Basic Electronics

Introductory Lecture Course for

Technology and Instrumentation in Particle Physics 2011

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Presented By

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Outline

- Session 1 – Preliminary Concepts, Passive Devices
  - Basic Concepts
  - Passive Components that Dissipate Energy (R's)
  - Basic Network Analysis Techniques
  - Passive Components that Store Energy (L's & C's)
  - Introduction to RL & RC Circuits
- Session 2 – Time-Varying Signals & Circuits
  - Time-Varying Signal Sources
  - Introduction to Fourier & Laplace Transforms
  - Principles of Frequency Domain Analysis
- Session 3 – Semiconductor Devices
  - Fundamentals of Semiconductors
  - Diodes
  - MOSFETs
  - Bipolar Junction Transistors
  - Semiconductor Circuits

A Definition

The field of electronics is both the science and the art of controlling the motion of charged particles through the precise manipulation of electric and magnetic fields, frequently through the use of very high levels of abstraction, to achieve the execution of work in a highly organized way, so as to realize physical functions at a level of structure and order not normally found in nature.

-G. Drake

(Disclaimer: Not from Webster's Dictionary…)

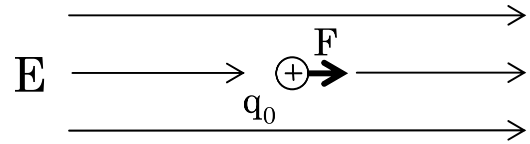
Session 1

Preliminary Concepts, Passive Devices

Preliminary Concepts

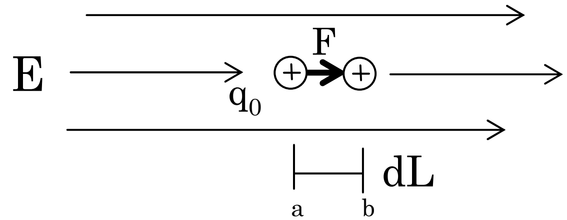
- The Electric Field
  - "A point charge in an electric field will experience a force that is proportional to the strength of the electric field" → Coulomb Force

$$\mathbf{F} = q_0\,\mathbf{E} = m\,\mathbf{a} \quad \rightarrow \quad \mathbf{a} = q_0\,\mathbf{E} / m$$



- Electrostatic Potential: Voltage
  - If a particle under the influence of a conservative force $\mathbf{F}$ is displaced by a distance dL, then there is a change in potential energy expressed by:

$$\mathbf{dU} = -\,\mathbf{F} \cdot \mathbf{dL}$$
$$= -\,q_0\,\mathbf{E} \cdot \mathbf{dL}$$



  - The ***voltage potential V*** is defined as the change in potential energy divided by value of the test charge

$$\mathrm{dV} = \mathbf{dU} / q_0 = -\,\mathbf{E} \cdot \mathbf{dL}$$

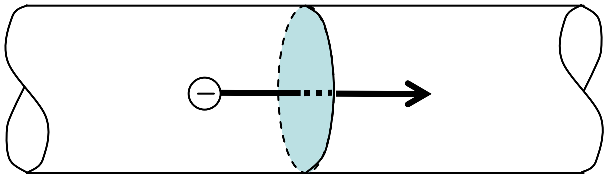
$$\mathrm{V} = -\int_a^b \mathbf{E} \cdot \mathbf{dL}$$

  - Units are in Volts   1 Volt = 1 Newton-Meter/Coulomb = 1 Joule/Coulomb

Preliminary Concepts

- Electrical Current Defined
  - ***Electrical current I*** is defined as the flow of charge carriers (rate) across an imaginary plane that is perpendicular to the flow

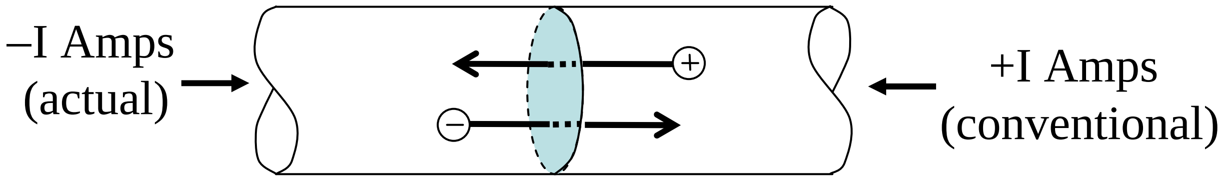
$$I = dQ / dt \cong \Delta Q / \Delta t \cong N\, e / \Delta t$$



- In electronics, charge carriers are generally electrons (e = -1.6E-19 Coulombs per electron)
- Units are in Coulombs/sec, or Amperes

  1 Coulomb/second = 1 Ampere

- Thanks to Benjamin Franklin, (remember the kite in the thunderstorm?), electronics uses "*conventional current flow,*"
  - The direction of conventional current is the opposite of the direction that electrons are actually flowing → Are they equivalent? …



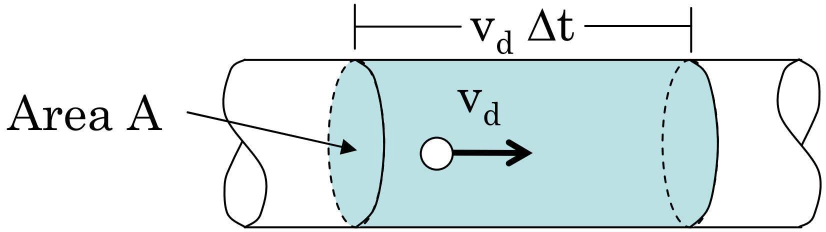
Preliminary Concepts

- Current Density Defined
  - Consider charge carriers q in a volume V moving with drift velocity v_d:
    - If the density of charge carriers is n [#/unit volume], in a time Δt, the charge ΔQ that passes through area A is given by:

$$\Delta Q = n\ q\ v_d\ A\ \Delta t$$

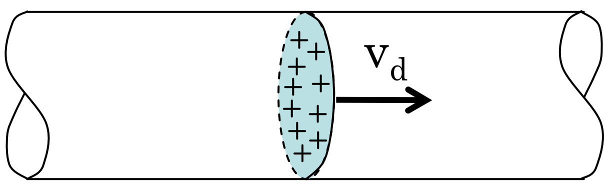
So that

$$I = \Delta Q\ /\ \Delta t = n\ q\ v_d\ A$$



  - ***Current density J*** is defined as the density of charge carriers flowing through a unit area, infinitely thin

$$J = I\ /\ A = \Delta Q\ /\ (A\ \Delta t) = n\ q\ v_d$$



or, in vector form:

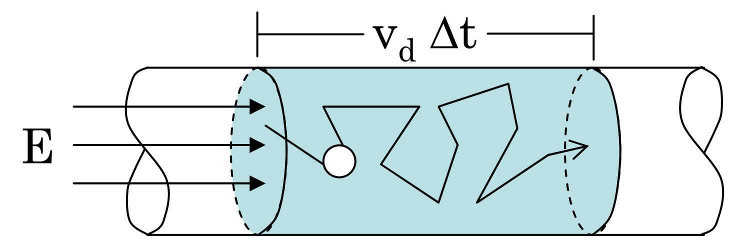
$\mathbf{J} = n\ q\ \mathbf{v_d}$ where the direction of $\mathbf{J}$ is in the direction of $\mathbf{v_d}$

Then, current I is given by:

$I = \int \mathbf{J} \cdot \mathbf{ň}\ dA$ where ň is the unit normal to A

Preliminary Concepts

- Relationship Between J and E
  - What determines drift velocity in a conductor, i.e. why not $v_d = c$?
    - Answer: scattering off atoms
    - For each segment, assume that charged particle starts from rest, and accelerates to a final velocity v_f



$$\mathbf{F} = q_0 \, \mathbf{E} = m \, \mathbf{a} \rightarrow \mathbf{a} = q_0 \, \mathbf{E} / m$$

$$v_f(t_f) = \int a \, dt = q_0 \, E \, t_f / m$$

- The average velocity for a given path is $v_f / 2$
- On average, the charged particle spends a certain time accelerating in the electric field before it is captured or scattered
  - Called the mean-free time τ
- The mean velocity, v_d, it given by

$$\mathbf{v_d} = q \, \mathbf{E} \, \tau / (2 \, m) \rightarrow \mathbf{J} = n \, q \, \mathbf{v_d} = n \, q \, \mathbf{E} \, \tau / (2 \, m)$$

Preliminary Concepts

- Define Conductivity σ

$$\mathbf{J} = n\,q\,\mathbf{E}\,\tau / (2\,m)$$

$\sigma = n\,q\,\tau / (2\,m)$ → property of the material: n & τ

$\mathbf{J} = \sigma\,\mathbf{E}$ → Units: Coulomb-sec/Kg-meter^3
= Siemens/meter

- Define Resistivity ρ

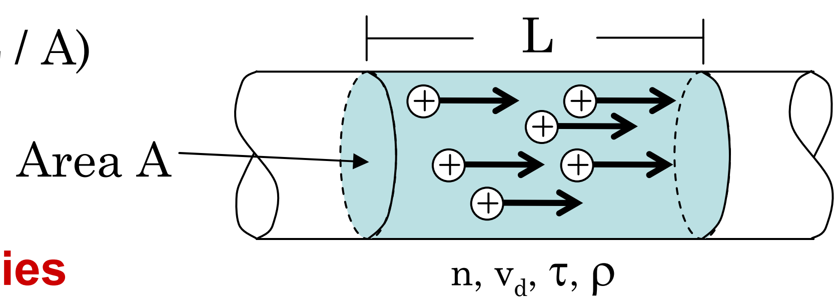
$\rho = 1 / \sigma = 2\,m / (n\,q\,\tau)$ → Units: Kg-meter^3 / Coulomb-sec
= Ohm-meter
(or often, Ohm-cm)

- Ohm's Law

$$J = \sigma\,E$$

$$I\ / A \cong \sigma\ dV / dL \cong \sigma\,\Delta V\ /\ L$$

$$\Delta V = I\ L\ / (\sigma\ A) = I\,(\rho\,L / A)$$



- Define Resistance

$$R = \rho\,L / A$$

⇨ **Only depends on physical properties**

Basic Circuit Concepts

- Ohm's Law

$\Delta V = I \, (\rho \, L / A)$

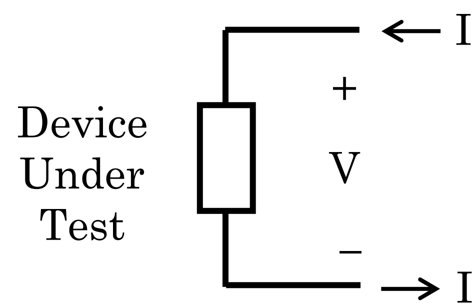
$V = I \, R$	- DC sources, resistive loads
$V(t) = I(t) \, R$	- AC sources, resistive loads
$V(t) = I(t) \, Z(t)$	- Time-dependent impedance $Z(t)$
$V(j\omega) = I(j\omega) \, Z(j\omega)$	- Frequency domain

- Define Conductance, Admittance

$G = 1 / R$	- Conductance
$I = V \, G$	- DC sources, resistive loads
$I(t) = V(t) \, G$	- AC sources, resistive loads
$Y(t) = 1 / Z(t)$	- Admittance $Y(t)$
$I(t) = V(t) \, Y(t)$	- Time-dependent impedance
$I(j\omega) = V(j\omega) \, Y(j\omega)$	- Frequency domain

Basic Circuit Concepts

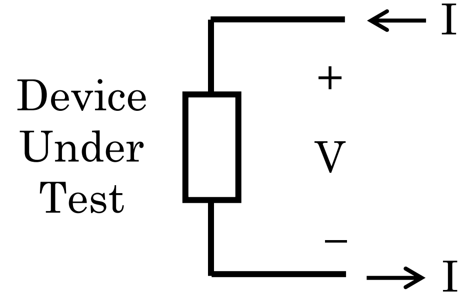
- Standard Convention – 2-Terminal Devices
  - Assign + and – designations to terminals
    - Arbitrary, but may have reasons to assign polarities in a particular way
  - Positive current enters + terminal
  - Positive current leaves – terminal
  - For 2-terminal devices, I_{IN} must equal I_{OUT}
  - At the end, might find that V or I (or both) are negative
    - That's OK – Just means that actual current or voltage is opposite of what was assumed

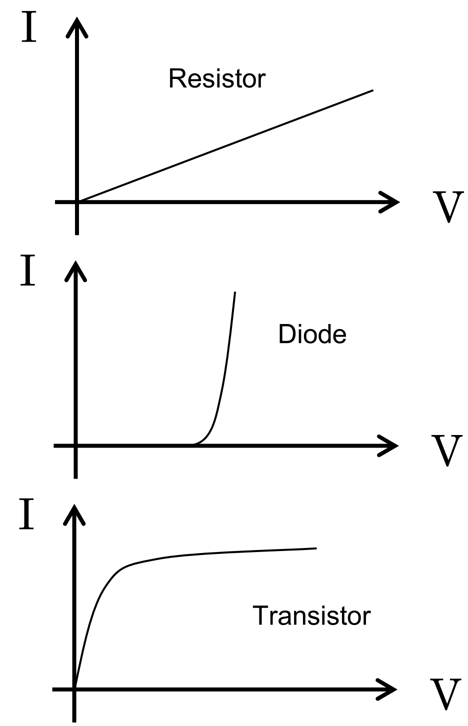


- Nomenclature
  - Upper case: DC Variables → V_O
  - Lower case: AC variables → $v_o(t)$

Basic Circuit Concepts

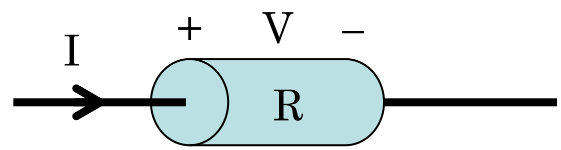
- I-V Characteristics
  - Often interested in the IV characteristics of the device under test
  - Plot I on Y-axis, V on x-axis
    - Reason will become clear when we get to semiconductors
  - The shape of the curve (in particular, the slope) tells something about the characteristics of the device
    → By inspection





Time-Invariant Passive Components

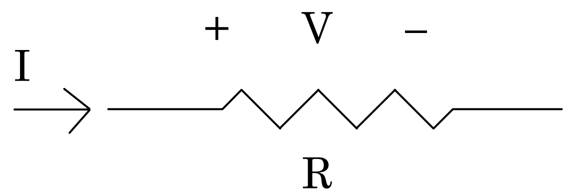
- Resistors
  - I-V Relationship



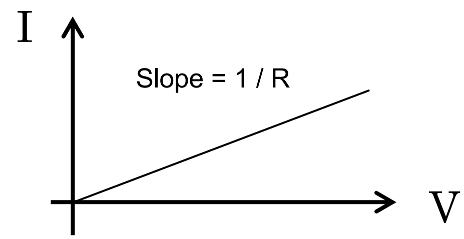
$$V = I\ R$$

$$I = V / R$$

  - Standard Symbol



  - I-V Characteristics
    - Straight line
    - Goes through (0,0)
    - Slope = $1/R$

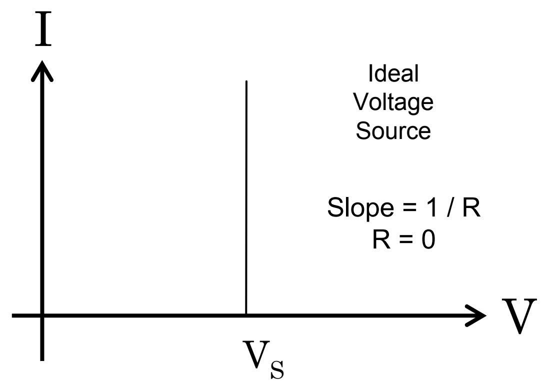


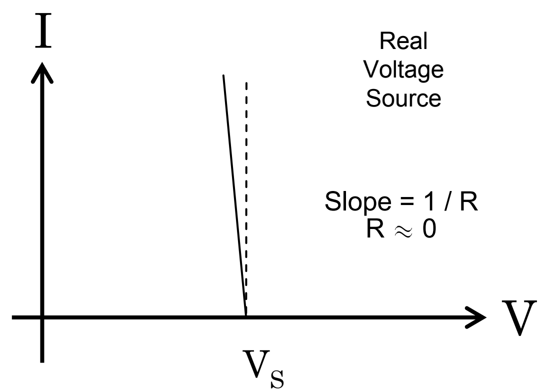
  - Properties
    - Resistors do not store energy, only dissipate → heat
    - Resistance is time invariant (to first order)
    - Resistance is often sensitive to temperature: $\rho = 2\ m / (n\ q\ \tau)$

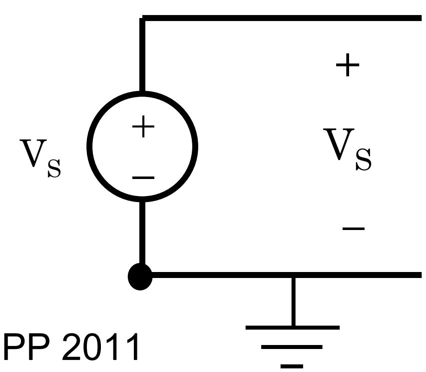
Time-Invariant (DC) Electrical Sources

- Voltage Sources
  - An ***ideal voltage source*** is defined as an electrical energy source that sources voltage at a constant value, independent of the load
    - Looks like a vertical line with infinite slope on the IV curve → **R = 0**
  - Real voltage sources have a finite slope
    - Outputs vary slightly depending on the load → More on this later
  - Properties of voltage sources
    - The value of the voltage is always referenced to one of the terminals, or to a designated reference point in the circuit
    - The voltages measured in a circuit are usually referenced to a single point, and may be one of the terminals of the source

      ⇨ Sometimes called: common, earth, "ground", 0-Volt Potential

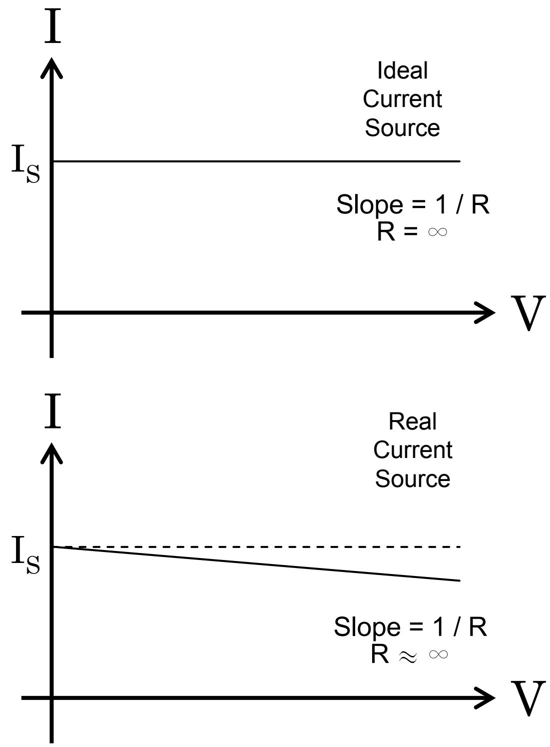


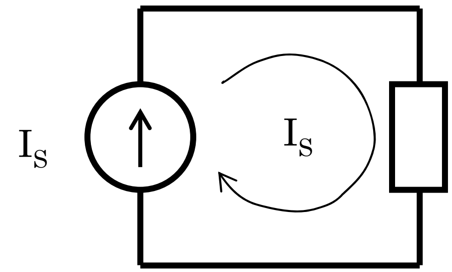




Time-Invariant (DC) Electrical Sources

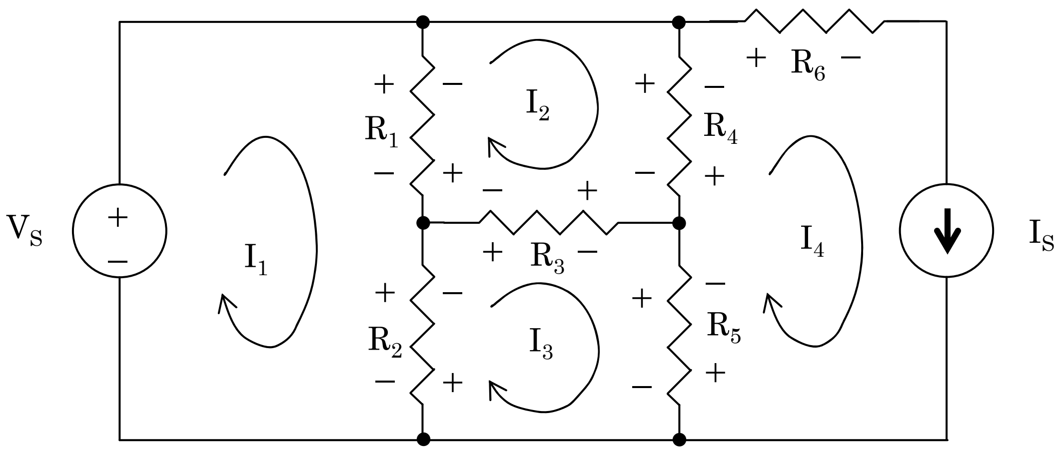
- Current Sources
  - An ***ideal current source*** is defined as an electrical energy source that sources current at a constant value, independent of the load
    - Looks like a horizontal line with zero slope on the IV curve → **R = ∞**
  - Real current sources have a small slope
    - Outputs vary slightly depending on the load
  - Properties of current sources
    - Currents always flow in loops
    - Current sources always have the source current returned to the source





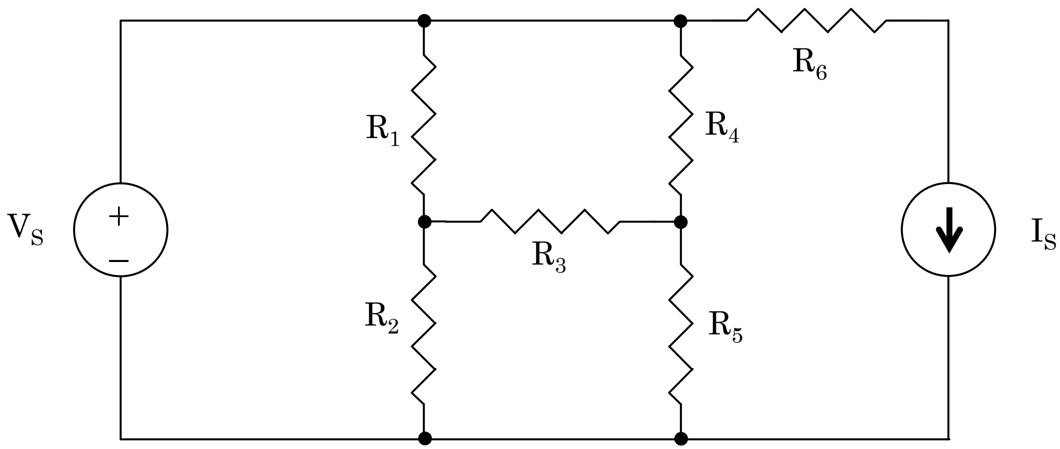
Basic Circuit Analysis Techniques

- Kirchoff's Voltage Law
  - ***"The sum of the voltage drops around a loop must equal zero."***
  - Use this principles to solve for loop currents in circuits:
    - For each window, draw a loop with an arbitrary direction
      - For current sources, draw only one current through the source
    - Assign polarities using standard convention:
      - Arrow into positive terminal = positive term
      - Arrow into negative terminal = negative term
    - Write equations for each loop
      - Sum voltage drops around each loop
      - Include contributions from neighboring loops
    - Solve system of equation → N equations, N unknowns

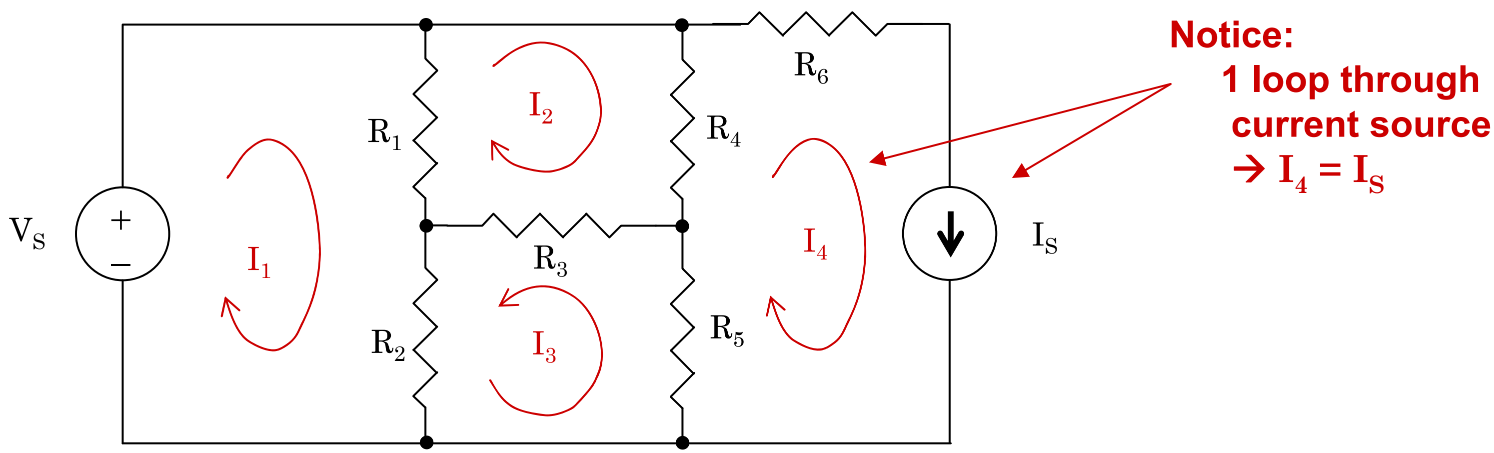


Basic Circuit Analysis Techniques

- Kirchoff's Loop Equations →Example 1

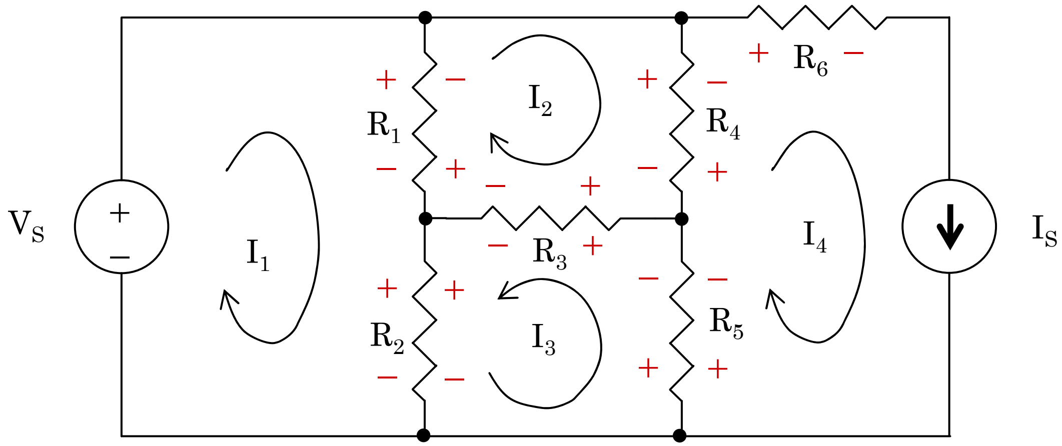


- Step 1: For each window, draw a loop with an arbitrary direction
  - For current sources, draw only one current through the source



Basic Circuit Analysis Techniques

- Kirchoff's Loop Equations →Example 1 (Continued)
  - Step 2: Assign polarities using standard convention:
    - Arrow into positive terminal = positive term
    - Arrow into negative terminal = negative term

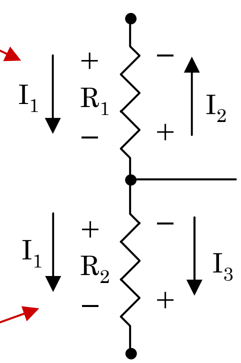


  - Step 3: Form terms for each loop based on direction of arrows

Currents oppose

For loop 1:

$V_{R1} = (I_1 - I_2)\ R_1$



For loop 2:

$V_{R1} = (I_2 - I_1)\ R_1$

For loop 1:

$V_{R2} = (I_1 + I_3)\ R_2$

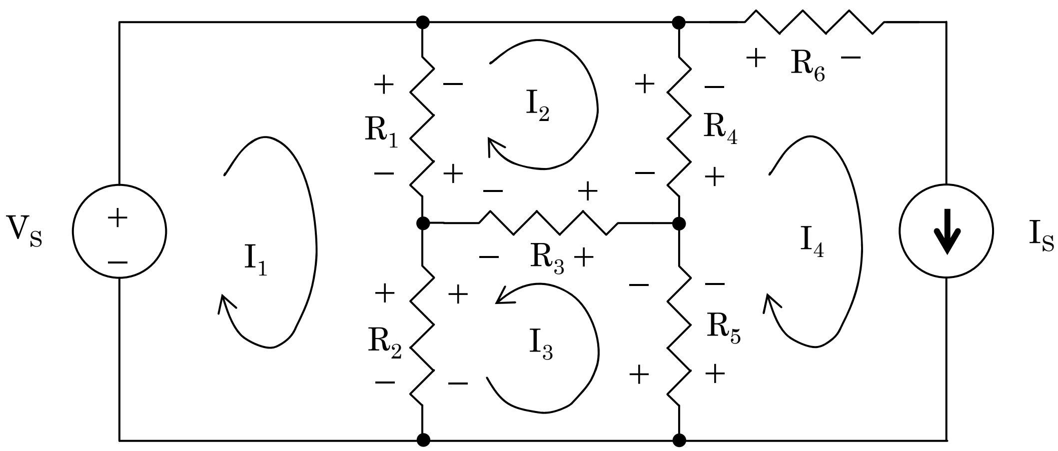
For loop 3:

$V_{R2} = (I_3 + I_1)\ R_2$

Currents add

Basic Circuit Analysis Techniques

- Kirchoff's Loop Equations →Example 1 (Continued)
  - Step 4: Write equations for each loop
    - Sum voltage drops around each loop
    - Include contributions from neighboring loops



Loop 1: $(I_1 - I_2)\,R_1 + (I_1 + I_3)\,R_2 - V_S = 0$

Loop 2: $(I_2 - I_1)\,R_1 + (I_2 + I_3)\,R_3 + (I_2 - I_4)\,R_4 = 0$

Loop 3: $(I_3 + I_1)\,R_2 + (I_3 + I_2)\,R_3 + (I_3 + I_4)\,R_5 = 0$

Loop 4: $I_4 = I_S$

Basic Circuit Analysis Techniques

- Kirchoff's Loop Equations → Example 1 (Continued)
  - Step 5: Solve system of equations → N equations, N unknowns
    - Rearrange:

$$
\begin{aligned}
(R_1 + R_2)\, I_1 &+ (-R_1)\, I_2 + (R_2)\, I_3 + (0)\, I_4 = V_S \\
(-R_1)\, I_1 &+ (R_1 + R_3 + R_4)\, I_2 + (R_3)\, I_3 + (-R_3)\, I_4 = 0 \\
(R_2)\, I_1 &+ (R_3)\, I_2 + (R_2 + R_3 + R_5)\, I_3 + (R_5)\, I_4 = 0 \\
(0)\, I_1 &+ (0)\, I_2 + (0)\, I_3 + I_4 = I_S
\end{aligned}
$$

    - Rearrange again, knowing $I_4 = I_S$:

$$
\begin{aligned}
(R_1 + R_2)\, I_1 + (-R_1)\, I_2 + (R_2)\, I_3 &= V_S \\
(-R_1)\, I_1 + (R_1 + R_3 + R_4)\, I_2 + (R_3)\, I_3 &= (R_3)\, I_S \\
(R_2)\, I_1 + (R_3)\, I_2 + (R_2 + R_3 + R_5)\, I_3 &= (-R_5)\, I_S
\end{aligned}
$$

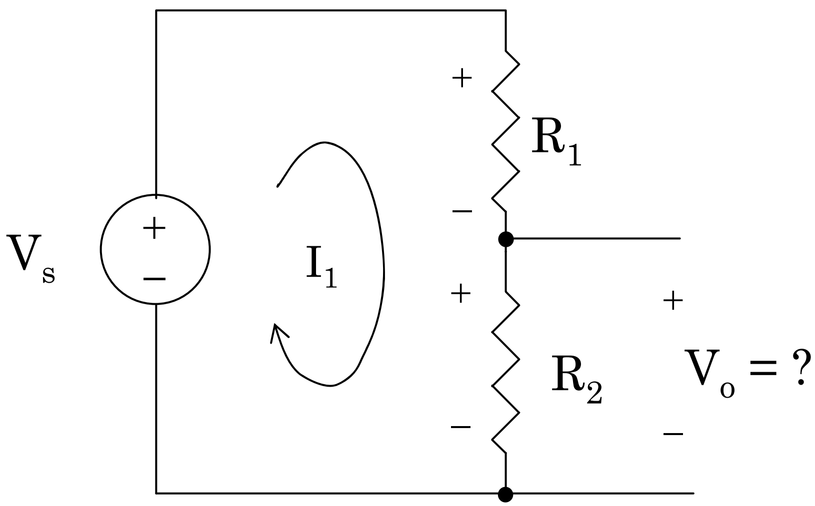
    - Matrix form:

$$
\begin{bmatrix} (R_1 + R_2) & (-R_1) & (R_3) \\ (-R_1) & (R_1 + R_3 + R_4) & (R_3) \\ (R_2) & (R_3) & (R_2 + R_3 + R_5) \end{bmatrix}
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}
=
\begin{bmatrix} V_S \\ (R_3)\, I_S \\ (-R_5)\, I_S \end{bmatrix}
$$

⇨ ***Solve for I_1, I_2, and I_3***

Basic Circuit Analysis Techniques

- Kirchoff's Loop Equations → Example 2
  - Voltage Divider



$I_1 R_1 + I_1 R_2 - V_s = 0$

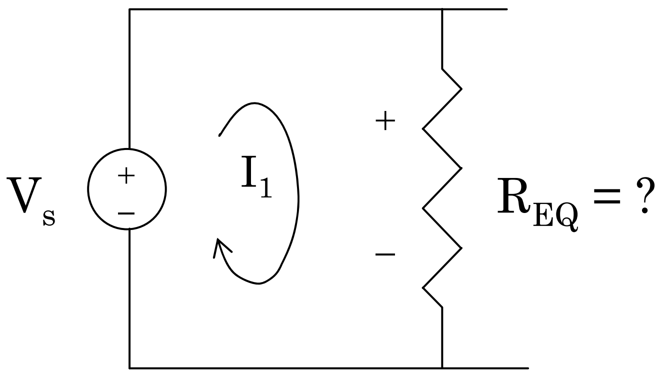
$I_1 = V_s / (R_1 + R_2)$

Then:

$V_1 = I_1 R_1, \; V_2 = I_1 R_2$

$\boxed{V_O = V_S \; R_2 / (R_1 + R_2)}$

  - Equivalent Circuit → Resistors in Series



$I_1 = V_s / (R_1 + R_2)$

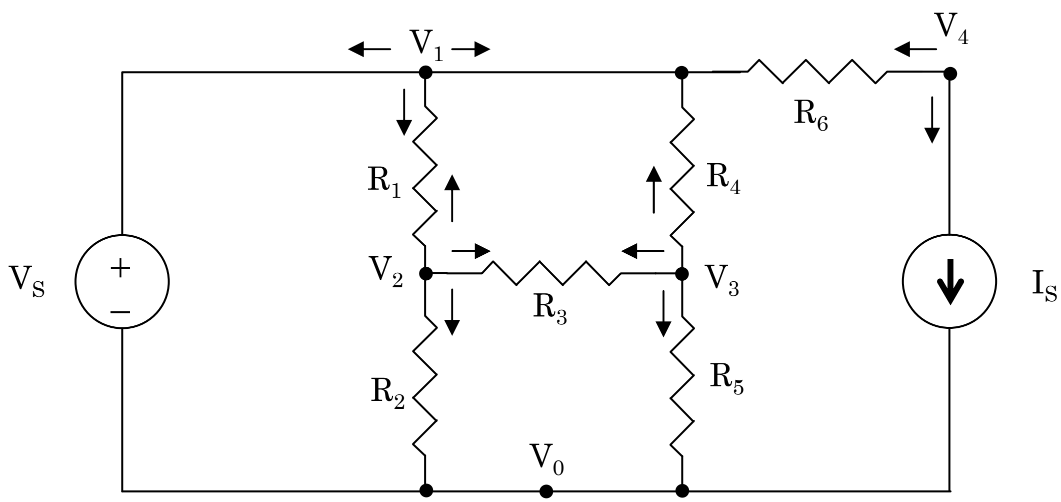
$R_{EQ} = V_S / I_1$

$\boxed{R_{EQ} = (R_1 + R_2)}$

⇨ ***Resistors in Series ADD***

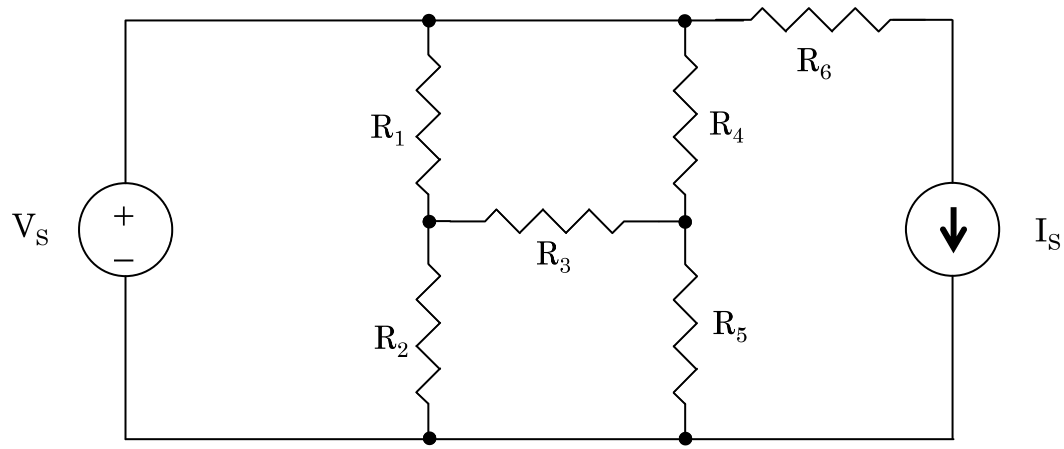
Basic Circuit Analysis Techniques

- Kirchoff's Current Law
  - ***"The sum of the currents at a node must equal zero."***
  - Use this principles to solve for node voltages in circuits:
    - Choose a reference node, and label in "Node 0"
      - Arbitrary, but may want to reference a voltage source to Node 0
    - For each node, assign a node voltage variable: $V_1, V_2, \ldots V_N$
    - Write equations for each node
      - Sum currents into each node
      - Currents from neighboring nodes defined as $(V_i - V_{i-1}) / R_{i,i-1}$
      - Nodes with current sources have current defined by the source
    - Solve system of equation → N equations, N unknowns

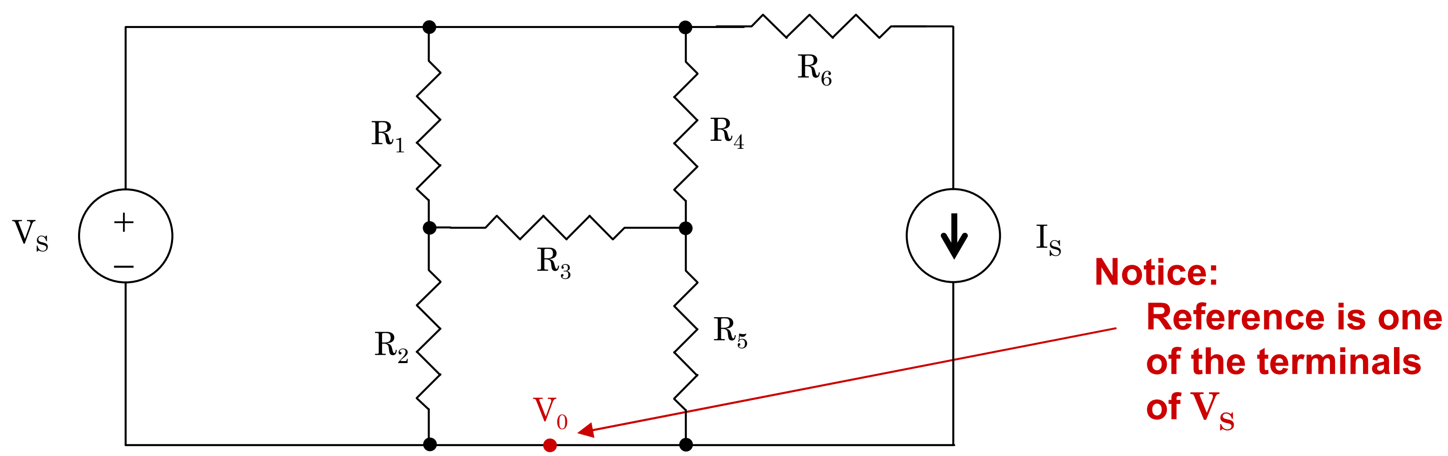


Basic Circuit Analysis Techniques

- Kirchoff's Node Equations →Example 3



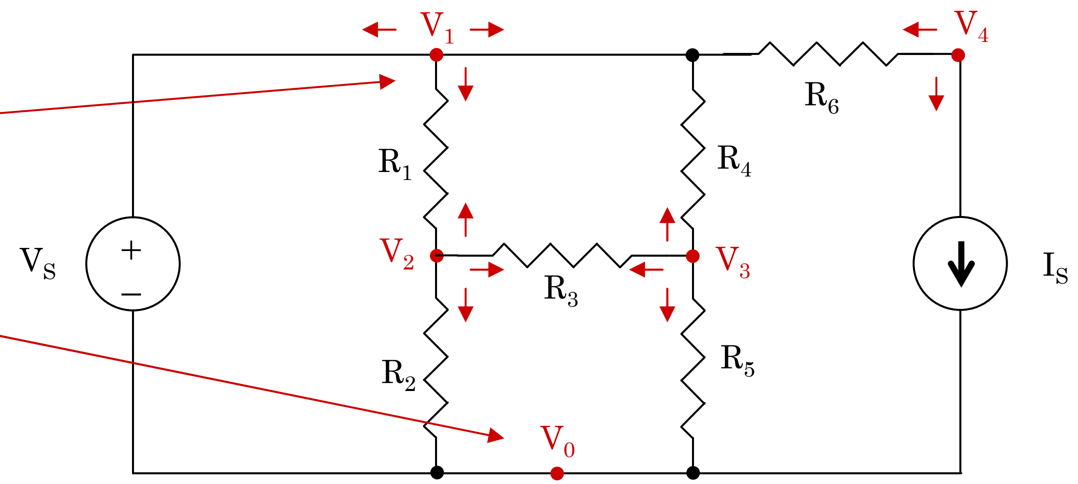
- Step 1: Choose a reference node and label it "Node 0"
  - Arbitrary, but may want to reference a voltage source



Basic Circuit Analysis Techniques

- Kirchoff's Node Equations →Example 3 (Continued)
  - Step 2: For each node, assign a node voltage variable: $V_1, V_2, \ldots V_N$

Notice:
$V_1 = V_S$
→ Get for free because of way Node 0 was defined



  - Step 3: Form terms for each node assuming <u>all currents leave node</u>

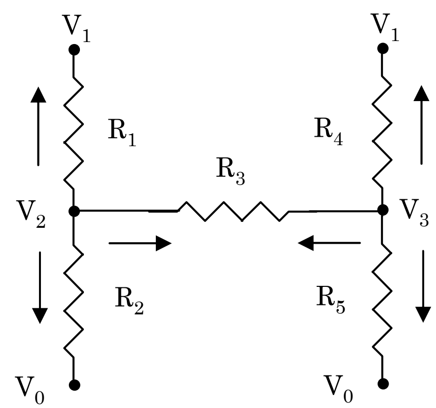
<u>For node 2:</u>

$I_{R1} + I_{R2} + I_{R3} = 0$

$I_{R1} = (V_2 - V_1) / R_1$

$I_{R2} = (V_2 - 0) / R_2$

$I_{R3} = (V_2 - V_3) / R_3$



<u>For node 3:</u>

$I_{R3} + I_{R4} + I_{R5} = 0$

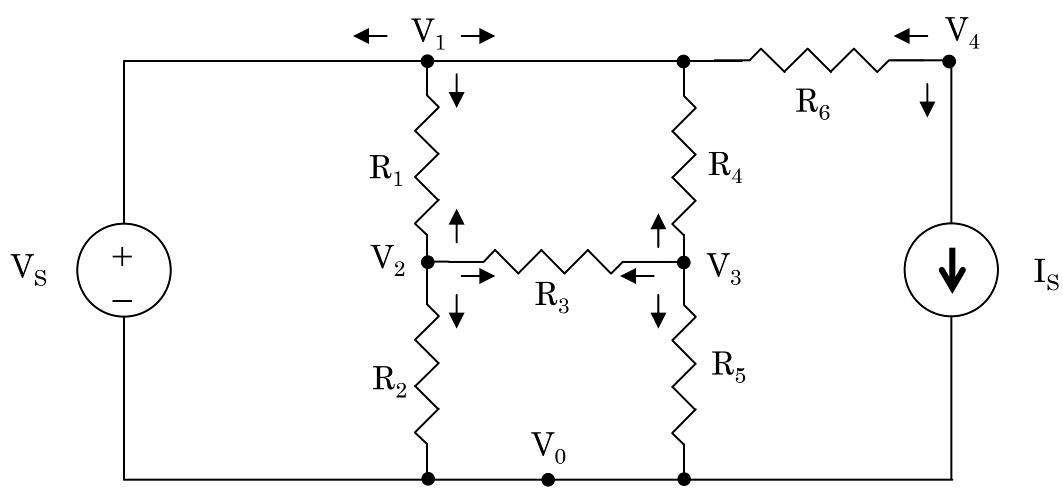
$I_{R3} = (V_3 - V_2) / R_3$

$I_{R4} = (V_3 - V_1) / R_4$

$I_{R5} = (V_3 - 0) / R_5$

Basic Circuit Analysis Techniques

- Kirchoff's Node Equations →Example 3 (Continued)
  - Step 3: Write equations for each node
    - Sum currents from each component connected to node
    - Simple form…



Node 1: $V_1 = V_S$

Node 2: $(V_2 - V_1) / R_1 + (V_2 - V_3) / R_3 + (V_2 - 0) / R_4 = 0$

Node 3: $(V_3 - V_1) / R_4 + (V_3 - V_2) / R_3 + (V_3 - 0) / R_5 = 0$

Loop 4: $(V_4 - V_1) / R_6 + I_S = 0$

Basic Circuit Analysis Techniques

- Kirchoff's Node Equations → Example 3 (Continued)
  - Step 3: Solve system of equations → N equations, N unknowns
    - Rearrange: ⇨ ***For convenience, will use $G = 1/R$***

$$
\begin{aligned}
V_1 &+ (0)\,V_2 + (-0)\,V_3 + (0)\,V_4 = V_S \\
(-G_1)\,V_1 &+ (G_1 + G_3 + G_4)\,V_2 + (-G_3)\,V_3 + (0)\,V_4 = 0 \\
(-G_4)\,V_1 &+ (-G_3)\,V_2 + (G_3 + G_4 + G_5)\,V_3 + (0)\,V_4 = 0 \\
(-G_6)\,V_1 &+ (0)\,V_2 + (0)\,V_3 + (G_6)\,V_4 = -I_S
\end{aligned}
$$

    - Rearrange again, knowing $V_1 = V_S$:

$$
\begin{aligned}
(G_1 + G_3 + G_4)\,V_2 + (-G_3)\,V_3 + (0)\,V_4 &= (G_1)\,V_S \\
(-G_3)\,V_2 + (G_3 + G_4 + G_5)\,V_3 + (0)\,V_4 &= (G_4)\,V_S \\
(0)\,V_2 + (0)\,V_3 + (G_6)\,V_4 &= -I_S + (G_6)\,V_S
\end{aligned}
$$

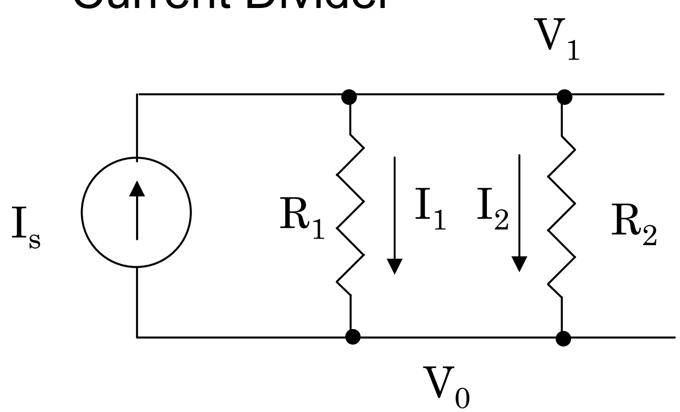
    - Matrix form:

$$
\begin{bmatrix} (G_1 + G_3 + G_4) & (-G_3) & 0 \\ (-G_3) & (G_3 + G_4 + G_5) & 0 \\ 0 & 0 & (G_6) \end{bmatrix}
\begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix}
=
\begin{bmatrix} (G_1)\,V_S \\ (G_4)\,V_S \\ (G_6)\,V_S - I_S \end{bmatrix}
$$

⇨ ***Solve for V_2, V_3, and V_4***

Basic Circuit Analysis Techniques

- Kirchoff's Node Equations → Example 4
  - Current Divider



$I_2 = ?$

$$V_1 / R_1 + V_1 / R_2 - I_s = 0$$

$$V_1 G_1 + V_1 G_2 - I_s = 0$$

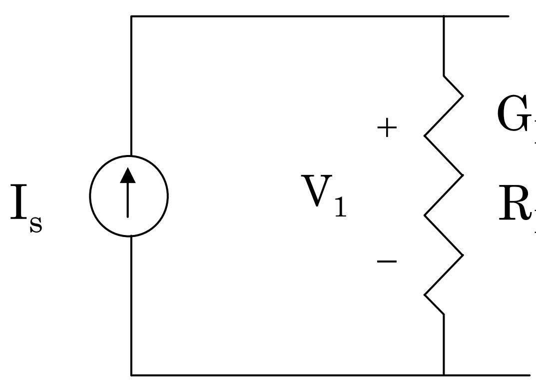
$$V_1 = I_s / (G_1 + G_2)$$

Then:

$$I_1 = V_1 \; G_1 = V_S \; G_1 / (G_1 + G_2)$$

$$I_2 = V_1 \; G_2 = V_S \; G_2 / (G_1 + G_2)$$

  - Equivalent Circuit → Resistors in Parallel



$G_{EQ} = ?$

$R_{EQ} = ?$

$$V_1 = I_s / (G_1 + G_2)]$$

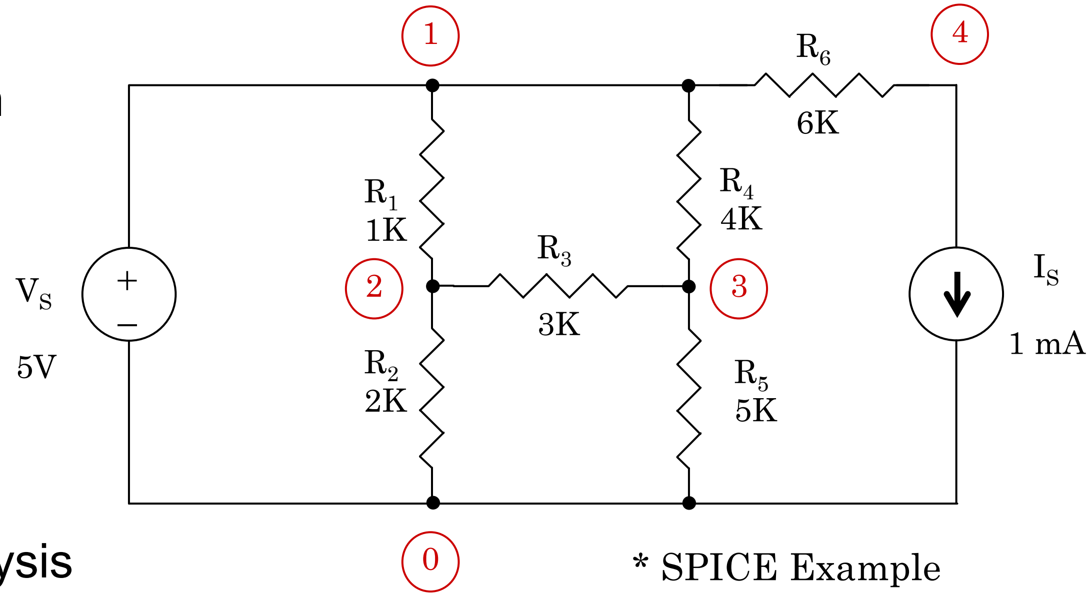
$$G_{EQ} = I_S / V_1$$

$$\boxed{\begin{aligned} G_{EQ} &= G_1 + G_2 \\ R_{EQ} &= R_1 R_2 / (R_1 + R_2) \end{aligned}}$$

⇨ ***Conductances in Parallel ADD***

Basic Circuit Analysis Techniques

- SPICE
  - Circuit simulation program
  - Simple coding
  - Text or graphical
  - Can perform wide range of analyses
  - Can use behavioral device models
  - Generally uses node analysis
  - Free versions or high-end full-blown simulators → $20K




```
* SPICE Example
VS  1  0  5V
R1  1  2  1K
R2  2  0  2K
R3  2  3  3K
R4  1  3  4K
R5  3  0  5K
R6  1  4  6K
IS  4  0  1MA
.DC
.SENS V(2)
.PRINT
.END
```

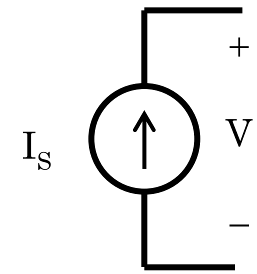
Power in Time-Invariant (DC) Circuits

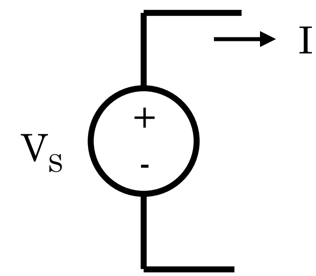
- ***Power*** is defined as:

$$P = dU / dt = d/dt \left[\int q\, V\, dq \right]$$
$$= dQ/dt\ V$$

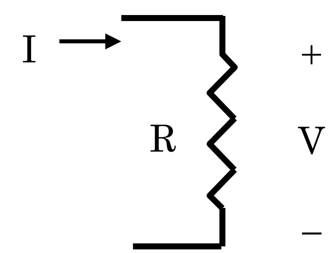
or: $$P = I \quad V$$

- Voltage across the device of interest
- Current flowing through the device of interest





- *This definition is only strictly true for time-invariant currents and voltages*
  - There is a correction for time-varying sources (to be covered later)



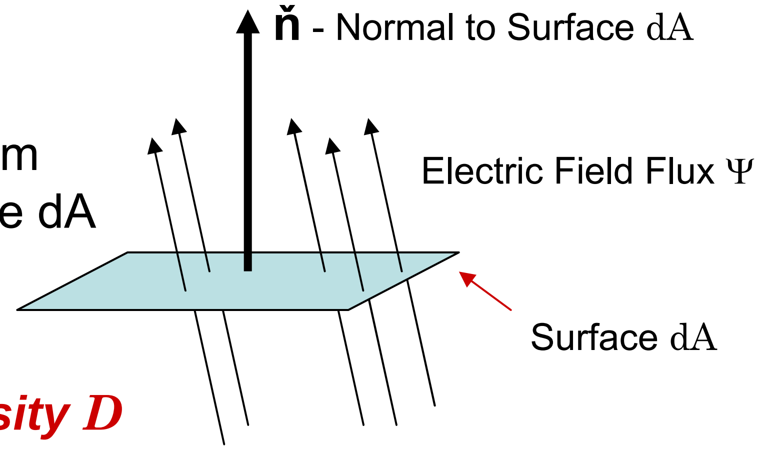
- Units are in Watts 1 Watt = 1 Joule/Second

- For Resistors

$$P = I \ V$$
$$= (V / R)\ V \rightarrow P = V^2 / R$$
$$= I \ (I \ R) \rightarrow P = I^2 \ R$$

Passive Components that Store Energy

- Capacitors
  - Electric Field Intensity $\mathbf{E}$

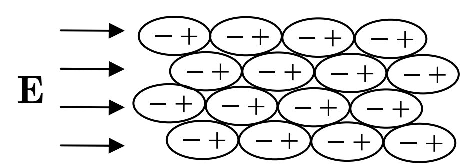


    - Consider an electric field in a medium that penetrates an imaginary surface dA
      - Lines of flux Ψ are created by the Electric Field $\mathbf{E}$
    - Define the ***Electric Field Flux Density D***

$$\mathbf{D} = \varepsilon\, \mathbf{E}$$

      - The flux density $\mathbf{D}$ depends on the medium
      - The units of D are in Coulombs / meter^2
      - ε is called the electric permittivity, and is dependent on the medium
      - In free space, $\varepsilon = \varepsilon_0 =$ 8.85E-12 Coulombs^2 / (Newton-meters^2)
    - Define ***"Dielectric Medium"*** as a medium that is intrinsically an insulator, but one in which the molecules can be polarized by an electric field → Bound charge in molecules
    - When polarization occurs, causes D to increase
    - Represent this by relative permittivity, or dielectric constant ε_r

$\varepsilon = \varepsilon_0 \ \varepsilon_r \ , \ \varepsilon_r \geqq 1$ depending on the material



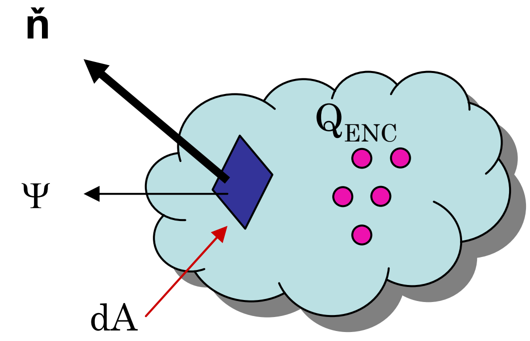
      - For ε_r large, material polarizes more easily, and increases $\mathbf{D}$

Passive Components that Store Energy

- Capacitors (Continued)
  - The Electric Field Flux Ψ through a differential surface dA is given by:

$$\Psi = \int_{\text{surface}} \mathbf{D} \cdot \check{\mathbf{n}} \; dA$$

Units: Coulombs



  - Gauss's Law
    - The total Electric Field Flux Ψ through a closed surface is given by:

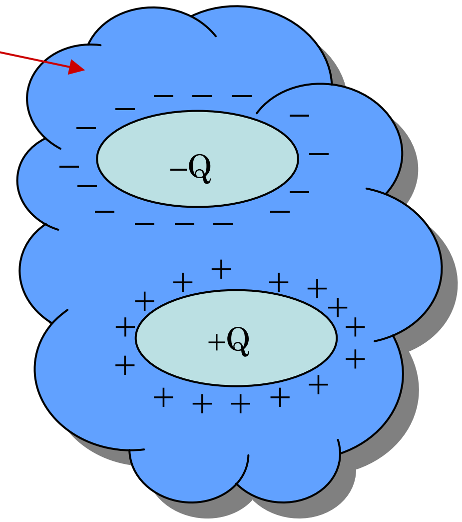
$$\Psi = \oint_{\substack{\text{closed}\\ \text{surface}}} \mathbf{D} \cdot \check{\mathbf{n}} \; dA = Q_{\text{enclosed}}$$

$$= \oint_{\substack{\text{closed}\\ \text{surface}}} \varepsilon_0 \, \varepsilon_r \, \mathbf{E} \cdot \check{\mathbf{n}} \; dA = Q_{\text{enclosed}}$$

⇨ Says that charge creates electric field flux

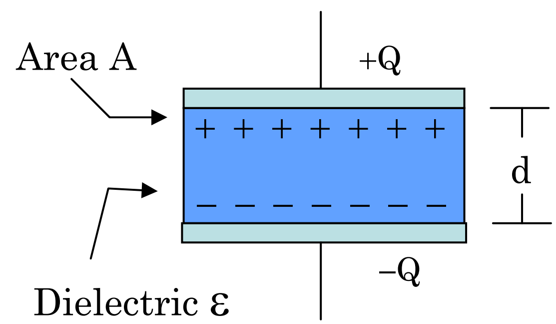
Passive Components that Store Energy

- Capacitors (Continued)
  - Define ***Capacitance C***
    - Suppose there are two conductors, immersed in a dielectric medium ε, and each has charge +Q and –Q respectively
    - There must exist an electric field between them
    - There must exist a potential difference between them
    - ***Capacitance is defined as the ratio of charge to the potential difference between the two conductors***



$$C = Q / V$$ Units: Farad = 1 Coulomb/Volt

$$C = \frac{\oint D \cdot \check{n} \, dA}{\int E \cdot dL}$$



- For a configuration with 2 parallel plates:

$$C \approx (D\ A) / (E\ d) = (\varepsilon E\ A) / (E\ L)$$

$$C = \varepsilon\ A / d = \varepsilon_0\ \varepsilon_r\ A / d$$

⇨ **Only depends on physical properties**

Passive Components that Store Energy

- Capacitors (Continued)
  - IV Relationship

$$Q = C\ V$$

$$d/dt\ Q = d/dt\ [C\ v(t)]$$

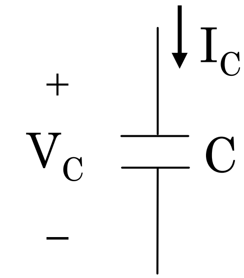
$$i(t) = C\ dv(t)/dt$$

$$v(t) = 1/C \int_0^t i(\tau)\, d\tau$$

⇨ Capacitor voltage cannot change instantaneously since $i(t)$ cannot be infinite

⇨ More on this later…

  - Symbol



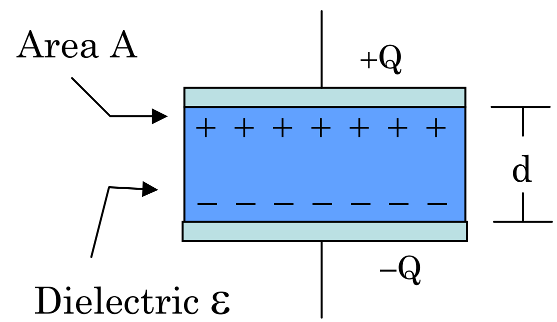
  - Energy Storage
    - The energy stored in the electric field due to static charge Q on the plates of a capacitor with voltage V between them is given by:

$$W = \int dW = \int_0^Q V\, dq = \int_0^Q q/C\ dq$$

$$W = ½\ Q^2 / C$$

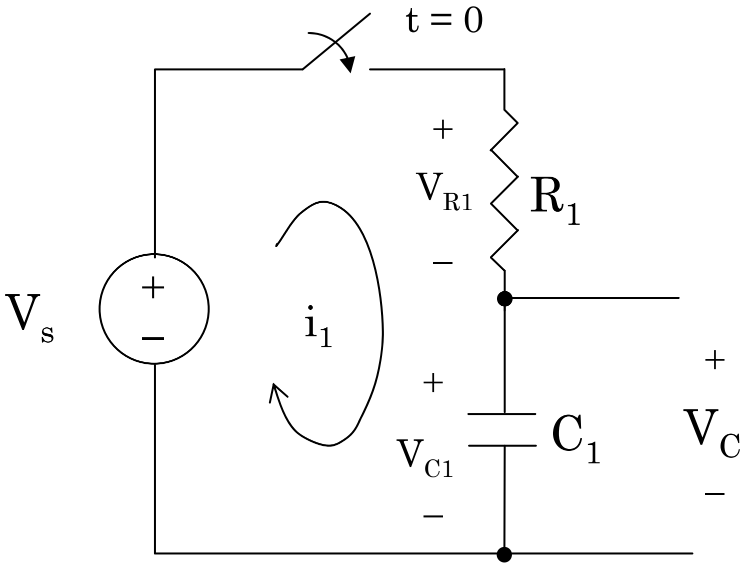
$$W = ½\ C\, V^2$$

Units: Joules



Passive Components that Store Energy

- Capacitors (Continued)
  - Simple RC Circuit
    - Assume that the capacitor is initially discharged
    - Let the switch close at time $t = 0$



Loop Equation:

$$V_{R1} + V_{C1} - V_s = 0$$

$$i_1 R_1 + [\, 1/C \int_0^t i_1(\lambda)\, d\lambda] = V_S$$

Solution has the form:

$$i_1(t) \;=\; a \;\; e^{-bt}$$

$$a\, R_1 \;\; e^{-bt} + [a \,/\, (-b\, C_1)] \;\; e^{-bt} + [a \,/\, (b\, C_1)] = V_S$$

At t = 0:

$$a\, R_1 - [a \,/\, (b\, C_1) + [a \,/\, (b\, C_1)] = V_S$$

$$a = V_S \,/\, R_1$$

At t = ∞:

$$a \,/\, (b\, C_1) = V_S$$

$$b = 1 \,/\, (R_1\, C_1)$$

Solution:

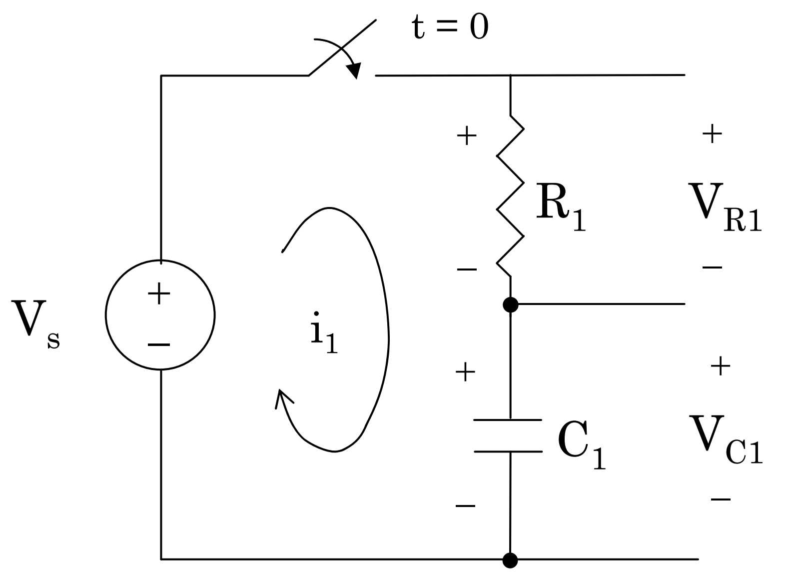
$$i_1(t) \;=\; V_S \,/\, R_1 \;\; e^{-t/R_1 C_1}$$

Passive Components that Store Energy

- Capacitors (Continued)
  - Simple RC Circuit (Continued)
    - Observations:
      - At time $t = 0$, $i(t) = V_S / R_1$
        - ⇨ Capacitor looks like a short circuit at $t = 0$
      - At time $t = \infty$, $i(t) = 0$
        - ⇨ Capacitor looks like an open circuit at $t = \infty$
      - Define Time Constant τ

        $\tau = R_1 C_1$

        - ⇨ For single capacitor circuits, the time constant is:
          C times the equivalent resistance seen by the capacitor
      - What about impedance $Z_C(t)$
        - ⇨ Difficult…
        - ⇨ Better done in frequency domain
        - ⇨ More later…



<u>Solution:</u>

$$i_1(t) = V_S / R_1 \; e^{-t/R_1C_1}$$

$$v_{R1}(t) = V_S \; e^{-t/R_1C_1}$$

$$v_{C1}(t) = V_S \,(1 - e^{-t/R_1C_1})$$

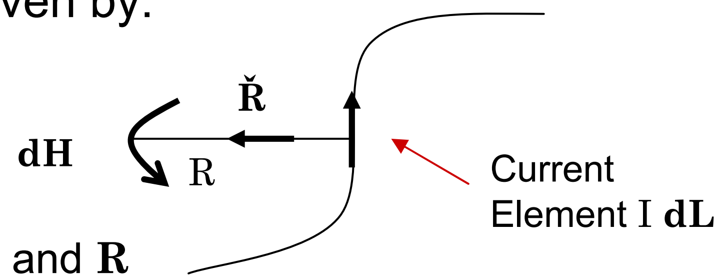
$$v_{C1}(t) = V_S \,(1 - e^{-t/\tau})$$

Passive Components that Store Energy

- Inductors
  - Biot-Savart Law: “A current $I\,\mathbf{dL}$ produces a magnetic field element $\mathbf{dH}$ which, at a distance R from the element is given by:

$$\mathbf{dH} = [\,I\,\mathbf{dL} \times \check{\mathbf{R}}] / [4\,\pi\,R^2]$$

Where $\check{\mathbf{R}} = \mathbf{R} / R$ is a unit vector



    - Cross product favors direction normal to $I\,\mathbf{dL}$ and $\mathbf{R}$

      ⇨ Says that currents produce magnetic fields

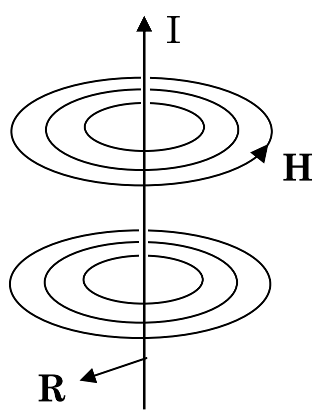
  - The total field at distance R from current element $I\,\mathbf{dL}$ is given by:

$$\mathbf{H(R)} = \oint_{\text{closed contour}} [\,I\,\mathbf{dL} \times \check{\mathbf{R}}] / [4\,\pi\,R^2]$$

    - For a long straight wire carrying current I, the magnetic field $\mathbf{H}$ at a distance R is given by:

$$\mathbf{H} = [I / 4\,\pi] \int_{-\infty}^{\infty} [\,\sin\theta\,dL\,] / [\,r^2]$$

$$= [I /(4\,\pi\,R)] \int_{0}^{\pi} \sin\theta\,d\theta \;=\; I / (2\,\pi\,R)$$

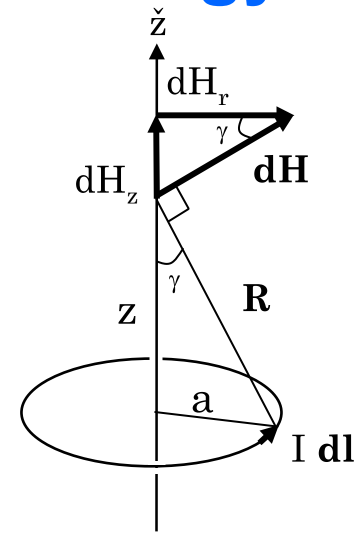


⇨ Magnetic field lines are continuous loops

Passive Components that Store Energy

- Inductors (Continued)
  - Example – Loop of wire
    - The contribution of a small segment dL to the magnetic field on the axis is given by:

$$\mathbf{dH(R)} = [\, \mathrm{I}\, \mathbf{dL} \times \check{\mathbf{R}}\,] \,/\, [4\,\pi\,\mathrm{R}^2]$$



    - Since the radial components cancel by symmetry, the magnetic field is in the z-direction:

$$\mathrm{H_z(R)} = \int_{\substack{\text{closed}\\\text{contour}}} \mathrm{dH_z} = [\,\mathrm{I}\sin\gamma\,] \,/\, [4\,\pi\,\mathrm{R}^2] \oint \mathrm{dL}$$

    - Using geometry:

$$\sin\gamma = \mathrm{a} \,/\, (\mathrm{a}^2 + \mathrm{z}^2), \qquad \oint \mathrm{dL} = 2\,\pi\,\mathrm{a}, \qquad \mathrm{R}^2 = (\mathrm{a}^2 + \mathrm{z}^2),$$

    - Combining and simplifying:

$$\mathrm{H_z(R)} = \mathrm{I\,a}^2 \,/\, [2\,(\mathrm{a}^2 + \mathrm{z}^2)^{3/2}\,]$$

    - At the center of the loop, for $\mathrm{z} = 0$:

$$\mathrm{H_z(a)} = \mathrm{I} \,/\, (2\,\mathrm{a}\,)$$

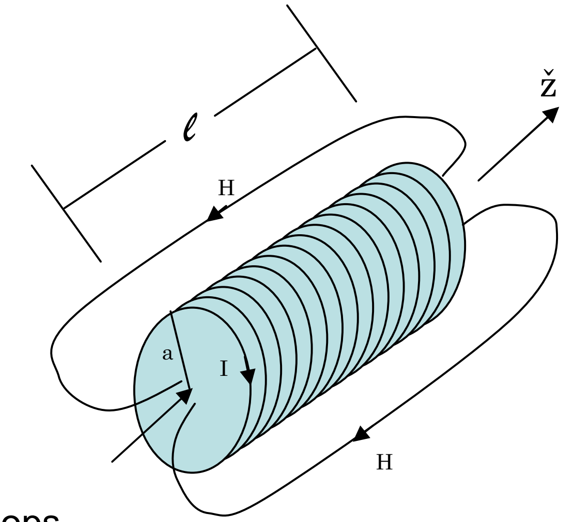
⇨ Units are in Ampere-turns / meter (# turns = 1 for a single loop)

Passive Components that Store Energy

- Inductors (Continued)
  - Example – Long solenoid
    - The contribution of each loop of wire to the field at the center of the coil (z axis) was found to be:

$$H_z(R) = I\,a^2 \,/\, [2\,(a^2 + z^2)^{3/2}\,]$$

    - Let the solenoid have N turns, diameter a, and and length ℓ
      - We can regard the solenoid as a stack of current loops
      - Instead of I, each loop contributes [$N\,I\,dz\,/\,\ell$] at the center of the solenoid



    - Treating this as an element $\mathbf{dH_Z}$ and integrating dz over the length of the coil ℓ

$$dH_z = \frac{a^2}{2\,(a^2 + z^2)^{3/2}}\;\frac{N\,I\,dz}{\ell}$$

$$H_z = \int dHz = \int_{-\ell/2}^{+\ell/2} \frac{N\;I\;a^2\,dz}{2\;\ell\;(a^2 + z^2)^{3/2}} = \frac{N\;I}{(4a^2 + \ell^{\,2})^{1/2}}$$

    - For $\ell >> a$,

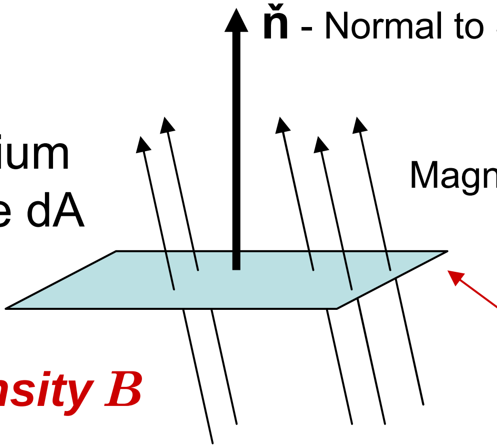
$$H_z = N\,I\,/\,\ell$$

⇨True on z axis

⇨ Approximately uniform through center of coil

Passive Components that Store Energy

- Inductors (Continued)
  - Magnetic Field Intensity $\mathbf{H}$
    - Consider an magnetic field in a medium that penetrates an imaginary surface dA
      - Lines of flux Φ are created by the Magnetic Field H

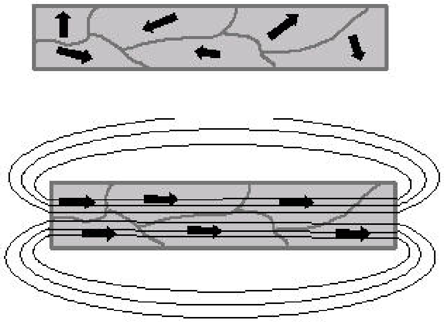


    - Define the ***Magnetic Field Flux Density B***

$$\mathbf{B} = \mu\ \mathbf{H}$$

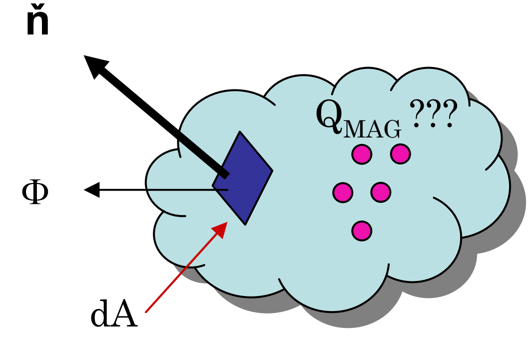
      - The flux density $\mathbf{B}$ depends on the medium
      - The units of B are in Webers / meter^2 or Tesla
      - μ is called the magnetic permeability, and is dependent on the medium
      - In free space, $\mu = \mu_0 = 4\pi$ E-12 Henrys / meter
    - Define ***"Magnetic Medium"*** as a medium that can be magnetized
      - Magnetic domains in material can be aligned by an external H field
      - Examples: iron, nickel, cobalt, steel, mu-metal
    - When magnetization occurs, causes B to increase
    - Represent this by relative permeability μ_r

      $\mu = \mu_0\ \mu_r$, $\mu_r \geqq 1$ depending on the material
      - For μ_r large, material polarizes more easily, and increases $\mathbf{B}$



Passive Components that Store Energy

- Inductors (Continued)
  - The total Magnetic Field Flux Φ through a differential surface dA is given by:



$$\Phi = \int_{\text{surface}} \mathbf{B} \cdot \check{\mathbf{n}} \; dA$$

Units: Webers

  - Gauss's Law for Magnetic Fields
    - The total Magnetic Field Flux Φ through a closed surface is given by:

$$\Phi = \oint_{\substack{\text{closed} \\ \text{surface}}} \mathbf{B} \cdot \check{\mathbf{n}} \; dA = Q_{\text{enclosed}} = 0$$

$$= \oint_{\substack{\text{closed} \\ \text{surface}}} \mu_0 \, \mu_r \, \mathbf{H} \cdot \check{\mathbf{n}} \; dA = 0$$

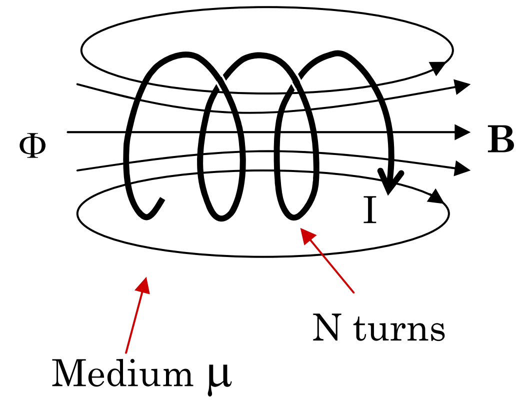
⇨ There is no free magnetic charge

⇨ Magnetic flux lines are loops

⇨ All flux lines that enter a closed surface must also leave through that surface

Passive Components that Store Energy

- Inductors (Continued)
  - Define ***self inductance L***
    - Suppose there exists a conductor, carrying a current I, in a magnetic medium μ
    - There must exist an magnetic field from the current
    - ***Inductance is defined as the ratio of the flux linkage Λ = N Φ of the circuit, to the current that created the flux***



$$L = N\Phi / I$$

Units: Henrys = 1 Weber-turn/Ampere

$$L = \frac{N \int B \cdot \check{n} \, dA}{I}$$

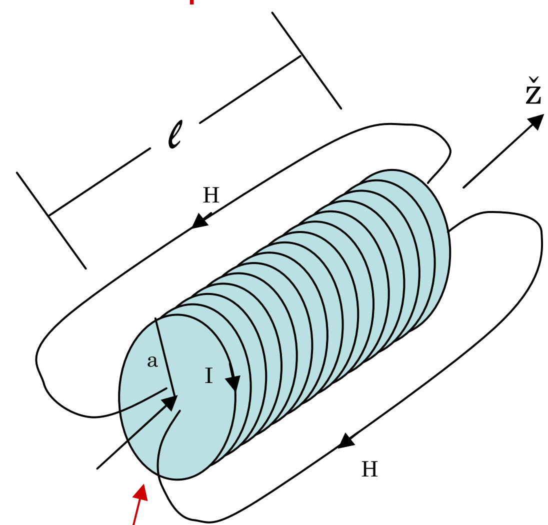
- For a solenoid, inside the coil:

$$L \approx (N B A) / I = (N \mu H A) / I$$

$$H \approx N I / \ell$$

$$L = \mu N^2 A / \ell$$

⇨ **Only depends on physical properties**



Passive Components that Store Energy

- Inductors (Continued)
  - Faraday's Law:
    - A change in magnetic flux produces a voltage that opposes the change

$$V = -\,d\Phi / dt$$

  - IV Relationship

$$V = -\,d\Phi / dt$$

$$= d/dt \int_{\text{surface}} \mathbf{B} \cdot \check{\mathbf{n}}\ dA \approx d/dt\ [\,B\ \ A\,]$$

$$\approx d/dt\ [\,I\ \ L\,]$$

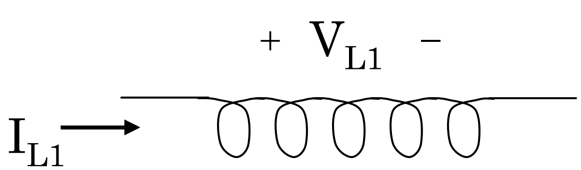
$$v(t) = L\ \ di(t)/dt$$

⇨ Inductor current cannot change instantaneously since $v(t)$ cannot be infinite

$$i(t) = 1/L \int_0^t v(\tau)\ d\tau$$

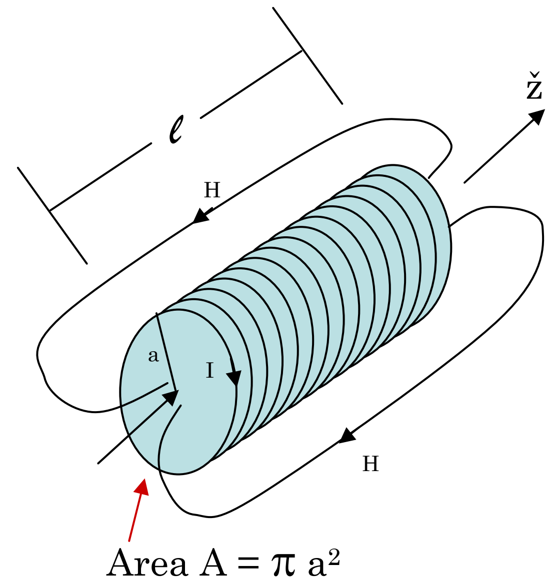
⇨ More on this later…

  - Symbol



Passive Components that Store Energy

- Inductors (Continued)
  - Energy Storage
    - The energy stored in the magnetic field due the current in an inductor is given by the average of the power in the inductor



$$W = \int_0^t P_L(\tau)\ d\tau = \int_0^t v_L(\tau)\ i_L(\tau)\ d\tau$$

$$= \int_0^t [\ L\ di_L(\tau) / d\tau\ ]\ [\ i_L(\tau)\ ]\ d\tau$$

$$= \int_0^t [\ L\ di_L(\tau) / d\tau\ ]\ [\ i_L(\tau)\ ]\ d\tau$$

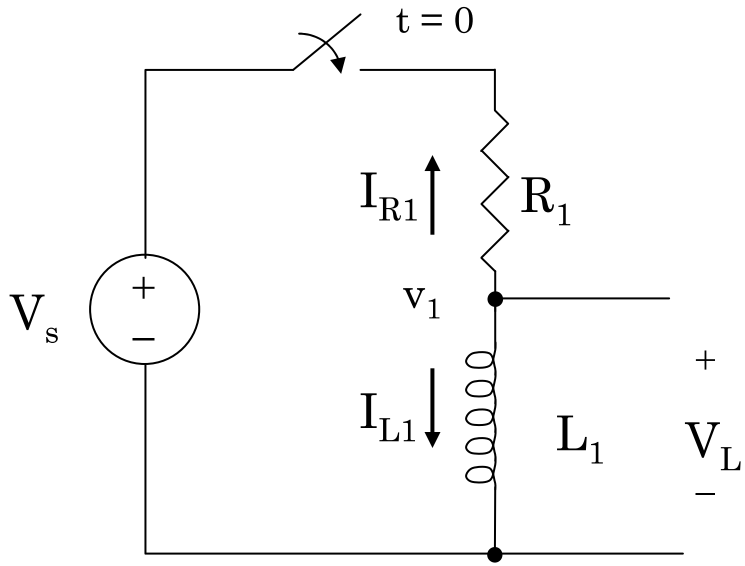
$$= L \int_0^t i_L(\tau)\ di_L$$

$$W = \tfrac{1}{2}\ L\ I^2$$

Units: Joules

Passive Components that Store Energy

- Inductors (Continued)
  - Simple RL Circuit
    - Assume that the capacitor is initially discharged
    - Let the switch close at time $t = 0$



Node Equation:

$$I_{R1} + I_{L1} = 0$$

$$[V_1 - V_S] / R_1 + [\ 1/L \int_0^t v_1(\lambda)\ d\lambda] = 0$$

Solution has the form:

$$v_1(t) = a\ e^{-bt}$$

$$a / R_1\ e^{-bt} + [a / (-b\ L_1)]\ e^{-bt} + [a / (b\ L_1)] = V_S / R_1$$

At t = 0:

$$A / R_1 - [a / (b\ L_1) + [a / (b\ L_1)] = V_S / R_1$$

$$a = V_S$$

At t = ∞:

$$a / (b\ L_1) = V_S / R_1$$

$$b = R_1 / L_1$$

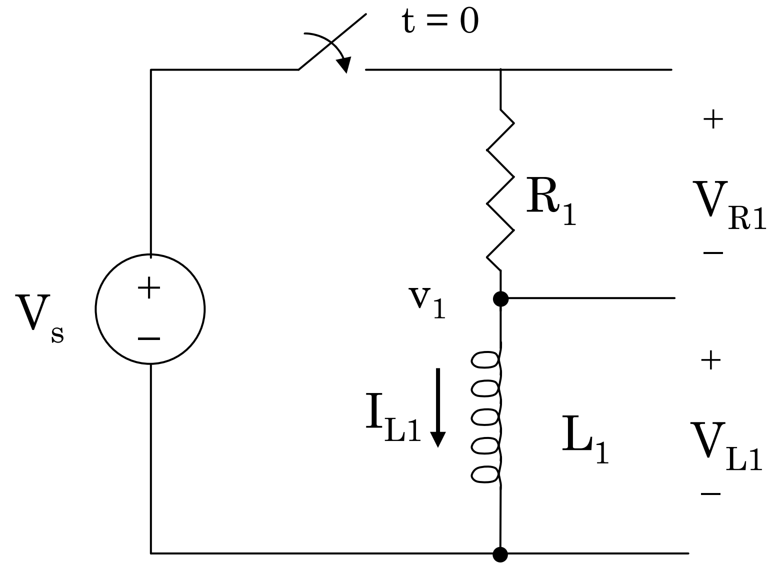
Solution:

$$v_1(t) = V_S\ e^{-t\ R_1 / L_1}$$

Passive Components that Store Energy

- Inductors (Continued)
  - Simple RL Circuit (Continued)
    - Observations:
      - At time $t = 0$, $i(t) = 0$
        - ⇨ Inductor looks like an open circuit at $t = 0$
      - At time $t = \infty$, $i(t) = V_S / R_1$
        - ⇨ Inductor looks like a short circuit at $t = \infty$
      - Define Time Constant τ

        $\tau = L_1 / R_1$
        - ⇨ For single inductor circuits, the time constant is: L divided by the equivalent R seen by the inductor
      - What about impedance $Z_L(t)$
        - ⇨ Difficult…
        - ⇨ Better done in frequency domain
        - ⇨ More later…



Solution:

$$v_{L1}(t) = V_S \; e^{-tR_1/L_1}$$

$$v_{R1}(t) = V_S \; (1 - e^{-tR_1/L_1})$$

$$i_{L1}(t) = V_S / R_1 \; (1 - e^{-tR_1/L_1})$$

$$i_{L1}(t) = V_S / R_1 \; (1 - e^{-t/\tau})$$

Passive Components that Store Energy

- Concluding Remark

  - Solving differential equations by hand this way is a lot of work…

  - Fortunately, there are tools to help make this easier:
    - Fourier Transforms
    - Laplace Transforms
    - ⇨ Frequency Domain → Next Session