Basic Electronics

Introductory Lecture Course for

Technology and Instrumentation in Particle Physics 2011

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Presented By

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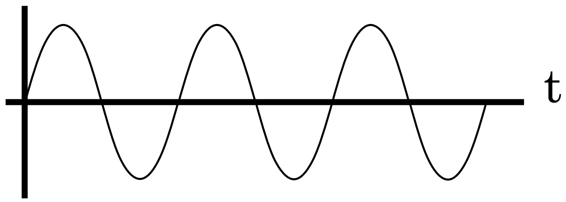
Session 2

Session 2

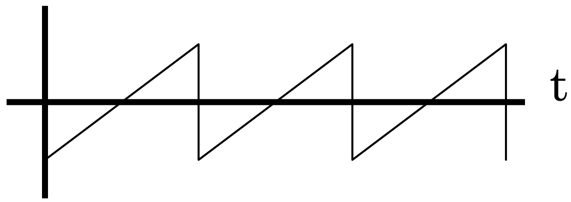
Time-Varying Signals & Circuits

Time-Varying, Periodic Sources

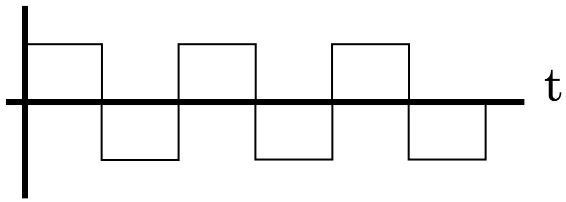
- Periodic Time-Varying Sources → ***"Steady State"***
  - Sinusoidal



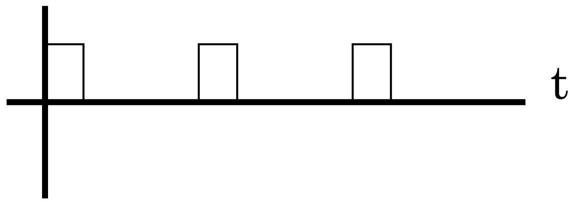
  - Ramp



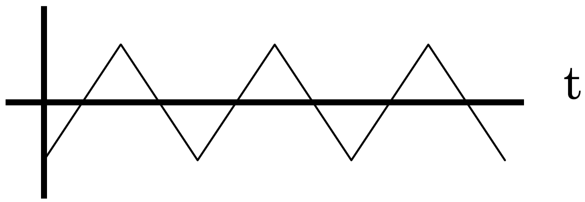
  - Square Wave



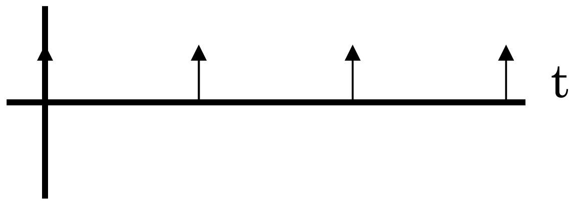
  - Pulse Train



  - Triangle Wave



  - Comb Function

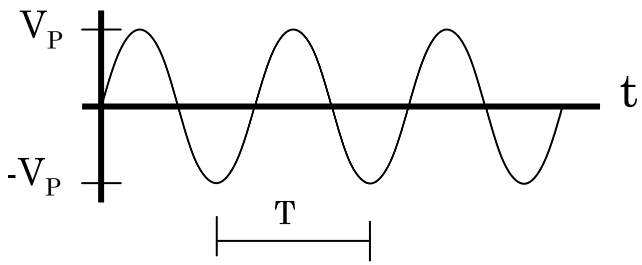


⇨ ***Can be Voltage Sources or Current Sources***

⇨ ***Depends on….. Source Impedance!***

Sinusoidal Sources

- Sinusoidal Sources
  - Characterized by frequency ω ($\omega = 2\pi f$), phase ϕ, period T ($T = 1 / f$), and peak amplitude V_p



$$v_O(t) = V_P \sin(\omega t + \phi)$$

  - Is the average value important?

$$V_{AVG} = 1/T \int_0^T v_O(t)\,dt = 1/T \int_0^T V_P \sin(\omega t)\,dt = 0$$

⇨ ***No, not really…***

  - We often use the ***"Effective Value,"*** or RMS

$$V_{EFF} = \sqrt{1/T \int_0^T v_O^2(t)\,dt} = \sqrt{1/T \int_0^T V_P^2 \sin^2(\omega t)\,dt}$$

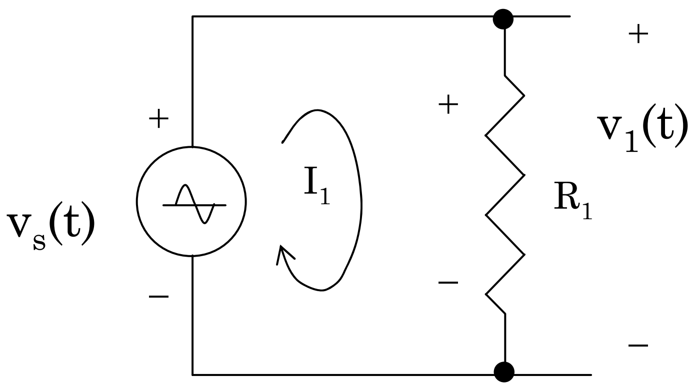
$$= V_P / \sqrt{2}$$

⇨ ***This IS useful…***

Images from Wikipedia

Resistive Circuits with Steady State Sources

- Consider again a resistive network, but this time with a sinusoidal source



$v_s(t) = V_p \sin(\omega_O \; t)$

Just as before, we can use Kirchoff's Laws:

$i_1(t)\, R_1 - v_s(t) = 0$

$i_1(t) = v_s(t) / R_1$

$v_O(t) = v_S(t)$

What is the instantaneous power consumed by R_1?

$$p_1(t) = v_1(t)\; i_1(t)$$
$$= V_p \sin(\omega_o\, t)\; [V_p \sin(\omega_o\, t)] / R_1$$
$$= [V_p^{\,2} / R_1]\; \sin^2(\omega_o\, t)$$

What is the average power consumed by R_1?

$$P_{AVG} = 1/T \int_0^T p_1(t)\, dt$$
$$= 1/T \int_0^T [V_P^{\,2}/R_1] \sin^2(\omega_o\, t)\, dt$$

$$P_{AVG} = ½\, V_P^{\,2} / R_1$$
$$= V_{EFF}^{\,2} / R_1, \text{ since } V_{EFF} = V_P / \sqrt{2}$$

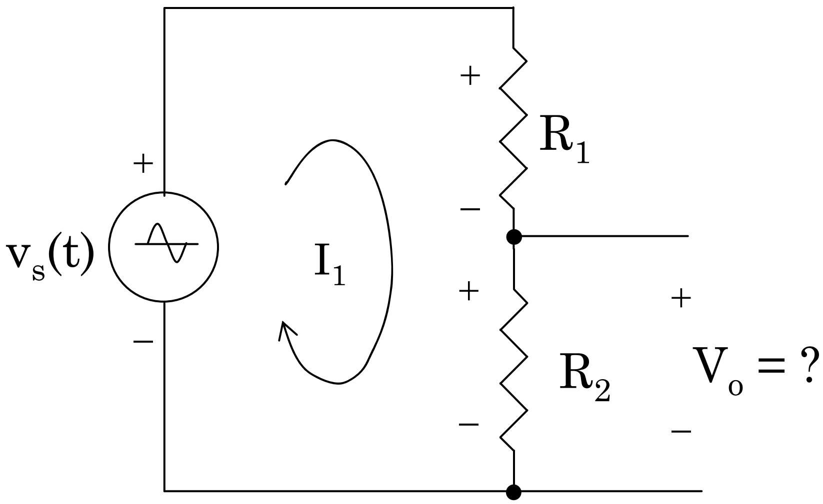
In general,

$$P_{AVG} = V_{EFF} \;\; I_{EFF}$$

⇨ ***Says that Effective Values of a sinusoid produce average power equivalent to comparable DC values***

Resistive Circuits with Steady State Sources

- In general, can perform the same analysis with any resistive network



$v_s(t) = V_p \sin(\omega_O\ t)$

<u>Voltage Divider – From before:</u>

$I_1 R_1 + I_1 R_2 - V_s = 0$

$I_1 = V_s / (R_1 + R_2)$

Then:

$V_1 = I_1 R_1,\ \ V_2 = I_1 R_2$

$V_O = V_S\ R_2 / (R_1 + R_2)$

<u>Inserting the time-varying source:</u>

$v_o(t) = V_p\ [R_2 / (R_1 + R_2)]\ \sin(\omega_o\ t)$

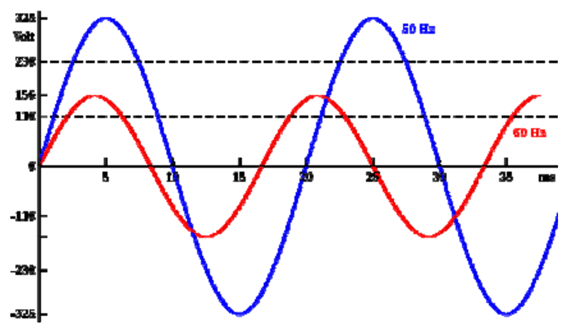
$P_{AVG} = V_{EFF}\ \ I_{EFF}$

$= ½\ V_p\ [R_2 / (R_1+R_2)]\ \ [V_p / (R_1+R_2)]$

$P_{AVG} = ½\ V_P^2\ [R_2 / (R_1 + R_2)^2]$

Introduction to Steady State (AC) Analysis

- Sinusoidal Sources
  - Why are sinusoids important?
    - Reason #1: Our electrical grid works on sinusoidal power
      - Famous shoot-out between Nikola Tesla and Thomas Edison, 1893 Worlds Fair in Chicago (Tesla won, but died a pauper…)

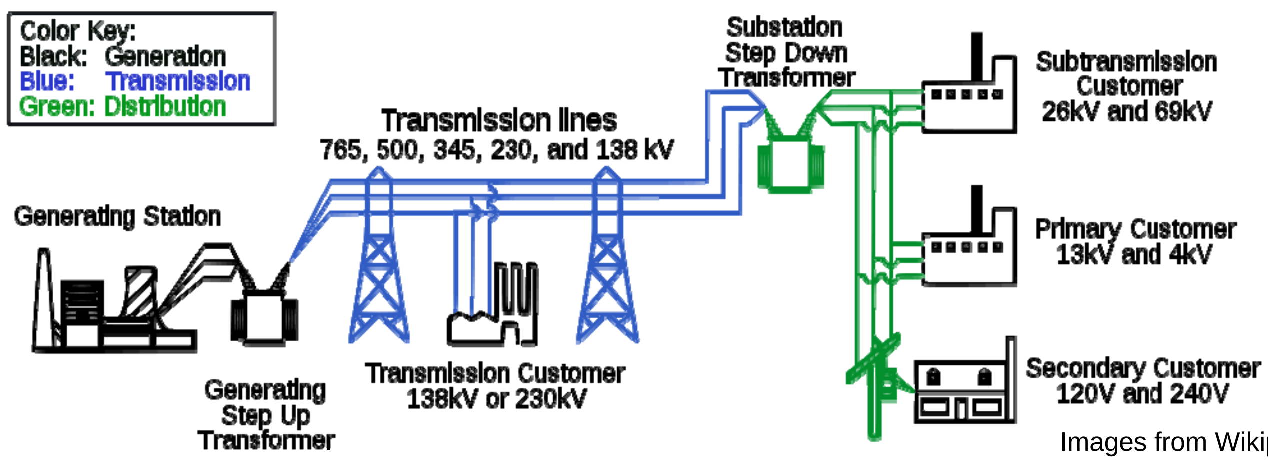


<u>In the USA:</u>

f = 60 Hz

V_{EFF} = 120V

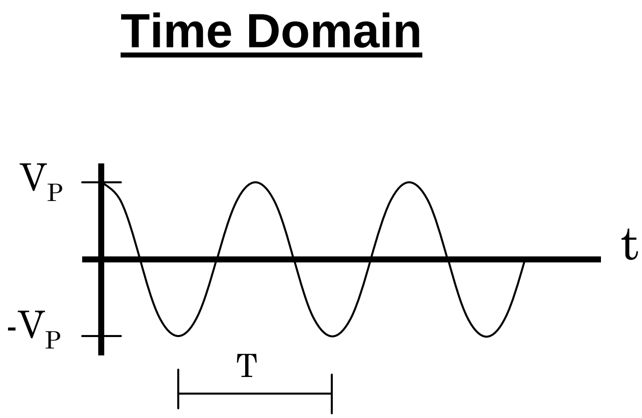
V_P = 170V



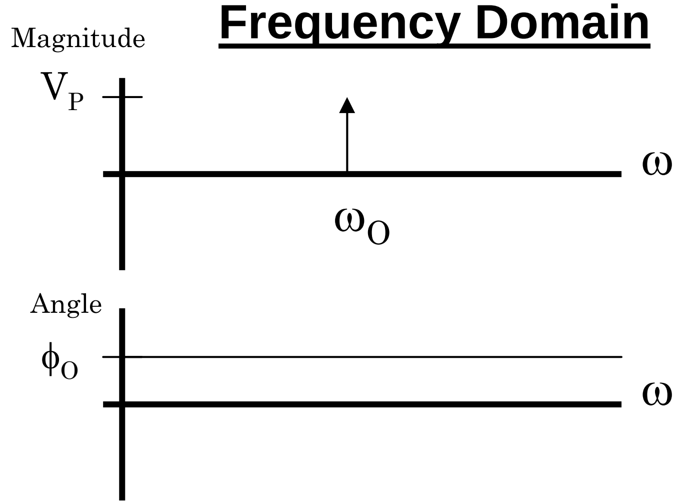
Images from Wikipedia

Introduction to Steady State (AC) Analysis

- Sinusoidal Sources (Continued)
  - Why are sinusoids important (Cont.)?
    - Reason #2: Convenient representation in the ***Frequency Domain***



$$v(t) = V_P \cos(\omega_O t + \phi_O)$$



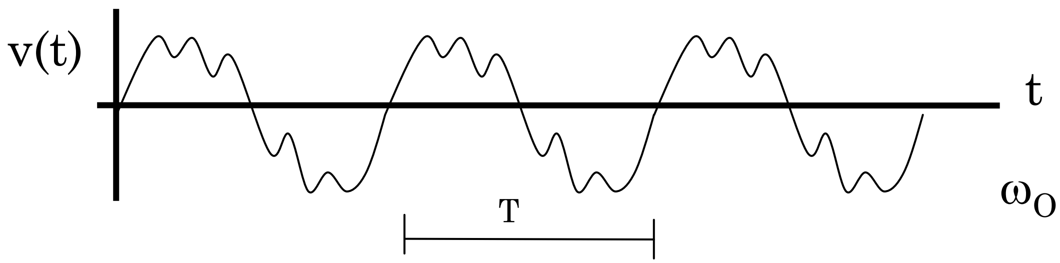
- Define ***phasors***: $F(\omega) = \text{Mag} \angle \text{Angle}$

If: $v(t) = V_P \cos(\omega_O t + \phi_O)$ Phasor form: $V(\omega) = V_P \angle \phi_O$

⇨ ***Has connection with Complex Number Analysis…*** ⇨ ***More on this later***

Introduction to Steady State (AC) Analysis

- Sinusoidal Sources (Continued)
  - Why are sinusoids important (Cont.)?
    - Reason #3: Any periodic waveform can be represented as an infinite sum of sinusoids, with frequencies that are multiples of the fundamental frequency ω_O **→ *Fourier Series***
      - For any periodic waveform $V(t)$, with fundamental frequency ω_O



$$\omega_O = (2\pi) / T$$

$$v(t) = \sum_{n=-\infty}^{+\infty} E_n \, e^{(jn\omega_o t)} \text{, where } j = \sqrt{-1}$$

      - The coefficients E_n are given by:

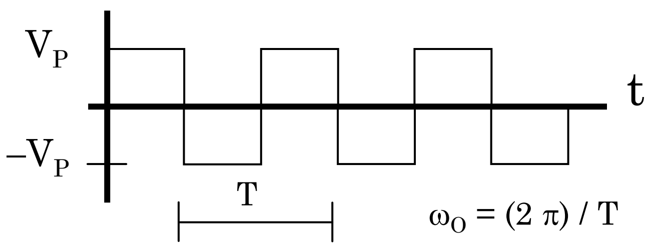
$$E_n = 1/T \int_{-T/2}^{T/2} V(t) \; e^{-jn\omega_o t} \; dt$$

⇨ ***$n\omega_0$ are the harmonic frequencies of v(t)***

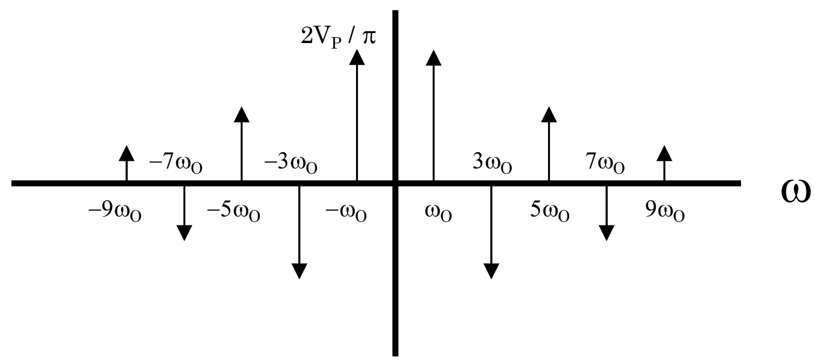
Introduction to Steady State (AC) Analysis

- Sinusoidal Sources (Continued)
  - Why are sinusoids important (Cont.)?
    - Example: Fourier Series of a Square Wave

Time Domain



Frequency Domain



$$V(t) = \sum_{n=-\infty}^{+\infty} E_n \, e^{(jn\omega_o t)} \qquad \text{where } j = \sqrt{-1}$$

$$\text{where } E_n = (2V_P / n\pi) \sin(n\pi / 2)$$

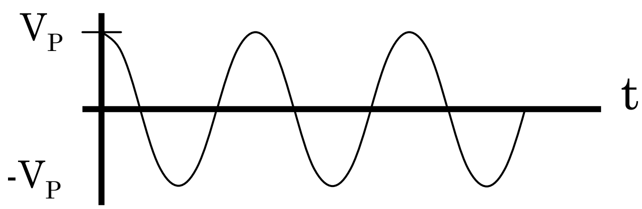
⇨ ***Notice that only the odd harmonics of v(t) are present in this particular example…***

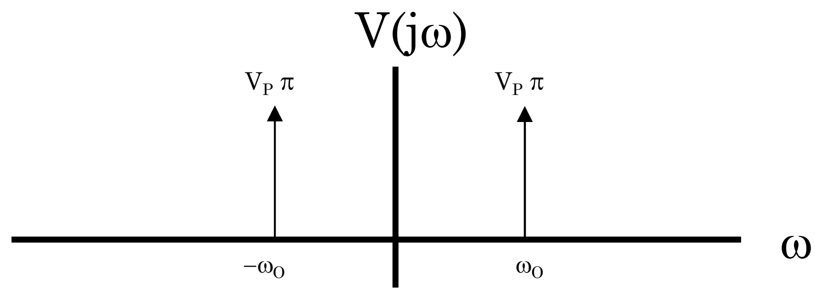
Introduction to Steady State (AC) Analysis

- More generally, use the Fourier Transform
  - For a periodic function v(t) with period T, the Fourier Transform is defined as:

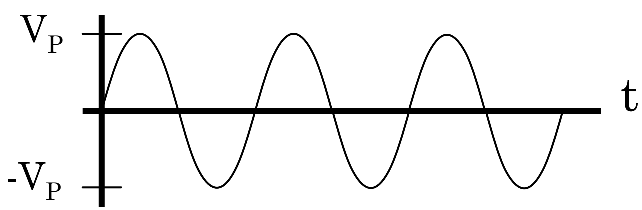
$$\mathscr{F}[v(t)] = V(j\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt \qquad \text{where } j = \sqrt{-1}$$

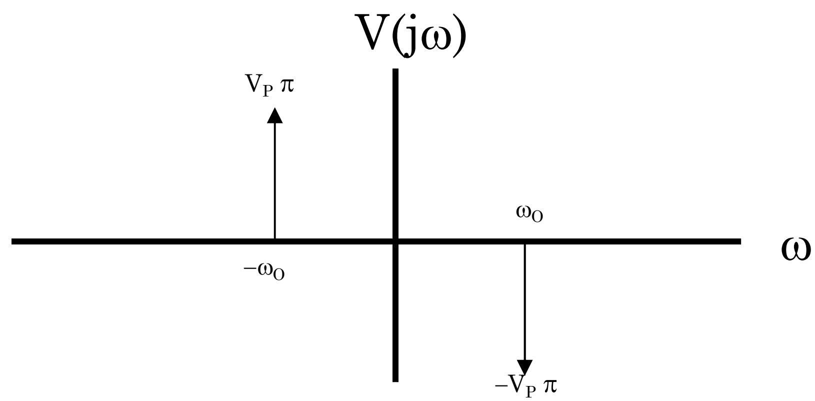
  - Some important Transforms
    - $v(t) = V_P \cos(\omega_O t)$





    - $v(t) = V_P \sin(\omega_O t)$





⇨ ***Generally, we will ignore the negative frequency space…***

Introduction to Steady State (AC) Analysis

- Fourier Transform Properties
  - For a periodic function $v(t)$ with period T, the Fourier Transform is defined as:

$$\mathscr{F}[v(t)] = V(j\omega) = \int_{-\infty}^{\infty} v(t)\ e^{-j\omega t}\ dt \qquad \text{where } j = \sqrt{-1}$$

  - The Inverse Fourier Transform is given as:

$$\mathscr{F}^{-1}[V(j\omega)] = v(t) = 1/(2\pi) \int_{-\infty}^{\infty} V(j\omega)\ e^{j\omega t}\ d\omega$$

  - An important Transform property that we will use:

$$\mathscr{F}[d/dt\ v(t)] = j\omega\ V(j\omega)$$

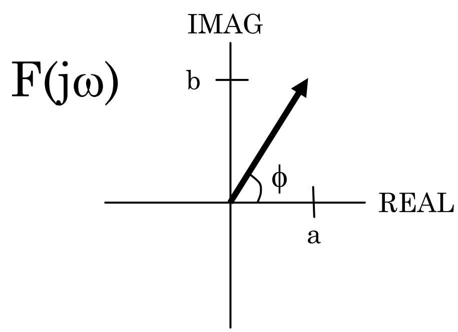
⇨ ***Fourier Transforms generally generate Complex Numbers…***

Introduction to Steady State (AC) Analysis

- A Review of Complex Numbers
  - For

$$F(j\omega) = a_{REAL} + j\, b_{IMAG}$$

  - Think of Complex Plane



$$\left| F(j\omega) \right| = \sqrt{a_{REAL}^{2} + b_{IMAG}^{2}}$$

$$\phi = \tan^{-1}\left[b_{IMAJ} / a_{REAL}\right]$$

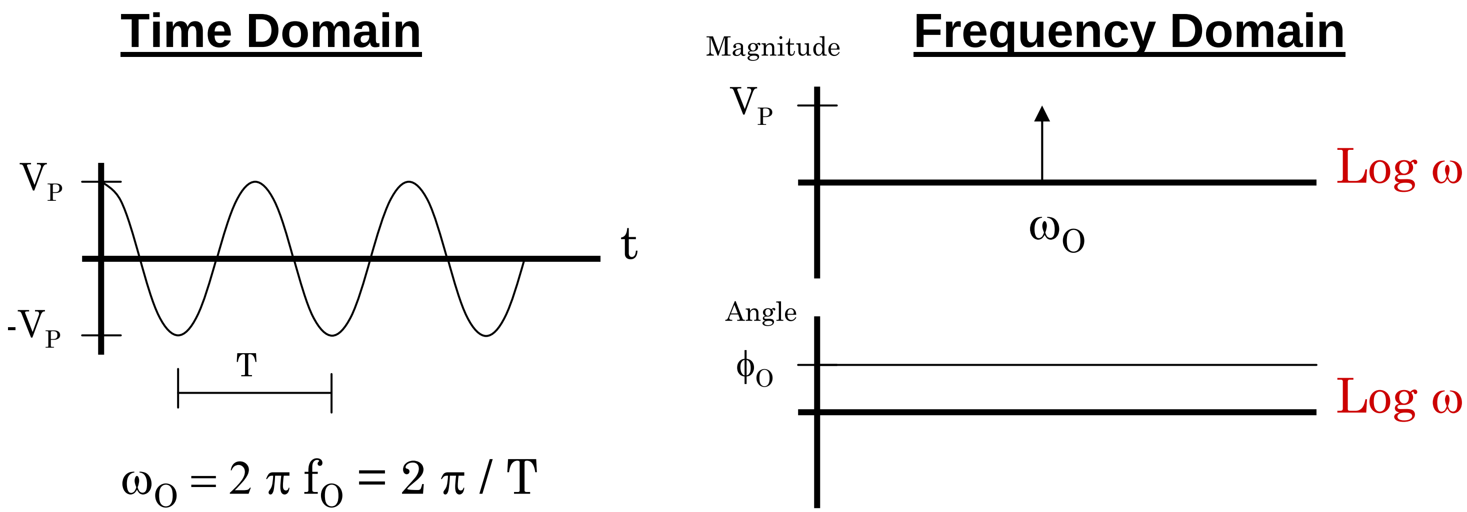
  - These are related to Phasors

$$\text{Phasor form: } F(j\omega) = \left|F(jw)\right| \angle \phi$$

$$F(j\omega) = \text{Mag} \angle \text{Angle}$$

Frequency Domain Analysis

- Basic Principles
  - We will generally use sinusoidal excitation to evaluate the performance of circuits, and sweep the frequency over a range of interest



  - In general, when energy storage elements are involved, we will calculate circuit response in the ***frequency domain***
  - Use Complex Analysis → REAL & IMAG → Phasor forms
  - Use Fourier Transforms

Frequency Domain Analysis

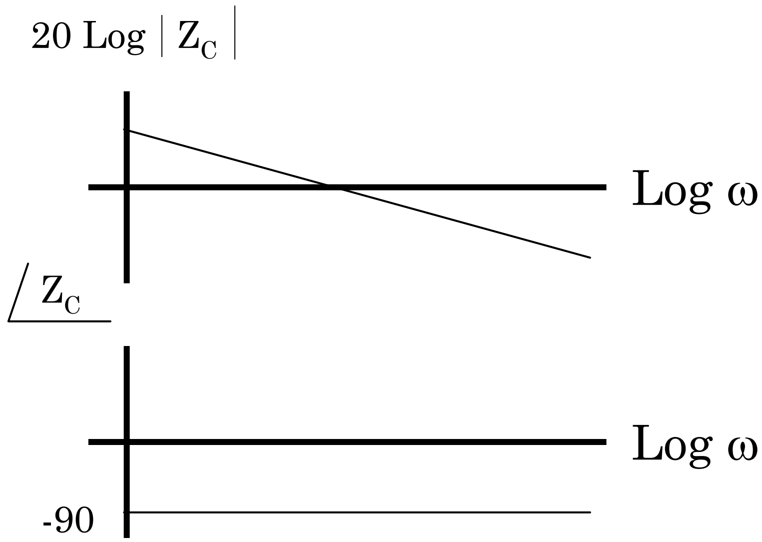
- Capacitors Revisited

$$i(t) = C\ dv(t) / dt$$

$$\mathscr{F}[i(t)] = \mathscr{F}[\ C\ dv(t) / dt\ ]$$

$$I(j\omega) = j\omega\ C\ V(j\omega)$$

$$Z_C(j\omega) = V(j\omega) / I(j\omega) = 1 / (j\omega\ C)$$



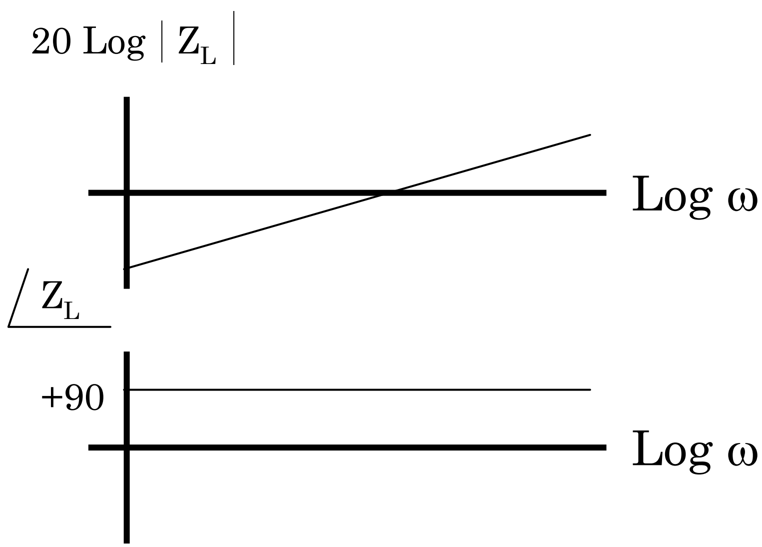
- Inductors Revisited

$$v(t) = L\ di(t) / dt$$

$$\mathscr{F}[v(t)] = \mathscr{F}[\ L\ di(t) / dt\ ]$$

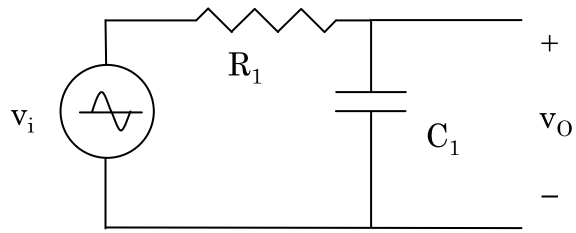
$$V(j\omega) = j\omega\ L\ I(j\omega)$$

$$Z_L(j\omega) = V(j\omega) / I(j\omega) = j\omega\ L$$



Frequency Domain Analysis

- Example 1 – Simple RC Circuit



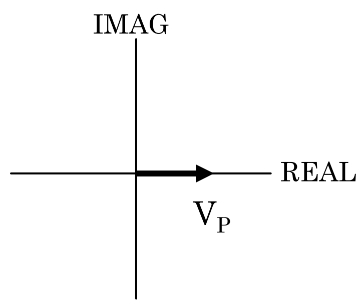
$v_i\,(t) = V_p \cos(\,\omega_O\ \ t)$

$v_O\,(t) = ?$

- First, express $v_i(t)$ in complex form

$$v_i(t) = V_P \cos(\omega_O t)$$

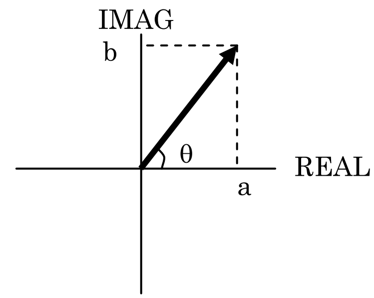
$$\left| V_i(j\omega) \right| = V_P$$



Phasor form: $V_i(j\omega) = V_P \underline{/0}$

Complex Numbers

$$F(j\omega) = a + j\,b$$



$$\text{Mag} = \sqrt{a^2 + b^2}$$

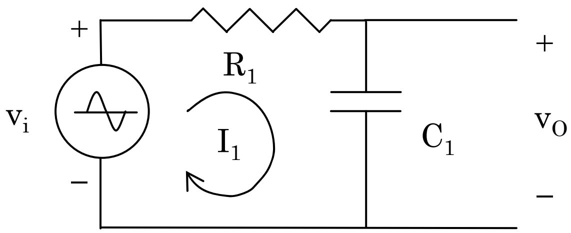
$$\text{Angle} = \tan^{-1}[b / a]$$

Phasor Notation:

$$F(j\omega) = \text{Mag}\ \underline{/\text{Angle}}$$

Frequency Domain Analysis

- Example 1 (Cont.)



$$v_i\,(t) = V_p \cos(\,\omega_O\;\; t)$$

- Next, express C_1 as an impedance in complex form

$$Z_{C1}(j\omega) = 1\,/\,(\,j\;\omega\;C_1)$$

- Find $V_O(j\omega)$ using Kirchoffs' Voltage Law

$$I_1\;\;R_1\;\;+\;\;I_1\;\;Z_{C1}(j\omega) - V_i\;(j\omega) = 0$$

<u>Solution → Voltage Divider:</u>

$$V_O(j\omega)\;\;=\;\;\frac{V_i\;(j\omega)\;Z_{C1}(j\omega)}{R_1 + Z_{C1}(j\omega)}$$

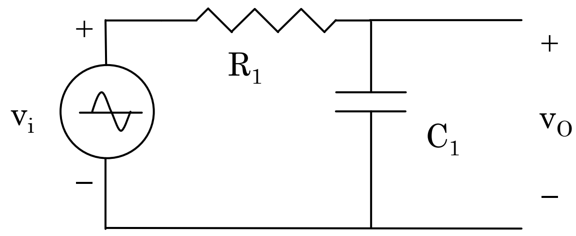
$$V_O(j\omega)\;\;=\;\;\frac{V_i\;(j\omega)\;[\;1\,/\,(\;j\;\omega\;C_1)\;]}{R_1 + [\;1\,/\;\;(j\;\omega\;C_1)\;]}$$

<u>Rearrange, using complex algebra:</u>

$$V_O(j\omega)\;\;=\;\;\frac{V_i\;(j\omega)}{j\;\omega\;R_1\;C_1\;\;+1}$$

Frequency Domain Analysis

- Example 1 (Cont.)



$$v_i(t) = V_p \cos(\omega_O t)$$

- Next, express in Phasor Form

$$V_O(j\omega) = \frac{V_i(j\omega)}{j\omega_o R_1 C_1 + 1}$$

$$= \frac{V_P \angle 0}{\sqrt{(\omega_o R_1 C_1)^2 + 1} \angle \tan^{-1}[\omega_o R_1 C_1]}$$

$$= \frac{V_P \angle -\tan^{-1}[\omega_o R_1 C_1]}{\sqrt{(\omega_o R_1 C_1)^2 + 1}}$$

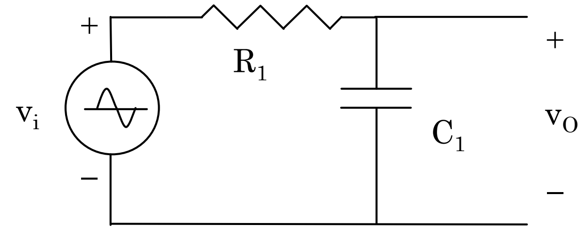
Solution:

$$V_O(j\omega) = \frac{V_P}{\sqrt{(\omega_o R_1 C_1)^2 + 1}} \angle -\tan^{-1}[\omega_o R_1 C_1]$$

$$v_O(t) = \frac{V_P}{\sqrt{(\omega_o R_1 C_1)^2 + 1}} \cos(\omega_o t - \tan^{-1}[\omega_o R_1 C_1])$$

Frequency Domain Analysis

- Example 1 (Cont.)
  - Try some numbers



$v_i(t) = V_p \cos(\omega_O\ t)$

$$v_O(t) = \frac{V_P}{\sqrt{(\omega_o R_1 C_1)^2 + 1}} \cos(\omega_o t - \tan^{-1}[(\omega_o R_1 C_1])$$

a) Let $R_1 = 1\ K\Omega$, $C_1 = 1\ \mu F$,
$V_P = 1V$, $\omega_o = 10$ rad/sec

Solution: $v_O(t) \approx 1 \cos(\omega_o t - 0^o)$ ← **Almost no change → $v_o(t) \approx v_i(t)$**

b) Let $R_1 = 1\ K\Omega$, $C_1 = 1\ \mu F$,
$V_P = 1V$, $\omega_o = 1000$ rad/sec → X100

Solution: $v_O(t) = 0.707 \cos(\omega_o t - 45^o)$ ← **Moderate change → $v_o(t)$ decreasing**

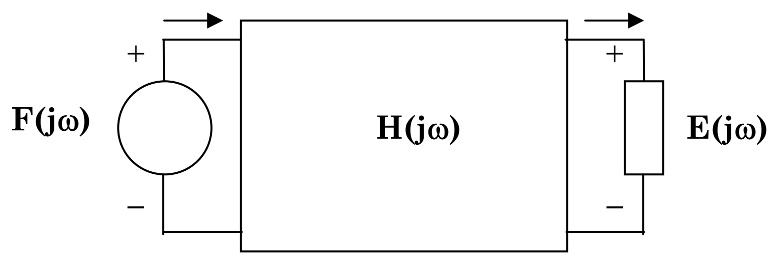
c) Let $R_1 = 1\ K\Omega$, $C_1 = 1\ \mu F$,
$V_P = 1V$, $\omega_o = 100{,}000$ rad/sec → X 10,000

Solution: $v_O(t) = 0.099 \cos(\omega_o t - 84.2^o)$ ← **Big change → $v_o(t)$ → 0**

⇨ ***We could plug in numbers all day long, but let's find a better way…***

Frequency Domain Analysis

- A useful tool to aid in analyzing circuits in the frequency domain is the concept of a ***Transfer Function***
  - Suppose have a network that contains R's, L's, C's and even transistors
  - Suppose have a single input $F(j\omega)$ (called a Forcing Function)
    - Could be either a voltage or a current
  - Suppose have a single output variable of interest $E(j\omega)$ that responds in some way to the input
    - Could also be either a voltage or a current
  - Denote the network as $H(j\omega)$



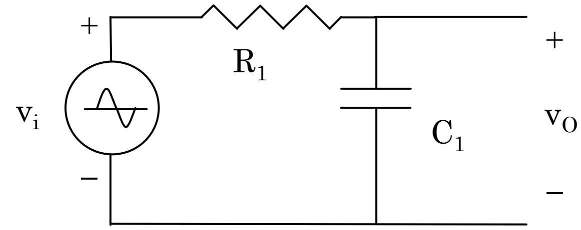
  - Then

$$H(j\omega) = \frac{E(j\omega)}{F(j\omega)} = \frac{\text{Output of Interest}}{\text{Forcing Function}}$$

⇨ ***We will use this concept extensively in Frequency Domain Analysis***

Frequency Domain Analysis

- Back to Example 1:
  - Back to Frequency Domain



$R_1 = 1\ K\Omega,\ C_1 = 1\ \mu F$

$$v_O(t) = \frac{V_P}{\sqrt{(\omega_o R_1 C_1)^2 + 1}} \cos(\omega_o t - \tan^{-1}[(\omega_o R_1 C_1])$$

$$V_O(j\omega) = \frac{V_i(j\omega)}{\sqrt{(\omega_o R_1 C_1)^2 + 1}} \angle -\tan^{-1}[(\omega_o R_1 C_1]$$ ⇨ **Phasor Form**

To find general solution, plot $V_O(j\omega) / V_i(j\omega)$ → ***Transfer Function*** ⇨ $H(j\omega) = \frac{\text{Output}(j\omega)}{\text{Input}(j\omega)}$

$$\frac{V_O(j\omega)}{V_i(j\omega)} = \frac{1}{\sqrt{(\omega R_1 C_1)^2 + 1}} \angle -\tan^{-1}[(\omega R_1 C_1] = \text{Mag} \angle \text{Angle}$$

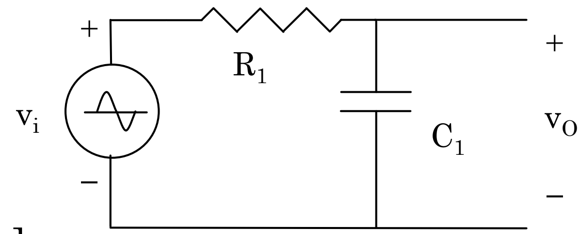
⇨ **Now, plot $|V_o / V_i|$ vs. ω, and $\angle V_o / V_i$ vs. ω**

(Actually, plot 20 Log$|V_o / V_i|$ vs. Log ω, and $\angle V_o / V_i$ vs. Log ω)

⇨ ***Called a Bode Plot***

Frequency Domain Analysis

- Example 1 (Cont.)
  - Plotting Transfer Functions

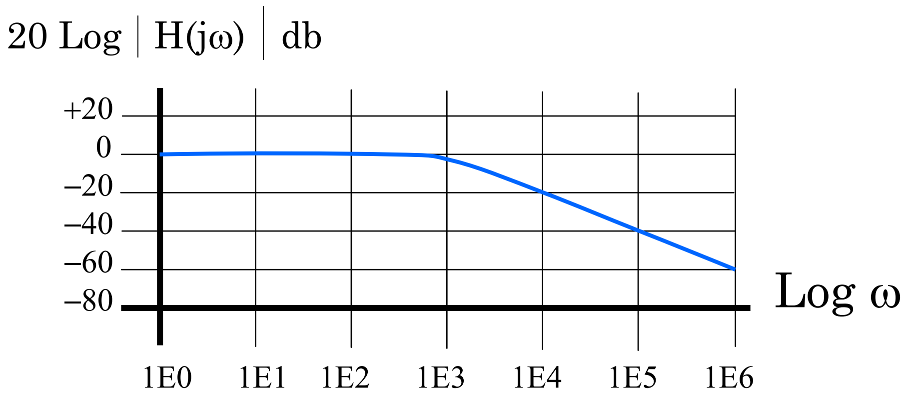


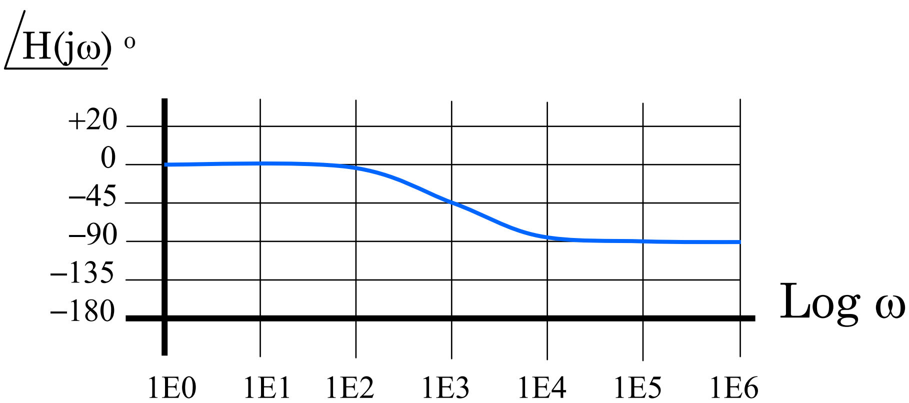
$R_1 = 1\ K\Omega,\ C_1 = 1\ \mu F$

$$H(j\omega) = \frac{V_O(j\omega)}{V_i(j\omega)} = \frac{1}{\sqrt{(\omega R_1 C_1)^2 + 1}} \angle -\tan^{-1}[(\omega R_1 C_1)]$$

$$|H(j\omega)| = \left[\frac{1}{\sqrt{(\omega R_1 C_1)^2 + 1}}\right]$$

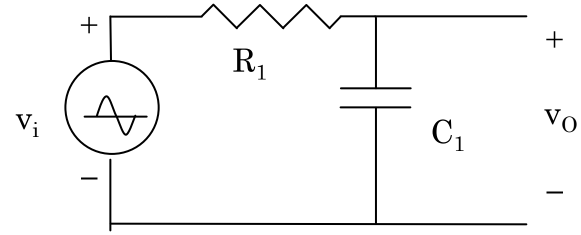
$$\angle H(j\omega) = -\tan^{-1}[(\omega R_1 C_1)]$$





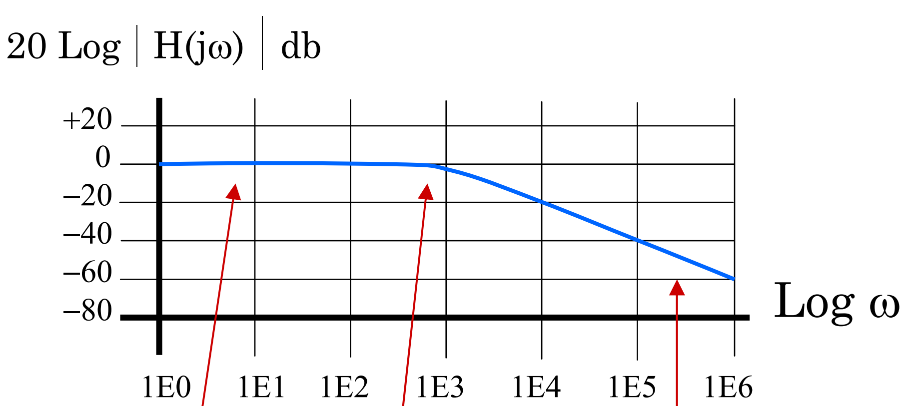
Frequency Domain Analysis

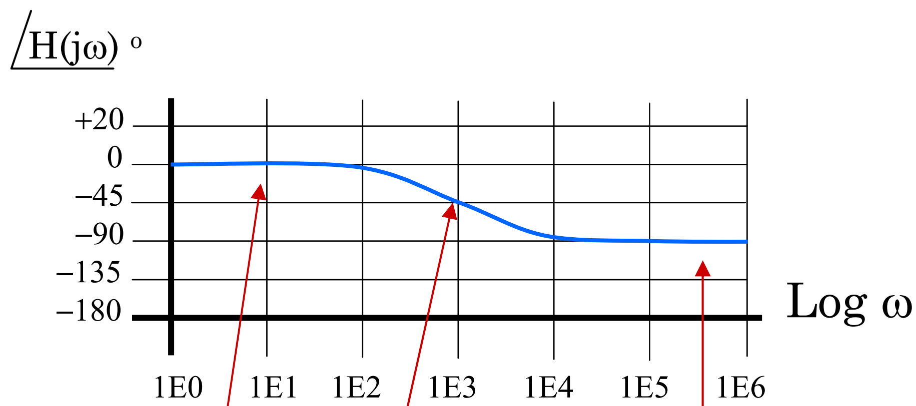
- Example 1 (Cont.)
  - Examining Bode Plots for features



$$H(j\omega) = \frac{V_O(j\omega)}{V_i(j\omega)} = \frac{1}{\sqrt{(\omega R_1 C_1)^2 + 1}} \;\angle -\tan^{-1}[(\omega R_1 C_1)]$$

$R_1 = 1\ \mathrm{K}\Omega,\ C_1 = 1\ \mu\mathrm{F}$





Response starts out flat

At critical frequency, see inflection point

Response falls at -20 db/decade

What is that frequency?

$\omega = 1 / R_1 C_1$

⇨ Called the "pole" frequency

⇨ Actual: –3 db

Response starts out flat

At critical frequency, see half-way point

Response ends flat, with -90° phase shift

What is that frequency?

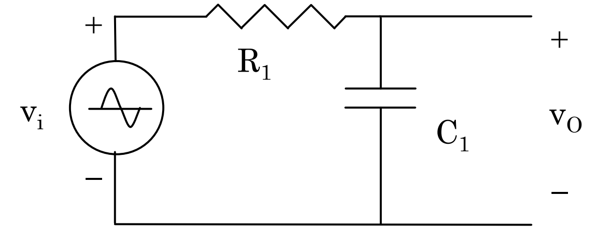
$\omega = 1 / R_1 C_1$

⇨ Again, the "pole" frequency

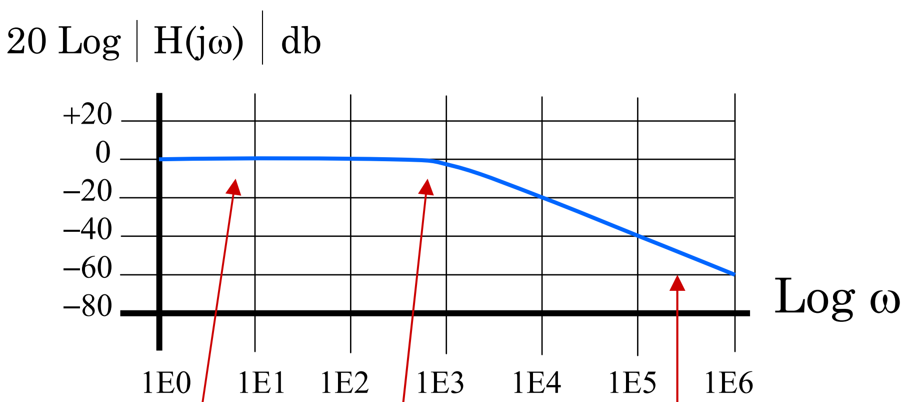
⇨ Actual: –45 °

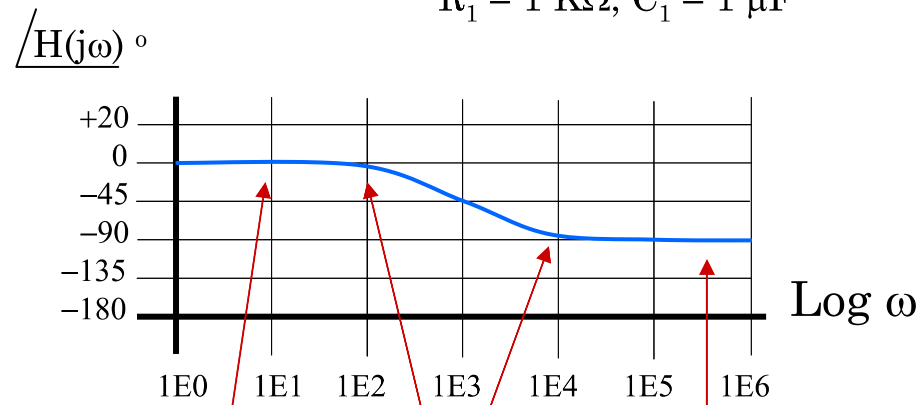
Frequency Domain Analysis

- Example 1 (Cont.)
  - General Guidelines for Bode Plots
    (Note: The following is true ONLY for single pole circuits (1 energy storage element)



$R_1 = 1\ K\Omega,\ C_1 = 1\ \mu F$





Straight line starts out flat at 0 db

Inflection point at:

$\omega = 1 / R_1 C_1$

Actual curve is -3 db down from intersection of the two straight lines

Straight line falls at -20 db/decade

Straight line starts out flat at 0 °

Inflection points at:

$\omega = 0.1 / R_1 C_1$

$\omega = 10 / R_1 C_1$

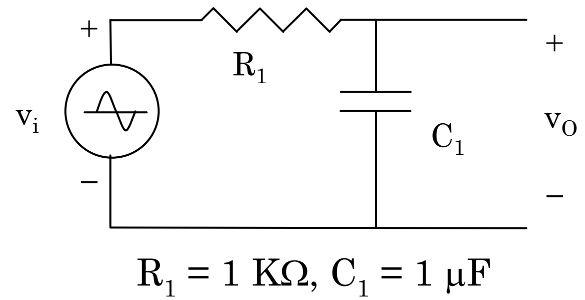
Curve = -45 ° at:

$\omega = 1 / R_1 C_1$

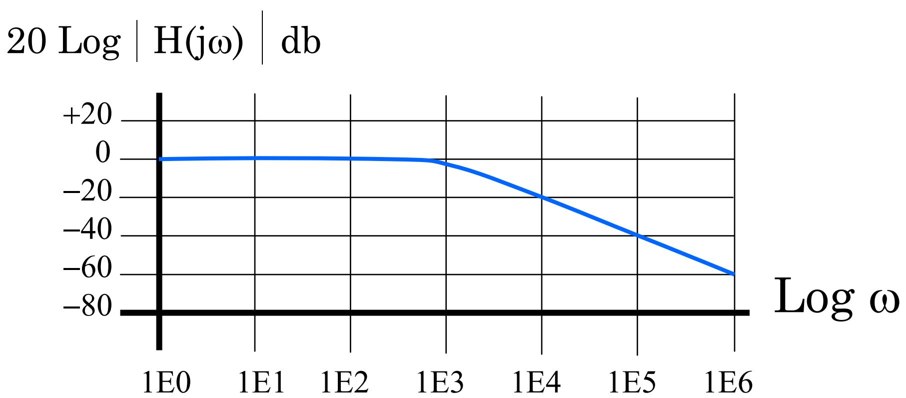
Straight line ends flat, with -90° phase shift

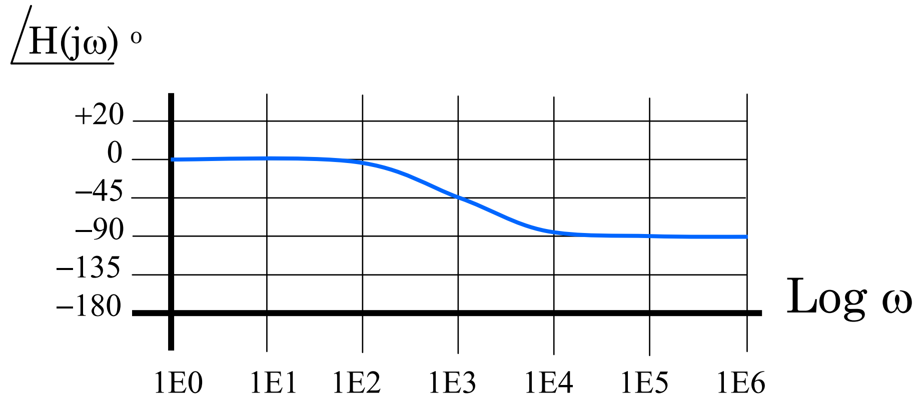
Frequency Domain Analysis

- Example 1 (Cont.)
  - This configuration is known as a ***"Low-Pass Filter"***
    - Low frequencies are passed with 0 db attenuation (→ Gain = 1)
    - High frequencies are attenuated
    - Filter frequency = pole frequency



$$\omega_o = 1 / R_1 C_1$$

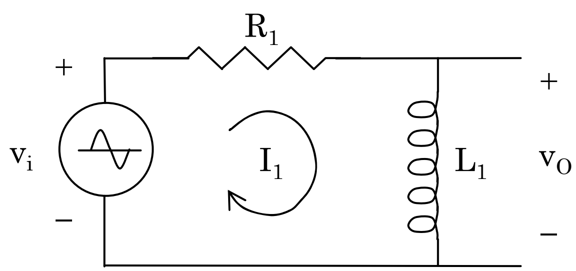




⇨ ***You can create Bode Plots almost by inspection !!!***

Frequency Domain Analysis

- Example 2 – Simple RL Circuit



$v_i\ (t) = V_p \cos(\ \omega_O\ \ t)$
$v_O\ (t) = ?$

- Find $V_O(j\omega)$ using Kirchoffs' Voltage Law → Voltage Divider

$$I_1\ R_1\ +\ I_1\ Z_{L1}(j\omega) - V_i\ (j\omega) = 0$$

$$V_O(j\omega)\ =\ \frac{V_i\ (j\omega)\ Z_{L1}(j\omega)}{R_1 + Z_{L1}(j\omega)}$$

- Insert expression for impedance of an inductor:

$$Z_{L1}(j\omega) =\ \ j\ \omega\ L_1$$

$$V_O(j\omega)\ =\ \frac{V_i\ (j\omega)\ j\ \omega\ L_1}{R_1 + j\ \omega\ L_1}\ =\ \frac{V_i\ (j\omega)\ j\ \omega\ L_1\ /\ R_1}{1 + j\ \omega\ L_1\ /\ R_1}$$

- Transfer Function:

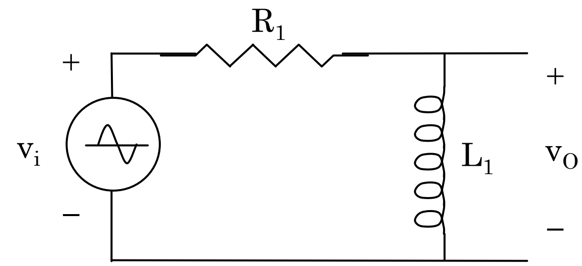
Now have frequency content in the numerator

$$\frac{V_O(j\omega)}{V_i(j\omega)}\ =\ \frac{\omega\ L_1\ /\ R1\ \ \angle 90}{\sqrt{(\omega\ L_1\ /\ R_1)^2 + (1)^2}\ \angle \tan^{-1}\ [\ (\omega\ L_1\ /\ R_1\ ]}$$

Denominator is ~ the same as before
→ Have a pole at $\omega = R_1\ /\ L_1$

Frequency Domain Analysis

- Example 2 (Cont.)
  - Try some numbers: Let $R_1 = 100\ \Omega$, $L_1 = 100$ mH

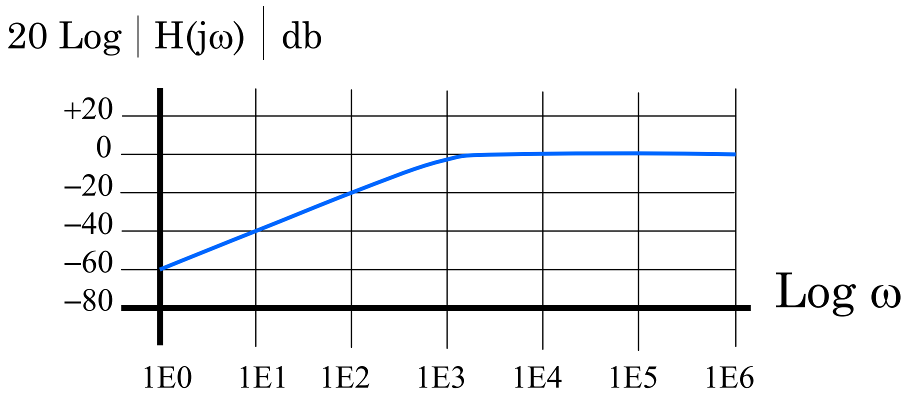


$$v_i(t) = V_p \cos(\omega_O\ t)$$

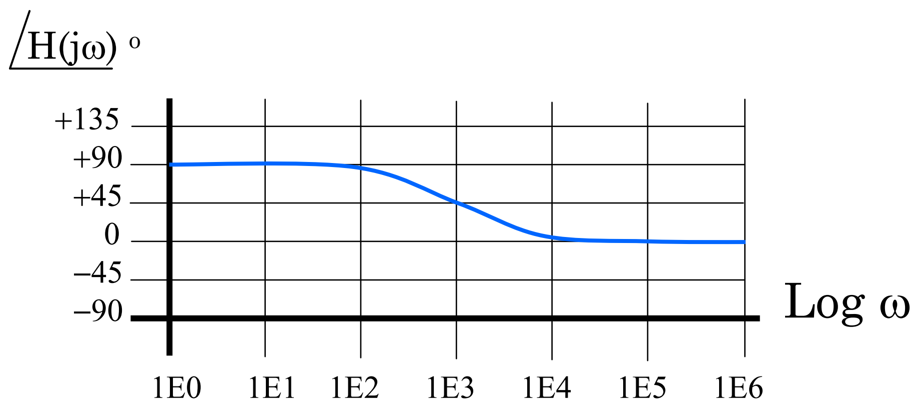
$$\frac{V_O(j\omega)}{V_i(j\omega)} = \frac{\omega L_1 / R1 \ \angle 90}{\sqrt{(\omega L_1 / R_1)^2 + (1)^2}\ \angle \tan^{-1}[(\omega_o L_1 / R_1]}$$

$$\omega_o = R_1 / L_1 = 1000 \text{ rad/sec}$$

$$|H(j\omega)| = \left[\frac{\omega L_1 / R1}{\sqrt{(\omega L_1 / R_1)^2 + (1)^2}}\right]$$

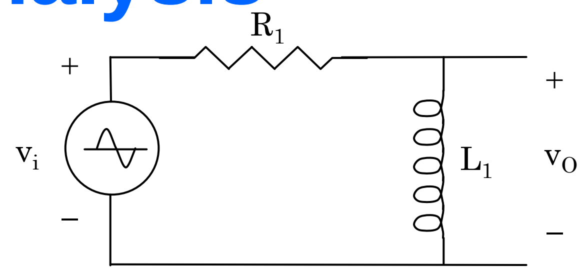


$$\angle H(j\omega) = 90 - \tan^{-1}[(\omega_o L_1 / R_1]$$

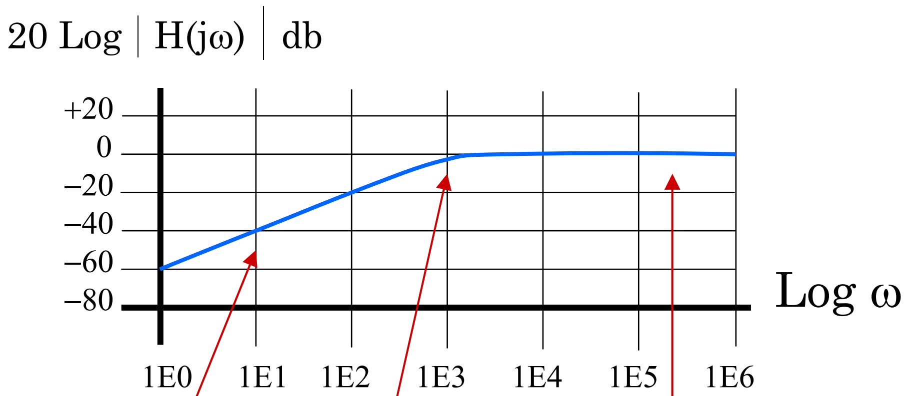


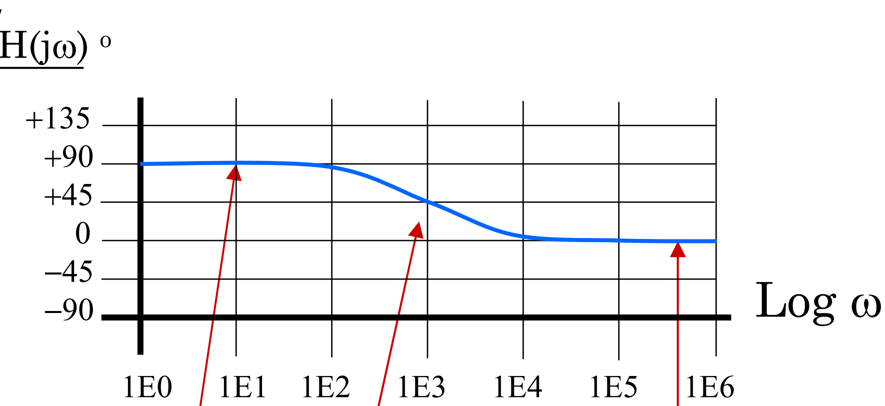
Frequency Domain Analysis

- Example 2 (Cont.)
  - Examining Bode Plots for features



$$\frac{V_O(j\omega)}{V_i(j\omega)} = \frac{\omega L_1 / R1 \;\angle 90}{\sqrt{(\omega L_1 / R_1)^2 + (1)^2} \;\angle \tan^{-1} [(\omega_o L_1 / R_1]}$$





Response rises at +20 db/decade

Response ends flat

At critical frequency, see inflection point

What is that frequency?

$\omega = R_1 / L_1$

⇨ **It's a "pole" frequency**

⇨ **Actual: –3 db**

Response starts out flat With +90º Phase shift

Response ends flat

At critical frequency, see half-way point

What is that frequency?

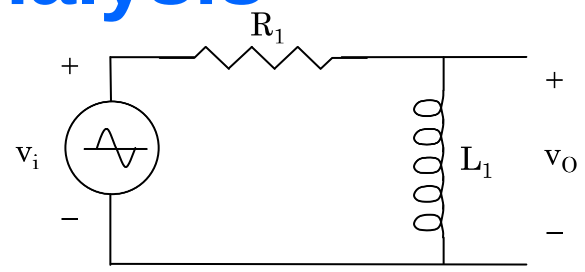
$\omega = 1 / R_1 C_1$

⇨ **Again, the "pole" frequency**

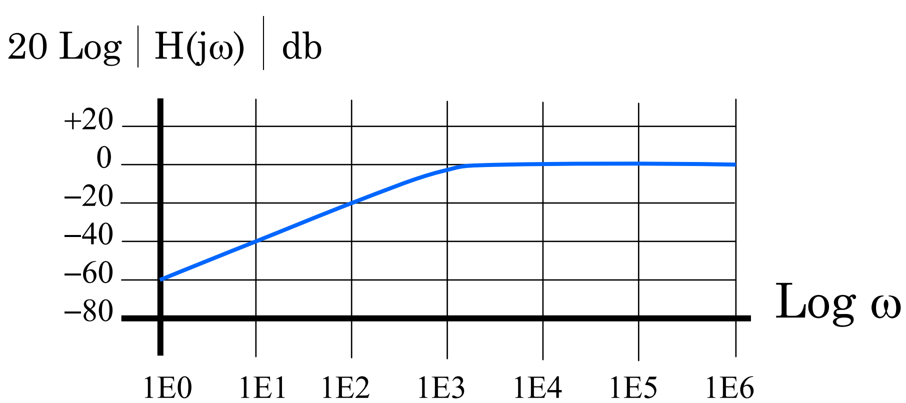
⇨ **Actual: –45 º**

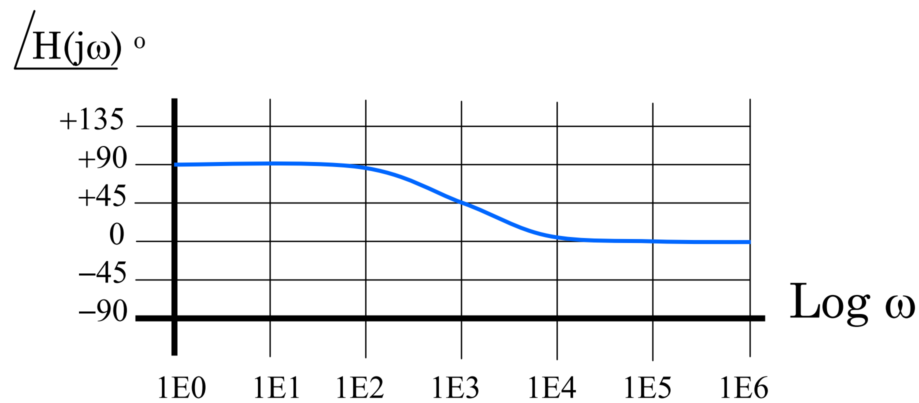
Frequency Domain Analysis

- Example 2 (Cont.)
  - This configuration is known as a ***"High-Pass Filter"***
    - High frequencies are passed with 0 db attenuation (→ Gain = 1)
    - Low frequencies are attenuated
    - Filter frequency = pole frequency $\omega_o = R_1 / L_1$
    - Also has a Zero at 0 frequency



$$v_i\,(t) = V_p \cos(\,\omega_O\ t)$$





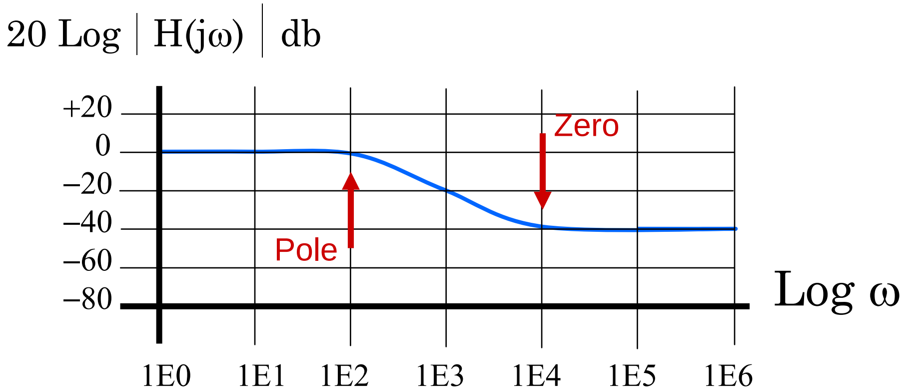
Frequency Domain Analysis

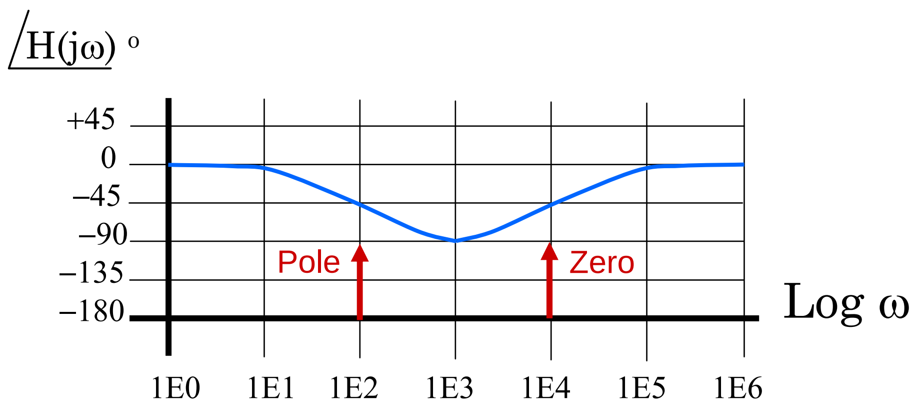
- General Features of Transfer Functions & Bode Plots
  - For a general transfer function $H(j\omega)$, express as:

$$H(j\omega) = \frac{V_O(j\omega)}{V_i(j\omega)} = \frac{A + jB}{F + jG} = \frac{A}{F}\,\frac{1 + j\,B/A}{1 + j\,G/F} = \frac{A}{F}\,\frac{\sqrt{(B/A)^2 + (1)^2}\;\angle \tan^{-1}[B/A]}{\sqrt{(G/F)^2 + (1)^2}\;\angle \tan^{-1}[G/F]}$$

Complex Numbers → **1 + j B/A Form** → **Phasors – Magnitude & Phase**

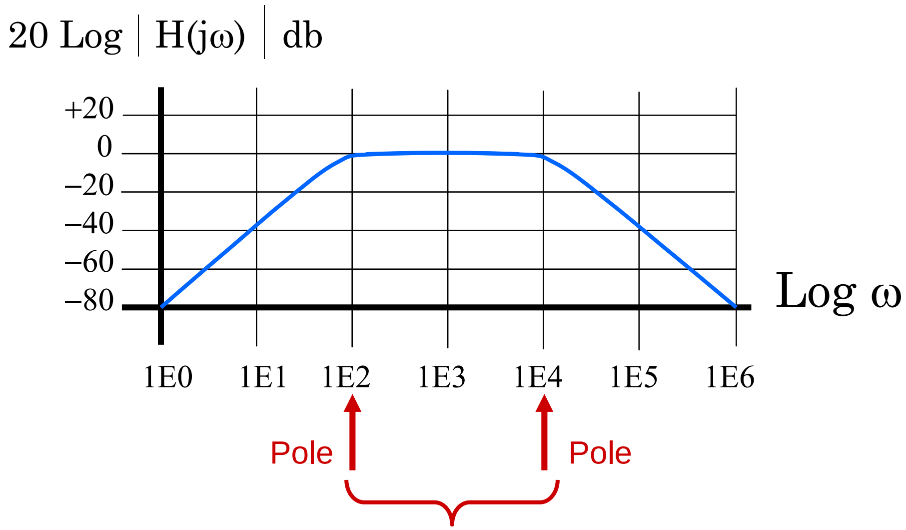
- The denominators give the ***Poles*** of the Transfer Function ⇨ **Where G/F = 1**
  - Magnitude changes by –20 db/decade
  - Phase lag → phase changes by -90^o
- The numerators give the ***Zeros*** of the Transfer Function ⇨ **Where B/A = 1**
  - Magnitude changes by +20 db/decade
  - Phase Lead → Phase changes by $+90^o$





Frequency Domain Analysis

- General Features of Transfer Functions & Bode Plots (Cont.)
  - Bandwidth
    - Generally concerned with points in frequency where the response begins to fall off
      ⇨ **Look for – 3db points**
    - Consider a typical amplifier



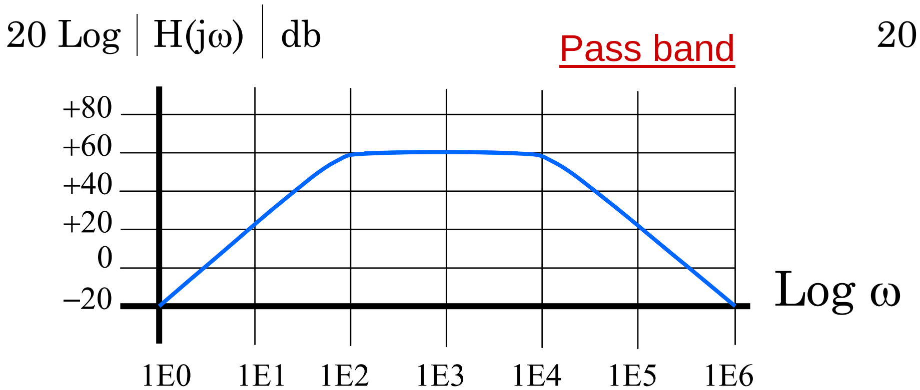
⇨ **For this example, the Bandwidth would be stated as: 100 rad/sec to 10K rad/sec (16 Hz to 1600 Hz)**

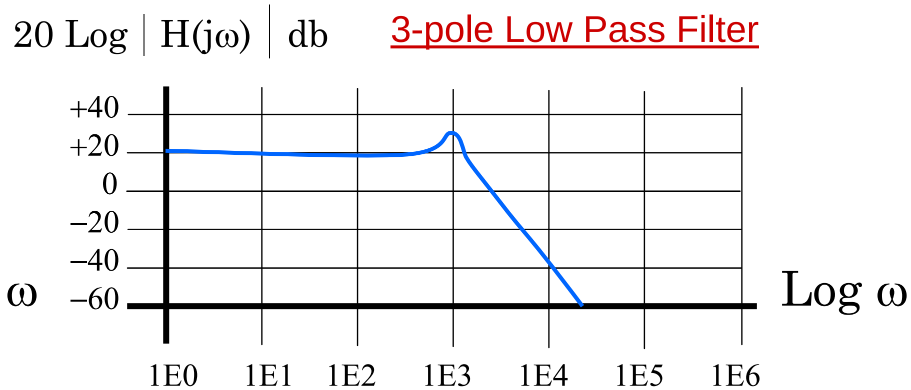
  - The pass band is defined as the range of frequencies where the response is flat

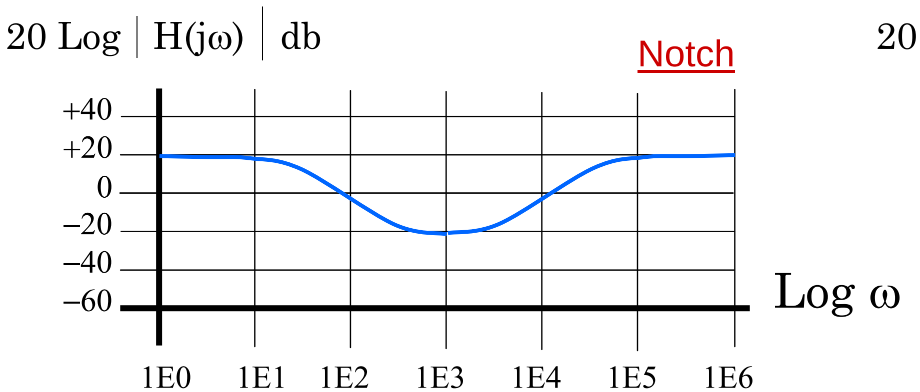
⇨ ***Exercise: Suppose that stereo has a frequency response of 20 Hz to 20 KHz, and a maximum gain of 30 db. Can you draw the frequency plot?***

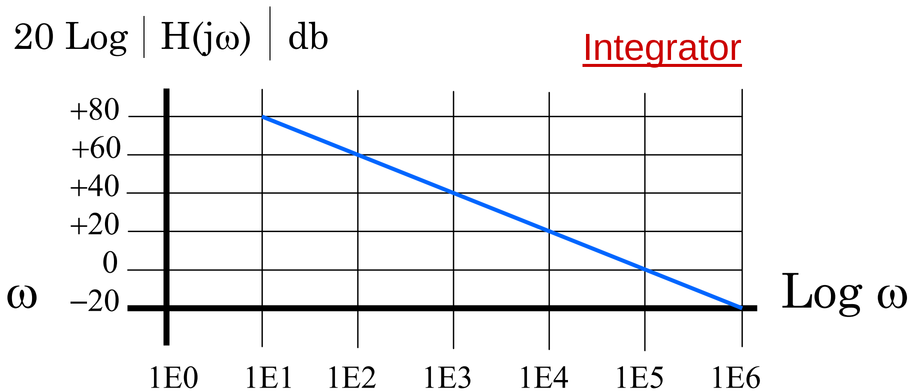
Filters

- Analog Filters
  - Many types
  - Most use “active” components (i.e. op amps), and have Gain
  - A few examples:



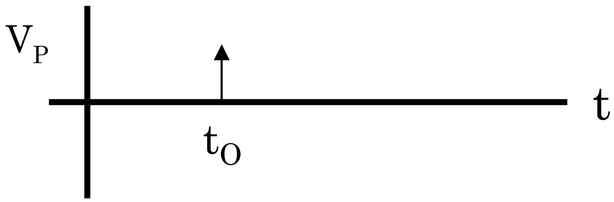




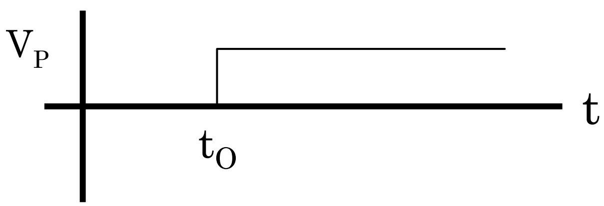


Aperiodic Signal Sources

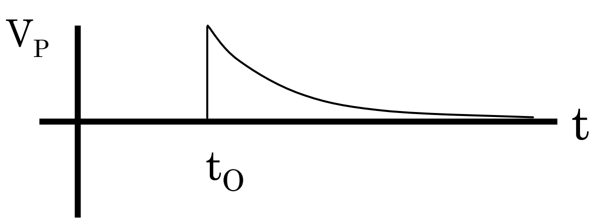
- Aperiodic Sources
  - Impulse $v(t) = V_p\,\delta(t - t_O)$



  - Step $v(t) = V_p\,u(t - t_O)$



  - Exponential $v(t) = V_p\;e^{-a(t-to)}\,u(t-t_o)$



- For aperiodic signals,
  → ***Laplace Transform***

$$\mathscr{L}[v(t)] = V(s) = \int_{-\infty}^{\infty} v(t)\;e^{-st}\;dt$$

$$\mathscr{L}^{-1}[V(s)] = v(t) = \int_{C-j\infty}^{C+j\infty} V(s)\;e^{st}\;ds$$

where $s = a + j\omega$

- Two important properties:

$$\mathscr{L}[d/dt\;v(t)] = s\,V(s) - v(0^-)$$

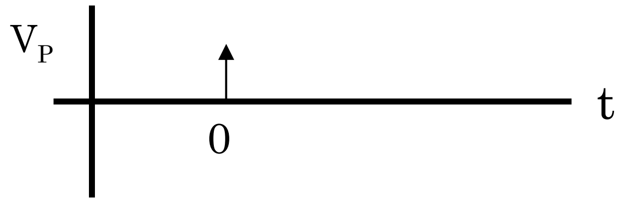
$$\mathscr{L}[\int_0^t v(\lambda)\;d\lambda\,] = V(s)\,/\,s$$

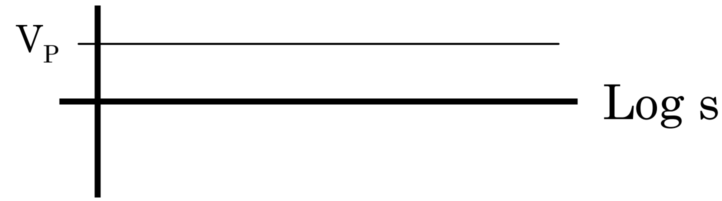
Aperiodic Signal Sources

- Aperiodic Sources

Time Domain **Frequency Domain**

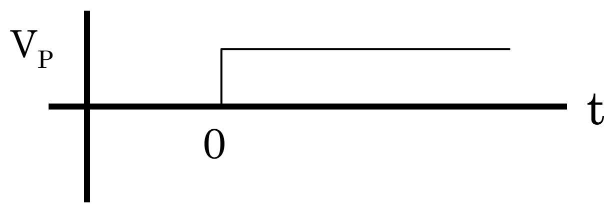
- Impulse $v(t) = V_p\,\delta(t)$ $\mathscr{L}[v(t)] = V_p$

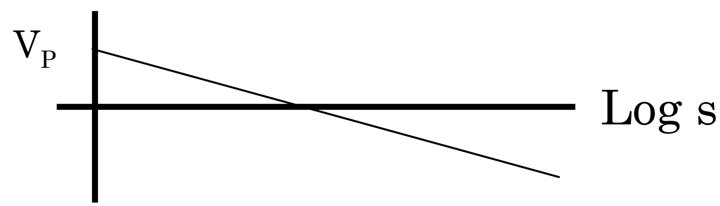




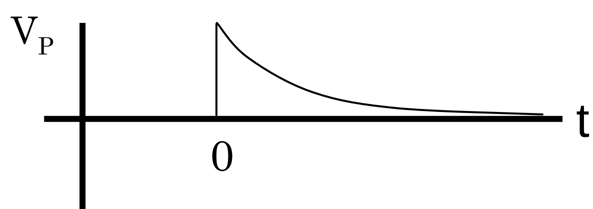
- Step

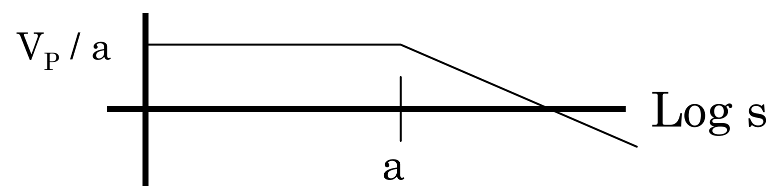
$v(t) = V_p\,u(t)$ $\mathscr{L}[v(t)] = V_p / s$





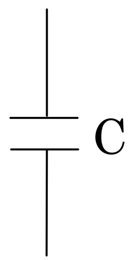
- Exponential $v(t) = V_p\,e^{-at}\,u(t)$ $\mathscr{L}[v(t)] = V_p / (s + a)$





Aperiodic Signal Sources

- What about Impedances Z_C and Z_L?
  - Replace $j\omega \rightarrow s$
  - Capacitors Revisited



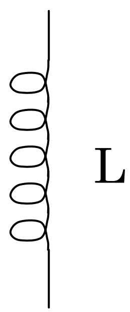
$$i(t) = C\, dv(t) / dt$$

$$\mathcal{L}[i(t)] = \mathcal{L}[\, C\, dv(t) / dt \,]$$

$$I(s) = s\, C\, V(s)$$

$$\boxed{Z_C(s) = V(s) / I(s) = 1 / (s\, C)}$$

  - Inductors Revisited



$$v(t) = L\, di(t) / dt$$

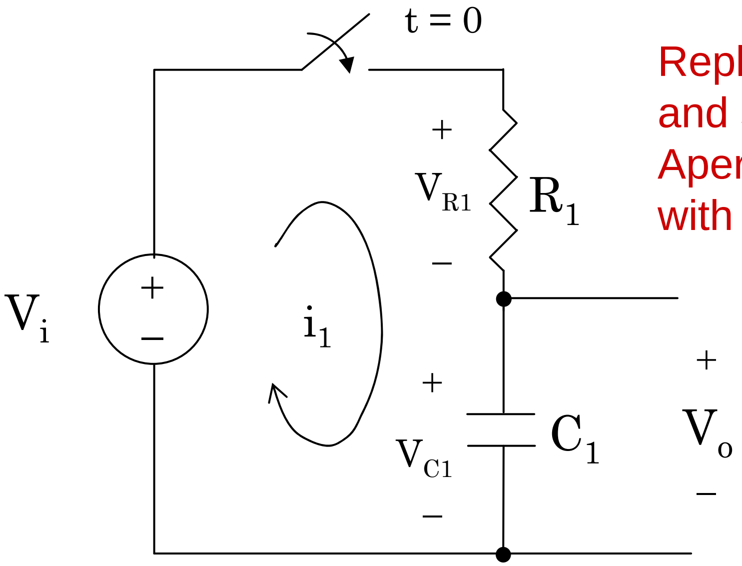
$$\mathcal{L}[v(t)] = \mathcal{L}[\, L\, di(t) / dt \,]$$

$$V(s) = s\, L\, I(s)$$

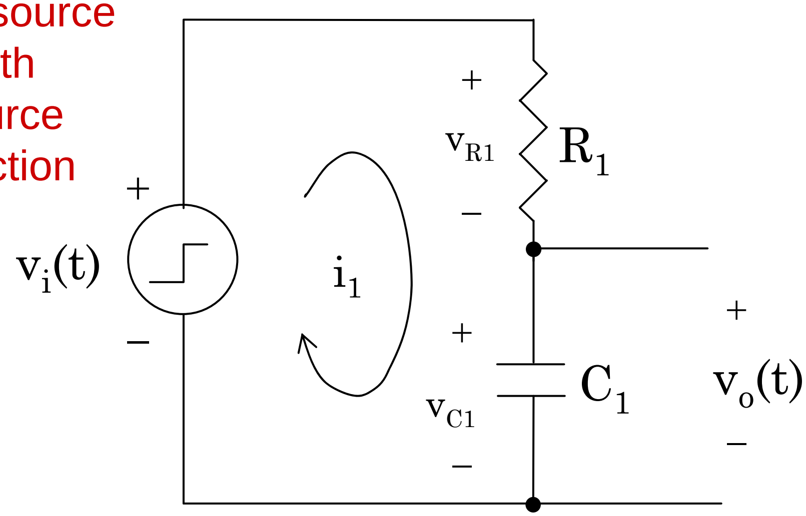
$$\boxed{Z_L(s) = V(s) / I(s) = s\, L}$$

Aperiodic Signal Sources

- Recall RC circuit



Replace DC source and switch with Aperiodic source with step function



$$\mathscr{L}[V_P\, u(t)] = V_p / s$$

$$Z_{C1}(s) = 1 / (s\, C_1)$$

Replace jω with s → Laplace Transform

$$V_O(s) = \frac{V_i(s)\, Z_{C1}(s)}{R_1 + Z_{C1}(s)} = \frac{V_P}{s\,(R_1\, s\, C_1 + 1)} = \underbrace{\frac{V_P}{s} - \frac{V_P\,[\,1 / R_1\, C_1]}{[R_1\, s\, C_1 + 1]}}_{\text{Partial fraction expansion}} = \frac{V_P}{s} - \frac{V_P}{s + R_1\, C_1}$$

$$\mathscr{L}^{-1}[V_O(s)] = V_P\,[\,1 - e^{-t/R1C1}]\, u(t)$$

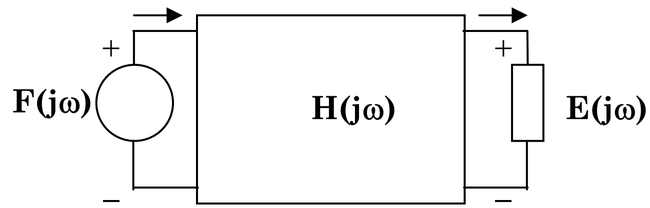
Partial fraction expansion

⇨ **Easier, yes?...**

Two Basic Circuit Principles

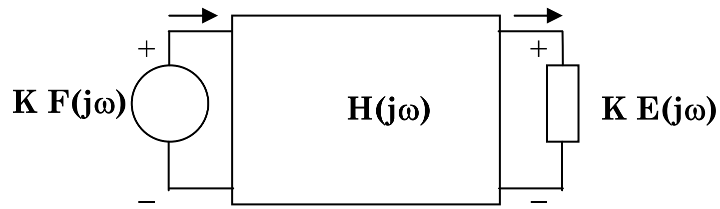
- Linearity
  - If a system H is linear, and has response E_o to forcing function input F_i, such that:

$$E_o(j\omega) = H(j\omega)\, F_i(j\omega)$$



then if the forcing function is multiplied by a constant factor K (a real number), the output responds as:

$$H(j\omega)\, [\, K\, F_i(j\omega)] = K\, E_o(j\omega)$$



  - Networks that contain resistors, capacitors, and inductors are linear networks
  - ***Networks that contain semiconductor devices may or may not be linear***
    - Depends on how the semiconductors are biased or being used

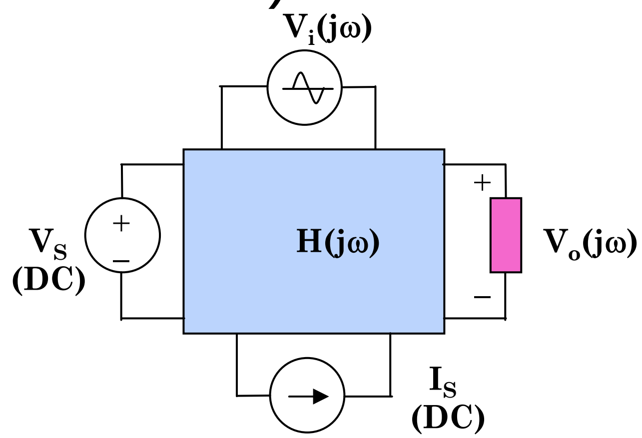
⇨ ***More on this in the next session***

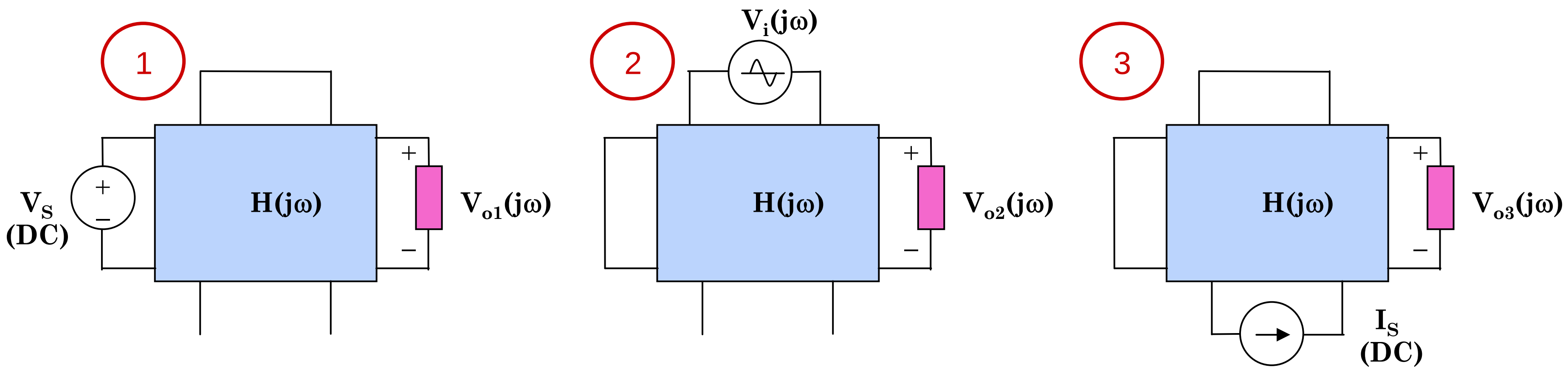
Two Basic Circuit Principles

- Superposition
  - If any linear network contains several independent sources (voltage sources or current sources), the quantity of interest (voltage across a component or current through a component) may be calculated by analyzing the circuit with one source at a time, with the other sources made "dead":
    - Voltage sources are replaced by short circuits (→ 0 impedance)
    - Current sources are replaced by open circuits (→ infinite impedance)
  - The complete response then is obtained by adding together the individual responses

Two Basic Circuit Principles

- Superposition (Continued)
  - Example:





- Then: $V_o(j\omega) = V_{o1}(j\omega) + V_{o2}(j\omega) + V_{o3}(j\omega)$

⇨ ***Will use this idea for analyzing amplifier circuits with DC & AC sources***

Time-Varying Circuits

- Concluding Remarks
  - Background presented here is the basis for all of modern communications
    - How can you have 500 cable channels and mixed internet on a single coaxial cable?...

      Answer: Because superposition works…

  - It is also the primary method by which analog circuits are designed and analyzed