



Science and  
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# Collider arc with skew $Q$ and vFFA

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# Two exotic options not discussed in MAP

Collider arc with skew quadrupoles

Vertical excursion FFA for muon acceleration

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Collider arc with skew quadrupoles

Vertical excursion FFA for muon acceleration

# Reasons of skew quadrupole collider arc

- Flexible **momentum compaction factor**
  - Without exciting non-zero harmonic of the dispersion function
  - Without reverse bending
- **Spreading out radiation** due to neutrinos by wiggling (wobbling) orbit in vertical as well as in horizontal.
  - Angle of wiggling orbit is a function of optics, not by mechanical, i.e. easy to adjust different configurations.
- No design of the low-beta insertion yet, but it should be a simple 45 rotation of the conventional one to start.

# Excite periodicity of the dispersion close to tune

## J-PARC synchrotron

Momentum compaction factor is dominated by non-zero harmonics of the dispersion.

$$\alpha_1 = \frac{1}{L} \int_0^L \frac{D_x}{\rho} ds = \frac{Q_x}{L} \int_0^{2\pi} \frac{\beta_x D_x}{\rho} d\phi$$

$$D_x(s) = \beta_x^{1/2}(s) Q_x^2 \sum_k \frac{a_k e^{jk\phi}}{Q_x^2 - k^2}$$

$$a_k = \frac{1}{2\pi} \int_0^{2\pi} \frac{\beta_x^{3/2}}{\rho} e^{-jk\phi} d\phi$$

$$\alpha_1 = \frac{2\pi Q_x^3}{L} \sum_k \frac{|a_k^2|}{Q_x^2 - k^2}$$

$$\sim \frac{1}{Q_x^2} \quad |a_0^2| = \frac{L}{2\pi Q_x^3}$$

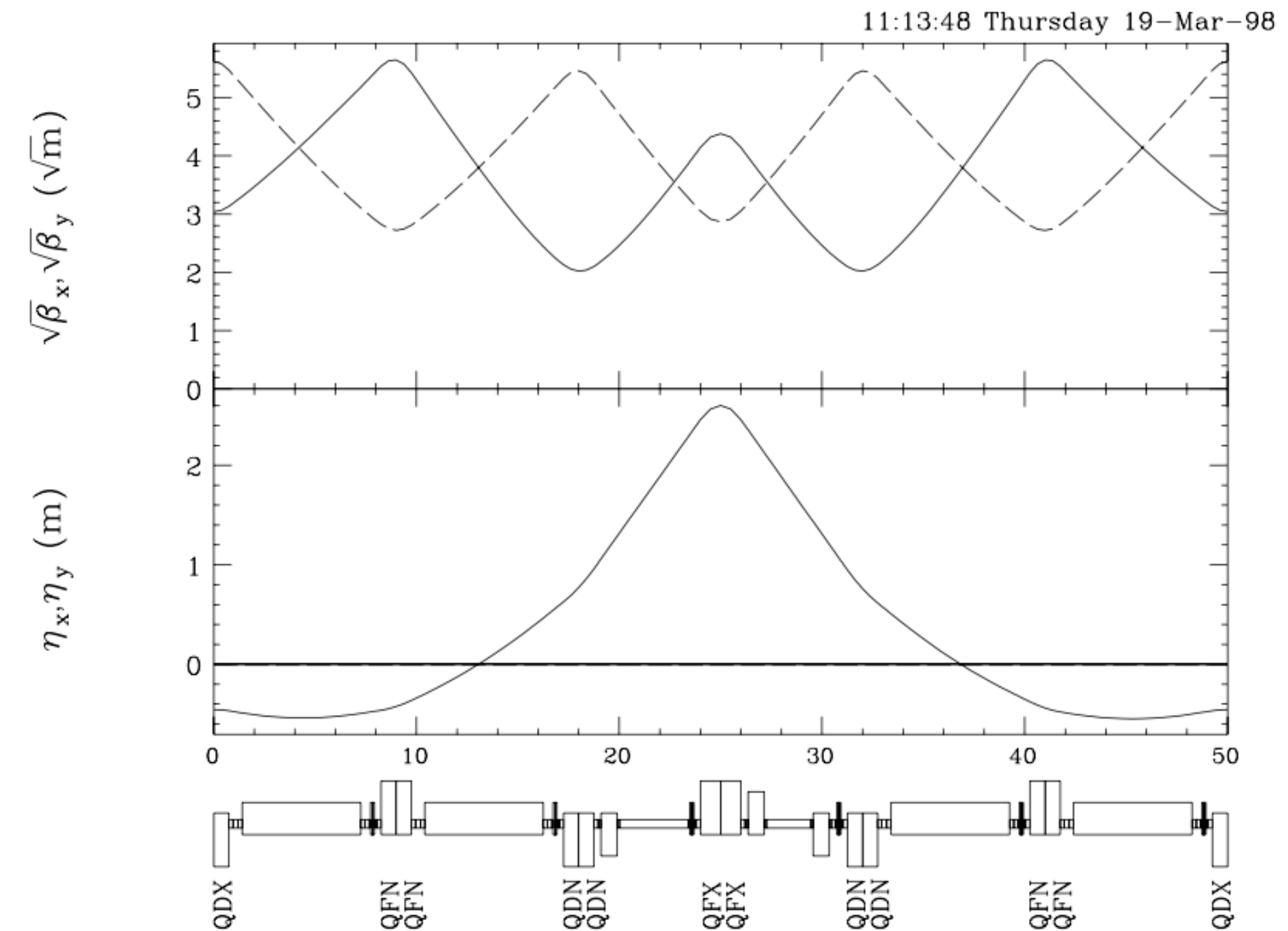


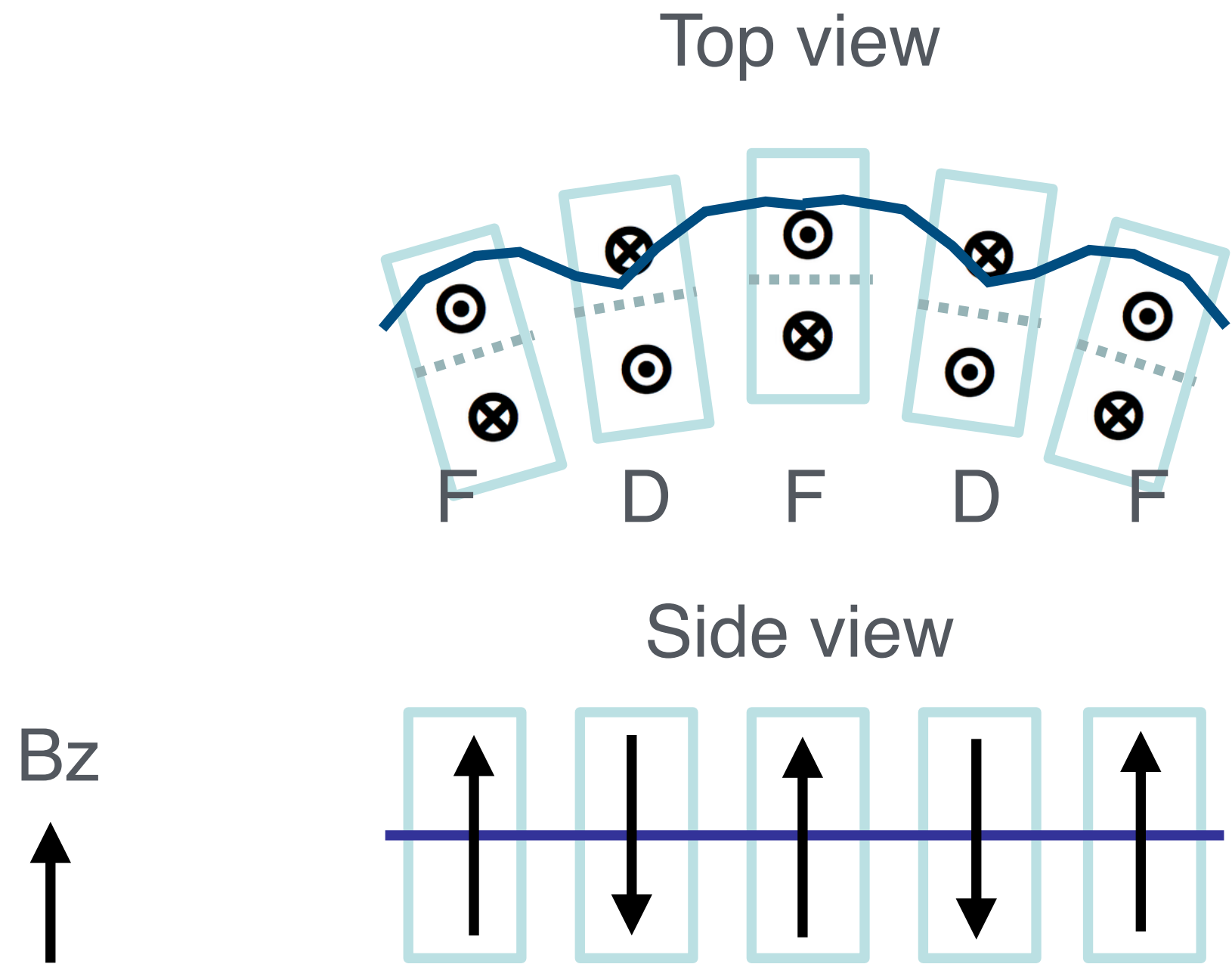
Figure 1: Beam optics functions of the module in arc section of the JHF 50 GeV main ring.  $\beta_x^{1/2}$ :solid line,  $\beta_y^{1/2}$ :dashed line.

# Radial shift of normal quadrupole FODO

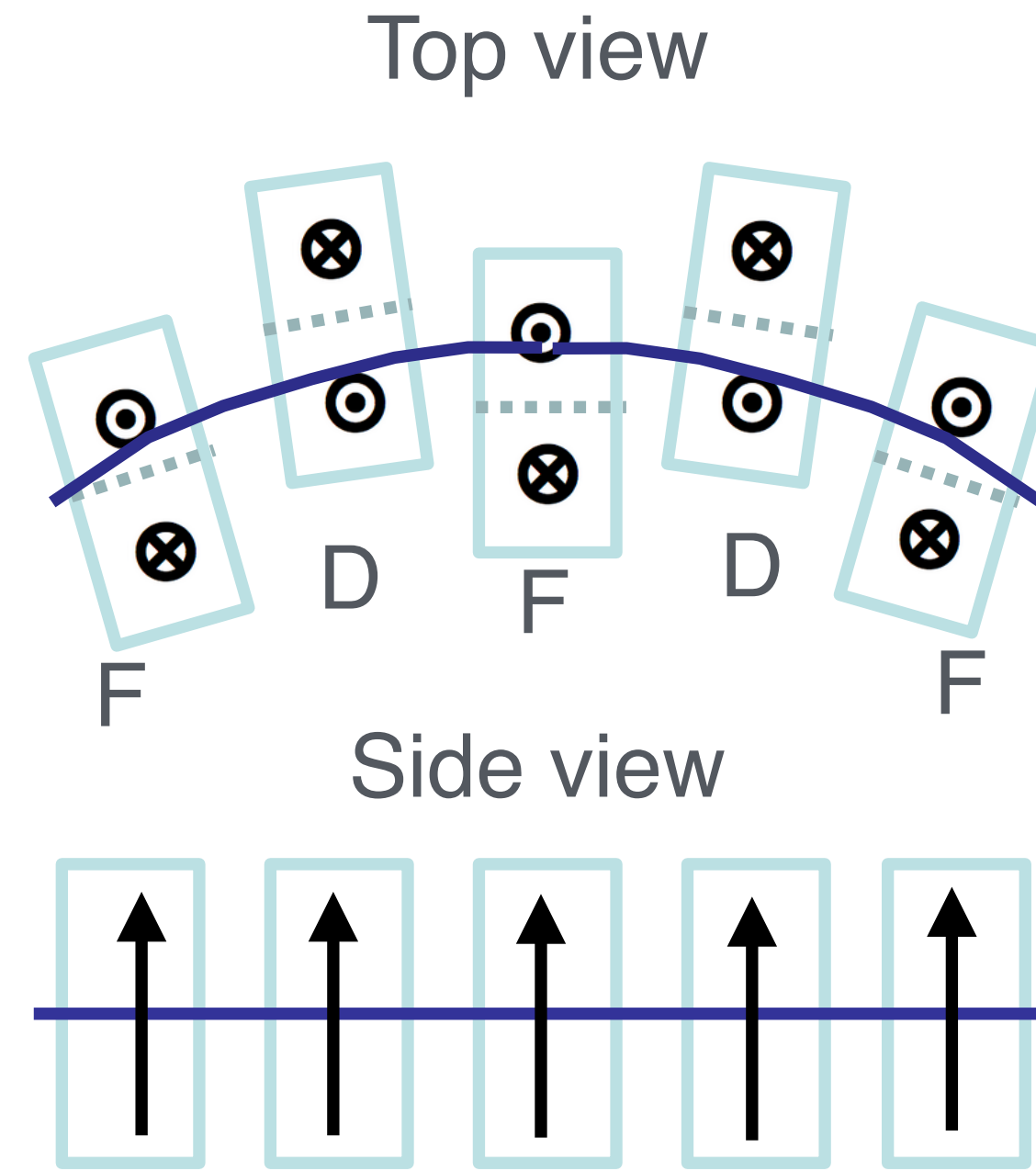
## *non-scaling FFA*

- Dispersion function  $D_x$  can be small in ns-FFA lattice configuration.
- **Dispersion action function  $H$**  is minimum in ns-FFA so that momentum compaction factor is zero.

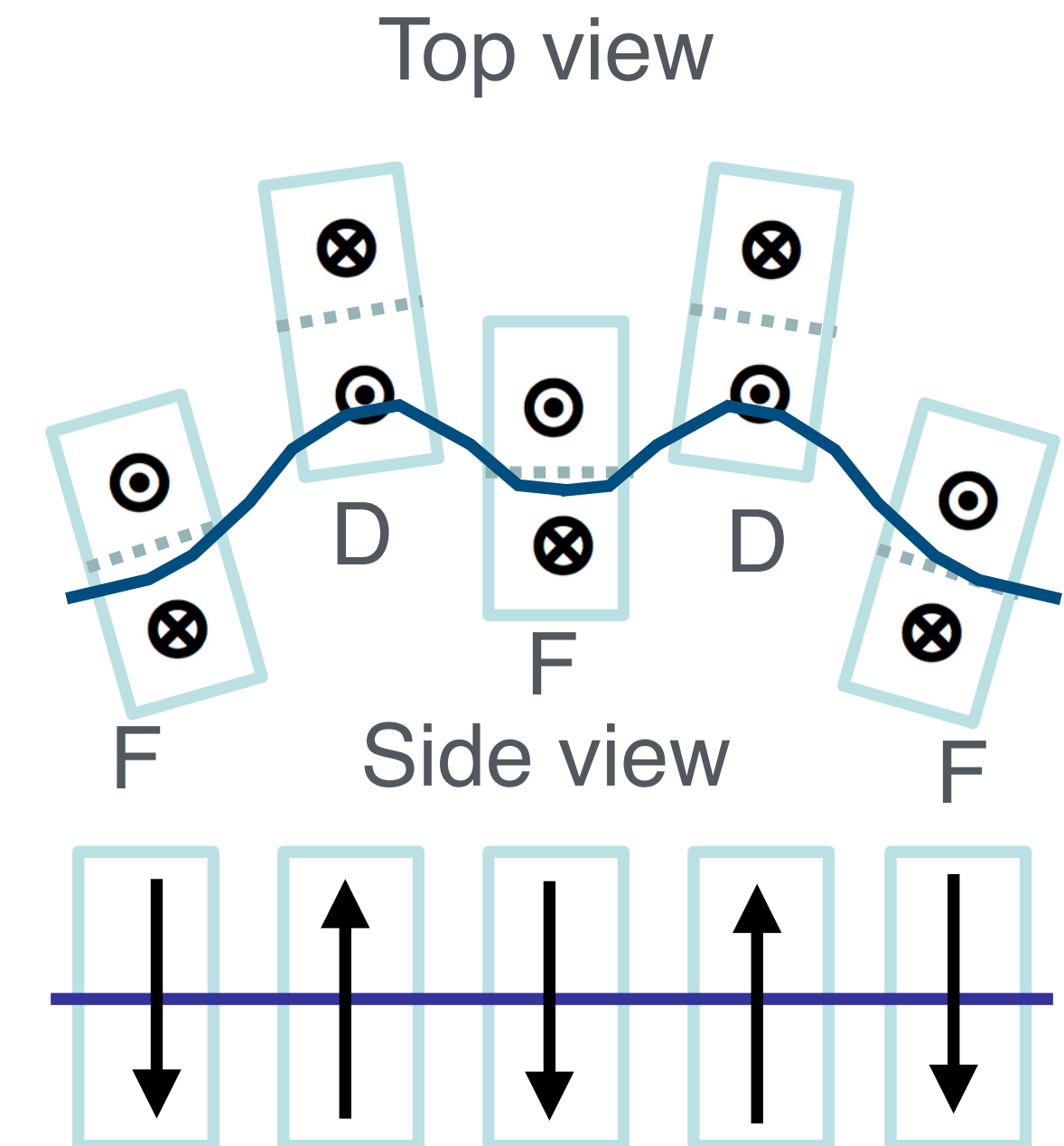
### Scaling FFA like (large alpha)



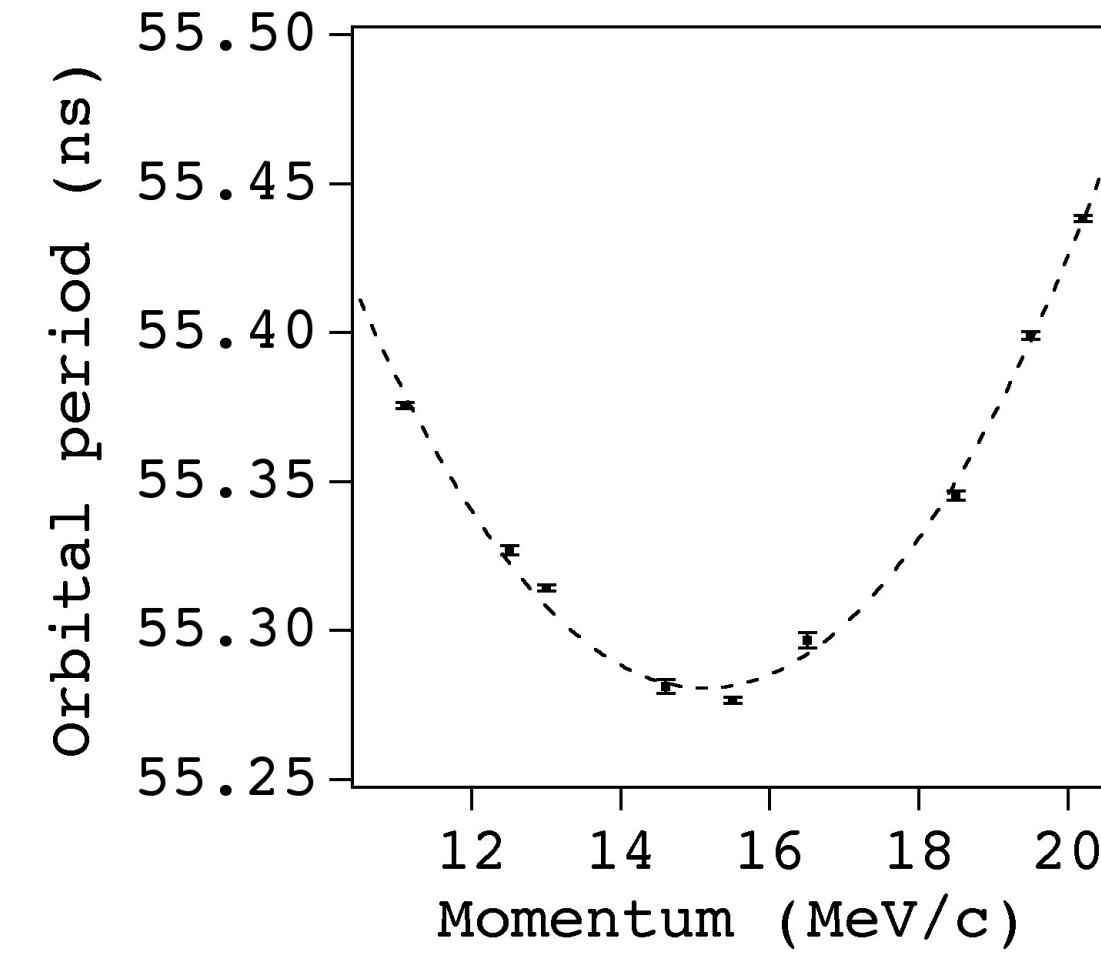
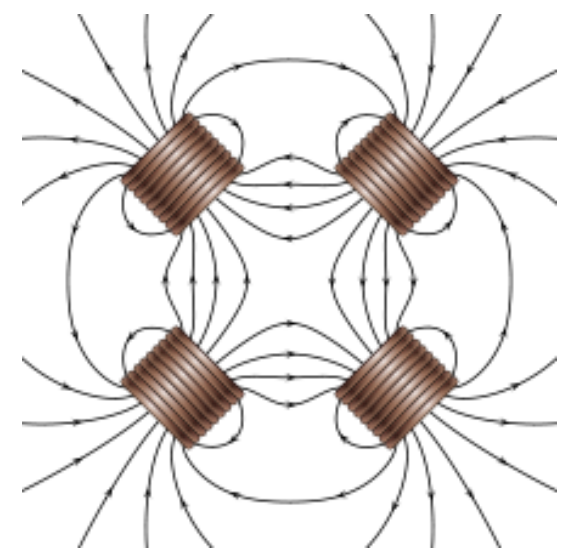
### Combined function



### Non scaling FFA (small alpha)



### Normal quadrupole



“EMMA comm.”, Nature Physics Vol. 8, No. 3 (2012).

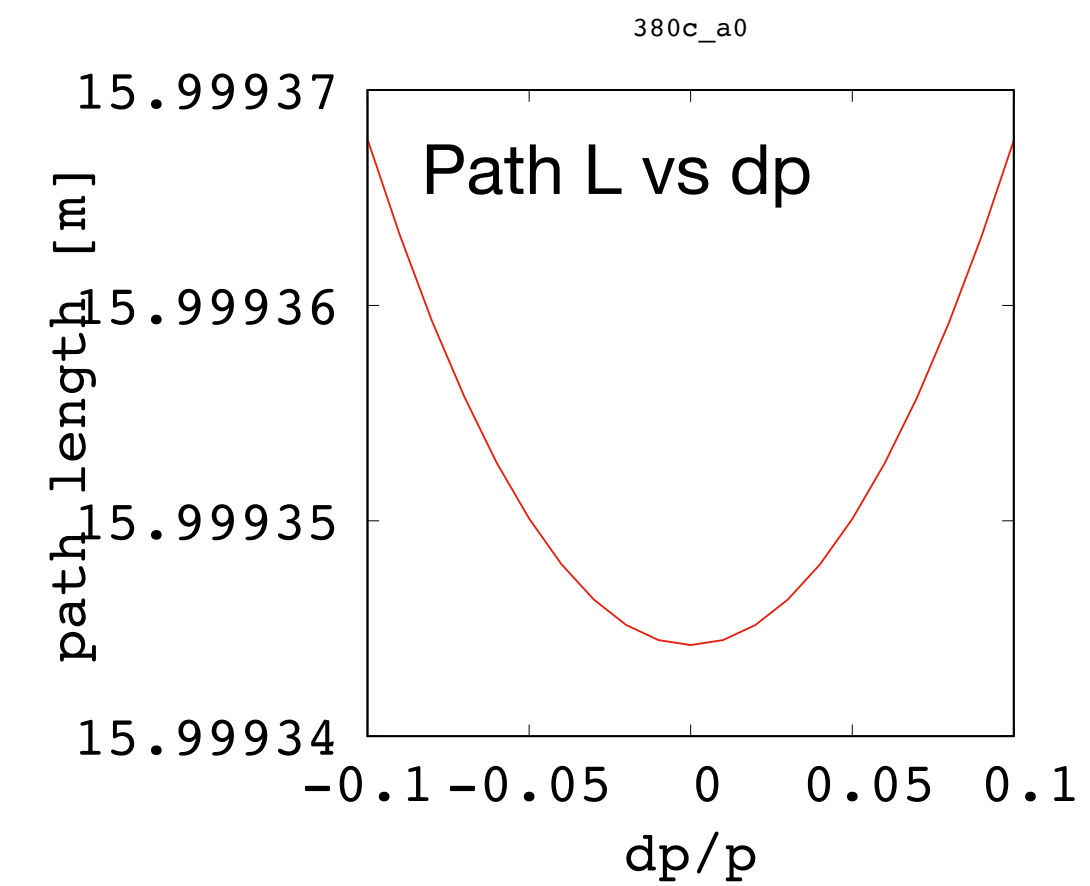
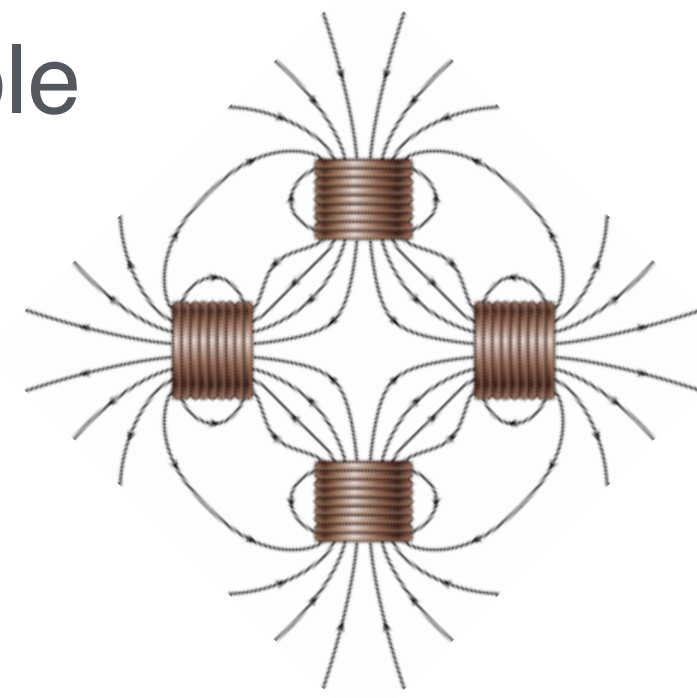


# Vertical shift of skew quadrupole FODO

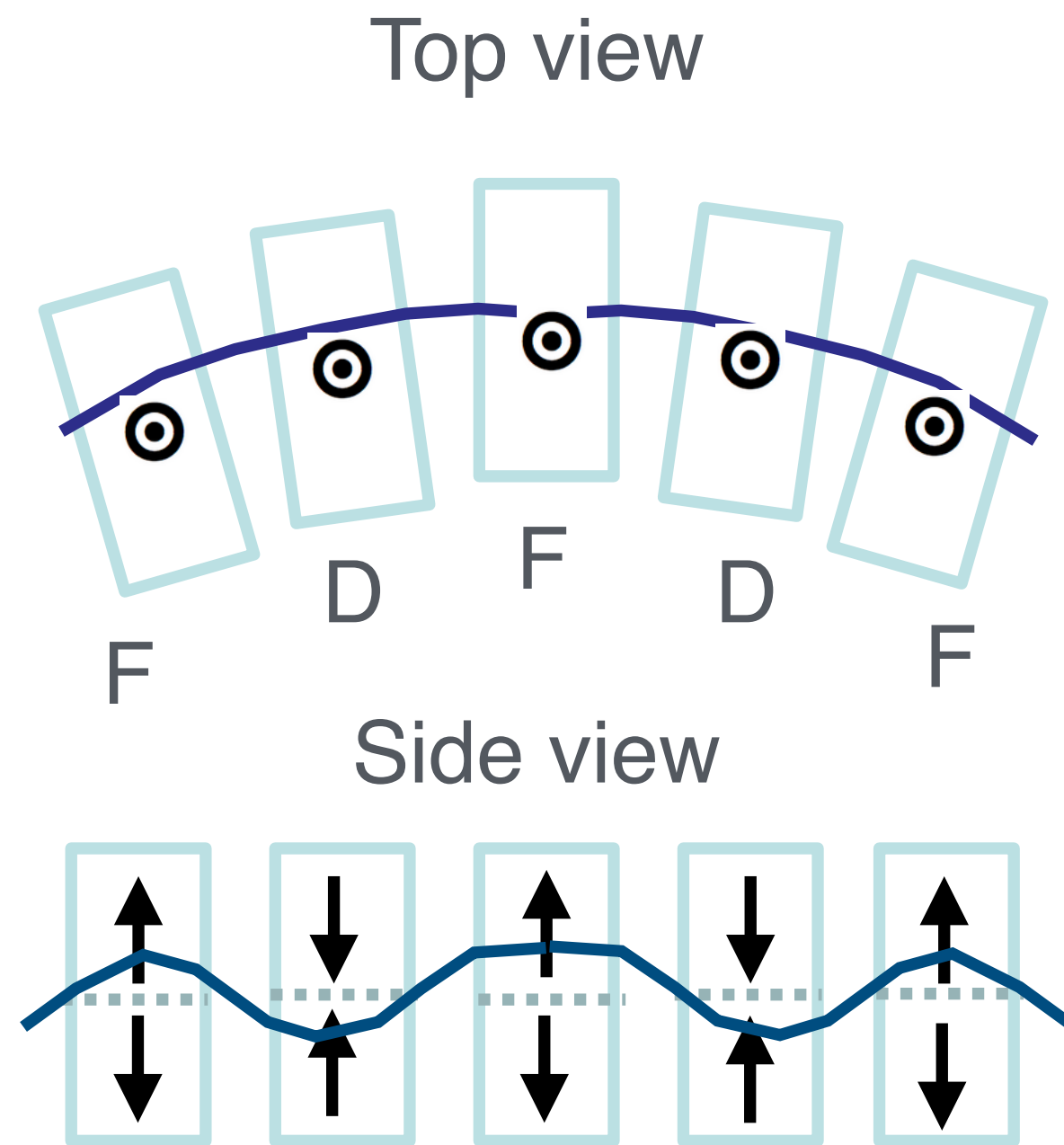
## *no reverse bending*

- Vertical bending field changes its sign below or above the mid-plane.
- We can eliminate reverse bend.

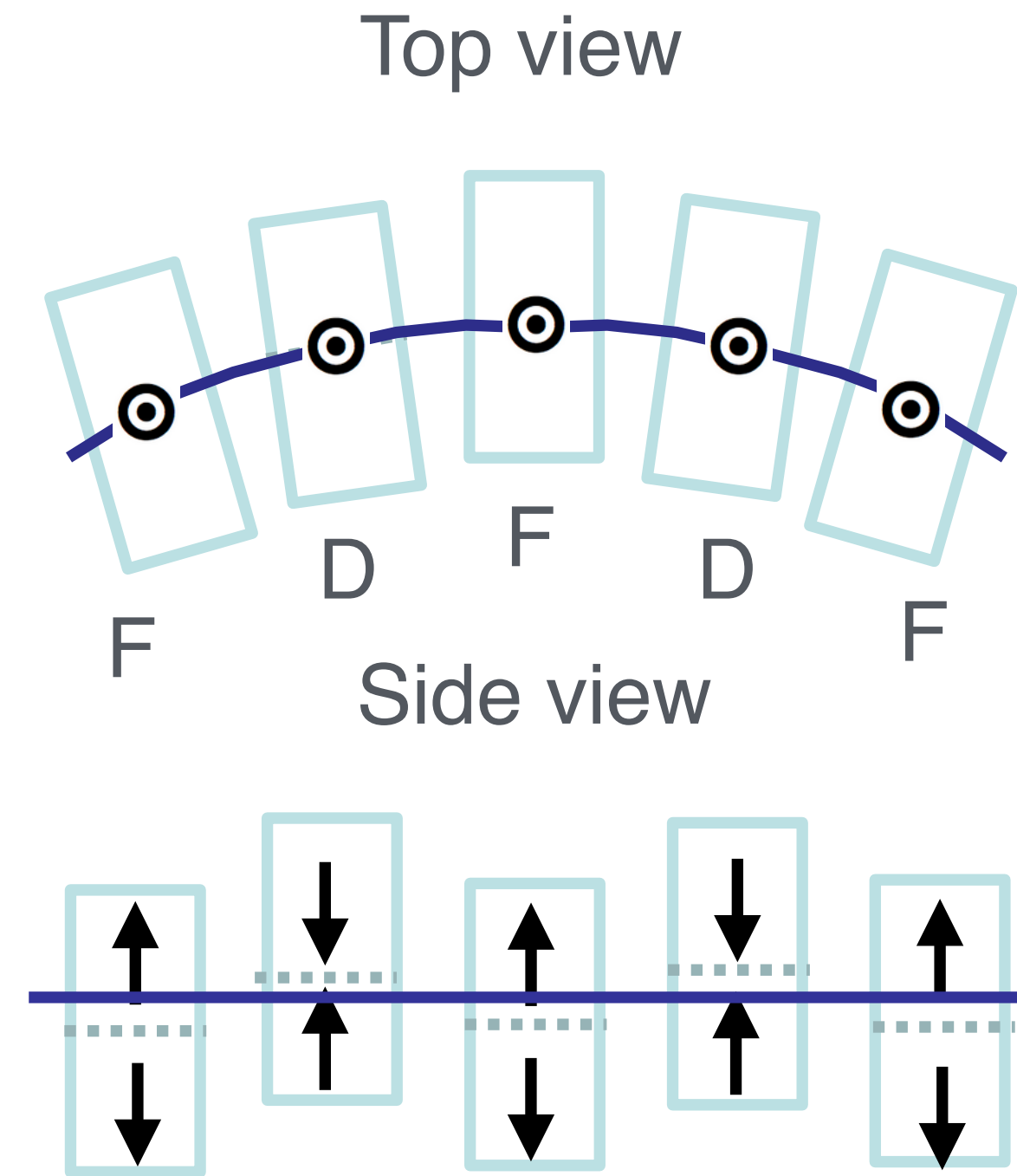
Skew quadrupole



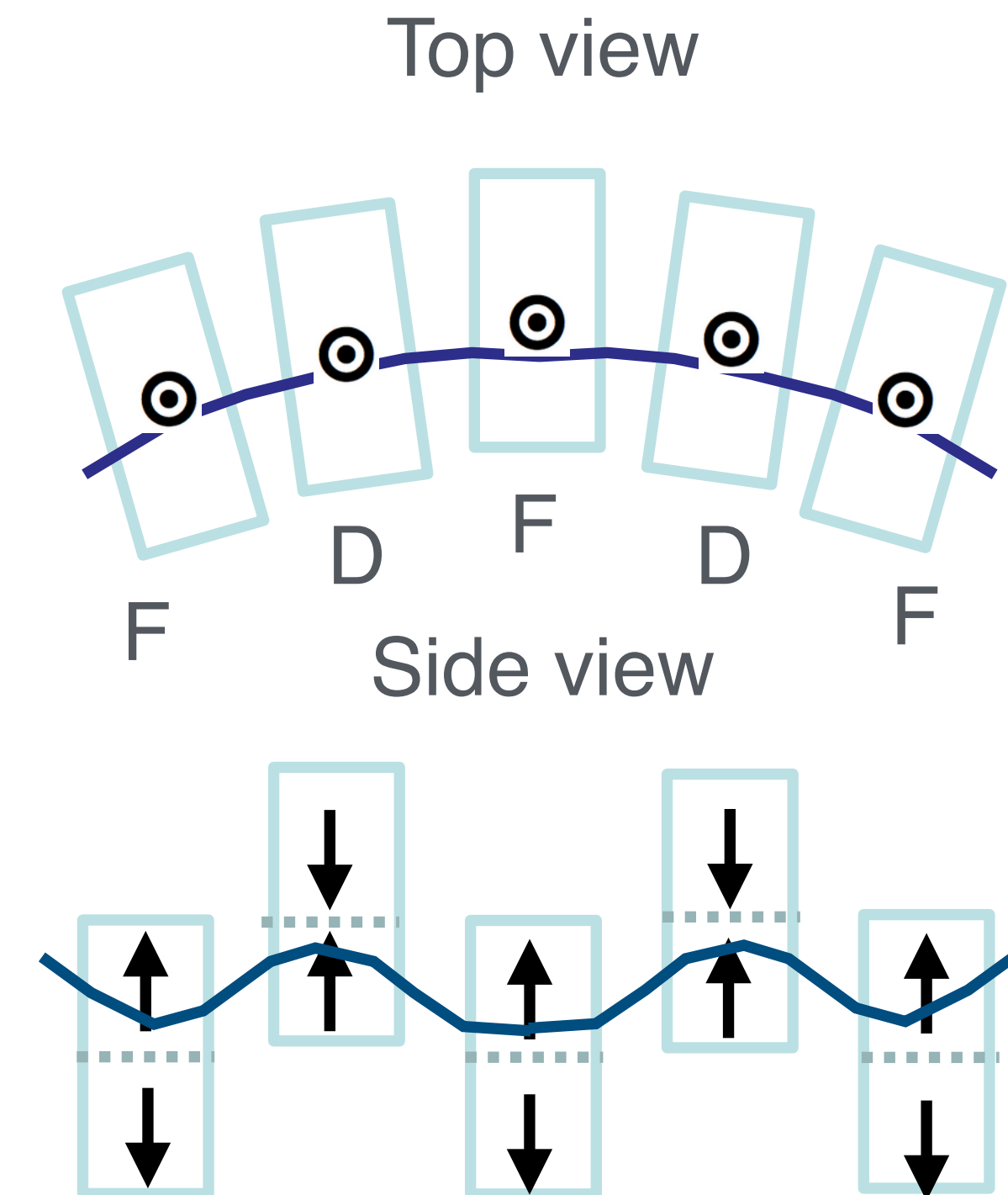
Combined function (large alpha)



Combined function

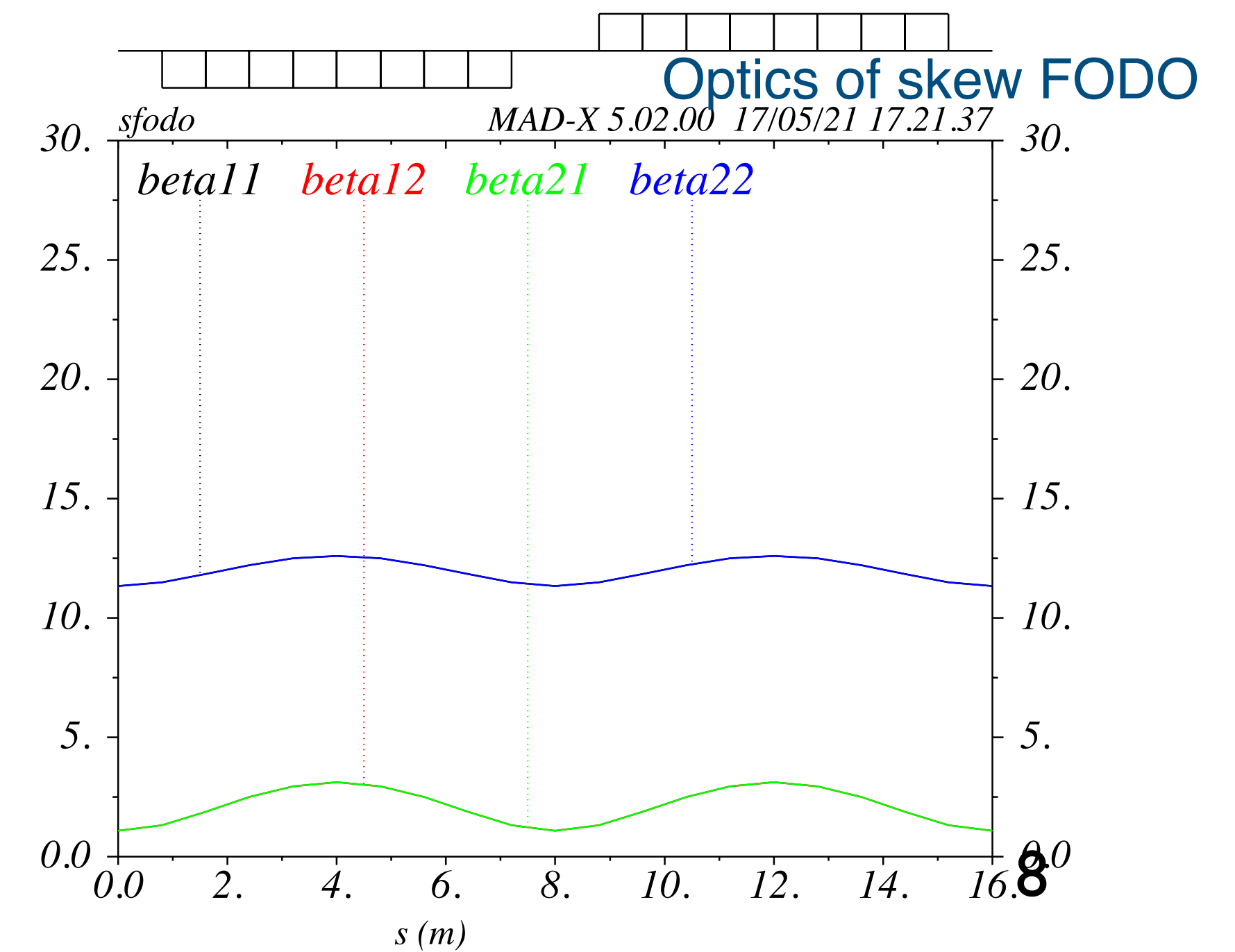
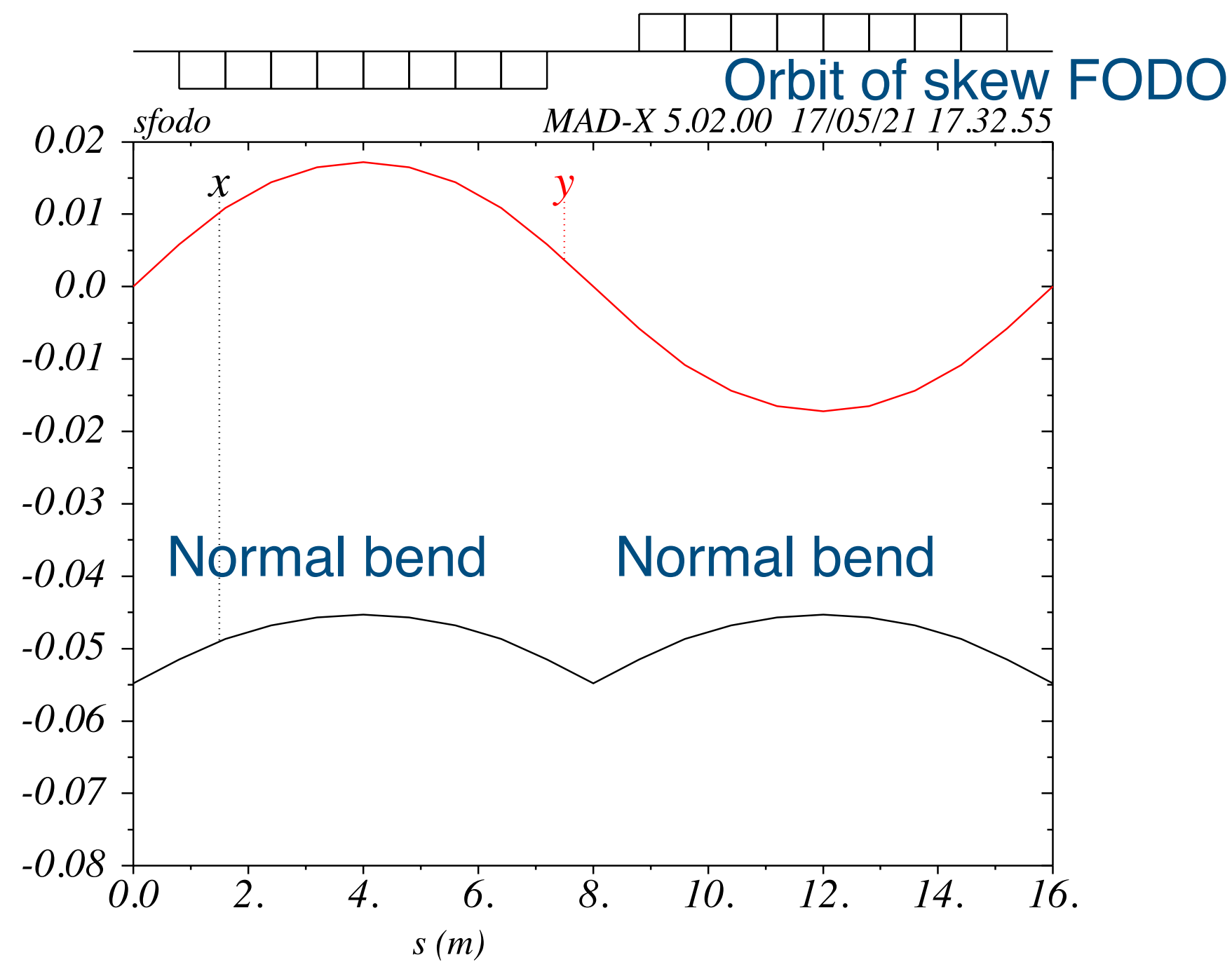
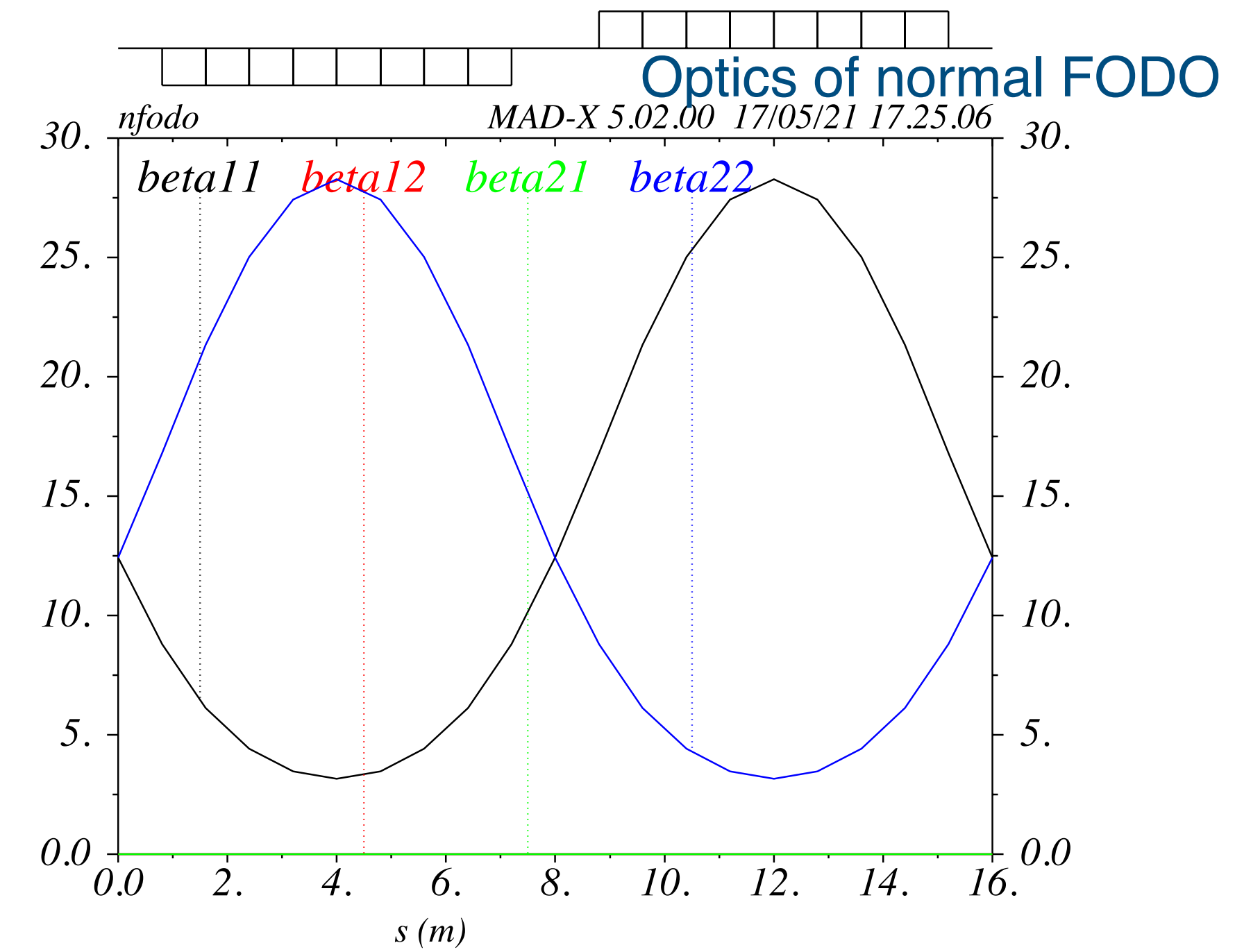
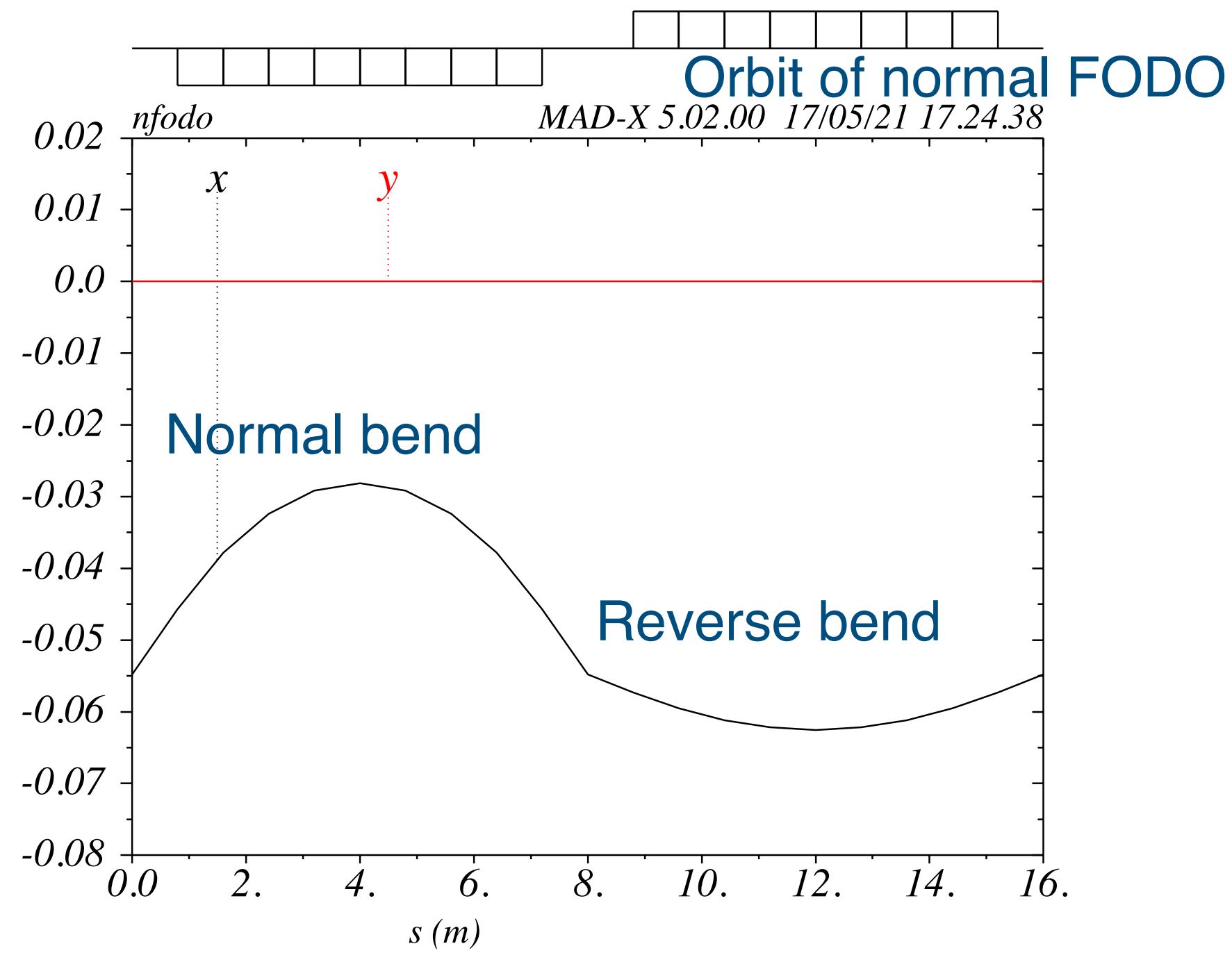


Combined function (small alpha)



# MADX results

## normal Q vs skew Q

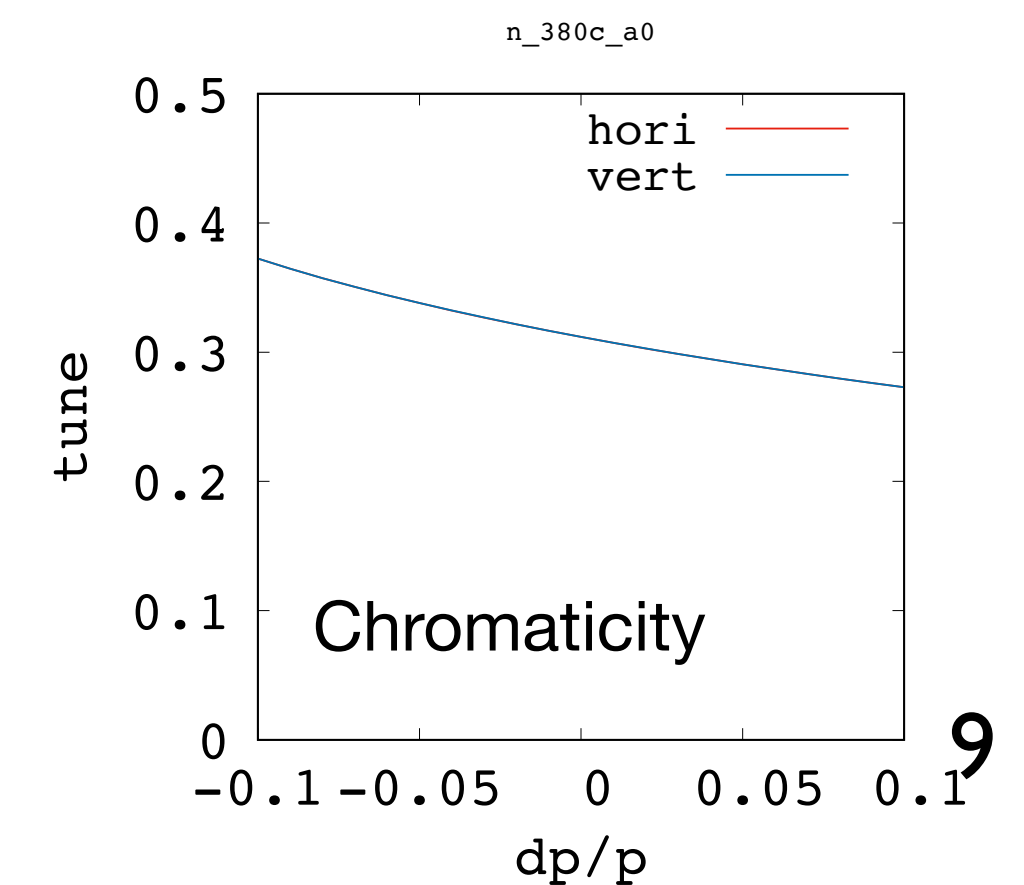
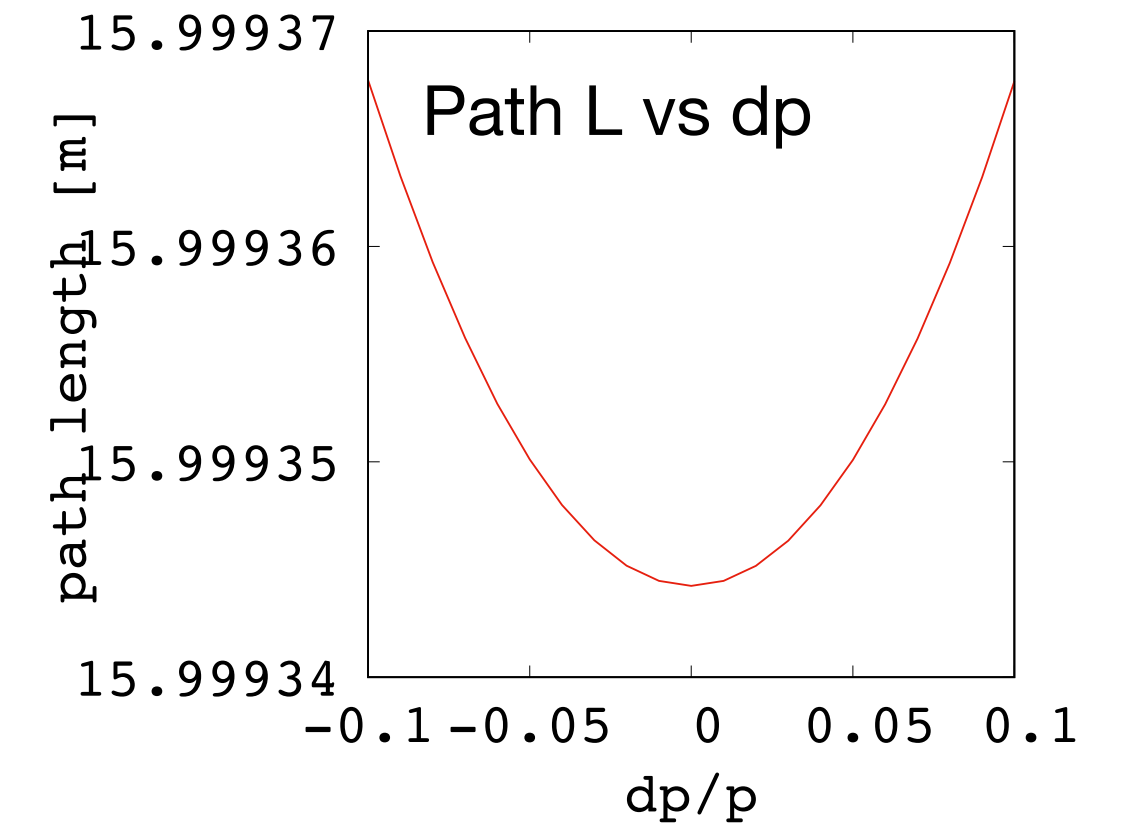
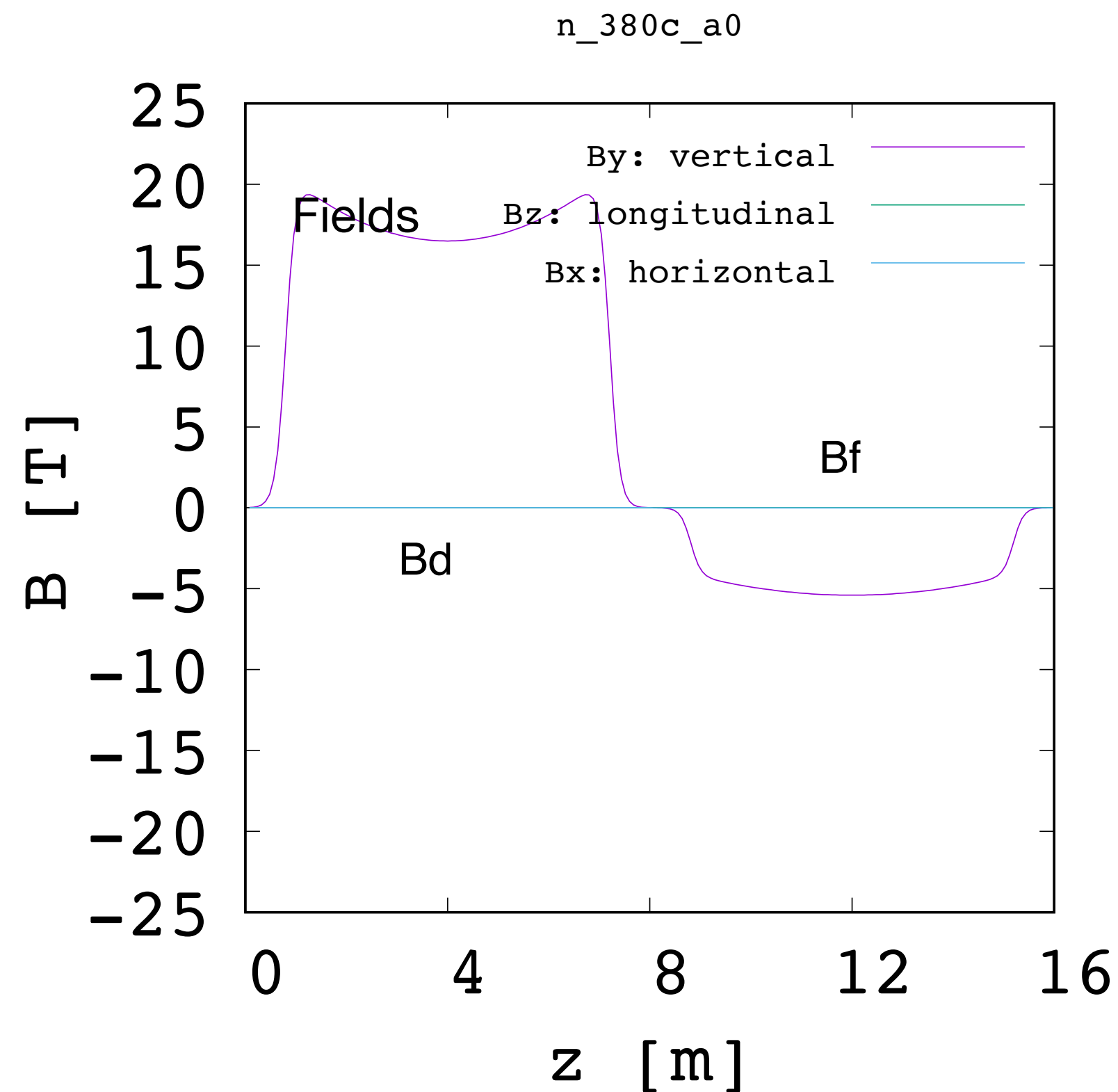
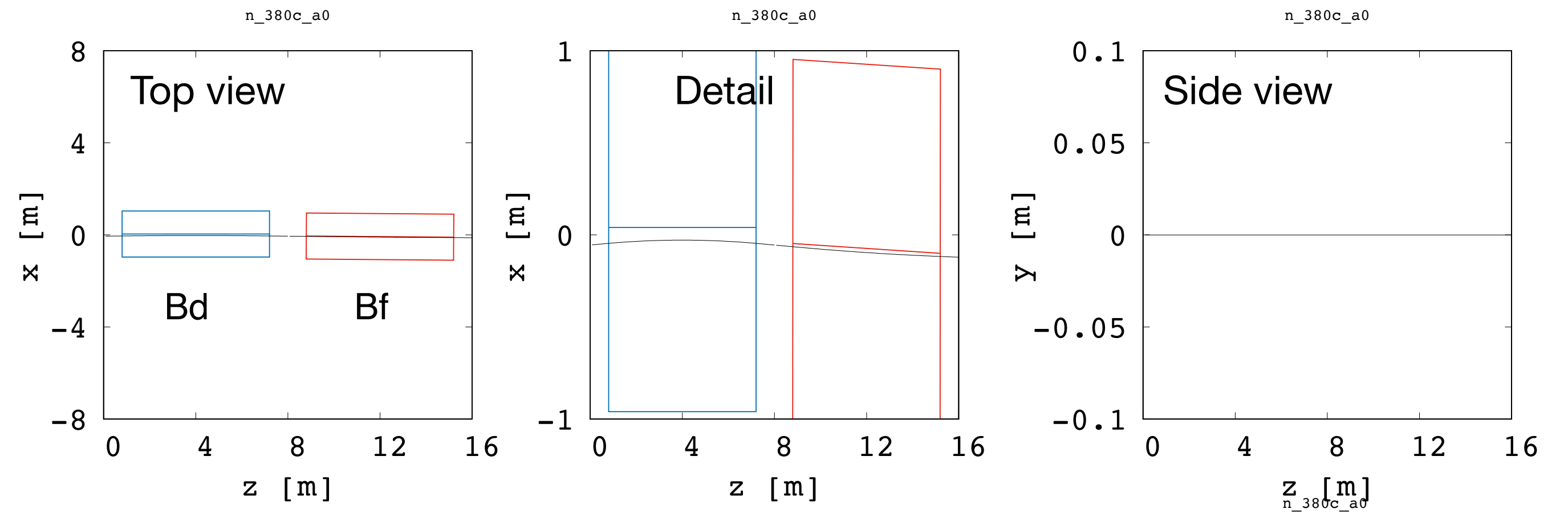




# 1.5 TeV collider ring

*momentum comp=0, arc only*

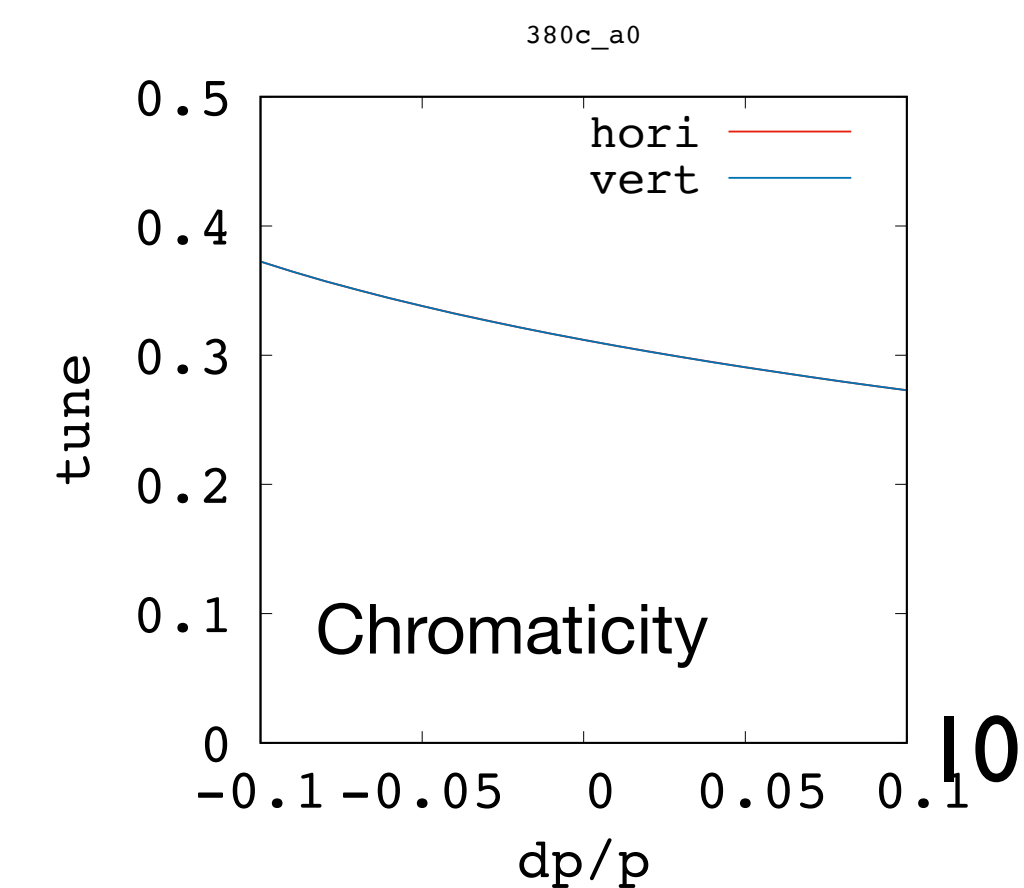
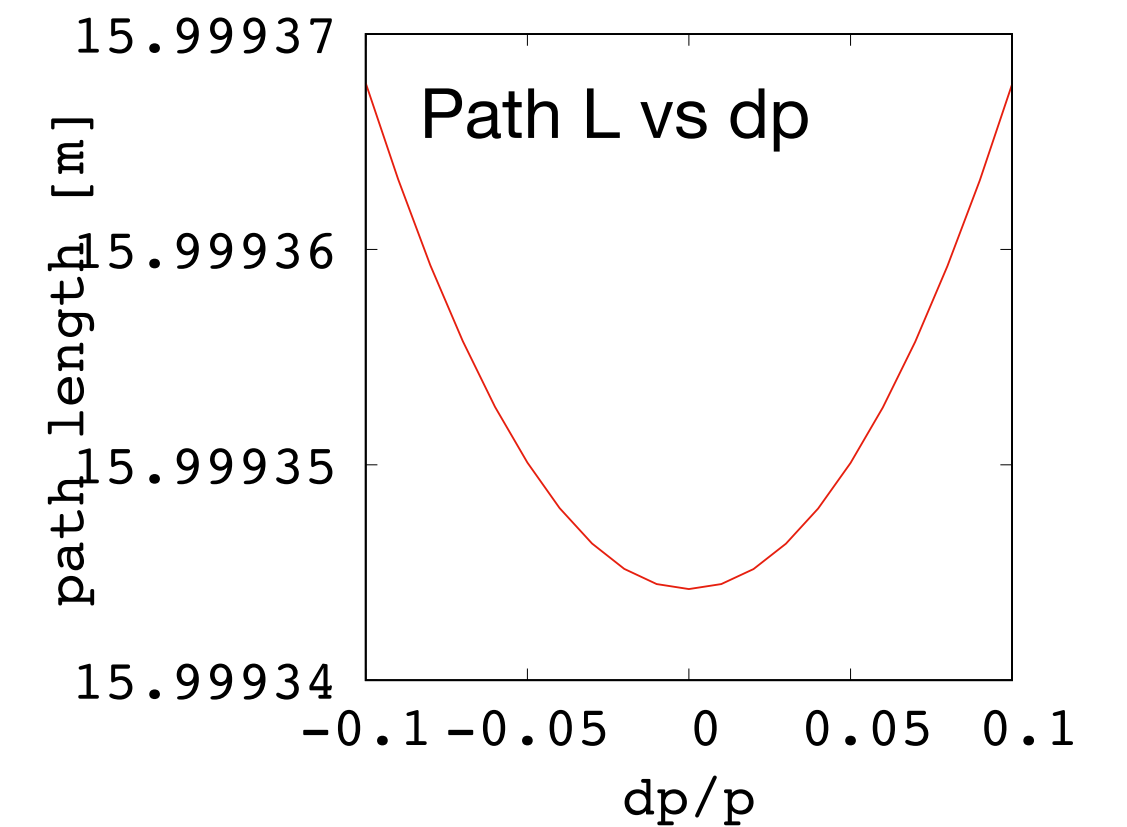
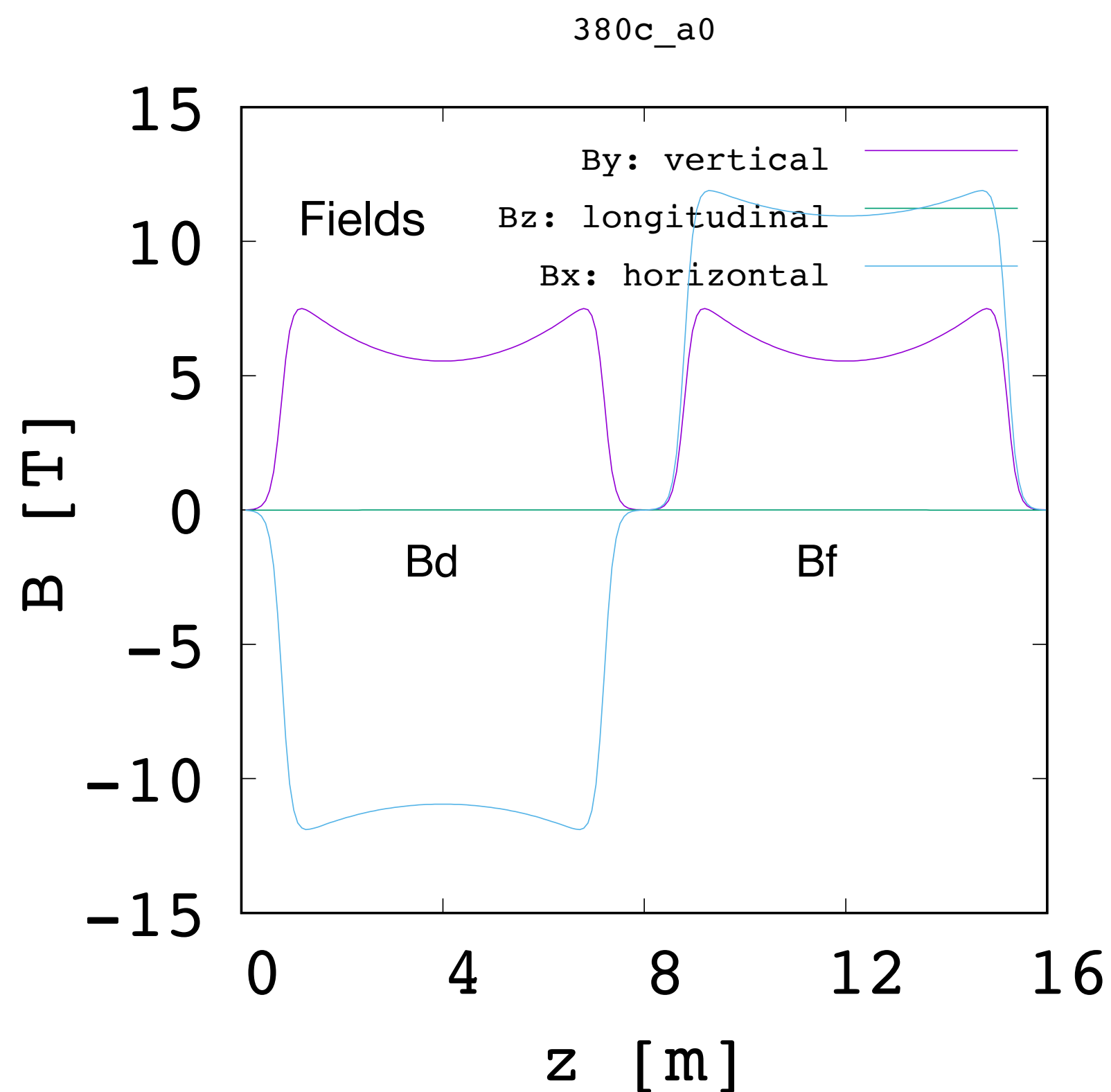
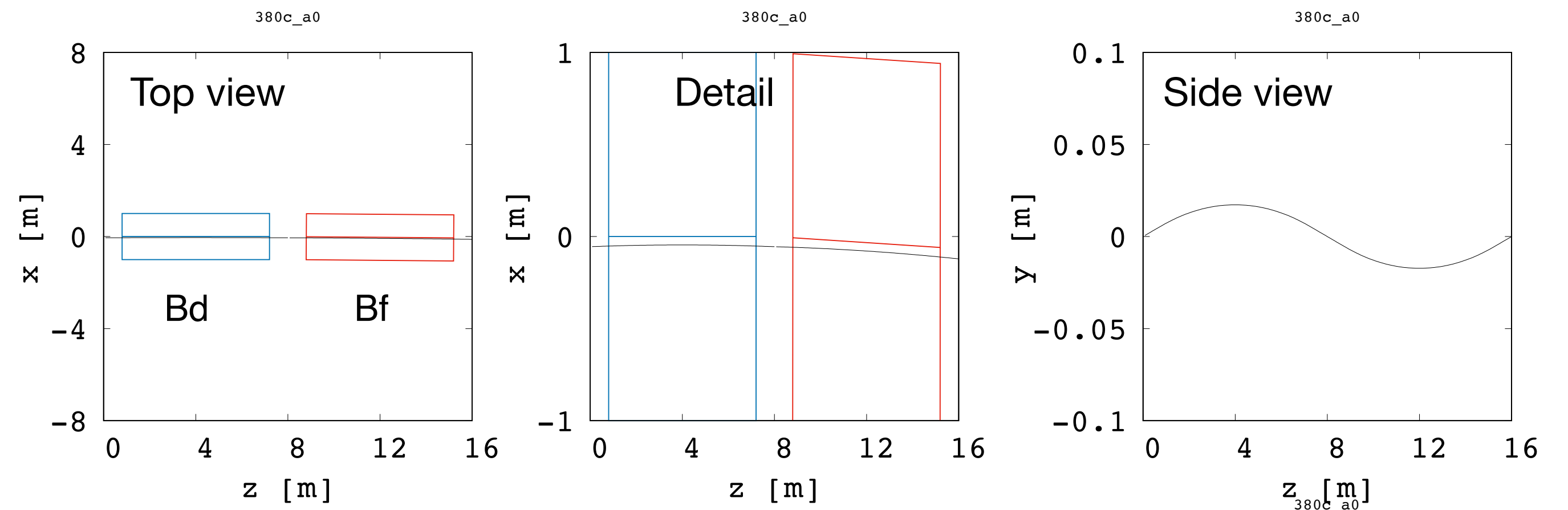
	Normal FODO
Energy	1.5 TeV
Momentum compaction	0
Circumference	6080 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	380
Maximum field	20 T
Field gradient	240 T/m
Cell tune	0.3131 / 0.3131



# 1.5 TeV collider ring

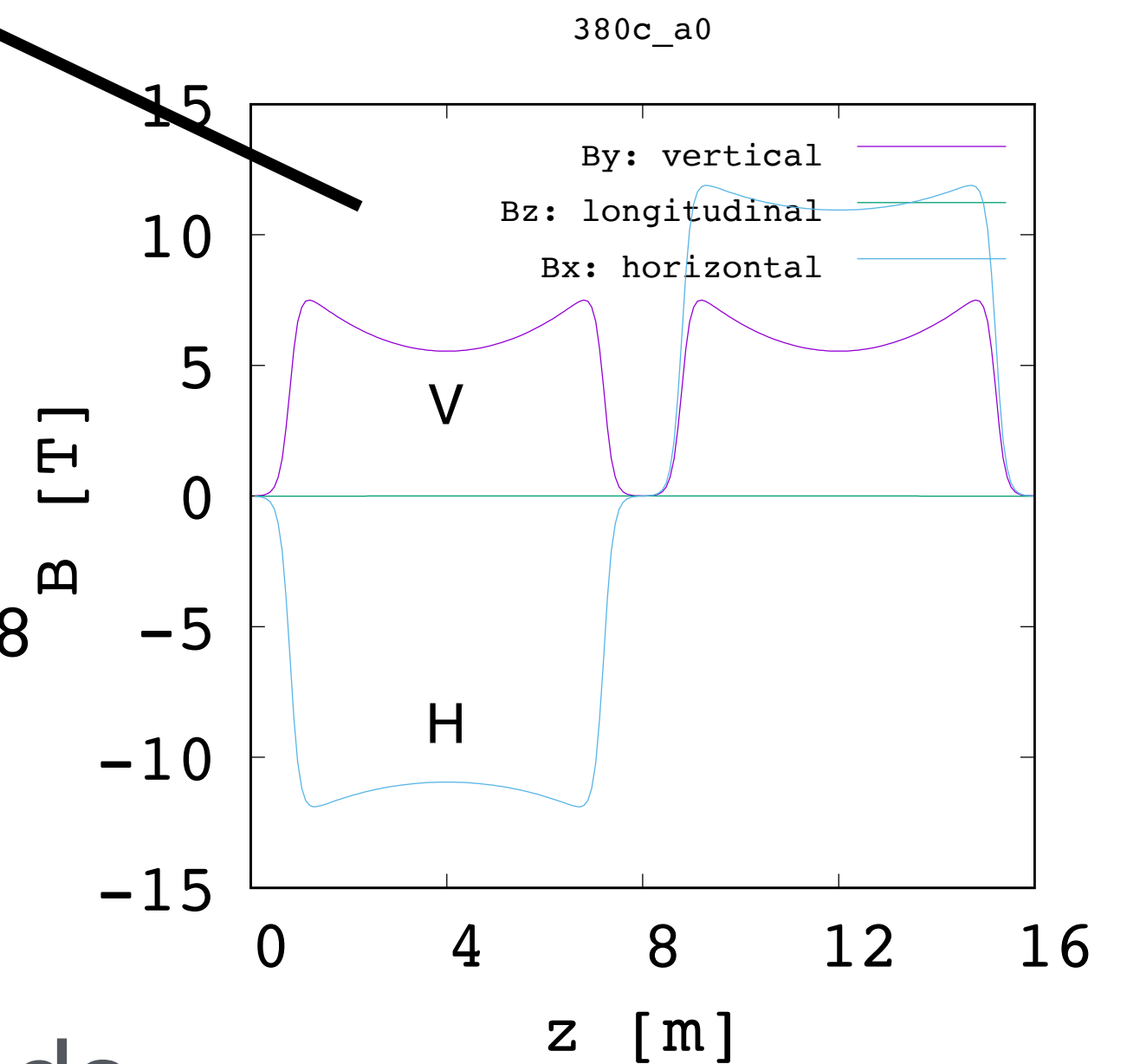
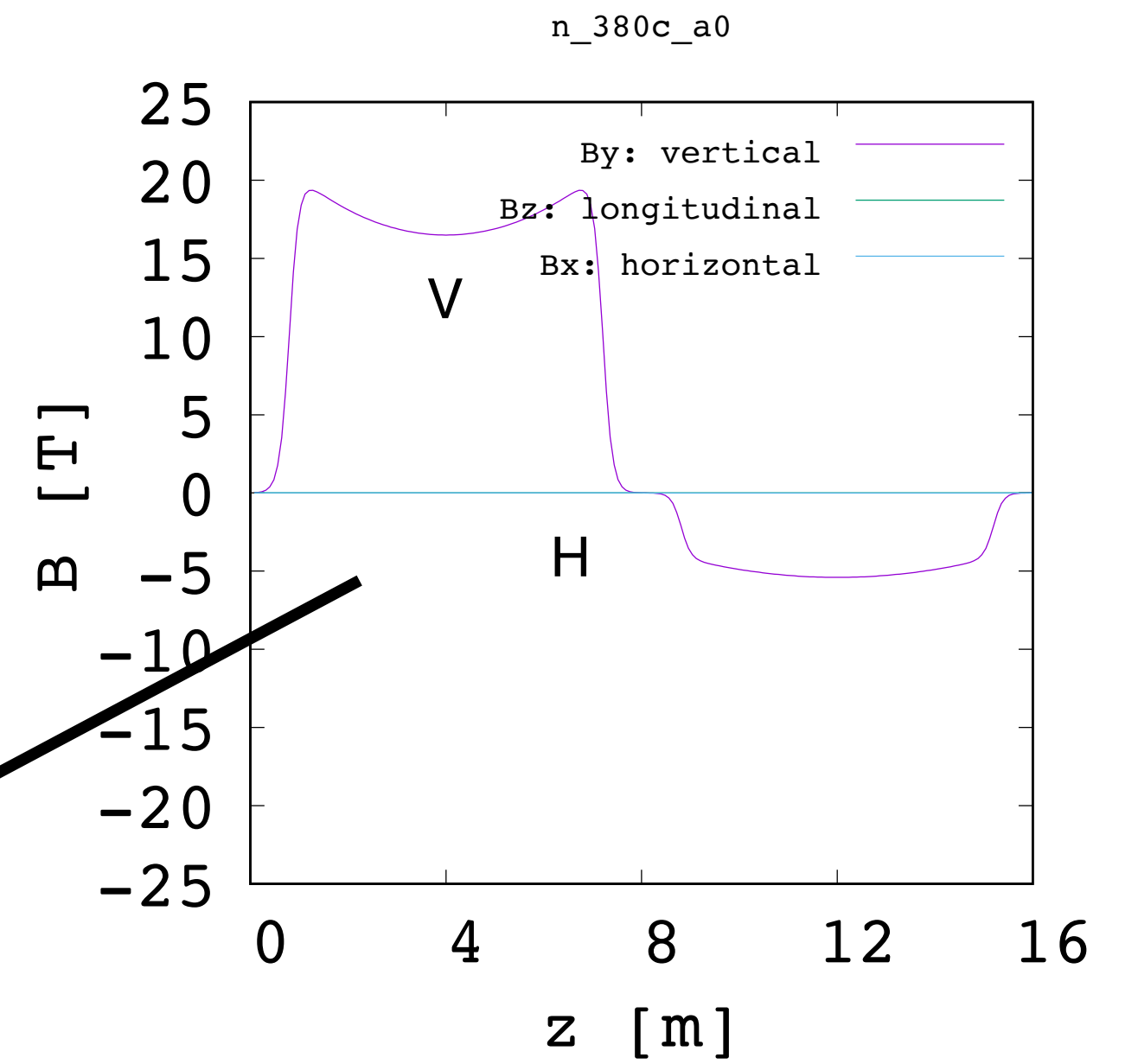
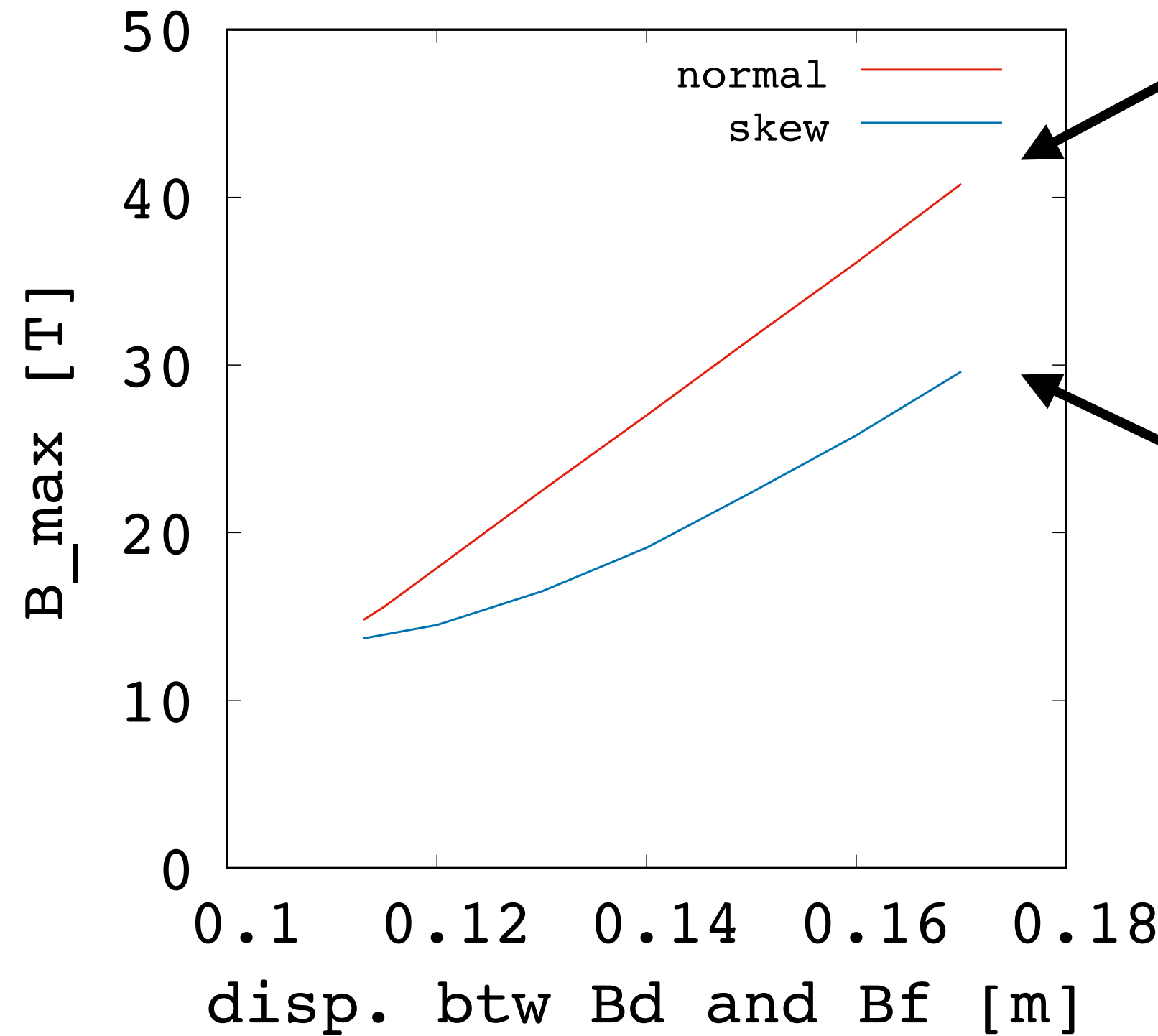
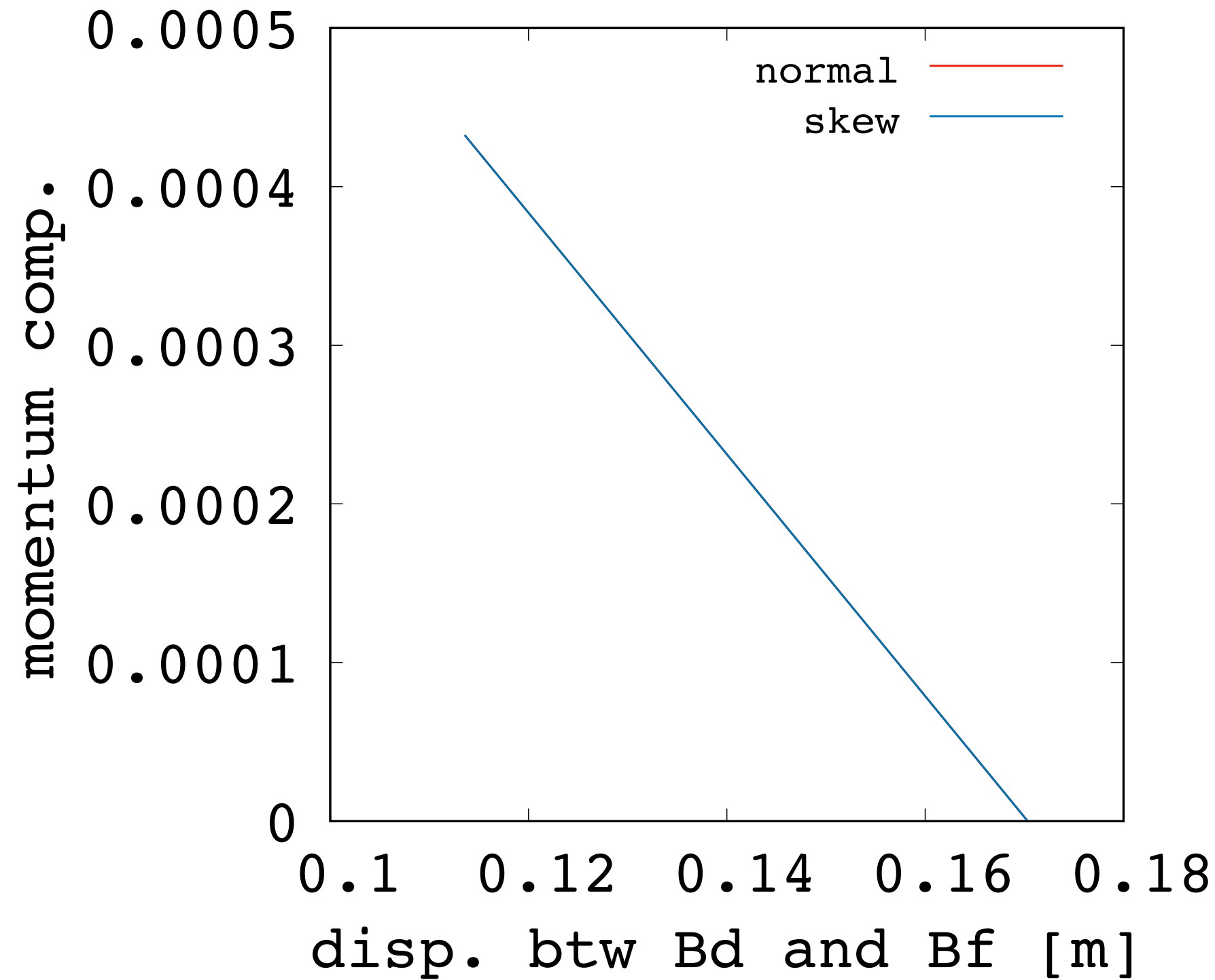
*momentum comp=0, arc only*

	<b>Skew FODO</b>
Energy	1.5 TeV
<b>Momentum compaction</b>	0
<b>Circumference</b>	6080 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	380
<b>Maximum field</b>	<b>14 T</b>
Field gradient	240 T/m
Cell tune	0.3131 / 0.3131



# 1.5 TeV collider ring

## *momentum compactor and maximum field strength*

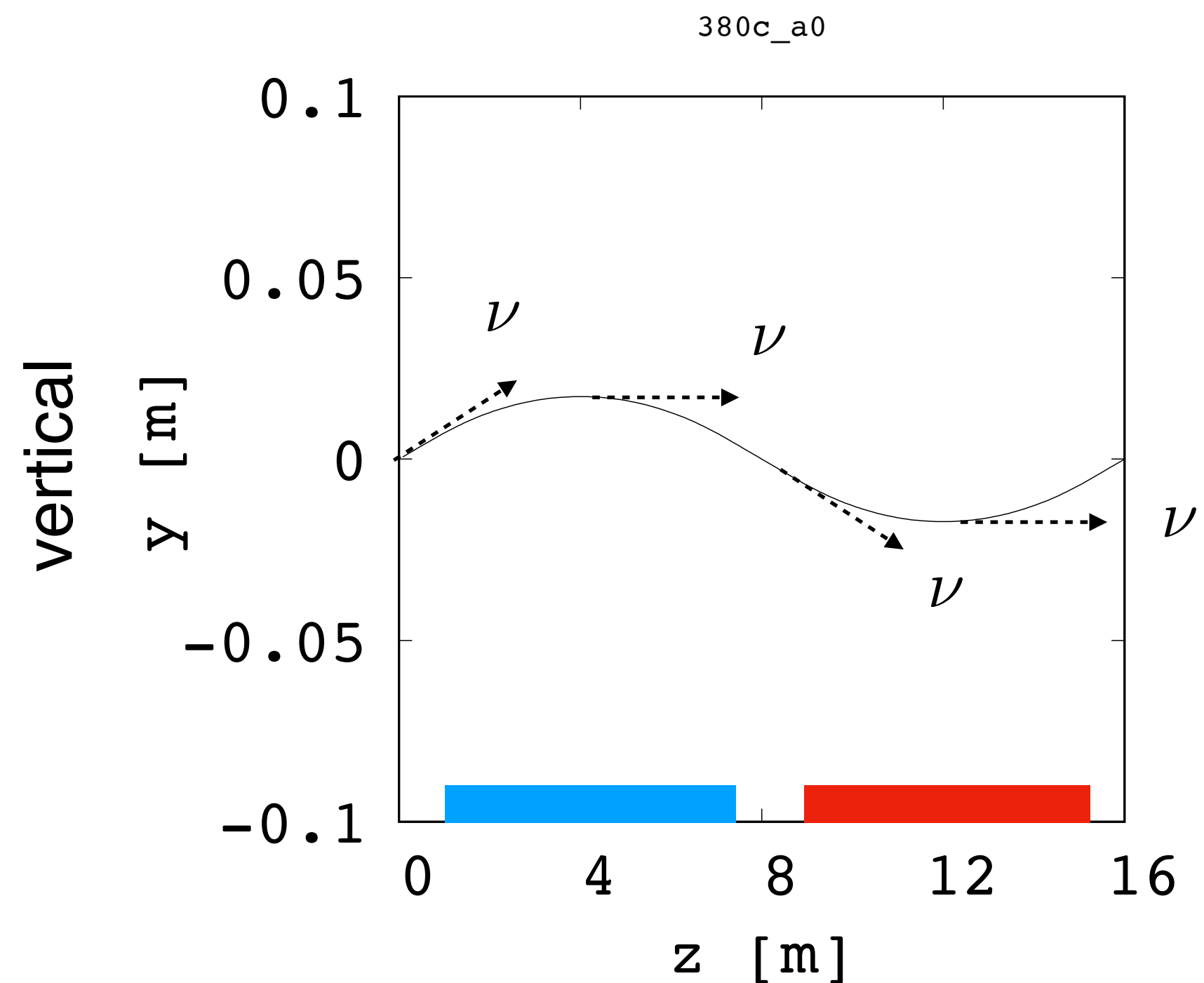


Skew FODO does not need negative bends.

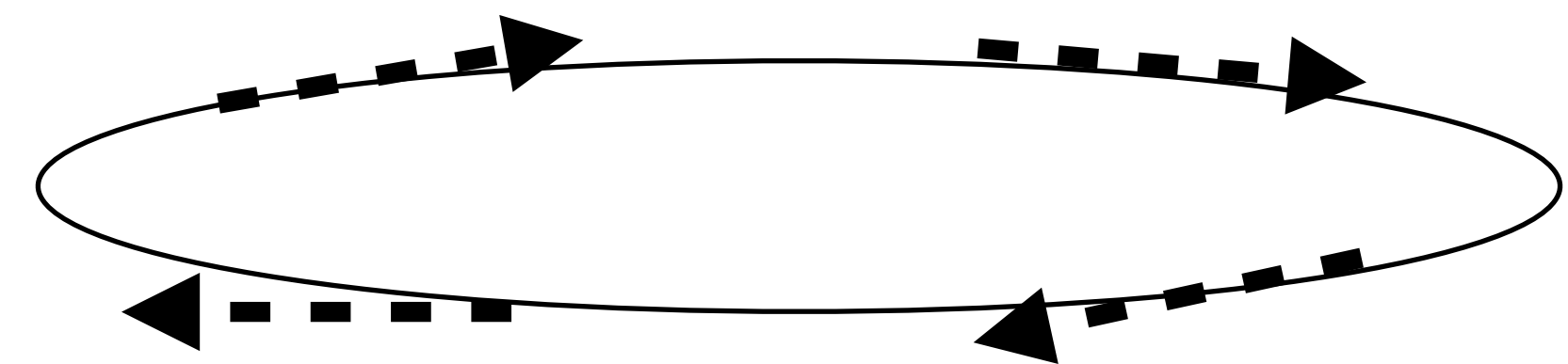
# Radiation mitigation

*orbit is wiggling in vertical direction by horizontal dipole*

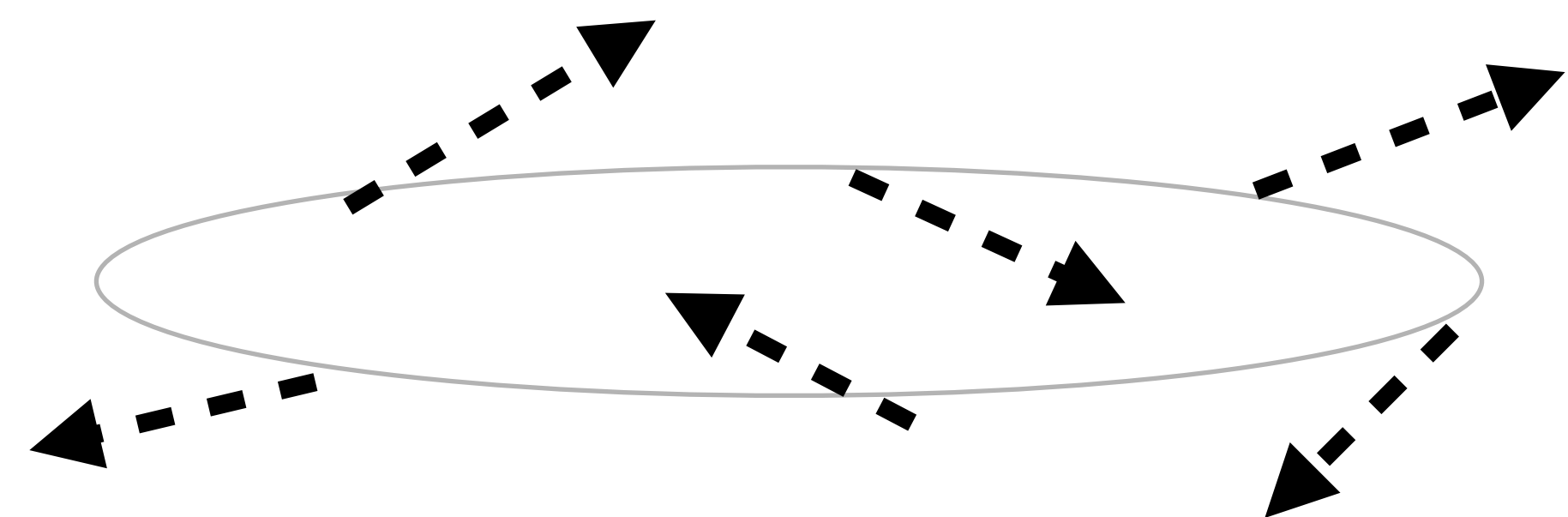
- Beam orbit is not constrained on a horizontal plane.
- Vertical wiggling angle is e.g. +/- 8 mrad (see below, but depend on cell length).



Planar ring

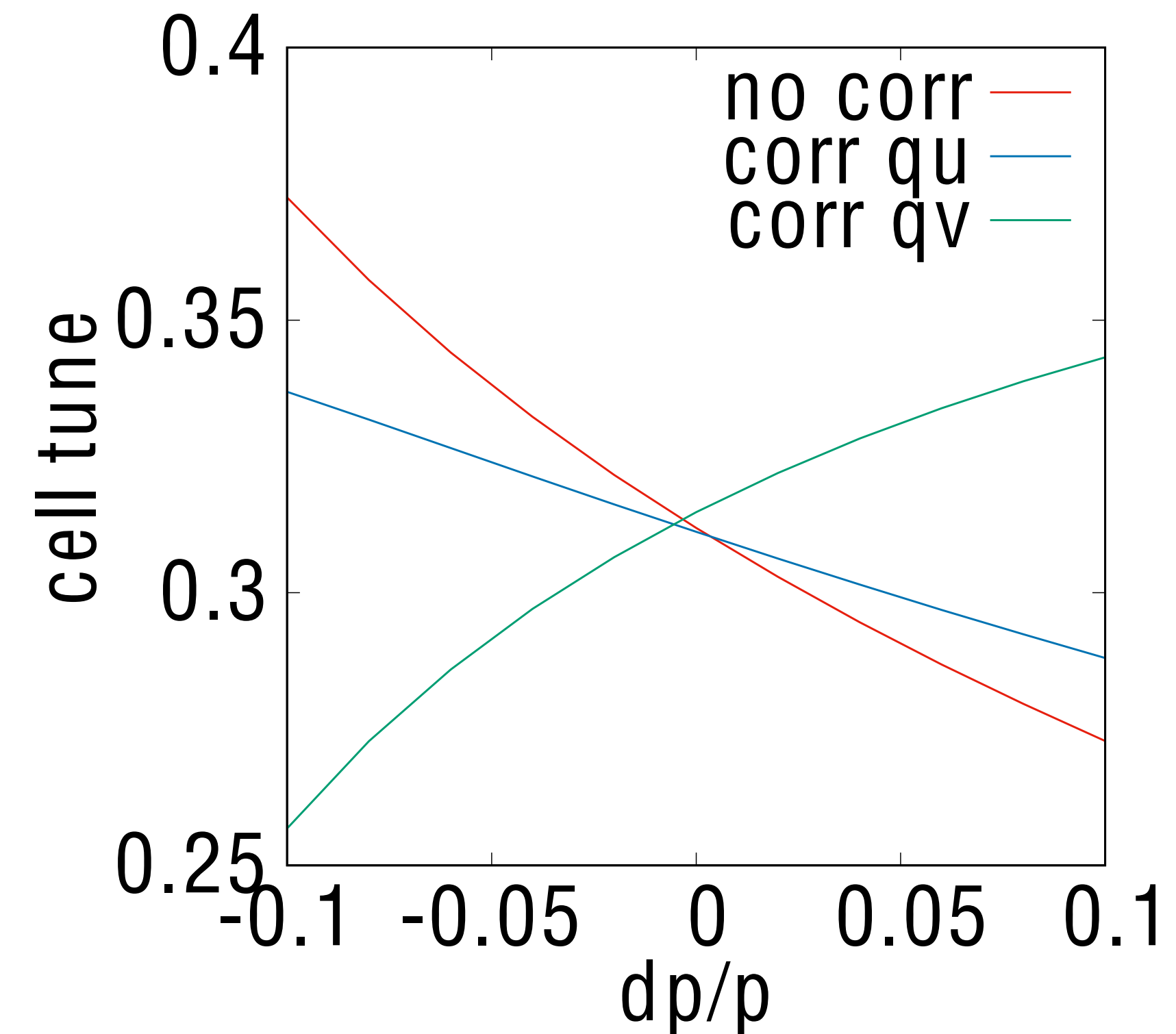
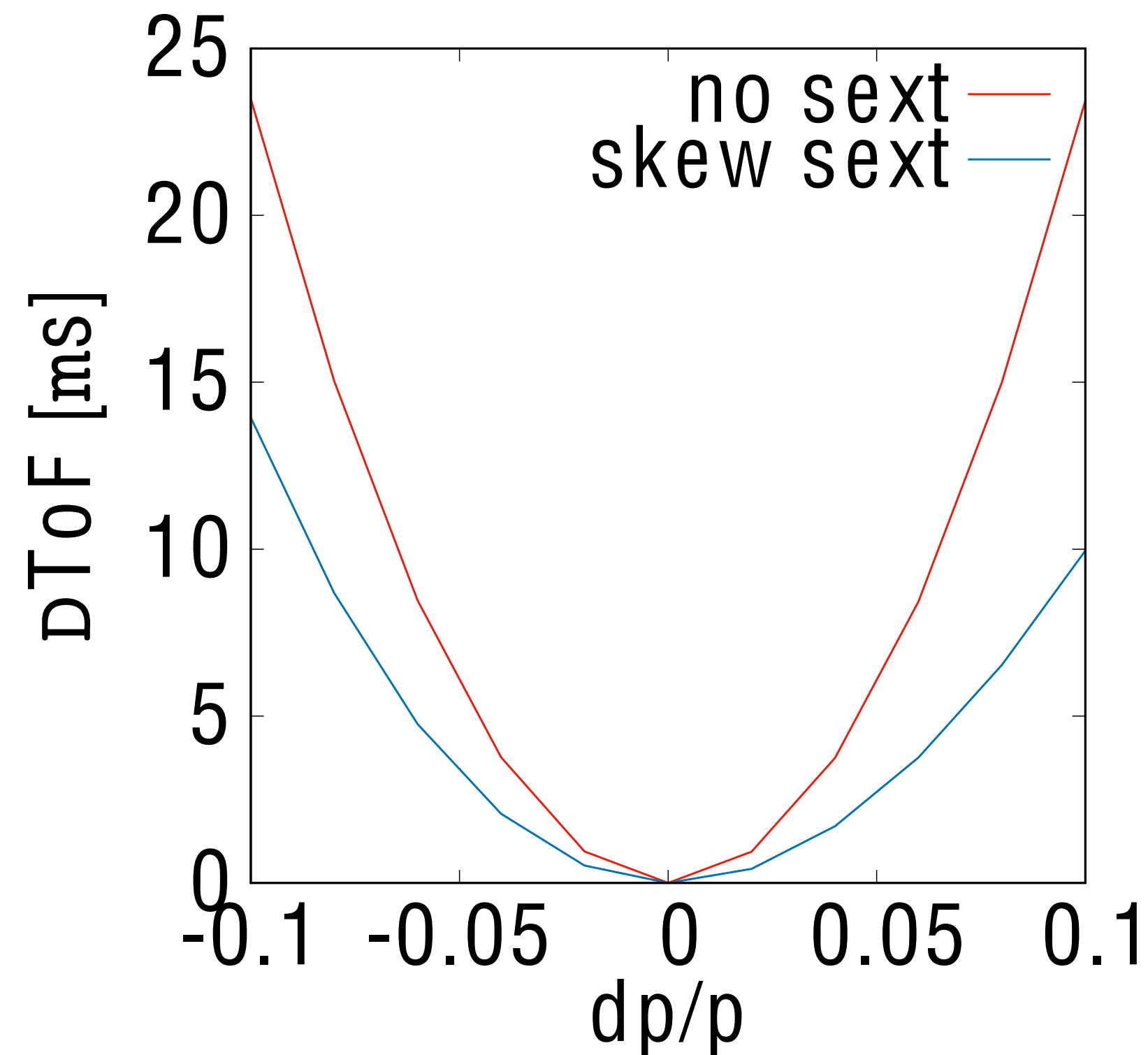


Non-planar ring



# Correction of higher order MC and chromaticity with skew sextupole

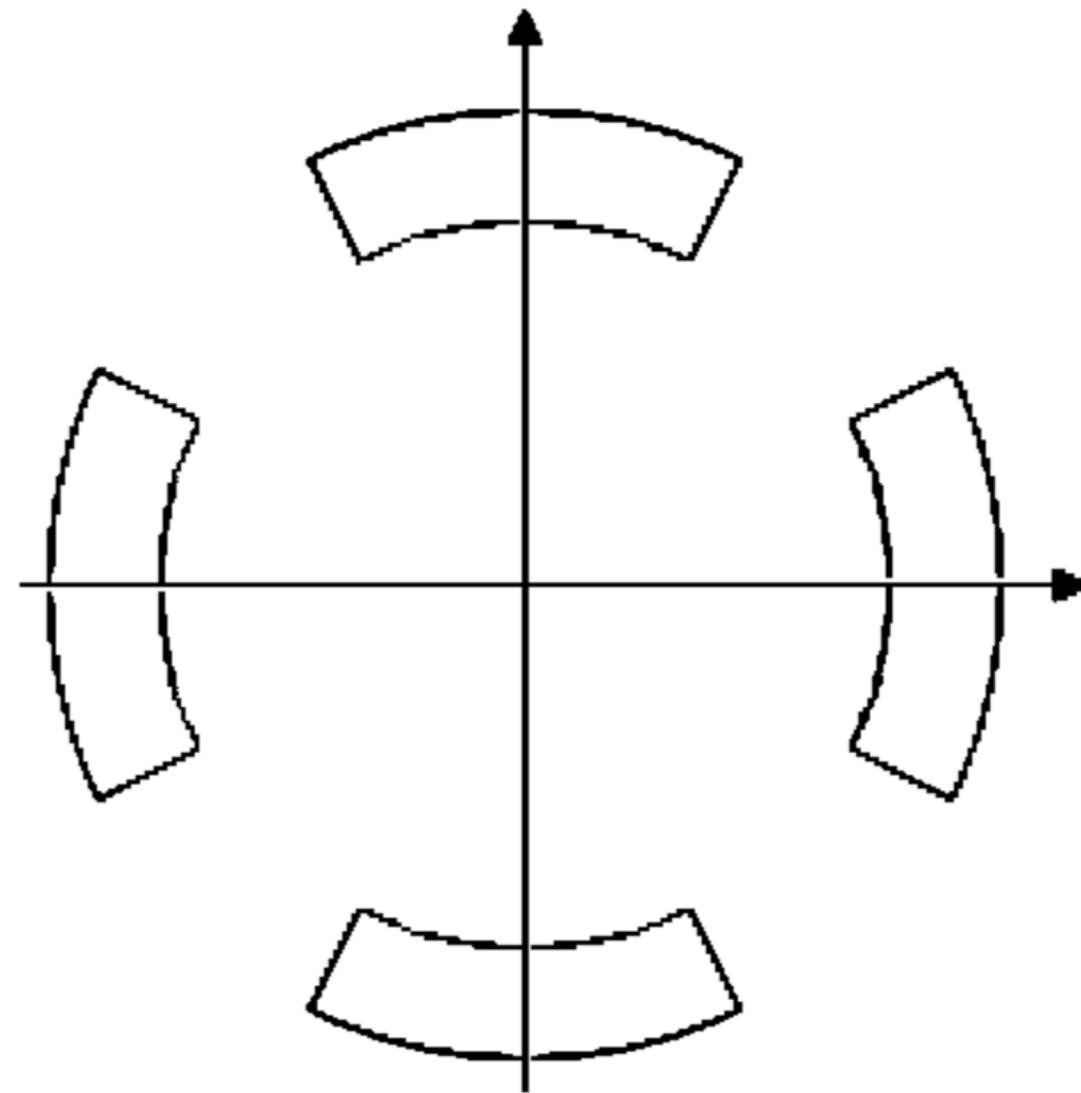
Skew sextupole control higher order momentum compaction factor and chromaticity at the same time.



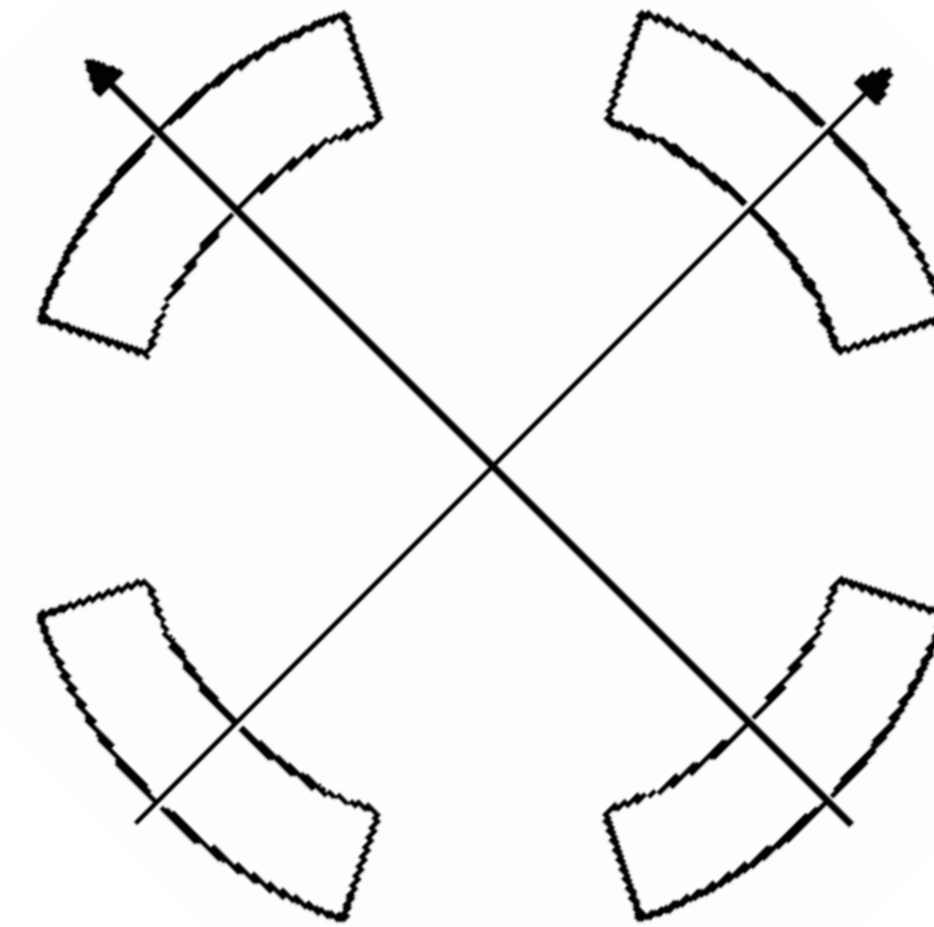


# Superconducting coil

Normal quad



Skew quad



Does the gap in horizontal plane  
(**open mid-plane**) help?

- **Full collider arc optics design with combined skew quadrupole magnets**
  - Control of the momentum compaction factor and mitigation of radiation due to neutrino decaying from muons can be achieved by a lattice whose main elements are skew quadrupoles with vertical displacement.
  - It is a promising novel design concept, but needs more investigation on the effects of nonlinear components, tuneability of the orbit and optics.
  - Low beta section is not designed which to be compatible with the arc. Simple 45 rotated low beta is a good start.
  - R&D aims for the completion of conceptual design in 2025.

# Two exotic options not discussed in MAP

Collider arc with skew quadrupoles

Vertical excursion FFA for muon acceleration

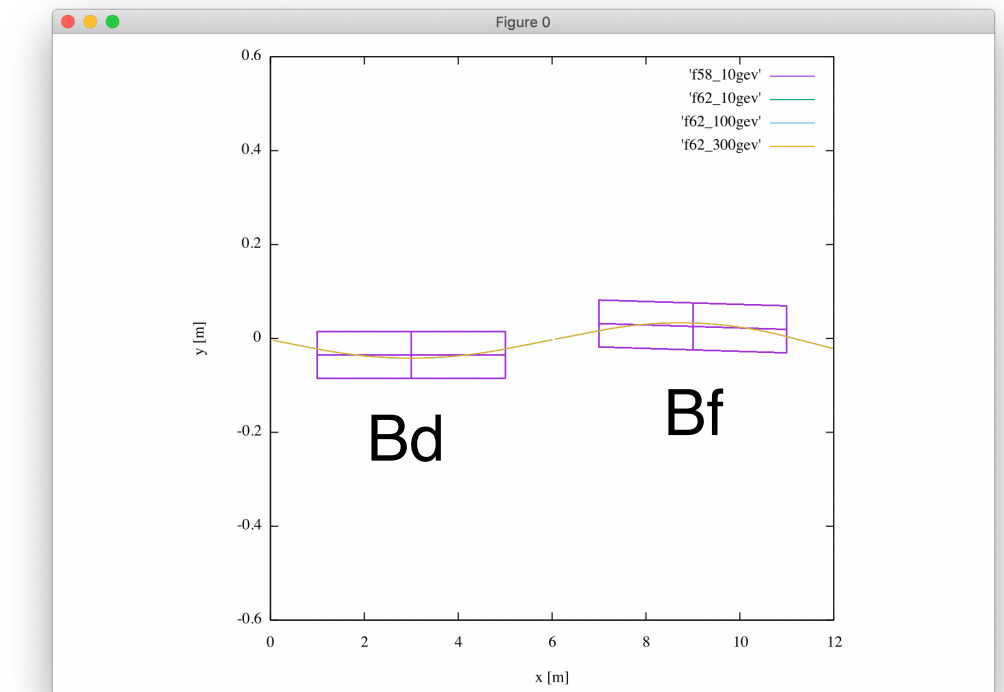
# Reasons for vertical excursion FFA (vFFA)

- **DC magnet:** no need to ramp according to the beam momentum.
- **Isochronous operation:** no need to modulate RF frequency according to the beam momentum.
- The beam orbit moves up when the beams are accelerated.

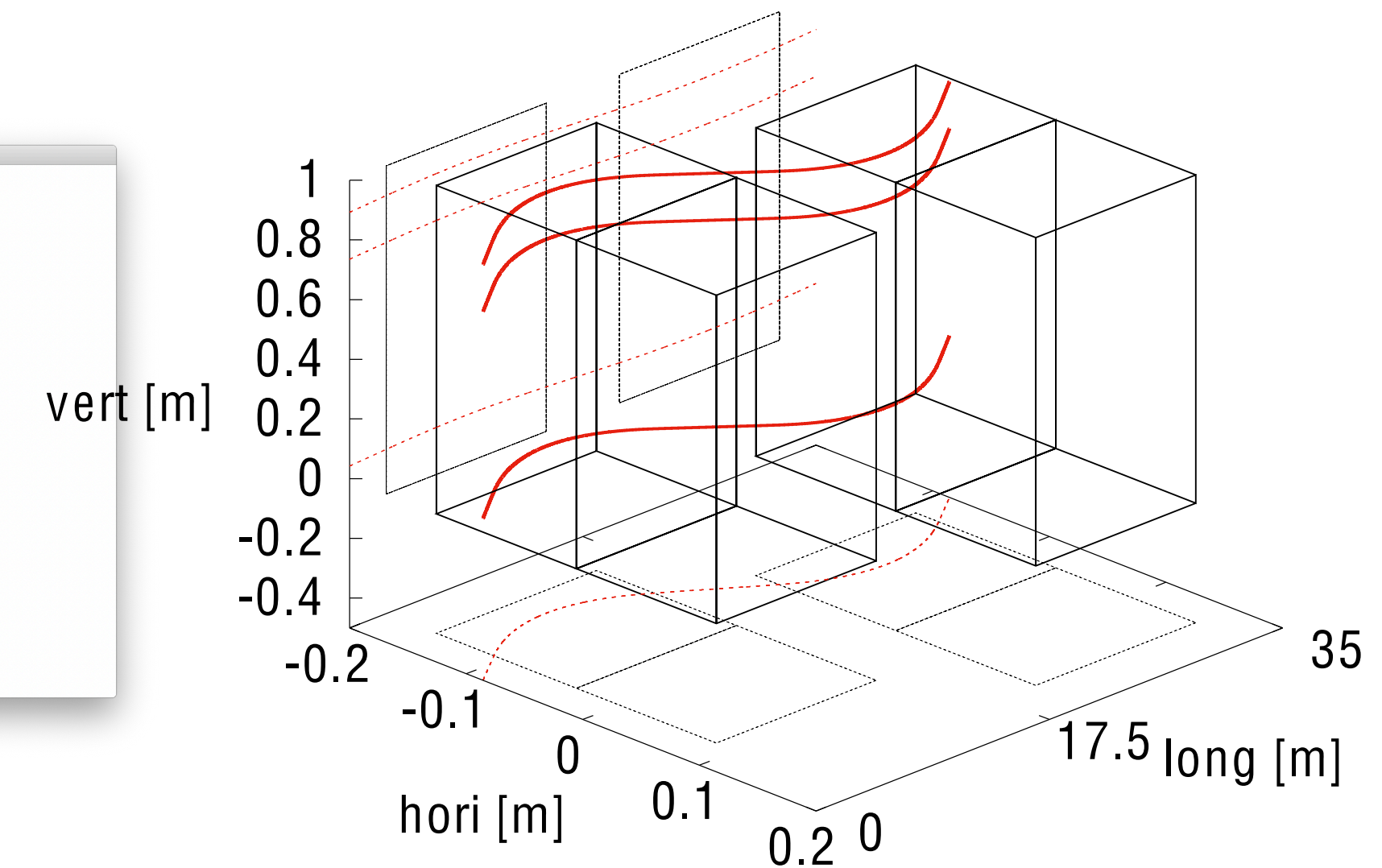
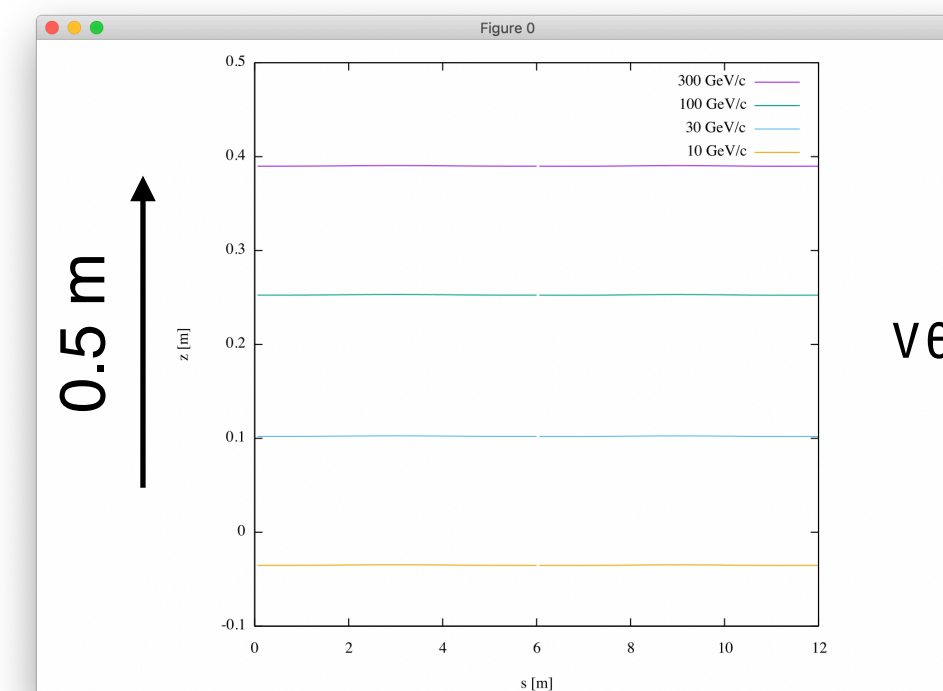
$$B = B_0 \exp(my)$$

$m$  : field index  
 $y$  : vertical

Top view

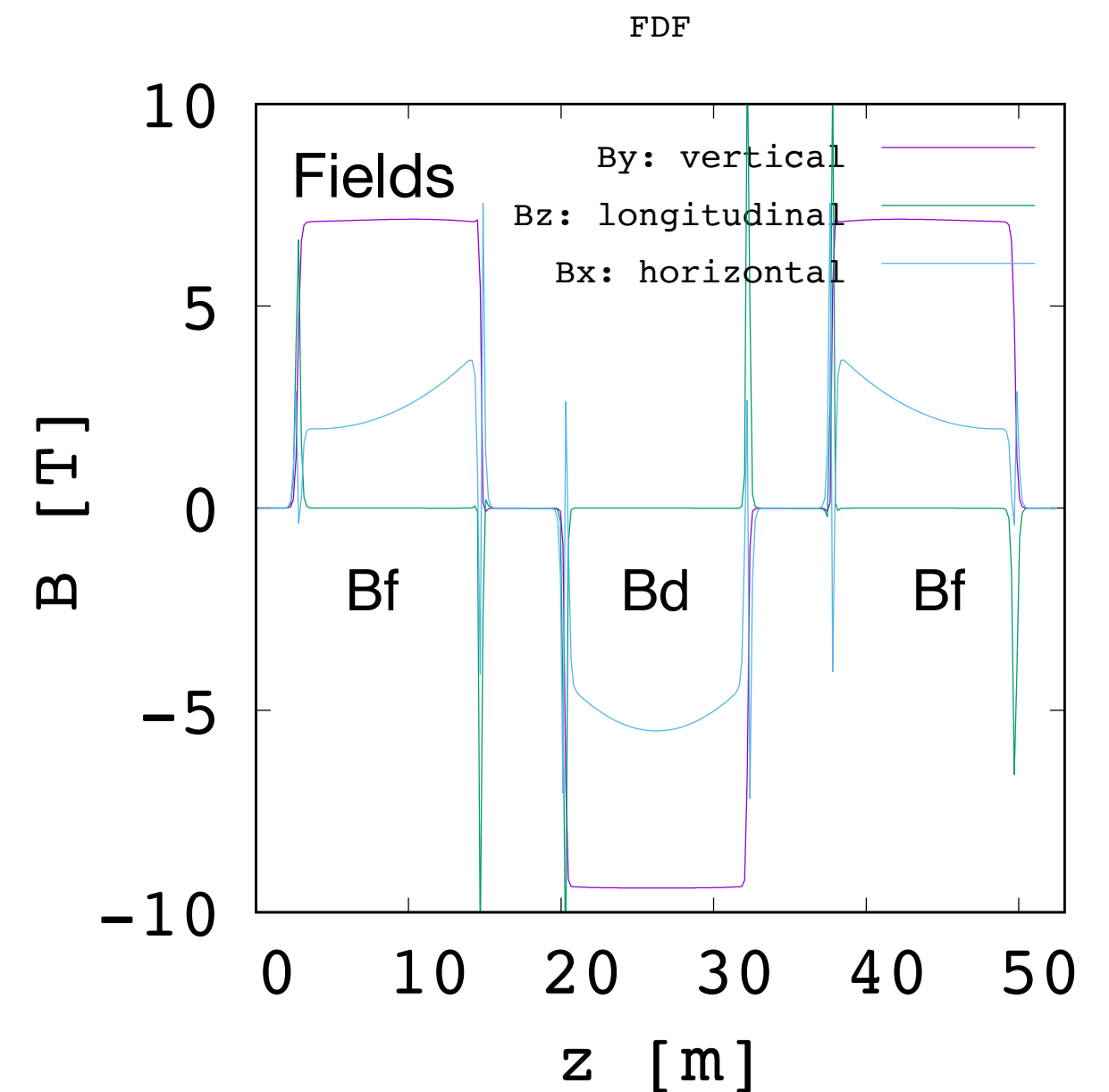
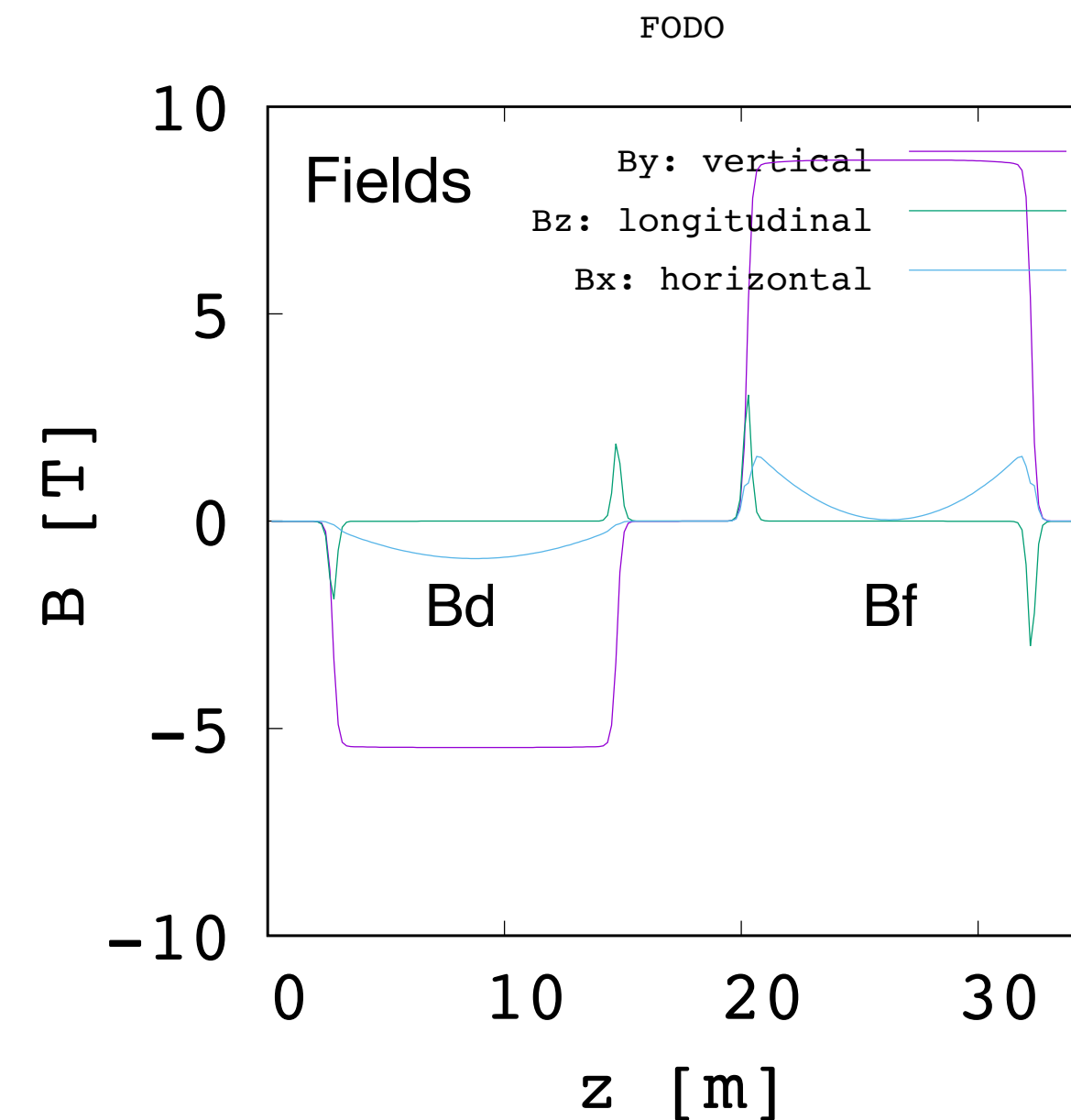
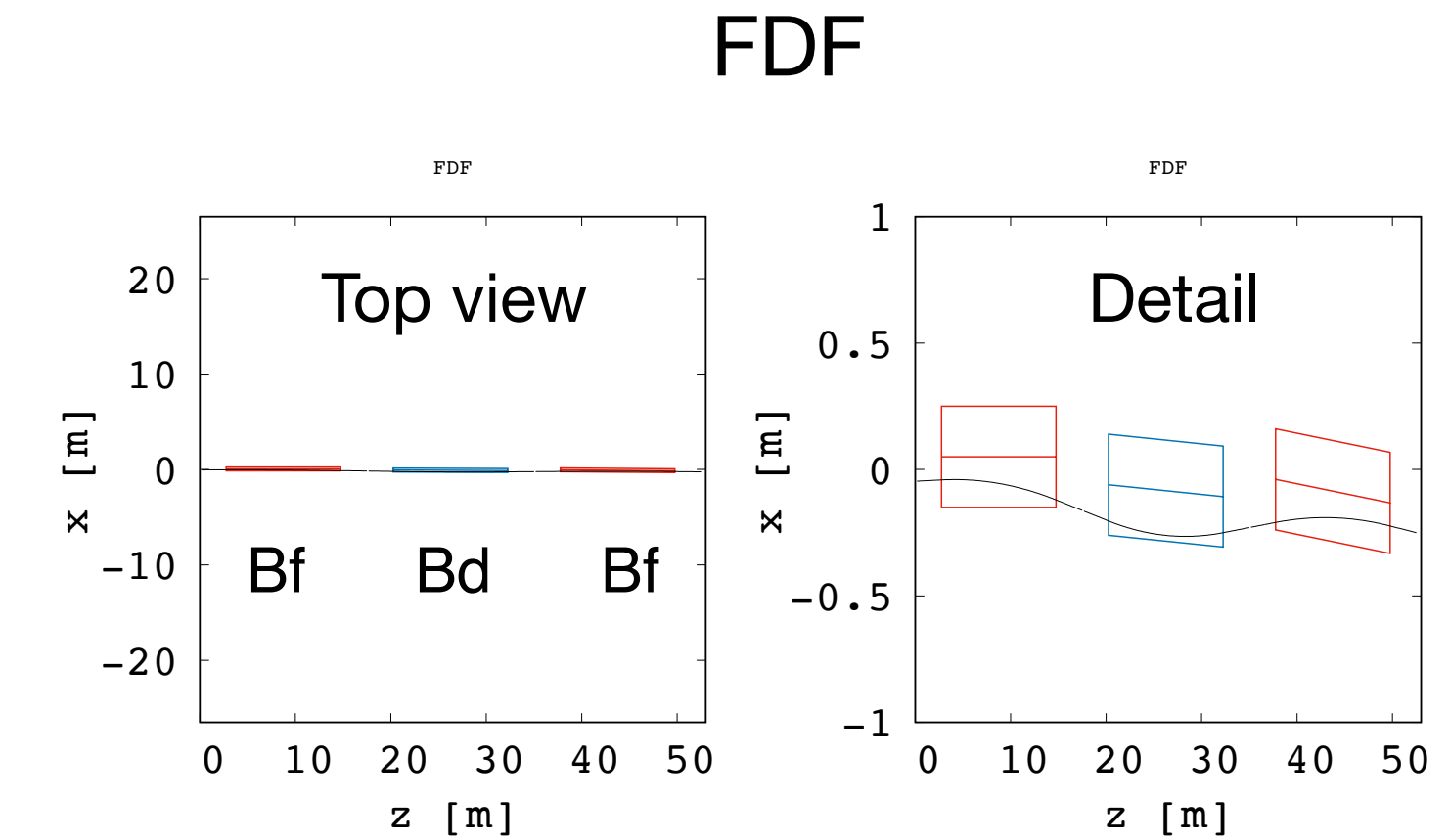
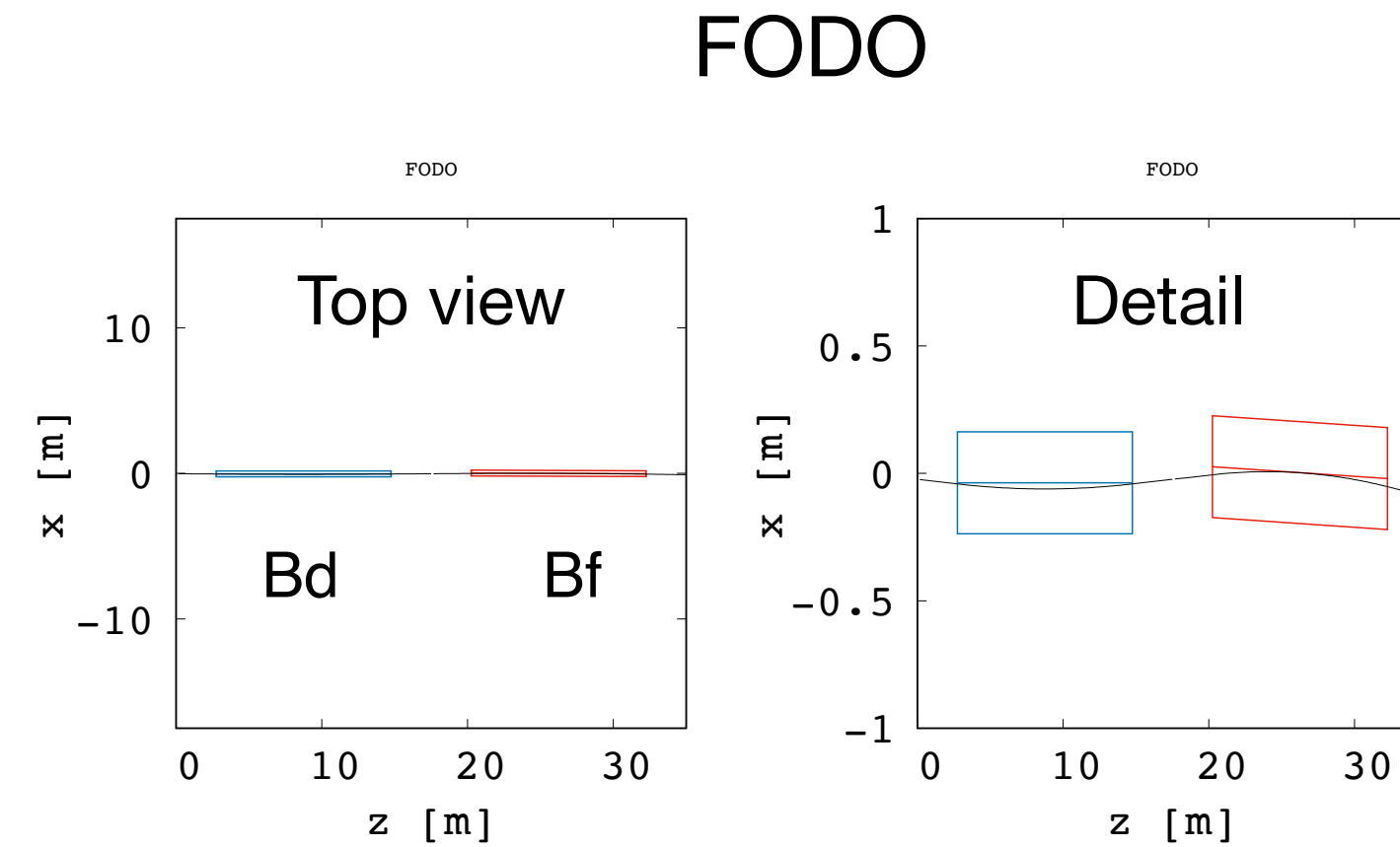


Side view



# Example: 1.5 TeV accelerator in similar size of LHC tunnel

	FODO	FDF
Energy	50 GeV to 1.5 TeV	50 GeV to 1.5 TeV
Cell length	35 m	52.5 m
Magnet length	2 x 15 m	3 x 15 m
# of cell	810	540
Maximum field	8.7 T	10.6 T
Field index m	6.8	3.0
Orbit excursion	0.50 m	1.13 m
Cell tune	0.3957 / 0.0861	0.3510 / 0.1515



- Reduction of reverse bending is one of optimisation targets.

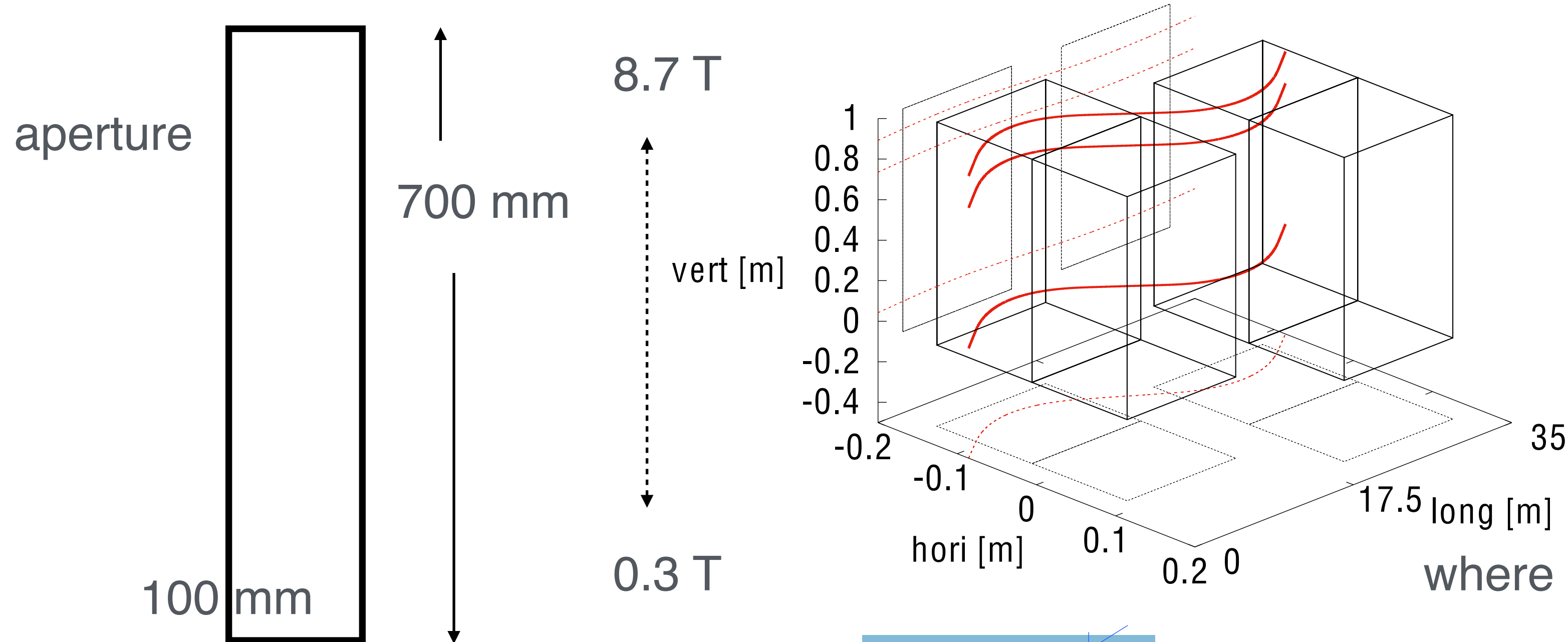


# Critical issues (1)

## orbit excursion makes a large aperture magnet.

- DC but large aperture (in vertical only) magnet.

- 3D magnetic field increase exponentially.



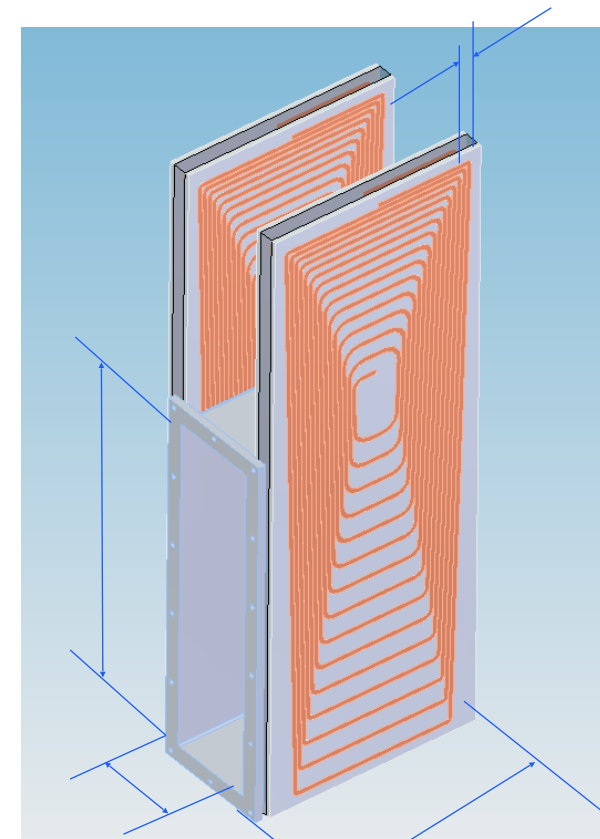
$$B_x(x, y, z) = B_0 \exp(my) \sum_{i=0}^N b_{xi}(z) x^i,$$

$$B_y(x, y, z) = B_0 \exp(my) \sum_{i=0}^N b_{yi}(z) x^i,$$

$$B_z(x, y, z) = B_0 \exp(my) \sum_{i=0}^N b_{zi}(z) x^i.$$

where  $m = (1/B) (\partial B / \partial y)$

Prototype at STFC/RAL



$$b_{x0}(z) = 0,$$

$$b_{y0}(z) = g(z),$$

$$b_{z0}(z) = \frac{1}{m} \frac{dg}{dz},$$

$$b_{x,i+1}(z) = -\frac{1}{i+1} \left( m b_{yi} + \frac{db_{zi}}{dz} \right),$$

$$b_{y,i+2}(z) = \frac{m}{i+2} b_{x,i+1},$$

$$b_{z,i+2}(z) = \frac{1}{i+2} \frac{db_{x,i+1}}{dz}.$$

# Critical issues (2)

## *reverse bend*

- The present lattice design has large fraction of reverse bending field which increases the overall circumference of the ring.
- Reduction of reverse bend is the high priority item of vFFA optics.
- e.g. **vFFA without reverse bend** magnets (replaced by edge focusing).

STEPHEN BROOKS

Phys. Rev. ST Accel. Beams 16, 084001 (2013)

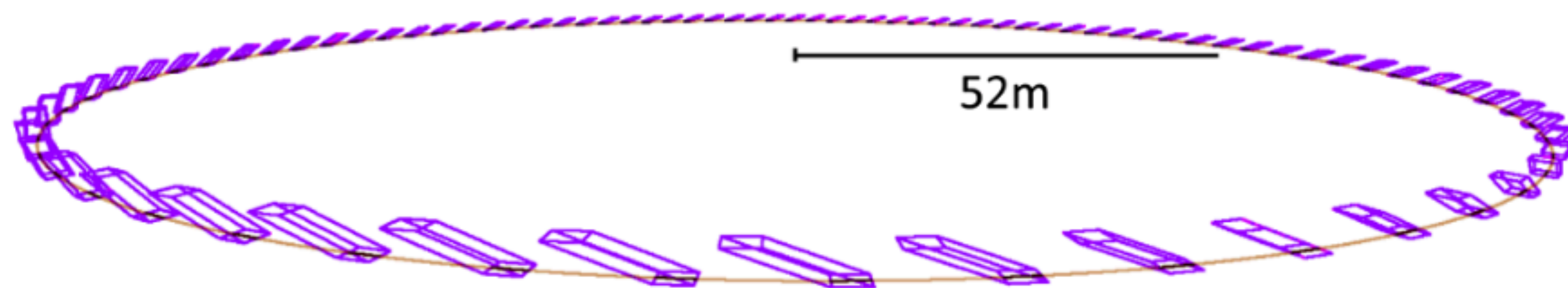
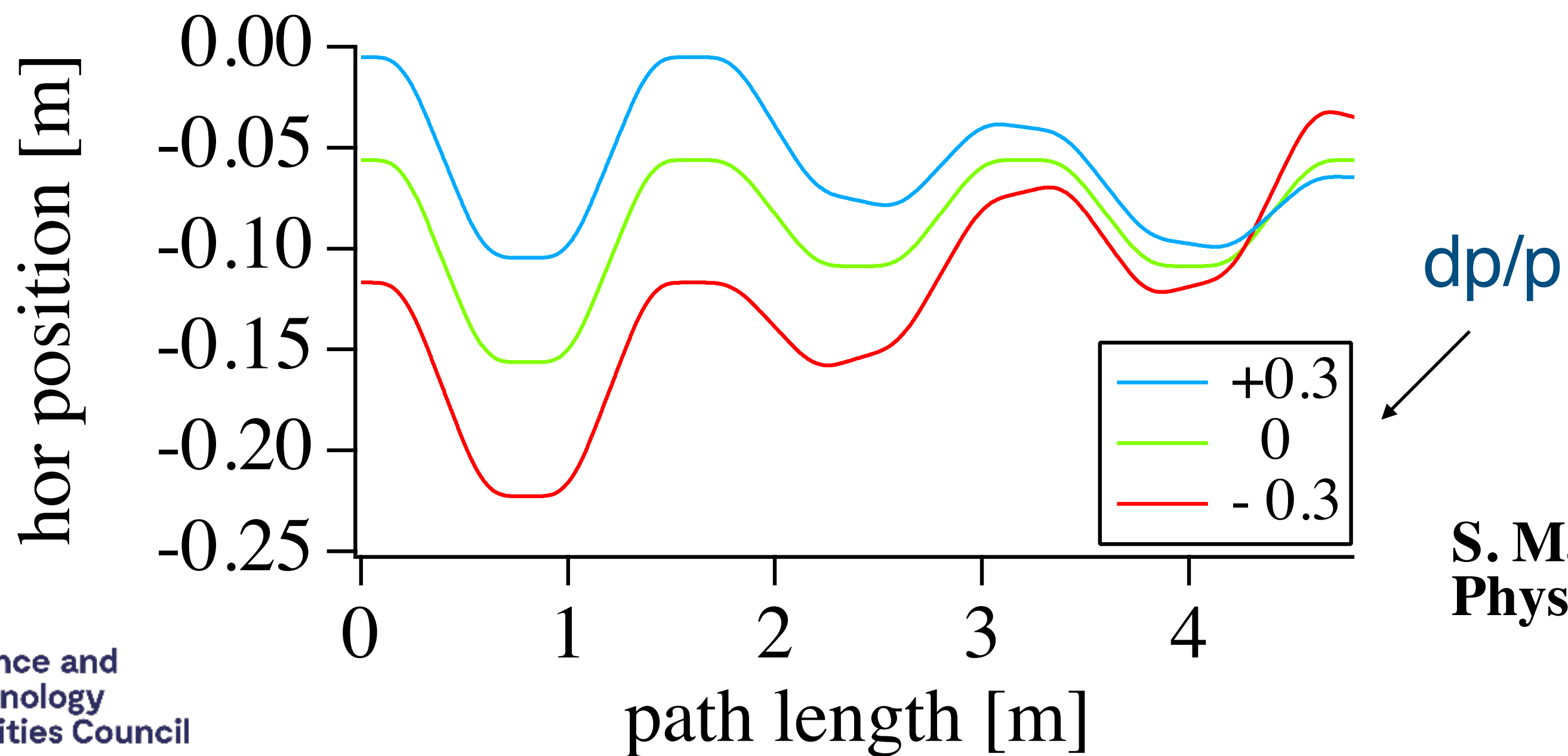


FIG. 8. Perspective view of the 12 GeV ring.

# Critical issues (3)

## *orbit excursion at RF cavity*

- High frequency RF (1.3 GHz) does not have a wide enough aperture ( $\sim 0.5$  m) to accommodate orbit excursion of high and low momentum beams.
- Conventional FFA has a way to **reduce the dispersion function locally**, which makes the orbit excursion small or zero locally.
- The same technique is under investigation in vFFA.



S. Machida, R. Fenning  
Phys. Rev. ST Accel. Beams 13, 084001

- **Optimisation of vFFA accelerator lattice**
  - DC magnet and isochronous operation are the main reasons to consider a vFFA for muon acceleration.
  - Relatively large circumference and orbit excursion of the order of  $\sim$  meter are two main issues of the vFFA based muon accelerator.
  - Reverse bend could be reduced or eliminated, for example, by edge focusing. Dispersion suppressor could be considered which was successfully designed in a conventional horizontal excursion FFA.
  - R&D aims for the completion of conceptual design in 2025.

# Thank you for your attention



# Summary

	Normal FODO		Skew FODO	
Energy	1.5 TeV	1.5 TeV	1.5 TeV	1.5 TeV
<b>Momentum compaction</b>	4.32 x 10 <sup>-4</sup>	→ 0	4.32 x 10 <sup>-4</sup>	→ 0
<b>Circumference</b>	2880 m	→ 6080 m	2880 m	→ 6080 m
Cell length	16 m	16 m	16 m	16 m
Magnet length	2 x 6.4 m	2 x 6.4 m	2 x 6.4 m	2 x 6.4 m
# of cell	180	380	180	380
<b>Maximum field</b>	14 T	→ 20 T	14 T	→ 14 T
Field gradient	240 T/m	240 T/m	240 T/m	240 T/m
Cell tune	0.3119 / 0.3119	0.3131 / 0.3131	0.3119 / 0.3119	0.3131 / 0.3131

Low beta insertion will be with skew quadrupoles even with up and down straight section.