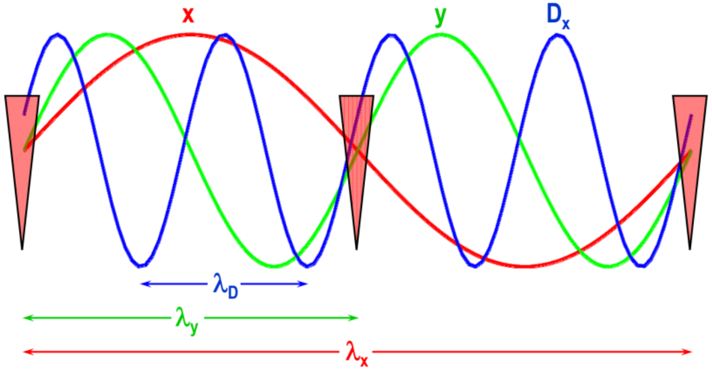
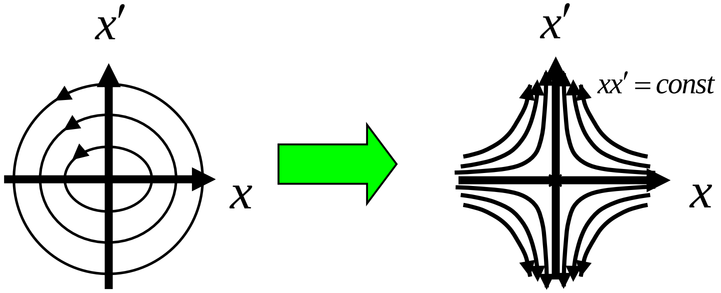
Parametric-resonance ionization cooling (PIC)

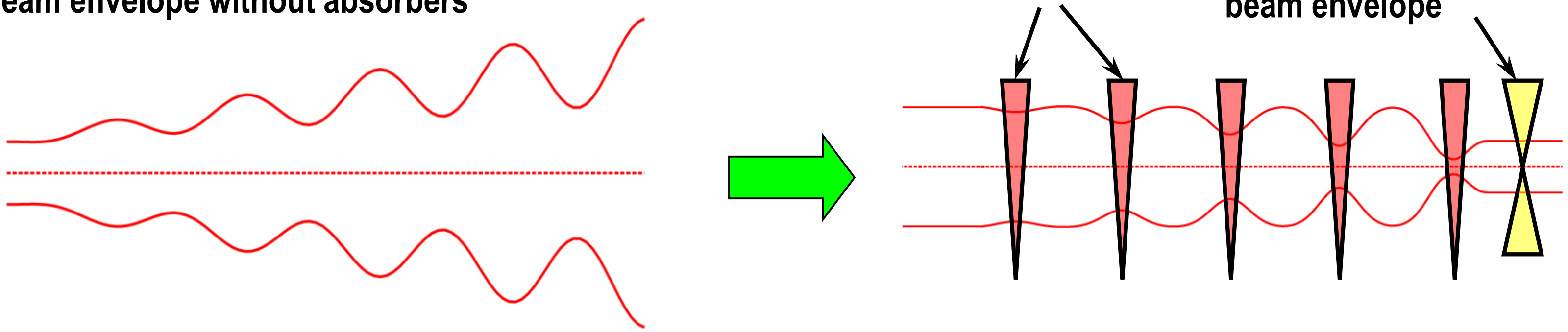
- Half-integer parametric resonances induced in cooling channel
- Enables order of magnitude equilibrium emittance reduction



Correlated optics for periodic focusing at absorber positions



Half-integer resonance at absorber positions drives reduction in x, growth in x'



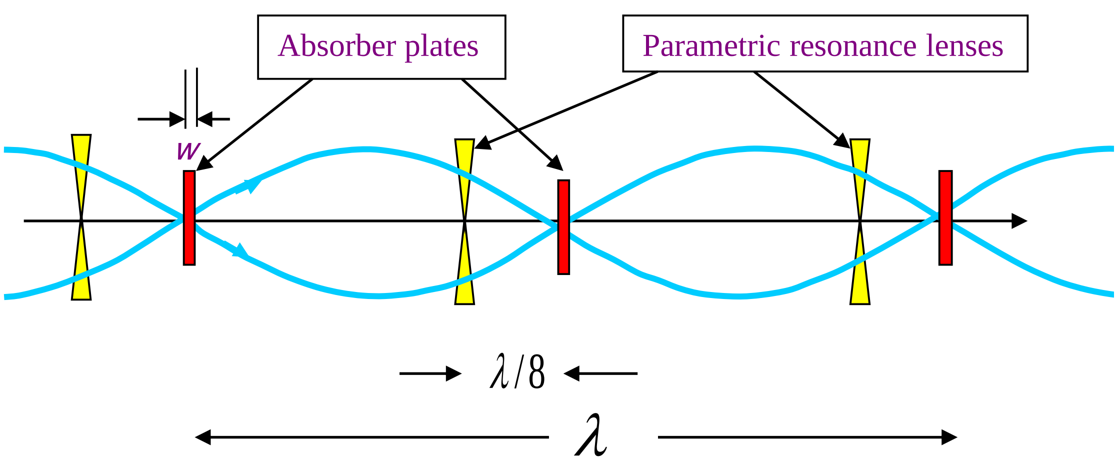
Beam envelope without absorbers

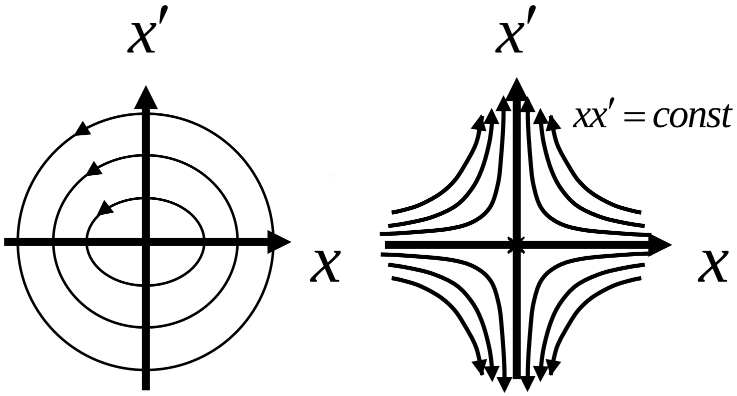
Absorbers

Optics to restore parallel beam envelope

Ionization cooling occurs at absorber plates, and RF cavities restore longitudinal momentum

PIC possible parameters





- **Equilibrium angular spread and beam size at absorber**

$$\theta_a^2 = \frac{3}{2}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu}, \quad \sigma_a = \frac{1}{2\sqrt{3}}\theta_a w$$

- **Equilibrium emittance**

$$\varepsilon_n = \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w$$

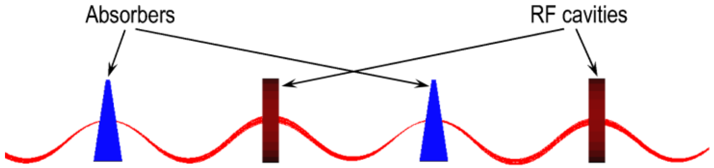
improvement by a factor of

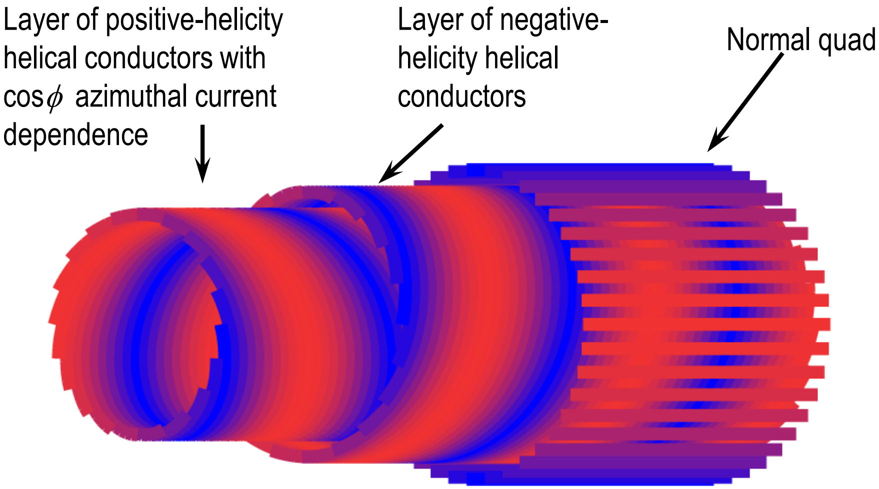
$$\frac{\pi}{\sqrt{3}}\frac{w}{\lambda} = \frac{\pi}{2\sqrt{3}}\frac{\gamma'_{acc}}{\gamma'_{abs}}$$

Parameter	Unit	Initial	Final
Muon beam momentum, p	MeV/c	250	250
Number of particles per bunch, N_b	10^{10}	1	1
Be (Z = 4) absorber thickness, w	mm	20	2
Normalized transverse emittance (rms), $\varepsilon_x = \varepsilon_y$	µm	230	23
Beam size at absorbers (rms), $\sigma_a = \sigma_x = \sigma_y$	mm	0.7	0.1
Angular spread at absorbers (rms), $\theta_a = \theta_x = \theta_y$	mrad	130	130
Momentum spread (rms), $\Delta p/p$	%	2	2
Bunch length (rms), σ_z	mm	10	10

Twin helix implementation

- Two equal-strength opposite-helicity helical dipole harmonics +
  Straight quad to redistribute horizontal and vertical focusing
- Orbit in horizontal plane + uncoupled horizontal, vertical motion
- $\lambda_D = \lambda \Rightarrow \lambda_x = 2\lambda_y = 4\lambda \Rightarrow \nu_x = 0.25, \nu_y = 0.5$

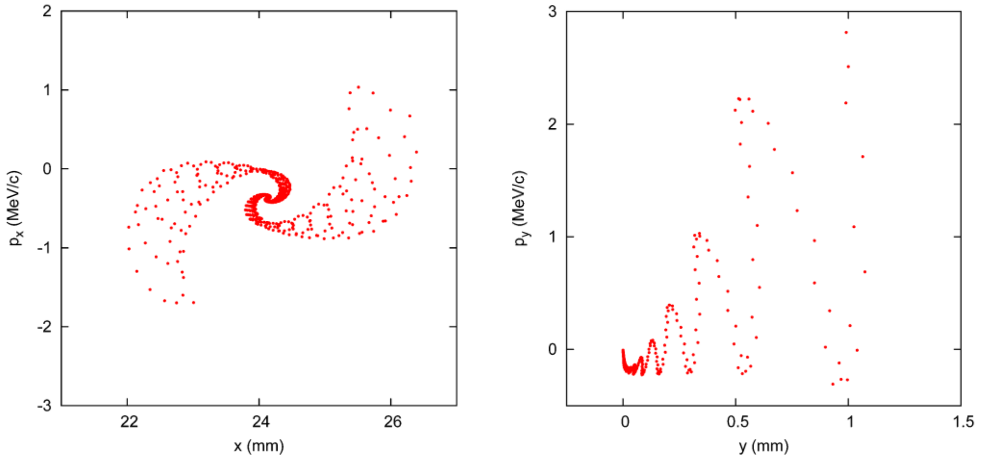



Layer of positive-helicity helical conductors with $\cos\phi$ azimuthal current dependence

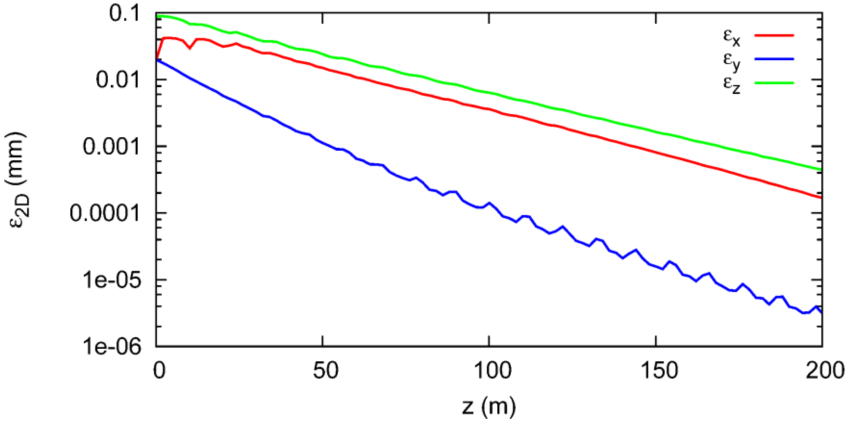
Layer of negative-helicity helical conductors

Normal quad

“Twisted” Meyer magnet



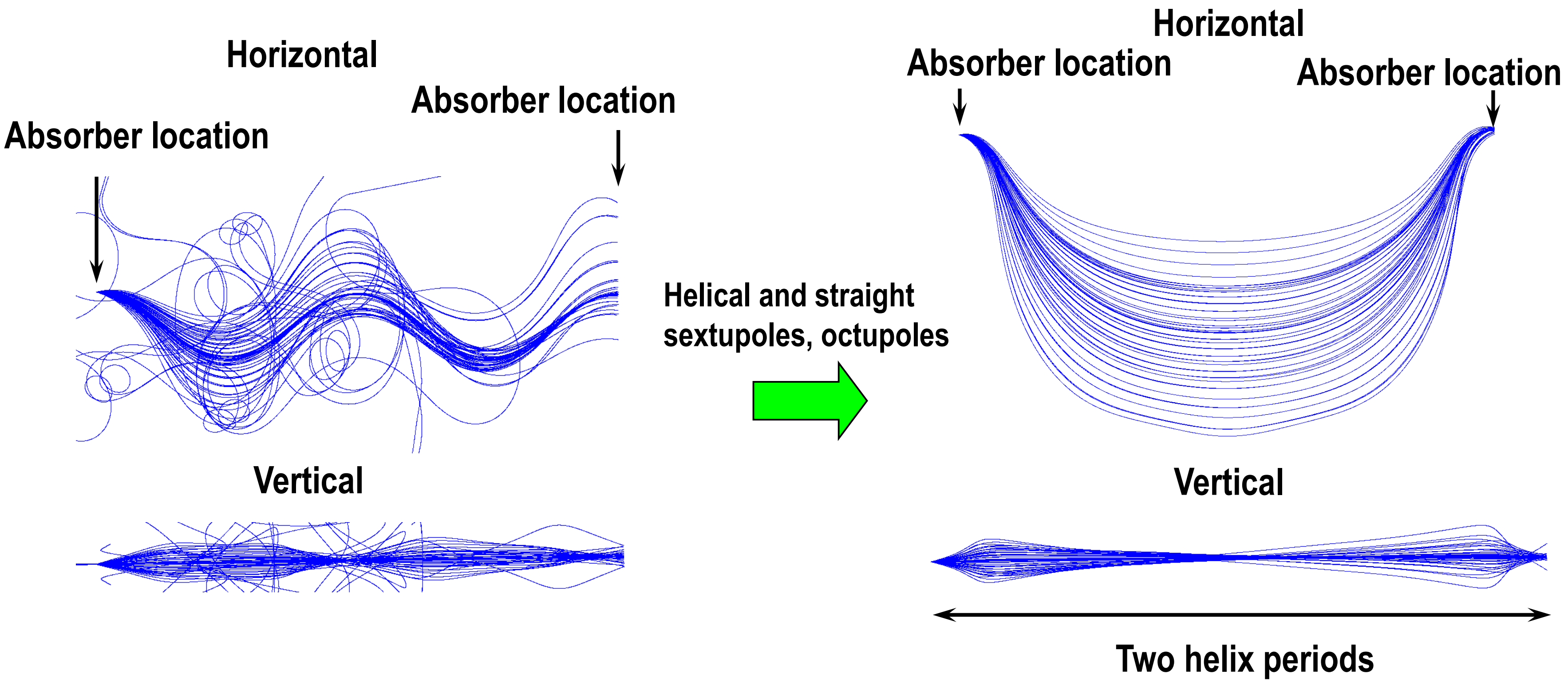
Phase space behavior with induced parametric resonance



Emittance evolution, no stochastics or energy straggling

Twin helix challenges

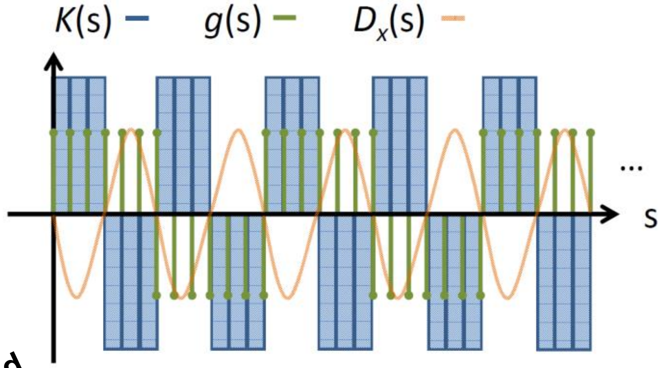
- Beam aberrations cause beam blowup at focal points



- Under correlated optics conditions, continuous harmonically-varying multipoles excite nonlinear resonances
- Aberration compensation is difficult with limited multipole choices

Skew PIC implementation

- Skew quads in PIC channel for strong x-y coupling → correlated optics for radial motion
- Betatron tunes shifted away from resonant values → easier aberration compensation



uncoupled $\lambda_{\mathrm{Dx}} = \lambda \Rightarrow \lambda_{\mathrm{x}} = 2\lambda_{\mathrm{y}} = 2\lambda \Rightarrow \nu_{\mathrm{x}} = 1.0,\ \nu_{\mathrm{y}} = 2.0$

Skew PIC theory

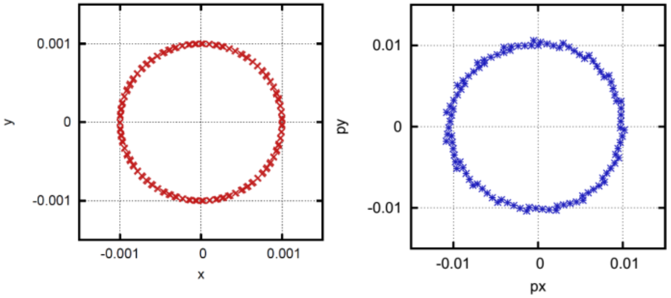
$$x'' + [K^2(s) - n]x + g(s)y = K(s)\delta$$
$$y'' + ny + g(s)x = 0$$

$$\begin{pmatrix} x_f \\ y_f \\ x'_f \\ y'_f \end{pmatrix} = \mathrm{M} \begin{pmatrix} x_i \\ y_i \\ x'_i \\ y'_i \end{pmatrix}, \quad \mathrm{M} = \begin{pmatrix} M & 0 \\ L & N \end{pmatrix}, \quad \det(\mathrm{M}) = \det(M)\cdot\det(N) = 1$$

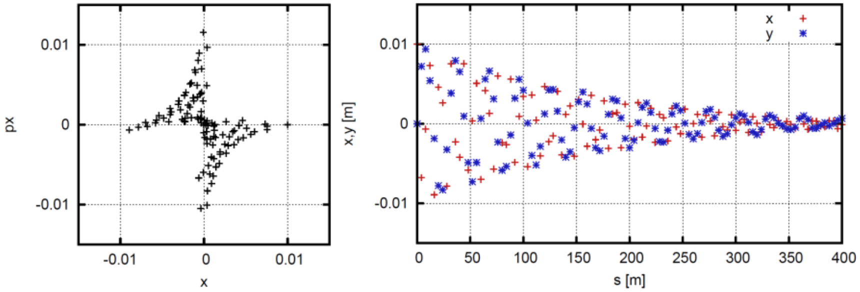
$\det(M) = \det(N) = 1$ **for stability of linear motion**

$$\mathrm{M} = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix}, \quad M = N = \begin{pmatrix} \cos(4\theta) & -\sin(4\theta) \\ \sin(4\theta) & \cos(4\theta) \end{pmatrix}$$

$$\tan\theta = \frac{K^2 - 2n - \sqrt{(K^2 - 2n)^2 + 4g^2}}{2g}$$



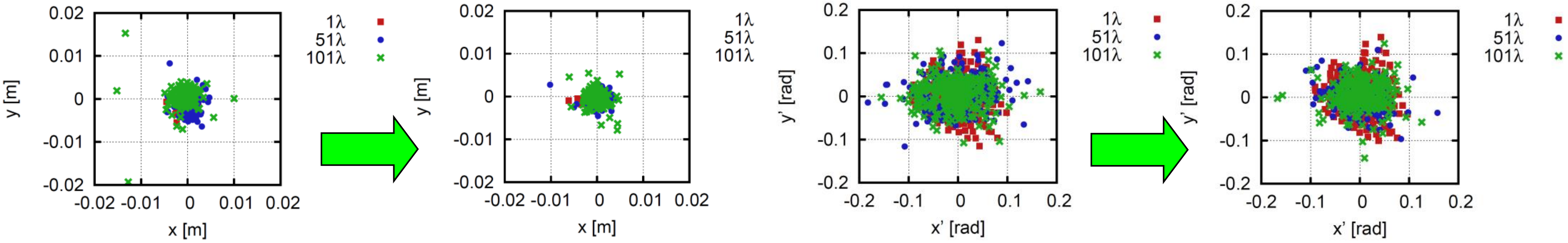
Rotational phase space behavior



Simultaneous x, y focusing with induced parametric resonance

Skew PIC challenges

- Dynamic aperture optimization easier than in normal PIC, but still challenging

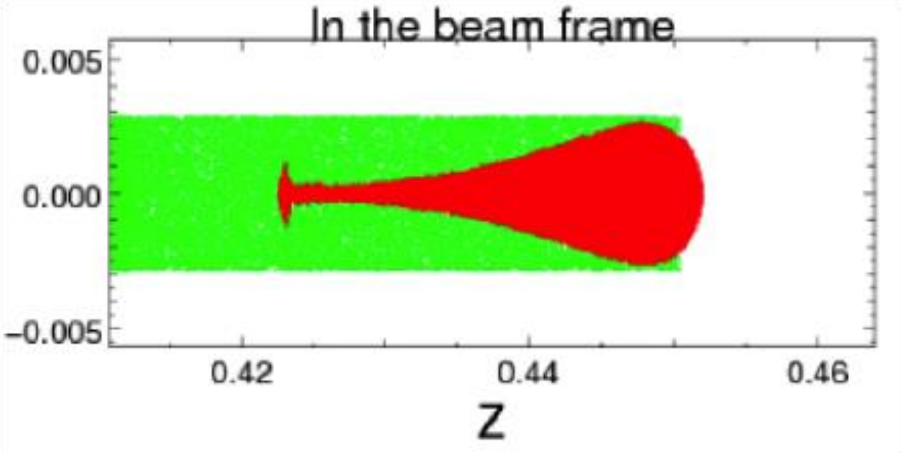


x,y and x',y' phase space including sextupole, octupole, decapole harmonics

- Able to stabilize particle motion within ±90 mrad without damping and ±120 mrad with damping
- However, ±120 mrad is ~$1\sigma_\theta$
- Serious problem with amplitude-dependent time of flight for large θ when longitudinal motion is included

Plasma channel with PIC

- Strong focusing would help alleviate many of the problems
- Consider plasma focusing in gas-filled RF cavities (K. Yonehara)
- Idea supported by initial simulations



- Proposal submitted on this topic