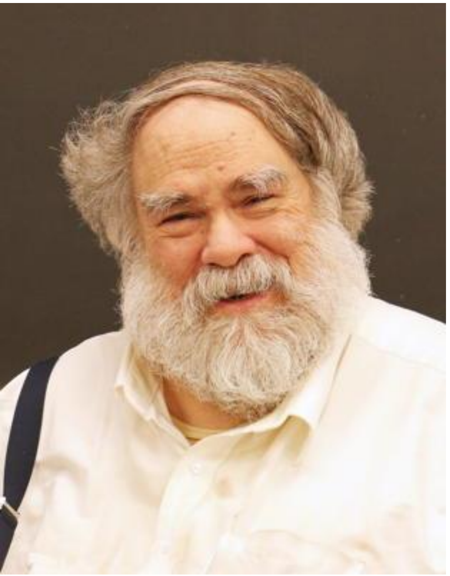
Approaches to Final "Cooling"



David Neuffer
Fermilab
Don Summers, Terry Hart Ole Miss
R. Palmer, H. Sayed, BNL

Outline

- Final Cooling for a Collider
  - Intro , options
- Final Cooling Simulations –
  - H. Sayed
- Other Final Cooling scenarios and variants
- Final scenario variations
  - w /D. Summers & T. Hart
  - round to flat and slicing ….
- Emittance exchanges
  - Wedges …..

Final cooling

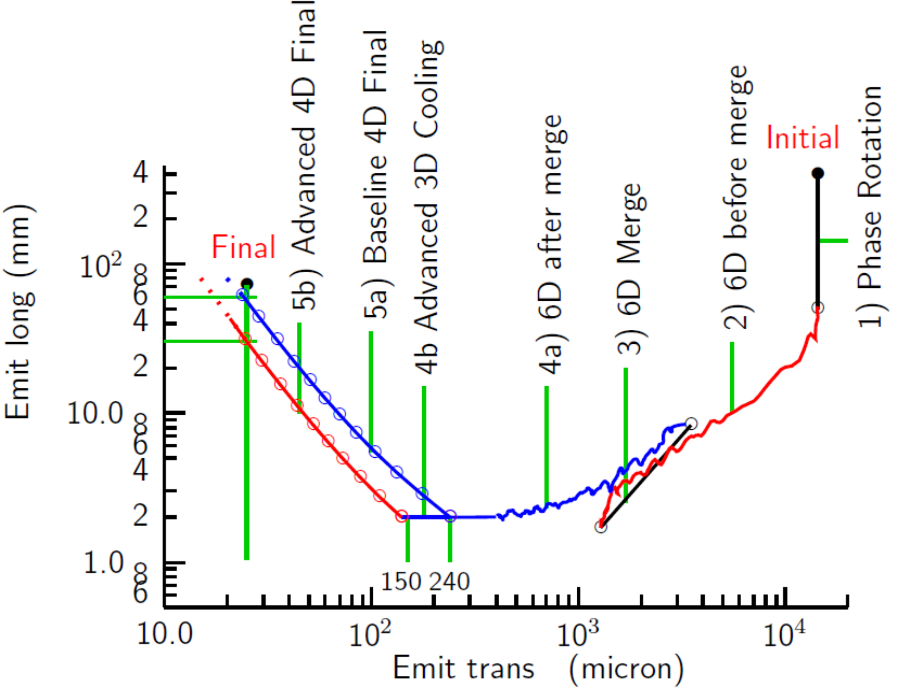
- Baseline High energy collider has final “cooling”
  - – $\varepsilon_{x}, \varepsilon_{y}$: 0.0003 → 0.00003m
  - – ε_{L} : 0.001 → 0.1m
    - Mostly emittance exchange…
- Outline
  - – Baseline scenario
  - – Simulation
  - – Variation
    - Can we use the round to flat beam “emittance exchange” ? –
      - – to change the rules
    - cool, rotate, slice (transverse) recombine (longitudinal)

- **For high-energy collider, we want transverse emittance as small as possible**

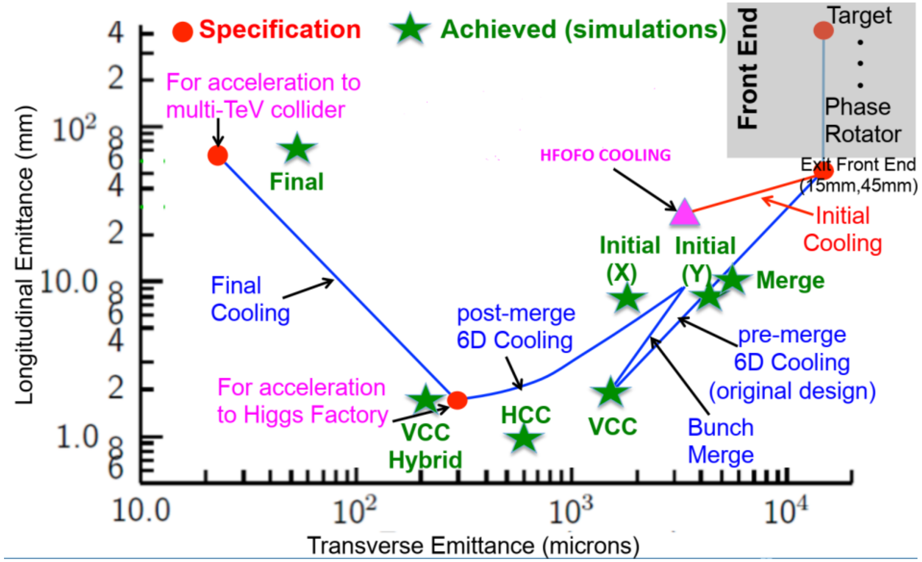
- **Ionization cooling equations get you to $\varepsilon_t = 0.0002$ m (1984)**

$$\varepsilon_{N,eq} \cong \frac{\beta_t E_s^2}{2\beta m c^2 L_R (dE/ds)} \qquad \beta_t \cong \frac{2P_\mu \ (GeV/c)}{0.3B}$$

- **Minimize ε_t by large B, small P_μ**



$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta\gamma\ \beta_\perp}{2}\frac{d\left\langle \theta_{rms}{}^2\right\rangle}{ds} = -\frac{g_\perp}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E}$$



Palmer scenario (2011)

- “Baseline” Muon Collider final cooling stages
  - No actual cooling – emittance exchange
  - High magnetic fields
  - Impossible “rf”

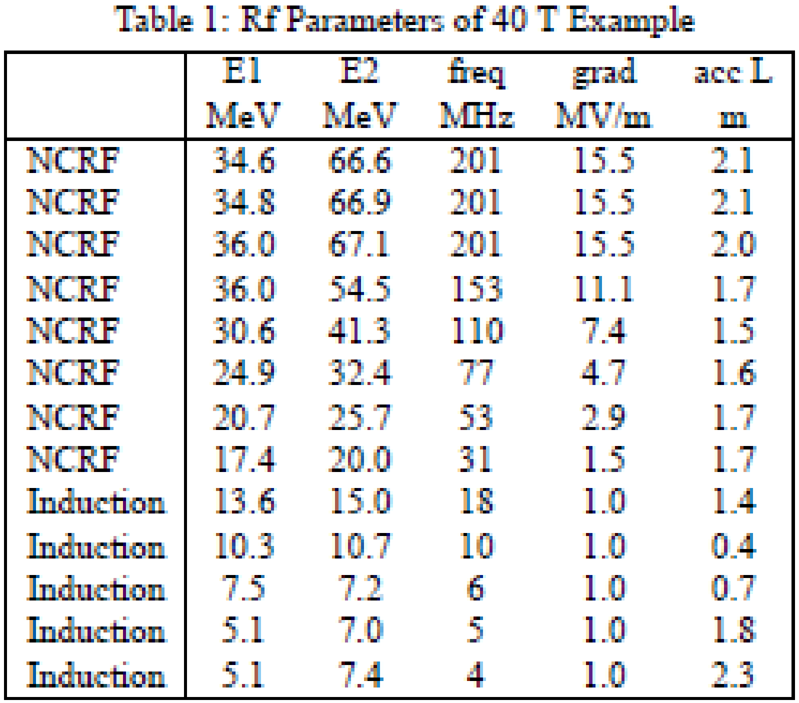
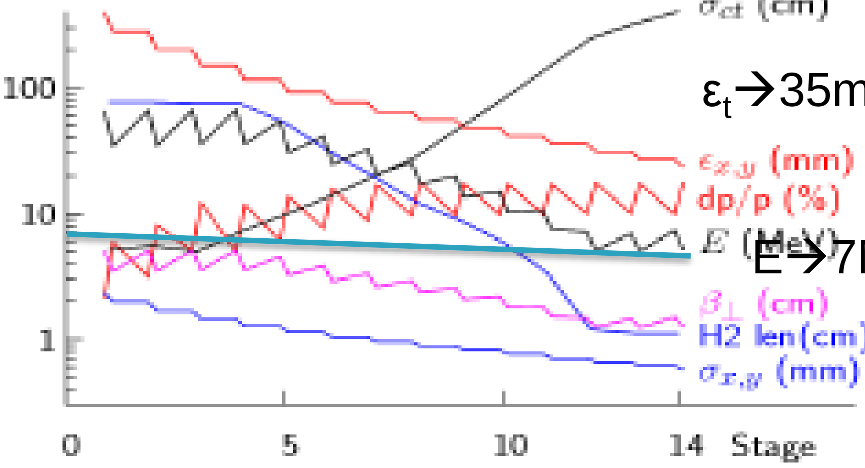


Table 1: Rf Parameters of 40 T Example

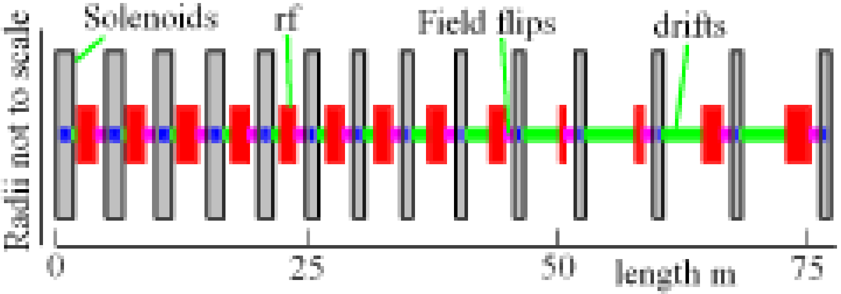
	E1 MeV	E2 MeV	freq MHz	grad MV/m	acc L m
NCRF	34.6	66.6	201	15.5	2.1
NCRF	34.8	66.9	201	15.5	2.1
NCRF	36.0	67.1	201	15.5	2.0
NCRF	36.0	54.5	153	11.1	1.7
NCRF	30.6	41.3	110	7.4	1.5
NCRF	24.9	32.4	77	4.7	1.6
NCRF	20.7	25.7	53	2.9	1.7
NCRF	17.4	20.0	31	1.5	1.7
Induction	13.6	15.0	18	1.0	1.4
Induction	10.3	10.7	10	1.0	0.4
Induction	7.5	7.2	6	1.0	0.7
Induction	5.1	7.0	5	1.0	1.8
Induction	5.1	7.4	4	1.0	2.3



7cm→400

ϵ_t→35mm

E→7MeV



Final Cooling

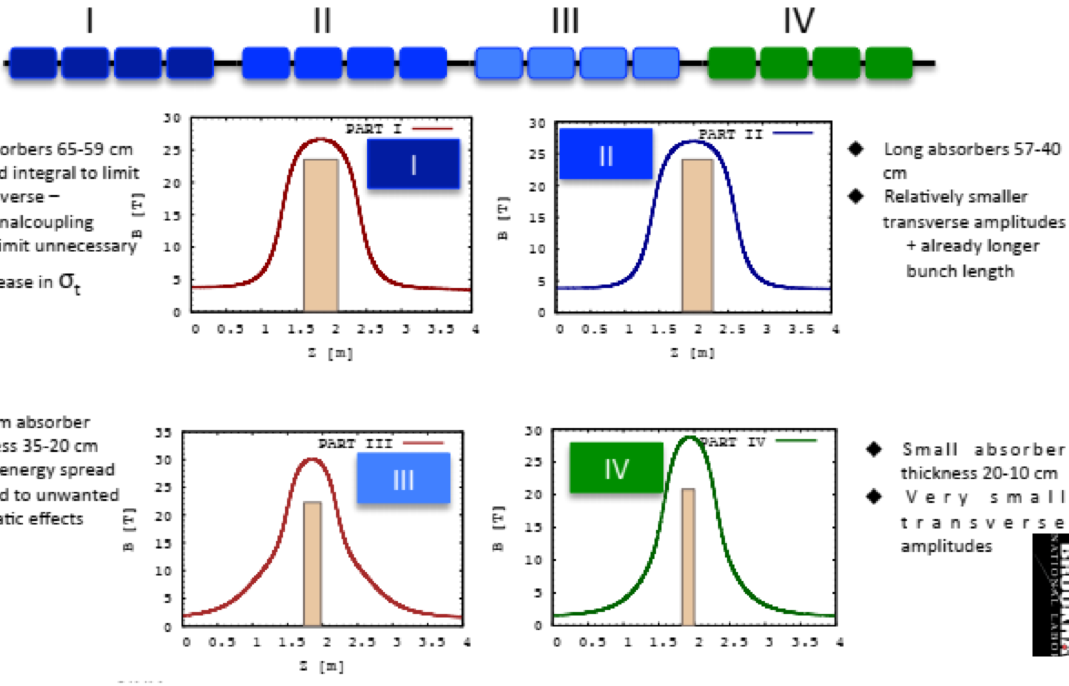
H. Sayed et al.

- **135m long**
- **Consists of 16 stages**
  - –130→110→90→70 MeV/c
    - 62 MeV → 21 MeV
  - –B: 25→30 T

$$\beta_t \cong \frac{2P_\mu\,(GeV/c)}{0.3B}$$

- **Parameter changes**
  - Rf: 325 → 10 MHz
  - σ_z : 5 cm →180cm
- **Some field flips**

Phys. Rev. ST Accel. Beams **18**, 091001

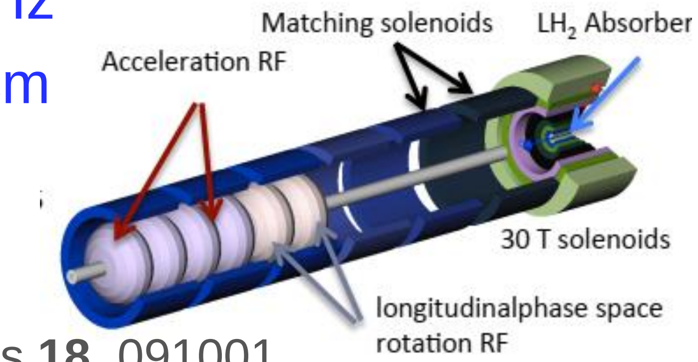


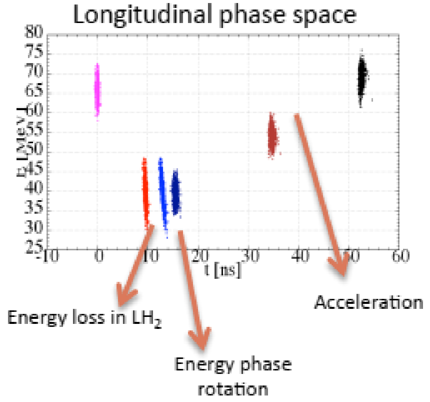
- Long absorbers 65-59 cm
- Limit field integral to limit the transverse – longitudinalcoupling → Limit unnecessary increase in σ_t

- Long absorbers 57-40 cm
- Relatively smaller transverse amplitudes + already longer bunch length

- Medium absorber thickness 35-20 cm
- Larger energy spread will lead to unwanted chromatic effects

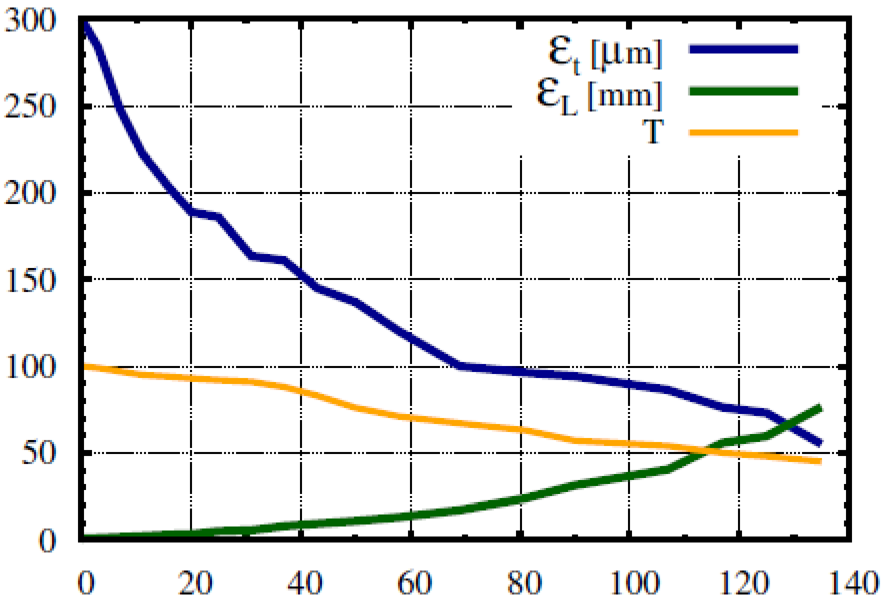
- Small absorber thickness 20-10 cm
- Very small transverse amplitudes

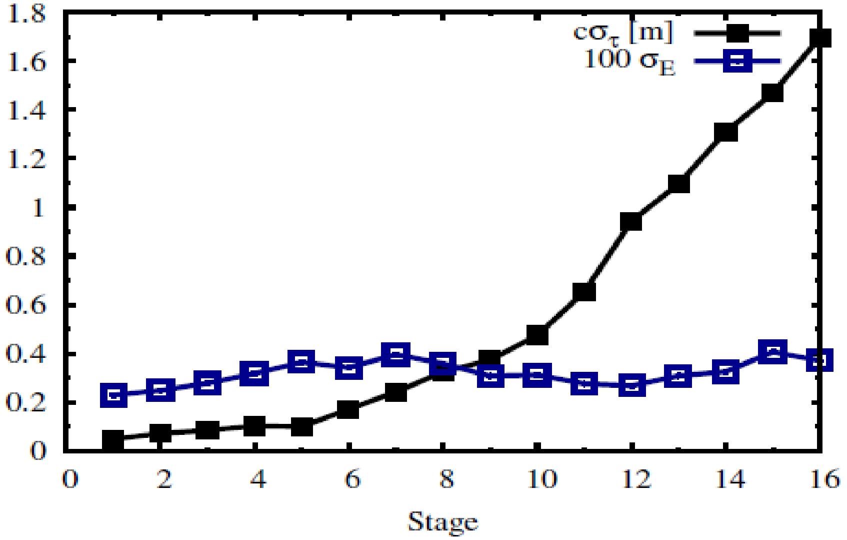




Simulation results

- **System is ~135m long**
  - $\varepsilon_{t,N}$: 300 →55 $\times 10^{-6}$ m
  - ε_L : 1.5→75mm
    - not quite specs
  - Transmission ~ 50%
- **First part has best cooling**
  - After that, emittance exchange with some heating
- **Can improve by larger B**
  - Also go to smaller P_μ

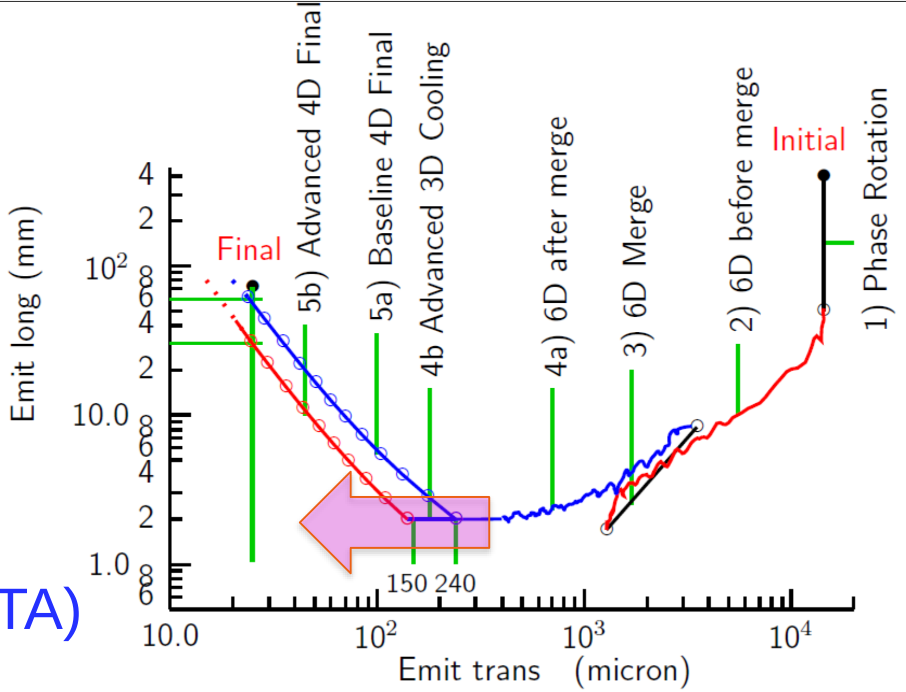




Stages

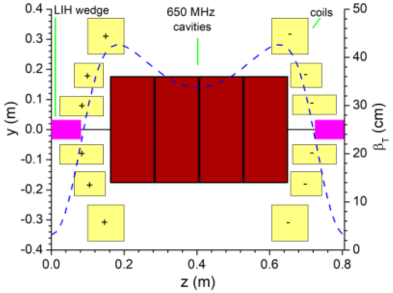
Extend Cooling with Advanced Methods…

- **Parametric resonance IC**
  - Derbenev, Morozov
- **Use Li lens for cooling**
  - $\varepsilon_{t,N}$ → <0.0001m
- **Plasma lenses**
- **Optical stochastic cooling**
  - First demonstration (2021, IOTA)
- **Extend to higher B fields**
  - RFOFO-D. Summers, T. Hart
- **Phase space manipulations**
  - Slice x and/or y, drift, recombine,
- **Emittance Exchange**
  - wedges

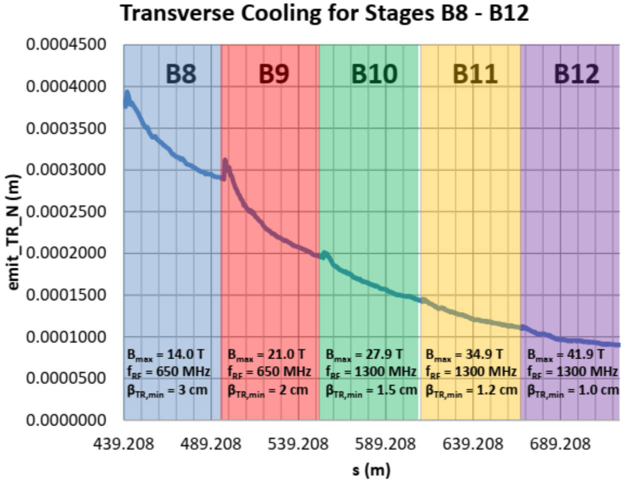


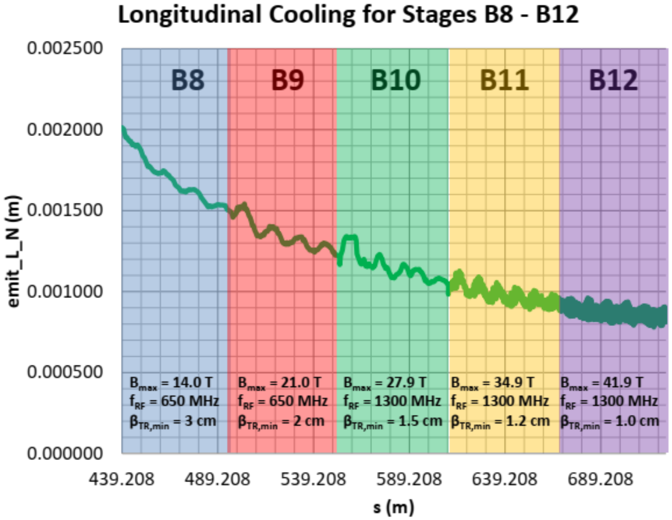
Extend Rectilinear channel with large B –D. Summers, T. Hart (2021)

- **Extend Rectilinear channel with 21T, 28, 35, 42 T**



- **$\varepsilon_t \rightarrow$ 0.0001 m, $\varepsilon_L \rightarrow$ 0.0008 m**
- **Cooler beam into “Final Cooler”**


Transverse Cooling for Stages B8 - B12

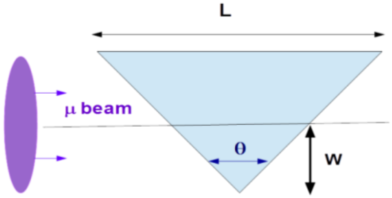

Longitudinal Cooling for Stages B8 - B12

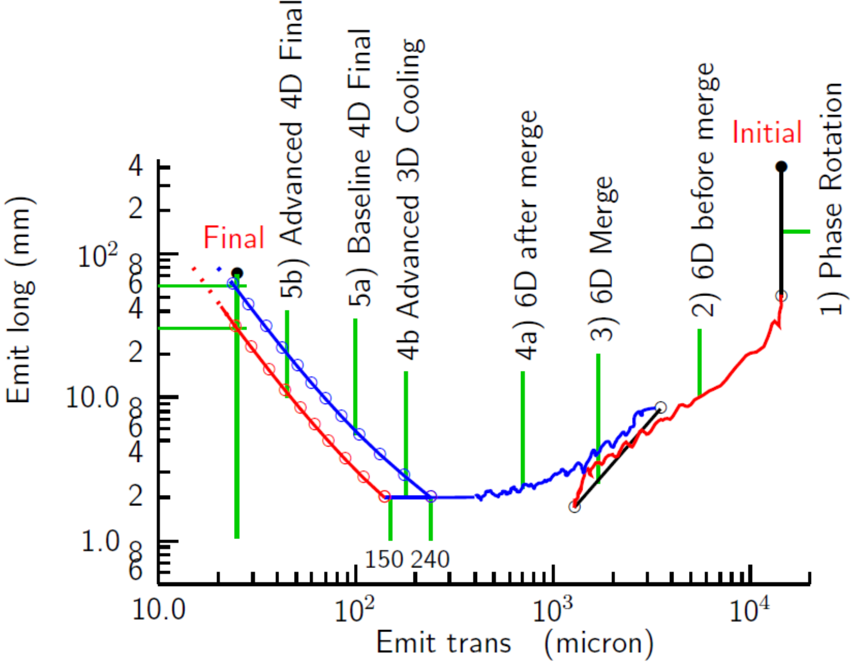
Wedges for Final Cooling

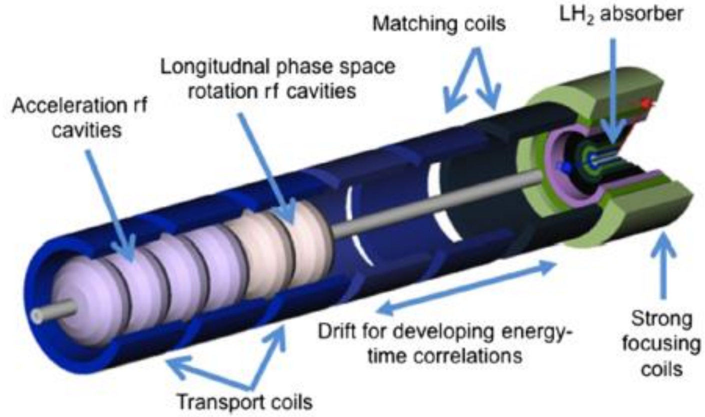
- TeV Collider wants small ε
  - ε → 25 μm or less
- Baseline final cooling is low…..

  Mostly emittance exchange

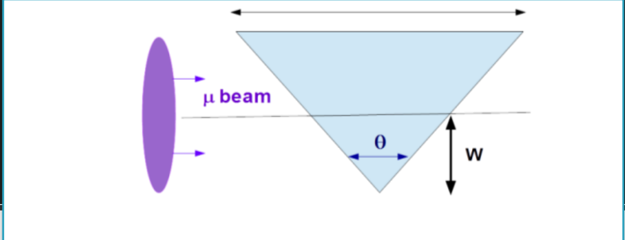
  Consider
- Can do most of this with wedge absorbers …





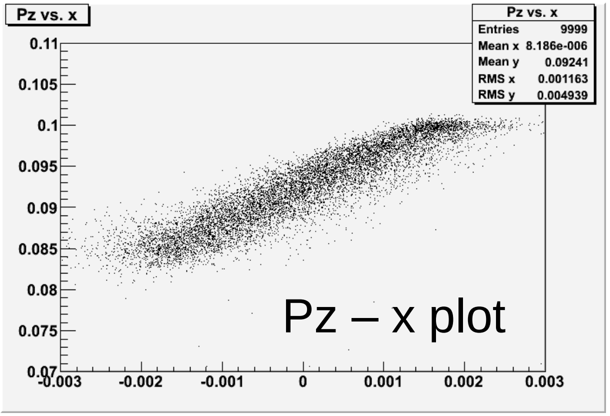


Numerical example



- **Wedge parameters**
  - Diamond, w=1.75mm, θ = 100°(4.17mm thick at center)

z(cm)	P_z	ε_x(μ)	ε_y	ε_L(mm	σ_E MeV	6-D ε increase
0	100	97	95.5	1.27	0.46	1.0
0.4	96.4	33.4	96.3	4.55	1.64	1.24
0.8	92.4	**22.7**	96.5	8.94	3.22	1.65

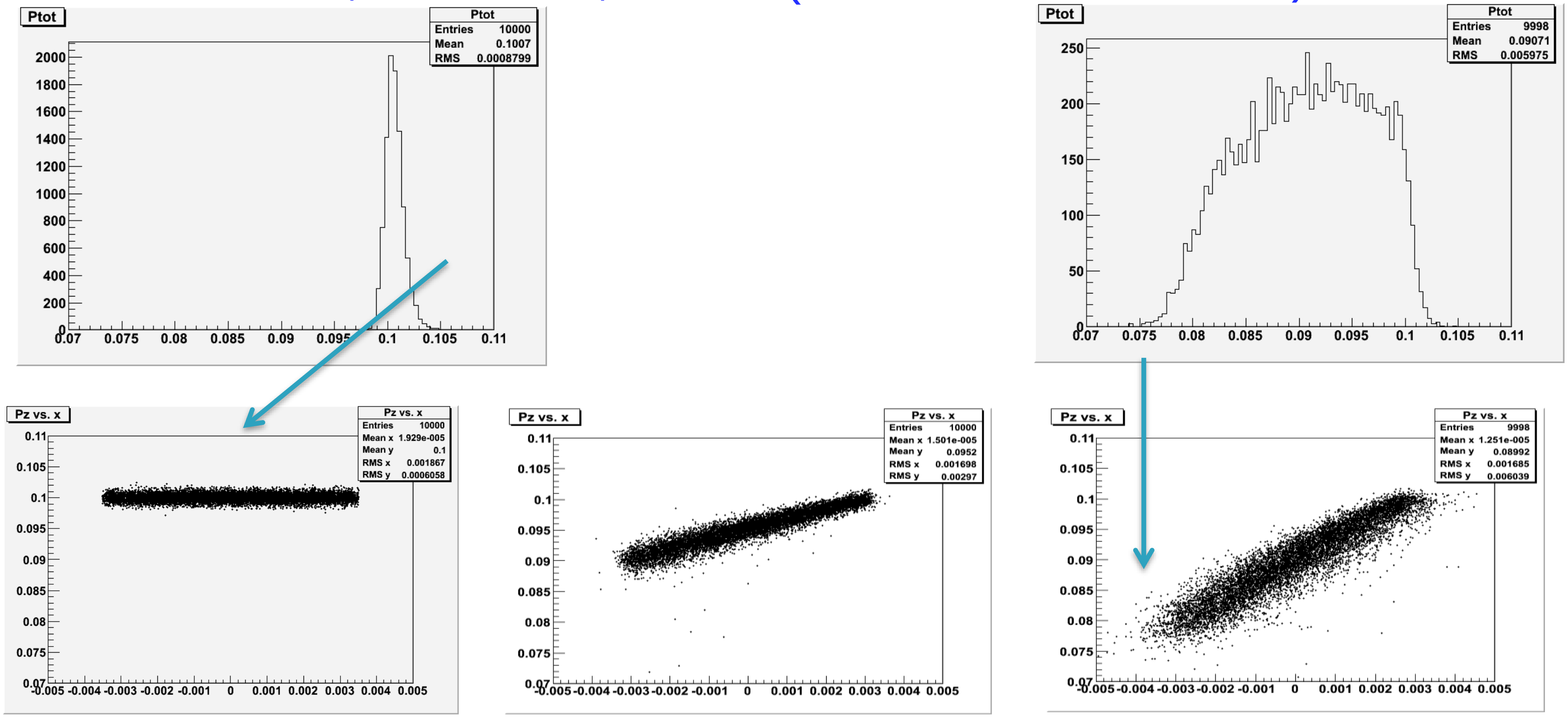


  - reduces ε_x by factor of 4.3, ε_L increases by factor of 7.0
    - **first half of wedge more efficient than second half …**
- **Second wedge**
  - if rematched to same optics (P_z → 100 MeV/c, σ_E→0.46 MeV)
    - ε_x : 23 →27μ; ε_y :97 → 23 μ

Wedge Simulations

z(cm)	P_z	ε_x(μ)	ε_y	ε_L (mm)	σ_E MeV	6-D ε increase
0	100	129	127	1.0	0.50	1.0
0.6	95.2	40.4	130	4.03	1.95	1.29
1.2	90.0	**25.0**	127	7.9	3.87	1.54

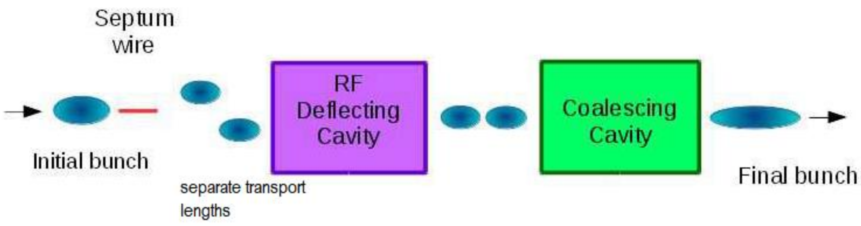
- **Beam, wedge parameters**
  - β_t=2.6cm, ε_t =130μ
  - Diamond, w=3.0mm, θ = 85°(5.6mm thick at center)



Emittance exchange: Slice and dice

- **Slice beam transversely**

- **Drift separated beams**
- **Combine longitudinally**



Two-stage transverse split

- **Schemes with relatively large numbers of bunch splittings possible**
  - D. Summers – “Potato slicer”
  - 16 → 1

Solenoidal Cooling:
Beams are not round

- **In solenoid:**
  - Eigen modes are not:
    - $\{x, p_x\}$, $\{y, p_y\}$
  - Drift, Cyclotron modes
- **Only cyclotron mode is cooled**
  - Field flip exchanges C, D
- **Without flips, emittances become “flat”**
  - $\varepsilon_1\varepsilon_2 = \varepsilon_k \varepsilon_c = (\varepsilon_t - \ell)(\varepsilon_t + \ell)$
  - → $\varepsilon_x \varepsilon_y$
- **Optimum final cooling state may be a flat beam**

Canonical Coordinates:

- Cyclotron mode

$$\begin{pmatrix} \kappa_1 \\ \kappa_2 \end{pmatrix} = \sqrt{\frac{c}{eB}} \begin{pmatrix} k_y \\ k_x \end{pmatrix} = \sqrt{\frac{c}{eB}} \begin{pmatrix} p_y + \frac{eB}{2c} x \\ p_x - \frac{eB}{2c} y \end{pmatrix}$$

- Drift mode

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \sqrt{\frac{eB}{c}} \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \sqrt{\frac{eB}{c}} \begin{pmatrix} \frac{x}{2} - \frac{c}{eB} p_y \\ \frac{y}{2} + \frac{c}{eB} p_x \end{pmatrix}$$

References

A. Burov, S. Nagaitsev, A. Shemyakin, PRSTAB 3 094002 (2000)

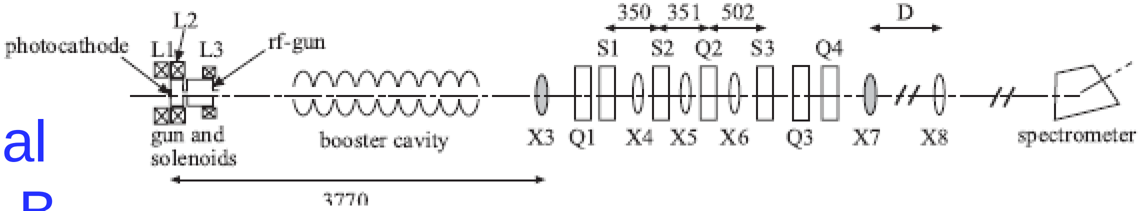
A. Burov, S. Nagaitsev, Y. Derbenev, Phys. Rev. E 66, 016503 (2002)

Round to Flat beam transform

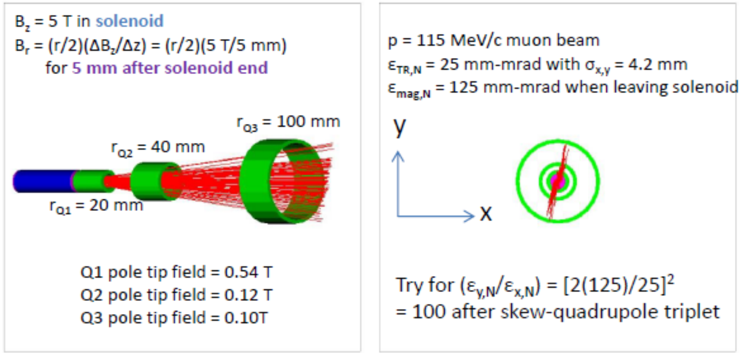
- **Beam has large angular momentum L from non-flip**
  - means beam internally has asymmetric emittance

$$\varepsilon_{4D} = \varepsilon_T^2 = \varepsilon_+ \varepsilon_- = \left(\varepsilon_P + L\right)\left(\varepsilon_P - L\right)$$

- **Beam is in same format as in electron source**
  - Beam cooled to thermal properties within large B



- **Round to Flat beam transform**
  - Demonstrated at FNAL (electron injector)
  - ~3 skew quads +
  - ε_+, ε_- → ε_x, ε_y


B_z = 5 T in solenoid
$B_r = (r/2)(\Delta B_z/\Delta z) = (r/2)(5\ \mathrm{T}/5\ \mathrm{mm})$
for 5 mm after solenoid end

Q1 pole tip field = 0.54 T
Q2 pole tip field = 0.12 T
Q3 pole tip field = 0.10T

p = 115 MeV/c muon beam
$\varepsilon_{TR,N}$ = 25 mm-mrad with $\sigma_{x,y}$ = 4.2 mm
$\varepsilon_{mag,N}$ = 125 mm-mrad when leaving solenoid

Try for $(\varepsilon_{y,N}/\varepsilon_{x,N}) = [2(125)/25]^2$
= 100 after skew-quadrupole triplet

Summary

- **Final Cooling:**
  - Baseline system
    - High-field solenoid, H absorbers at low energy
    - Low-frequency rf → induction Linac
  - In simulation, (almost) meets design goal
  - Can be improved
- **Alternatives for improvements should be explored**

- **Optimum final emittance likely to be asymmetric**
  - $\varepsilon_x < \varepsilon_y$

Will be important research topic

Bernd Stechauner, CERN tech. student

Cooling within solenoids

- **Ionization cooling**
  - Absorbers within solenoids
    - Cools k_1, k_2
  - Cyclotron mode is preferentially cooled
  - With

$$\varepsilon_x = \sqrt{\langle x^2 \rangle \langle {p_x}^2 \rangle - \langle x\, p_x \rangle^2}$$

    - and

$$\ell = \frac{1}{2} \langle x p_y - y p_x \rangle$$

then:

$$\varepsilon_1 \varepsilon_2 = \varepsilon_x \varepsilon_y - \ell^2$$

  - Typically (at $\varepsilon_x = \varepsilon_y = \varepsilon_t$)
    - $\varepsilon_1 \varepsilon_2 = \varepsilon_k\, \varepsilon_c = (\varepsilon_t - \ell)\, (\varepsilon_t + \ell)$

- **With field flips:**
  - k_1, k_2 and d_1, d_2 change identities with each flip
  - Both modes are equally damped
    - Angular momentum is damped
- **Without field flips**
  - One mode is preferentially cooled
  - Canonical angular momentum not damped