about χ (2)

Two possible estimators:

1) optimise linearity: best accuracy but not best sensitivity

2) optimise resolution: best sensitivity but not best accuracy

Accuracy (also trueness): how much the mean value is close to true value

Sensitivity (also precision or resolution): how much the measurements are close to the mean value

I don't see any way to guarantee you get both at same time

possible χ estimations

Possible estimations of χ

1) Optimise accuracy: $\left\langle \frac{S-\chi\cdot C}{1-\chi} \right\rangle = E \quad \Leftrightarrow \quad \chi = \frac{E-\langle S\rangle}{E-\langle C\rangle} = \frac{\langle E-S\rangle}{\langle E-C\rangle}$

2) Optimise sensitivity (minimise variance):

$$\sigma_{DR}^2 = \langle E_{DR}^2 \rangle - \langle E_{DR} \rangle^2 = \left\langle \left(\frac{S-\chi\cdot C}{1-\chi} \right)^2 \right\rangle - \left\langle \frac{S-\chi\cdot C}{1-\chi} \right\rangle^2$$

$$= \frac{\sigma_S^2 - 2\cdot\chi\cdot cov(S,C) + \chi^2\cdot\sigma_C^2}{(1-\chi)^2}$$

The first one contains E, the second one nothing knows about E ...

continuing ...

possible χ estimations (2)

Requiring $\frac{d}{d\chi}\sigma_{DR}^2 = 0$ and defining $\rho_{S,C} \equiv corr(S,C) \equiv \frac{cov(S,C)}{\sigma_S \cdot \sigma_C}$

you obtain: $\chi_{min} = \frac{\frac{\sigma_S}{\sigma_C} \cdot \left(\rho_{S,C} - \frac{\sigma_S}{\sigma_C}\right)}{1 - \frac{\sigma_S}{\sigma_C} \cdot \rho_{S,C}}$, positive for $\frac{\sigma_S}{\sigma_C} < \rho_{S,C} < \frac{\sigma_C}{\sigma_S}$

Hard to believe may knows something about method-1 result!

Resolution becomes: $\frac{1}{\sigma_{DR\,min}^2} = \left(\frac{1}{\sigma_S^2} - 2 \cdot \frac{\rho_{S,C}}{\sigma_S \cdot \sigma_C} + \frac{1}{\sigma_C^2}\right) \cdot \frac{1}{1 - \rho_{S,C}^2}$

making explicit the limit for *em* showers (for which $corr(S,C) = 0$):

$$\lim_{\rho_{S,C} \to 0} \chi_{min} \to -\frac{\sigma_S^2}{\sigma_C^2} \quad , \quad \frac{1}{\sigma_{DR\,min}^2} \to \frac{1}{\sigma_S^2} + \frac{1}{\sigma_C^2}$$

other considerations

Other limits:

$$\lim_{\rho_{S,C} \to \pm 1} \quad \chi_{min} \to \pm \frac{\sigma_S}{\sigma_C} \quad , \quad \sigma_{DR\,min} \to 0$$

Other expression for $\sigma_{DR\,min}$:

$$\sigma^2_{DR\,min} = \frac{\sigma^2_S - \chi^2_{min} \cdot \sigma^2_C}{1 - \chi^2_{min}}$$

Covariance:

$$cov(S, C) = \frac{\sigma^2_S + \chi_{min} \cdot \sigma^2_C}{1 + \chi_{min}}$$

my present understanding about χ

1) optimising resolution has many drawbacks for getting X value:

→ it may even become negative

→ it should depend on E and $<f_{em}>$

→ it may also depend on particle type

→ it may give unpredictable results

2) only sensible way:

$X = <E-S> / <E-C>$ → matches definition = $(1-h/e_S)/(1-h/e_C)$

→ it should be stable

→ it should be independent of E and $<f_{em}>$

→ it should be independent of particle type

••• TO BE ASSESSED •••