Optimally sensitive observables for global EFT fits

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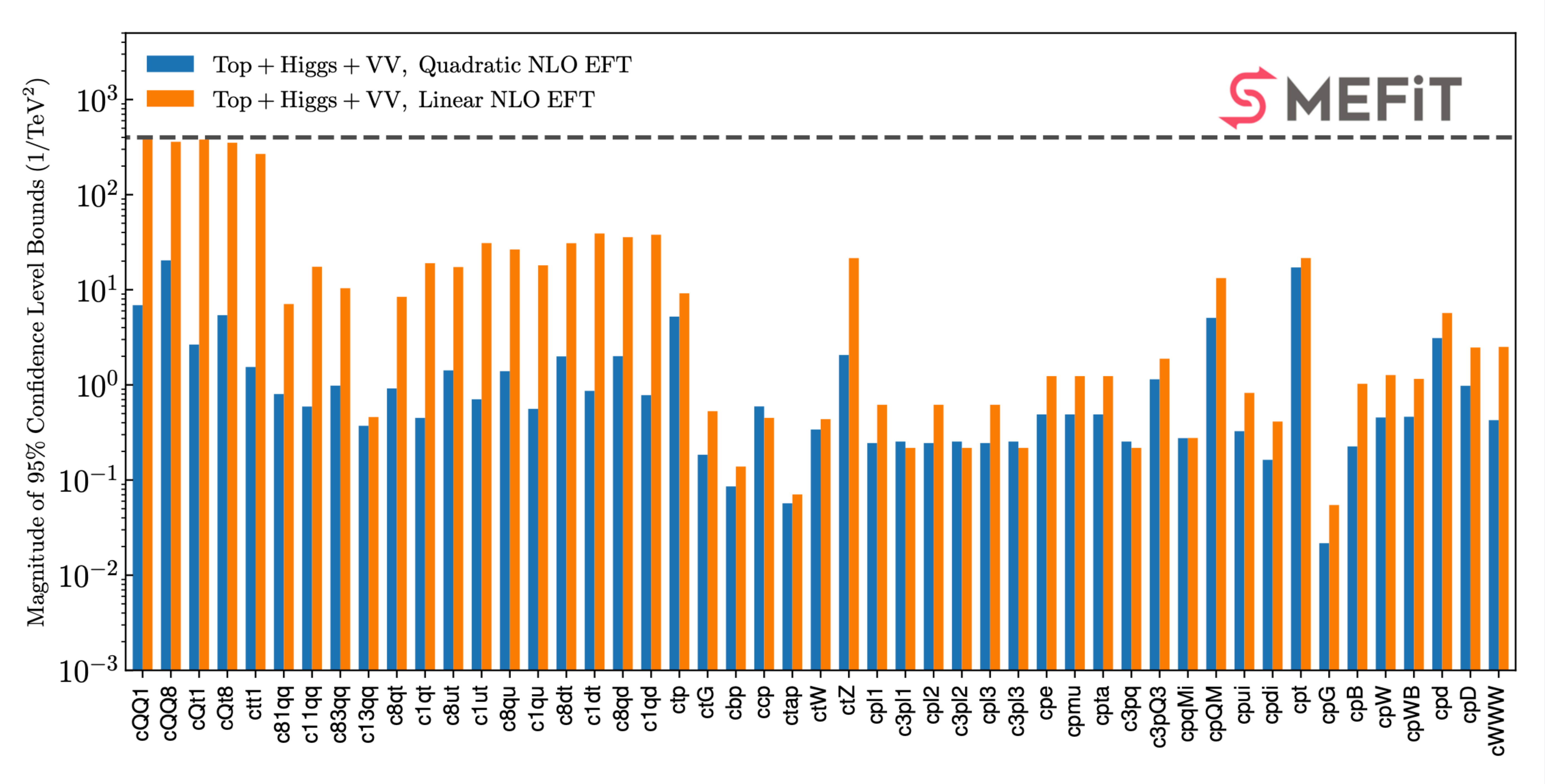
NWO

Introduction

- Status of the global EFT program: **Top + Higgs + diboson** data
- Based on traditional unfolded cross section distributions
- Can one construct observables **specifically designed to constrain EFT operators**?



See also the next talk by Ken Mimasu (Fitmaker)!

J.J. Ethier et al. [2105.00006]

Introduction

Key question: given a collider process, how can one define **optimal observables** with the highest sensitivity to EFT coefficients?

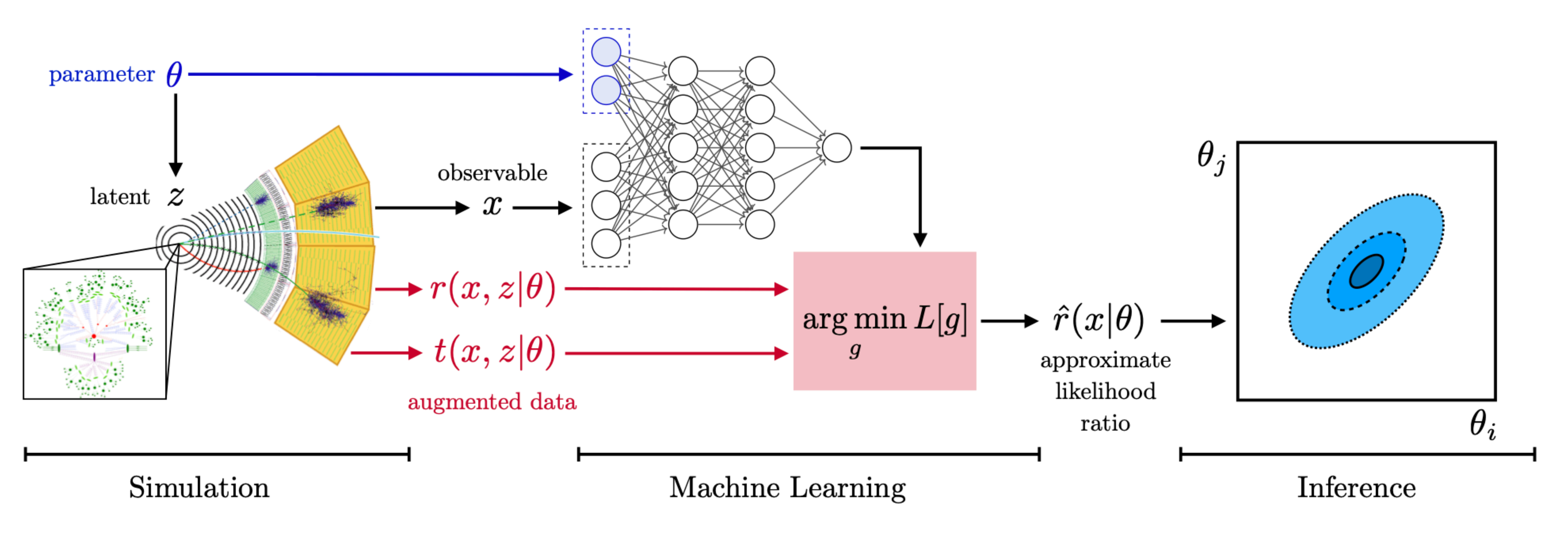
- We lose information in the process of binning
- To what degree can binned analyses achieve statistically optimal bounds?
- Even for bins, the precise choice of binning is not clear

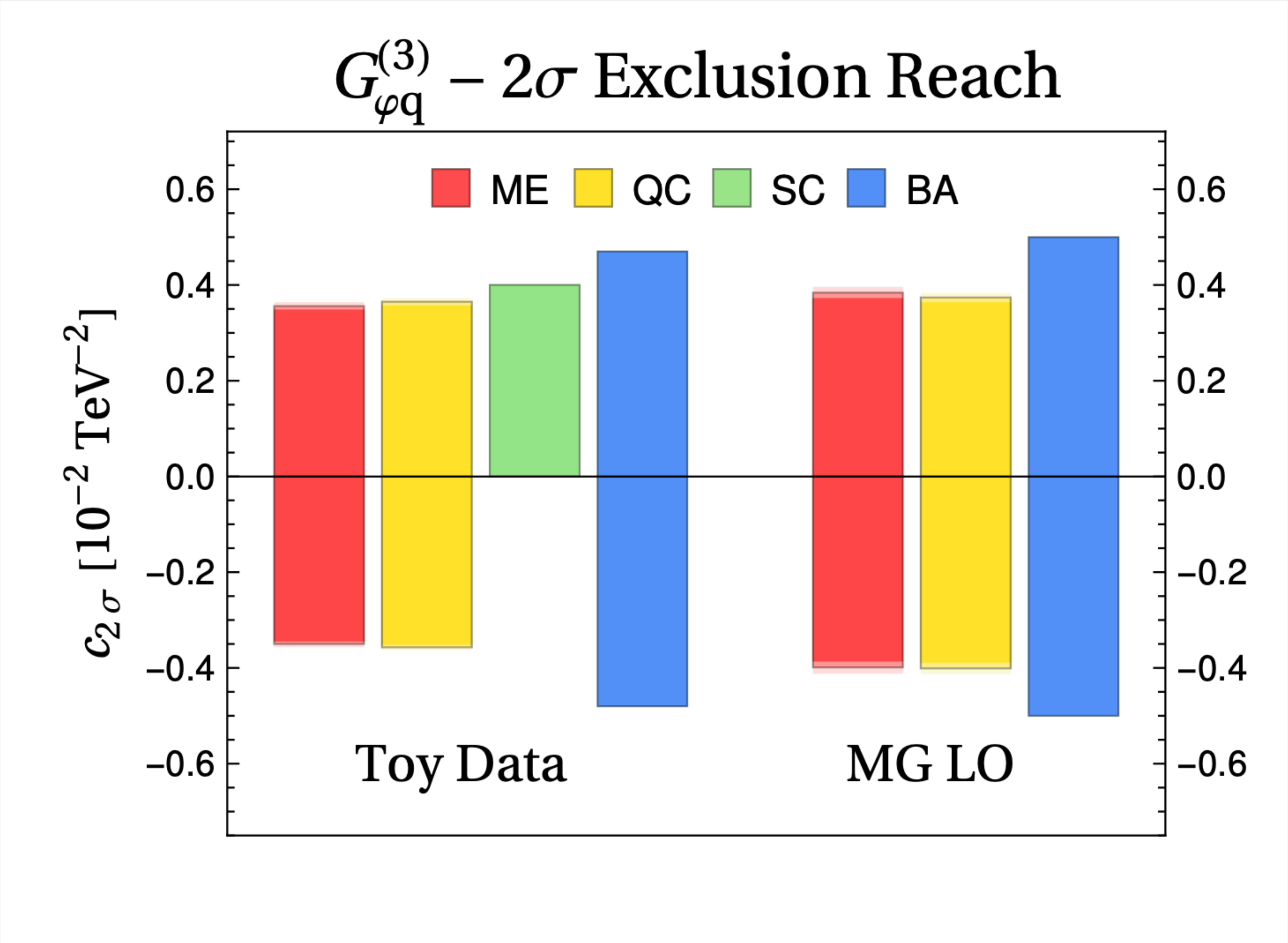
Goal: develop statistically optimal observables and integrate them into global EFT fits

Related work

- The **likelihood** (ratio) as central object
- Matrix Element Method (MEM): transfer functions
- Parameterise the likelihood ratio with **Neural Networks**
- Current studies are limited to a **small number of EFT coefficients**

S. Chen, A. Glioti, G. Panico, A. Wulzer [2007.10356]





J. Brehmer, K. Cranmer [2010.06439]

Finding optimal observables

Neyman-Pearson: the most powerful statistical test at fixed size (significance level) between two *simple* hypotheses H_0 and H_1 is the (log) likelihood ratio:

$$t_c(D) \equiv \log \frac{\mathscr{L}(H_1 \mid D)}{\mathscr{L}(H_0 \mid D)}$$

- Any other test statistic has less power, i.e. gives suboptimal bounds
- No longer applies in case of systematics: profile likelihood ratio (WIP)

Finding optimal observables

Key idea: train a NN classifier to learn the *extended* likelihood ratio

SM hypothesis

Number of events

$$t_c \equiv \log \frac{\mathscr{L}(H_1 | D)}{\mathscr{L}(H_0 | D)} = \nu^{\text{eft}} - \nu^{\text{sm}} - \sum_{i=1}^{n} \log \frac{d\sigma(x_i, c)}{d\sigma(x_i, 0)}$$

Extended likelihood ratio

EFT hypothesis (null)

Expected number of events under the SM

Cross section ratio

The events x_i can be invariant masses, rapidities, scattering angles, p_T, ...

Binary classifier

- Train a **classifier** by minimising the cross entropy (or the quadratic loss) loss functional

$$L[f(x)] = -\int dx \frac{d\sigma_0}{dx} \log(1-f) - \int dx \frac{d\sigma_1}{dx} \log f$$

which gives

The choice of loss functional is not unique!

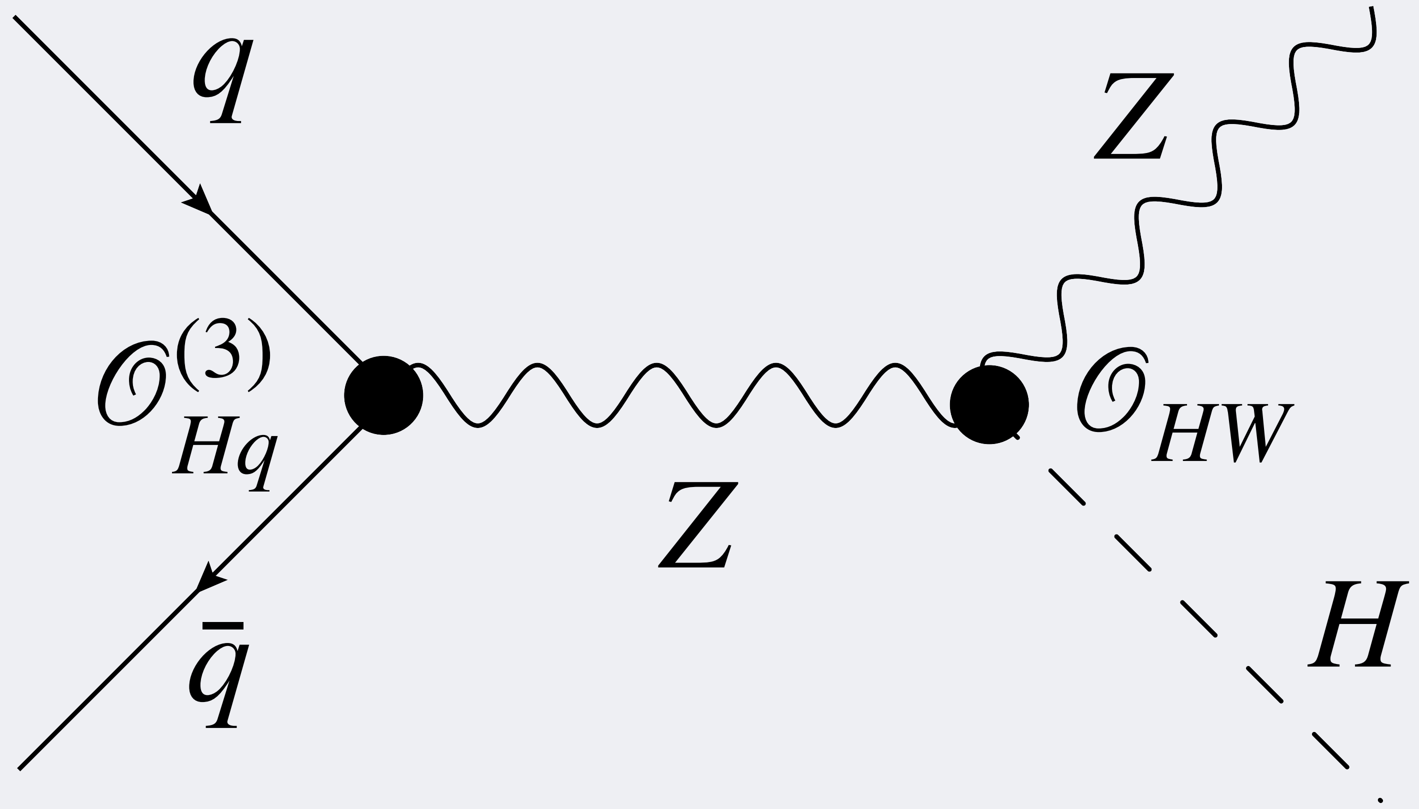
$$\frac{\delta L}{\delta f(x')} = \frac{d\sigma_0}{1-f} - \frac{d\sigma_1}{f} = 0 \implies \hat{f} = \frac{1}{1 + d\sigma_0/d\sigma_1}$$

- This is a **one-to-one estimator** of the likelihood ratio!

VH production

- Need access to underlying truth to assess the NN accuracy
- Efficient **pipeline** using **FeynRules, SMEFTsim, FeynArts / FormCalc** to obtain analytical predictions
- LO parton level, but the method is applicable to any final state
- Study $\mathcal{O}_{HW}, \mathcal{O}_{HWB}, \mathcal{O}_{HB}, \mathcal{O}_{HD}$ and $\mathcal{O}_{Hq}^{(3)}$ up to $\mathcal{O}\left(\Lambda^{-4}\right)$ differential in the rapidity and invariant mass m_{VH}

$$\frac{d\sigma}{dm_{VH}dY} = \frac{2m_{VH}}{s}\left[\sum_f f_f\left(x_1, Q\right) f_{\bar{f}}\left(x_2, Q\right) \hat{\sigma}_{q\bar{q}\to VH}\right]$$



VH production: FormCalc

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dim6QuadSimp = FullSimplify[dim6Quad]
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$$\frac{1}{216\, cth^6\, MZ2\, (MZ2 - S)^2\, S^2\, sth^6\, T^2\, U^2\, \Lambda^4}$$

$$\pi v^2 \Big(-48\, Alfa2\, cHq3\, cth^2\, \pi\, S\, sth^2\, T^2\, U^2 \Big(3\, cHq3\, (MZ2 - S)\, S\, \big(MZ2^2 + MZ2\, (S - T - U) + T\, U\big) + cHW\, cth^2 \big(-3\, S + 2\, (MZ2 + S)\, sth^2\big) \big(MZ2^3 + MH2\, (3\, MZ2 - 2\, S - T)\, T + MH2^2\, (-MZ2 + T) + MZ2^2\, (6\, S - 3\, U) -$$

$$MH2\, (MZ2 + T)\, U + S\, T\, (S + T + U) + MZ2 \big(S^2 - (T - 2\, U)\, (T + U) - S\, (5\, T + U)\big)\big) + sth \big(cHWB\, cth \big(-MZ2 - 2\, S + 2\, (MZ2 + S)\, sth^2\big) + cHB\, sth \big(2\, cth^2\, (-MZ2 + S) + S \big(-3 + 4\, sth^2\big)\big)\big)$$

$$\big(MZ2^3 + MH2\, (3\, MZ2 - 2\, S - T)\, T + MH2^2\, (-MZ2 + T) + MZ2^2\, (6\, S - 3\, U) - MH2\, (MZ2 + T)\, U + S\, T\, (S + T + U) + MZ2 \big(S^2 - (T - 2\, U)\, (T + U) - S\, (5\, T + U)\big)\big)\Big)\, v^2 +$$

$$Alfa \Big(24\, cHq3\, cth^4\, (MZ2 - S)\, S\, sth^4 \big(cHW\, cth^2 \big(-3\, S + 4\, MZ2\, sth^2\big) + sth \big(-cHWB\, cth \big(S + MZ2 \big(2 - 4\, sth^2\big)\big) + cHB\, sth \big(4\, cth^2\, (-MZ2 + S) + S \big(-3 + 4\, sth^2\big)\big)\big)\big)\, T^2\, U^2 \big(MZ2^3 + MH2\, (3\, MZ2 - 2\, S - T)\, T +$$

$$MH2^2\, (-MZ2 + T) + MZ2^2\, (6\, S - 3\, U) - MH2\, (MZ2 + T)\, U + S\, T\, (S + T + U) + MZ2 \big(S^2 - (T - 2\, U)\, (T + U) - S\, (5\, T + U)\big)\big) + 8\, cth^4\, sth^4 \big(cHW^2\, cth^4 \big(9\, S^2 - 24\, MZ2\, S\, sth^2 + 32\, MZ2^2\, sth^4\big) +$$

$$2\, cHW\, cth^2\, sth \big(cHWB\, cth \big(3\, S\, (2\, MZ2 + S) - 8\, MZ2\, (2\, MZ2 + S)\, sth^2 + 32\, MZ2^2\, sth^4\big) + cHB\, sth \big(4\, cth^2\, (MZ2 - S) \big(3\, S - 8\, MZ2\, sth^2\big) + S \big(9\, S - 12\, (MZ2 + S)\, sth^2 + 32\, MZ2\, sth^4\big)\big)\big) +$$

$$sth^2 \big(cHWB^2\, cth^2 \big(5\, S^2 + 8\, MZ2^2 \big(1 - 2\, sth^2\big)^2 + 4\, MZ2\, S \big(-1 + 2\, sth^2\big)\big) + cHB^2\, sth^2 \big(32\, cth^4\, (MZ2 - S)^2 - 8\, cth^2\, (MZ2 - S)\, S \big(-3 + 8\, sth^2\big) + S^2 \big(9 - 24\, sth^2 + 32\, sth^4\big)\big) +$$

$$2\, cHB\, cHWB\, cth\, sth \big(S \big(6\, MZ2 + 3\, S + 4\, (-7\, MZ2 + S)\, sth^2 + 32\, MZ2\, sth^4\big) + 4\, cth^2\, (-MZ2 + S) \big(S + MZ2 \big(-4 + 8\, sth^2\big)\big)\big)\big)\big)\, T^2\, U^2$$

$$\big(MZ2^3\, (4\, S + T - 2\, U) - (MH2 - S)^2\, T\, (MH2 - S - T - U) + MZ2^2 \big(4\, S^2 + MH2\, T - (T - 2\, U)\, (T + U) - S\, (3\, T + 2\, U)\big) - MZ2 \big(MH2^2\, (T - 2\, U) + S\, (3\, S\, T + 2\, (T - U)\, U) + 2\, MH2\, (S\, (-2\, T + U) + U\, (T + U))\big)\big)\Big) +$$

$$9\, cHq3^2\, S^2 \Big(8\, Alfa2\, \pi^2\, T^2\, U^2 \big(MZ2^2 + MZ2\, (S - T - U) + T\, U\big)\, v^4 - cth^4\, (MZ2 - S)^2\, sth^4$$

$$\big(-8\, T^2\, U^2 \big(MZ2^2 + MZ2\, (S - T - U) + T\, U\big) - \big(2\, S\, T^2\, U^2 - 3\, MH2\, MZ2^2 \big(T^2 + U^2\big) - MZ2^3 \big(T^2 + U^2\big) + 2\, MZ2\, T\, U \big(T^2 + U^2\big) + MZ2^2\, (S + T + U) \big(T^2 + U^2\big)\big)\, v^2\, yu^2\big)\Big)\Big)$$

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dim6LinSimp = FullSimplify[dim6Lin]
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$$\frac{1}{108\, cth^6\, MZ2\, (MZ2 - S)^2\, S\, sth^6\, T^2\, U^2\, \Lambda^2}$$

$$\pi v^2 \Big(-4\, Alfa2\, \pi\, T^2\, U^2 \Big(cHW\, cth^4\, sth^2 \big(-9\, S + 12\, (MZ2 + S)\, sth^2 - 32\, MZ2\, sth^4\big) \big(MZ2^3 + MH2\, (3\, MZ2 - 2\, S - T)\, T + MH2^2\, (-MZ2 + T) + MZ2^2\, (6\, S - 3\, U) - MH2\, (MZ2 + T)\, U + S\, T\, (S + T + U) +$$

$$MZ2 \big(S^2 - (T - 2\, U)\, (T + U) - S\, (5\, T + U)\big)\big) - cth^2\, sth^3 \big(cHWB\, cth \big(6\, MZ2 + 3\, S + 4\, (-7\, MZ2 + S)\, sth^2 + 32\, MZ2\, sth^4\big) + cHB\, sth \big(-4\, cth^2\, (MZ2 - S) \big(-3 + 8\, sth^2\big) + S \big(9 - 24\, sth^2 + 32\, sth^4\big)\big)\big)$$

$$\big(MZ2^3 + MH2\, (3\, MZ2 - 2\, S - T)\, T + MH2^2\, (-MZ2 + T) + MZ2^2\, (6\, S - 3\, U) - MH2\, (MZ2 + T)\, U + S\, T\, (S + T + U) + MZ2 \big(S^2 - (T - 2\, U)\, (T + U) - S\, (5\, T + U)\big)\big) +$$

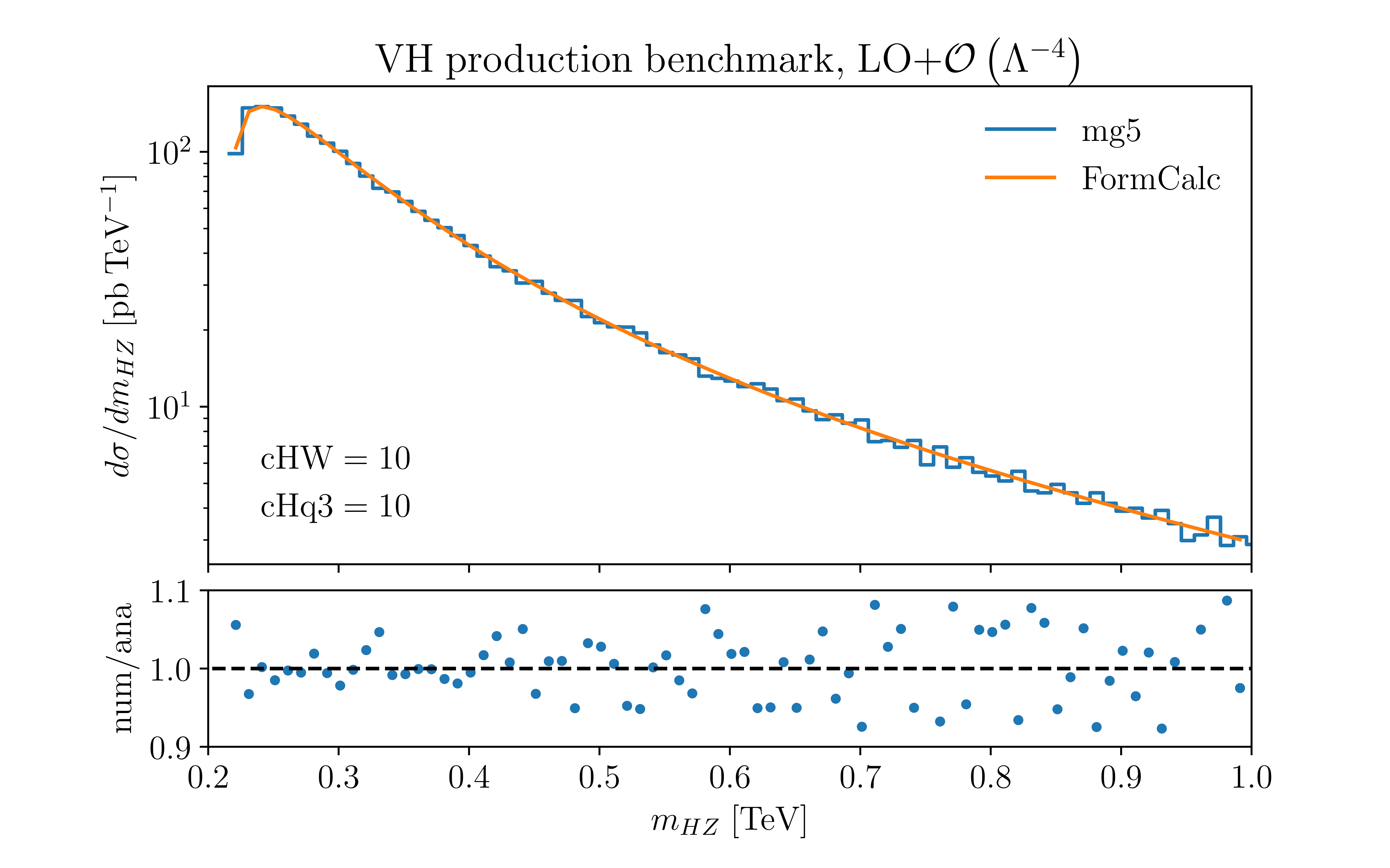
$$6\, cHq3\, S \big(-3 + 4\, sth^2\big) \big(MZ2^2 + MZ2\, (S - T - U) + T\, U\big) \big(cth^2\, (-MZ2 + S)\, sth^2 + Alfa\, \pi\, v^2\big)\Big) +$$

$$3\, Alfa\, cHq3\, cth^4\, (MZ2 - S)^2\, S\, sth^4 \Big(MZ2^2 \big(-3 + 4\, sth^2\big)\, (MZ2 - S - T)\, T^2 + MZ2\, T \big(8\, (MZ2 - S)\, (MZ2 + 2\, S)\, sth^2 + 3\, MZ2\, T - 4\, (3\, MZ2 + 4\, S)\, sth^2\, T + 2 \big(3 - 4\, sth^2\big)\, T^2\big)\, U +$$

$$\big(MZ2^2\, (MZ2 - S) \big(-3 + 4\, sth^2\big) + MZ2 \big(3\, MZ2 - 4\, (3\, MZ2 + 4\, S)\, sth^2\big)\, T + 2 \big(3\, S - 8\, MZ2\, sth^2\big)\, T^2\big)\, U^2 -$$

$$MZ2 \big(-3 + 4\, sth^2\big)\, (MZ2 + 2\, T)\, U^3 + 3\, MH2\, MZ2^2 \big(\big(-3 + 4\, sth^2\big)\, T^2 + 8\, sth^2\, T\, U + \big(-3 + 4\, sth^2\big)\, U^2\big)\Big)\, yu^2\Big)$$

VH production: benchmark



Training the likelihood ratio

We separate the learning problem by exploiting the structure inherent to the EFT parameter space:

$$r(x, \mathbf{c}) = 1 + c_1\alpha_1(x) + c_2\alpha_2(x) + c_1^2\beta_{11}(x) + c_1c_2\beta_{12}(x) + \beta_{22}c_2^2$$

1. Train the linear coefficient functions in parallel
2. Switch on quadratic corrections and train the quadratic coefficients
3. The cross terms can finally be extracted

New: this allows for efficient scaling (quadratically) and parallel training for n EFT parameters

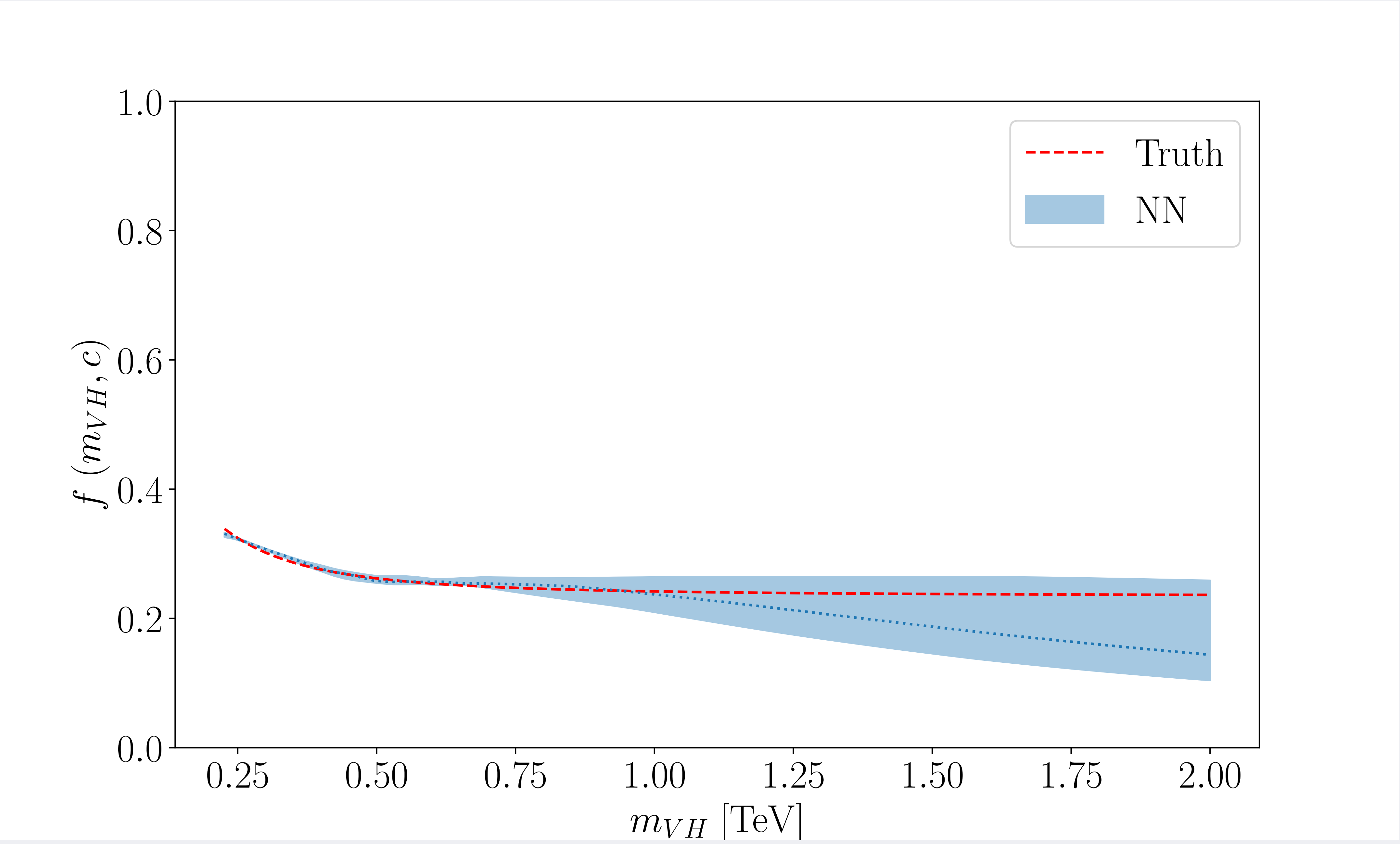
NN model uncertainties

- We systematically assess the **model uncertainties** associated to the NN parameterisation of the likelihood ratio

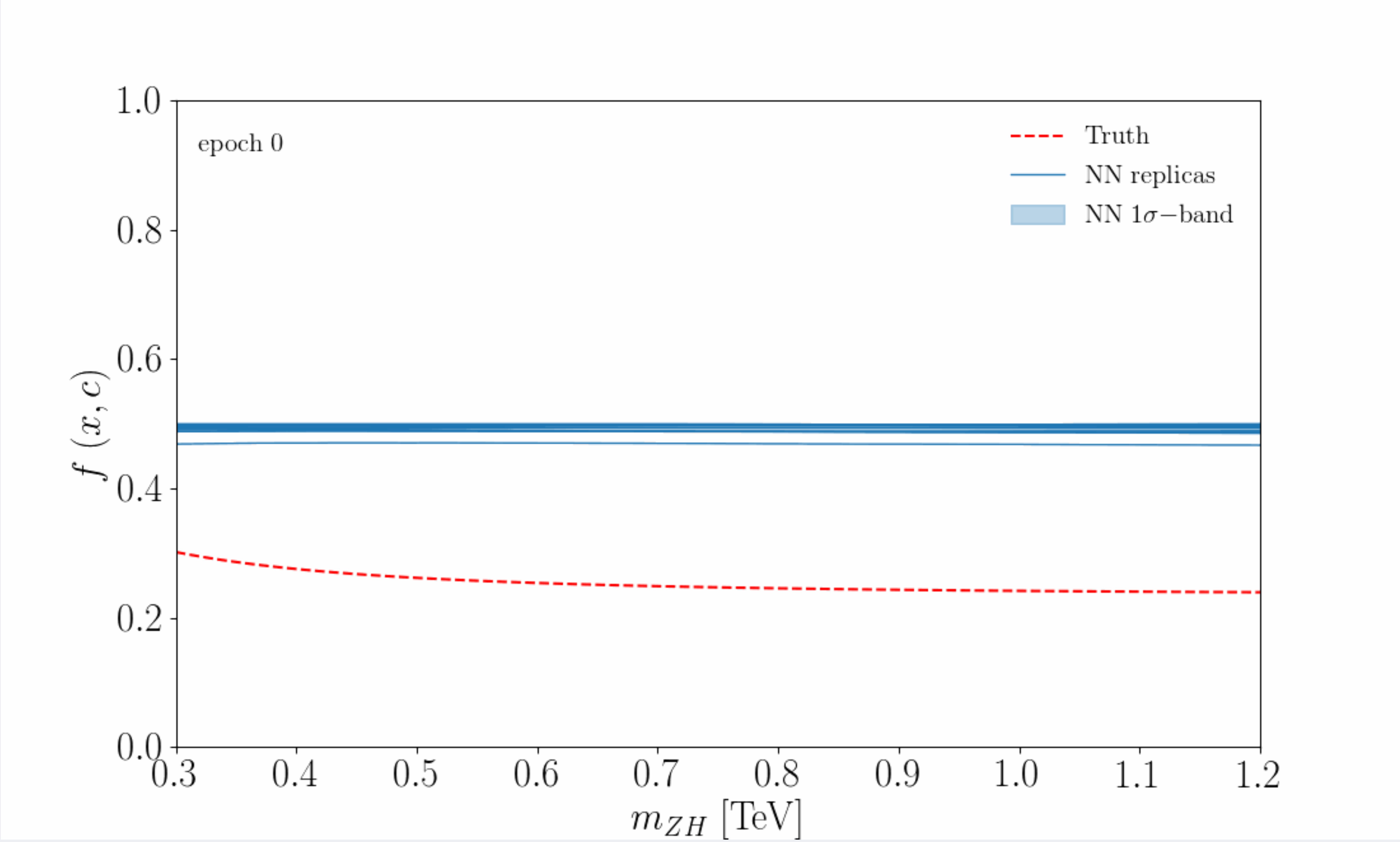
- **Replica**: an independent MC training set to propagate the error to the space of models

- Train 30 independent replicas **in parallel**

- Translate to the error on the Wilson coefficients

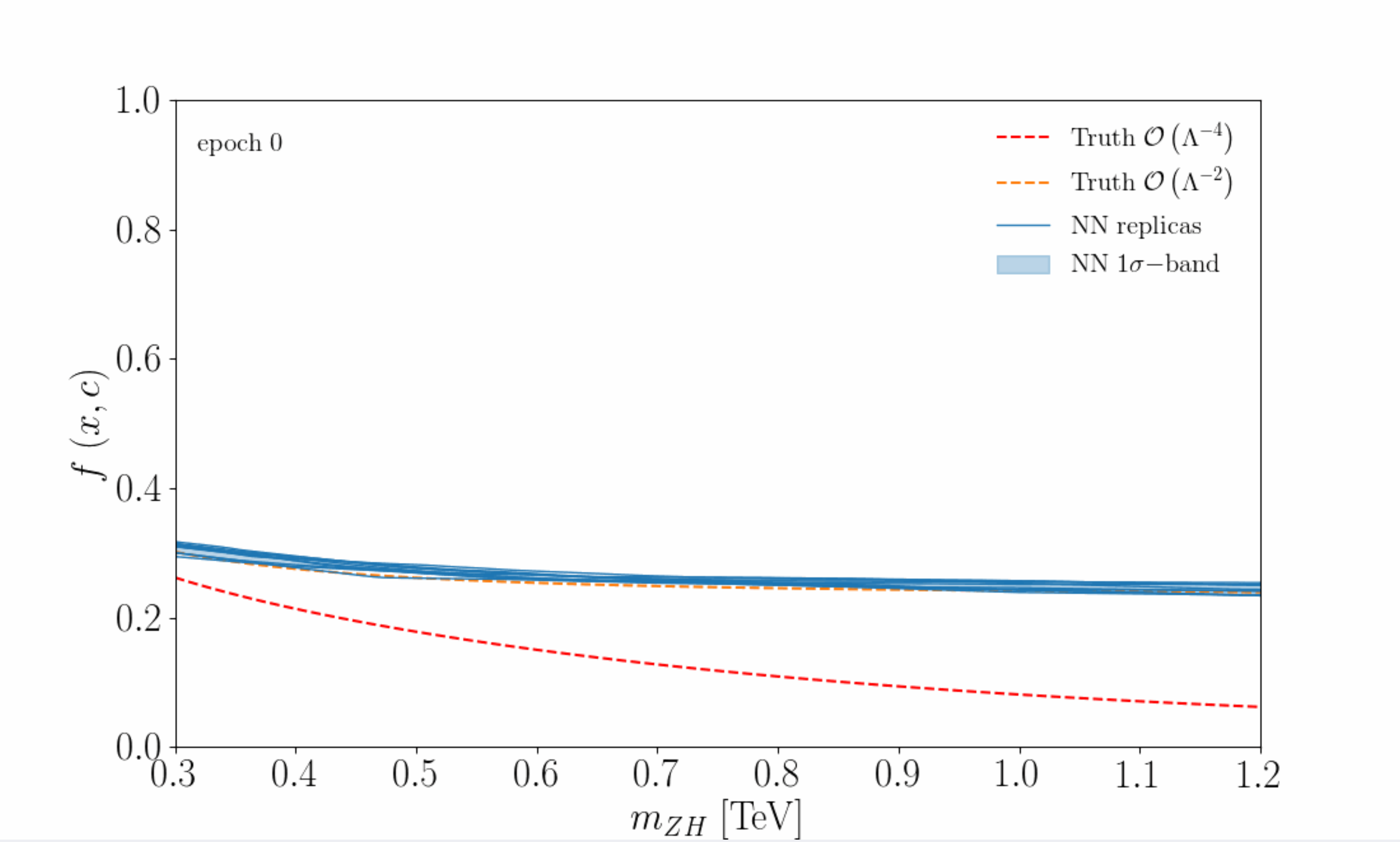


Seeing the training at work



- Trained on 30 replicas
- 100K events in SM and EFT
- Cross validation
- Architecture: {2, 5x30, 1} with ReLU activation functions
- Standardised training data to zero mean and unit variance

Adding quadratic corrections



- Trained on 30 replicas
- 100K events in SM and EFT
- Cross validation
- Architecture: {2, 5x30, 1} with ReLU activation functions
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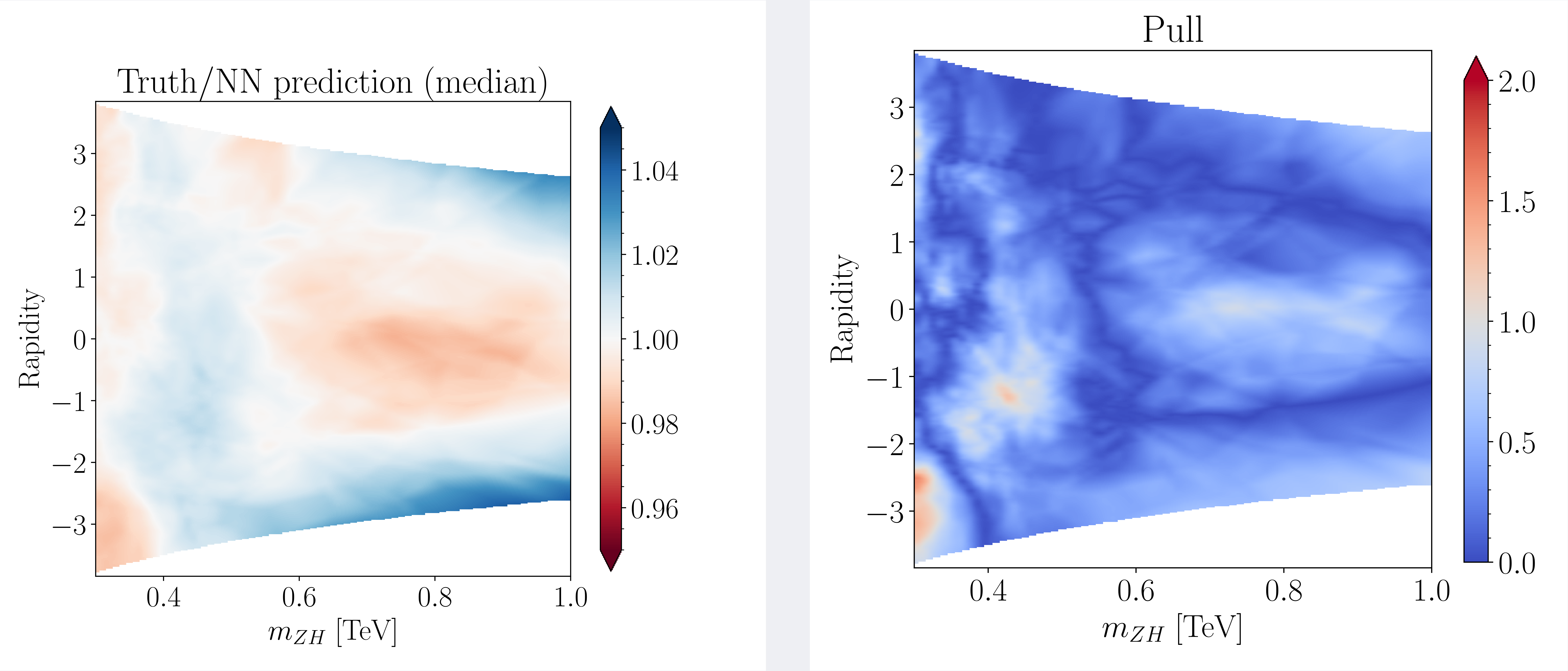
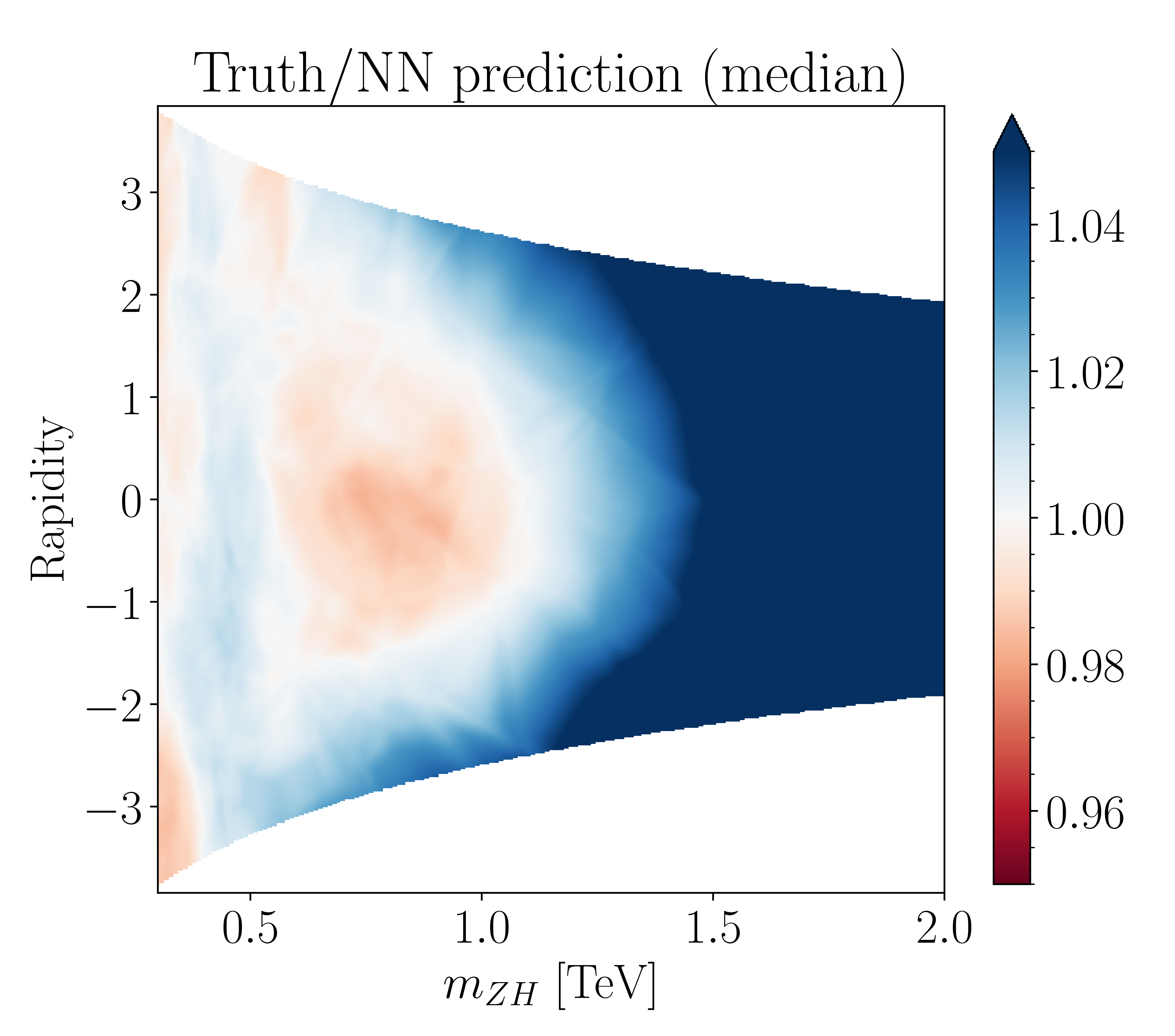
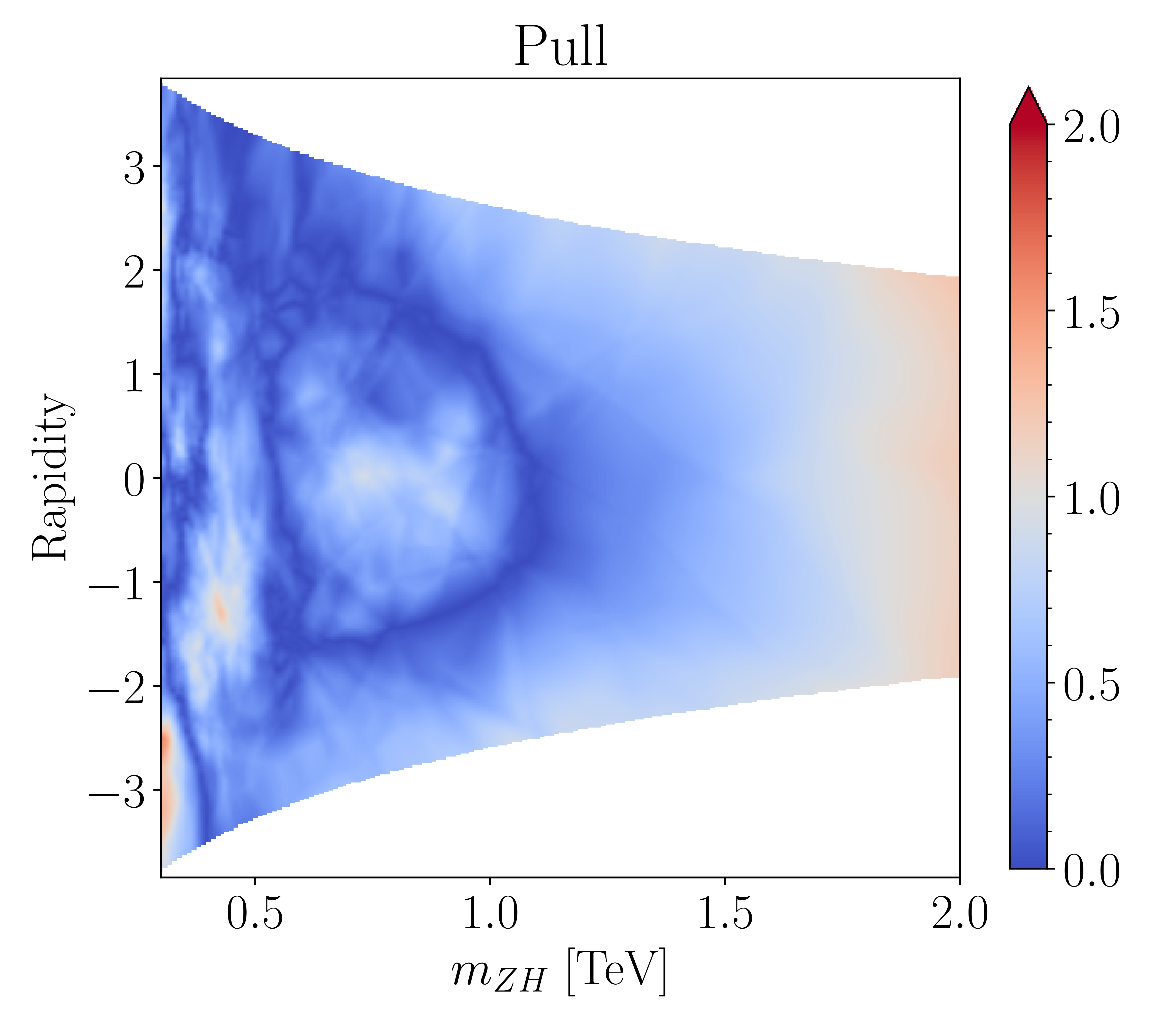
Training performances

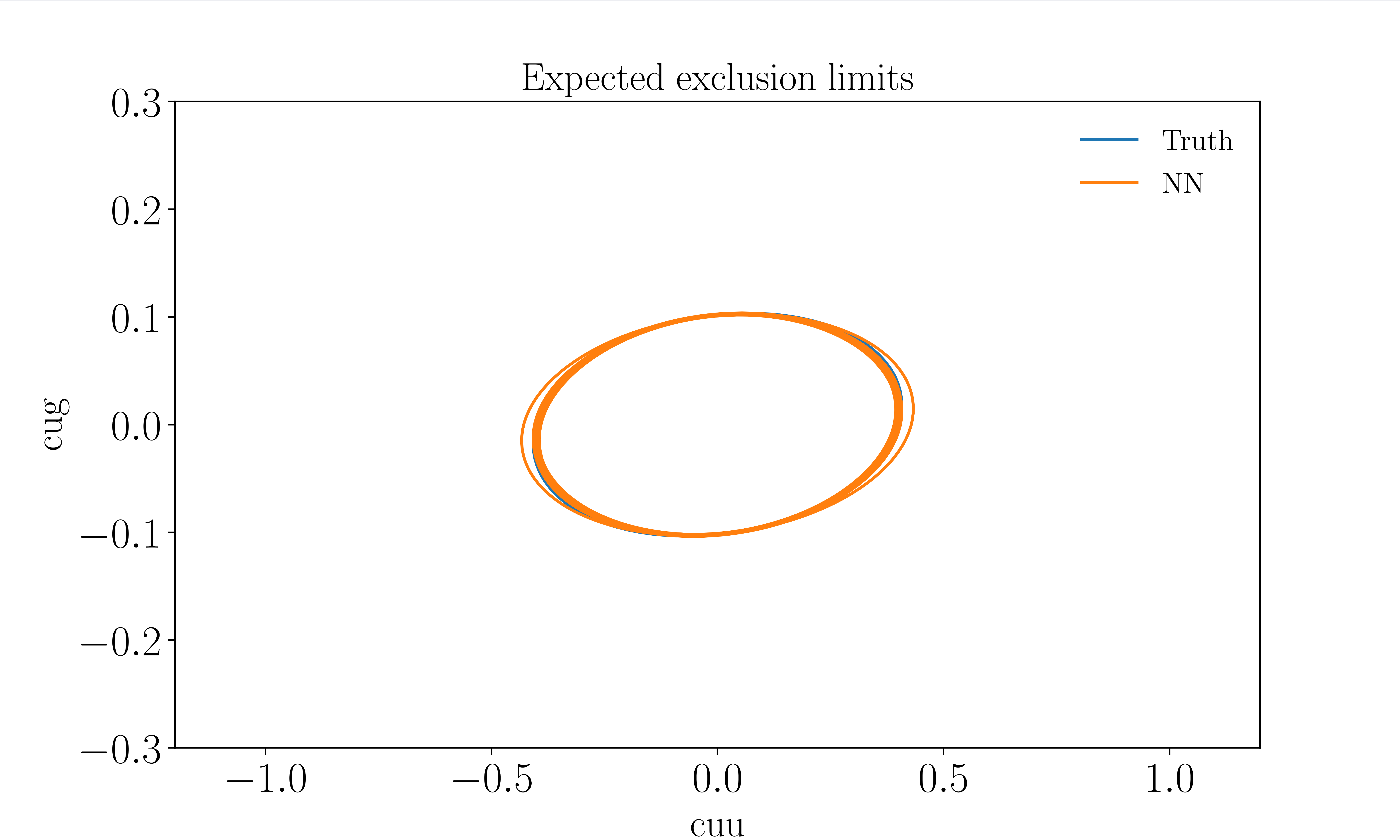


Training performances





Limits: NN versus Truth



Summary

Key question: given a collider process, how can one define **optimal observables** with the highest sensitivity to EFT coefficients?

- Train a surrogate of the **likelihood ratio**
- **Efficient scaling** properties to n EFT parameters necessary for global EFT fits
- Good reconstruction **performances** throughout phase space
- Outlook: Include **systematics** with the profile likelihood ratio

Thank you!