Gravitational Waves from the Early Universe

Lecture 1B: Gravitational Waves, Experiments

Kai Schmitz (CERN)

Chung-Ang University, Seoul, South Korea | June 2-4



GE0600 IGO Hanford LIGO Livingston AGRA Operational Planned **Gravitational Wave Observatories** 

[ligo.caltech.edu]

Ground-based GW laser interferometers:

Generation 1: GEO600, LIGO, TAMA, Virgo Generation 2: Advanced LIGO (Laser Interferometer Gravitational-Wave Observatory) / Virgo Generation 2.5: KAGRA (Kamioka Gravitational Wave Detector), underground and cryogenic

#### On the eve of multifrequency GW astronomy



Plus: AEDGE, AION, MAGIS, TaiJi, TianQin, ... Plus: Future measurements of CMB polarization and spectral distortions

## **Outline Lecture 1B**

- 1. GW interferometers
- 2. Experimental sensitivity
- 3. Pulsar timing arrays
- 4. Cosmic microwave background
- 5. Summary

## Transient GW signals

## GW passing a Michelson interferometer



[Ballmer, Mandic: Ann. Rev. Nucl. Part. Sci. 65 (2015) 555]

## Chirp signal in the LIGO / Virgo detectors



[ligo.caltech.edu]

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#### [ligo.caltech.edu

0 0 0 0 0

Signal seen by the detector: Convolute  $h_{ij}$  with impulse response  $R^{ij}$  (detector geometry)

$$s(t) = \int_{-\infty}^{\infty} dt' \int d^3 x' R^{ij}(t', x') h_{ij}(t - t', x - x')$$
(1)

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Decompose incoming GW into plane-wave contributions with definite f, p, and n:

$$h_{ij}(t, \boldsymbol{x}) = \sum_{p=+,\times} \int_{-\infty}^{\infty} df \int d^2 \boldsymbol{n} \, h_p(f, \boldsymbol{n}) \, e_{ij}^p(\boldsymbol{n}) \, e^{2\pi i f(t - \boldsymbol{n} \boldsymbol{x})} \tag{2}$$

[ligo.caltech.edu

## Detector response to the signal

Signal seen by the detector in the frequency domain:

$$\tilde{s}(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \, s(t) \, e^{-2\pi i f t} = \sum_{p=+,\times} \int d^2 n \, R_p(f, n) \, h_p(f, n) \tag{3}$$

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[Cornish, Romano: 1608.06889]

Antenna patterns: Graphs of  $|R_p(f, n)|$  as functions n at fixed f.

Can be computed based on changes in the light-travel time between test masses at the end of the interferometer arms.

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Signal response / detector transfer function: Average over the square of the antenna patterns.

$$\mathcal{R} = rac{1}{2} \sum_{p} rac{1}{4\pi} \int d^2 n \; |R_p(f, \boldsymbol{n})|^2$$
 (4)

Quantifies loss in sensitivity due to the fact that, on average, GWs do not arrive from the optimal direction.

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Challenge: Signal looks like another form of noise. Therefore, extract SGWB signal from the noisy background based on: spectral properties, temporal modulations, null channels, etc.



0 0 0 0

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Single detector: Require  $S_h \gtrsim D_{\text{noise}}/\mathcal{R}$  for detection. Detector network: Cross-correlate signal from detector pairs.

$$S_{IJ} = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' d_I(t) Q_{IJ}(t-t') d_J(t') \quad (7)$$

with filter function  $Q_{IJ}$  (depends only on t - t', highly localized in time). Match  $Q_{IJ}$  so as to maximize the SNR.

Expectation value of  $S_{IJ}$ , assuming uncorrelated detector noise,  $\langle n_I n_J \rangle = 0$ :

$$\langle S_{IJ} 
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0 0 0 0 0

(8)

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Fourier-transformed filter  $\widetilde{Q}_{IJ}$  and overlap reduction function  $\Gamma_{IJ}$  (generalization of  $\mathcal{R}$ ):

$$\Gamma_{IJ}(f) = \frac{1}{2} \sum_{p} \frac{1}{4\pi} \int d^2 \boldsymbol{n} \, R_p^I(f, \boldsymbol{n}) \, R_p^{J*}(f, \boldsymbol{n}) \tag{9}$$

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Root mean square of the noise  $N_{IJ} = S_{IJ} - \langle S_{IJ} \rangle$  (in the weak-signal approximation):

$$\langle N_{IJ}^2 \rangle^{1/2} = \left[ \langle S_{IJ}^2 \rangle - \langle S_{IJ} \rangle^2 \right]^{1/2} = \left[ \frac{T}{4} \int_{-\infty}^{\infty} df \left| \widetilde{Q}_{IJ}(f) \right|^2 D_{\text{noise}}^I(f) D_{\text{noise}}^J(f) \right]^{1/2}$$
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Optimal filter that maximizes the signal-to-noise ratio  $\varrho_{IJ} = \langle S_{IJ} \rangle / \langle N_{IJ}^2 \rangle^{1/2}$ :

$$\widetilde{Q}_{IJ}(f) \propto \frac{\Gamma_{IJ}(f) S_h(f)}{D^I_{\text{noise}}(f) D^J_{\text{noise}}(f)}$$
(11)

Note:  $\widetilde{Q}_{IJ}$  requires knowledge of the signal one intends to measure o template banks

#### Overlap reduction functions



Normalization such that  $\gamma_{IJ}(f=0) = 1$  for a pair of identical, co-located, co-aligned detectors with opening angle  $\delta$  between their two interferometer arms:

$$\gamma_{IJ}(f) = \frac{5}{\sin^2 \delta} \, \Gamma_{IJ} \tag{12}$$

For a network of  $I, J = 1, \cdots, N_{det}$  detectors:

$$\varrho = \left[\sum_{J>I} \varrho_{IJ}^2\right]^{1/2}, \qquad \varrho_{IJ} = \left[2T \int_{\Delta f} df \frac{\Gamma_{IJ}^2(f) S_h^2(f)}{D_{\text{noise}}^I(f) D_{\text{noise}}^J(f)}\right]^{1/2} \tag{13}$$

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Effective strain noise power spectrum  $S^{\rm eff}_{\rm noise}$  for the entire network:

$$S_{\text{noise}}^{\text{eff}}(f) = \left[\sum_{J>I} \frac{\Gamma_{IJ}^2(f)}{D_{\text{noise}}^I(f) D_{\text{noise}}^J(f)}\right]^{-1}$$

0 0 0 0 0

(14)

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Express both signal and noise in terms of a GW energy density power spectrum:

$$\Omega_{\text{signal}}(f) = \frac{2\pi^2}{3H_0^2} f^3 S_h(f) \,, \quad \Omega_{\text{noise}}(f) = \frac{2\pi^2}{3H_0^2} f^3 S_{\text{noise}}^{\text{eff}}(f) \,. \tag{15}$$

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$$\left| \varrho = \left[ 2 T \int_{\Delta f} df \left( \frac{\Omega_{\text{signal}}(f)}{\Omega_{\text{noise}}(f)} \right)^2 \right]^{1/2} \propto \sqrt{N_{\text{det}}(N_{\text{det}} - 1) N_{\text{bin}} T \,\delta f} \right|$$
(16)

Integration over time and frequency boosts SNR by many orders of magnitude!

0 0 0 0 0

(14)

#### Effective strain noise power spectra



Instantaneous sensitivity, no integration over time and frequency

## Sensitivity curves<sup>1</sup> for the two-detector network aLIGO Hanford + aLIGO Livingston:<sup>2</sup>



[Romano, Thrane: 1310.5300

 $^1\text{All}$  curves expressed in terms of  $\Omega$ : the GW energy density spectrum in units of the critical energy density  $\rho_c.$   $^2\text{Based on slightly obsolete data for <math display="inline">D_{\text{noise}}$ : https://dcc.ligo.org/LIGO-T0900288/public

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Black curve: Detector noise spectrum

 $D_{\text{noise}}(f)$ 

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Red curve: Effective strain noise

 $S_{\text{noise}}^{\text{eff}}(f) = D_{\text{noise}}(f) / |\Gamma(f)|$ 

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Red curve: Effective strain noise  $S^{\rm eff}_{\rm noise}(f) = D_{\rm noise}(f)/\left|\Gamma(f)\right|$ 

Green curve: Rescaled effective strain noise

 $S_{\text{noise}}^{\text{eff}}(f)/\sqrt{N_{\text{det}}(N_{\text{det}}-1) T \,\delta f}$ 

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## Power-law-integrated sensitivity curves



Sensitivity integrated over time and frequency, starting point of phenomenological studies

Use an array of pulsars across the Milky Way to construct a galaxy-sized GW detector!<sup>3</sup>



B. Saxton for nrao.edul



<sup>3</sup>First *indirect* detection of GWs from the orbital decay of the Hulse–Taylor binary (pulsar + neutron star).

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Pulsars: Highly magnetized rotating dead stars (usually neutron stars but also white dwarfs)

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0 0 0 0

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- $\,\circ\,$  Beamed radio pulses emitted from magnetic poles  $\rightarrow$  cosmic lighthouses

Ultra-precise clocks in the sky! Look for tiny distortions caused by nanohertz GWs.

<sup>3</sup>First *indirect* detection of GWs from the orbital decay of the Hulse–Taylor binary (pulsar + neutron star).

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Residuals in pulse times of arrival (TOAs):

$$R^{(i)} = \mathrm{TOA}_{\mathrm{SSB}}^{(i)} - \mathrm{TOA}_{\mathrm{Model}}^{(i)}$$

- Measure TOAs on earth and convert to TOAs at the solar-system barycenter (SSB)
- Compare to timing models for each pulsar (frequency and derivatives, position, proper motion, binary dynamics, relativistic effects, ...)

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Sensitive to GWs in the frequency range: from  $f_{\min} \sim 1/T$  to  $f_{\max} \sim 1/\Delta T$ Existing PTA Collaborations:



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Integrate metric perturbation along the geodesic of the pulse  $\rightarrow$  shift in pulsation frequency:

$$\frac{\Delta\nu(t)}{\nu} = -H^{ij} \left[ h_{ij}(t, \boldsymbol{x}_e) - h_{ij}(t - D/c, \boldsymbol{x}_p) \right], \qquad R(t) = \int_0^t dt' \, \frac{\Delta\nu(t')}{\nu} \tag{17}$$

Two contributions: earth term, pulsar term ( $H^{ij}$ : geometrical factor; D: pulsar distance)

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Louise Mayor for physicsworld.com

Cross-correlate the timing residuals of a pair of pulsars that is separated by an angle  $\psi$  in the sky:

$$\langle R_I R_J \rangle \propto \Gamma_{IJ}(\psi) = \frac{\zeta_{IJ}(\psi)}{12\pi^2 f^2}$$
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Earth term results in characteristic correlation:

$$\zeta_{IJ}(\psi) = \frac{1}{2} \left[ \delta_{IJ} + 1 + c_{\psi} \left( 3 \ln c_{\psi} - \frac{1}{2} \right) \right]$$
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with  $c_{\psi} = (1 - \cos \psi) / 2$ . Hellings–Downs curve! [Hellings, Downs: Astrophys. J. 265 (1983) L39]]

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Hallmark signature of a SGWB signal: Quadrupole correlation among timing residuals. Other systematic effects typically lead to monopole or dipole correlations (see Lecture 3B).

#### Imprint of primordial GWs on the CMB



[National Astronomical Observatory of Japan, gwpo.nao.ac.jp]

Probe GWs with oscillation periods of billions of years! (Inflation, topological defects, ...)

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Temperature anisotropies 
 Polarization anisotropies 
 Spectral distortions



CMB: Baby picture of the early Universe, surface of last scattering after recombination

4. Cosmic microwave background

18/21



CMB: Baby picture of the early Universe, surface of last scattering after recombination Highly isotropic, temperature anisotropies:

$$\Theta = \frac{\Delta T}{\bar{T}} \sim 10^{-5}, \qquad \bar{T} \simeq 2.725 \,\mathrm{K} \quad (20)$$



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Expand in spherical harmonics:

$$\Theta(n) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} \Theta_{\ell m} Y_{\ell m}(n) \qquad (21)$$





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Tensor perturbations induce  $C_{\ell}^{TT}$  through the Sachs–Wolfe effect (gravitational redshift):

$$\Theta = -\int_{\text{CMB}}^{\text{today}} d\lambda \, h'_{ij}(\eta, x) \, n^i n^j$$
 (23)



Tensor perturbations  $\rightarrow$  quadrupole temperature anisotropy  $\rightarrow$  Thomson scattering results in linear CMB polarization



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Kamionkowski, Kovetz: Annual Review of Astronomy and Astrophysics 54 (2016) 227



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Strength of tensor perturbations on CMB scales in terms of tensor-to-scalar ratio r:

$$r = \frac{\mathcal{P}_h(k_*)}{\mathcal{P}_{\mathcal{R}}(k_*)}, \qquad \langle \mathcal{R}\mathcal{R}^* \rangle = \frac{2\pi^2}{k^3} \,\delta^{(3)} \,\mathcal{P}_{\mathcal{R}}(k), \qquad \langle h_{ij}h_{ij}^* \rangle = \frac{2\pi^2}{k^3} \,\delta^{(3)} \,\mathcal{P}_h(k) \tag{25}$$

Best limit to date (PLANCK and BICEP/Keck):  $r(k_* = 0.05 \,\mathrm{Mpc}^{-1}) < 0.044 \,\mathrm{at} \, 95 \%$  C.L.

## Spectral distortions



## CMB: Best blackbody spectrum in nature

#### Spectral distortions



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Tensor perturbations dissipate; energy transfer to photons at  $z \lesssim 2 \times 10^6$  when  $n_{\gamma}$  constant leads to  $\mu$ -distortion: Bose–Einstein distribution with  $\mu \neq 0$ :

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# Future CMB spectrometers:

Bridge the gap between cosmological and astrophysical GW searches!

 $f \sim 10^{-15} \cdots 10^{-9} \,\mathrm{Hz}$ 

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[Kite, Ravenni, Patil, Chluba: 2010.00040]



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End of Lecture 1B. Thanks a lot for your attention!