

Exact top-quark mass dependence in hadronic Higgs production

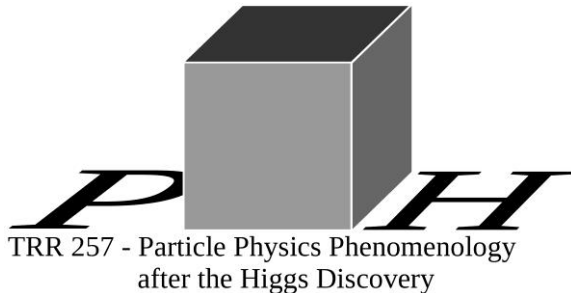
Marco Niggetiedt

in collaboration with M. Czakon, R.V. Harlander and J. Klappert

Institute for Theoretical Particle Physics and Cosmology

RWTH Aachen University

Based on [PRL 127 \(2021\), 162002](#)



Higgs 2021
Stony Brook (virtual)
October 21st 2021

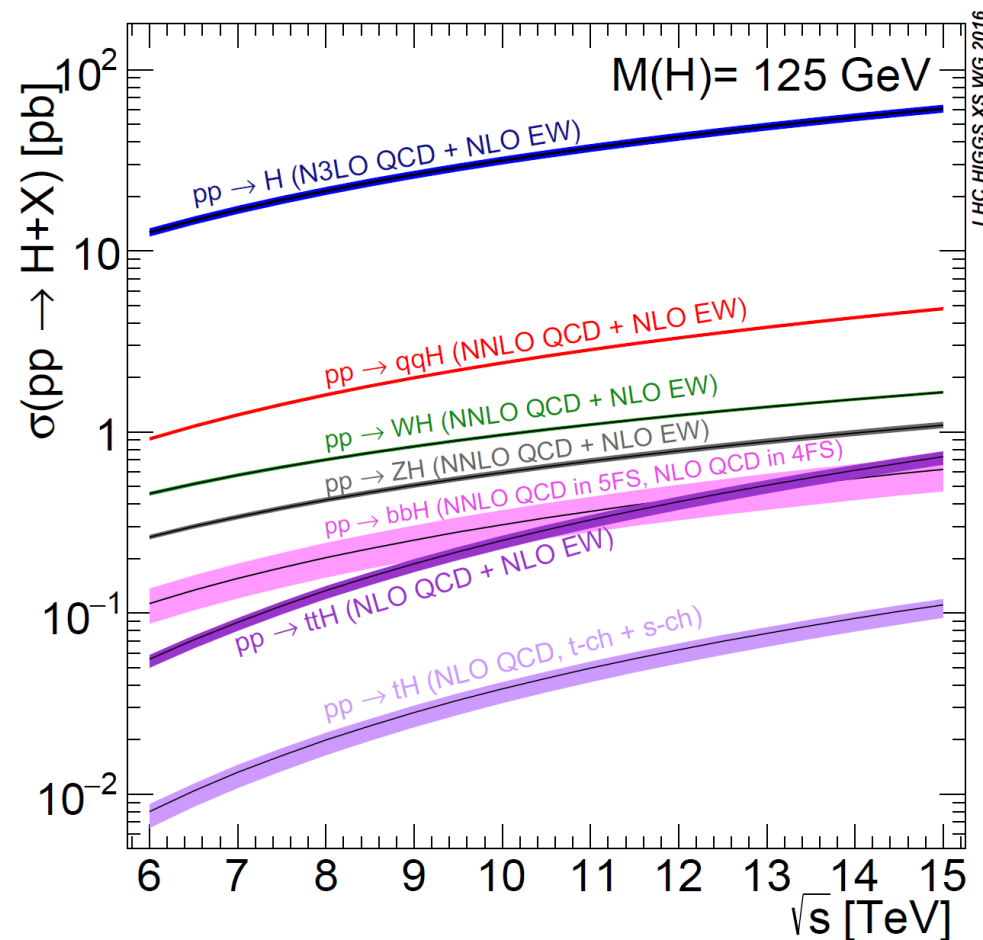
Motivation

- Gluon fusion is the predominant Higgs-boson production mode at the LHC
- Higgs-boson plays unique role in the SM:
 - Only scalar particle
 - Only particle with Yukawa interactions to fermions

Handbook of LHC Higgs cross sections:

4. Deciphering the nature of the Higgs sector

Report of the LHC Higgs Cross Section Working Group '16



LHC @13 TeV

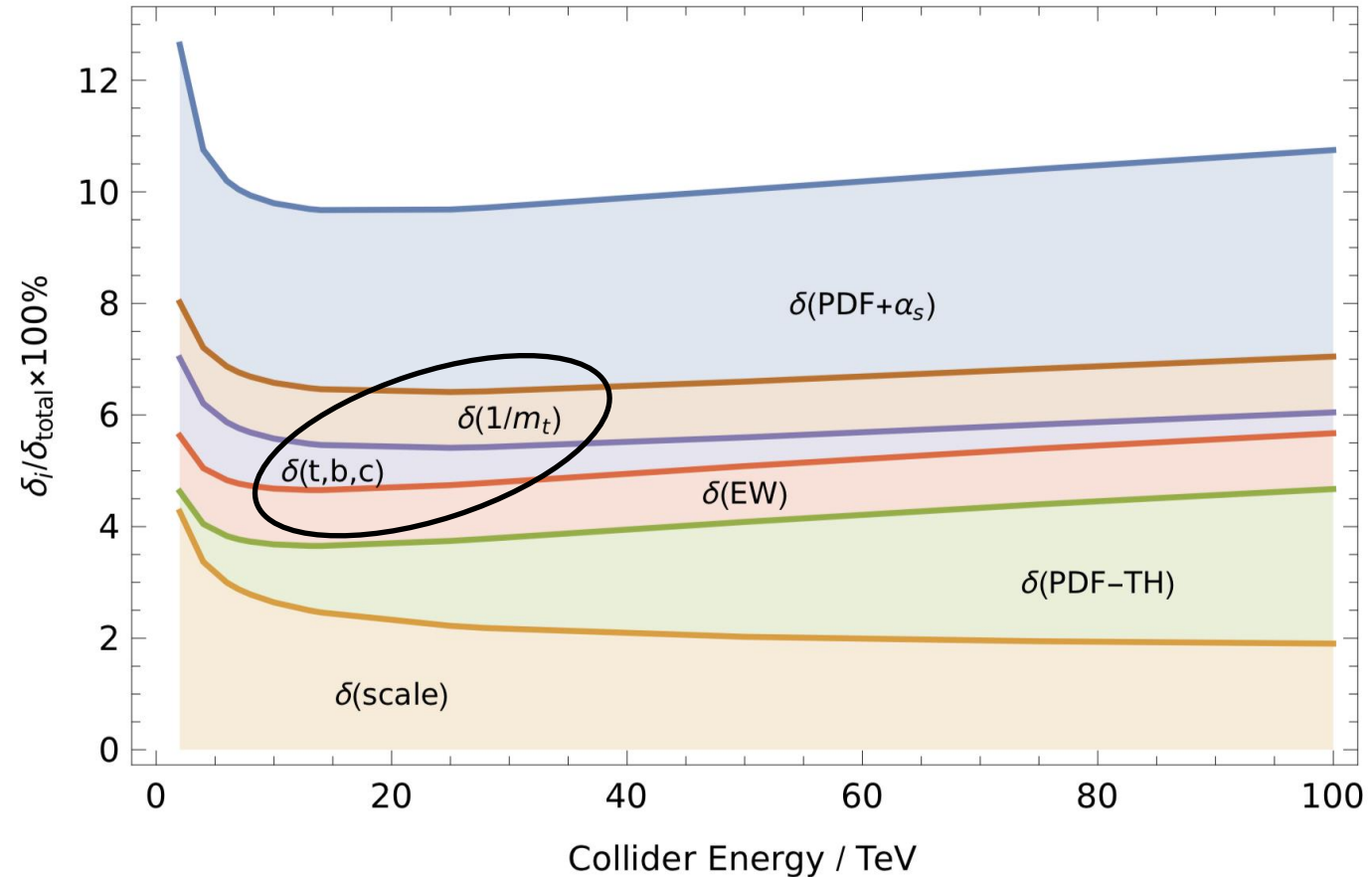
$$\sigma = 48.58 \text{ pb}^{+2.22 \text{ pb} (+4.56\%)}_{-3.27 \text{ pb} (-6.72\%)} \text{ (theory)} \pm 1.56 \text{ pb} (3.20\%) \text{ (PDF}+\alpha_s\text{)}.$$

Theory uncertainties

- $\delta(\text{scale})$ and $\delta(\text{PDF-TH})$ due to missing higher-order terms in $\hat{\sigma}$ and PDFs [Anastasiou, et al. '15](#)
- $\delta(\text{trunc})$ has been removed [Mistlberger '18](#)
- $\delta(\text{EW})$ was addressed recently
[Bonetti, Melnikov, Tancredi '18](#)
[Anastasiou, del Duca, et al. '19](#)
[Becchetti, Bonciani, et al. '21](#)
- $\delta(t,b,c)$ and $\delta(1/m_t)$ related to quark mass effects

$\delta(\text{scale})$	$\delta(\text{trunc})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t, b, c)$	$\delta(1/m_t)$
+0.10 pb -1.15 pb	± 0.18 pb	± 0.56 pb	± 0.49 pb	± 0.40 pb	± 0.49 pb
+0.21% -2.37%	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$

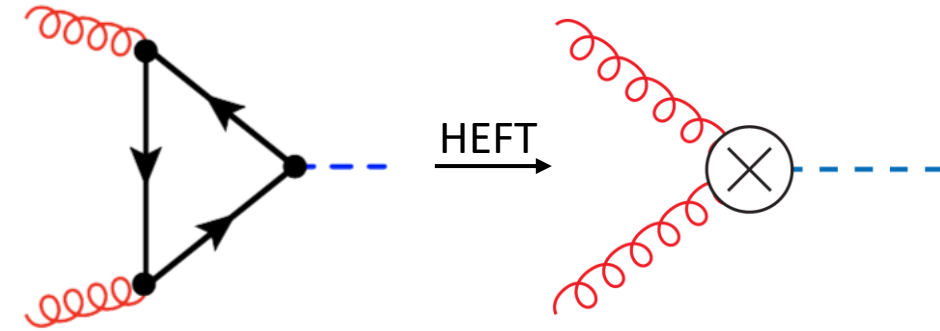
Higgs Physics at the HL-LHC and HE-LHC Report from Working Group 2 on the Physics of the HL-LHC, and Perspectives at the HE-LHC '19



Handbook of LHC Higgs cross sections:
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Contributions to σ_{tot}

48.58 pb =	16.00 pb	(+32.9%)	(LO, rEFT)
	+ 20.84 pb	(+42.9%)	(NLO, rEFT)
	− 2.05 pb	(−4.2%)	((t, b, c), exact NLO)
	+ 9.56 pb	(+19.7%)	(NNLO, rEFT)
	+ 0.34 pb	(+0.7%)	(NNLO, 1/m _t)
	+ 2.40 pb	(+4.9%)	(EW, QCD-EW)
	+ 1.49 pb	(+3.1%)	(N ³ LO, rEFT)



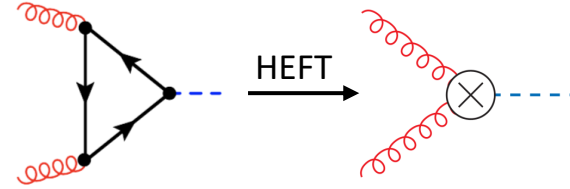
"Born-improved" total cross section:

$$\sigma_{\text{HEFT}}^{\text{HO}} = \left(\frac{\sigma^{\text{HO}}}{\sigma^{\text{LO}}} \right)_{M_t \rightarrow \infty} \sigma^{\text{LO}}$$

- Gluon-fusion is induced by quark loops
 - NLO result available for arbitrary quark masses [Graudenz, Spira, Zerwas '93](#)
 - Radiative corrections beyond NLO restricted to top-loop induced terms [Anastasiou, Melnikov '02](#)
[Harlander, Kilgore '02](#)
[Ravindran, Smith, van Neerven '03](#)
- Dominant effect of top-loop induced terms can be accounted for in HEFT approximation

HEFT

- Introduce effective Higgs-gluon vertex
→ reduce number of **loops** by one
→ reduce number of **scales** by one



- Very good agreement with exact result at NLO

→ Remarkable, because

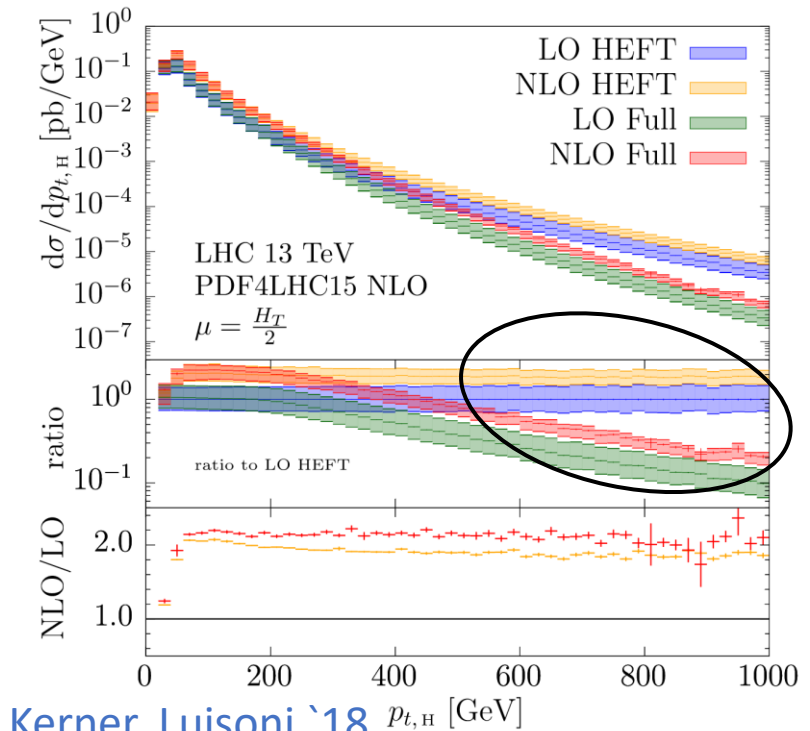
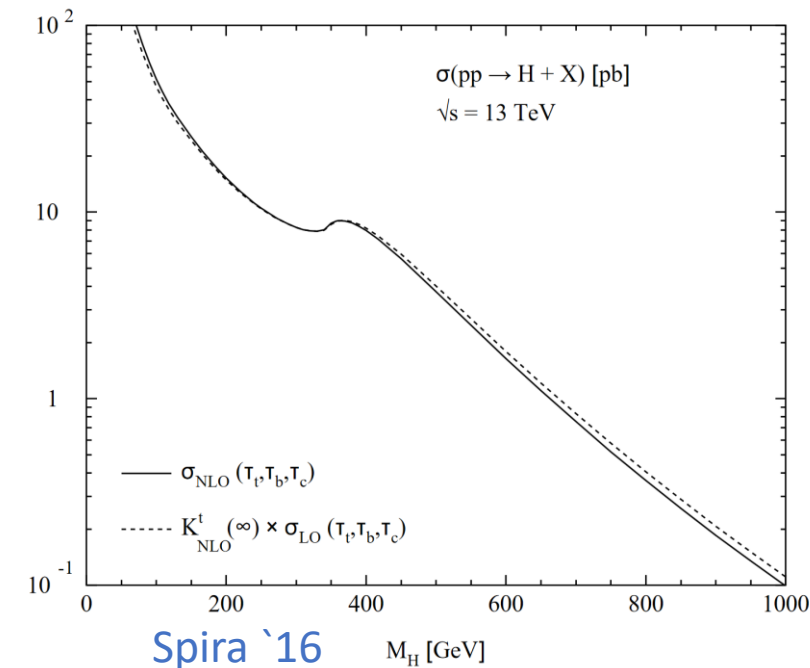
- M_t being the largest scale is invalid over large range of $\sqrt{\hat{s}}$
- $M_t \rightarrow \infty$ is applied to more than 50% of total cross section
- HEFT fails to capture top-mass effects for partonic quark channels

- Qualitative explanation:

- Suppression of large- \hat{s} region by PDFs
- Dominance of the soft region

- Only estimate of top-mass effects beyond HEFT at NNLO based on combination of $1/M_t$ -expansion with leading terms in large- \hat{s} limit

→ Eliminate this uncertainty with exact calculation of top-quark mass effects



Ingredients

- 3-loop virtual corrections (✓)



Davies, Gröber, et al. '19
Czakon, MN '20

- Double-real corrections (✓)



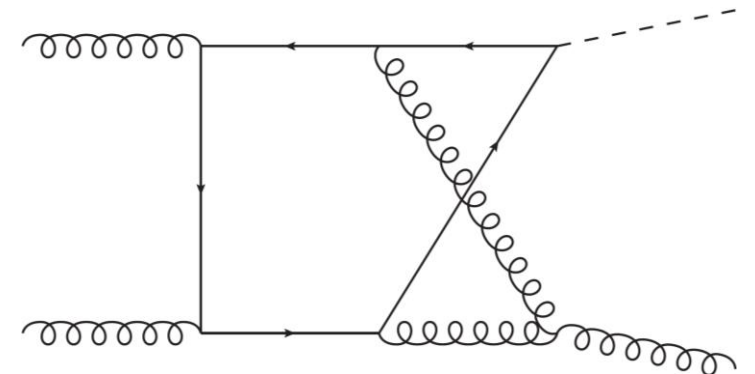
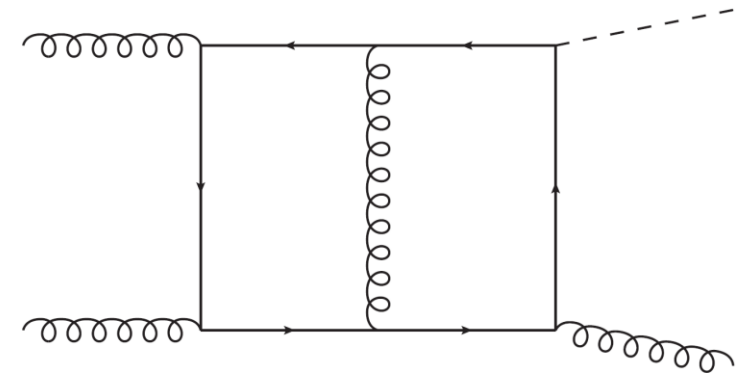
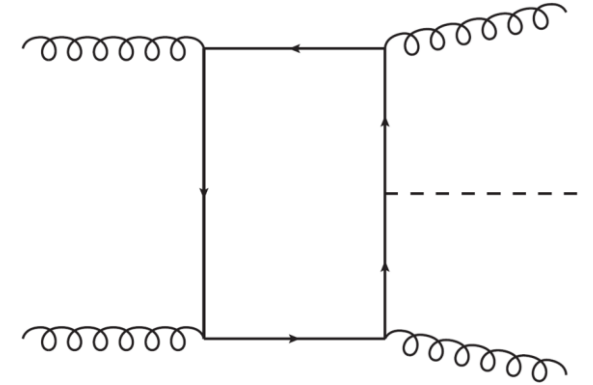
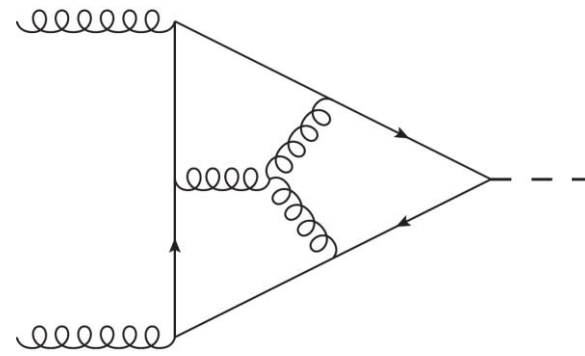
del Duca, Kilgore, et al. '01

- 2-loop real-virtual corrections:

- 4 scales (m_q^2, s, t, u or m_H^2)
+ dimension $d = 4 - 2\varepsilon$
- 282 diagrams for $gg \rightarrow Hg$
- 49 diagrams for $q\bar{q} \rightarrow Hg$

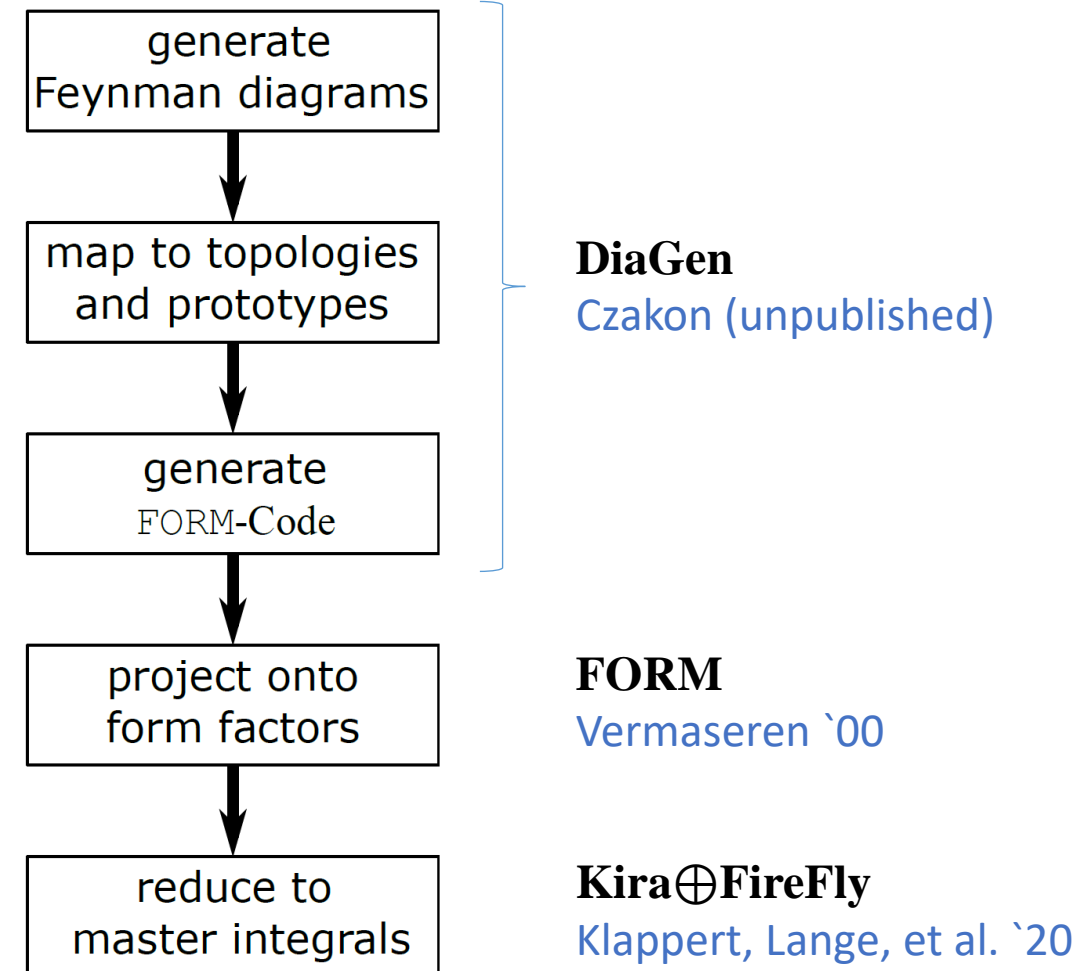
→ Master integrals have been computed

Frellesvig, Hidding, et al. '19



Workflow of the computation

- Get rid of tensor/colour structure to end up with a linear combination of scalar integrals with rational function coefficients in front
- Reduce the scalar integrals to a linearly independent set of master integrals (MI) (447 master integrals for $gg \rightarrow Hg$)
- Reduction is highly non-trivial since rational coefficients depend on 5 variables!
→ Exploit finite fields to reconstruct symbolic coefficients from numerical probes of the system of equations



Details of the reduction

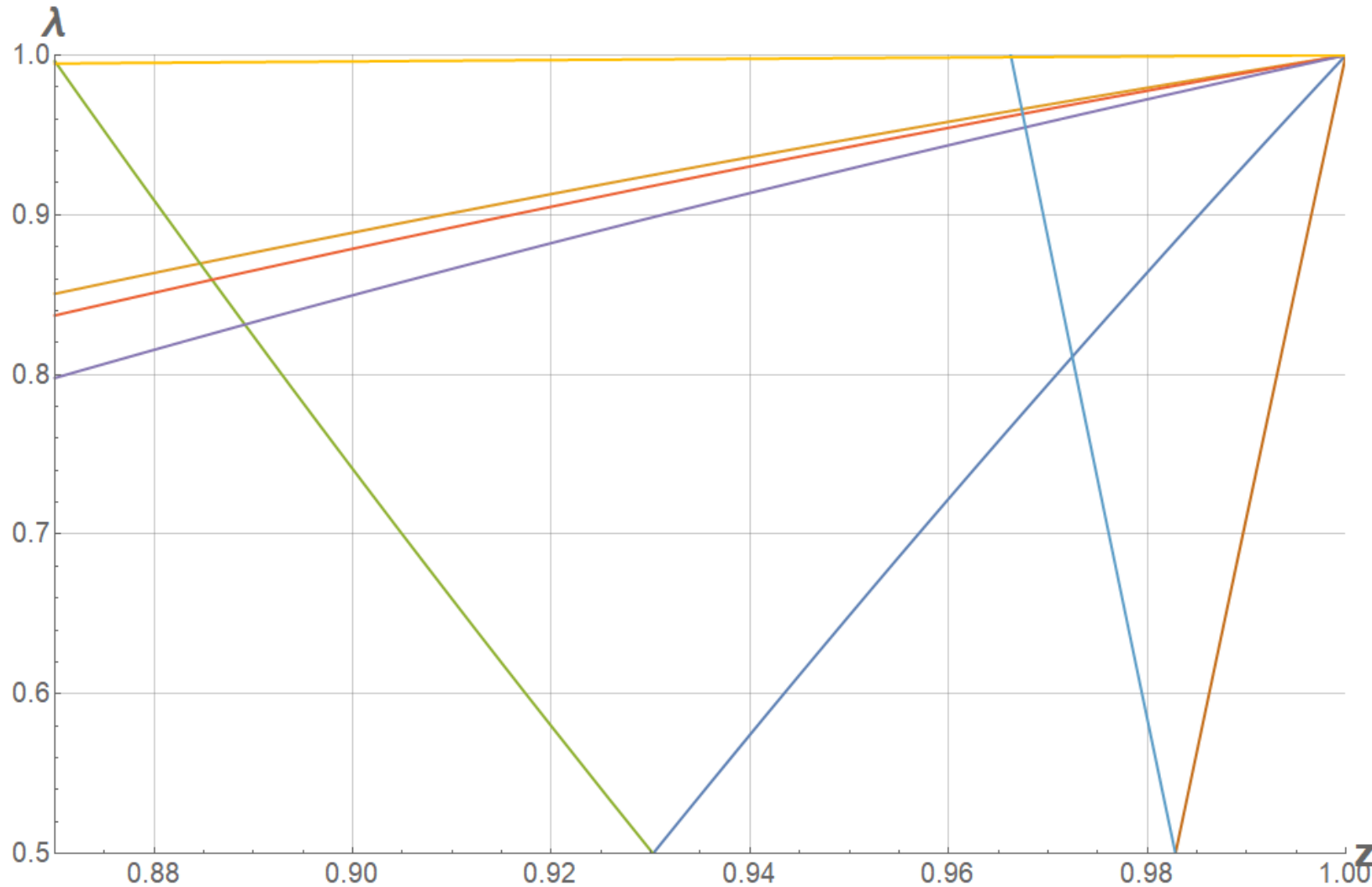
- Run reduction once with symbolic d and m_q^2 , kinematics are set to rational numbers
 - arrive at the default basis of dotted masters
- Perform an automatized (heuristic) search for a basis, which minimizes the size of differential equations in m_q^2 by shifting the dots
- Reduce with full symbolic dependence to this set of master integrals
- Additionally, shift dimension of (almost) all masters with seven propagators from $d = 4 - 2\varepsilon$ to $d = 6 - 2\varepsilon$
 - New basis guarantees faster reduction
 - Better numerical stability

Computing the MI

- Refrain from computing the MI analytically
- Two popular methods for calculating MI numerically:
 - Sector decomposition
 - Via differential equations
- Take the derivative of the MI with respect to the kinematic invariants and reduce the result to obtain a system of differential equations
- Provide boundary conditions in the limit $m_q^2 \rightarrow \infty$ and let the system evolve

Details of the computation

Above top-quark threshold



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$m_t^2 / m_H^2 = 23/12$$

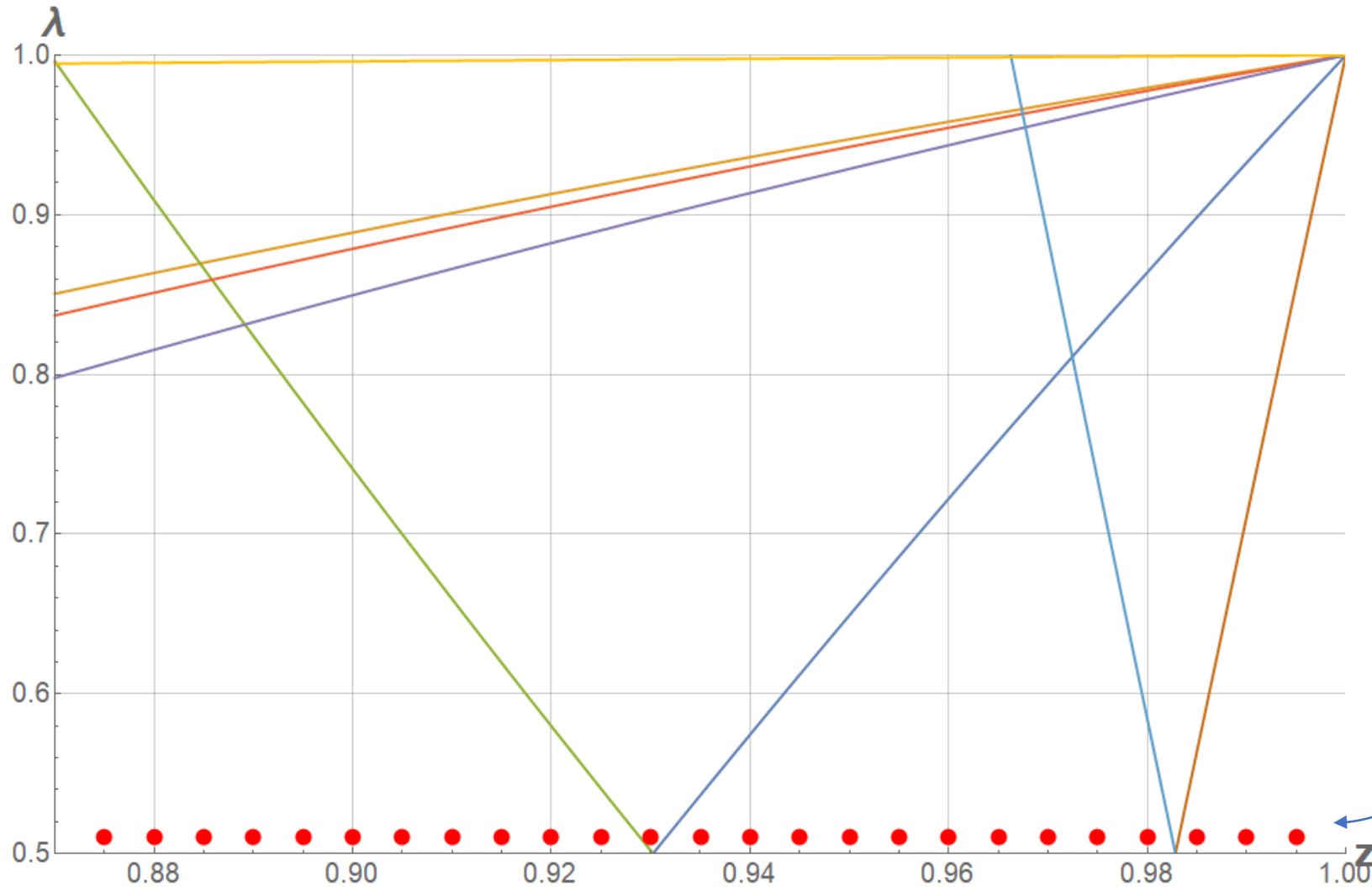
Range of parameters:

- $\lambda \in (0, 1)$
- $z \in (0, 1)$

Poles of
differential
equations
in λ

Details of the computation

Above top-quark threshold



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

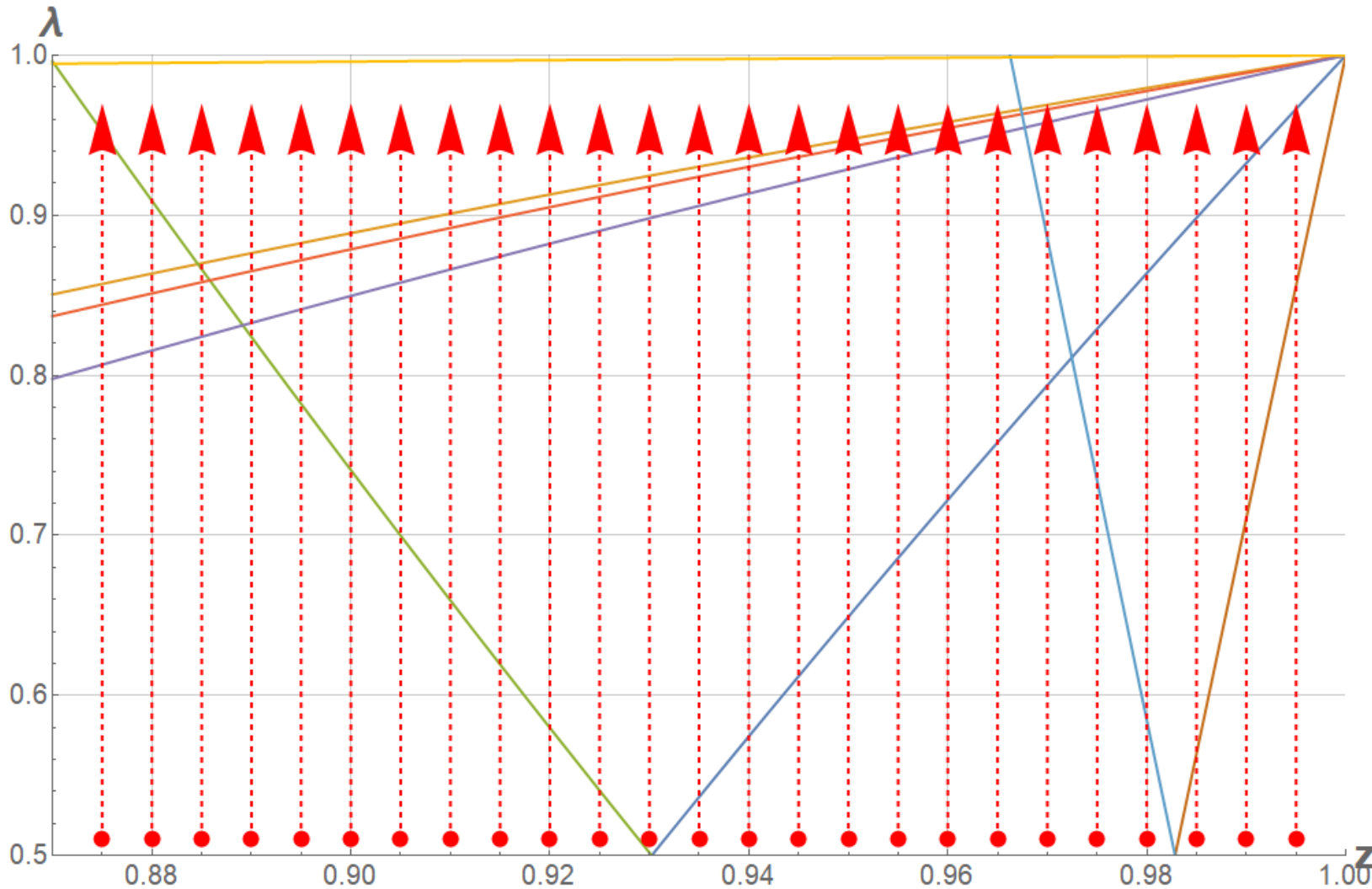
$$m_t^2 / m_H^2 = 23/12$$

Integrate differential equations in m_q^2 from boundaries at $m_q^2 \rightarrow \infty$ to top-quark mass

Boundaries for numerical integration

Details of the computation

Above top-quark threshold



$$z = 1 - m_H^2 / \hat{s}$$

$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$m_t^2 / m_H^2 = 23/12$$

Integrate differential equations in λ to the edge at $\lambda \sim 1$

Collect numerical samples for MI along straight integration contours

Details of the computation

- Problem is symmetric with respect to $\lambda = 1/2$
 - reflect numerical samples to cover entire region above top-quark threshold
- Below threshold:
 - Construct large-mass expansion up to order $(1/m_q^2)^{40}$ with full symbolic dependence
 - Method to obtain expansion coefficients inspired by interpolation techniques
 - Sufficient to cover region below top-quark threshold

Subtraction for $gg \rightarrow gH$

$$z = 1 - m_H^2 / \hat{s}$$

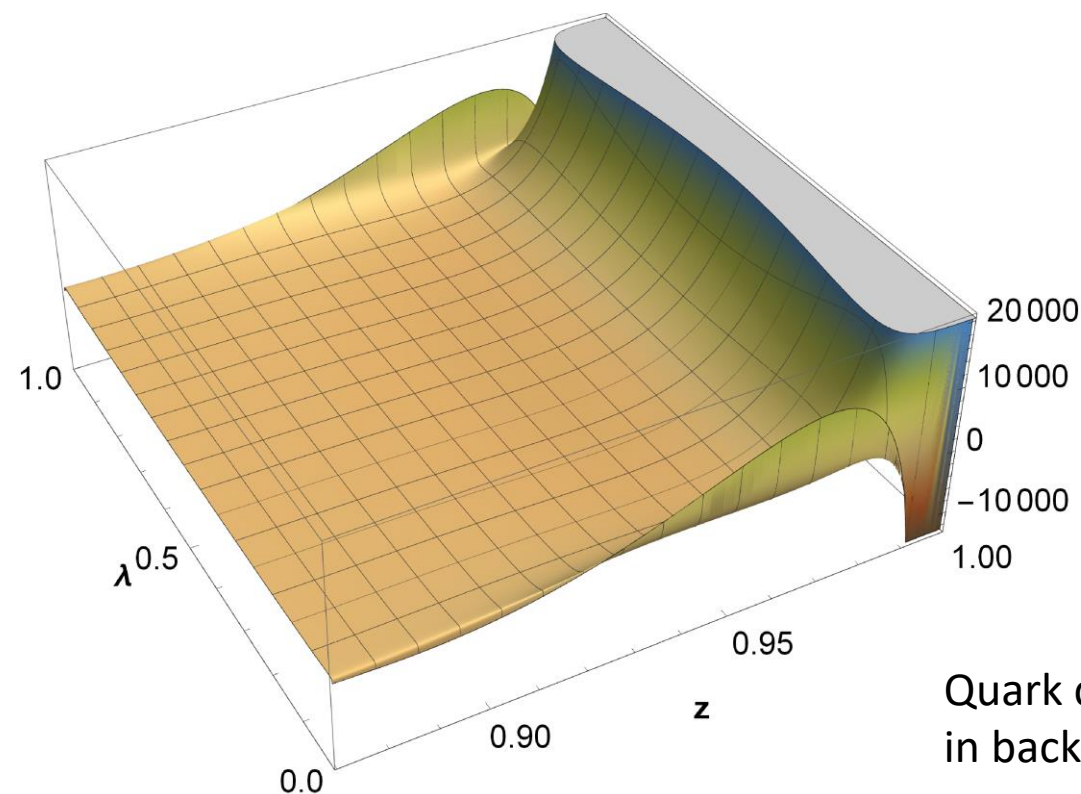
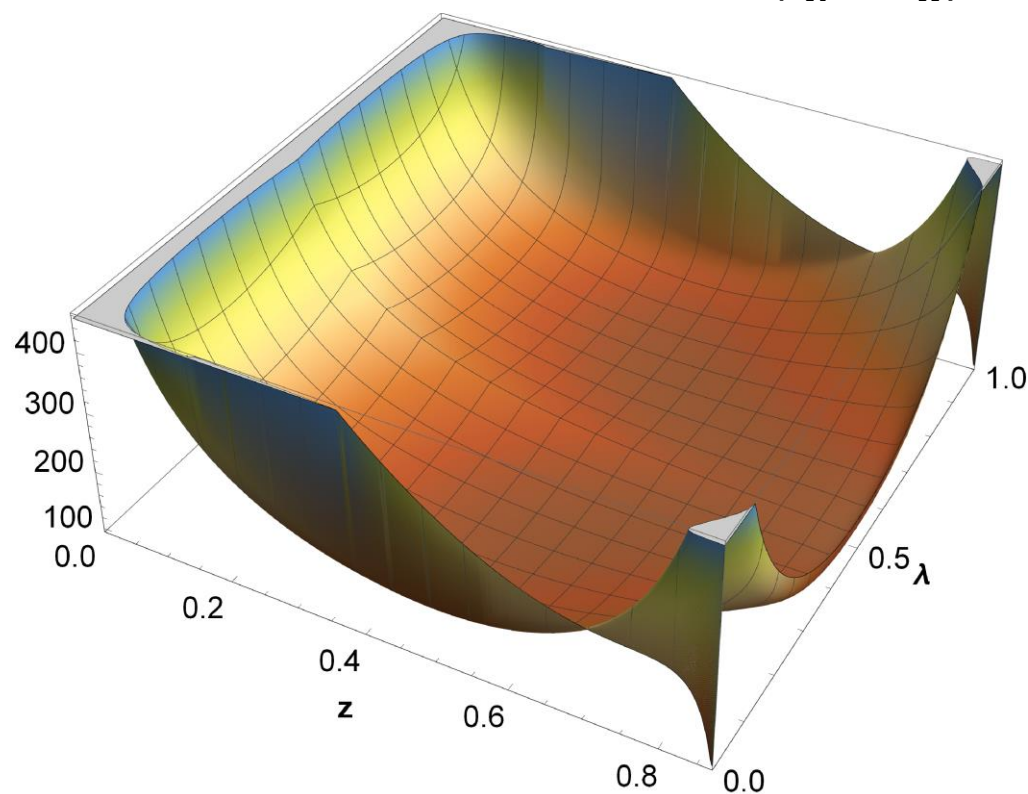
$$\lambda = \hat{t} / (\hat{t} + \hat{u})$$

$$m_t^2 / m_H^2 = 23/12$$

- Directly evaluate difference between HEFT and the exact result:

$$\langle M_{\text{exact}}^{(1)} | M_{\text{exact}}^{(2)} \rangle \Big|_{\text{regulated}} \equiv \langle M_{\text{exact}}^{(1)} | M_{\text{exact}}^{(2)} \rangle - \left[\langle M_{\text{HEFT}}^{(1)} | M_{\text{HEFT}}^{(2)} \rangle + \frac{8\pi\alpha_s}{\hat{t}} \left\langle P_{gg}^{(0)} \left(\frac{\hat{s}}{\hat{s} + \hat{u}} \right) \right\rangle \langle F^{(1)} | (F_{\text{exact}}^{(2)} - F_{\text{HEFT}}^{(2)}) \rangle \right]$$

- Real part of the regulated quantity at $\mu_R = m_H/2$:



- Integrate in λ and convolute with PDFs to obtain contribution to σ_{tot}
- Subtraction term and other contributions are computed with Monte Carlo methods using **Stripper** [Czakon \(unpublished\)](#) 12

Results

- Effects of a finite top-quark mass on the total hadronic Higgs-boson production cross section for the LHC
 - PDF set: NNPDF31_nnlo_as_0118
 - $\mu_R = \mu_F = m_H/2$
 - $M_H = 125 \text{ GeV} \Rightarrow M_t \approx 173.055 \text{ GeV}$

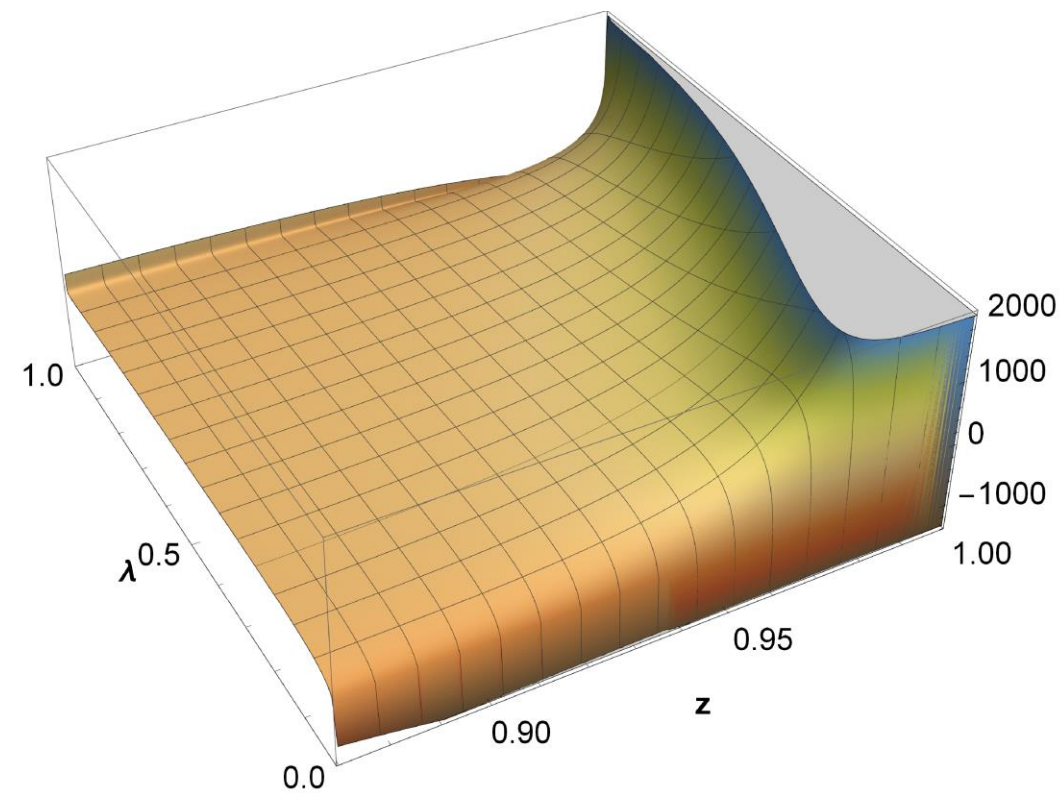
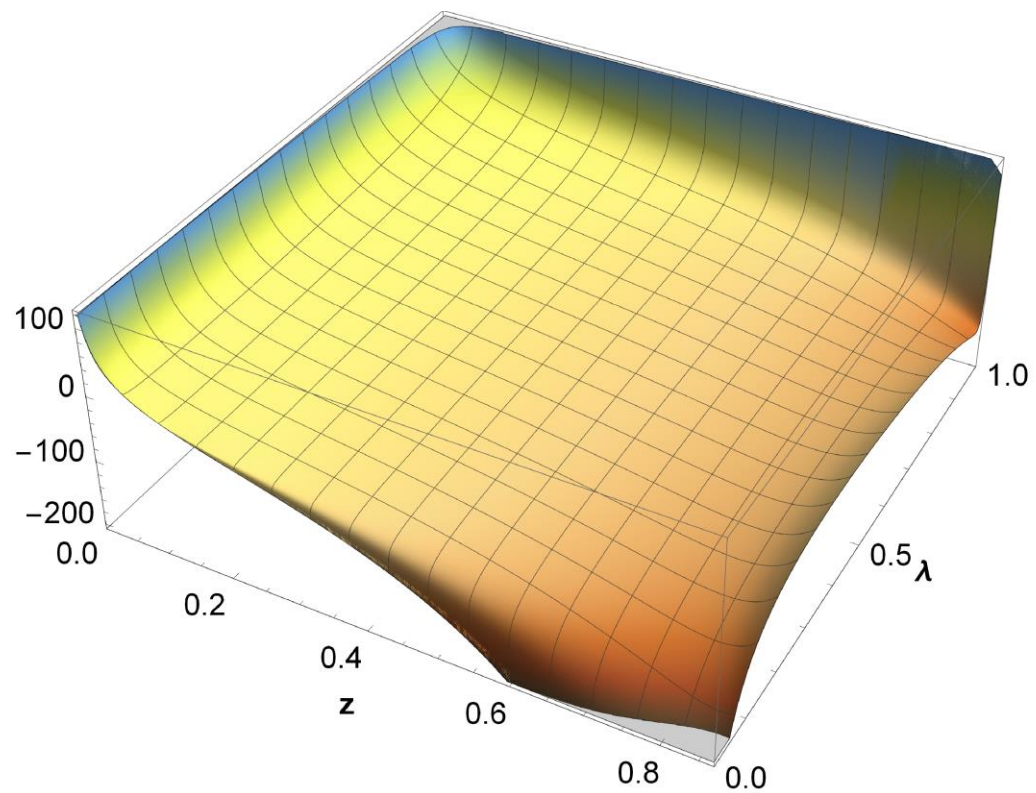
channel	$\sigma_{\text{HEFT}}^{\text{NNLO}}$ [pb] $\mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3) + \mathcal{O}(\alpha_s^4)$	$(\sigma_{\text{exact}}^{\text{NNLO}} - \sigma_{\text{HEFT}}^{\text{NNLO}})$ [pb] $\mathcal{O}(\alpha_s^3) \qquad \mathcal{O}(\alpha_s^4)$		$(\sigma_{\text{exact}}^{\text{NNLO}} / \sigma_{\text{HEFT}}^{\text{NNLO}} - 1)$ [%]
$\sqrt{s} = 8 \text{ TeV}$				
gg	$7.39 + 8.58 + 3.88$	$+0.0353$	$+0.0879 \pm 0.0005$	$+0.62$
qg	$0.55 + 0.26$	-0.1397	-0.0021 ± 0.0005	-18
qq	$0.01 + 0.04$	$+0.0171$	-0.0191 ± 0.0002	-4
total	$7.39 + 9.15 + 4.18$	-0.0873	$+0.0667 \pm 0.0007$	-0.10
$\sqrt{s} = 13 \text{ TeV}$				
gg	$16.30 + 19.64 + 8.76$	$+0.0345$	$+0.2431 \pm 0.0020$	$+0.62$
qg	$1.49 + 0.84$	-0.3696	-0.0115 ± 0.0010	-16
qq	$0.02 + 0.10$	$+0.0322$	-0.0501 ± 0.0006	-15
total	$16.30 + 21.15 + 9.79$	-0.3029	$+0.1815 \pm 0.0023$	-0.26

Conclusions

- The hadronic Higgs production cross section including the full top-quark mass dependence was computed
- Slight decrease relative to the result in HEFT
 - -0.26% at 13 TeV
 - -0.10% at 8 TeV
- The result confirms and eliminates the uncertainty estimate from the lack of knowledge of the exact top-quark mass effects
- Same techniques can be applied to compute bottom-quark mass effects

Backup

$$qg \rightarrow qH$$



$$q\bar{q} \rightarrow gH$$

