Gravitational Waves from the Early Universe

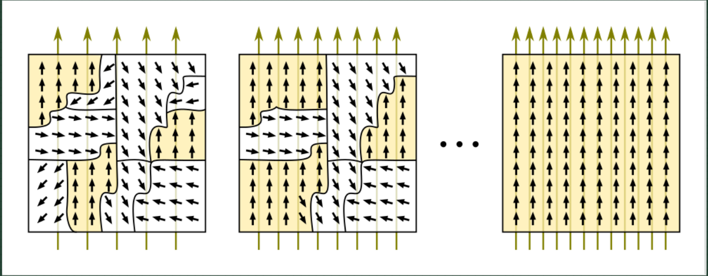
Lecture 3A: Cosmic Defects

Kai Schmitz (CERN)

Chung-Ang University, Seoul, South Korea | June 2 – 4

Magnetic domains in a ferromagnet

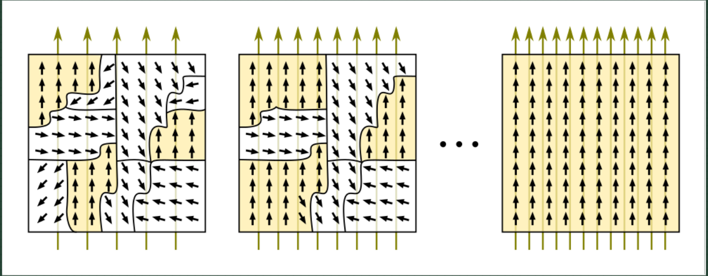
[wikimedia.org]



Magnetization in a ferromagnet:

Magnetic domains in a ferromagnet

[wikimedia.org]

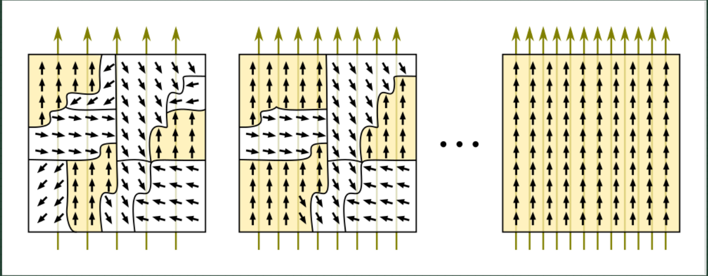


Magnetization in a ferromagnet:

- Phase transition at the Curie temperature: paramagnet $\rightarrow$ ferromagnet

Magnetic domains in a ferromagnet

[wikimedia.org]

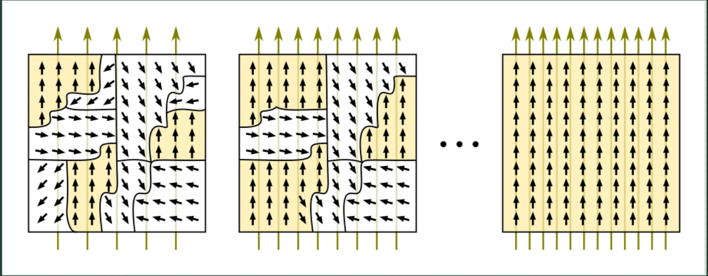


Magnetization in a ferromagnet:

- Phase transition at the Curie temperature: paramagnet $\rightarrow$ ferromagnet
- Magnetic dipoles align spontaneously due to exchange interaction

Magnetic domains in a ferromagnet

[wikimedia.org]

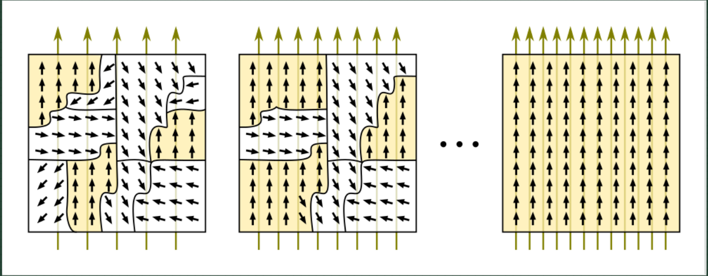


Magnetization in a ferromagnet:

- Phase transition at the Curie temperature: paramagnet $\rightarrow$ ferromagnet
- Magnetic dipoles align spontaneously due to exchange interaction
- Translation and rotation invariance spontaneously broken

Magnetic domains in a ferromagnet

[wikimedia.org]

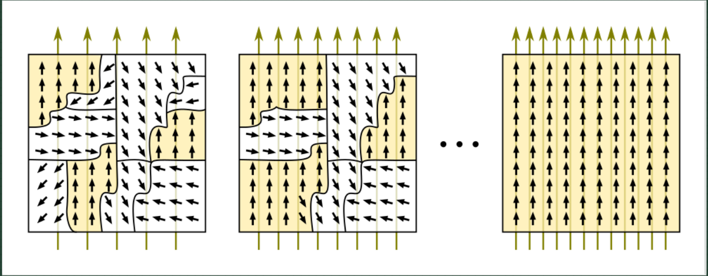


Magnetization in a ferromagnet:

- Phase transition at the Curie temperature: paramagnet $\rightarrow$ ferromagnet
- Magnetic dipoles align spontaneously due to exchange interaction
- Translation and rotation invariance spontaneously broken
- Magnetic domains, regions of uniform magnetization, separated by domain walls

Magnetic domains in a ferromagnet

[wikimedia.org]

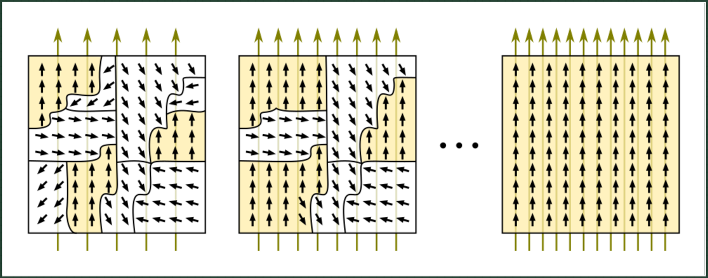


Magnetization in a ferromagnet:

- Phase transition at the Curie temperature: paramagnet $\rightarrow$ ferromagnet
- Magnetic dipoles align spontaneously due to exchange interaction
- Translation and rotation invariance spontaneously broken
- Magnetic domains, regions of uniform magnetization, separated by domain walls
- Domain walls are stable, unless an external force (magnetic field) is applied

Magnetic domains in a ferromagnet

[wikimedia.org]

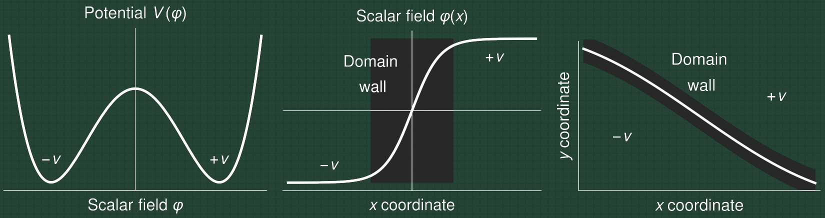


Magnetization in a ferromagnet:

- Phase transition at the Curie temperature: paramagnet $\rightarrow$ ferromagnet
- Magnetic dipoles align spontaneously due to exchange interaction
- Translation and rotation invariance spontaneously broken
- Magnetic domains, regions of uniform magnetization, separated by domain walls
- Domain walls are stable, unless an external force (magnetic field) is applied

Similar phenomenology after phase transitions in the early Universe!

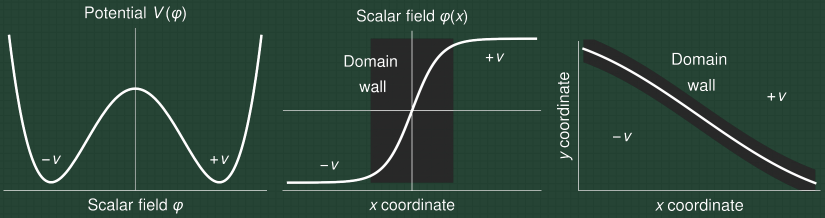
Domain walls in cosmology



Spontaneous breaking of a global $\mathbb{Z}_2$ symmetry by a real scalar field φ:

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - v^2\right)^2 \tag{1}$$

Domain walls in cosmology

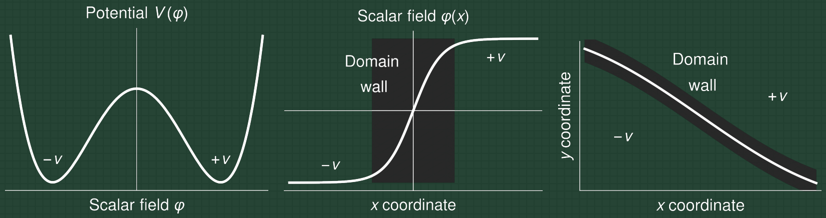


Spontaneous breaking of a global $\mathbb{Z}_2$ symmetry by a real scalar field φ:

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - v^2\right)^2 \tag{1}$$

- $\langle\varphi\rangle = \pm v$ randomly distributed on length scales larger than the causal horizon

Domain walls in cosmology

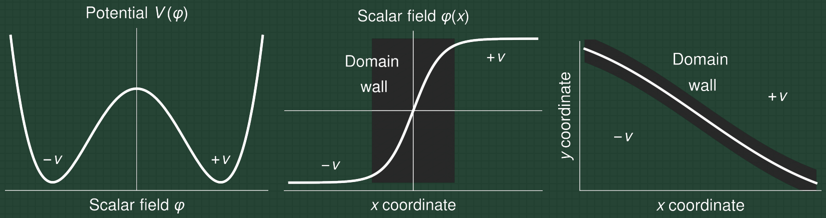


Spontaneous breaking of a global $\mathbb{Z}_2$ symmetry by a real scalar field φ:

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - v^2\right)^2 \tag{1}$$

- $\langle\varphi\rangle = \pm v$ randomly distributed on length scales larger than the causal horizon
- Different domains separated by domain walls where φ transitions from $-v$ to $+v$

Domain walls in cosmology

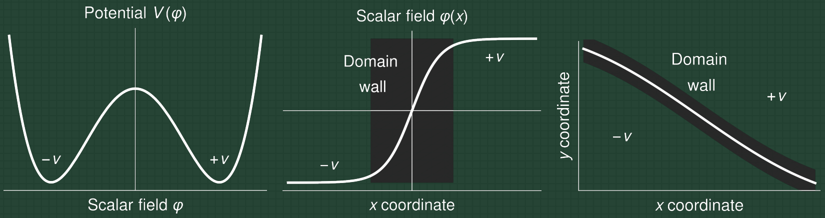


Spontaneous breaking of a global $\mathbb{Z}_2$ symmetry by a real scalar field φ:

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - v^2\right)^2 \tag{1}$$

- $\langle\varphi\rangle = \pm v$ randomly distributed on length scales larger than the causal horizon
- Different domains separated by domain walls where φ transitions from $-v$ to $+v$
- $\mathbb{Z}_2$ symmetry remains unbroken at the center of the domain wall: $\varphi = 0$

Domain walls in cosmology

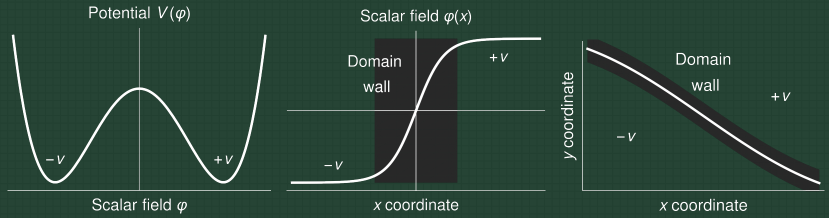


Spontaneous breaking of a global $\mathbb{Z}_2$ symmetry by a real scalar field φ:

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - v^2\right)^2 \tag{1}$$

- $\langle\varphi\rangle = \pm v$ randomly distributed on length scales larger than the causal horizon
- Different domains separated by domain walls where φ transitions from $-v$ to $+v$
- $\mathbb{Z}_2$ symmetry remains unbroken at the center of the domain wall: $\varphi = 0$
- Domain wall carries gradient + potential energy per unit surface $\sim \lambda^{1/2} v^3$

Domain walls in cosmology

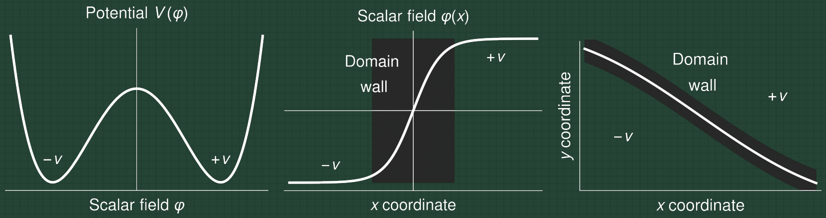


Spontaneous breaking of a global $\mathbb{Z}_2$ symmetry by a real scalar field φ:

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - v^2\right)^2 \tag{1}$$

- $\langle\varphi\rangle = \pm v$ randomly distributed on length scales larger than the causal horizon
- Different domains separated by domain walls where φ transitions from $-v$ to $+v$
- $\mathbb{Z}_2$ symmetry remains unbroken at the center of the domain wall: $\varphi = 0$
- Domain wall carries gradient + potential energy per unit surface $\sim \lambda^{1/2} v^3$
- Stable on cosmological time scales if $V(-v) = V(+v)$ to high accuracy

Domain walls in cosmology



Spontaneous breaking of a global $\mathbb{Z}_2$ symmetry by a real scalar field φ:

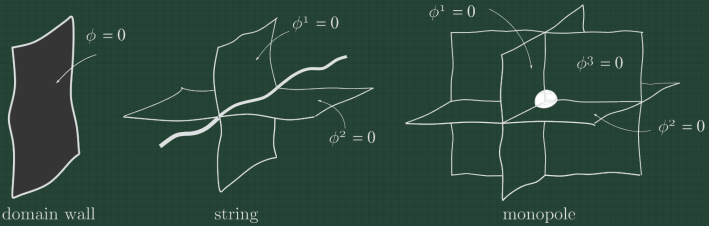
$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - v^2\right)^2 \tag{1}$$

- $\langle\varphi\rangle = \pm v$ randomly distributed on length scales larger than the causal horizon
- Different domains separated by domain walls where φ transitions from $-v$ to $+v$
- $\mathbb{Z}_2$ symmetry remains unbroken at the center of the domain wall: $\varphi = 0$
- Domain wall carries gradient + potential energy per unit surface $\sim \lambda^{1/2} v^3$
- Stable on cosmological time scales if $V(-v) = V(+v)$ to high accuracy

Intriguing scenario: GWs from a cosmic defect network after a cosmological phase transition!

Topological defects

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]

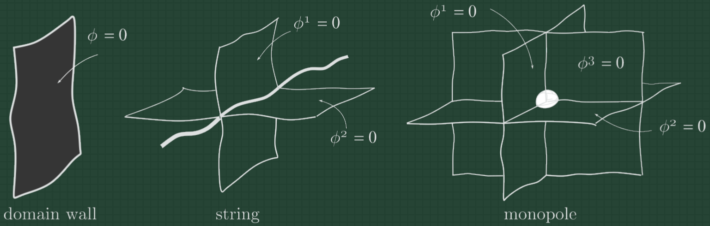


Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2 , \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T \tag{2}$$

Topological defects

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]



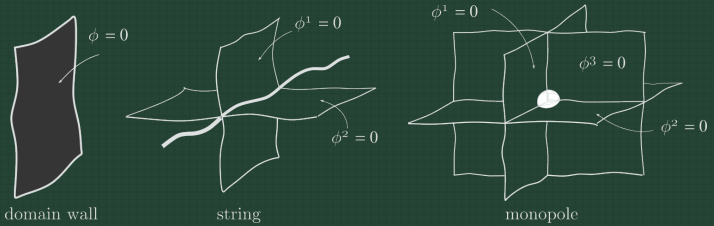
Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2 , \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T \tag{2}$$

- Scalar fields transform under $SO(N)$ global or local gauge symmetry

Topological defects

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]



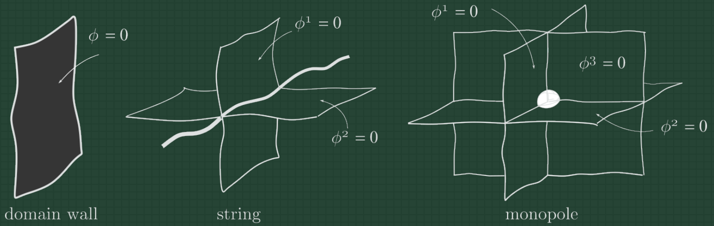
Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2, \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T \tag{2}$$

- Scalar fields transform under $SO(N)$ global or local gauge symmetry
- $\mathbb{Z}_2 \rightarrow$ domain walls; $U(1) \cong SO(2) \rightarrow$ cosmic strings; $SU(2) \cong SO(3) \rightarrow$ monopoles

Topological defects

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]



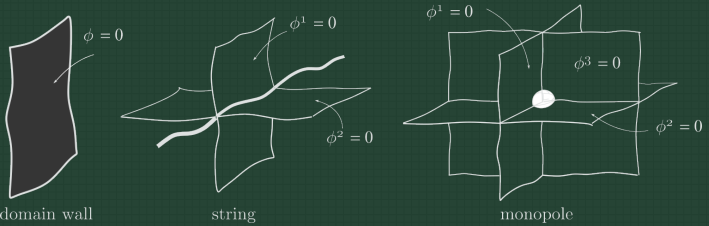
Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2, \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T \tag{2}$$

- Scalar fields transform under $SO(N)$ global or local gauge symmetry
- $\mathbb{Z}_2 \to$ domain walls; $U(1) \cong SO(2) \to$ cosmic strings; $SU(2) \cong SO(3) \to$ monopoles
- Solitonic solutions of the classical equations of motion for the gauge and Higgs fields

Topological defects

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]



Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

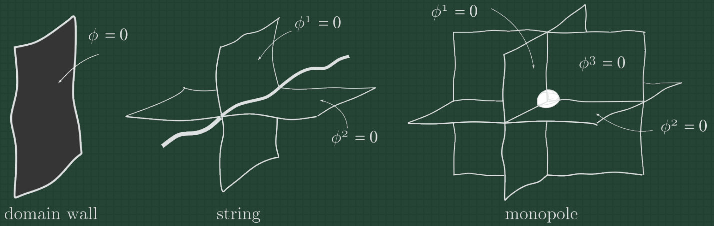
$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2, \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T \tag{2}$$

- Scalar fields transform under $SO(N)$ global or local gauge symmetry
- $\mathbb{Z}_2 \to$ domain walls; $U(1) \cong SO(2) \to$ cosmic strings; $SU(2) \cong SO(3) \to$ monopoles
- Solitonic solutions of the classical equations of motion for the gauge and Higgs fields

General description in terms of homotopy group $\pi_n(\mathcal{M})$ of the vacuum manifold $\mathcal{M}$:

Topological defects

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]



Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2 , \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T \tag{2}$$

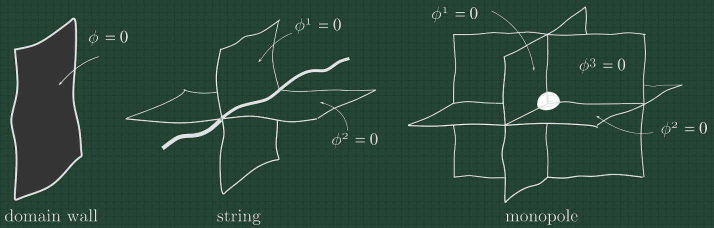
- Scalar fields transform under $SO(N)$ global or local gauge symmetry
- $\mathbb{Z}_2 \to$ domain walls; $U(1) \cong SO(2) \to$ cosmic strings; $SU(2) \cong SO(3) \to$ monopoles
- Solitonic solutions of the classical equations of motion for the gauge and Higgs fields

General description in terms of homotopy group $\pi_n(\mathcal{M})$ of the vacuum manifold $\mathcal{M}$:

- $\pi_n(\mathcal{M})$ counts number of topologically inequivalent maps $S^n \to \mathcal{M}$

Topological defects

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]



Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2, \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T \tag{2}$$

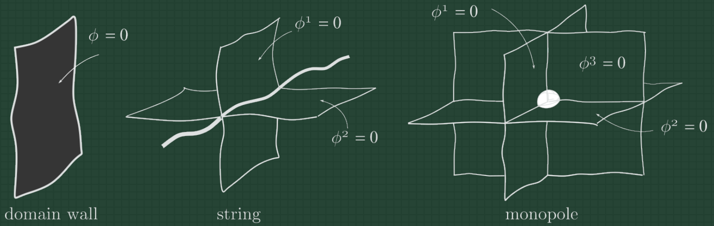
- Scalar fields transform under $SO(N)$ global or local gauge symmetry
- $\mathbb{Z}_2 \to$ domain walls; $U(1) \cong SO(2) \to$ cosmic strings; $SU(2) \cong SO(3) \to$ monopoles
- Solitonic solutions of the classical equations of motion for the gauge and Higgs fields

General description in terms of homotopy group $\pi_n(\mathcal{M})$ of the vacuum manifold $\mathcal{M}$:

- $\pi_n(\mathcal{M})$ counts number of topologically inequivalent maps $S^n \to \mathcal{M}$
- Nontrivial $\pi_n(\mathcal{M}) \to$ domain walls ($n = 0$), cosmic strings ($n = 1$), monopoles ($n = 2$)

Topological defects

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]



Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2 , \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T \tag{2}$$

- Scalar fields transform under $SO(N)$ global or local gauge symmetry
- $\mathbb{Z}_2 \to$ domain walls; $U(1) \cong SO(2) \to$ cosmic strings; $SU(2) \cong SO(3) \to$ monopoles
- Solitonic solutions of the classical equations of motion for the gauge and Higgs fields

General description in terms of homotopy group $\pi_n(\mathcal{M})$ of the vacuum manifold $\mathcal{M}$:

- $\pi_n(\mathcal{M})$ counts number of topologically inequivalent maps $S^n \to \mathcal{M}$
- Nontrivial $\pi_n(\mathcal{M}) \to$ domain walls ($n = 0$), cosmic strings ($n = 1$), monopoles ($n = 2$)
- In addition, there is a whole zoo of composite defects, non-topological defects, etc.

Outline Lecture 3A

1. Cosmic defects

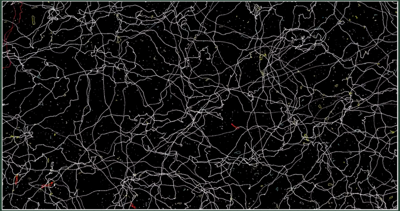
2. General defect networks

3. Cosmic strings

4. Summary

Scaling regime

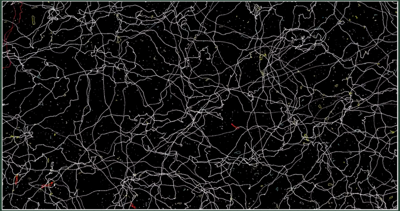
[wikimedia.org]



Cosmic defect networks approach a scaling regime sufficiently long after their formation:

Scaling regime

[wikimedia.org]

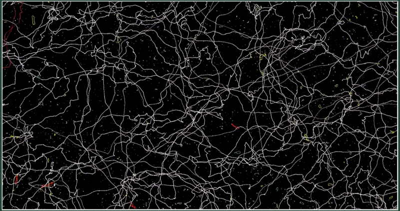


Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions

Scaling regime

[wikimedia.org]

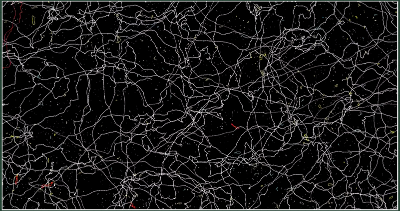


Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion

Scaling regime

[wikimedia.org]

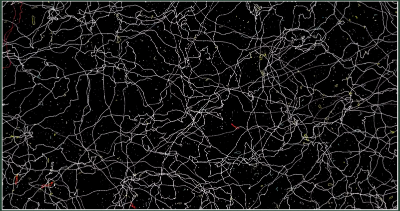


Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

Scaling regime

[wikimedia.org]



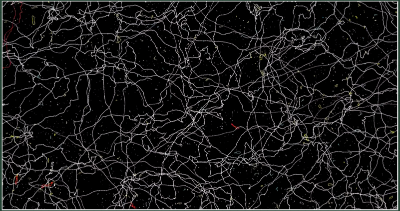
Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

Number and energy density of defect networks during scaling:

Scaling regime

[wikimedia.org]



Cosmic defect networks approach a scaling regime sufficiently long after their formation:

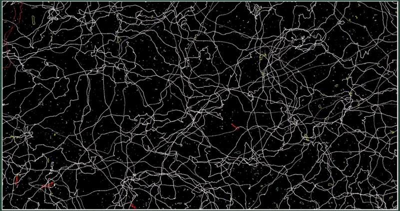
- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

Number and energy density of defect networks during scaling:

- Typical separation between defects, L, grows like the Hubble radius, $L \propto H^{-1}$

Scaling regime

[wikimedia.org]

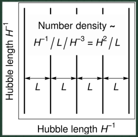


Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

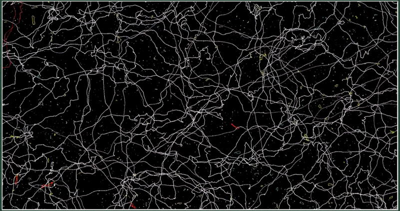
Number and energy density of defect networks during scaling:

- Typical separation between defects, L, grows like the Hubble radius, $L \propto H^{-1}$
- Domain walls: $n_{\rm walls} \sim H^2/L$ and $\rho_{\rm walls} \sim \sigma/L \propto \sigma\, H \rightarrow \Omega_{\rm walls}$ grows like H^{-1}



Scaling regime

[wikimedia.org]

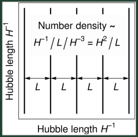


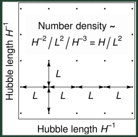
Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

Number and energy density of defect networks during scaling:

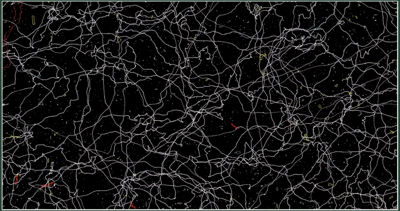
- Typical separation between defects, L, grows like the Hubble radius, $L \propto H^{-1}$
- Domain walls: $n_{\rm walls} \sim H^2/L$ and $\rho_{\rm walls} \sim \sigma/L \propto \sigma\, H \rightarrow \Omega_{\rm walls}$ grows like H^{-1}
- Cosmic strings: $n_{\rm strings} \sim H/L^2$ and $\rho_{\rm strings} \sim \mu/L^2 \propto \mu\, H^2 \rightarrow \Omega_{\rm strings} = \mathrm{const}$





Scaling regime

[wikimedia.org]

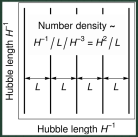


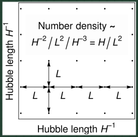
Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

Number and energy density of defect networks during scaling:

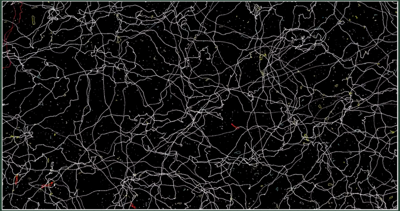
- Typical separation between defects, L, grows like the Hubble radius, $L \propto H^{-1}$
- Domain walls: $n_{\rm walls} \sim H^2/L$ and $\rho_{\rm walls} \sim \sigma/L \propto \sigma\, H \;\rightarrow\; \Omega_{\rm walls}$ grows like H^{-1}
- Cosmic strings: $n_{\rm strings} \sim H/L^2$ and $\rho_{\rm strings} \sim \mu/L^2 \propto \mu\, H^2 \;\rightarrow\; \Omega_{\rm strings} = {\rm const}$
- Monopoles: Redshift like matter, $\rho_{\rm monopoles} = M\, n_{\rm monopoles} \propto a^{-3}$





Scaling regime

[wikimedia.org]

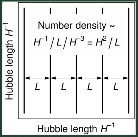


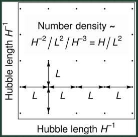
Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

Number and energy density of defect networks during scaling:

- Typical separation between defects, L, grows like the Hubble radius, $L \propto H^{-1}$
- Domain walls: $n_{\rm walls} \sim H^2/L$ and $\rho_{\rm walls} \sim \sigma/L \propto \sigma\, H \rightarrow \Omega_{\rm walls}$ grows like H^{-1}
- Cosmic strings: $n_{\rm strings} \sim H/L^2$ and $\rho_{\rm strings} \sim \mu/L^2 \propto \mu\, H^2 \rightarrow \Omega_{\rm strings} = {\rm const}$
- Monopoles: Redshift like matter, $\rho_{\rm monopoles} = M\, n_{\rm monopoles} \propto a^{-3}$

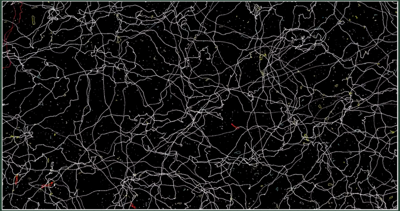




Domain walls and monopoles can “overclose” the Universe:

Scaling regime

[wikimedia.org]

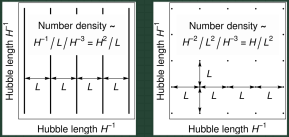


Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

Number and energy density of defect networks during scaling:

- Typical separation between defects, L, grows like the Hubble radius, $L \propto H^{-1}$
- Domain walls: $n_{\mathrm{walls}} \sim H^2/L$ and $\rho_{\mathrm{walls}} \sim \sigma/L \propto \sigma\, H \to \Omega_{\mathrm{walls}}$ grows like H^{-1}
- Cosmic strings: $n_{\mathrm{strings}} \sim H/L^2$ and $\rho_{\mathrm{strings}} \sim \mu/L^2 \propto \mu\, H^2 \to \Omega_{\mathrm{strings}} = \mathrm{const}$
- Monopoles: Redshift like matter, $\rho_{\mathrm{monopoles}} = M\, n_{\mathrm{monopoles}} \propto a^{-3}$

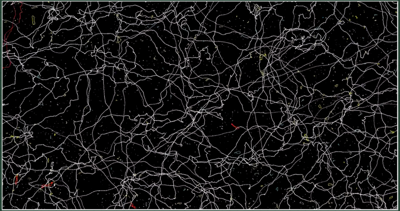


Domain walls and monopoles can “overclose” the Universe:

- Make domain walls unstable by introducing ΔV bias

Scaling regime

[wikimedia.org]

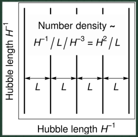


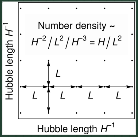
Cosmic defect networks approach a scaling regime sufficiently long after their formation:

- Attractor solution, dynamics become independent of the initial conditions
- Self-similar evolution, crucial properties remain constant despite the expansion
- Indicated by analytical arguments and confirmed by numerical simulations

Number and energy density of defect networks during scaling:

- Typical separation between defects, L, grows like the Hubble radius, $L \propto H^{-1}$
- Domain walls: $n_{\mathrm{walls}} \sim H^2/L$ and $\rho_{\mathrm{walls}} \sim \sigma/L \propto \sigma\, H \rightarrow \Omega_{\mathrm{walls}}$ grows like H^{-1}
- Cosmic strings: $n_{\mathrm{strings}} \sim H/L^2$ and $\rho_{\mathrm{strings}} \sim \mu/L^2 \propto \mu\, H^2 \rightarrow \Omega_{\mathrm{strings}} = \mathrm{const}$
- Monopoles: Redshift like matter, $\rho_{\mathrm{monopoles}} = M\, n_{\mathrm{monopoles}} \propto a^{-3}$





Domain walls and monopoles can "overclose" the Universe:

- Make domain walls unstable by introducing ΔV bias
- Dilute to negligible density during cosmic inflation

Analytical estimates

Scaling regime during radiation domination results in a flat GW spectrum (lecture 2A):

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_c}\left(\frac{a}{a_0}\right)^4 \rho_{\mathrm{tot}}(T)\left[\frac{1}{\rho_{\mathrm{tot}}}\frac{d\rho_{\mathrm{GW}}}{d\ln f}\right]_T , \qquad a^4\rho_{\mathrm{tot}}(T) \approx \mathrm{const} \tag{3}$$

Analytical estimates

Scaling regime during radiation domination results in a flat GW spectrum (lecture 2A):

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\left(\frac{a}{a_0}\right)^4 \rho_{\rm tot}(T)\left[\frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm GW}}{d\ln f}\right]_T , \qquad a^4\rho_{\rm tot}(T) \approx \text{const} \tag{3}$$

General form of the unequal-time correlator Π based on dimensional analysis:

$$\left\langle \Pi_p(\eta,\boldsymbol{k})\,\Pi_q^*(\zeta,\boldsymbol{\ell})\right\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}\,\delta_{pq}\,\Pi(k,\eta,\zeta)\,, \qquad \Pi\left(k,\eta,\zeta\right) = \frac{v^4}{\sqrt{\eta\zeta}}\,\frac{U\left(k\eta,k\zeta\right)}{a^2\left(\eta\right)a^2\left(\zeta\right)} \tag{4}$$

Analytical estimates

Scaling regime during radiation domination results in a flat GW spectrum (lecture 2A):

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\left(\frac{a}{a_0}\right)^4 \rho_{\rm tot}(T)\left[\frac{1}{\rho_{\rm tot}}\,\frac{d\rho_{\rm GW}}{d\ln f}\right]_T\,, \qquad a^4\rho_{\rm tot}(T) \approx \text{const} \tag{3}$$

General form of the unequal-time correlator Π based on dimensional analysis:

$$\left\langle \Pi_p(\eta,\boldsymbol{k})\,\Pi_q^*(\zeta,\boldsymbol{\ell})\right\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}\,\delta_{pq}\,\Pi(k,\eta,\zeta)\,, \qquad \Pi\left(k,\eta,\zeta\right) = \frac{v^4}{\sqrt{\eta\zeta}}\,\frac{U\left(k\eta,k\zeta\right)}{a^2\left(\eta\right)a^2\left(\zeta\right)} \tag{4}$$

- Π has mass dimension 5, is proportional to an even power of v and symmetric in η and ζ

Analytical estimates

Scaling regime during radiation domination results in a flat GW spectrum (lecture 2A):

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\left(\frac{a}{a_0}\right)^4 \rho_{\rm tot}(T)\left[\frac{1}{\rho_{\rm tot}}\,\frac{d\rho_{\rm GW}}{d\ln f}\right]_T\,, \qquad a^4\rho_{\rm tot}(T) \approx {\rm const} \tag{3}$$

General form of the unequal-time correlator Π based on dimensional analysis:

$$\left\langle \Pi_p(\eta,\boldsymbol{k})\,\Pi_q^*(\zeta,\boldsymbol{\ell})\right\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}\,\delta_{pq}\,\Pi(k,\eta,\zeta)\,, \qquad \Pi\left(k,\eta,\zeta\right) = \frac{v^4}{\sqrt{\eta\zeta}}\,\frac{U\left(k\eta,k\zeta\right)}{a^2\left(\eta\right)a^2\left(\zeta\right)} \tag{4}$$

- Π has mass dimension 5, is proportional to an even power of v and symmetric in η and ζ
- No dependence on absolute scale k, only on k in relation to the horizon, $k\eta = k/(aH)$

Analytical estimates

Scaling regime during radiation domination results in a flat GW spectrum (lecture 2A):

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\left(\frac{a}{a_0}\right)^4 \rho_{\rm tot}(T)\left[\frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm GW}}{d\ln f}\right]_T\,, \qquad a^4\rho_{\rm tot}(T) \approx \text{const} \tag{3}$$

General form of the unequal-time correlator Π based on dimensional analysis:

$$\left\langle \Pi_p(\eta,\boldsymbol{k})\,\Pi_q^*(\zeta,\boldsymbol{\ell})\right\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}\,\delta_{pq}\,\Pi(k,\eta,\zeta)\,, \qquad \Pi\left(k,\eta,\zeta\right) = \frac{v^4}{\sqrt{\eta\zeta}}\,\frac{U\left(k\eta,k\zeta\right)}{a^2\left(\eta\right)a^2\left(\zeta\right)} \tag{4}$$

- Π has mass dimension 5, is proportional to an even power of v and symmetric in η and ζ
- No dependence on absolute scale k, only on k in relation to the horizon, $k\eta = k/(aH)$
- Details contained in $U(x_1,x_2)$; typically peaked at $x_1 = x_2 = x$; scales like x^{-p}, $p > 2$

Analytical estimates

Scaling regime during radiation domination results in a flat GW spectrum (lecture 2A):

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_c}\left(\frac{a}{a_0}\right)^4 \rho_{\mathrm{tot}}(T)\left[\frac{1}{\rho_{\mathrm{tot}}}\frac{d\rho_{\mathrm{GW}}}{d\ln f}\right]_T , \qquad a^4 \rho_{\mathrm{tot}}(T) \approx \mathrm{const} \tag{3}$$

General form of the unequal-time correlator Π based on dimensional analysis:

$$\left\langle \Pi_p(\eta, \boldsymbol{k})\, \Pi_q^*(\zeta, \boldsymbol{\ell})\right\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}\,\delta_{pq}\,\Pi(k,\eta,\zeta)\,, \qquad \Pi\left(k,\eta,\zeta\right) = \frac{v^4}{\sqrt{\eta\zeta}}\,\frac{U\left(k\eta, k\zeta\right)}{a^2\left(\eta\right)a^2\left(\zeta\right)} \tag{4}$$

- Π has mass dimension 5, is proportional to an even power of v and symmetric in η and ζ
- No dependence on absolute scale k, only on k in relation to the horizon, $k\eta = k/(aH)$
- Details contained in $U(x_1, x_2)$; typically peaked at $x_1 = x_2 = x$; scales like x^{-p}, $p > 2$

Recall, general solution for the GW signal from a sub-horizon source during RD:

$$\Omega_{\mathrm{GW}}(f) = \frac{k^3}{6\pi^2\, a_0^4\, m_{\mathrm{Pl}}^4 H_0^2}\int_{\eta_{\mathrm{ini}}}^{\eta_{\mathrm{fin}}} d\eta\, a^3(\eta)\int_{\zeta_{\mathrm{ini}}}^{\zeta_{\mathrm{fin}}} d\zeta\, a^3(\zeta)\cos\left[k(\eta-\zeta)\right]\Pi(k,\eta,\zeta) \tag{5}$$

Analytical estimates

Scaling regime during radiation domination results in a flat GW spectrum (lecture 2A):

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\left(\frac{a}{a_0}\right)^4 \rho_{\rm tot}(T)\left[\frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm GW}}{d\ln f}\right]_T , \qquad a^4\rho_{\rm tot}(T) \approx \text{const} \tag{3}$$

General form of the unequal-time correlator Π based on dimensional analysis:

$$\left\langle \Pi_p(\eta, \boldsymbol{k})\,\Pi_q^*(\zeta, \boldsymbol{\ell})\right\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}\,\delta_{pq}\,\Pi(k,\eta,\zeta)\,, \qquad \Pi\left(k,\eta,\zeta\right) = \frac{v^4}{\sqrt{\eta\zeta}}\,\frac{U\left(k\eta, k\zeta\right)}{a^2\left(\eta\right)a^2\left(\zeta\right)} \tag{4}$$

- Π has mass dimension 5, is proportional to an even power of v and symmetric in η and ζ
- No dependence on absolute scale k, only on k in relation to the horizon, $k\eta = k/(aH)$
- Details contained in $U(x_1, x_2)$; typically peaked at $x_1 = x_2 = x$; scales like x^{-p}, $p > 2$

Recall, general solution for the GW signal from a sub-horizon source during RD:

$$\Omega_{\rm GW}(f) = \frac{k^3}{6\pi^2\,a_0^4\,m_{\rm Pl}^4 H_0^2}\int_{\eta_{\rm ini}}^{\eta_{\rm fin}} d\eta\,a^3(\eta)\int_{\zeta_{\rm ini}}^{\zeta_{\rm fin}} d\zeta\,a^3(\zeta)\cos\left[k(\eta-\zeta)\right]\Pi(k,\eta,\zeta) \tag{5}$$

Scale factor during RD, follows from $\eta = 1/(aH)$ and general solution for $H(a)$:

$$a\left(\eta\right) = \Omega_{\rm r}^{1/2}\left(\frac{g_*}{g_*^0}\right)^{1/2}\left(\frac{g_{*,s}^0}{g_{*,s}}\right)^{2/3} a_0^2 H_0\,\eta \tag{6}$$

Shape of the GW spectrum

GW signal emitted during RD: Indeed approximately flat, proportional to v^4:

$$\Omega_{\rm GW}(f) = \frac{\Omega_{\rm r}}{6\pi^2}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}}\right)^{4/3}\left(\frac{v}{m_{\rm Pl}}\right)^4 \mathcal{F} \tag{7}$$

Shape of the GW spectrum

GW signal emitted during RD: Indeed approximately flat, proportional to v^4:

$$\Omega_{\rm GW}(f) = \frac{\Omega_{\rm r}}{6\pi^2}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}}\right)^{4/3}\left(\frac{v}{m_{\rm Pl}}\right)^4 \mathcal{F} \tag{7}$$

where the function $\mathcal{F}$ approaches a constant in the limit $k\eta_{\rm fin}, k\zeta_{\rm fin} \gg 1$:

$$\mathcal{F} = \int_{\eta_{\rm ini}}^{\eta_{\rm fin}} d\eta \int_{\zeta_{\rm ini}}^{\zeta_{\rm fin}} d\zeta\, \sqrt{\eta\zeta}\, k^3 \cos\left[k(\eta-\zeta)\right] U\left(k\eta, k\zeta\right) \quad \longrightarrow \quad \text{const} \tag{8}$$

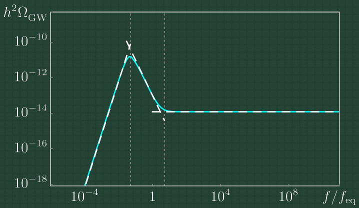
Shape of the GW spectrum

GW signal emitted during RD: Indeed approximately flat, proportional to v^4:

$$\Omega_{\rm GW}(f) = \frac{\Omega_{\rm r}}{6\pi^2}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}}\right)^{4/3}\left(\frac{v}{m_{\rm Pl}}\right)^4 \mathcal{F} \tag{7}$$

where the function $\mathcal{F}$ approaches a constant in the limit $k\eta_{\rm fin}, k\zeta_{\rm fin} \gg 1$:

$$\mathcal{F} = \int_{\eta_{\rm ini}}^{\eta_{\rm fin}} d\eta \int_{\zeta_{\rm ini}}^{\zeta_{\rm fin}} d\zeta\, \sqrt{\eta\zeta}\, k^3 \cos\left[k(\eta-\zeta)\right] U\left(k\eta, k\zeta\right) \quad \longrightarrow \quad \text{const} \tag{8}$$



[Figueroa, Hindmarsh, Lizarraga, Urrestilla: 2007.03337]

GW signal emitted after matter-radiation equality:

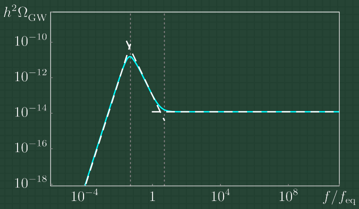
Shape of the GW spectrum

GW signal emitted during RD: Indeed approximately flat, proportional to v^4:

$$\Omega_{\rm GW}(f) = \frac{\Omega_{\rm r}}{6\pi^2}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}}\right)^{4/3}\left(\frac{v}{m_{\rm Pl}}\right)^4 \mathcal{F} \tag{7}$$

where the function $\mathcal{F}$ approaches a constant in the limit $k\eta_{\rm fin}, k\zeta_{\rm fin} \gg 1$:

$$\mathcal{F} = \int_{\eta_{\rm ini}}^{\eta_{\rm fin}} d\eta \int_{\zeta_{\rm ini}}^{\zeta_{\rm fin}} d\zeta\, \sqrt{\eta\zeta}\, k^3 \cos\left[k(\eta-\zeta)\right] U\left(k\eta, k\zeta\right) \quad \longrightarrow \quad \text{const} \tag{8}$$



[Figueroa, Hindmarsh, Lizarraga, Urrestilla: 2007.03337]

GW signal emitted after matter-radiation equality:

- f^{-2} decline between f_0 and $f_{\rm eq}$, similarly as in the case of primordial GWs from inflation

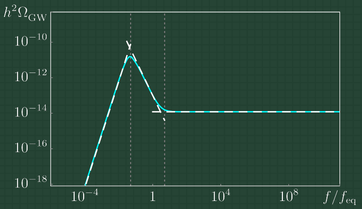
Shape of the GW spectrum

GW signal emitted during RD: Indeed approximately flat, proportional to v^4:

$$\Omega_{\rm GW}(f) = \frac{\Omega_{\rm r}}{6\pi^2}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}}\right)^{4/3}\left(\frac{v}{m_{\rm Pl}}\right)^4 \mathcal{F} \qquad (7)$$

where the function $\mathcal{F}$ approaches a constant in the limit $k\eta_{\rm fin}, k\zeta_{\rm fin} \gg 1$:

$$\mathcal{F} = \int_{\eta_{\rm ini}}^{\eta_{\rm fin}} d\eta \int_{\zeta_{\rm ini}}^{\zeta_{\rm fin}} d\zeta\, \sqrt{\eta\zeta}\, k^3 \cos\left[k(\eta-\zeta)\right] U\left(k\eta, k\zeta\right) \quad \longrightarrow \quad \text{const} \qquad (8)$$



[Figueroa, Hindmarsh, Lizarraga, Urrestilla: 2007.03337]

GW signal emitted after matter-radiation equality:

- f^{-2} decline between f_0 and $f_{\rm eq}$, similarly as in the case of primordial GWs from inflation
- f^3 rise on super-horizon scales, $f \lesssim f_0$, as expected from causality (lecture 2B)

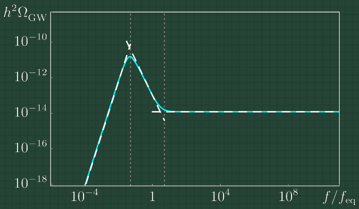
Shape of the GW spectrum

GW signal emitted during RD: Indeed approximately flat, proportional to v^4:

$$\Omega_{\rm GW}(f) = \frac{\Omega_{\rm r}}{6\pi^2}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}}\right)^{4/3}\left(\frac{v}{m_{\rm Pl}}\right)^4 \mathcal{F} \tag{7}$$

where the function $\mathcal{F}$ approaches a constant in the limit $k\eta_{\rm fin}, k\zeta_{\rm fin} \gg 1$:

$$\mathcal{F} = \int_{\eta_{\rm ini}}^{\eta_{\rm fin}} d\eta \int_{\zeta_{\rm ini}}^{\zeta_{\rm fin}} d\zeta\, \sqrt{\eta\zeta}\, k^3 \cos\left[k(\eta-\zeta)\right] U\left(k\eta, k\zeta\right) \quad \longrightarrow \quad \text{const} \tag{8}$$



[Figueroa, Hindmarsh, Lizarraga, Urrestilla: 2007.03337]

GW signal emitted after matter-radiation equality:

- f^{-2} decline between f_0 and $f_{\rm eq}$, similarly as in the case of primordial GWs from inflation
- f^3 rise on super-horizon scales, $f \lesssim f_0$, as expected from causality (lecture 2B)
- Irreducible SGWB for any network of local / global, topological / nontopological defects

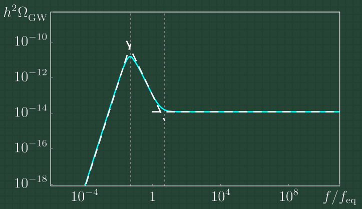
Shape of the GW spectrum

GW signal emitted during RD: Indeed approximately flat, proportional to v^4:

$$\Omega_{\rm GW}(f) = \frac{\Omega_{\rm r}}{6\pi^2}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}}\right)^{4/3}\left(\frac{v}{m_{\rm Pl}}\right)^4 \mathcal{F} \qquad (7)$$

where the function $\mathcal{F}$ approaches a constant in the limit $k\eta_{\rm fin}, k\zeta_{\rm fin} \gg 1$:

$$\mathcal{F} = \int_{\eta_{\rm ini}}^{\eta_{\rm fin}} d\eta \int_{\zeta_{\rm ini}}^{\zeta_{\rm fin}} d\zeta\,\sqrt{\eta\zeta}\,k^3 \cos\left[k(\eta-\zeta)\right] U\left(k\eta, k\zeta\right) \quad \longrightarrow \quad \text{const} \qquad (8)$$



[Figueroa, Hindmarsh, Lizarraga, Urrestilla: 2007.03337]

GW signal emitted after matter-radiation equality:

- f^{-2} decline between f_0 and $f_{\rm eq}$, similarly as in the case of primordial GWs from inflation
- f^3 rise on super-horizon scales, $f \lesssim f_0$, as expected from causality (lecture 2B)
- Irreducible SGWB for any network of local / global, topological / nontopological defects

Figure: GW spectrum from global defects after spontaneous $O(4)$ symmetry breaking, effects due to changing number of DOFs scaled out. Lattice computation of $U\left(k\eta, k\zeta\right)$.

Motivation

GWs from cosmic strings are particularly interesting:

Motivation

GWs from cosmic strings are particularly interesting:

- No overclosure problem; constant energy fraction in the scaling regime, $\Omega_{\mathrm{strings}} \approx \mathrm{const}$

Motivation

GWs from cosmic strings are particularly interesting:

- No overclosure problem; constant energy fraction in the scaling regime, $\Omega_{\rm strings} \approx \rm const$
- Well motivated, expected in many BSM models: $U(1)_{B-L}$, $U(1)_{\rm PQ}$, ...

Motivation

GWs from cosmic strings are particularly interesting:

- No overclosure problem; constant energy fraction in the scaling regime, $\Omega_{\text{strings}} \approx \text{const}$
- Well motivated, expected in many BSM models: $U(1)_{B-L}$, $U(1)_{\text{PQ}}$, ...
- Extra mechanism of GW emission: GWs from oscillating cosmic-string loops

Motivation

GWs from cosmic strings are particularly interesting:

- No overclosure problem; constant energy fraction in the scaling regime, $\Omega_{\rm strings} \approx \text{const}$
- Well motivated, expected in many BSM models: $U(1)_{B-L}$, $U(1)_{\rm PQ}$, ...
- Extra mechanism of GW emission: GWs from oscillating cosmic-string loops

---

Origin of cosmic strings:

Motivation

GWs from cosmic strings are particularly interesting:

- No overclosure problem; constant energy fraction in the scaling regime, $\Omega_{\text{strings}} \approx \text{const}$
- Well motivated, expected in many BSM models: $U(1)_{B-L}$, $U(1)_{\text{PQ}}$, ...
- Extra mechanism of GW emission: GWs from oscillating cosmic-string loops

---

Origin of cosmic strings:

- Emerge from a phase transition when $\pi_1(\mathcal{M})$, the fundamental group of the vacuum manifold $\mathcal{M}$, is nontrivial; that is, when $\mathcal{M}$ is not simply connected

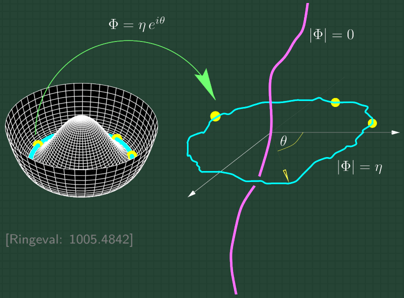
Motivation

GWs from cosmic strings are particularly interesting:

- No overclosure problem; constant energy fraction in the scaling regime, $\Omega_{\mathrm{strings}} \approx \mathrm{const}$
- Well motivated, expected in many BSM models: $U(1)_{B-L}$, $U(1)_{\mathrm{PQ}}$, ...
- Extra mechanism of GW emission: GWs from oscillating cosmic-string loops

---

Spontaneous $U(1)$ symmetry breaking in a Mexican-hat scalar potential:



[Ringeval: 1005.4842]

String tension / energy per unit length:
$\mu \sim v^2$, or equivalently $G\mu \sim (v/m_{\mathrm{Pl}})^2$

Origin of cosmic strings:

- Emerge from a phase transition when $\pi_1(\mathcal{M})$, the fundamental group of the vacuum manifold $\mathcal{M}$, is nontrivial; that is, when $\mathcal{M}$ is not simply connected
- Global / local cosmic strings from breaking a global / local $U(1)$ symmetry

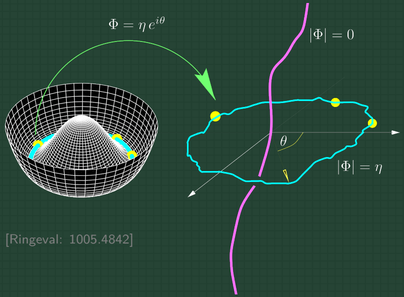
Motivation

GWs from cosmic strings are particularly interesting:

- No overclosure problem; constant energy fraction in the scaling regime, $\Omega_{\text{strings}} \approx \text{const}$
- Well motivated, expected in many BSM models: $U(1)_{B-L}$, $U(1)_{\text{PQ}}$, ...
- Extra mechanism of GW emission: GWs from oscillating cosmic-string loops

---

Spontaneous $U(1)$ symmetry breaking in a Mexican-hat scalar potential:



[Ringeval: 1005.4842]

String tension / energy per unit length:
$\mu \sim v^2$, or equivalently $G\mu \sim (v/m_{\text{Pl}})^2$

Origin of cosmic strings:

- Emerge from a phase transition when $\pi_1(\mathcal{M})$, the fundamental group of the vacuum manifold $\mathcal{M}$, is nontrivial; that is, when $\mathcal{M}$ is not simply connected
- Global / local cosmic strings from breaking a global / local $U(1)$ symmetry
- Local cosmic strings carry magnetic flux, $\Phi = 2\pi/e$, with $U(1)$ gauge coupling e

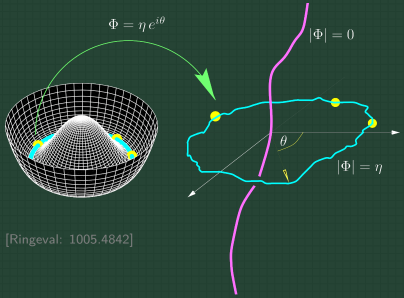
Motivation

GWs from cosmic strings are particularly interesting:

- No overclosure problem; constant energy fraction in the scaling regime, $\Omega_{\text{strings}} \approx \text{const}$
- Well motivated, expected in many BSM models: $U(1)_{B-L}$, $U(1)_{\text{PQ}}$, ...
- Extra mechanism of GW emission: GWs from oscillating cosmic-string loops

---

Spontaneous $U(1)$ symmetry breaking in a Mexican-hat scalar potential:



[Ringeval: 1005.4842]

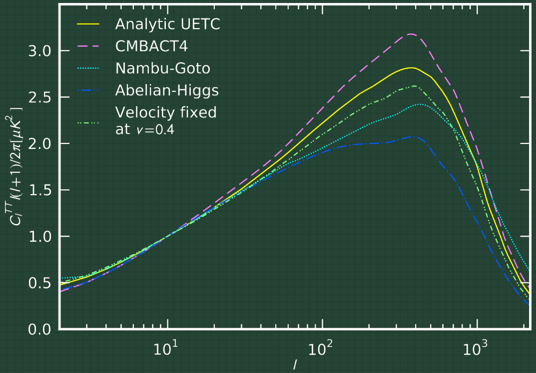
String tension / energy per unit length:
$\mu \sim v^2$, or equivalently $G\mu \sim (v/m_{\text{Pl}})^2$

Origin of cosmic strings:

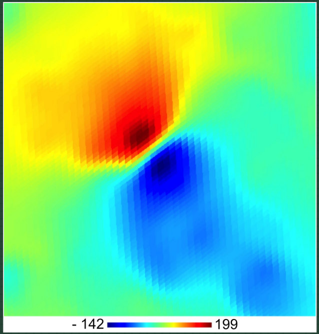
- Emerge from a phase transition when $\pi_1(\mathcal{M})$, the fundamental group of the vacuum manifold $\mathcal{M}$, is nontrivial; that is, when $\mathcal{M}$ is not simply connected
- Global / local cosmic strings from breaking a global / local $U(1)$ symmetry
- Local cosmic strings carry magnetic flux, $\Phi = 2\pi/e$, with $U(1)$ gauge coupling e
- Abrikosov vortex in Ginzburg–Landau theory of superconductivity

Phenomenology

[Charnock, Avgoustidis, Copeland, Moss: 1603.01275]



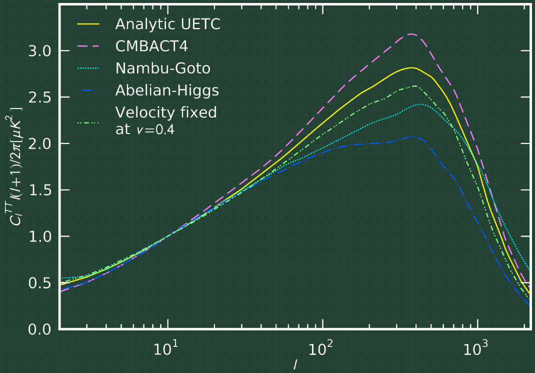
[PLANCK Collaboration: 1303.5085]



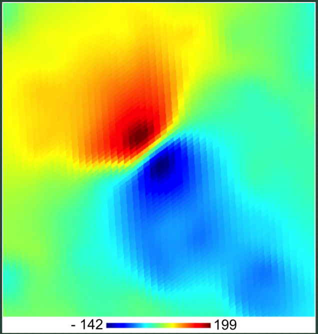
- Strings constantly source metric perturbations $\to$ contribution to CMB power spectra

Phenomenology

[Charnock, Avgoustidis, Copeland, Moss: 1603.01275]



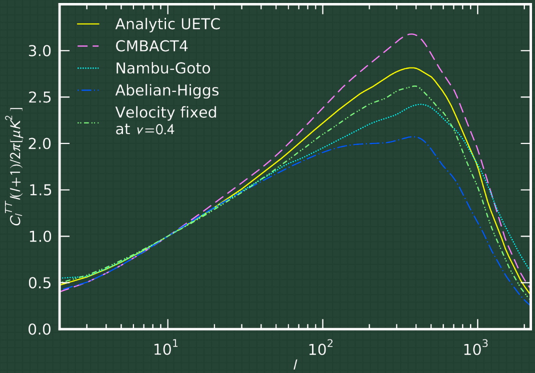
[PLANCK Collaboration: 1303.5085]



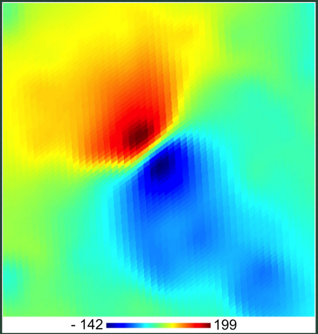
- Strings constantly source metric perturbations $\to$ contribution to CMB power spectra
- Gott–Kaiser–Stebbins effect: line-like discontinuity caused by string in line of sight

Phenomenology

[Charnock, Avgoustidis, Copeland, Moss: 1603.01275]



[PLANCK Collaboration: 1303.5085]



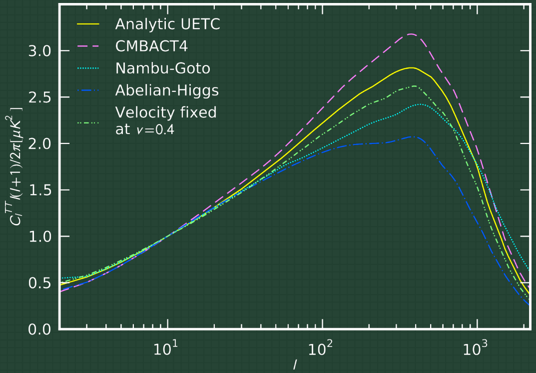
- Strings constantly source metric perturbations $\rightarrow$ contribution to CMB power spectra
- Gott–Kaiser–Stebbins effect: line-like discontinuity caused by string in line of sight

Upper bound on the cosmic-string tension from CMB data:

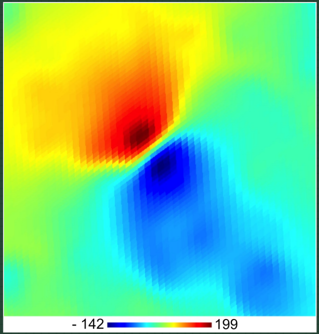
$$G\mu \simeq 10^{-7} \left(\frac{v}{10^{16}\,\mathrm{GeV}} \right)^2 \lesssim 10^{-7} \tag{9}$$

Phenomenology

[Charnock, Avgoustidis, Copeland, Moss: 1603.01275]



[PLANCK Collaboration: 1303.5085]



- Strings constantly source metric perturbations $\rightarrow$ contribution to CMB power spectra
- Gott–Kaiser–Stebbins effect: line-like discontinuity caused by string in line of sight

Upper bound on the cosmic-string tension from CMB data:

$$G\mu \simeq 10^{-7} \left(\frac{v}{10^{16}\,\mathrm{GeV}}\right)^2 \lesssim 10^{-7} \tag{9}$$

Other signatures: Lensing events, emission of cosmic rays, radio bursts, and of course GWs

Theoretical description

Abelian Higgs model: Field theory of a complex scalar and a $U(1)$ vector field

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - (D_\mu\phi)^*\,(D^\mu\phi) - \frac{\lambda}{4}\left(|\phi|^2 - v^2\right) \tag{10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + ie\,A_\mu$.

Theoretical description

Abelian Higgs model: Field theory of a complex scalar and a $U(1)$ vector field

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - \left(D_\mu\phi\right)^*\left(D^\mu\phi\right) - \frac{\lambda}{4}\left(|\phi|^2 - v^2\right) \tag{10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + ie\,A_\mu$. Nielsen–Olesen string solution:

$$\phi\left(r,\varphi,z\right) = v\,f\left(rm_A\right)e^{in\varphi}\,, \qquad \boldsymbol{A}\left(r,\varphi,z\right) = \frac{n}{er}\,g\left(rm_A\right)\boldsymbol{e}_\varphi \tag{11}$$

with some complicated functions f and g; $f, g \to 0$ as $r \to 0$ and $f, g \to 1$ as $r \to \infty$.

Theoretical description

Abelian Higgs model: Field theory of a complex scalar and a $U(1)$ vector field

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (D_\mu \phi)^* (D^\mu \phi) - \frac{\lambda}{4} \left(|\phi|^2 - v^2 \right) \tag{10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + ie\, A_\mu$. Nielsen–Olesen string solution:

$$\phi(r, \varphi, z) = v\, f(r m_A)\, e^{in\varphi}\,, \qquad \boldsymbol{A}(r, \varphi, z) = \frac{n}{er}\, g(r m_A)\, \boldsymbol{e}_\varphi \tag{11}$$

with some complicated functions f and g; $f, g \to 0$ as $r \to 0$ and $f, g \to 1$ as $r \to \infty$.

Radial extent of the Nielsen–Olesen string is microscopically small:

- Magnetic core $\sim 1/m_A$, with vector-boson mass $m_A = \sqrt{2}\, ev$
- Scalar core $\sim 1/m_H$, with Higgs-boson mass $m_H = \sqrt{2\lambda}\, v$

Theoretical description

Abelian Higgs model: Field theory of a complex scalar and a $U(1)$ vector field

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - \left(D_\mu\phi\right)^*\left(D^\mu\phi\right) - \frac{\lambda}{4}\left(\left|\phi\right|^2 - v^2\right) \tag{10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + ie\,A_\mu$. Nielsen–Olesen string solution:

$$\phi\left(r,\varphi,z\right) = v\,f\left(rm_A\right)e^{in\varphi}\,, \qquad \boldsymbol{A}\left(r,\varphi,z\right) = \frac{n}{er}\,g\left(rm_A\right)\boldsymbol{e}_\varphi \tag{11}$$

with some complicated functions f and g; $f,g \to 0$ as $r \to 0$ and $f,g \to 1$ as $r \to \infty$.

Radial extent of the Nielsen–Olesen string is microscopically small:

- Magnetic core $\sim 1/m_A$, with vector-boson mass $m_A = \sqrt{2}\,ev$
- Scalar core $\sim 1/m_H$, with Higgs-boson mass $m_H = \sqrt{2\lambda}\,v$

Alternative description in terms of the Nambu–Goto action:

$$S_{\mathrm{NG}} = -\mu\int d^2\sigma\,\sqrt{-\gamma}\,, \qquad \gamma_{ab} = g_{\mu\nu}\,\frac{dx^\mu}{d\sigma^a}\,\frac{dx^\nu}{d\sigma^b} \tag{12}$$

Theoretical description

Abelian Higgs model: Field theory of a complex scalar and a $U(1)$ vector field

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (D_\mu \phi)^* (D^\mu \phi) - \frac{\lambda}{4} \left(|\phi|^2 - v^2\right) \tag{10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + ie\, A_\mu$. Nielsen–Olesen string solution:

$$\phi(r, \varphi, z) = v\, f(r m_A)\, e^{in\varphi}\,, \qquad \boldsymbol{A}(r, \varphi, z) = \frac{n}{er}\, g(r m_A)\, \boldsymbol{e}_\varphi \tag{11}$$

with some complicated functions f and g; $f, g \to 0$ as $r \to 0$ and $f, g \to 1$ as $r \to \infty$.

Radial extent of the Nielsen–Olesen string is microscopically small:

- Magnetic core $\sim 1/m_A$, with vector-boson mass $m_A = \sqrt{2}\, ev$
- Scalar core $\sim 1/m_H$, with Higgs-boson mass $m_H = \sqrt{2\lambda}\, v$

Alternative description in terms of the Nambu–Goto action:

$$S_{\mathrm{NG}} = -\mu \int d^2\sigma \sqrt{-\gamma}\,, \qquad \gamma_{ab} = g_{\mu\nu} \frac{dx^\mu}{d\sigma^a} \frac{dx^\nu}{d\sigma^b} \tag{12}$$

- Action of an exactly one-dimensional featureless (super)string with tension μ

Theoretical description

Abelian Higgs model: Field theory of a complex scalar and a $U(1)$ vector field

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \left(D_\mu \phi\right)^* \left(D^\mu \phi\right) - \frac{\lambda}{4} \left(|\phi|^2 - v^2\right) \tag{10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + ie\, A_\mu$. Nielsen–Olesen string solution:

$$\phi\left(r, \varphi, z\right) = v\, f\left(r m_A\right) e^{in\varphi}\,, \qquad \boldsymbol{A}\left(r, \varphi, z\right) = \frac{n}{er}\, g\left(r m_A\right) \boldsymbol{e}_\varphi \tag{11}$$

with some complicated functions f and g; $f, g \rightarrow 0$ as $r \rightarrow 0$ and $f, g \rightarrow 1$ as $r \rightarrow \infty$.

Radial extent of the Nielsen–Olesen string is microscopically small:

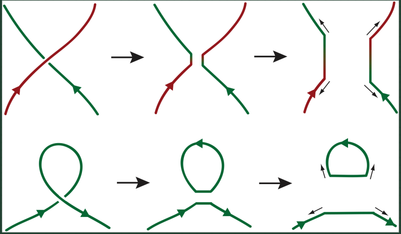
- Magnetic core $\sim 1/m_A$, with vector-boson mass $m_A = \sqrt{2}\, ev$
- Scalar core $\sim 1/m_H$, with Higgs-boson mass $m_H = \sqrt{2\lambda}\, v$

Alternative description in terms of the Nambu–Goto action:

$$S_{\mathrm{NG}} = -\mu \int d^2\sigma \sqrt{-\gamma}\,, \qquad \gamma_{ab} = g_{\mu\nu} \frac{dx^\mu}{d\sigma^a} \frac{dx^\nu}{d\sigma^b} \tag{12}$$

- Action of an exactly one-dimensional featureless (super)string with tension μ
- No knowledge about heavy bosonic DOFs or underlying particle physics in general

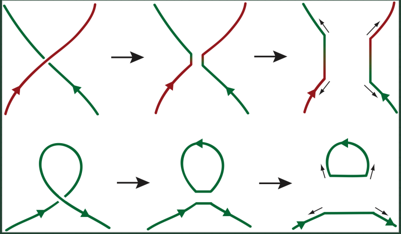
Cosmic-string loops



[Rocha: 0812.4020]

Interactions in the string network:

Cosmic-string loops

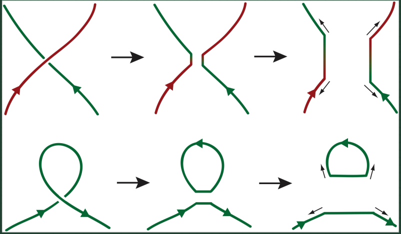


[Rocha: 0812.4020]

Interactions in the string network:

- Intercommutation events among two strings; exchange of string segments

Cosmic-string loops

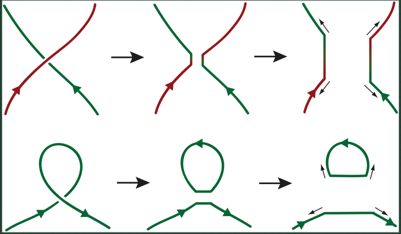


[Rocha: 0812.4020]

Interactions in the string network:

- Intercommutation events among two strings; exchange of string segments
- Loop production by single strings or pairs of strings; chopping off closed segments

Cosmic-string loops

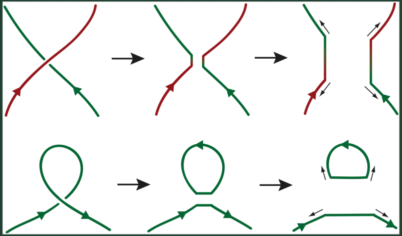


[Rocha: 0812.4020]

Interactions in the string network:

- Intercommutation events among two strings; exchange of string segments
- Loop production by single strings or pairs of strings; chopping off closed segments
- Interactions leave behind kinks traveling along the strings in opposite directions

Cosmic-string loops



[Rocha: 0812.4020]

Interactions in the string network:

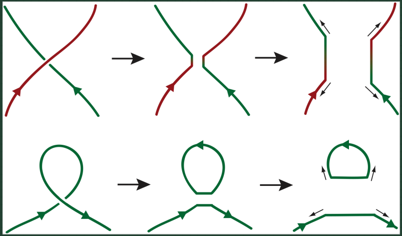
- Intercommutation events among two strings; exchange of string segments
- Loop production by single strings or pairs of strings; chopping off closed segments
- Interactions leave behind kinks traveling along the strings in opposite directions

Estimate GWs emitted by a cosmic-string loop by Einstein's quadrupole formula:

$$\dot{E}_{\mathrm{GW}} = \frac{G}{5c^5} \left\langle \dddot{Q}_{ij} \dddot{Q}_{ij} \right\rangle \,, \qquad Q_{ij}(t) = \int d^3\boldsymbol{x}\, \rho(t,\boldsymbol{x}) \left(x_i\, x_j - \frac{1}{3}\, r^2\, \delta_{ij} \right) \tag{13}$$

where $\dddot{Q}$ needs to be evaluated at the retarded time $t - r/c$.

Cosmic-string loops



[Rocha: 0812.4020]

Interactions in the string network:

- Intercommutation events among two strings; exchange of string segments
- Loop production by single strings or pairs of strings; chopping off closed segments
- Interactions leave behind kinks traveling along the strings in opposite directions

Estimate GWs emitted by a cosmic-string loop by Einstein's quadrupole formula:

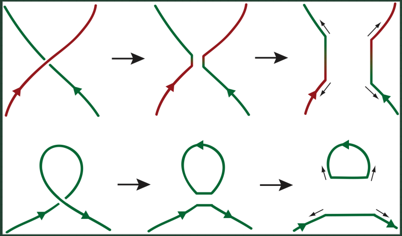
$$\dot{E}_{\mathrm{GW}} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle , \qquad Q_{ij}(t) = \int d^3\boldsymbol{x}\,\rho(t,\boldsymbol{x})\left(x_i\,x_j - \frac{1}{3}\,r^2\,\delta_{ij}\right) \tag{13}$$

where $\dddot{Q}$ needs to be evaluated at the retarded time $t - r/c$.

Consider a loop with

- Length ℓ, quadrupole moment $Q \sim M\ell^2$, mass $M = \mu\ell$, oscillation frequency $\omega \sim 1/\ell$

Cosmic-string loops



[Rocha: 0812.4020]

Interactions in the string network:

- Intercommutation events among two strings; exchange of string segments
- Loop production by single strings or pairs of strings; chopping off closed segments
- Interactions leave behind kinks traveling along the strings in opposite directions

Estimate GWs emitted by a cosmic-string loop by Einstein's quadrupole formula:

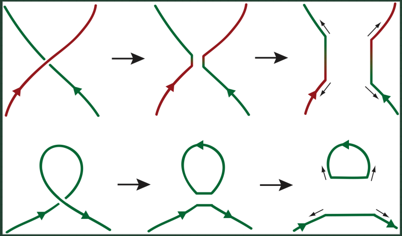
$$\dot{E}_{\rm GW} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle \,, \qquad Q_{ij}(t) = \int d^3\boldsymbol{x}\,\rho(t,\boldsymbol{x})\left(x_i\,x_j - \frac{1}{3}\,r^2\,\delta_{ij}\right) \tag{13}$$

where $\dddot{Q}$ needs to be evaluated at the retarded time $t - r/c$.

Consider a loop with

- Length ℓ, quadrupole moment $Q \sim M\ell^2$, mass $M = \mu\ell$, oscillation frequency $\omega \sim 1/\ell$
- Emitted power in GWs: $\dot{E} \sim G\,Q^2\omega^6 \sim G\,M^2\,\ell^4\,\ell^{-6} \sim G\mu^2$

Cosmic-string loops



[Rocha: 0812.4020]

Interactions in the string network:

- Intercommutation events among two strings; exchange of string segments
- Loop production by single strings or pairs of strings; chopping off closed segments
- Interactions leave behind kinks traveling along the strings in opposite directions

Estimate GWs emitted by a cosmic-string loop by Einstein's quadrupole formula:

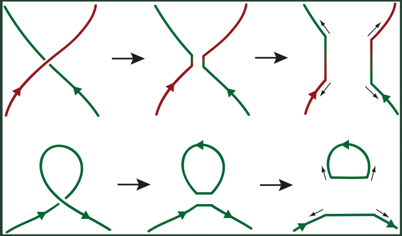
$$\dot{E}_{\mathrm{GW}} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle\,, \qquad Q_{ij}(t) = \int d^3\boldsymbol{x}\,\rho(t,\boldsymbol{x})\left(x_i\,x_j - \frac{1}{3}\,r^2\,\delta_{ij}\right) \tag{13}$$

where $\dddot{Q}$ needs to be evaluated at the retarded time $t - r/c$.

Consider a loop with

- Length ℓ, quadrupole moment $Q \sim M\ell^2$, mass $M = \mu\ell$, oscillation frequency $\omega \sim 1/\ell$
- Emitted power in GWs: $\dot{E} \sim G\,Q^2\omega^6 \sim G\,M^2\,\ell^4\,\ell^{-6} \sim G\mu^2$
- Captures correct parameter dependence, $\dot{E} = \Gamma\,G\mu^2$, with constant of proportionality Γ

Cosmic-string loops



[Rocha: 0812.4020]

Interactions in the string network:

- Intercommutation events among two strings; exchange of string segments
- Loop production by single strings or pairs of strings; chopping off closed segments
- Interactions leave behind kinks traveling along the strings in opposite directions

Estimate GWs emitted by a cosmic-string loop by Einstein's quadrupole formula:

$$\dot{E}_{\mathrm{GW}} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle\,, \qquad Q_{ij}(t) = \int d^3\boldsymbol{x}\,\rho(t,\boldsymbol{x})\left(x_i\,x_j - \frac{1}{3}\,r^2\,\delta_{ij}\right) \tag{13}$$

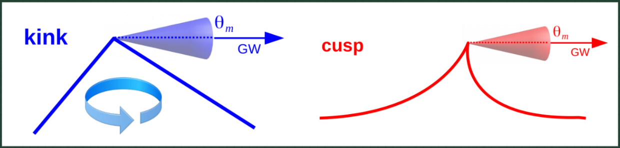
where $\dddot{Q}$ needs to be evaluated at the retarded time $t - r/c$.

Consider a loop with

- Length ℓ, quadrupole moment $Q \sim M\ell^2$, mass $M = \mu\ell$, oscillation frequency $\omega \sim 1/\ell$
- Emitted power in GWs: $\dot{E} \sim G\,Q^2\omega^6 \sim G\,M^2\,\ell^4\,\ell^{-6} \sim G\mu^2$
- Captures correct parameter dependence, $\dot{E} = \Gamma\,G\mu^2$, with constant of proportionality Γ
- Numerical simulations show that Γ is sharply peaked around $\Gamma \simeq 50$

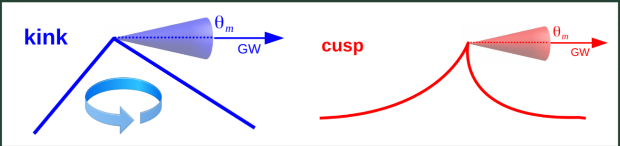
Kinks and cusps

[Pierre Auclair: talk on iap.fr]



Kinks and cusps

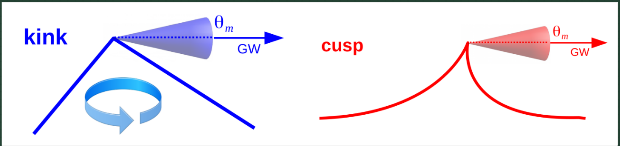
[Pierre Auclair: talk on iap.fr]



Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Kinks and cusps

[Pierre Auclair: talk on iap.fr]

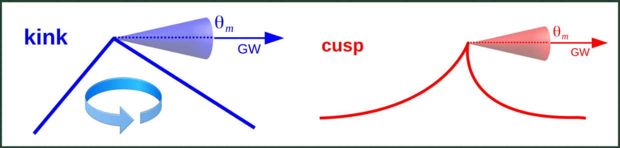


Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



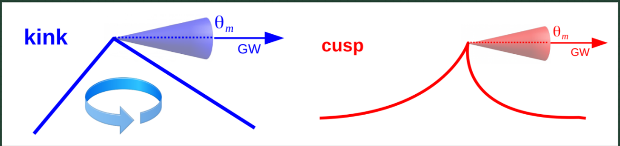
Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

- Cusps and kinks travel along the string at the speed of light

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



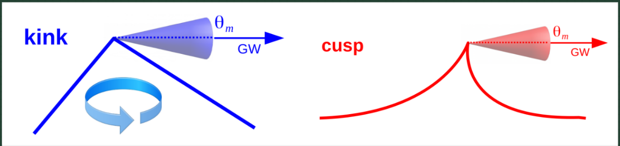
Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

- Cusps and kinks travel along the string at the speed of light
- Cusps, kinks, and kink–kink collisions emit bursts of GWs within a certain solid angle

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



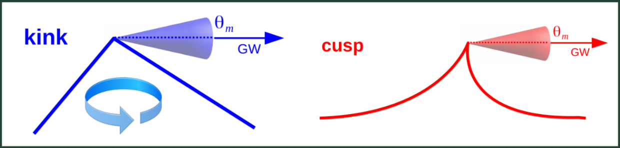
Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

- Cusps and kinks travel along the string at the speed of light
- Cusps, kinks, and kink–kink collisions emit bursts of GWs within a certain solid angle
- Superposition of individual GW bursts contributes to stochastic background

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

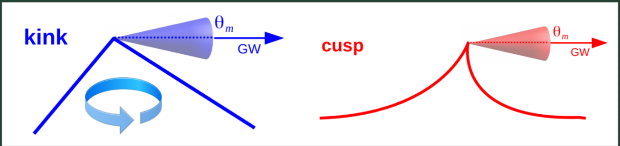
Cusps: String doubles back on itself

- Cusps and kinks travel along the string at the speed of light
- Cusps, kinks, and kink–kink collisions emit bursts of GWs within a certain solid angle
- Superposition of individual GW bursts contributes to stochastic background

On-going debate: What is the dominant energy loss mechanism of string loops?

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

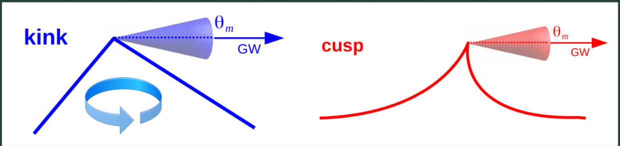
- Cusps and kinks travel along the string at the speed of light
- Cusps, kinks, and kink–kink collisions emit bursts of GWs within a certain solid angle
- Superposition of individual GW bursts contributes to stochastic background

On-going debate: What is the dominant energy loss mechanism of string loops?

- Nambu–Goto: GWs from loop oscillations + bursts from kinks and cusps only option

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

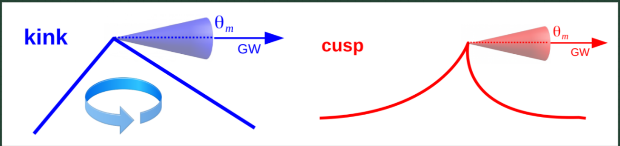
- Cusps and kinks travel along the string at the speed of light
- Cusps, kinks, and kink–kink collisions emit bursts of GWs within a certain solid angle
- Superposition of individual GW bursts contributes to stochastic background

On-going debate: What is the dominant energy loss mechanism of string loops?

- Nambu–Goto: GWs from loop oscillations + bursts from kinks and cusps only option
- Abelian-Higgs: Numerical simulations $\rightarrow$ efficient particle radiation, loops short-lived

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

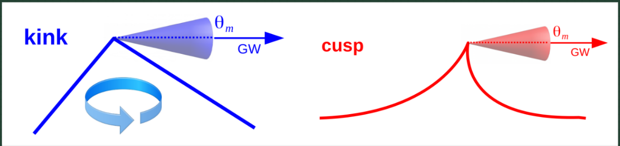
- Cusps and kinks travel along the string at the speed of light
- Cusps, kinks, and kink–kink collisions emit bursts of GWs within a certain solid angle
- Superposition of individual GW bursts contributes to stochastic background

On-going debate: What is the dominant energy loss mechanism of string loops?

- Nambu–Goto: GWs from loop oscillations + bursts from kinks and cusps only option
- Abelian-Higgs: Numerical simulations $\rightarrow$ efficient particle radiation, loops short-lived
- Many studies focus on GWs from Nambu–Goto strings (stronger signal), ...

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

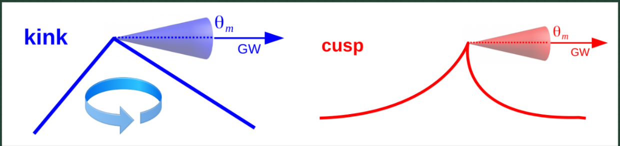
- Cusps and kinks travel along the string at the speed of light
- Cusps, kinks, and kink–kink collisions emit bursts of GWs within a certain solid angle
- Superposition of individual GW bursts contributes to stochastic background

On-going debate: What is the dominant energy loss mechanism of string loops?

- Nambu–Goto: GWs from loop oscillations + bursts from kinks and cusps only option
- Abelian-Higgs: Numerical simulations → efficient particle radiation, loops short-lived
- Many studies focus on GWs from Nambu–Goto strings (stronger signal), ...
- ... but more work is certainly needed to settle the remaining discrepancies!

Kinks and cusps

[Pierre Auclair: talk on iap.fr]



Kinks: Discontinuous jumps of the string tangent vector, formed in intercommutation events

Cusps: String doubles back on itself

- Cusps and kinks travel along the string at the speed of light
- Cusps, kinks, and kink–kink collisions emit bursts of GWs within a certain solid angle
- Superposition of individual GW bursts contributes to stochastic background

On-going debate: What is the dominant energy loss mechanism of string loops?

- Nambu–Goto: GWs from loop oscillations + bursts from kinks and cusps only option
- Abelian-Higgs: Numerical simulations $\to$ efficient particle radiation, loops short-lived
- Many studies focus on GWs from Nambu–Goto strings (stronger signal), ...
- ... but more work is certainly needed to settle the remaining discrepancies!

We will restrict ourselves to Nambu–Goto strings in the following.

Stochastic GW background

GW power emitted by a loop of length ℓ per frequency bin df (recall quadrupole formula):

$$P_{\mathrm{GW}}(\ell, f) = \frac{d^2 E}{dt df} = G\mu^2 \ell\, P_k \qquad (14)$$

Stochastic GW background

GW power emitted by a loop of length ℓ per frequency bin df (recall quadrupole formula):

$$P_{\mathrm{GW}}(\ell, f) = \frac{d^2E}{dt\,df} = G\mu^2 \ell\, P_k \tag{14}$$

where k labels the harmonic excitations of the loop, which oscillate at frequencies

$$f = \frac{2k}{\ell} \tag{15}$$

Stochastic GW background

GW power emitted by a loop of length ℓ per frequency bin df (recall quadrupole formula):

$$P_{\rm GW}\left(\ell, f\right) = \frac{d^2E}{dtdf} = G\mu^2 \ell\, P_k \tag{14}$$

where k labels the harmonic excitations of the loop, which oscillate at frequencies

$$f = \frac{2k}{\ell} \tag{15}$$

Dimensionless power spectrum describing the GW emission by the kth mode of a loop:

$$P_k = \frac{\Gamma}{\zeta\left(q\right)}\frac{1}{k^q}\,, \qquad \sum_{k=1}^{\infty} P_k = \frac{\Gamma}{\zeta\left(q\right)}\sum_{k=1}^{\infty}\frac{1}{k^q} = \Gamma \simeq 50 \tag{16}$$

where $q = 4/3$ for cusps, $q = 5/3$ for kinks, and $q = 2$ for kink–kink collisions at large k.

Stochastic GW background

GW power emitted by a loop of length ℓ per frequency bin df (recall quadrupole formula):

$$P_{\rm GW}\left(\ell, f\right) = \frac{d^2 E}{dt df} = G\mu^2 \ell\, P_k \tag{14}$$

where k labels the harmonic excitations of the loop, which oscillate at frequencies

$$f = \frac{2k}{\ell} \tag{15}$$

Dimensionless power spectrum describing the GW emission by the kth mode of a loop:

$$P_k = \frac{\Gamma}{\zeta\left(q\right)}\,\frac{1}{k^q}\,, \qquad \sum_{k=1}^{\infty} P_k = \frac{\Gamma}{\zeta\left(q\right)} \sum_{k=1}^{\infty} \frac{1}{k^q} = \Gamma \simeq 50 \tag{16}$$

where $q = 4/3$ for cusps, $q = 5/3$ for kinks, and $q = 2$ for kink–kink collisions at large k.

Weight $P_{\rm GW}$ with loop number density n, integrate over all lengths ℓ and emission times t:

$$\Omega_{\rm GW}\left(f\right) = \frac{G\mu^2}{\rho_c}\, f \int_0^{t_0} dt \left(\frac{a}{a_0}\right)^4 \int_0^{\infty} d\ell\,\ell\, n\left(\ell, t\right) P_{k'}\,, \qquad n(\ell, t) = \frac{d\#}{V d\ell} \tag{17}$$

where $k' = f'\ell/2$ with $f' = a_0/a\, f$ accounts for the frequency redshift after emission

Expressions for the GW signal

Rewrite time integral as an integral over redshift, replace integral over ℓ by discrete sum:

$$\boxed{\Omega_{\mathrm{GW}}(f) = \frac{G\mu^2}{\rho_c}\sum_{k=1}^{\infty} C_k(f)\, P_k\,, \qquad C_k(f) = \frac{2k}{f}\int_0^{\infty} dz\, \frac{n(\ell(z), t(z))}{H(z)(1+z)^6}} \tag{18}$$

where $\ell(z) = 2k/\left[(1+z)\,f\right]$ and the usual time–redshift relation $t(z)$.

Expressions for the GW signal

Rewrite time integral as an integral over redshift, replace integral over ℓ by discrete sum:

$$\boxed{\Omega_{\mathrm{GW}}(f) = \frac{G\mu^2}{\rho_c} \sum_{k=1}^{\infty} C_k(f)\, P_k\,, \qquad C_k(f) = \frac{2k}{f} \int_0^{\infty} dz\, \frac{n(\ell(z), t(z))}{H(z)(1+z)^6}} \tag{18}$$

where $\ell(z) = 2k/[(1+z)f]$ and the usual time–redshift relation $t(z)$.

- Total signal is superposition of GW emission from all oscillation modes

Expressions for the GW signal

Rewrite time integral as an integral over redshift, replace integral over ℓ by discrete sum:

$$\boxed{\Omega_{\rm GW}(f) = \frac{G\mu^2}{\rho_c} \sum_{k=1}^{\infty} C_k(f)\, P_k\,, \qquad C_k(f) = \frac{2k}{f} \int_0^{\infty} dz\, \frac{n(\ell(z)\,, t(z))}{H(z)\,(1+z)^6}} \tag{18}$$

where $\ell(z) = 2k/\left[(1+z)\, f\right]$ and the usual time–redshift relation $t(z)$.

- Total signal is superposition of GW emission from all oscillation modes
- In general, one needs to sum up to large values of k for a reliable prediction

Expressions for the GW signal

Rewrite time integral as an integral over redshift, replace integral over ℓ by discrete sum:

$$\boxed{\Omega_{\rm GW}\left(f\right) = \frac{G\mu^2}{\rho_{\rm c}} \sum_{k=1}^{\infty} C_k\left(f\right) P_k \,, \qquad C_k\left(f\right) = \frac{2k}{f} \int_0^{\infty} dz\, \frac{n\left(\ell\left(z\right), t\left(z\right)\right)}{H\left(z\right)\left(1+z\right)^6}} \tag{18}$$

where $\ell\left(z\right) = 2k/\left[\left(1+z\right) f\right]$ and the usual time–redshift relation $t\left(z\right)$.

- Total signal is superposition of GW emission from all oscillation modes
- In general, one needs to sum up to large values of k for a reliable prediction
- $\Omega_{\rm GW}\left(f\right)$ receives contributions from the kth modes of all loops of length $\ell(z)$ at z

Expressions for the GW signal

Rewrite time integral as an integral over redshift, replace integral over ℓ by discrete sum:

$$\boxed{\Omega_{\mathrm{GW}}\left(f\right) = \frac{G\mu^2}{\rho_c}\sum_{k=1}^{\infty} C_k\left(f\right)P_k\,, \qquad C_k\left(f\right) = \frac{2k}{f}\int_0^\infty dz\,\frac{n\left(\ell\left(z\right),t\left(z\right)\right)}{H\left(z\right)\left(1+z\right)^6}} \tag{18}$$

where $\ell\left(z\right) = 2k/\left[\left(1+z\right)f\right]$ and the usual time–redshift relation $t\left(z\right)$.

- Total signal is superposition of GW emission from all oscillation modes
- In general, one needs to sum up to large values of k for a reliable prediction
- $\Omega_{\mathrm{GW}}\left(f\right)$ receives contributions from the kth modes of all loops of length $\ell(z)$ at z
- All the nontrivial physics is contained in the loop number density n

Expressions for the GW signal

Rewrite time integral as an integral over redshift, replace integral over ℓ by discrete sum:

$$\boxed{\Omega_{\mathrm{GW}}\left(f\right) = \frac{G\mu^2}{\rho_{\mathrm{c}}} \sum_{k=1}^{\infty} C_k\left(f\right) P_k \,, \qquad C_k\left(f\right) = \frac{2k}{f} \int_0^{\infty} dz\, \frac{n\left(\ell\left(z\right), t\left(z\right)\right)}{H\left(z\right)\left(1+z\right)^6}} \tag{18}$$

where $\ell\left(z\right) = 2k/\left[\left(1+z\right) f\right]$ and the usual time–redshift relation $t\left(z\right)$.

- Total signal is superposition of GW emission from all oscillation modes
- In general, one needs to sum up to large values of k for a reliable prediction
- $\Omega_{\mathrm{GW}}\left(f\right)$ receives contributions from the kth modes of all loops of length $\ell(z)$ at z
- All the nontrivial physics is contained in the loop number density n

Alternative approach: Integrate over differential rate of GW bursts from cusps and kinks

$$\boxed{\Omega_{\mathrm{GW}}\left(f\right) = \frac{f^3}{\rho_{\mathrm{c}}} \int_{z_{\mathrm{min}}}^{\infty} dz \int_0^{\infty} d\ell\, h^2\left(\ell, z, f\right) \frac{d^2 R\left(z,\ell\right)}{dz d\ell}} \tag{19}$$

with burst rate per loop length + redshift bin, $d^2 R/(dz d\ell) \propto n$, and waveform amplitude h.

Expressions for the GW signal

Rewrite time integral as an integral over redshift, replace integral over ℓ by discrete sum:

$$\boxed{\Omega_{\mathrm{GW}}(f) = \frac{G\mu^2}{\rho_c}\sum_{k=1}^{\infty} C_k(f)\,P_k\,, \qquad C_k(f) = \frac{2k}{f}\int_0^\infty dz\,\frac{n(\ell(z),t(z))}{H(z)(1+z)^6}} \tag{18}$$

where $\ell(z) = 2k/[(1+z)f]$ and the usual time–redshift relation $t(z)$.

- Total signal is superposition of GW emission from all oscillation modes
- In general, one needs to sum up to large values of k for a reliable prediction
- $\Omega_{\mathrm{GW}}(f)$ receives contributions from the kth modes of all loops of length $\ell(z)$ at z
- All the nontrivial physics is contained in the loop number density n

Alternative approach: Integrate over differential rate of GW bursts from cusps and kinks

$$\boxed{\Omega_{\mathrm{GW}}(f) = \frac{f^3}{\rho_c}\int_{z_{\min}}^{\infty} dz\int_0^\infty d\ell\,h^2(\ell,z,f)\,\frac{d^2R(z,\ell)}{dz\,d\ell}} \tag{19}$$

with burst rate per loop length + redshift bin, $d^2R/(dz\,d\ell) \propto n$, and waveform amplitude h.

Typically, good agreement between both methods if parameters are properly matched!

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \frac{\partial}{\partial \ell} + 3H\right] n\left(t,\ell\right) = f\left(t,\ell\right) \tag{20}$$

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \frac{\partial}{\partial \ell} + 3H\right] n\left(t,\ell\right) = f\left(t,\ell\right) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu\left(t - t'\right)$

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \frac{\partial}{\partial \ell} + 3H\right] n\left(t,\ell\right) = f\left(t,\ell\right) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu \left(t - t'\right)$
- Number density scales like a^{-3} in consequence of the cosmic expansion

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \frac{\partial}{\partial \ell} + 3H\right] n\left(t, \ell\right) = f\left(t, \ell\right) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu \left(t - t'\right)$
- Number density scales like a^{-3} in consequence of the cosmic expansion
- Production of new loops from long strings described by loop production function f

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \frac{\partial}{\partial \ell} + 3H\right] n\left(t,\ell\right) = f\left(t,\ell\right) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu\left(t - t'\right)$
- Number density scales like a^{-3} in consequence of the cosmic expansion
- Production of new loops from long strings described by loop production function f

General solution, valid after the onset of the scaling regime at $t_{\rm ini}$:

$$n\left(t,\ell\right) = \int_{t_{\rm ini}}^{t} dt' \left[\frac{a\left(t'\right)}{a\left(t\right)}\right]^3 f\left(t',\ell'\right) , \qquad \ell' = \ell + \Gamma G\mu\left(t - t'\right) \tag{21}$$

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \, \frac{\partial}{\partial \ell} + 3H\right] n\,(t,\ell) = f\,(t,\ell) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu\,(t - t')$
- Number density scales like a^{-3} in consequence of the cosmic expansion
- Production of new loops from long strings described by loop production function f

General solution, valid after the onset of the scaling regime at t_{ini}:

$$n\,(t,\ell) = \int_{t_{\mathrm{ini}}}^{t} dt' \left[\frac{a\,(t')}{a\,(t)}\right]^3 f\left(t',\ell'\right) \,, \qquad \ell' = \ell + \Gamma G\mu \left(t - t'\right) \tag{21}$$

Models for the loop number density n and loop production function f:

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \frac{\partial}{\partial \ell} + 3H\right] n(t,\ell) = f(t,\ell) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu\,(t - t')$
- Number density scales like a^{-3} in consequence of the cosmic expansion
- Production of new loops from long strings described by loop production function f

General solution, valid after the onset of the scaling regime at t_{ini}:

$$n(t,\ell) = \int_{t_{\mathrm{ini}}}^{t} dt' \left[\frac{a(t')}{a(t)}\right]^3 f\left(t',\ell'\right) , \qquad \ell' = \ell + \Gamma G\mu \left(t - t'\right) \tag{21}$$

Models for the loop number density n and loop production function f:

- VOS: velocity-dependent one-scale model, analytical, calibrated by simulations

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \frac{\partial}{\partial \ell} + 3H\right] n\left(t, \ell\right) = f\left(t, \ell\right) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu\left(t - t'\right)$
- Number density scales like a^{-3} in consequence of the cosmic expansion
- Production of new loops from long strings described by loop production function f

General solution, valid after the onset of the scaling regime at $t_{\rm ini}$:

$$n\left(t, \ell\right) = \int_{t_{\rm ini}}^{t} dt' \left[\frac{a\left(t'\right)}{a\left(t\right)}\right]^3 f\left(t', \ell'\right) \,, \qquad \ell' = \ell + \Gamma G\mu\left(t - t'\right) \tag{21}$$

Models for the loop number density n and loop production function f:

- VOS: velocity-dependent one-scale model, analytical, calibrated by simulations
- BOS: Blanco-Pillado–Olum–Shlaer, large-scale numerical simulation of the network

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \frac{\partial}{\partial \ell} + 3H\right] n\left(t,\ell\right) = f\left(t,\ell\right) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu\left(t - t'\right)$
- Number density scales like a^{-3} in consequence of the cosmic expansion
- Production of new loops from long strings described by loop production function f

General solution, valid after the onset of the scaling regime at t_{ini}:

$$n\left(t,\ell\right) = \int_{t_{\text{ini}}}^{t} dt' \left[\frac{a\left(t'\right)}{a\left(t\right)}\right]^3 f\left(t',\ell'\right) \,, \qquad \ell' = \ell + \Gamma G\mu\left(t - t'\right) \tag{21}$$

Models for the loop number density n and loop production function f:

- VOS: velocity-dependent one-scale model, analytical, calibrated by simulations
- BOS: Blanco-Pillado–Olum–Shlaer, large-scale numerical simulation of the network
- LRS: Lorenz–Ringeval–Sakellariadou, numerical, focus on small-scale structure

Loop number density

Evolution of the loop number density described by a partial differential equation:

$$\left[\frac{\partial}{\partial t} - \Gamma G\mu \, \frac{\partial}{\partial \ell} + 3H\right] n\,(t,\ell) = f\,(t,\ell) \tag{20}$$

- Loops shrink due to GW emission: $\dot{\ell} = -\Gamma G\mu$ such that $\ell = \ell' - \Gamma G\mu\,(t - t')$
- Number density scales like a^{-3} in consequence of the cosmic expansion
- Production of new loops from long strings described by loop production function f

General solution, valid after the onset of the scaling regime at t_{ini}:

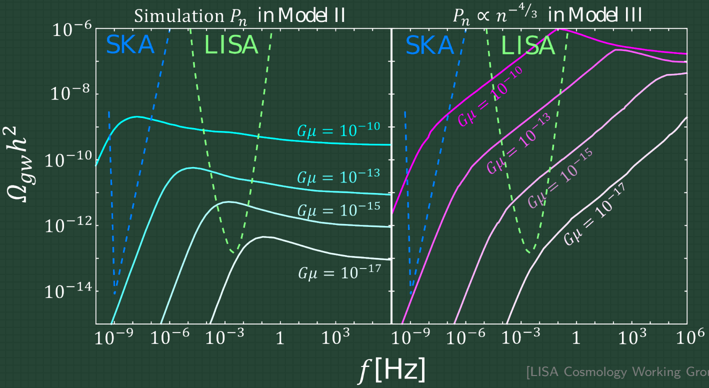
$$n\,(t,\ell) = \int_{t_{\text{ini}}}^{t} dt' \left[\frac{a\,(t')}{a\,(t)}\right]^3 f\left(t',\ell'\right)\,, \qquad \ell' = \ell + \Gamma G\mu\left(t - t'\right) \tag{21}$$

Models for the loop number density n and loop production function f:

- VOS: velocity-dependent one-scale model, analytical, calibrated by simulations
- BOS: Blanco-Pillado–Olum–Shlaer, large-scale numerical simulation of the network
- LRS: Lorenz–Ringeval–Sakellariadou, numerical, focus on small-scale structure

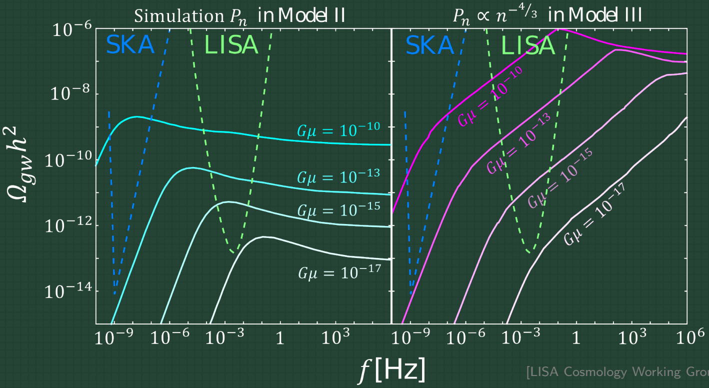
Good agreement between VOS and BOS; predictions of LRS model qualitatively different!

GW spectrum



[LISA Cosmology Working Group: to appear]

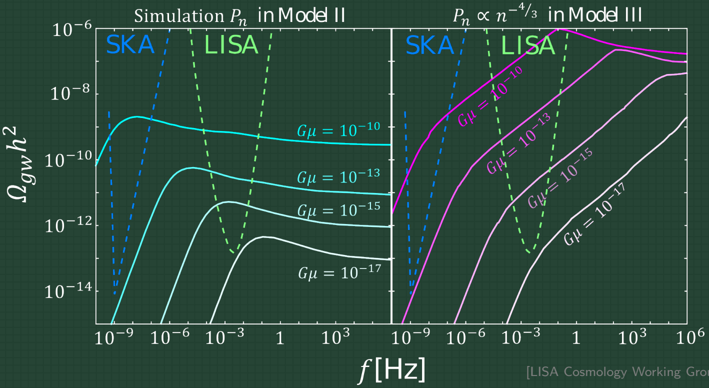
GW spectrum



[LISA Cosmology Working Group: to appear]

- BOS (left): Production and decay during RD, plus RD $\rightarrow$ MD, plus MD $\rightarrow$ MD

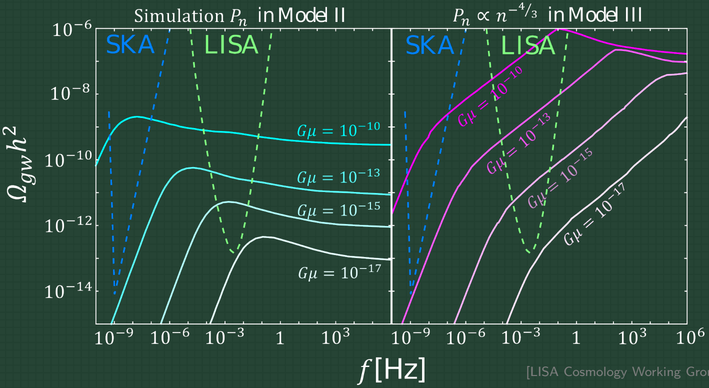
GW spectrum



[LISA Cosmology Working Group: to appear]

- BOS (left): Production and decay during RD, plus RD $\to$ MD, plus MD $\to$ MD
- LRS (right): Two extra contributions, small loops during RD and small loops during MD

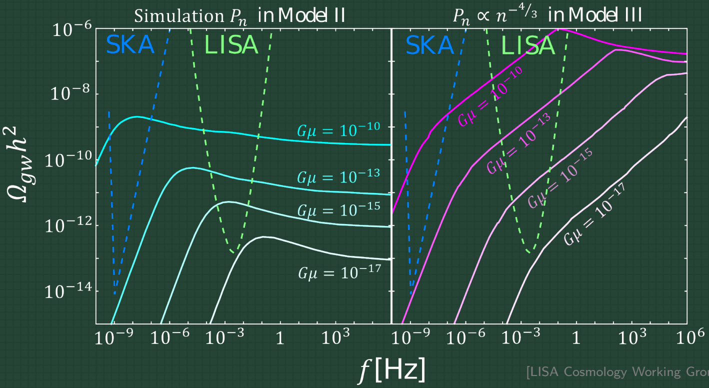
GW spectrum



[LISA Cosmology Working Group: to appear]

- BOS (left): Production and decay during RD, plus RD $\to$ MD, plus MD $\to$ MD
- LRS (right): Two extra contributions, small loops during RD and small loops during MD
- Common features: $f^{3/2}$ rise at low f, plateau at large f, LISA sensitive to $G\mu \gtrsim 10^{-17}$

GW spectrum



[LISA Cosmology Working Group: to appear]

- BOS (left): Production and decay during RD, plus RD $\to$ MD, plus MD $\to$ MD
- LRS (right): Two extra contributions, small loops during RD and small loops during MD
- Common features: $f^{3/2}$ rise at low f, plateau at large f, LISA sensitive to $G\mu \gtrsim 10^{-17}$
- LISA will be able to distinguish between both models (if no further theoretical progress)

Loop size

Key assumptions of the VOS model:

Loop size

Key assumptions of the VOS model:

1. Loops predominantly lose energy via GW emission

Loop size

Key assumptions of the VOS model:

1. Loops predominantly lose energy via GW emission
2. Initial loop size at production is a fixed fraction of characteristic length scale $L \sim H^{-1}$

Loop size

Key assumptions of the VOS model:

1. Loops predominantly lose energy via GW emission
2. Initial loop size at production is a fixed fraction of characteristic length scale $L \sim H^{-1}$

Loop production function, normalization fixed by the VOS model in the scaling regime:

$$f\,(t,\ell) \propto \frac{1}{\ell\, L^3}\,\delta\,(\ell - \alpha/H)\ , \qquad \alpha = \frac{\ell}{H^{-1}} = \text{const}\,, \qquad \alpha_L = \frac{\ell}{L} = \text{const} \tag{22}$$

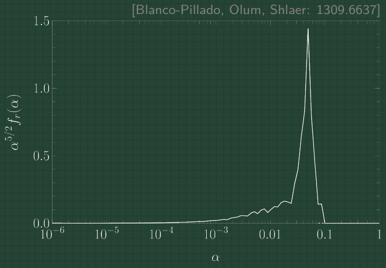
Loop size

Key assumptions of the VOS model:

1. Loops predominantly lose energy via GW emission
2. Initial loop size at production is a fixed fraction of characteristic length scale $L \sim H^{-1}$

Loop production function, normalization fixed by the VOS model in the scaling regime:

$$f(t,\ell) \propto \frac{1}{\ell\, L^3}\,\delta\left(\ell - \alpha/H\right)\,, \qquad \alpha = \frac{\ell}{H^{-1}} = \text{const}\,, \qquad \alpha_L = \frac{\ell}{L} = \text{const} \tag{22}$$



- Peak in the initial loop size at $\alpha \sim 0.01 \cdots 0.1$ well confirmed in numerical simulations

Loop size

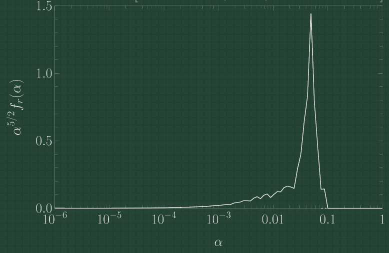
Key assumptions of the VOS model:

1. Loops predominantly lose energy via GW emission
2. Initial loop size at production is a fixed fraction of characteristic length scale $L \sim H^{-1}$

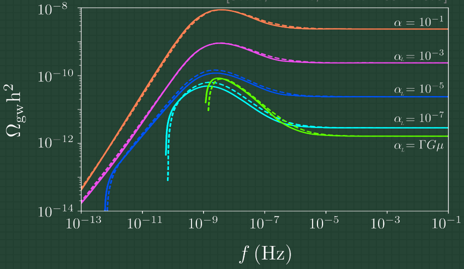
Loop production function, normalization fixed by the VOS model in the scaling regime:

$$f\,(t,\ell) \propto \frac{1}{\ell\, L^3}\,\delta\,(\ell - \alpha/H)\ , \qquad \alpha = \frac{\ell}{H^{-1}} = \mathrm{const}\,, \qquad \alpha_L = \frac{\ell}{L} = \mathrm{const} \tag{22}$$

[Blanco-Pillado, Olum, Shlaer: 1309.6637]



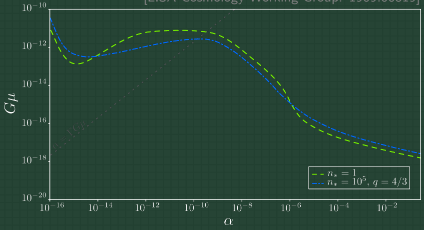
[Sousa, Avelino, Guedes: 2002.01079]



- Peak in the initial loop size at $\alpha \sim 0.01 \cdots 0.1$ well confirmed in numerical simulations
- GW signal decreases for smaller initial loop size, $\Omega_{\rm GW} \propto (\alpha\, G\mu)^{1/2}$, if $\alpha \gg \Gamma G\mu$

Reach of GW interferometers

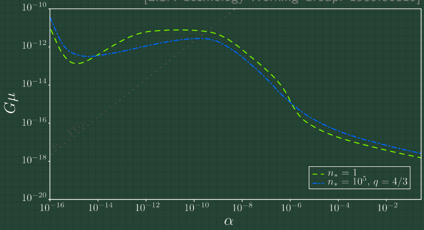
[LISA Cosmology Working Group: 1909.00819]



Present and future bounds on $G\mu$:

Reach of GW interferometers

[LISA Cosmology Working Group: 1909.00819]

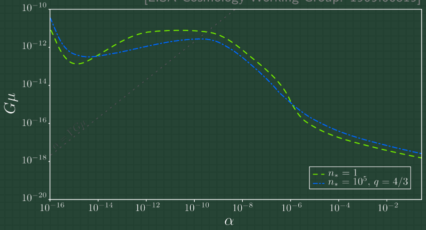


Present and future bounds on $G\mu$:

- CMB power spectra: $G\mu \lesssim 10^{-7}$

Reach of GW interferometers

[LISA Cosmology Working Group: 1909.00819]

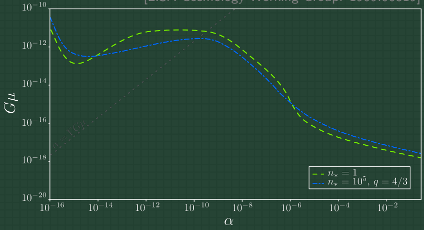


Present and future bounds on $G\mu$:

- CMB power spectra: $G\mu \lesssim 10^{-7}$
- LISA, VOS, $\alpha \sim 0.1$: $G\mu \lesssim 10^{-17}$

Reach of GW interferometers

[LISA Cosmology Working Group: 1909.00819]

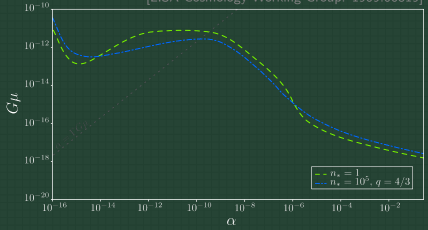


Present and future bounds on $G\mu$:

- CMB power spectra: $G\mu \lesssim 10^{-7}$
- LISA, VOS, $\alpha \sim 0.1$: $G\mu \lesssim 10^{-17}$
- LISA, VOS, any α: $G\mu \lesssim 10^{-11}$

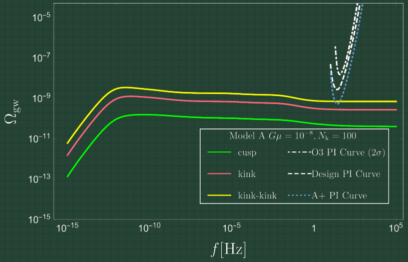
Reach of GW interferometers

[LISA Cosmology Working Group: 1909.00819]

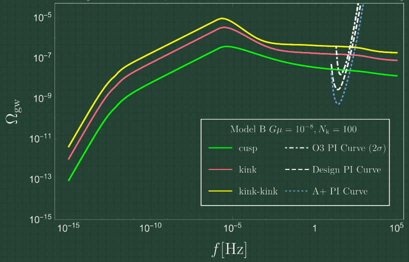


Present and future bounds on $G\mu$:

- CMB power spectra: $G\mu \lesssim 10^{-7}$
- LISA, VOS, $\alpha \sim 0.1$: $G\mu \lesssim 10^{-17}$
- LISA, VOS, any α: $G\mu \lesssim 10^{-11}$
- LIGO + Virgo O3, BOS: $G\mu \lesssim 10^{-7}$

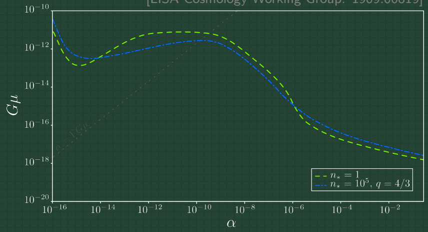


[LIGO + Virgo + Kagra Collaborations: 2101.12248]



Reach of GW interferometers

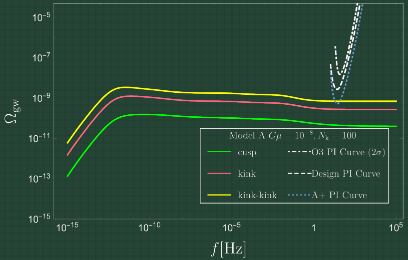
[LISA Cosmology Working Group: 1909.00819]

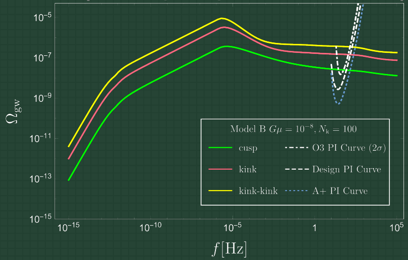


Present and future bounds on $G\mu$:

- CMB power spectra: $G\mu \lesssim 10^{-7}$
- LISA, VOS, $\alpha \sim 0.1$: $G\mu \lesssim 10^{-17}$
- LISA, VOS, any α: $G\mu \lesssim 10^{-11}$
- LIGO + Virgo O3, BOS: $G\mu \lesssim 10^{-7}$
- LIGO + Virgo O3, LRS: $G\mu \lesssim 10^{-14}$

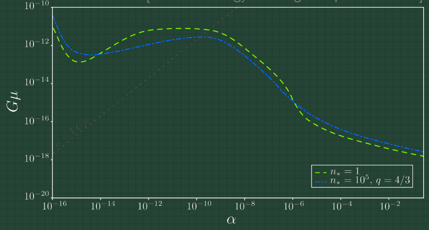
[LIGO + Virgo + Kagra Collaborations: 2101.12248]





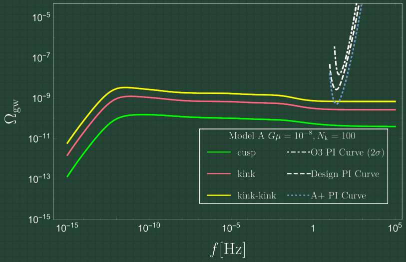
Reach of GW interferometers

[LISA Cosmology Working Group: 1909.00819]

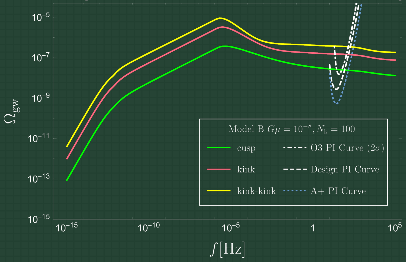


Present and future bounds on $G\mu$:

- CMB power spectra: $G\mu \lesssim 10^{-7}$
- LISA, VOS, $\alpha \sim 0.1$: $G\mu \lesssim 10^{-17}$
- LISA, VOS, any α: $G\mu \lesssim 10^{-11}$
- LIGO + Virgo O3, BOS: $G\mu \lesssim 10^{-7}$
- LIGO + Virgo O3, LRS: $G\mu \lesssim 10^{-14}$



[LIGO + Virgo + Kagra Collaborations: 2101.12248]



Huge discovery potential with existing, planned, and next-generation interferometers!

Pulsar timing arrays

PTA constraints on the string tension $G\mu$, loop size α, and intercommutation probability p:

Pulsar timing arrays

PTA constraints on the string tension $G\mu$, loop size α, and intercommutation probability p:

- $p < 1$ for cosmic superstrings in string theory (quantum nature, extra dimensions)

Pulsar timing arrays

PTA constraints on the string tension $G\mu$, loop size α, and intercommutation probability p:

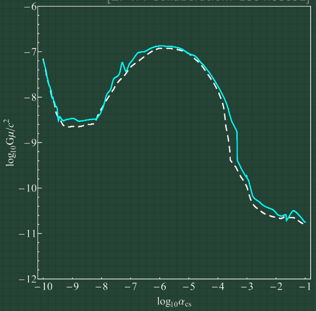
- $p < 1$ for cosmic superstrings in string theory (quantum nature, extra dimensions)
- String network loses energy less efficiently, stronger GW signal, $\Omega_{\mathrm{GW}} \propto p^{-0.6} \cdots p^{-1.0}$

Pulsar timing arrays

PTA constraints on the string tension $G\mu$, loop size α, and intercommutation probability p:

- $p < 1$ for cosmic superstrings in string theory (quantum nature, extra dimensions)
- String network loses energy less efficiently, stronger GW signal, $\Omega_{\mathrm{GW}} \propto p^{-0.6} \cdots p^{-1.0}$

[EPTA Collaboration: 1504.03692]



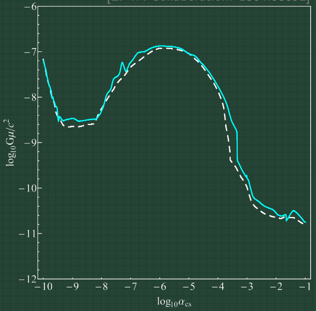
- EPTA 2015, VOS (Sanidas, Battye, Stappers: 1211.5042): $G\mu \lesssim 3.0 \times 10^{-11}$

Pulsar timing arrays

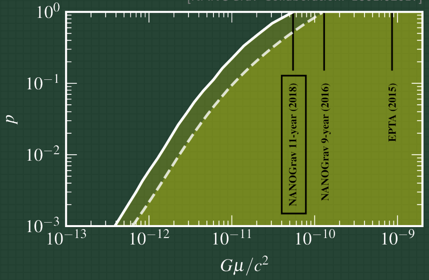
PTA constraints on the string tension $G\mu$, loop size α, and intercommutation probability p:

- $p < 1$ for cosmic superstrings in string theory (quantum nature, extra dimensions)
- String network loses energy less efficiently, stronger GW signal, $\Omega_{\rm GW} \propto p^{-0.6} \cdots p^{-1.0}$

[EPTA Collaboration: 1504.03692]



[NANOGrav Collaboration: 1801.02617]



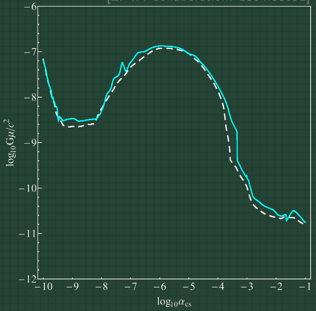
- EPTA 2015, VOS (Sanidas, Battye, Stappers: 1211.5042): $G\mu \lesssim 3.0 \times 10^{-11}$
- EPTA 2015, BOS: $G\mu \lesssim 8.6 \times 10^{-10}$

Pulsar timing arrays

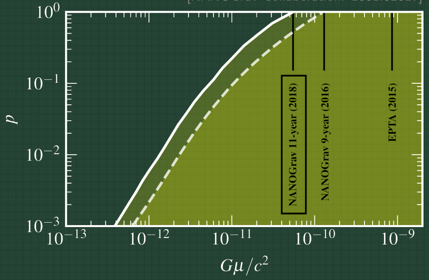
PTA constraints on the string tension $G\mu$, loop size α, and intercommutation probability p:

- $p < 1$ for cosmic superstrings in string theory (quantum nature, extra dimensions)
- String network loses energy less efficiently, stronger GW signal, $\Omega_{\mathrm{GW}} \propto p^{-0.6} \cdots p^{-1.0}$

[EPTA Collaboration: 1504.03692]



[NANOGrav Collaboration: 1801.02617]



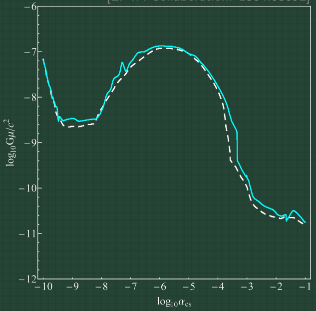
- EPTA 2015, VOS (Sanidas, Battye, Stappers: 1211.5042): $G\mu \lesssim 3.0 \times 10^{-11}$
- EPTA 2015, BOS: $G\mu \lesssim 8.6 \times 10^{-10}$
- NANOGrav 2016 (9-year data set), BOS, $p = 1$: $G\mu \lesssim 1.3 \times 10^{-10}$

Pulsar timing arrays

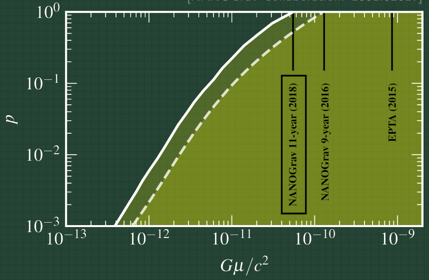
PTA constraints on the string tension $G\mu$, loop size α, and intercommutation probability p:

- $p < 1$ for cosmic superstrings in string theory (quantum nature, extra dimensions)
- String network loses energy less efficiently, stronger GW signal, $\Omega_{\rm GW} \propto p^{-0.6} \cdots p^{-1.0}$

[EPTA Collaboration: 1504.03692]



[NANOGrav Collaboration: 1801.02617]



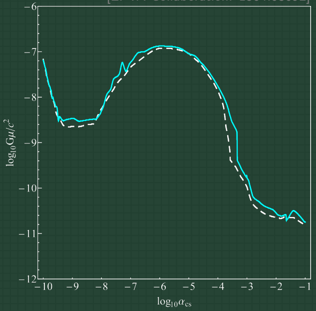
- EPTA 2015, VOS (Sanidas, Battye, Stappers: 1211.5042): $G\mu \lesssim 3.0 \times 10^{-11}$
- EPTA 2015, BOS: $G\mu \lesssim 8.6 \times 10^{-10}$
- NANOGrav 2016 (9-year data set), BOS, $p = 1$: $G\mu \lesssim 1.3 \times 10^{-10}$
- NANOGrav 2018 (11-year data set), BOS, $p = 1$: $G\mu \lesssim 5.3 \times 10^{-11}$

Pulsar timing arrays

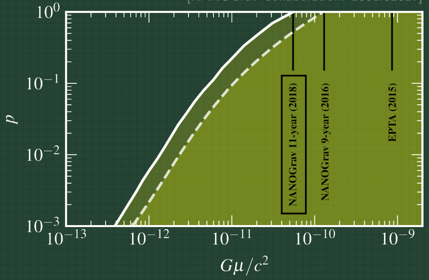
PTA constraints on the string tension $G\mu$, loop size α, and intercommutation probability p:

- $p < 1$ for cosmic superstrings in string theory (quantum nature, extra dimensions)
- String network loses energy less efficiently, stronger GW signal, $\Omega_{\mathrm{GW}} \propto p^{-0.6} \cdots p^{-1.0}$

[EPTA Collaboration: 1504.03692]



[NANOGrav Collaboration: 1801.02617]



- EPTA 2015, VOS (Sanidas, Battye, Stappers: 1211.5042): $G\mu \lesssim 3.0 \times 10^{-11}$
- EPTA 2015, BOS: $G\mu \lesssim 8.6 \times 10^{-10}$
- NANOGrav 2016 (9-year data set), BOS, $p = 1$: $G\mu \lesssim 1.3 \times 10^{-10}$
- NANOGrav 2018 (11-year data set), BOS, $p = 1$: $G\mu \lesssim 5.3 \times 10^{-11}$

PTAs yield the strongest constraints on the cosmic-string parameter space!

Summary Lecture 3A

Take-home messages:

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.
- Networks reach scaling regime: Attractor solution with H^{-1} as the only relevant scale.

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.
- Networks reach scaling regime: Attractor solution with H^{-1} as the only relevant scale.
- Irreducible background of GWs from any network, characteristic shape (f^3, f^{-2}, f^0).

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.
- Networks reach scaling regime: Attractor solution with H^{-1} as the only relevant scale.
- Irreducible background of GWs from any network, characteristic shape (f^3, f^{-2}, f^0).
- Cosmic strings are particularly well motivated: Extra GWs from loops, no overclosure.

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.
- Networks reach scaling regime: Attractor solution with H^{-1} as the only relevant scale.
- Irreducible background of GWs from any network, characteristic shape $(f^3,\ f^{-2},\ f^0)$.
- Cosmic strings are particularly well motivated: Extra GWs from loops, no overclosure.
- Two approaches: Abelian-Higgs and Nambu–Goto; particle versus GW emission?

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.
- Networks reach scaling regime: Attractor solution with H^{-1} as the only relevant scale.
- Irreducible background of GWs from any network, characteristic shape $(f^3,\ f^{-2},\ f^0)$.
- Cosmic strings are particularly well motivated: Extra GWs from loops, no overclosure.
- Two approaches: Abelian-Higgs and Nambu–Goto; particle versus GW emission?
- Compute GW signal based on averaged power spectrum P_n or superposition of bursts.

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.
- Networks reach scaling regime: Attractor solution with H^{-1} as the only relevant scale.
- Irreducible background of GWs from any network, characteristic shape (f^3, f^{-2}, f^0).
- Cosmic strings are particularly well motivated: Extra GWs from loops, no overclosure.
- Two approaches: Abelian-Higgs and Nambu–Goto; particle versus GW emission?
- Compute GW signal based on averaged power spectrum P_n or superposition of bursts.
- Analytical and numerical models for the loop number density: VOS, BOS, LRS.

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.
- Networks reach scaling regime: Attractor solution with H^{-1} as the only relevant scale.
- Irreducible background of GWs from any network, characteristic shape (f^3, f^{-2}, f^0).
- Cosmic strings are particularly well motivated: Extra GWs from loops, no overclosure.
- Two approaches: Abelian-Higgs and Nambu–Goto; particle versus GW emission?
- Compute GW signal based on averaged power spectrum P_n or superposition of bursts.
- Analytical and numerical models for the loop number density: VOS, BOS, LRS.
- Relevant parameters $G\mu$, α, p, etc.; constraints from CMB, interferometers, PTAs, etc.

Summary Lecture 3A

Take-home messages:

- Domain walls, strings, monopoles, etc. after phase transitions in the early Universe.
- Networks reach scaling regime: Attractor solution with H^{-1} as the only relevant scale.
- Irreducible background of GWs from any network, characteristic shape (f^3, f^{-2}, f^0).
- Cosmic strings are particularly well motivated: Extra GWs from loops, no overclosure.
- Two approaches: Abelian-Higgs and Nambu–Goto; particle versus GW emission?
- Compute GW signal based on averaged power spectrum P_n or superposition of bursts.
- Analytical and numerical models for the loop number density: VOS, BOS, LRS.
- Relevant parameters $G\mu$, α, p, etc.; constraints from CMB, interferometers, PTAs, etc.

End of Lecture 3A. Thanks a lot for your attention!