems require extensive monitoring and calibration
 - Parasitic collective effects and external noise can occur at the system level
- Much effort to develop meaning procedures which can be implemented efficiently
- These will differ for the various architectures due to the limitations imposed.

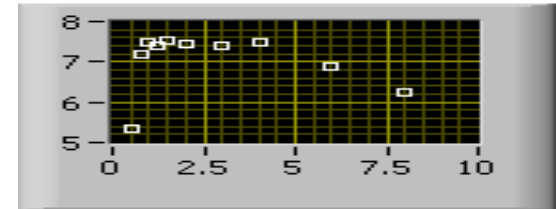
Binary Performance Measure



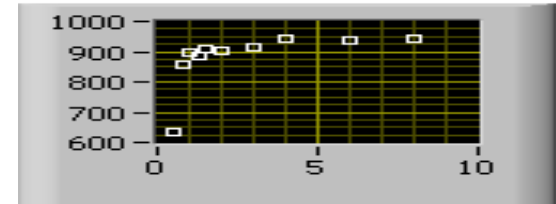
Chip 0



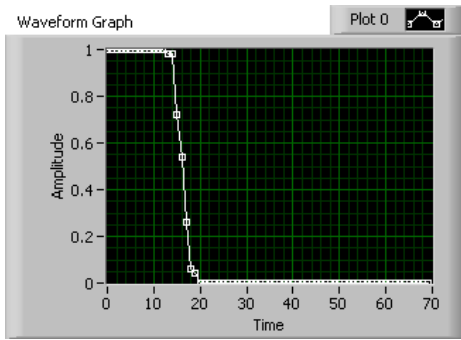
Chip 0



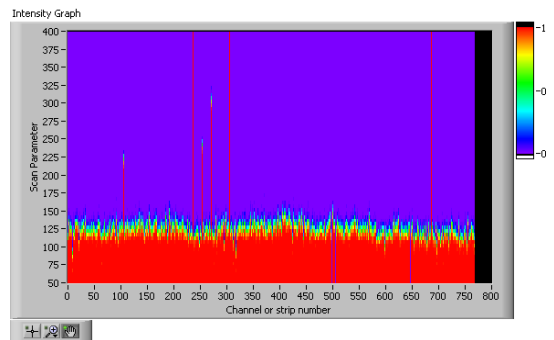
Chip 0



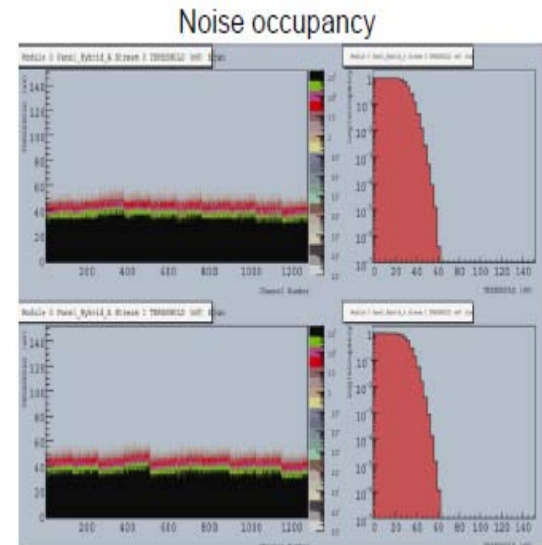
Full Response Study: Module



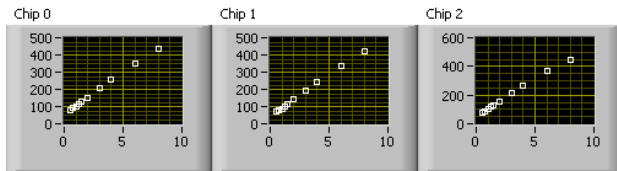
S-curve



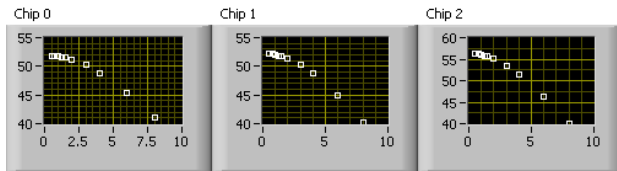
threshold



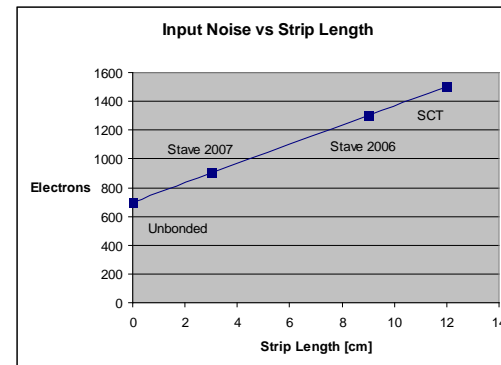
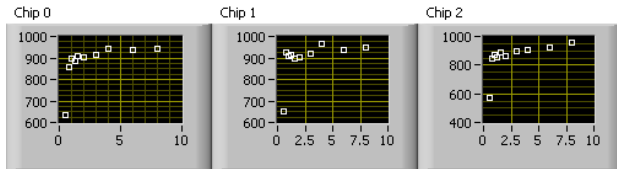
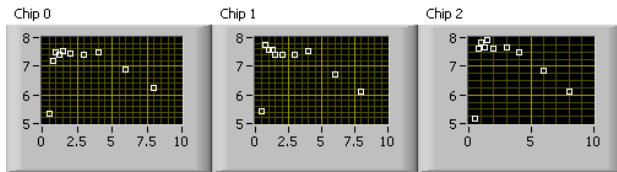
VT50



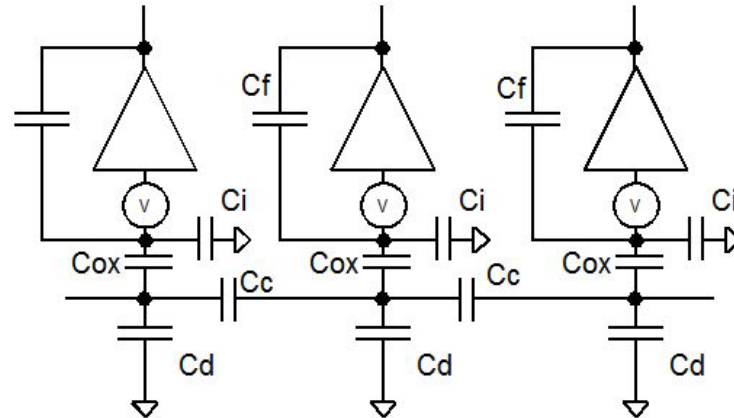
gain



noise



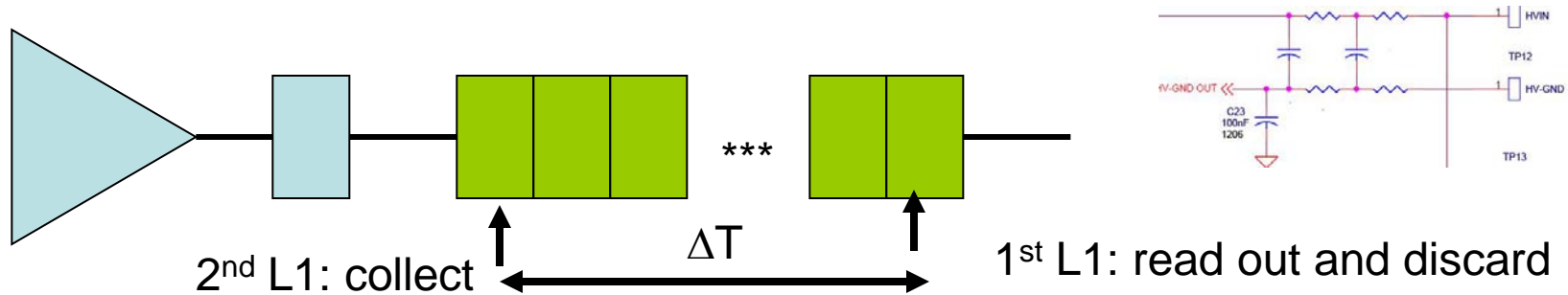
Noise and Correlations



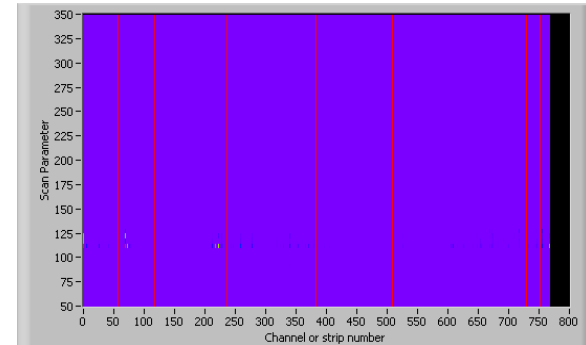
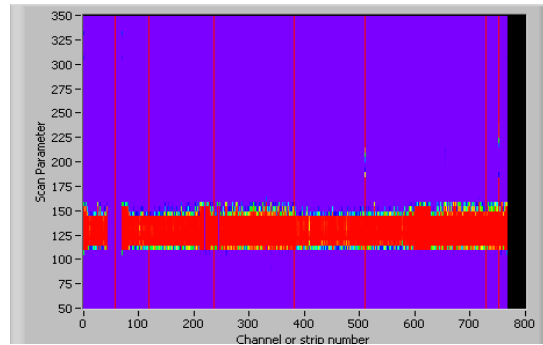
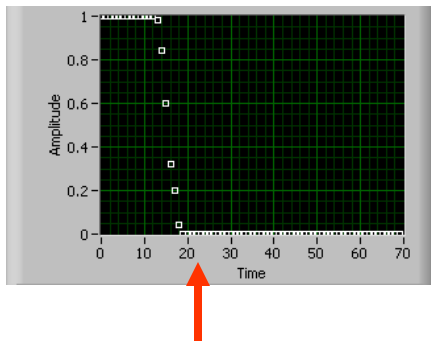
For analog and digital systems

- Interstrip capacitance C_c dominates
- Total noise charge at an input N is due to that channel and a coupled contribution from $N-1, +1$, with a **negative** correlation
- A useful statistic is to histogram the instantaneous difference between channels separated by $J= 1,2,3,....$ strips ($/\sqrt{2}$)
- For $J \sim 5$ the mean approaches 1 in a system with no extra noise

Noise Interference



- Deadtimeless systems: simultaneous integration & readout
- Test: Consider a 132 cell pipeline, collected at 40 MHz, issue a trigger, readout, discard, wait 132 cycles, issue a trigger, readout
- Operate at onset of low occupancy
- Vary grounding, shielding, and filtering configuration



High Density Packaging

- Electronic packaging is often the only “reducible” part of the detector mass
- Advances in packaging have allowed us to integrate increasing complexity into denser footprints
- Maintain necessary thermal performance with minimized mass and high reliability
- Key technologies are based upon commercial processes
- Avoid the homemade syndrome

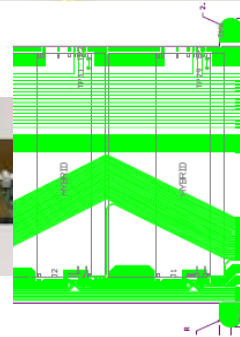
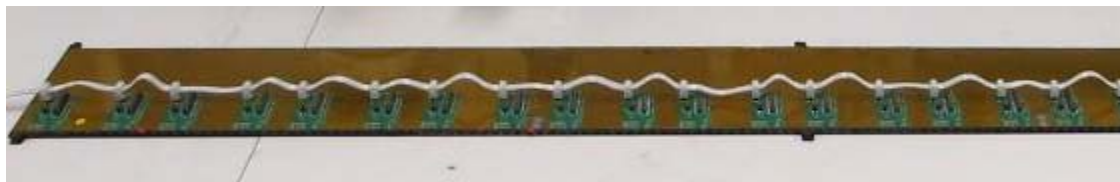
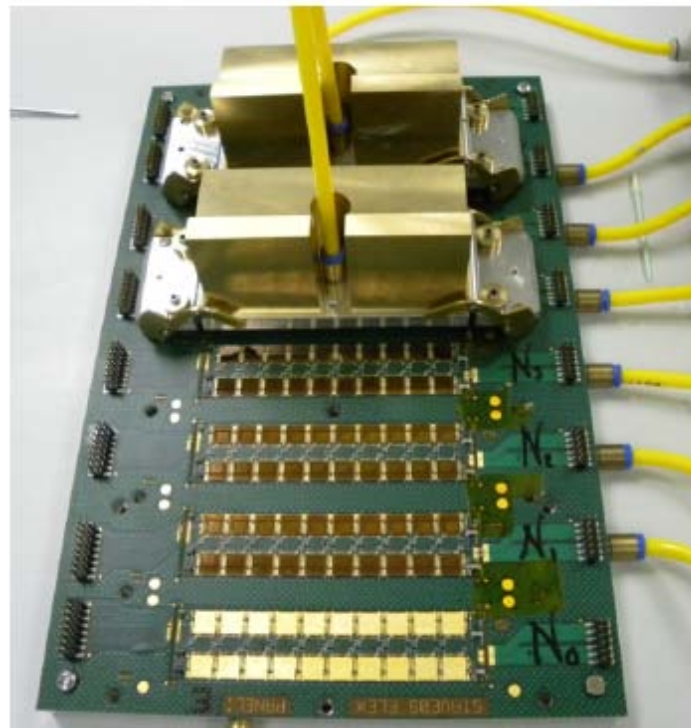
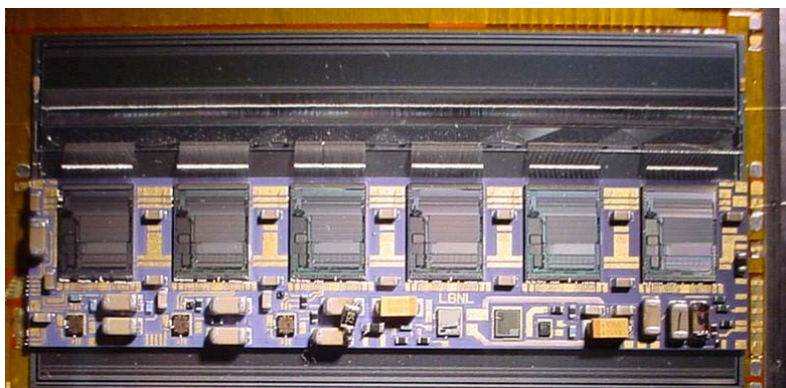
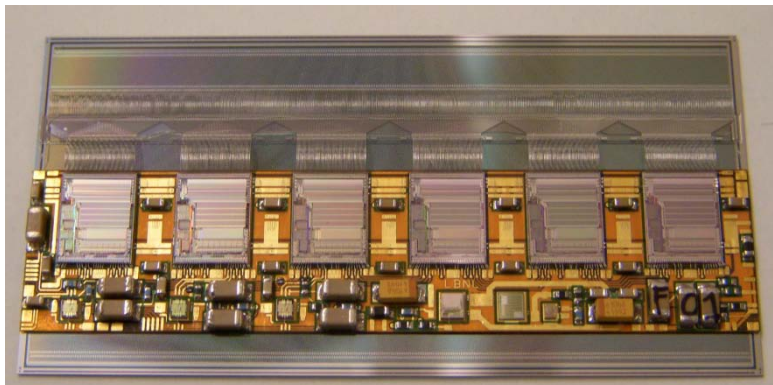
Key Technologies

- Surface mount technology (SMT), pick & place
- Flexible circuits
- High density multilayer PCB and flex
 - Trace widths/space approaching 25 μm
- Large area flexible circuits > 1 meter length
- Chip on Board (COB) and Chip on Flex
- Thin film on ceramic, glass, and polyimide
- Thick film on ceramic, BeO and AlN substrates
- Lamination onto high-TC carbon substrates

Electrical Materials

material	Resistivity ($\mu\Omega\text{cm}$)	dielectric constant	Xo(cm)	Thermal C. (W/m $^\circ$ K)	CTE (ppm)
Silicon		11.9	9.37	149	2.6
Aluminum	2.65		8.9	237	23.9
Copper	1.67		1.43	398	16.6
Gold	2.44		0.335	297	14.2
Carbon	1375		19.32	varies	
Kapton		3.4	28.4	0.2	~20
SiO ₂		3.9	10	1.1	
BeO	10 ²¹	6.6	14.4	230	8.3
AlN	>10 ²⁰	9	8.4	170	4.3
Al ₂ O ₃	>10 ²⁰	9.0	7.55	24	7.2
G-10		4.7	19.4	0.2	

Technology examples



June 10, 2011

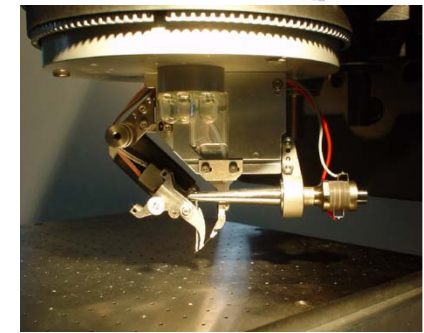
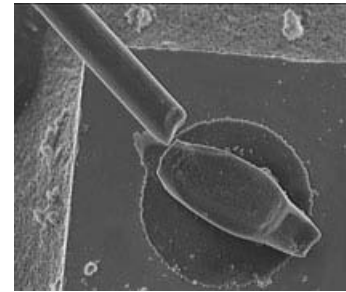
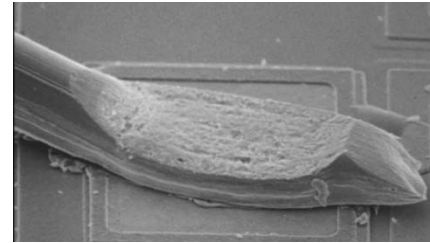
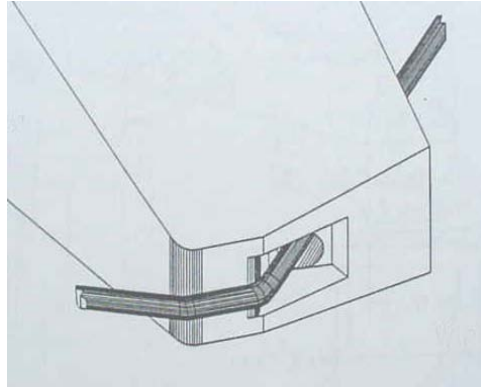
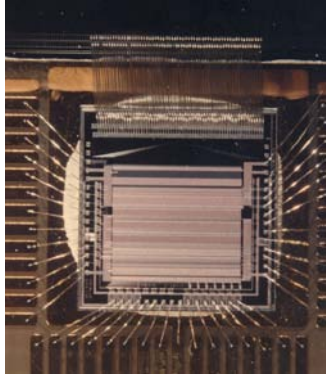
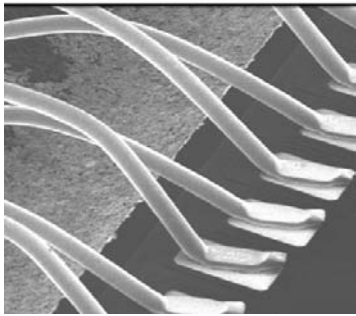
Silicon Detectors TIPP 2011

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Emerging Interconnects

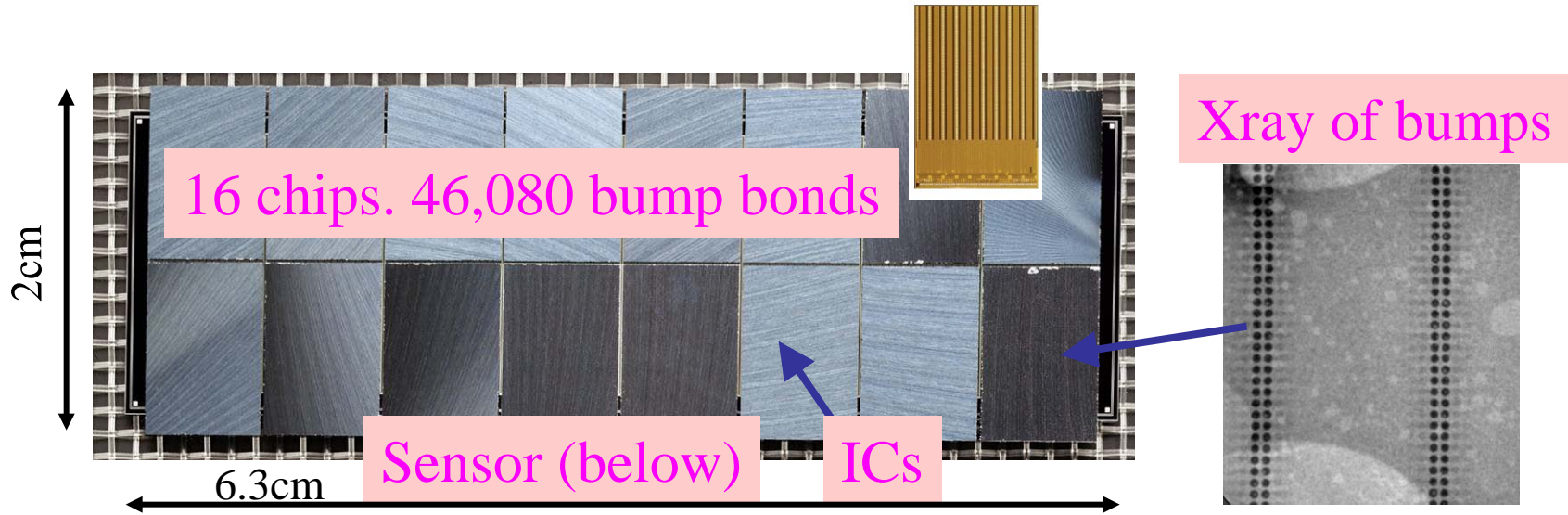
Attribute	Standard (Epoxy Glass or Polyimide)	HDI: Dense (Particle Filled Epoxy)	HDI: LCP (liquid crystal polymer)	HDI: PTFE (PTFE)
Line width	75 microns	25 microns	37.5 microns	25 microns
Line space	75 microns	25 microns	37.5 microns	33 microns
Via type	mechanical	laser	Laser	laser
Via diameter	200 microns	50 microns	50 microns	50 microns
Stacked vias	Build up only	Build up only	In 2010	In 2010
Capture pad diameter	400 microns	100 microns	110 microns	110 microns
Surface finish	E-less Ni / I Au, ENIG	Same	Same	Same
Solder mask	yes	yes	yes	no
Thickness	<1mm	0.4 - .7mm	0.5mm	0.5mm
Layers	10	12	4, 6 in 1 st article	11

Wirebonding



- Mainstay of microelectronic interconnection
- Typically uses 25 μm Al or Au wire, ultrasonic welding process
- Requires particular control of materials, cleanliness, and process
- Automated (5 bonds/sec) machines are commercially available and in widespread use in the HEP and related communities
- 75 μm pitch is achievable with good process control, a typical HEP “module” might contain ~ 5000 bonds

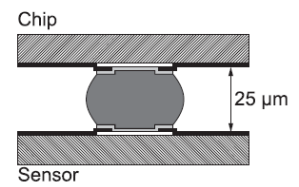
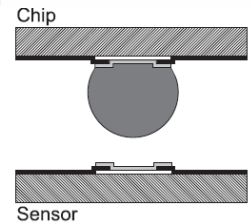
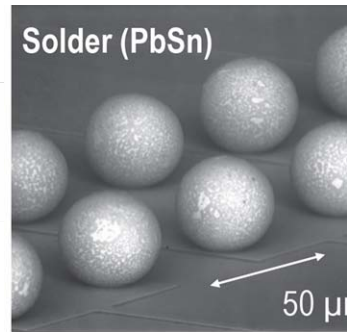
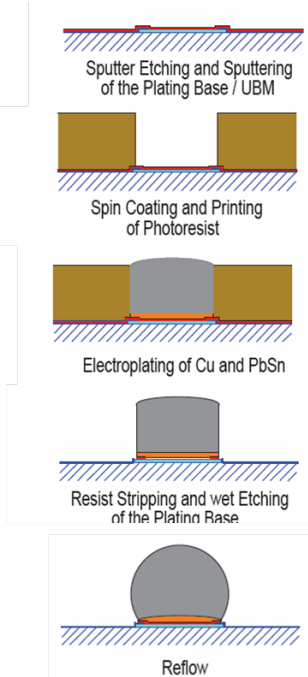
Bump or Flip Chip Bonding



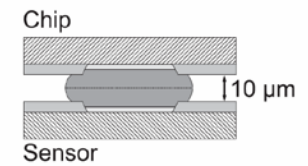
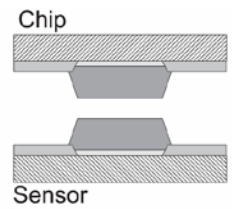
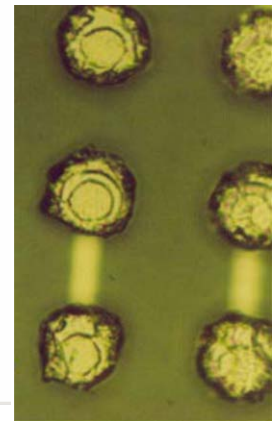
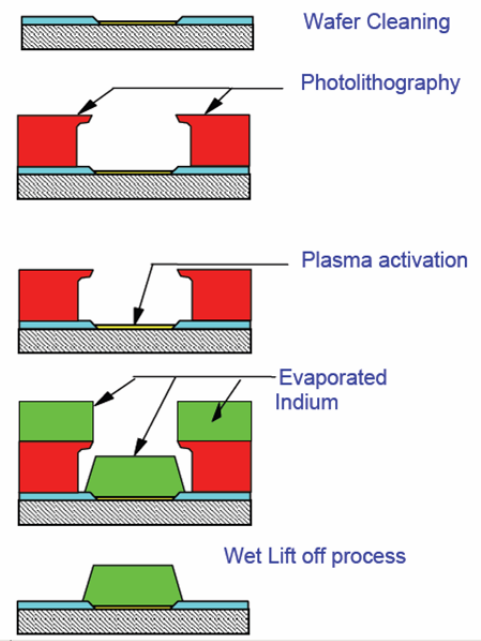
- Wirebonding is impractical for large 2D arrays
- ATLAS pixel cell is $50 \times 250 \mu\text{m}$
- At this density FE interconnect is made with a conductive “bump”
- This is an industrial process and requires expensive technology, therefore has not become “in-house”

Bump Bonding Processes

SOLDER BUMPING



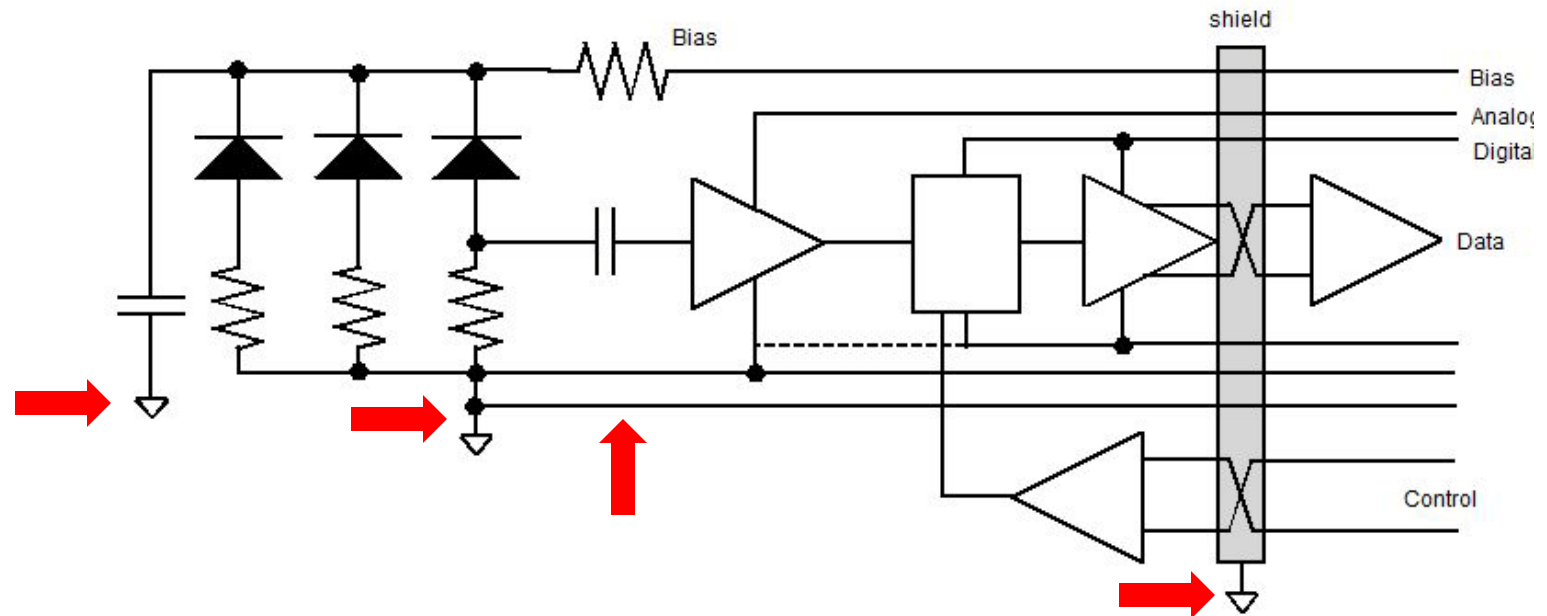
INDIUM BUMPING



Issues for Large Systems

- Grounding and Shielding
- Powering
- Bias
- Control and Data Transmission
 - Cables
 - AC Coupling
- Monitoring and fast/slow control
 - Interlocks, safety

Grounding and Shielding



- Front end is a mixed analog and digital system
- Data transmission and control introduces long range signals with drive
- Sensor has a capacitance of pf's, so mV of noise is an issue
- Rules apply but also thorough diagnostics before installation
- Trace down and control coupling paths and impedances
- Control locations where grounds are connected

Power Distribution

- Conventional wisdom stated that each “module” of tracker should be serviced independently
 - isolate single point failures
 - Avoid electrical interference and ensure low noise
- For large trackers this has led to a cabling (mass and access) limitation
- Future trackers may be larger by x5 or more
- An active R&D effort in alternative powering approaches



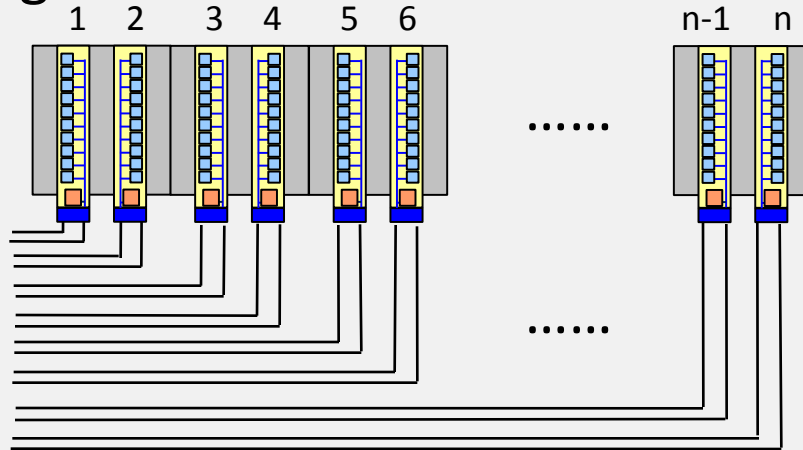
Powering Alternatives

- Independent Power
 - One cable with current I_{mod} , and V_{mod} for each module
- Serial Power
 - Reuse current, connecting N modules in series, one cable carries current I_{mod} , $V=NV_{\text{mod}}$
 - Practical implementation utilizes shunt regulation and active bypass protection, requires extensive AC coupling of control and data
- DC-DC Conversion
 - Step down voltage by factor R at each module, one cable carries $I_{\text{tot}}=Ni_{\text{mod}}/R$ at $V=Rv_{\text{mod}}$
 - Practical implementation utilizes switching converters either charge pumps or inductive
- These alternative approaches can be very efficient compared to linear regulation

Power architectures

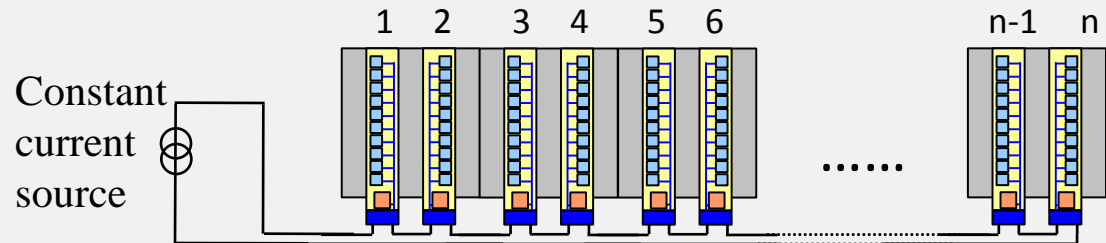
- Independent powering

Hybrid current = I
Number of hybrids = n
Total current = nI
Power lines = n



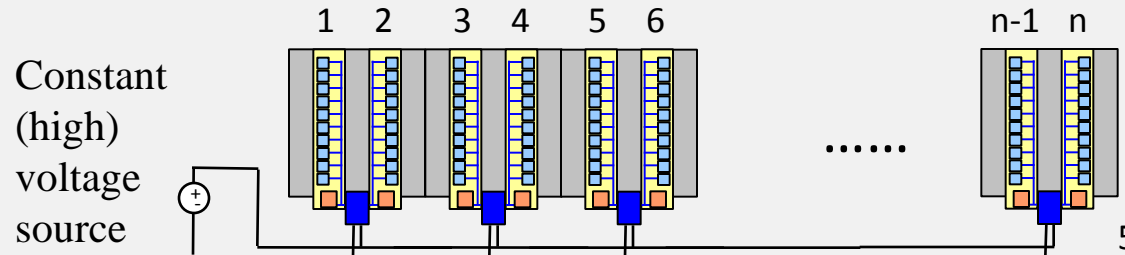
- Serial powering

Hybrid current = I
Number of hybrids = n
Total current = I
Power lines = 1

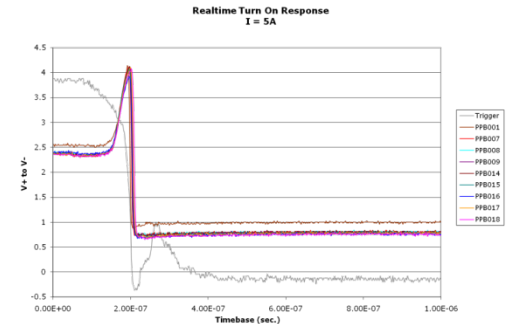
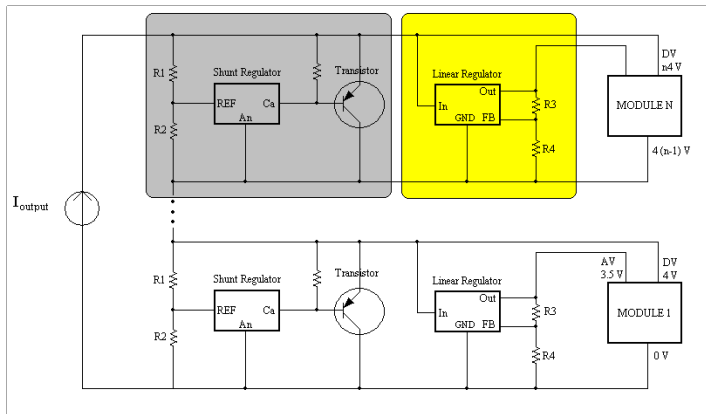


- DC-DC powering

Hybrid current = I
Number of hybrids = n
Total current = $n(I/r)$
Power lines = 1



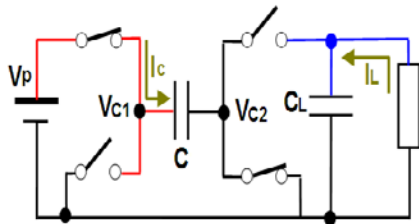
Serial Power



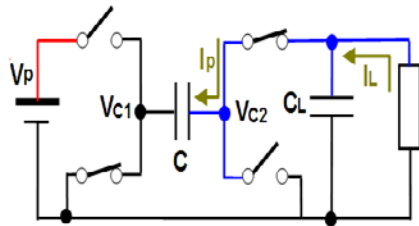
- Many variations on this have been studied in the R&D efforts, custom ASICs exist and are in development
- Large systems (>30 drops) have been operated with AC coupling
- Stable, low noise behavior obtained
- Failure recovery and control circuits have already been tested
- Most efficient when current per module is uniform

DC-DC Conversion

1. Charge

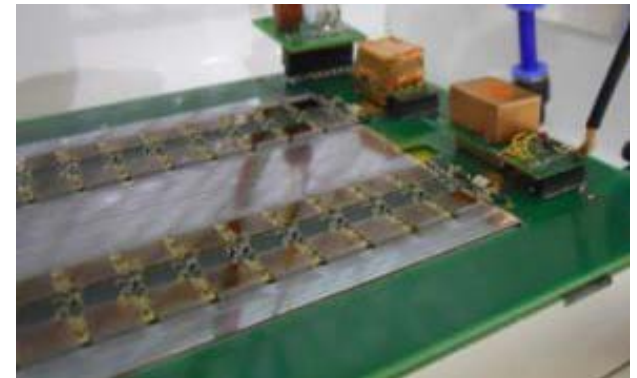
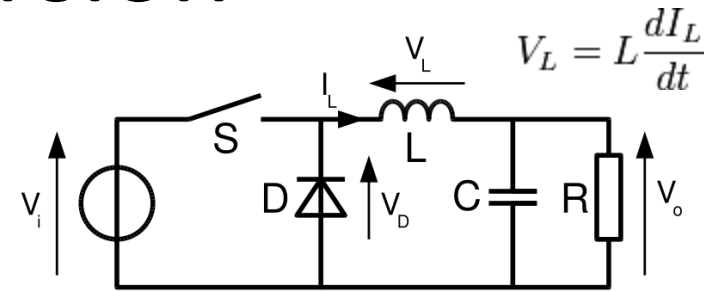


2. Pumping

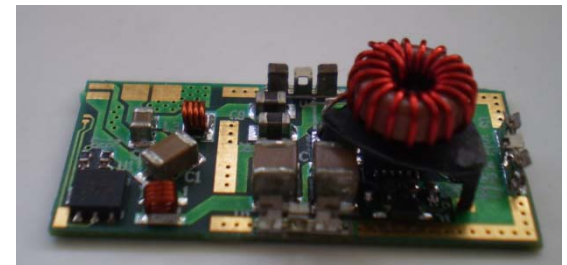


Charge pump

- DC-DC converters require high frequency clocks
- Realistic circuits have been operated in close proximity to sensor/modules with excellent noise performance, when adequately shielded.
- Main concern is the mass and size of required components

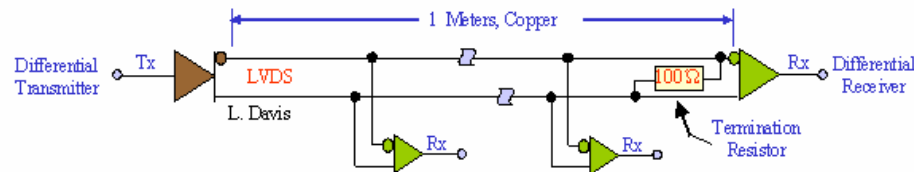


Buck converter with custom air core inductor



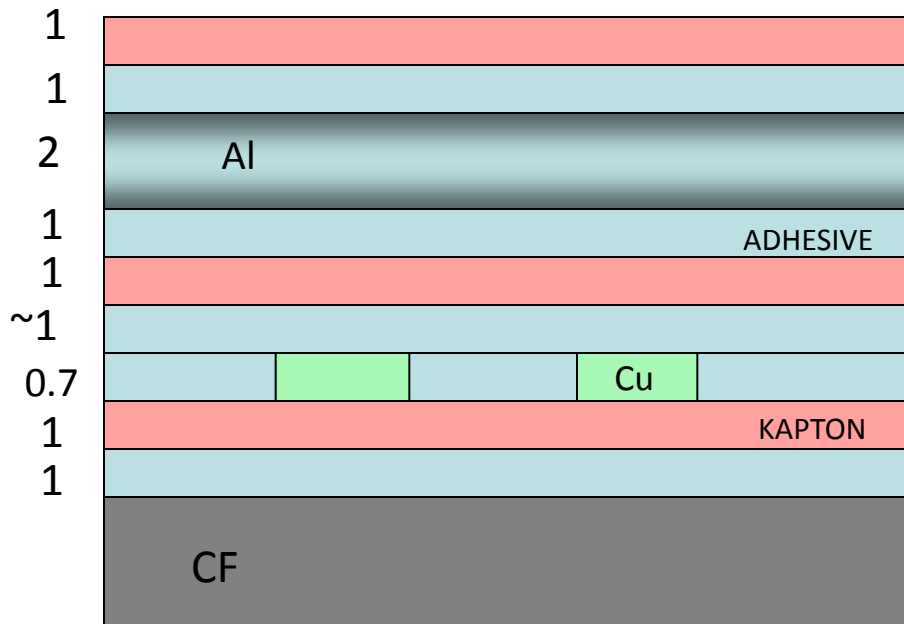
Control and Data Transmission

- Efficient low mass cabling requires a high degree of multiplexing
- Low power differential protocols – LVDS
- Clock and command distribution looks like a “multidrop” system (MLVDS and other variants)
- Bandwidth: fast clocks on copper 40-160 MHz or greater
- Transmission line structures in large flex circuits
- Much use of optical transmission, reliability concerns remain....
- Serially powered systems require AC coupling
- **Much of this looks like a departure from standard practice**



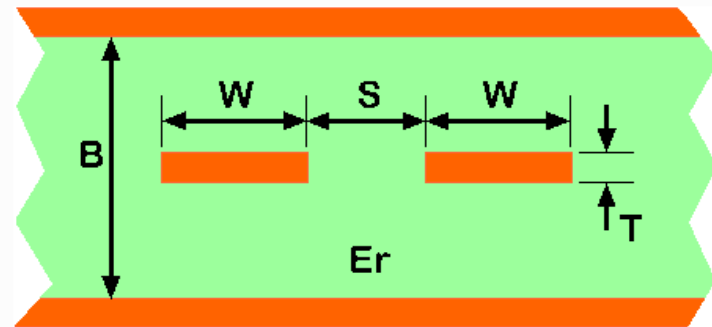
Bus Cable Geometry and Impedance

Materials: Al foil 2mil, Dupont LF0100, Shinetsu CA333 2 mils, Cu 18 um, Kapton 1 mil, Adhesive



>>Matches measured impedance

Differential Stripline Impedance Calculator



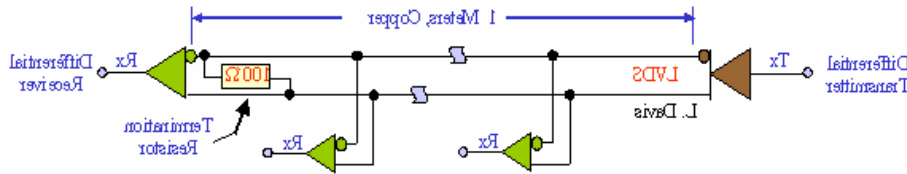
Notes:

- 1) Calculation assumes traces are centered vertically.
- 2) $S/T > 5.0$

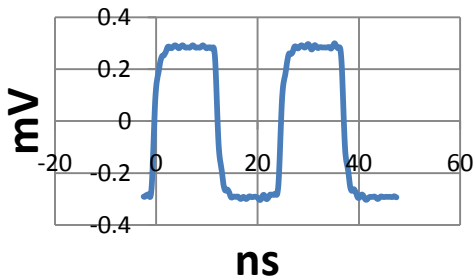
Enter dimensions:

Trace width (W)	<input type="text" value="3"/>	mils
Trace thickness (T)	<input type="text" value=".7"/>	mils
Trace spacing (S)	<input type="text" value="4"/>	mils
Distance between planes (B)	<input type="text" value="5"/>	mils
Relative Dielectric constant (Er)	<input type="text" value="3.6"/>	
<input type="button" value="Compute Z"/>		
Differential Trace Impedance	<input type="text" value="72.9"/>	ohms

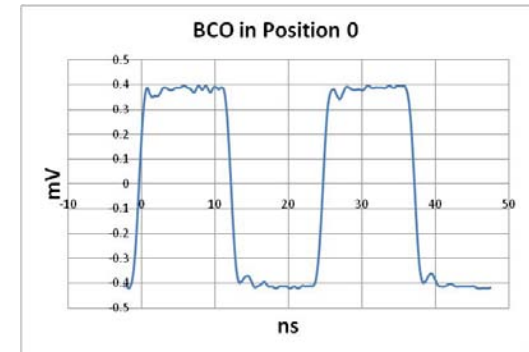
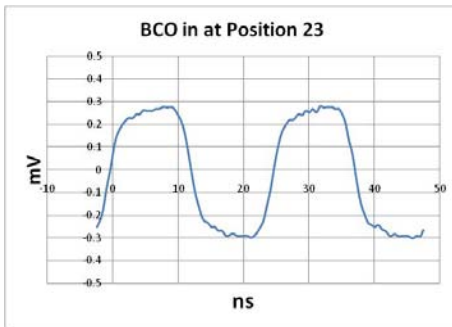
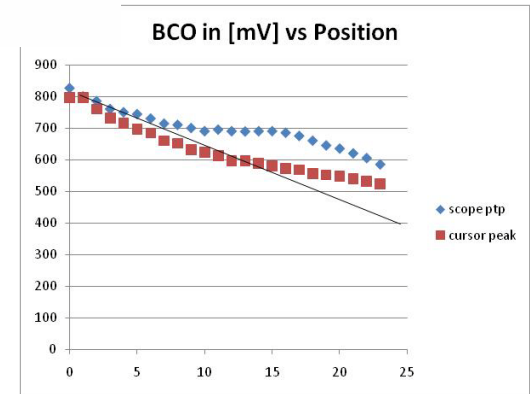
Signal Dispersion in a Large AC Coupled Multidrop System



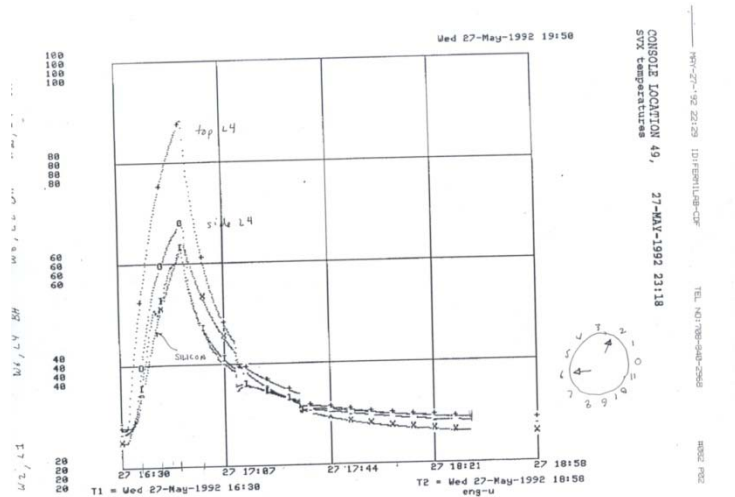
BCO out Position 2



AC coupled LVDS receiver, driver, controller



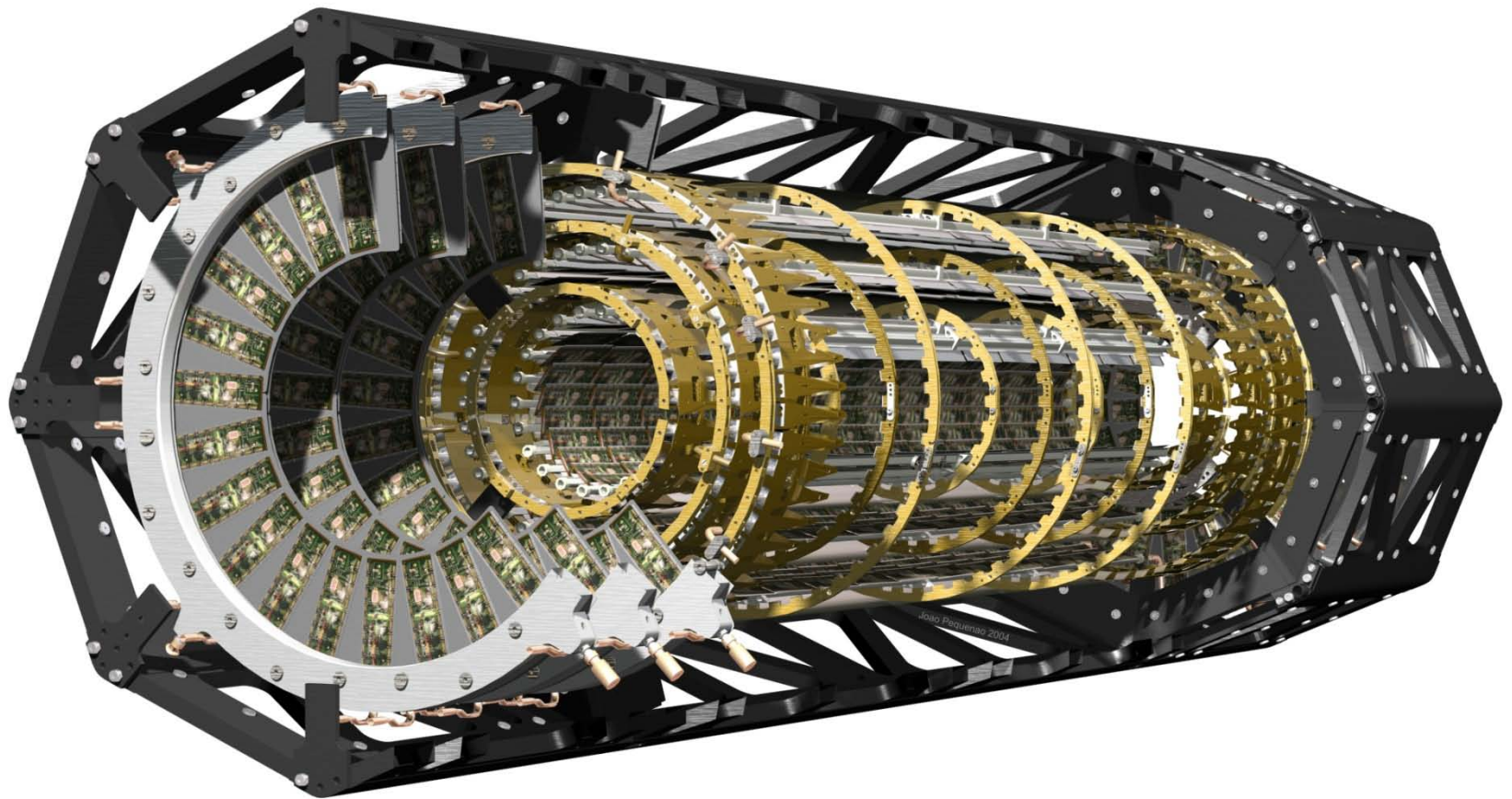
Monitoring, Fast/Slow Control



- Temperature, humidity, flow, radiation
- Voltages and currents
- Local on module monitoring, dedicated ASICs: fast
- Dedicated nearby process controllers
- Online logging
- Machine interlocks
- Service interlocks

Mechanical Aspects

- Precision tracker requires support structure which is stable and low in mass (X_o).
- There are two key materials/classes which come into consideration here
 - Beryllium: structural grade metal, low Z, metallic CTE, expensive, and hazardous to machine, probably impractical for very large structures
 - Carbon composites: tremendously flexible class of materials, reasonable X_o , good thermal properties, variable CTE



June 10, 2011

Silicon Detectors TIPP 2011

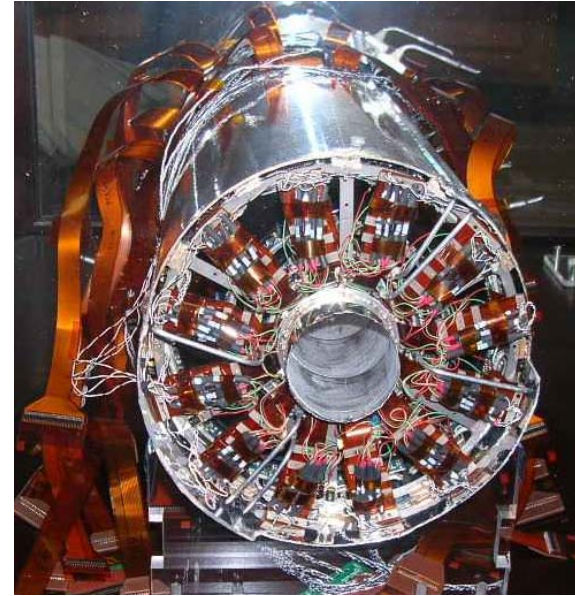
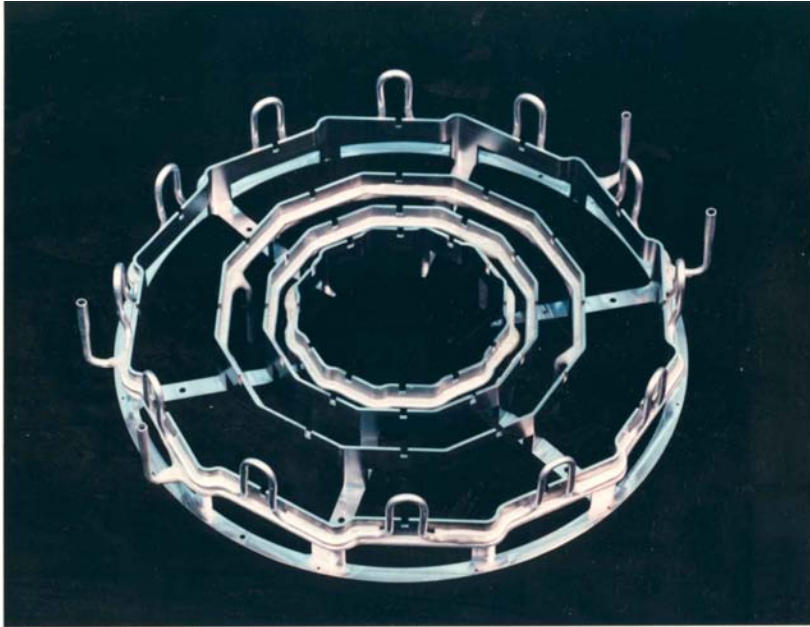
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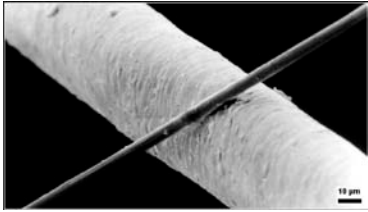
Mechanical Grade Materials

material	Density (gr/cc)	Xo (cm)	Young's Mod (GPa)	CTE (ppm/°K)	TC: W/m°K
Silicon	2.33	9.37	130-185	2.8-7.3	149
Beryllium	1.85	35.27	255	12.4	201
Aluminum	2.70	8.9	69	23.9	237
Stainless Steel	7.9	1.76	193	11.7	95
Titanium	4.54	3.56	116	8.5	
Carbon	2.21	19.32		0.6-4.3	
Carbon fiber frac 70-40%	~1.7-2	23-27	180-125 along fibre	Varies with layup	10's – several 100's
CF: K13D2U	2.2		135 Msi		800
POCO Graphite foam	0.5	~100	low	0.7	45/135 in/out
Boron Nitride	2.25	20.8		<1	250-300

Beryllium



- Bulkhead which supports vertex detector layers, radius ~ 7 cm
- Costly, precision machined component
- Modest temperature excursion from RT



Carbon Composites

- Carbon fiber “sheets” consist of filaments or woven layers impregnated with epoxy.
- By arranging layers in various “lay-ups” and configurations, a great variety of components can be created with enhanced mechanical and thermal properties.
- Other advanced carbon based materials can be combined in structure as well

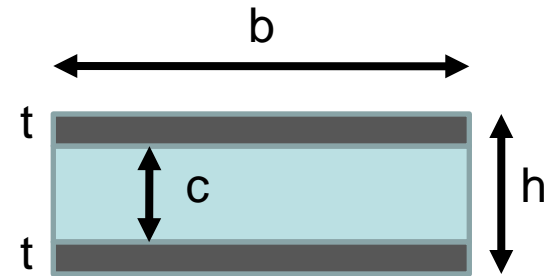
Basic Issues

- Sag in a uniform composite beam

$$\delta(\text{bending}) = \frac{wL^4}{384EI}$$

$$I = 2 \frac{bt^3}{12} + 2bt \left(\frac{c+t}{2} \right)^2 = \frac{bt^3}{6} + \frac{btc^2 + 2bct^2 + bt^3}{2} \approx btc \left(\frac{c}{2} + t \right)$$

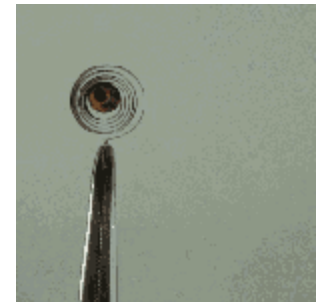
$$\delta = \frac{wL^4}{384Ebtc \left(\frac{c}{2} + t \right)}$$



- Stress in the “bi-material” strip

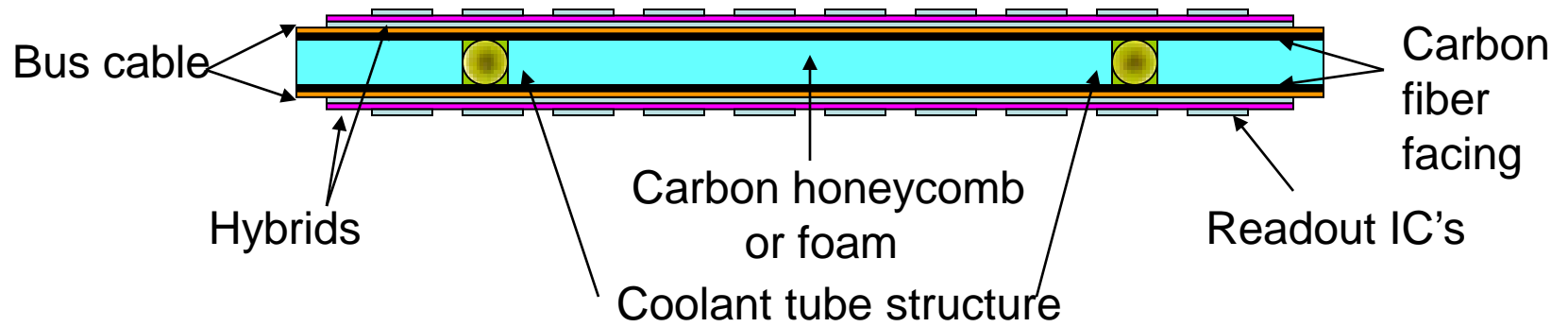
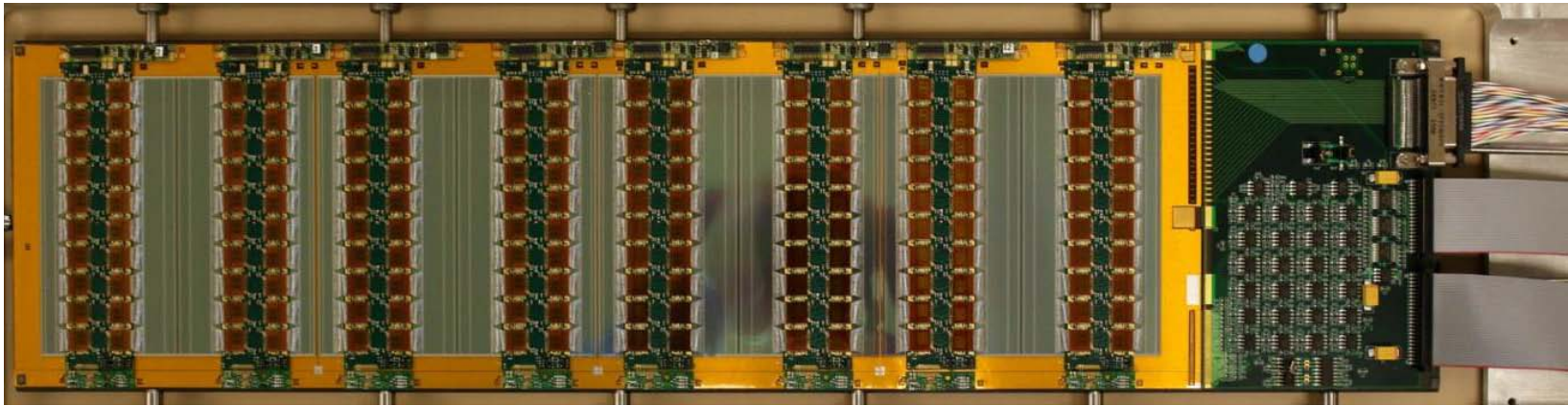
$$\kappa = \frac{6E_1E_2(h_1 + h_2)h_1h_2\epsilon}{E_1^2h_1^4 + 4E_1E_2h_1^3h_2 + 6E_1E_2h_1^2h_2^2 + 4E_1E_2h_2^3h_1 + E_2^2h_2^4}$$

$$\epsilon = (\alpha_1 - \alpha_2)\Delta T$$



Example: Carbon Fiber Beam

- Structure consists of 2 CF facings laminated on either side of a “soft” core w. embedded metal cooling lines: sandwich beam
- Symmetry keeps the structure flat
- Facing contains multiple sheets in order to tune mechanical and/or thermal properties
- Core may have enhanced thermal properties to improve cooling efficiency
- Subsequent (or co-)lamination of electrical circuitry and sensors
- Issue of stress between carbon and other unlike materials as composite is cooled from lamination temperature to RT and to operating temperature



Composites Facility

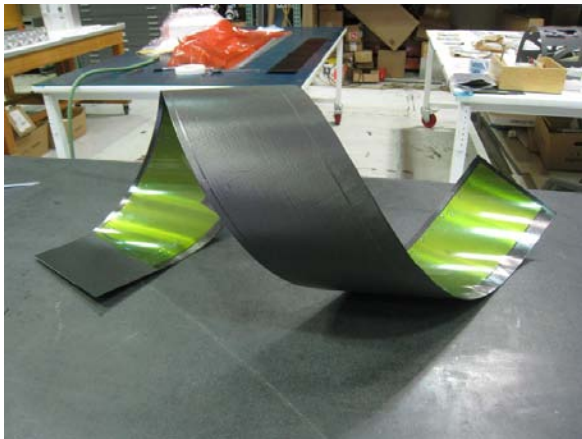
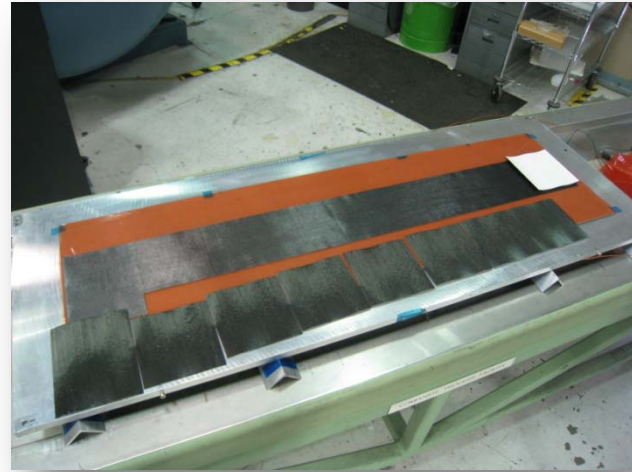
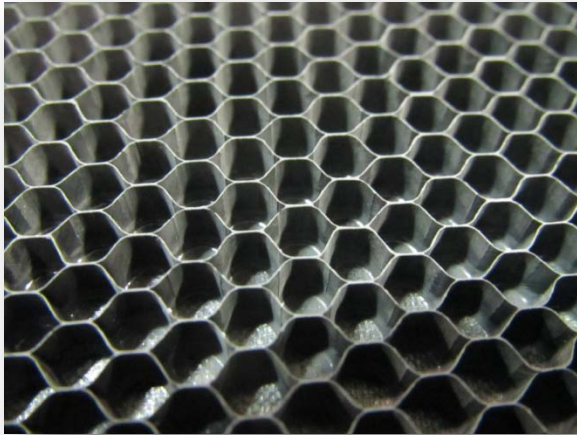


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Silicon Detectors TIPP 2011

Carl Haber LBNL

Some assembly steps



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Prototype Core Construction

BNL, Yale
LBNL
RAL
Oxford

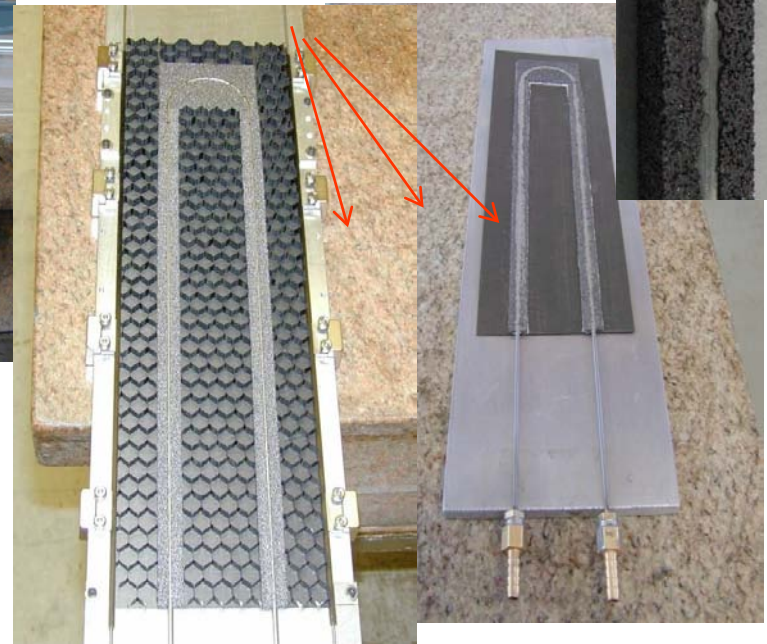


Honeycomb core

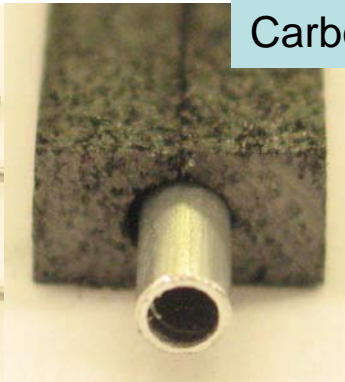
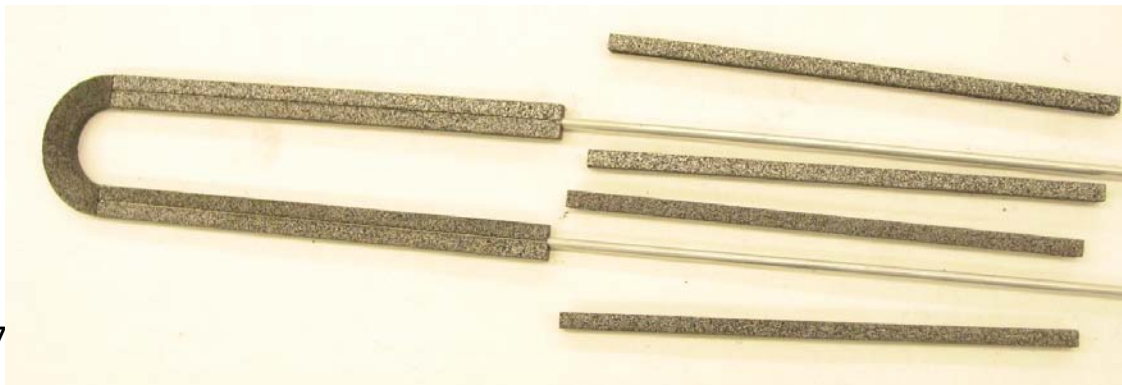
Carbon poco-foam



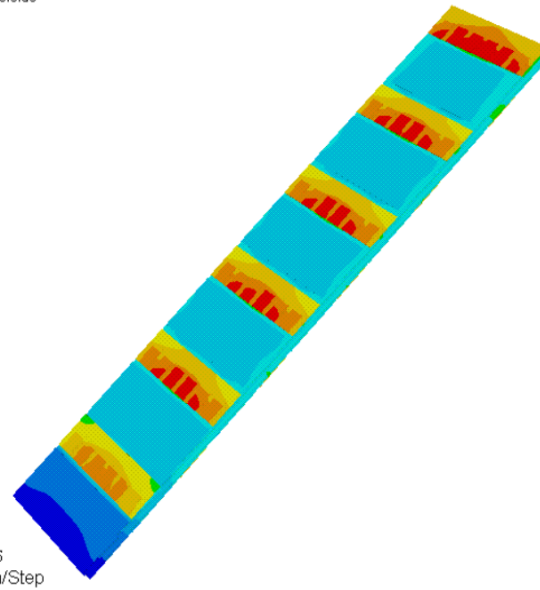
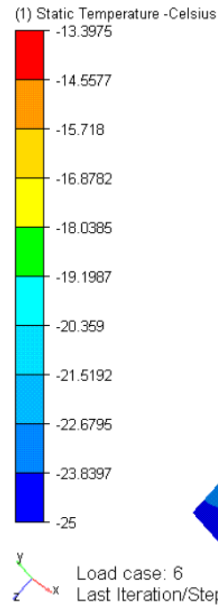
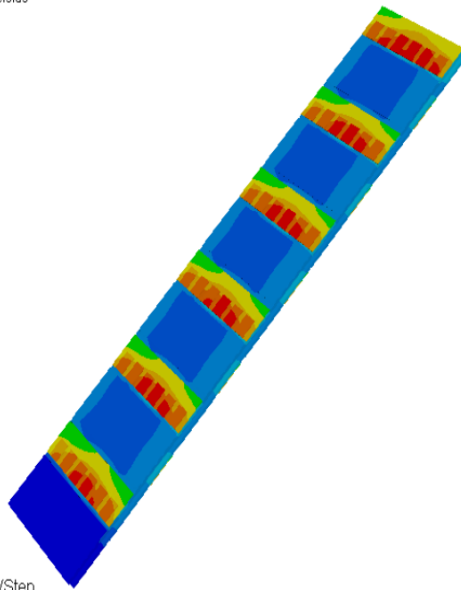
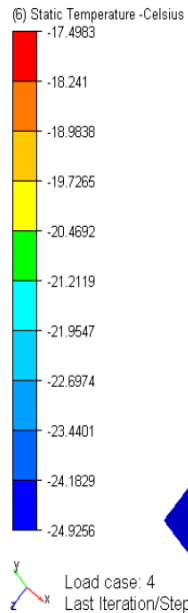
Facing



Carbon poco-foam

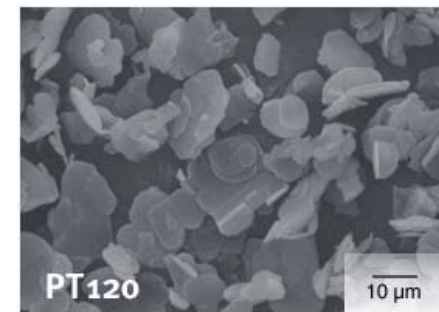
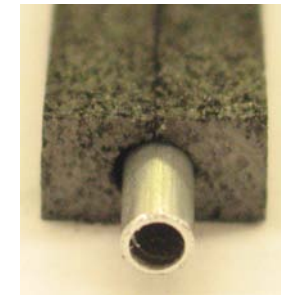
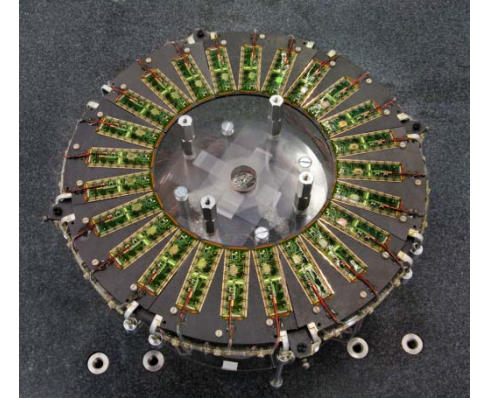
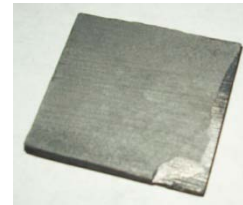


Example thermal performance for various material selections, calculated



Advanced Materials

- Processed carbons
 - Carbon-Carbon: CF reinforced C by pyrolysis
 - Pyrolytic Graphite: $TC > 1000$
- Graphite Foams: of varying density, conductivity
 - Pocofoam
 - Allcomp foam
- Boron Nitride: fillers
 - Varying particle size, shape
- Thermal adhesives: rigid, compliant, radiation hard
- Silicon Carbide: solid, foam, also an electrical material



Metrology

- Precision, non-contact mechanical measurement
- R&D, construction, in-situ alignment & monitoring
- Optical and touch probe CMM's
- ESPI/TV Holography
- Frequency Scan Interferometer (FSI)
- Laser rangefinding displacement sensor
- Confocal probe

Metrology

Technology	Application	Resolution	Interface	Ease/Speed
CMM-touch	Large objects	x/y/z ~ μm 's	commercial	Teach mode
CMM-optical	In plane location Small heights	x/y/z ~ μm 's	commercial	same
ESPI	Dynamics	x/y/z ~ μm 's	commercial	R&D tool
FSI	In-situ alignment Stability	One axis	custom	System design
Laser Displacement	Flexible heights R&D tests	z ~ μm 's	User defined	User defined, 1 KHz
Confocal Probe	Precision heights Small area	10-100 nm	Limited use commercial or user	User defined, 100 Hz–2 KHz

Metrology: Coordinate Measuring

Good for repetitive, feature driven surveying



Labels for the software interface:

- Title Bar
- Main Menu
- Target and Image Controls
- Image Window
- Toolbox
- Measurement Window
- Model Window or Surface Window
- Digital Readout (DRO) Window
- Illumination Controls
- Target Settings Window
- Tool and Target Icons
- Print/Edit Window (Advanced Part Routine Editor)

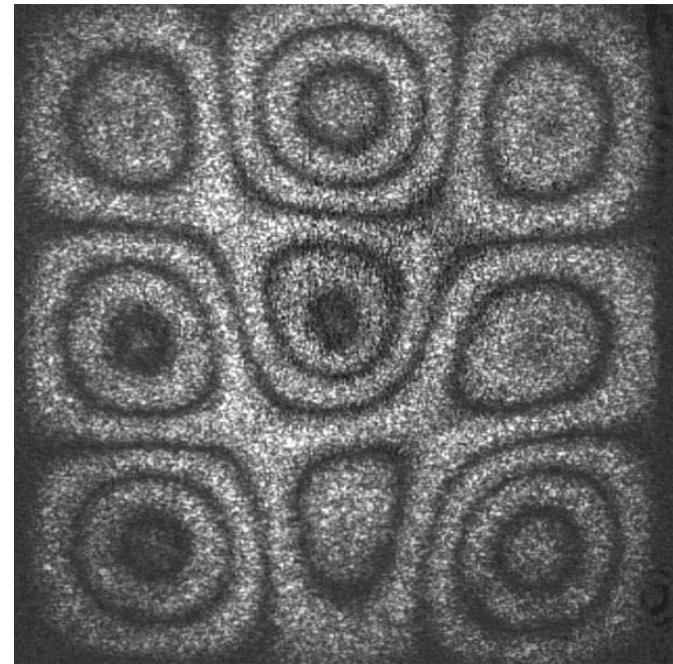
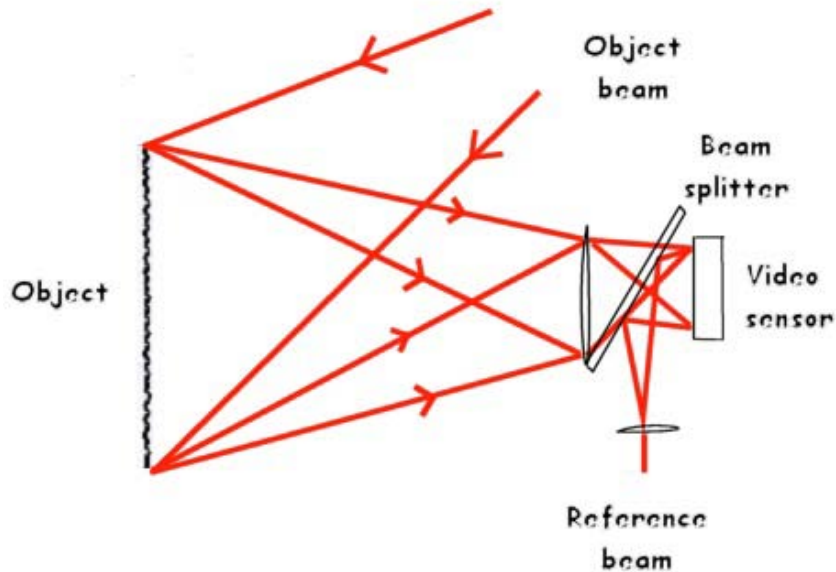
Step	Desc	Default	Program	Construct
1	Start			
2	Acquire			
3	Inspect			
4	Inspect			
5	Inspect			
6	Inspect			
7	Inspect			
8	Inspect			
9	Inspect			
10	Inspect			
11	Inspect			
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99	Inspect			
100	Inspect			



These devices have been used extensively in the development and construction of silicon trackers

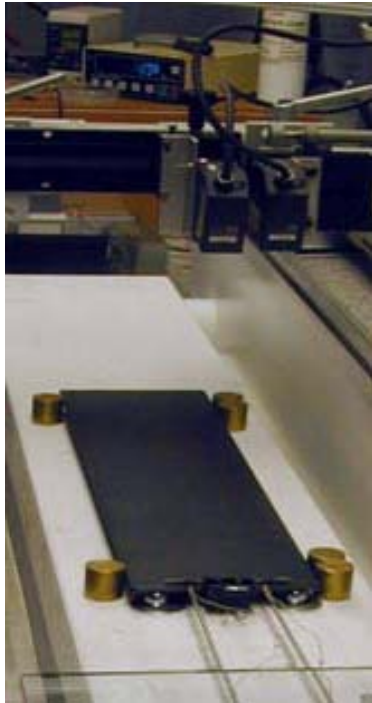
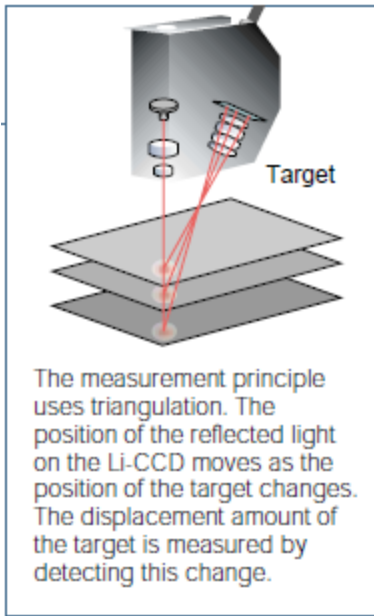
Metrology: ESPI

Electronic Speckle Pattern Interferometry
Also called TV Holography

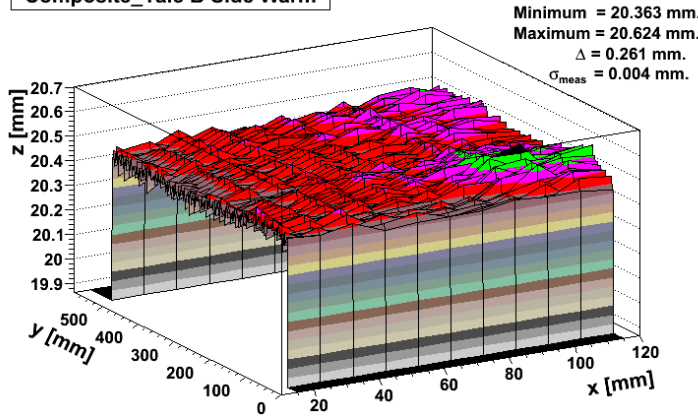


Modes of a clamped metal plate

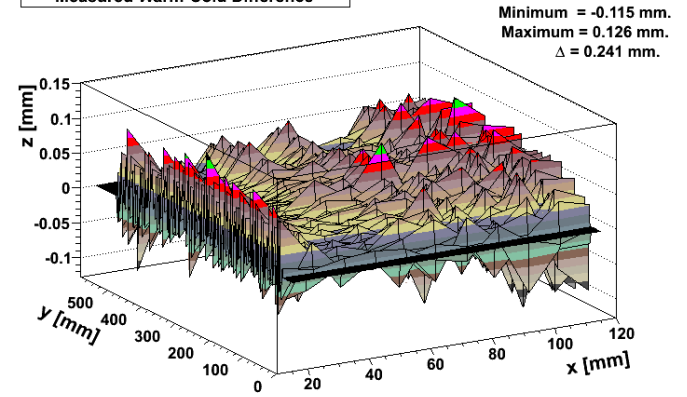
Metrology: Laser Displacement



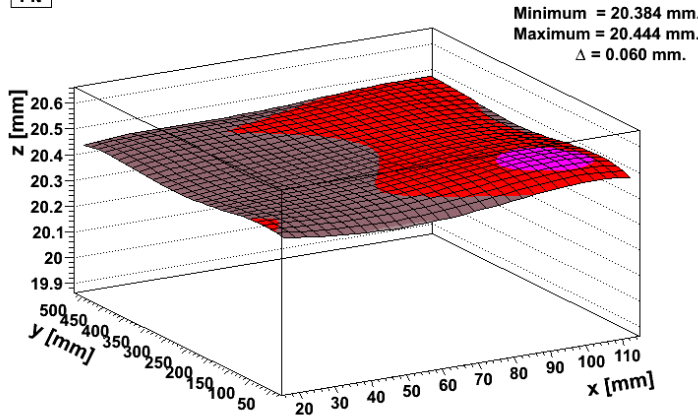
Composite_Yale B Side Warm



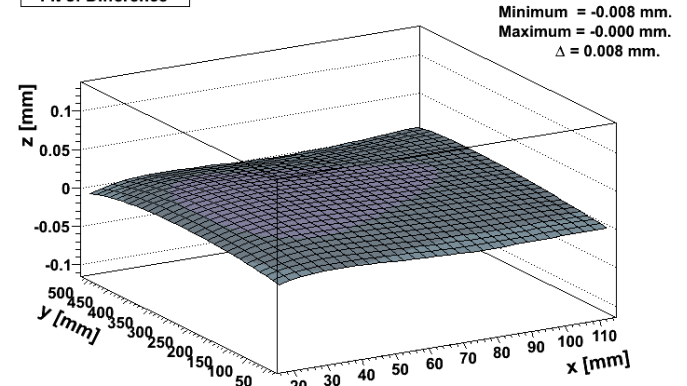
Measured Warm-Cold Difference



Fit



Fit of Difference



Warm

Warm – Cold
No significant distortion

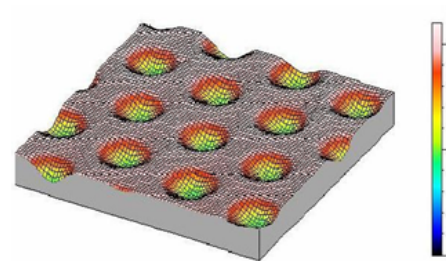
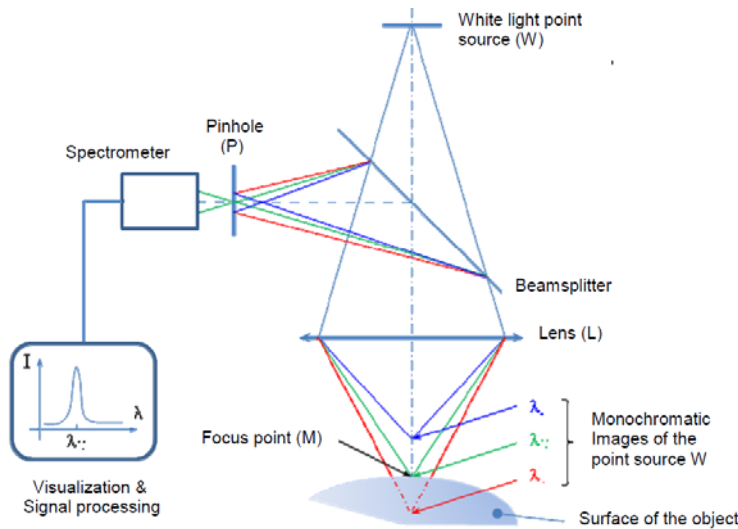
Metrology: Confocal Probe

Application 2 : Mold for microlens array

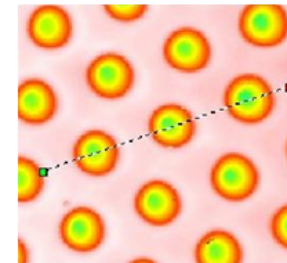
Aims :

- ▶ Quality control of a mold for fabricating microlens arrays,
- ▶ Control of the shape, the spacing and the position of individual array cells,
- ▶ Control of polishing quality.

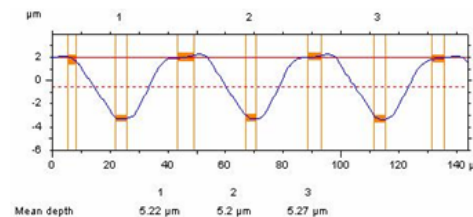
Click on image to improve definition



1 - Mold for microlens array (3D representation)



2 - Mold for microlens array (false color Altitude image)



3 - Profile through the mold along the line shown on image 2

Measurement parameters

Measurement system : MICROMESURE 2 Profilometer
 Controller : OP 20
 Optical pen : CHR 150
 Sample : Mold for microlens array
 Material : Metal
 Picture size : 150 μm x 150 μm
 Measurement pitch : 0.5 μm x 0.5 μm

Radiation Environment

- Primary source are collision products
 - High energy charged particles + neutrals
- Additional component due to “accidents”
- Primary field falls with radius as $\sim r^{-(1-2)}$
- Each interaction yields ~ 7 particles/angular unit: sum crossings and interactions
- Fluence and dose have increased $>10^4$ since mid-80's
 - Near future expect unprecedented dose due to increased luminosity and energy
 - 100 Mrad absorbed energy (units)
 - 10^{15} - 10^{16} particles/cm²
 - Compare to: space (~ 1 MRad), nuclear weapons ($\sim 10^{13}$)

Radiation Effects: Ionizing

- Incident particle interacts with atomic electrons
- Measure in energy absorbed (rads (Si))
- e/h pairs created, recombine or trap
- Transient effect
 - Actual signal formation
 - Single event upset condition in circuits
- Electronics: charge trapping at Si/SiO₂ interface (largely controlled by rad-hard circuit designs or thinner oxides)
- Detectors: surface effects, oxides

Electronics

- For presently operating systems commercial rad-hard CMOS has provided sufficient resistance.
- New chips use commercial deep submicron CMOS
 - Thin oxides provide automatic hardness, verified in test
 - Augment design rules with enclosed gate geometries to block radiation induced leakage paths
- Certain bipolar technologies are also rad-hard (analog)

Radiation Effects: Non-ionizing

- Incident particle interacts with nucleus
 - Displacement damage – permanent or slow to reverse
 - 2nd order effects as defects interact over time
- Depends upon particle type and energy
- Measure in particles/cm²

Radiation Effects: Detectors

- Damage to the periodic lattice creates mid-gap states
- Increased leakage
 - Shot noise
 - Power
 - Heat

$$I_L = \frac{en_i(\sigma v_{thermal} N_T)WA}{2}$$

n_i = intrinsic carrier concentration

σ = recombination cross section

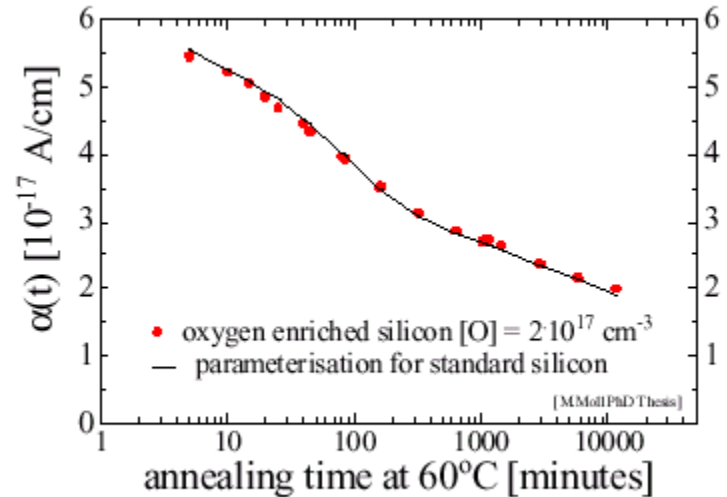
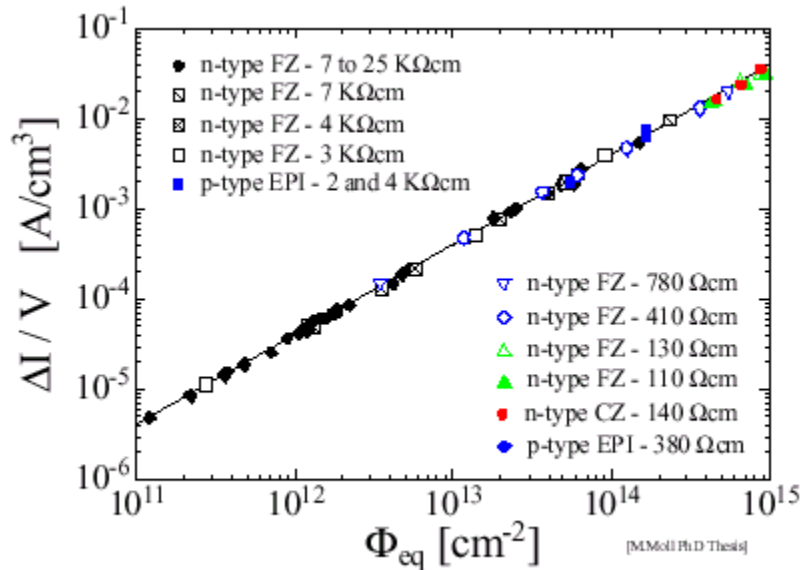
$v_{thermal}$ = carrier thermal velocity

N_T = trap density

A = junction area

$$I_L(T) \propto T^2 e^{-E_a/2kT}$$

Reverse current with fluence and time



$$\Delta I = \alpha V \Phi$$

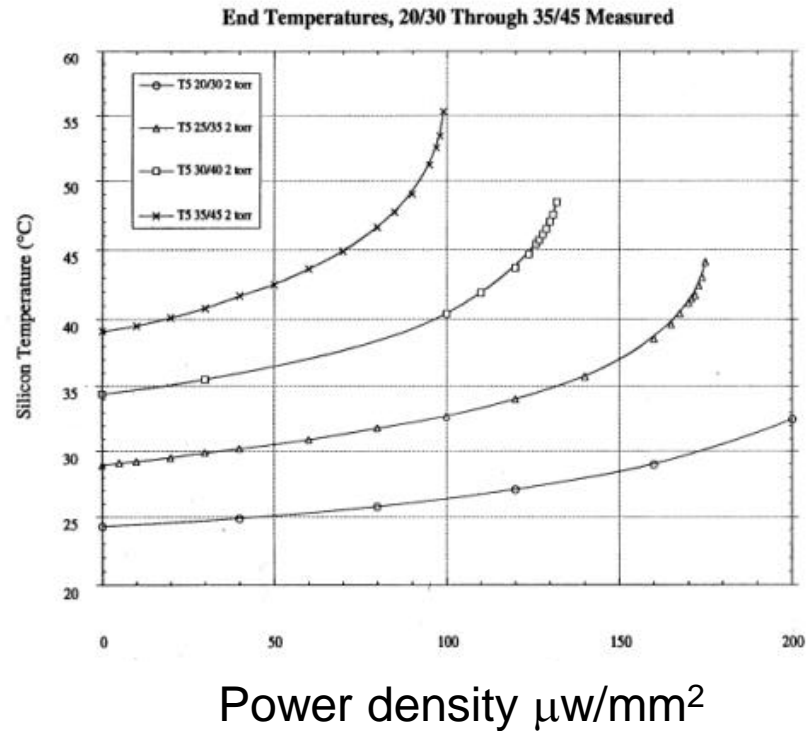
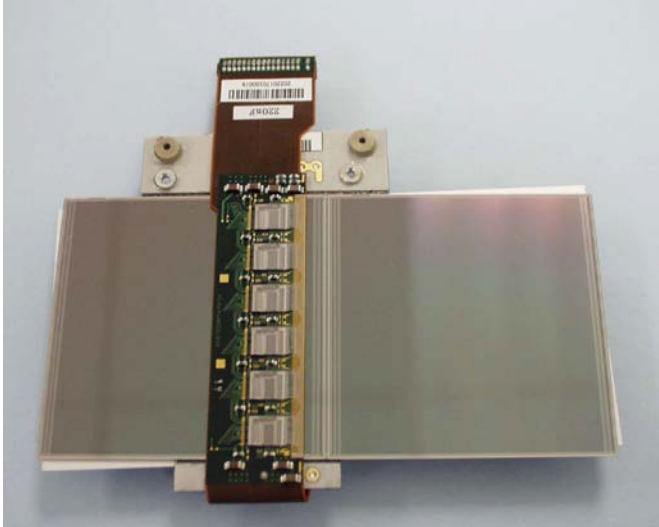
$$\text{Damage constant } \alpha \approx 2 - 3 \times 10^{-17} \frac{\text{Amp}}{\text{cm}}$$

$$\text{Volume } V \approx 2 \times 10^{-3} \text{ cm}^3$$

$$\text{Incident Flux } \Phi \approx 10^{14} - 10^{15} \text{ particles / cm}^2 \text{ @ LHC}$$

$$\Rightarrow \Delta I \approx 2 \mu\text{A} \text{ @ } 0^\circ \text{C} \quad (\text{current doubles every 7 degrees})$$

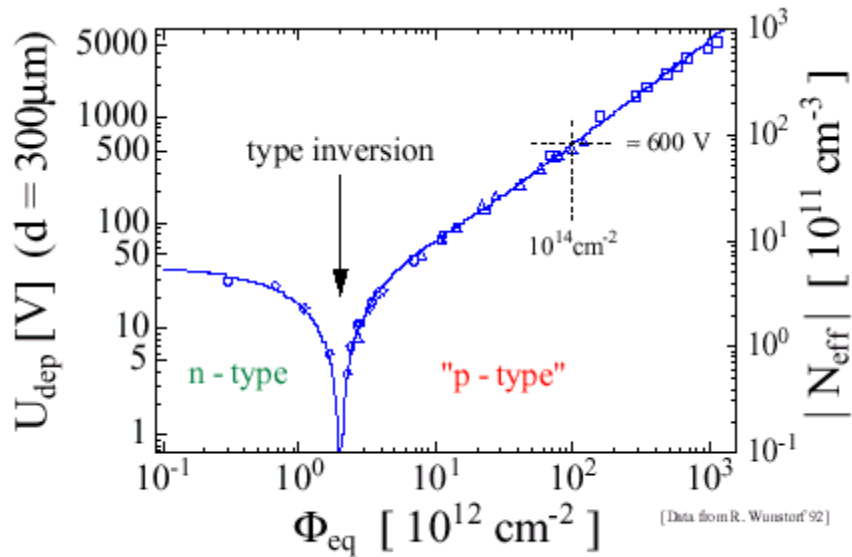
Thermal Run-away



Increased current \rightarrow Power dissipation \rightarrow Increased temperature \rightarrow Increased current

Change in effective acceptor concentration

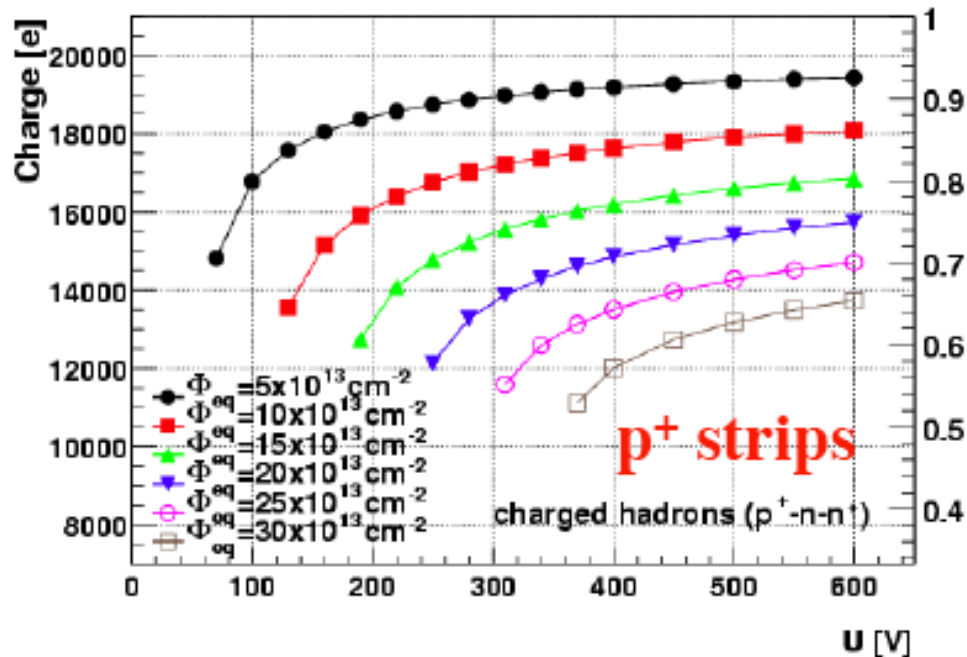
Effective space charge ($N_{\text{eff}} \Rightarrow V_{\text{fd}}$) with fluence and time



- Creation of acceptor states or removal of donor states
 - Effective change of resistivity
 - Type inversion: $n \rightarrow p$
 - Depletion voltage changes in proportion to $|N_{\text{eff}}| \rightarrow$ higher voltage operation required
 - Dramatic time and temperature dependence

Charge Collection

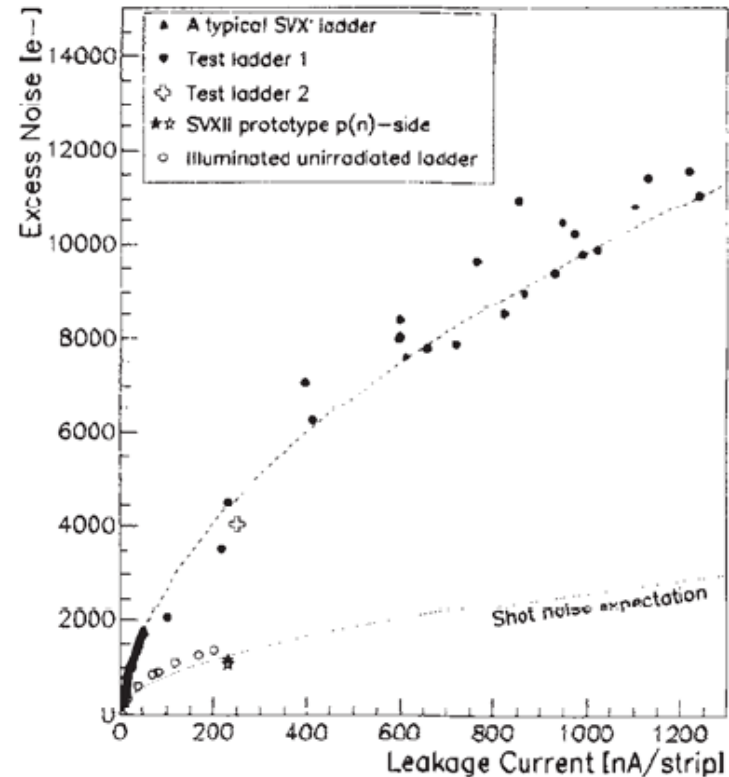
- Reduction in charge collection efficiency (CCE)
 - $N_{e,h}(t) = N_0 \exp(-t/\tau_{trap})$
 - Ratio of collection and charge trapping time constants evolves with fluence



Gregor
Kramberger,
Ljubljana

Excess Radiation Induced Noise

- An effect seen in AC coupled or double sided sensor which are biased with punchthrough structures (this approach has been widely abandoned)
- Functional dependence is like shot noise but magnitude is 4X too large
- Scales linearly with integration time
- Only induced by heavy particle flux
- Actual mechanism is not understood but phenomenology is consistent



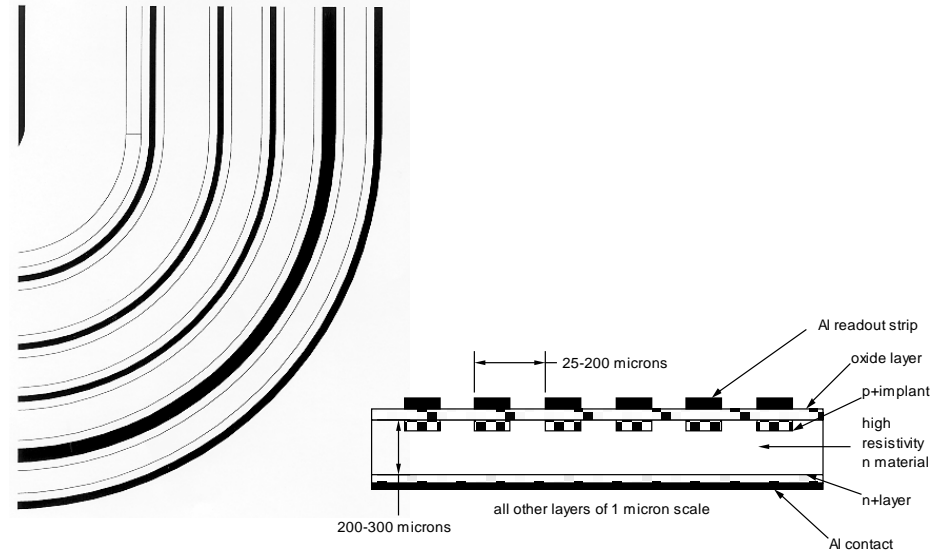
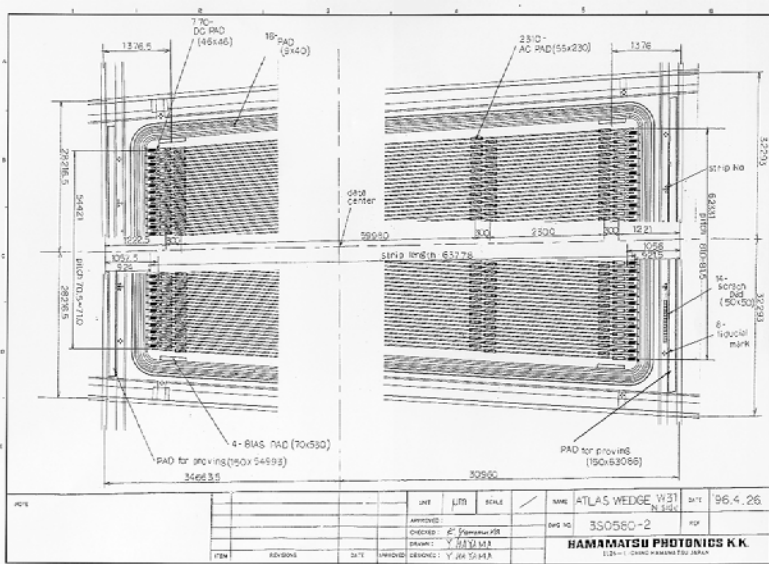
P. Azzi et al, NIM A 383 N.1 (1996) 155-158

Methods to Control Radiation Effects

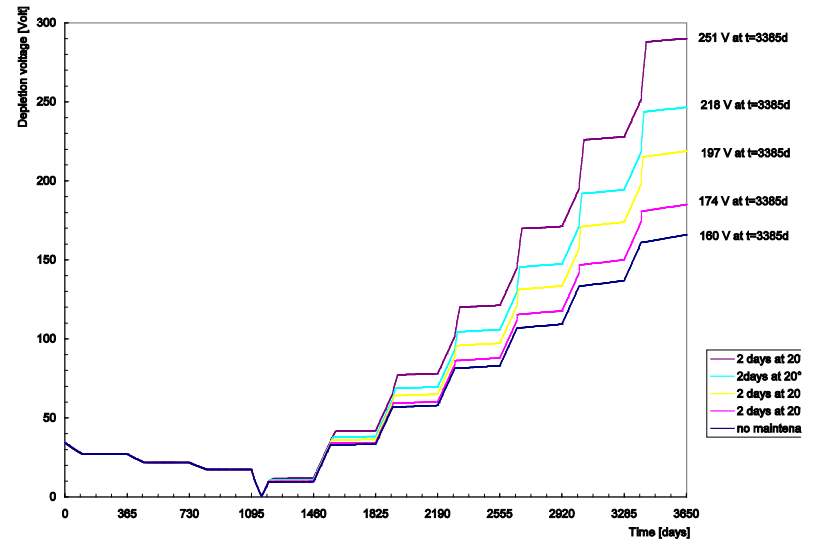
- Most represent some tradeoff
- Size matters
 - Smaller volumes generate less leakage current (but require more channels, power, heat...)
 - Thinner detectors deplete at lower voltage (usually means less signal)
- Temperature
 - Low temperature (-10 C) operation can “stabilize” reverse annealing for $< 10^{14}$
 - Reduce leakage current effects
- Integration time
 - Current noise is reduced for short shaping times at the expense of increased pre-amp noise, power.

- Biasing schemes
 - Reduce value of parallel biasing resistor to reduce voltage drop due to $I_{\text{Leak}}R_{\text{bias}}$ at the expense of increased thermal noise
- HV operation
 - Configure detectors to withstand higher voltage operation
 - Tolerate increased depletion voltage
 - Operate in partial depletion (collection issues)
- Low noise electronics
 - Tolerate reduced signal due to CCE and partial depletion
- Configuration
 - p in n substrate – simple, type inverts
 - n in n substrate – 2 sided process, can be operated in partial depletion after inversion
 - n in p – non-traditional process, does not invert

Strip detectors with multiple guard ring structures to tolerate HV=500 V operation



Simulation of 10 year operating scenarios for silicon tracking at the LHC
 $L = 10^{33}-10^{34}$

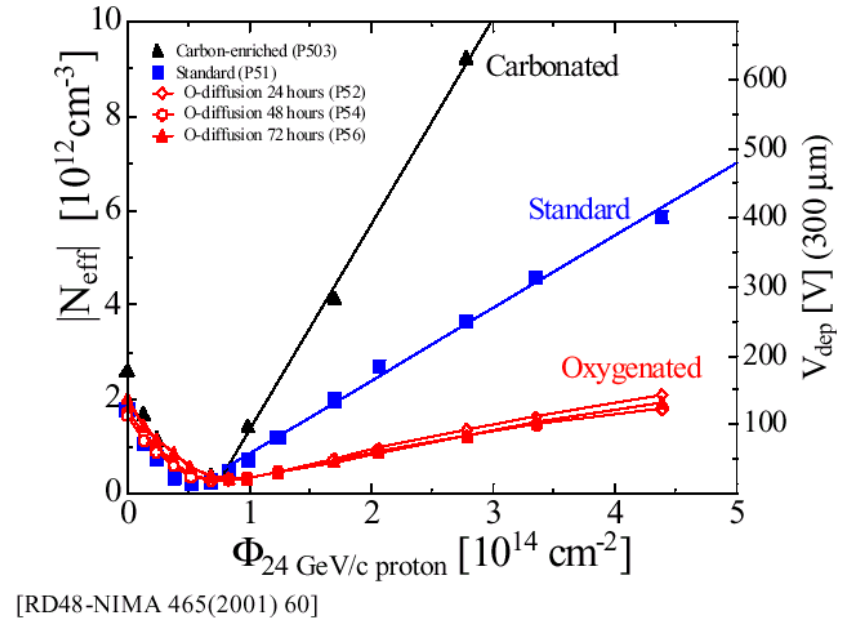


New developments

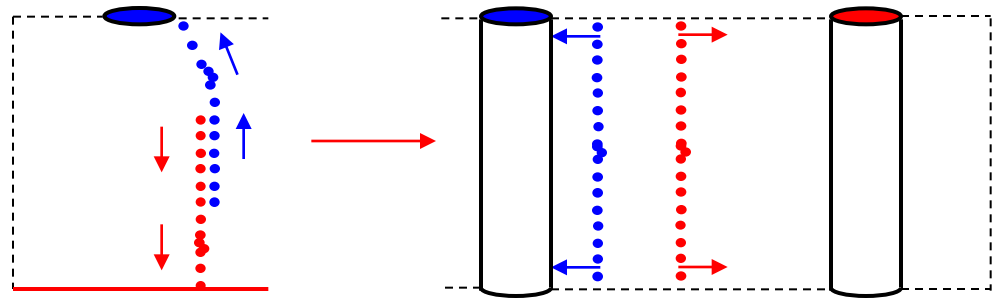
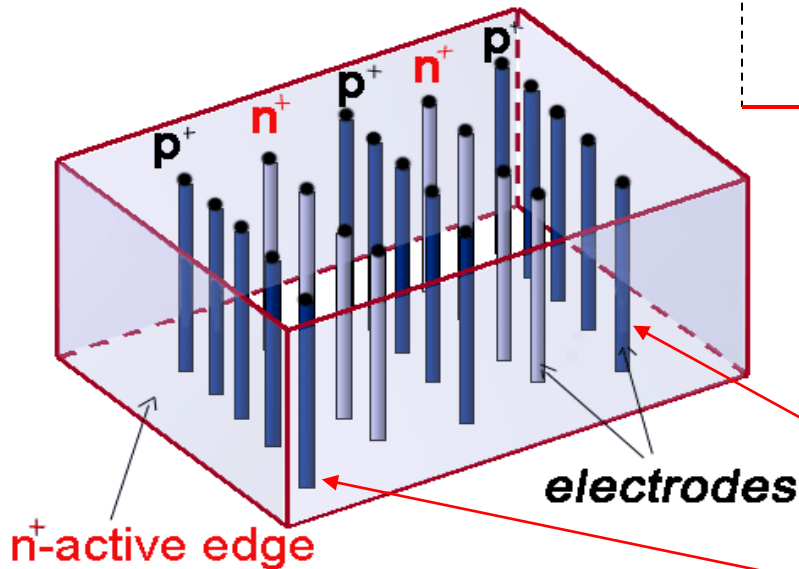
- Engineered materials
- New configurations, 3D electrodes, interleaved strips
- Cryogenics
- Alternate materials: Diamond, SiC,...
- RD efforts organized at CERN
 - RD42: development of diamond as detector
 - RD48: radiation damage to silicon
 - RD50: development of radiation resistant detectors
 - RD39: cryogenic detectors and systems
 - <http://rdXY.web.cern.ch/rdXY>

Engineered Silicon

- Microscopic understanding of damage mechanisms, defects, and kinetics
 - Modeling
 - Measurements
 - Time and temperature dependence
- Engineer the silicon for greater radiation resistance



3D Detectors



3D silicon detectors were proposed in 1995
 — by S. Parker, and active edges in 1997 by C. Kenney.

Combine traditional VLSI processing and MEMS (Micro Electro Mechanical Systems) technology.

Electrodes are processed inside the detector bulk instead of being implanted on the Wafer's surface.

The edge is an electrode! Dead volume at the Edge < 2 microns! Essential for
 -Large area coverage
 -Forward physics

1. NIMA 395 (1997) 328
2. IEEE Trans Nucl Sci 464 (1999) 1224
3. IEEE Trans Nucl Sci 482 (2001) 189
4. IEEE Trans Nucl Sci 485 (2001) 1629
5. IEEE Trans Nucl Sci 48 6 (2001) 2405
6. CERN Courier, Vol 43, Jan 2003, pp 23-26
7. NIMA 509 (2003)86-91

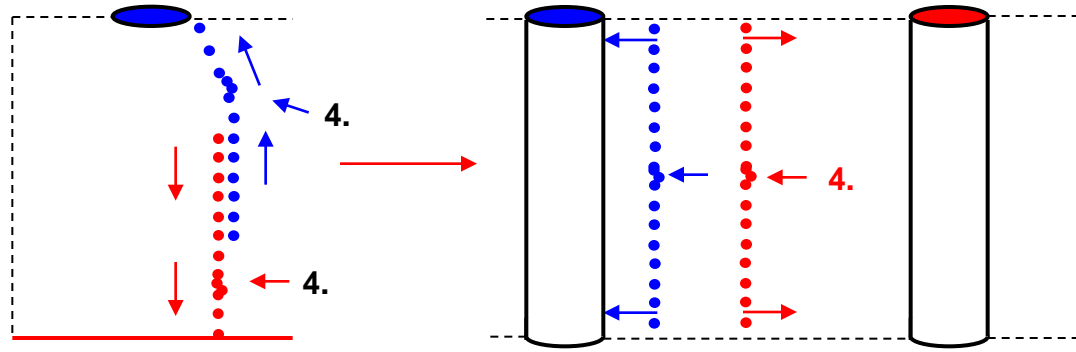
S.Parker

June 10, 2011

Silicon Detectors TIPP 2011

Carl Haber LBNL

Speed: planar → 3D



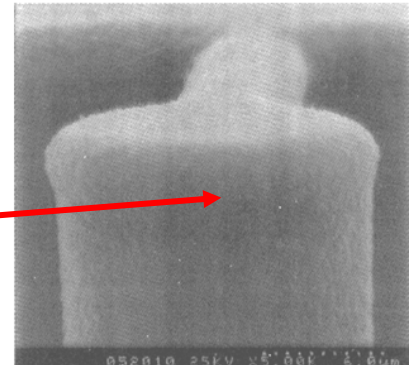
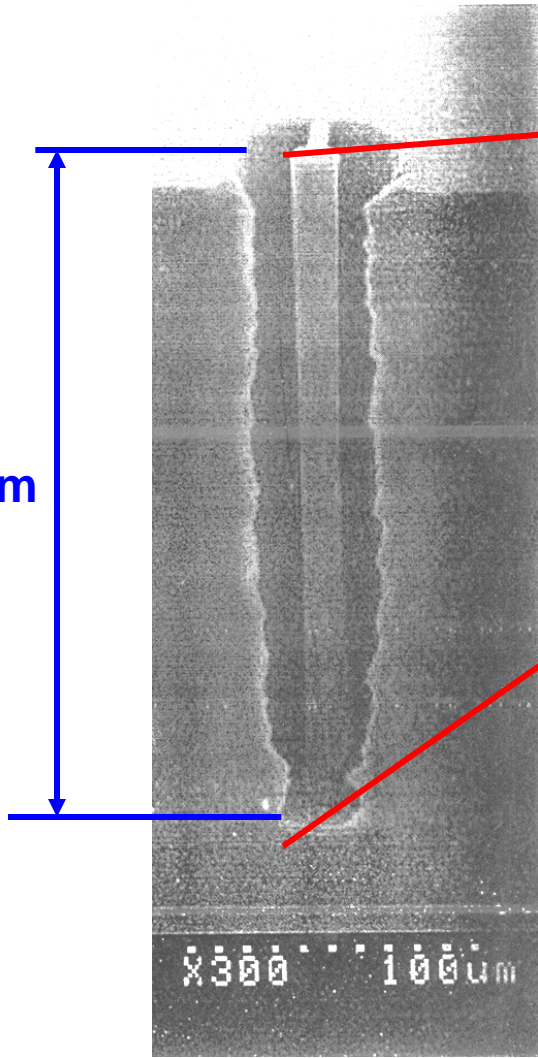
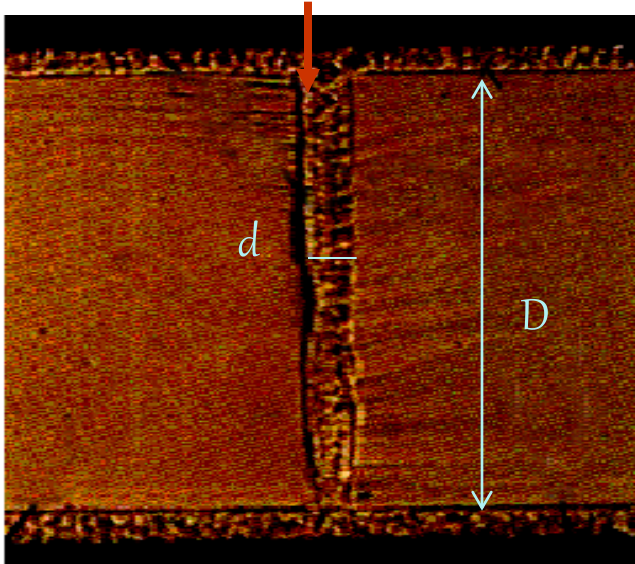
- | | | |
|--|--|---|
| <p>1. 3D lateral cell size can be smaller than wafer thickness, so</p> | → | <p>1. shorter collection distance</p> |
| <p>2. in 3D, field lines end on cylinders rather than on circles, so</p> | → | <p>2. higher average fields for any given maximum field (price: larger electrode capacitance)</p> |
| <p>3. most of the signal is induced when the charge is close to the electrode, where the electrode solid angle is large, so planar signals are spread out in time as the charge arrives, and</p> | → | <p>3. 3D signals are concentrated in time as the track arrives</p> |
| <p>4. Landau fluctuations along track arrive sequentially and may cause secondary peaks (see next slide)</p> | → | <p>4. Landau fluctuations arrive nearly simultaneously</p> |
| <p>5. if readout has inputs from both n+ and p+ electrodes,</p> | - - - - - → | <p>5. drift time corrections can be made</p> |
| <p>6. for long, narrow pixels and fast electronics,</p> | - - - - - → | <p>6. track locations within the pixel can be found</p> |

S.Parker

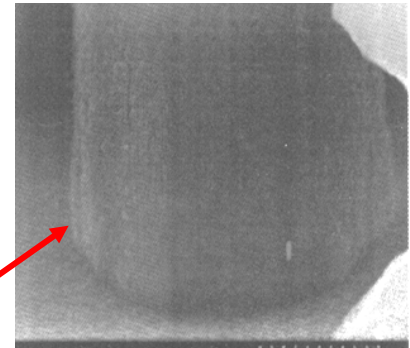
Examples of etching and coating with polysilicon.

An early test structure by Julie Segal, etched and coated (middle, right), showing conformal nature of poly coat. →

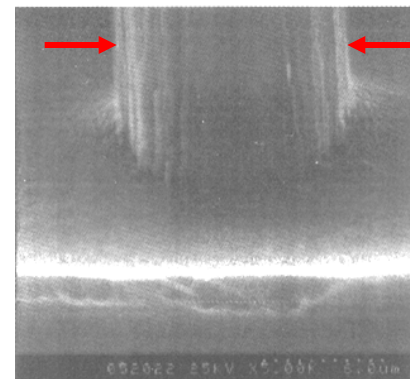
An **electrode hole, filled**, broken (accidentally) in a plane through the axis, showing grain structure (below). The surface poly is later etched off. **290 μm**



coated, top



coated, bottom



12 μm

uncoated

Alternate Materials

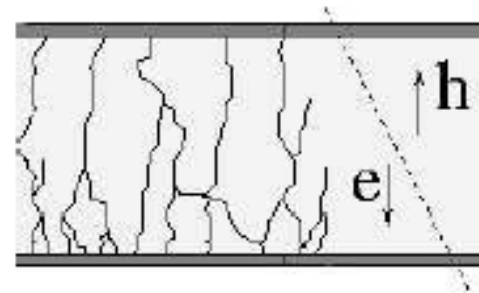
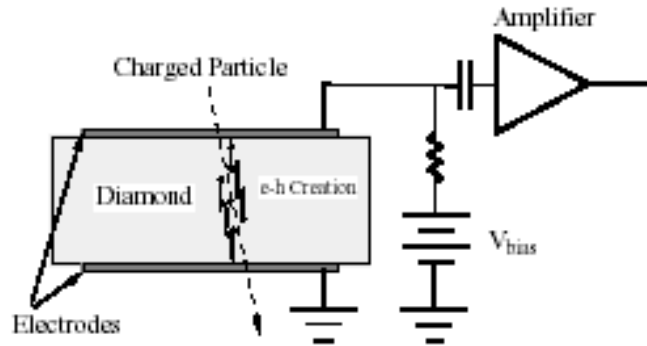
- Very active R&D effort for >10 years
 - <http://rd48.web.cern.ch/RD48/>
- Most work has been on pCVD diamond material
 - Significant improvement in charge collection
- New results on single crystal materials – but small samples
- Issue of industrial capacity vs silicon

Property	Diamond	4H-SiC	Si
Band Gap [eV]	5.5	3.3	1.12
Breakdown field [V/cm]	10 ⁷	4×10 ⁶	3×10 ⁵
Resistivity [Ω-cm]	> 10 ¹¹	10 ¹¹	2.3×10 ⁵
Intrinsic Carrier Density [cm ⁻³]	< 10 ³		1.5×10 ¹⁰
Electron Mobility [cm ² V ⁻¹ s ⁻¹]	1800	800	1350
Hole Mobility [cm ² V ⁻¹ s ⁻¹]	1200	115	480
Saturation Velocity [km/s]	220	200	82
Mass Density [g cm ⁻³]	3.52	3.21	2.33
Atomic Charge	6	14/6	14
Dielectric Constant	5.7	9.7	11.9
Displacement Energy [eV/atom]	43	25	13-20
Energy to create e-h pair [eV]	13	8.4	3.6
Radiation Length [cm]	12.2	8.7	9.4
Spec. Ionization Loss [MeV/cm]	4.69	4.28	3.21
Ave. Signal Created/100 μm [e]	3600	5100	8900
Ave. Signal Created/0.1% X ₀ [e]	4400	4400	8400

- Low dielectric constant - low capacitance
- Large bandgap - low leakage current
- Large energy to create an eh pair - small signal

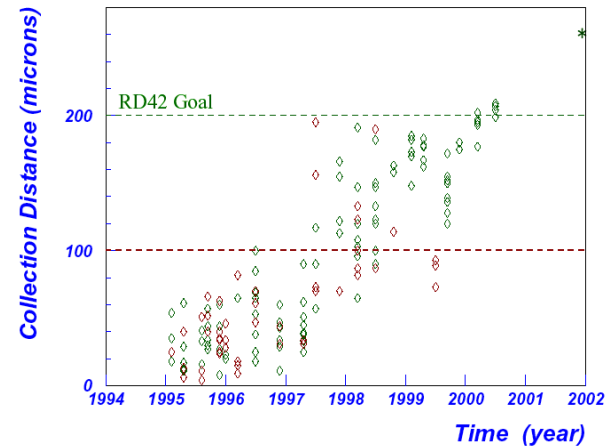
Characterization of Diamond:

Signal formation



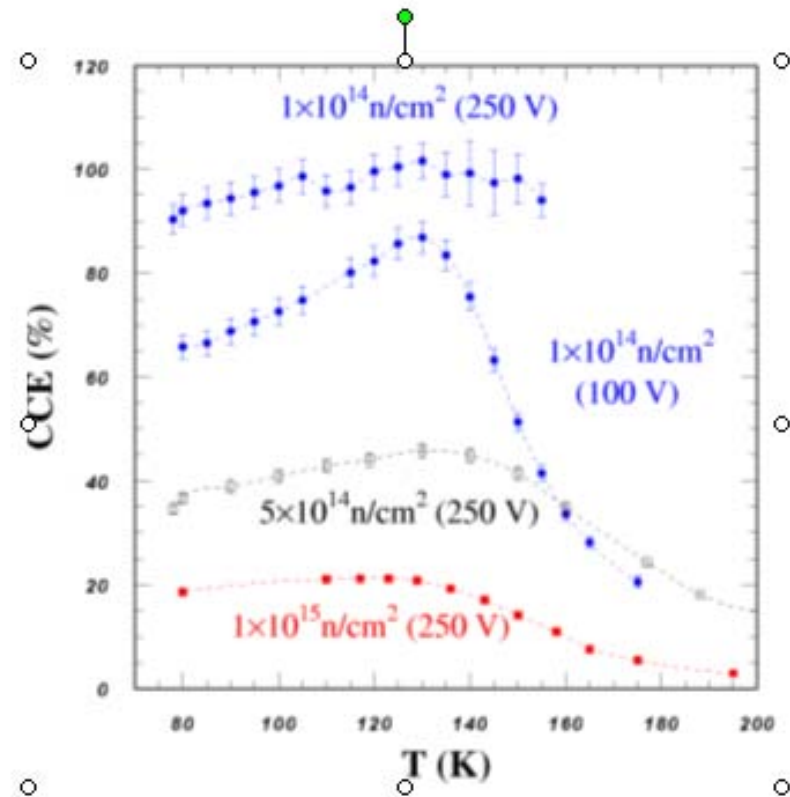
- $Q = \frac{d}{t} Q_0$ where d = collection distance = distance e-h pair move apart
- $d = (\mu_e \tau_e + \mu_h \tau_h) E$
- $d = \mu E \tau$
 with $\mu = \mu_e + \mu_h$
 and $\tau = \frac{\mu_e \tau_e + \mu_h \tau_h}{\mu_e + \mu_h}$

Charge Collection in DeBeers CVD Diamond



Cryogenic operation

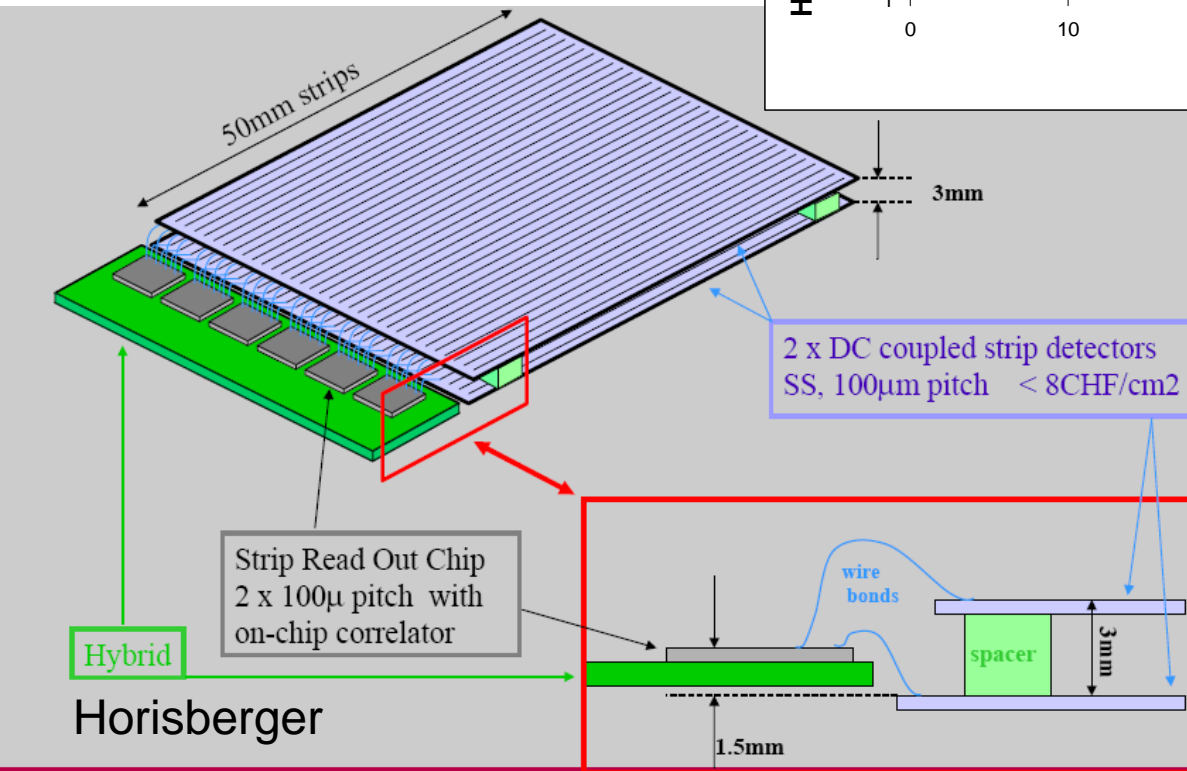
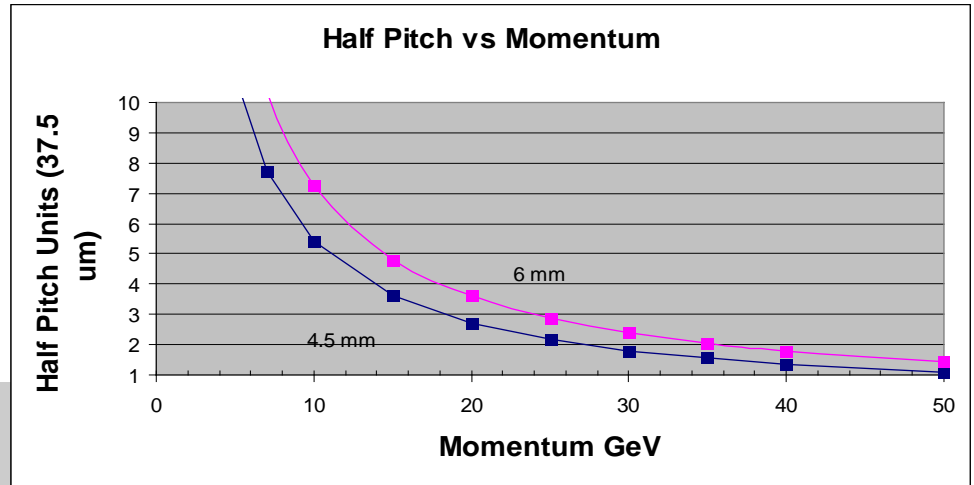
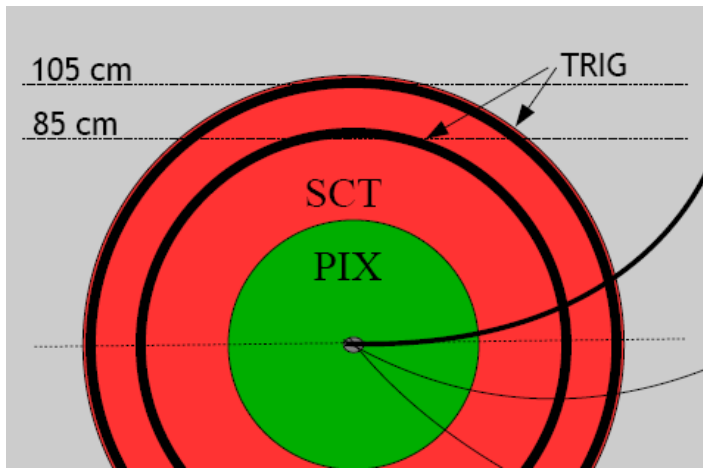
- Palmieri et al (1998) recovery of lost CCE at cryogenic temperatures
- “Lazarus Effect” due to freeze-out of traps
- R&D activity centered at CERN (RD39)
- Practical difficulty for “low mass” tracker if substantial cryogenic engineering and infrastructure is required.



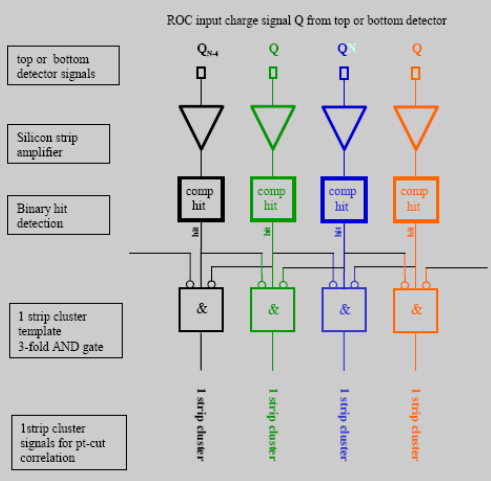
Technology Development Shopping List

- Radiation resistance of silicon and other solids
- Radiation hard electronics – smaller feature size, larger IC's
- Signal processing and circuit design
- Pixel architectures, monolithic and active pixel sensors
- Alternative powering schemes. serial power, DC-DC conversion
- Real time fast trigger processors
- Large area and precision low mass mechanics
- Alignment and survey technology (metrology, lasers, sensors)
- Low mass electrical and mechanical components including discretes & substrates
- Cooling technology – materials, coolants, delivery systems
- Finite element thermal and mechanical simulations
- Pattern recognition and data reduction methods
- Reliability and redundancy methods
- Large area, fine line, lithographic methods
- Robotic methods for assembly and test
- Wireless data transfer
- Optical readout methods

Real time momentum trigger



See WIT2010
pub in JINST



Conclusions

- Huge progress ~30 years to build silicon trackers using a broad suite of advanced technologies
- Much significant science done, and to be done, with these devices
- Progress in understanding and compensating for effects of radiation over a range of 10^4
- R&D underway for next generation



June 10, 2011

Silicon Detectors TIPP 2011

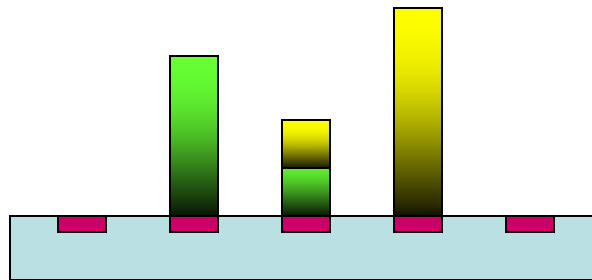
Carl Haber LBNL

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EXTRA SLIDES

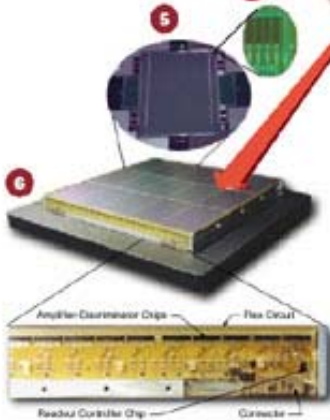
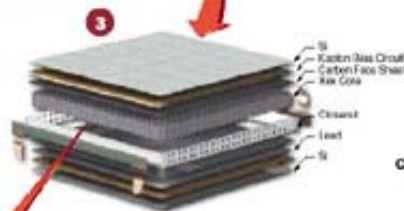
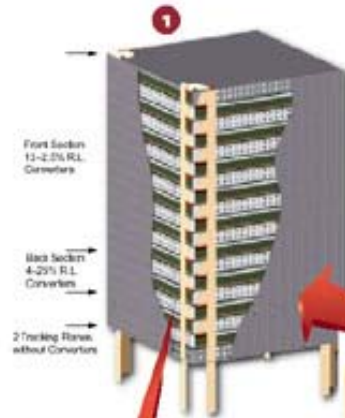
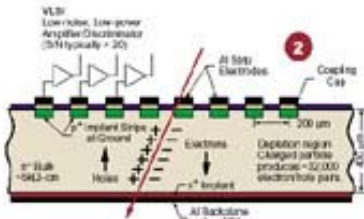
Multi-hit performance

- Binary response (hit or no hit), on pitch p , two hit separation requires an empty element.
 - Wide pitch \rightarrow most hits are single element, separation = $2p$
 - Narrow pitch \rightarrow double element hits, separation = $3p$
- Analog response: can use local minima in a merged cluster



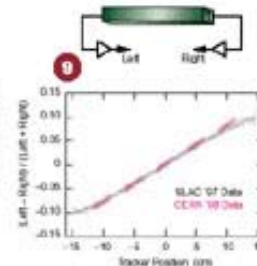
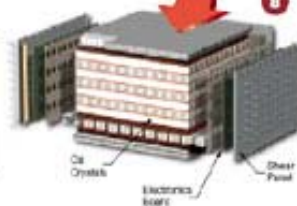
Tracker

1. Tracker tower: stack of 19 trays with 18 x,y detection planes, enclosed in C walls.
2. Si strip detector cross section.
3. Exploded view of a tracker tray.
4. Si strips, bias resistors, and bonding pads.
5. 4" Si wafer, with a BTM detector surrounded by test structures.
6. Complete tracker tray of the BTM, with Si detectors on the top and bottom faces and readout electronics on two sides.



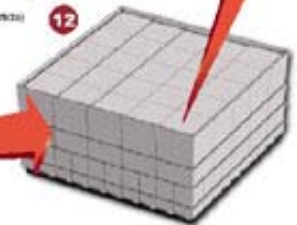
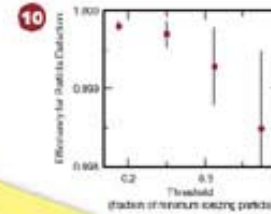
Calorimeter

7. Assembled BTM CAL module.
8. CAL compression cell design.
9. CAL beam-test results: Position measurement from left-right light asymmetry.



The Anticoincidence Shield

10. ACD beam-test results: efficiency to detect a minimum-ionizing particle versus the discriminator threshold. The required efficiency is 0.9997.
11. ACD scintillator tile, with waveshifting fiber readout.
12. The LAT enclosed in the ACD veto shield.



GLAST LAT/Foldout C Instrument

Key Features:

- Low Aspect Ratio—Wide Field of View
- Large Energy Reach, Excellent PSF
- Proven Detector Technologies
- Large Detector Performance Margins
- Modularity, Redundancy
- No Consumables

Instrument Detector Technologies

Tracker (TKR):

- Silicon Microstrip Detectors
- High efficiency
 - High signal/noise
 - Robust, Rad-Hard, Low Voltage
 - Widespread use in space and HEP

Calorimeter (CAL):

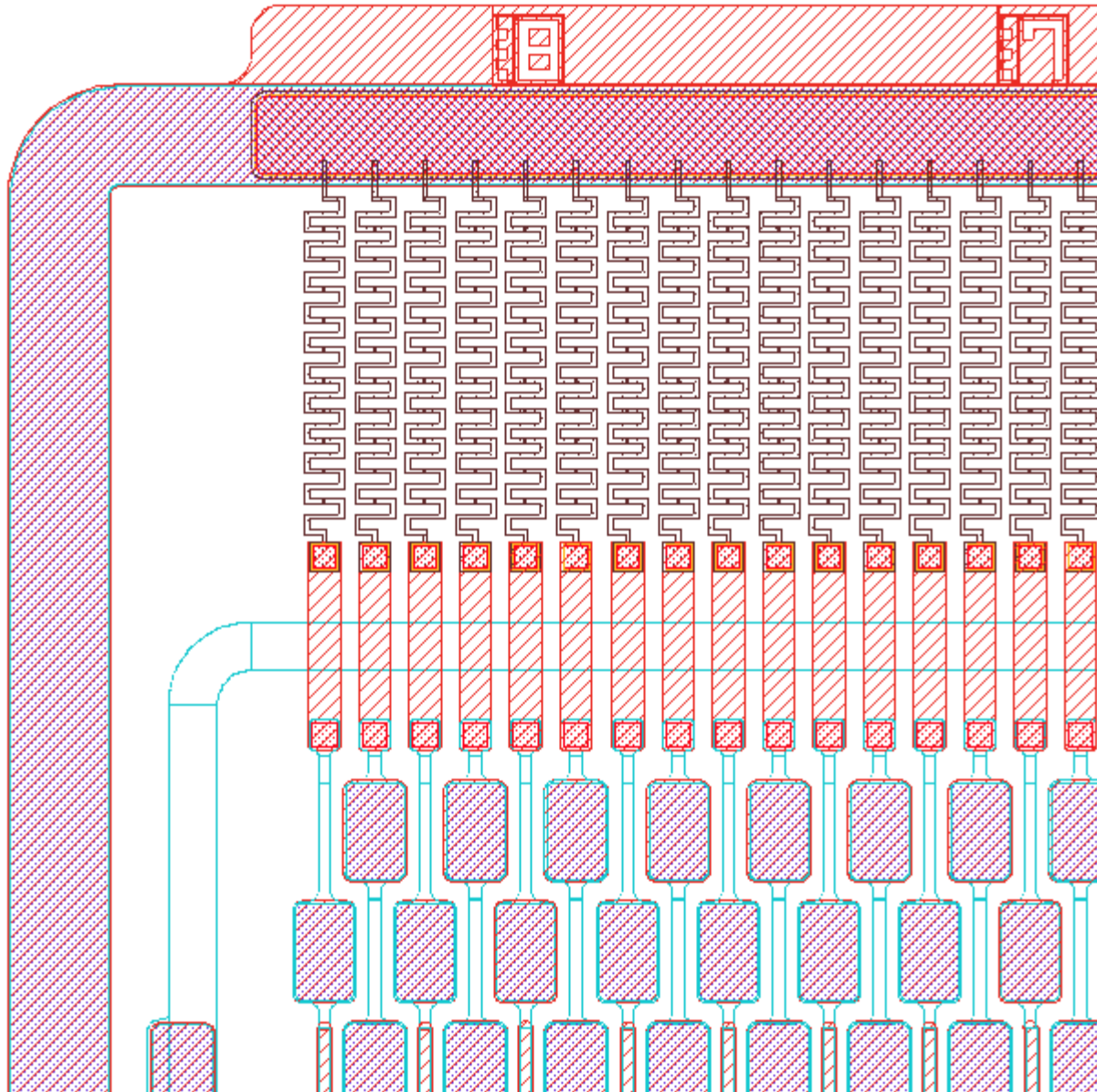
- Cesium-Iodide crystals; PIN diode readout
- Excellent energy resolution over wide range
 - High signal/noise
 - Hodoscopic array gives good position resolution and shower leakage correction
 - Widespread use in space and HEP

Anticoincidence Detectors (ACD):

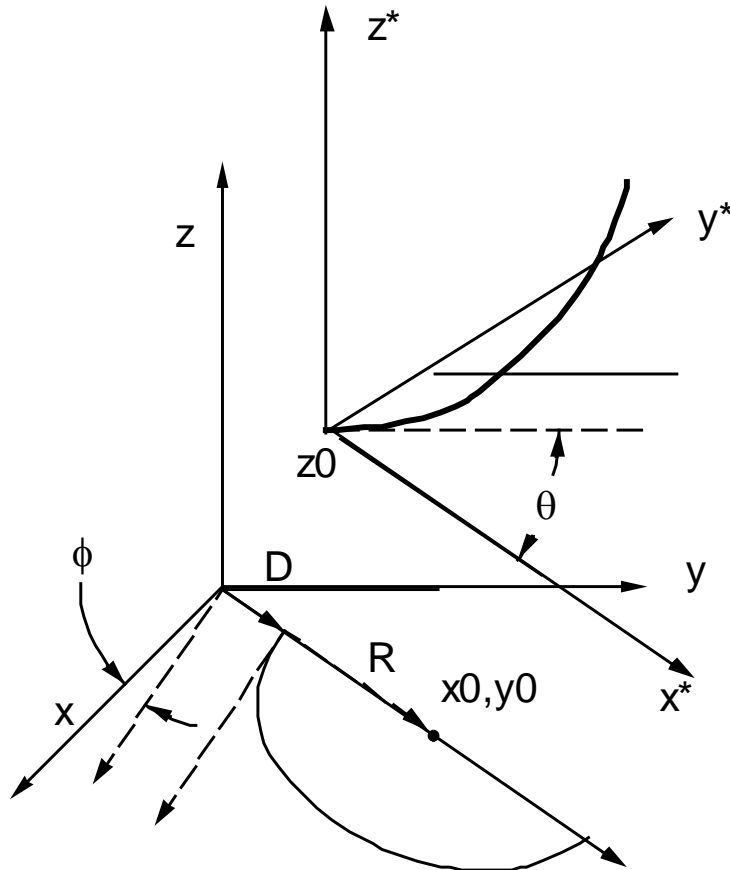
- plastic scintillator tiles; waveshifting-fiber/PMT readout.
- High efficiency
 - High segmentation to avoid self veto
 - Widespread use in space and HEP

Lower Mass

- Large area and precision low mass mechanics
- Alignment technology (lasers, sensors)
 - Drop stiffness requirements in favor of active monitoring and feedback (lesson from the telescope builders).
- Low mass electrical and mechanical components including discretely & substrates
 - Power distribution schemes, serial power, DC-DC conversion, less redundancy, grounding issues
 - Technologies for hybrid circuits – thick, thin films, laminates
- Cooling technology – materials, coolants, delivery systems
 - High thermal conductivity materials
 - High pressure CO₂
 - Cooling integrated with FE electronics
 - Reduced power consumption



Trajectory



$$\begin{aligned} x &= x_0 + R \cos \lambda \\ y &= y_0 + R \sin \lambda \\ z &= z_0 + R \lambda \tan \theta \end{aligned}$$

- Charged particle in a magnetic field $B=Bz$
- 3D Helix : 5 parameters
 - $C = \text{half curvature}$
($1(\text{sgn})/R$)
 - $z_0 = \text{offset}$
 - $D = \text{signed impact parameter (distance of closest approach)}$
 - Azimuth $\phi = \text{angle of track at closest approach}$
 - $\theta = \text{dip angle}$

Momentum Resolution

Simple case: Measure sagitta s of track with radius R , over projected arc length L (cm, KGauss, MeV/c), assuming $R \gg L$

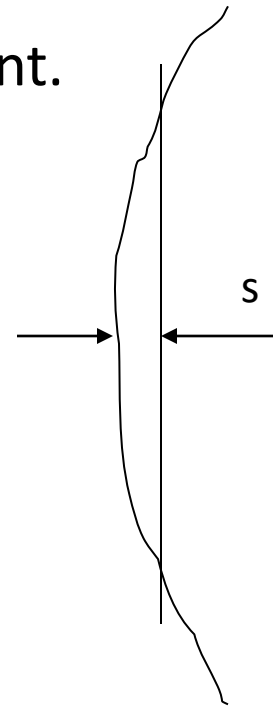
$$p = \frac{0.3BR}{\cos \theta} = \frac{0.3BL^2}{8s \cos \theta} \quad \text{using } R = \frac{L^2}{8s} \Rightarrow \left(\frac{\Delta p}{p} \right)_{\text{sagitta}} = \frac{8p \Delta s}{0.3BL^2 \cos \theta}$$

where Δs is the error on the sagitta measurement.

Effect of material: multiple scattering

$$(\Delta s)^2 = \frac{\sigma_{MCS}^2}{16} \frac{L^2}{3 \cos^2 \theta} \Rightarrow \left(\frac{\Delta p}{p} \right)_{MCS} = \frac{52.8}{B \sqrt{L X_0} \cos \theta}$$

$$\frac{\Delta p}{p}_{TOTAL} = \left(\left(\frac{\Delta p}{p} \right)_{\text{sagitta}}^2 + \left(\frac{\Delta p}{p} \right)_{MCS}^2 \right)^{\frac{1}{2}}$$



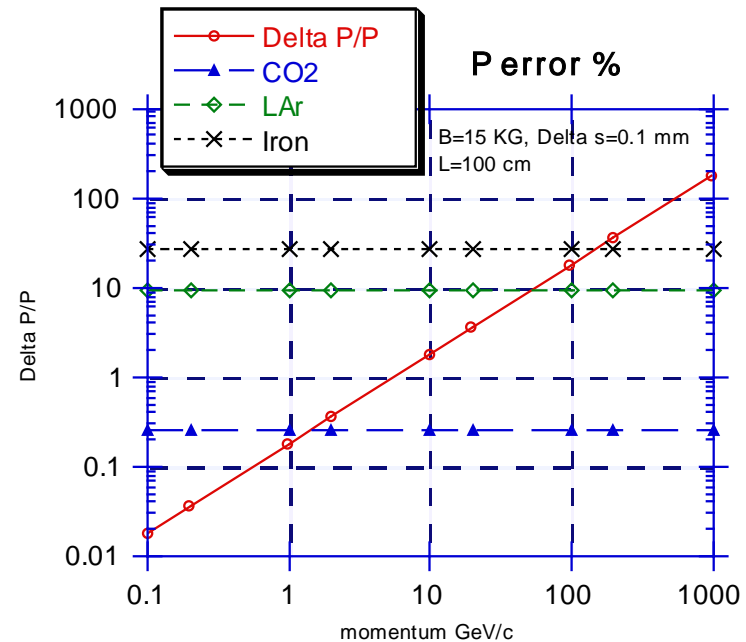
Effect of Material

$$\left(\frac{\Delta p}{p}\right)_{sagitta} = \frac{8p\Delta s}{0.3BL^2 \cos \theta}$$

$$\left(\frac{\Delta p}{p}\right)_{MCS} = \frac{52.8}{B\sqrt{LX_0} \cos \theta}$$

$$\frac{\Delta p}{p}_{TOTAL} = \left(\left(\frac{\Delta p}{p}\right)_{sagitta}^2 + \left(\frac{\Delta p}{p}\right)_{MCS}^2 \right)^{\frac{1}{2}}$$

- Minimize sagitta error
- Maximize B,L
- Minimize material



Vertex Resolution

$x_1, x_2 =$ measurement planes

$y_1, y_2 =$ measured points, with errors δy

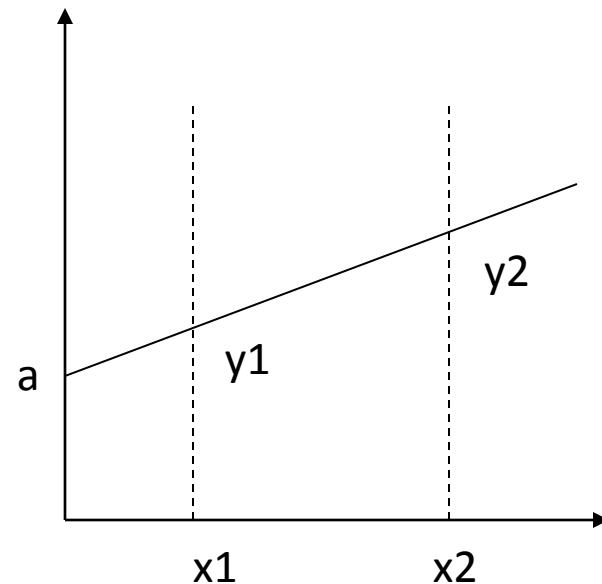
$$y = a + bx$$

$$b = \text{slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_1 - y_2}{\Delta x}$$

$$a = \text{intercept} = \frac{1}{2}(y_1 + y_2) - \frac{1}{2}(y_1 - y_2) \left(\frac{x_1 + x_2}{\Delta x} \right) = \bar{y} - b\bar{x}$$

$$(\delta b)^2 = \left(\frac{\partial b}{\partial y_1} \right)^2 (\delta y)^2 + \left(\frac{\partial b}{\partial y_2} \right)^2 (\delta y)^2 \Rightarrow \delta b = \frac{\sqrt{2}\delta y}{\Delta x}$$

$$\delta a = \frac{\delta y}{2} \sqrt{1 + \frac{8\bar{x}}{\Delta x}}$$



for good resolution on angles (f and q) and intercepts (d, z_0)

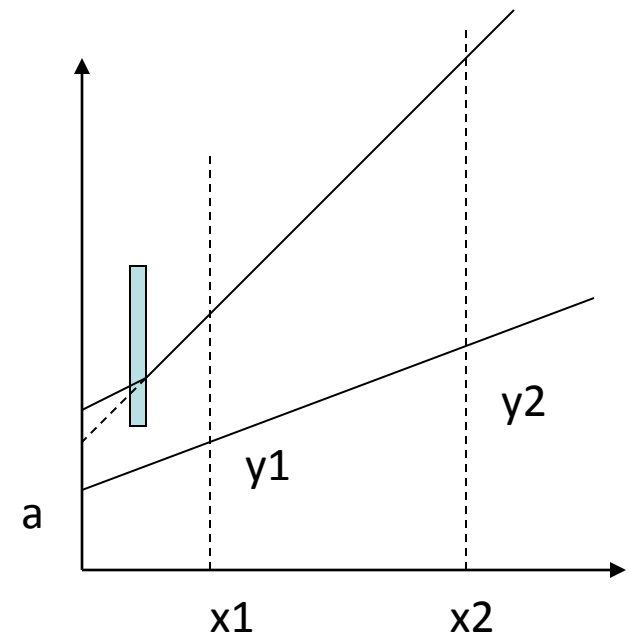
- Precision track point measurements
- Maximize separation between planes for good resolution on intercepts
- Minimize extrapolation - first point close to interaction

Effect of Material

$x_1, x_2 =$ measurement planes

$y_1, y_2 =$ measured points, with errors δy

$$\delta a = \frac{\delta y}{2} \sqrt{1 + \frac{8\bar{x}}{\Delta x}}$$



for good resolution on angles (f and q) and intercepts (d, z_0)

- Precision track point measurements
- Maximize separation between planes for good resolution on intercepts
- Minimize extrapolation - first point close to interaction
- Material inside 1st layer should be at minimum radius (multiple scattering)