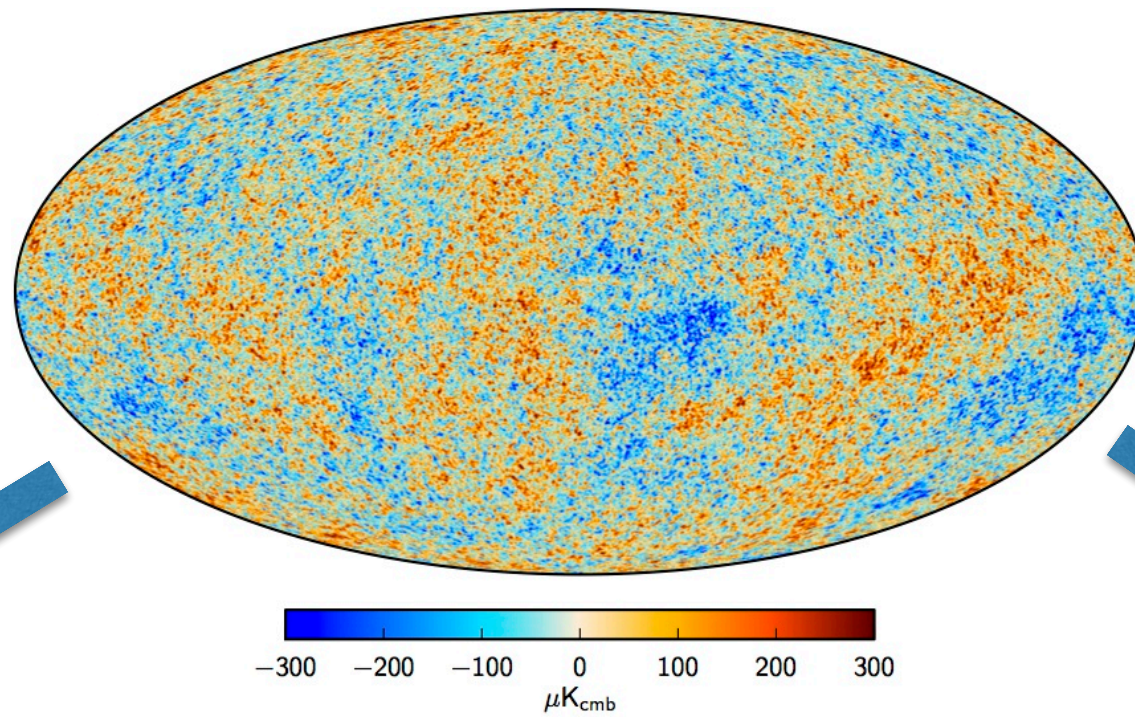
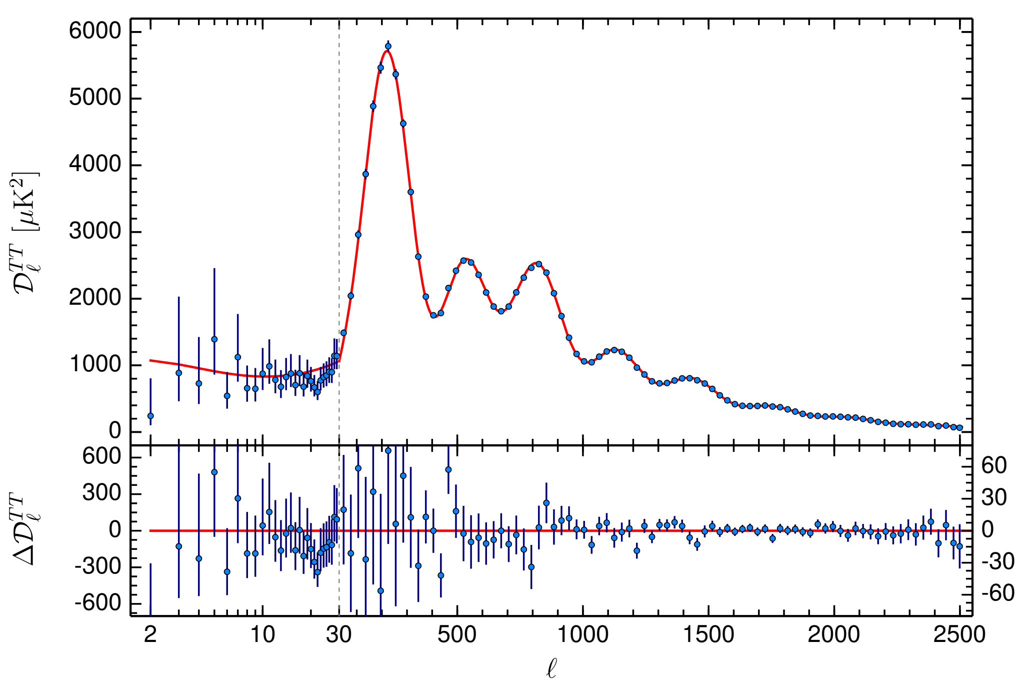
Dark Matter Interactions in Condensed Matter Systems

Yoni Kahn (UIUC)

BSM PANDEMIC seminar, 5/4/21

Dark matter exists!

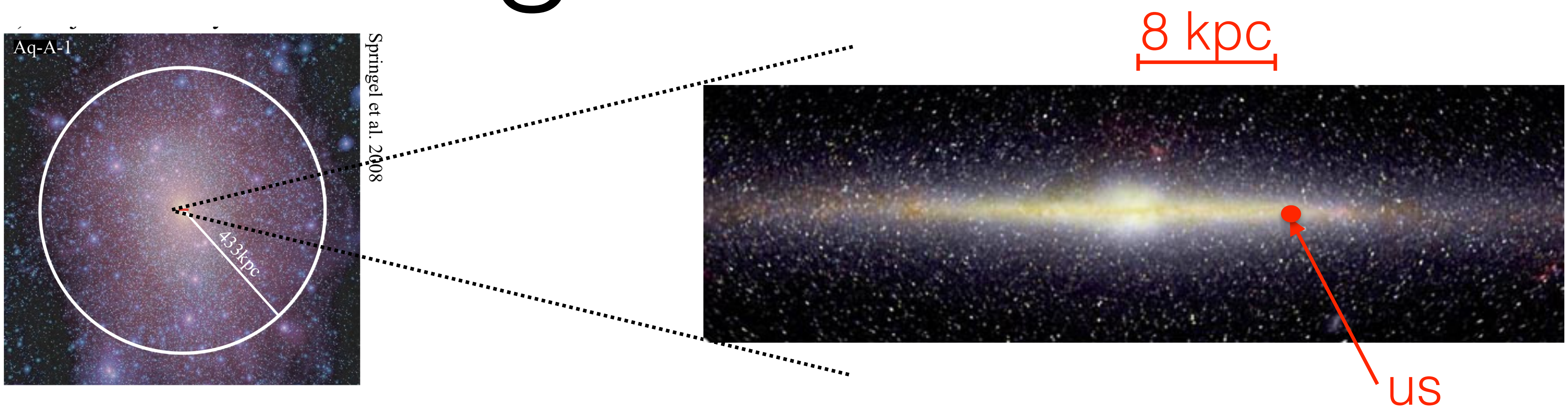




Parameter	[1] *Planck* TT+lowP
$\Omega_b h^2$	0.02222 ± 0.00023
$\Omega_c h^2$	0.1197 ± 0.0022
$100\theta_{MC}$	1.04085 ± 0.00047
τ	0.078 ± 0.019
$\ln(10^{10} A_s)$	3.089 ± 0.036
n_s	0.9655 ± 0.0062
H_0	67.31 ± 0.96
Ω_m	0.315 ± 0.013
σ_8	0.829 ± 0.014
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014

We have never observed a dark matter particle.

Setting the scales



Local measurements of stars tell us:

$$\not{m}_{\rm DM} v_{\rm DM}^2 \sim \not{m}_{\rm DM} \frac{GM(<R)}{R}$$

$$\rho_{\rm DM} \sim 0.3 \ {\rm GeV/cm^3}$$

$$v_{\rm DM} \sim 10^{-3} c \quad (\sim v_\odot \sim v_{\rm esc})$$

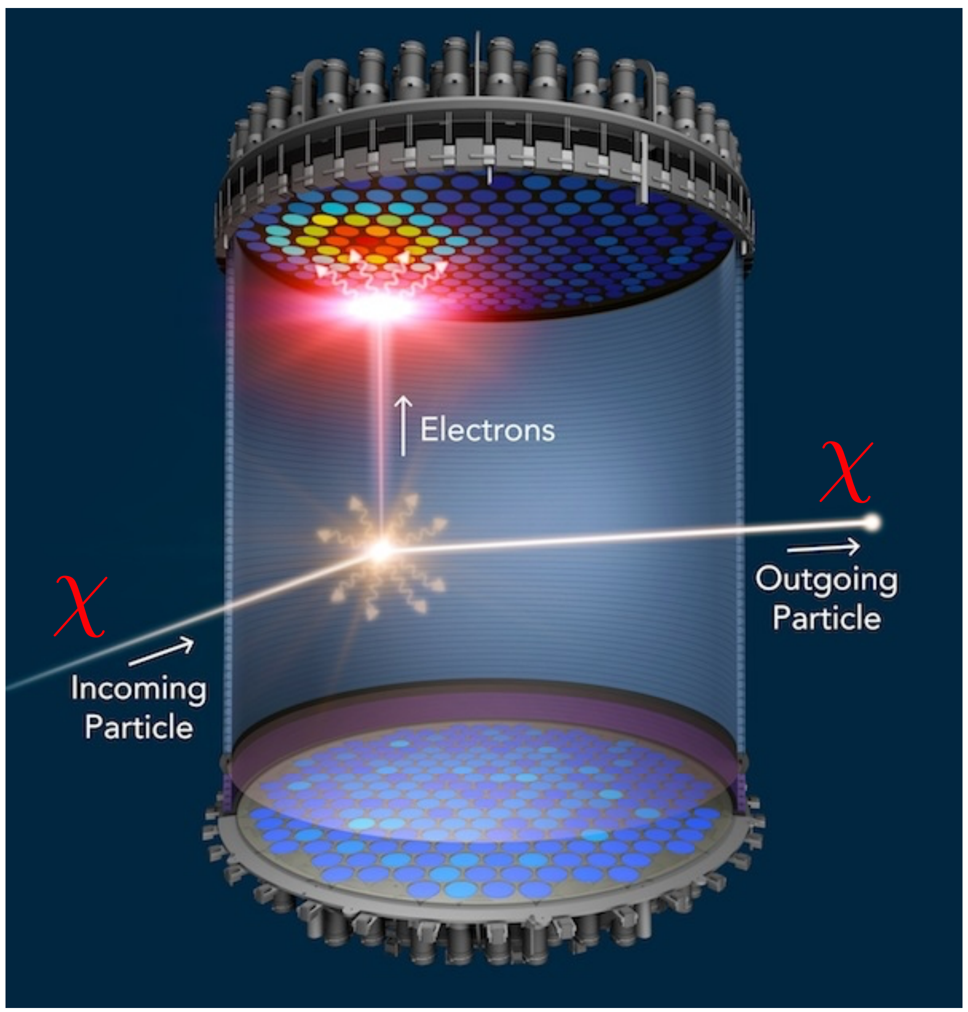
GeV-mass DM with weak-interaction cross sections:

$$n_{\rm DM} \sigma v_{\rm DM} \sim 10^{-26}/{\rm s}$$

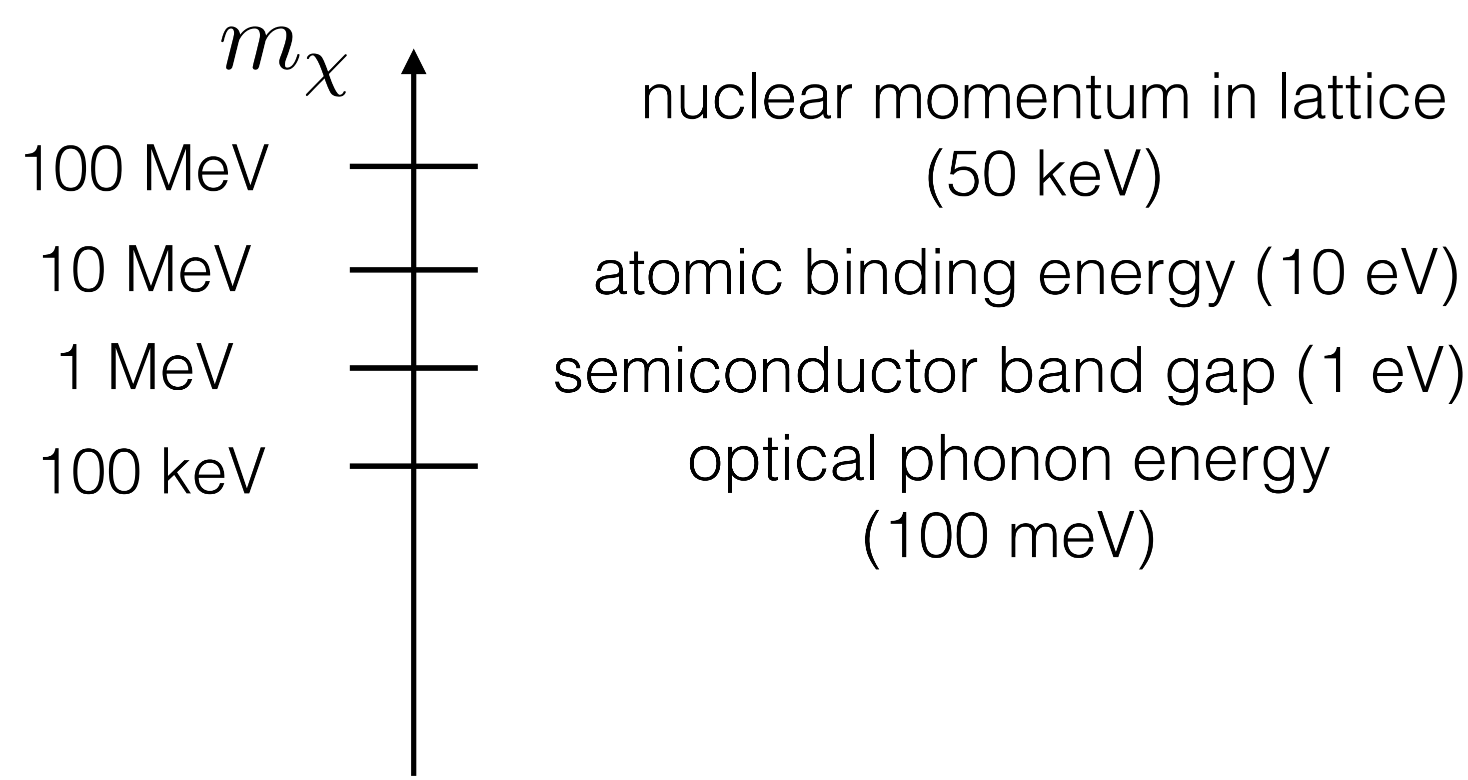
need Avogadro's number of targets to see anything

Is your detector a bag of free particles?

$$v_\chi \sim 10^{-3} \implies p_\chi \simeq 10^{-3} m_\chi, \; E_\chi \simeq 10^{-6} m_\chi$$



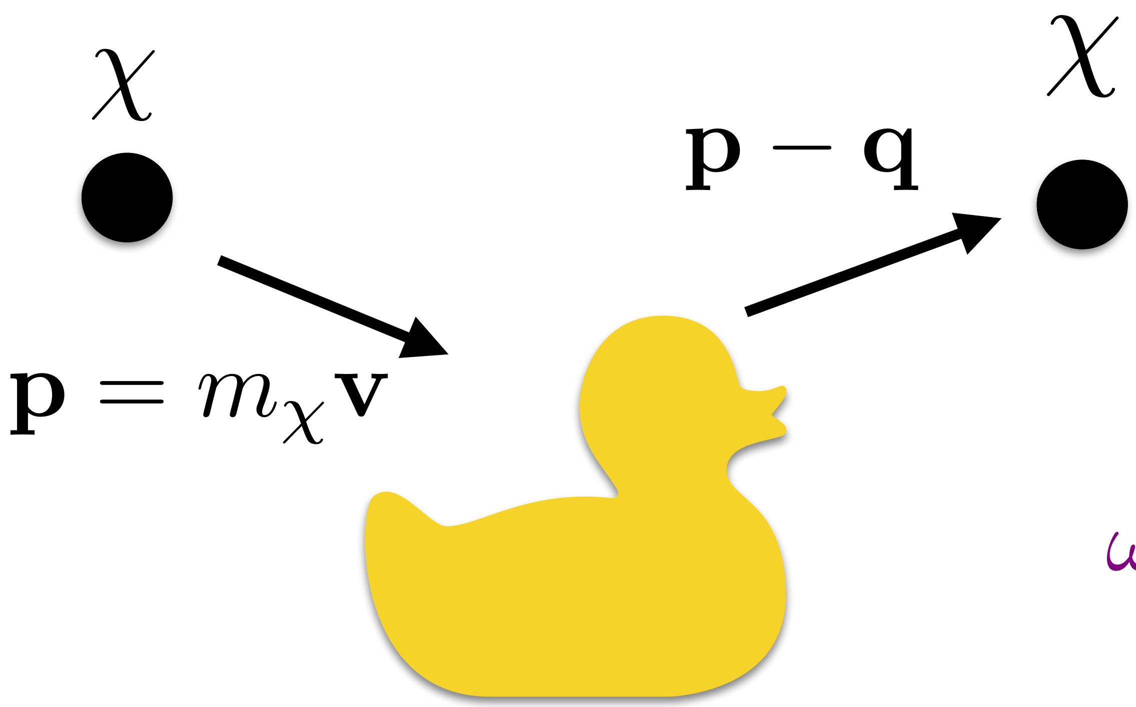
"Free" = E_χ and p_χ are the largest scales in the system.



Condensed matter physics is mandatory for sub-GeV DM!

 [Goodman & Witten PRD 1985; Drukier, Freese, Spergel PRD 1986; image credit LZ collab.]

Response functions



(insert your favorite detector here)

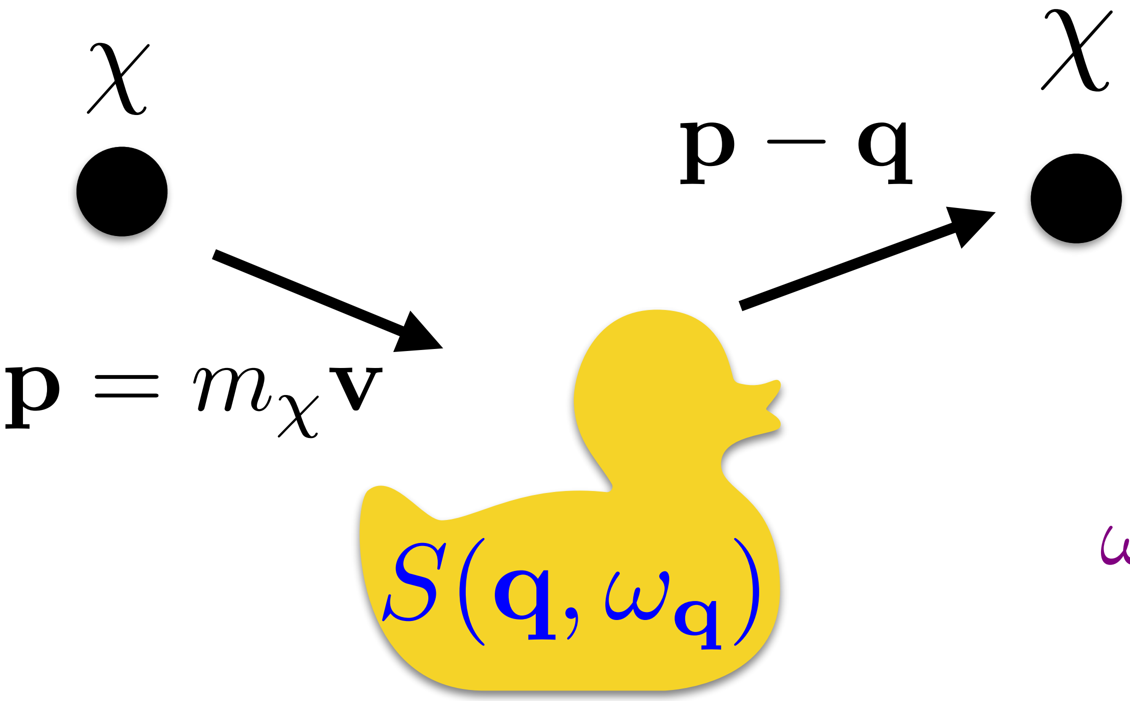
Energy deposited by DM:

$$\omega_{\mathbf{q}} = \frac{\mathbf{p}^2}{2m_\chi} - \frac{(\mathbf{p}-\mathbf{q})^2}{2m_\chi} = \mathbf{q}\cdot\mathbf{v} - \frac{q^2}{2m_\chi}$$

Require energy conservation, but **not** momentum conservation

if your target is not a free particle, it is not a momentum eigenstate!

Response functions



does the target have an eigenstate at $\omega_\mathbf{q}$?

Energy deposited by DM:

$$\omega_\mathbf{q} = \frac{\mathbf{p}^2}{2m_\chi} - \frac{(\mathbf{p}-\mathbf{q})^2}{2m_\chi} = \mathbf{q}\cdot\mathbf{v} - \frac{q^2}{2m_\chi}$$

Require energy conservation, but **not** momentum conservation

$$R \sim \int d^3\mathbf{v}\, f(\mathbf{v}) \int d^3\mathbf{q}\, F^2(\mathbf{q}) S(\mathbf{q}, \omega_\mathbf{q})$$

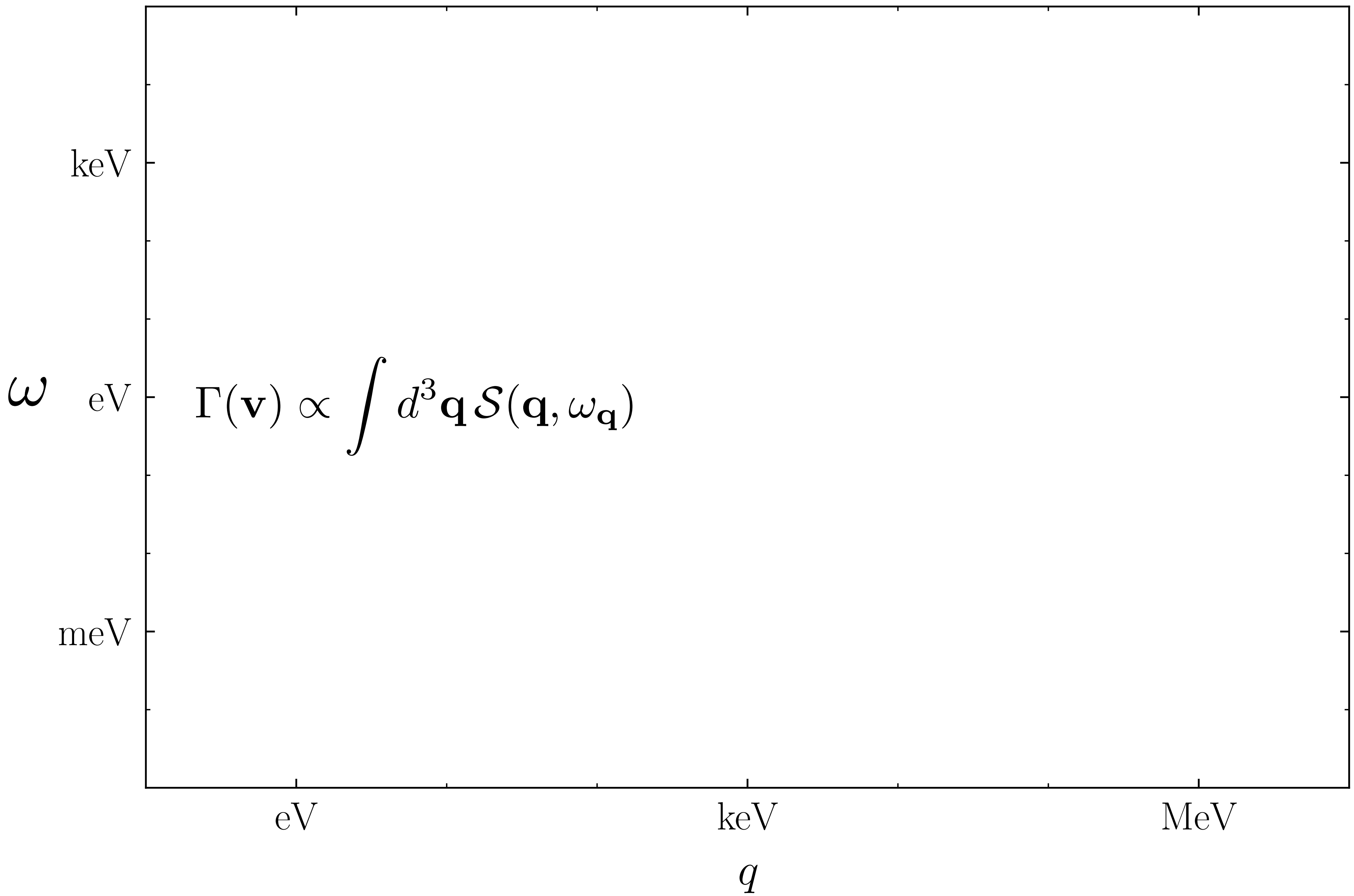
DM properties

Material properties

General framework that works for **any** many-body system

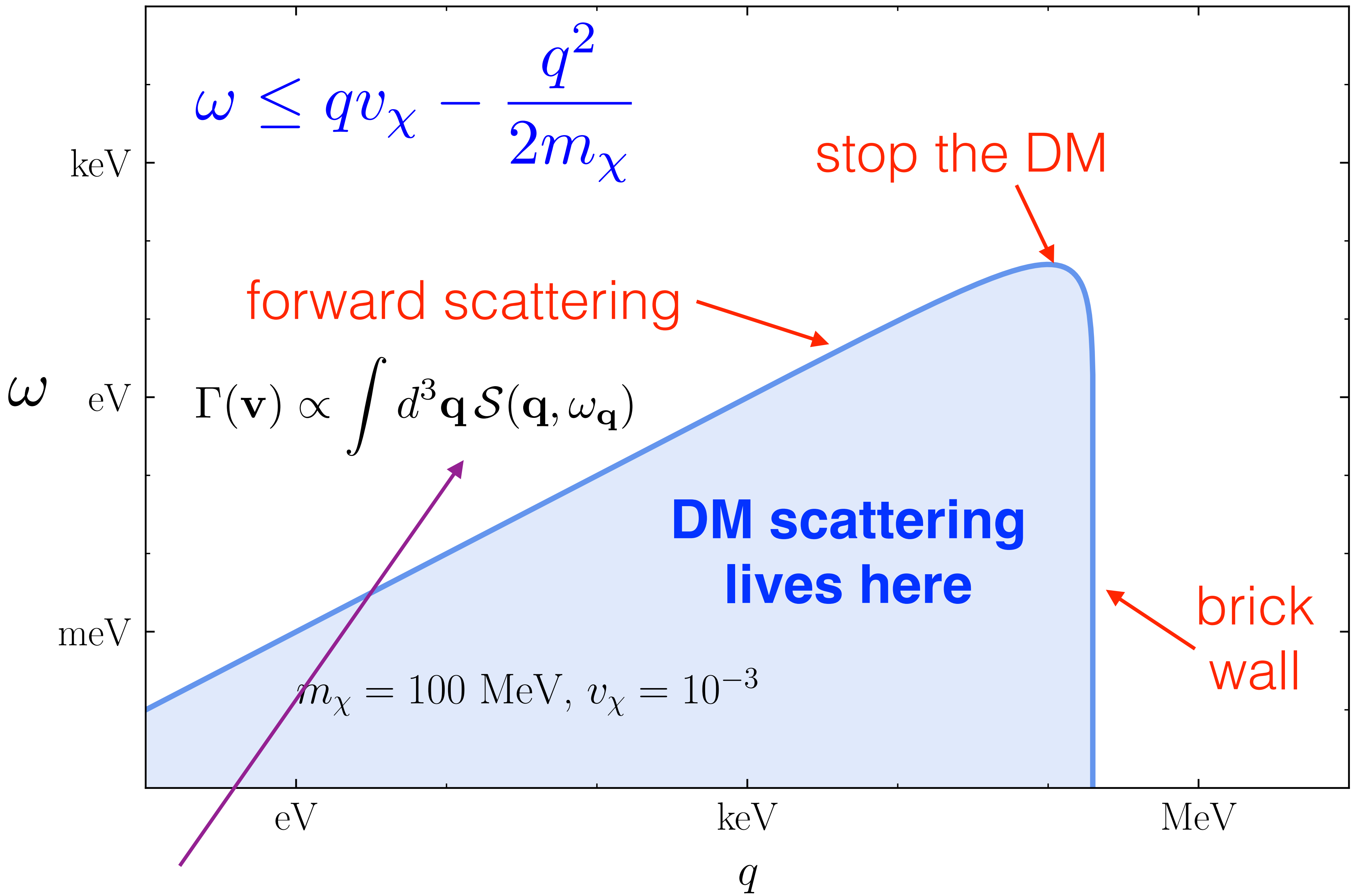
[Trickle, Zhang, Zurek, Inzani, Griffin, JHEP 2020; Berggren, Hochberg, **YK**, Kurinsky, Lehmann, Yu, arXiv:2101.08263]

Sub-GeV DM kinematics



$$\Gamma(\mathbf{v}) \propto \int d^3\mathbf{q}\, \mathcal{S}(\mathbf{q}, \omega_{\mathbf{q}})$$

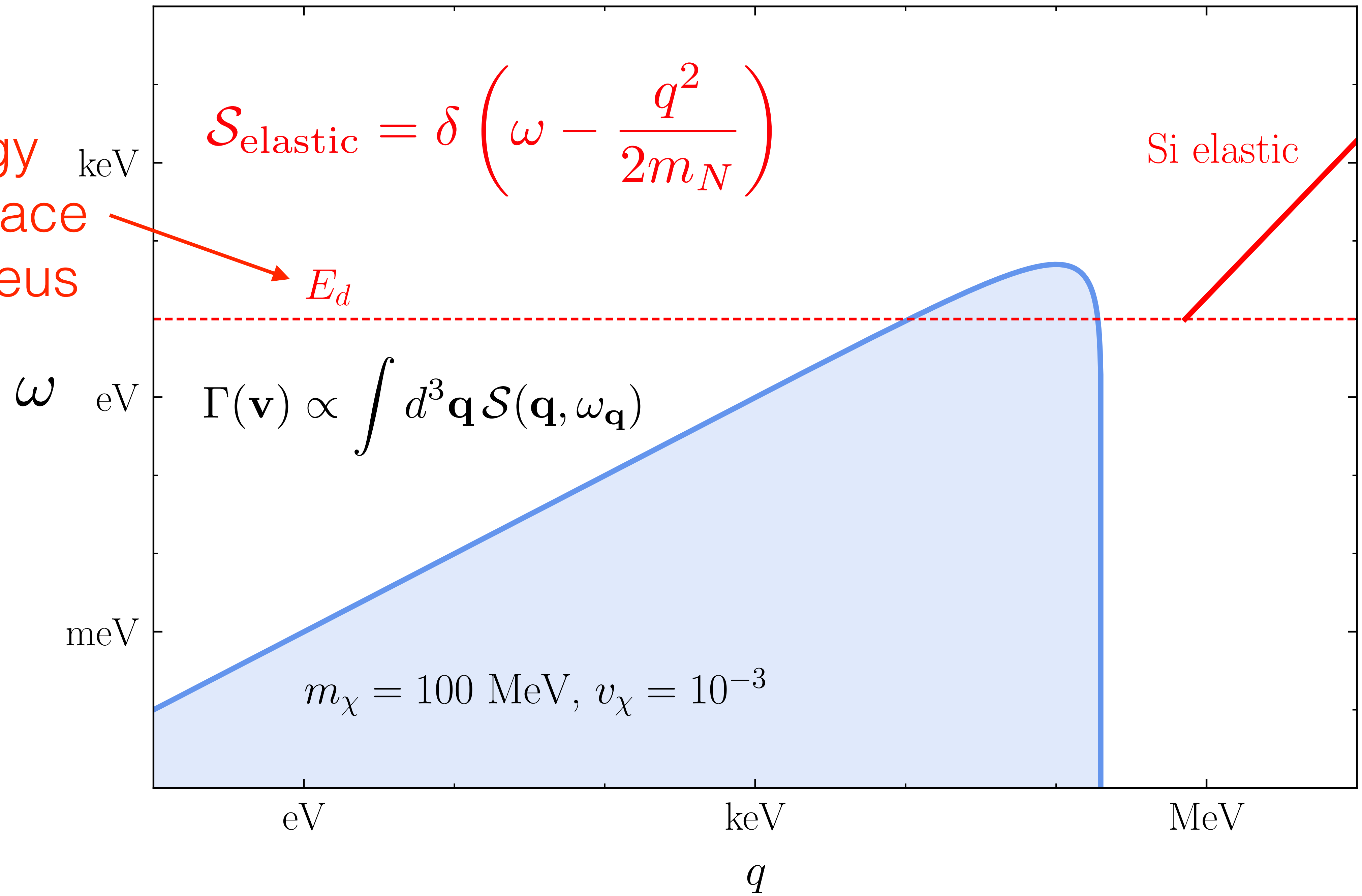
Sub-GeV DM kinematics



Goal: maximize the response function inside the DM parabola

Sub-GeV DM kinematics

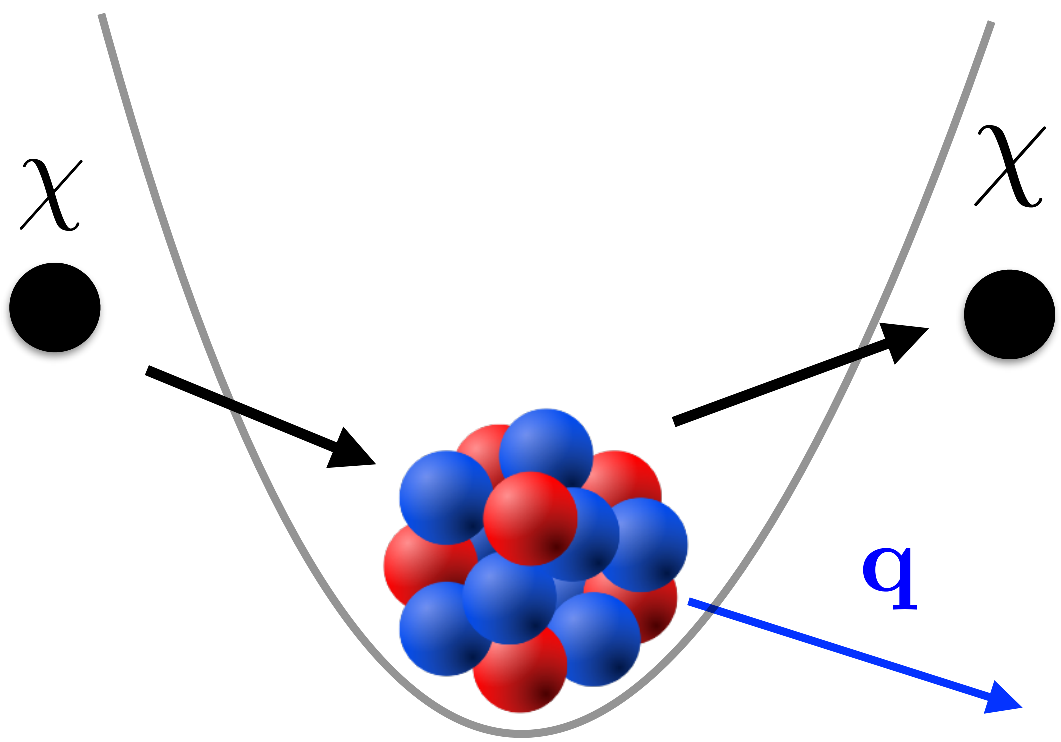
energy to displace a nucleus



Delta function response = free particle

How can we see sub-GeV DM with solid-state detectors?

A very simple toy model



A single nucleus in a harmonic potential:

$$\phi_0(\mathbf{p}) = (\pi m_N \omega_0)^{-1/4} e^{-\mathbf{p}^2/{q_0}^2}$$

$$q_0 = \sqrt{2 m_N \omega_0}$$

(56 keV for Si)

$$V(r) \approx \frac{1}{2} m_N \omega_0^2 r^2,$$

optical phonon energy ~ 60 meV

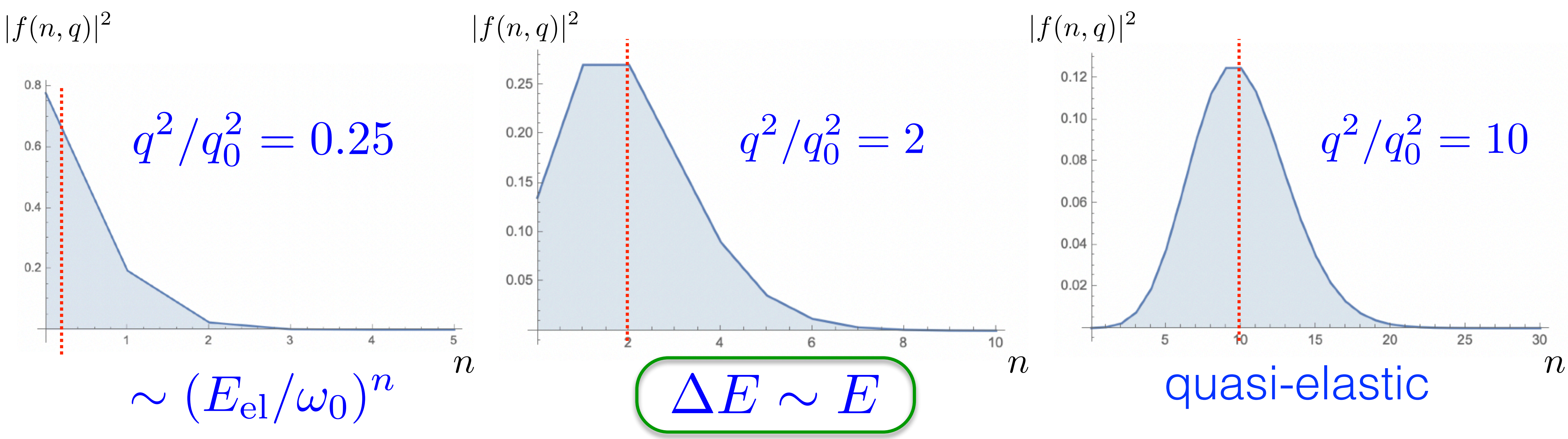
$$S(\mathbf{q}, \omega_{\mathbf{q}}) = \sum_f |\langle f | e^{i\mathbf{q}\cdot\mathbf{x}_N} | 0 \rangle|^2 \delta(E_f - E_i) \equiv \sum_n |f(n, \mathbf{q})|^2 \delta(n\omega_0 - \omega_{\mathbf{q}})$$

Poissonian phonons

In a harmonic oscillator model, you can compute everything analytically:

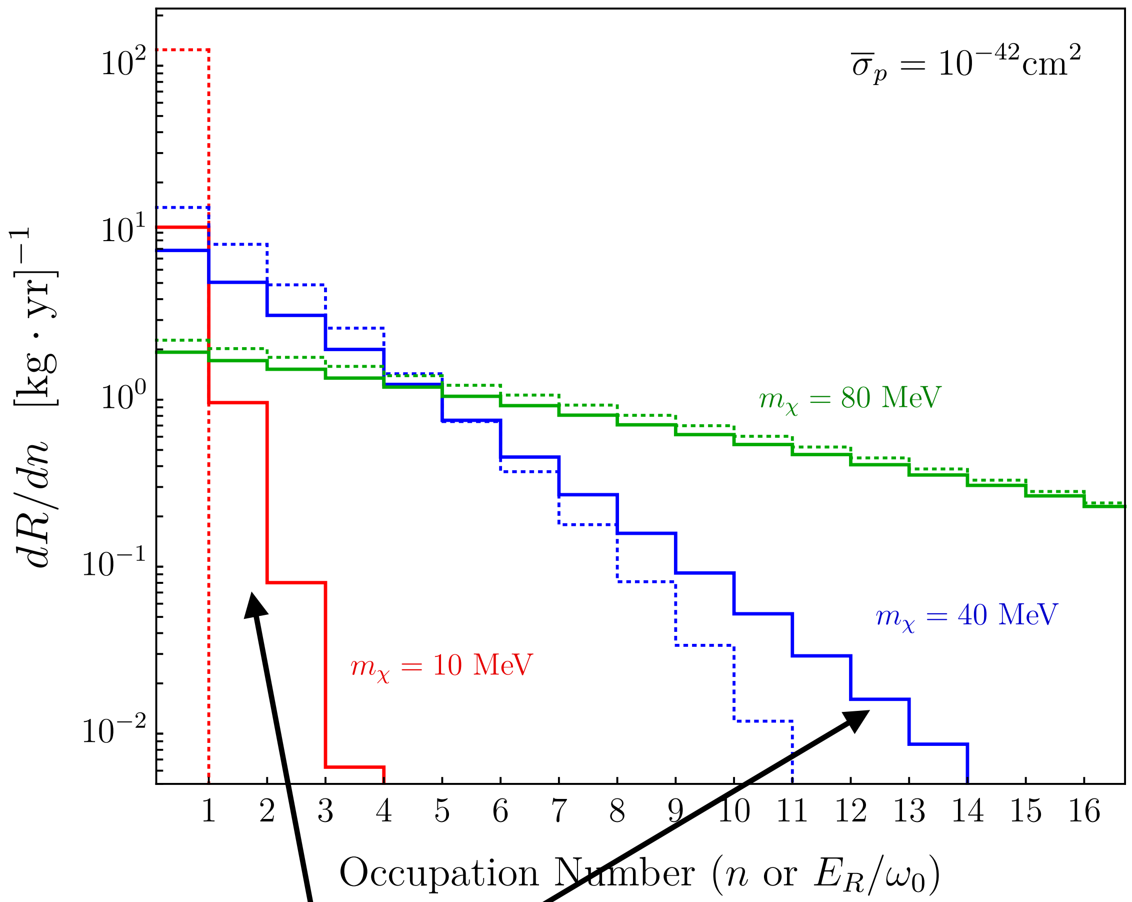
$$|f(n,q)|^2 \equiv \frac{1}{4\pi}\int d\Omega_{\mathbf{q}}|f(n,\mathbf{q})|^2 = \frac{1}{4\pi}\frac{1}{n!}\left(\frac{q}{q_0}\right)^{2n} e^{-q^2/q_0^2}$$

A Poisson distribution! With mean $\lambda = q^2/q_0^2 = q^2/(2m_N\omega_0) = E_{\rm el}/\omega_0$



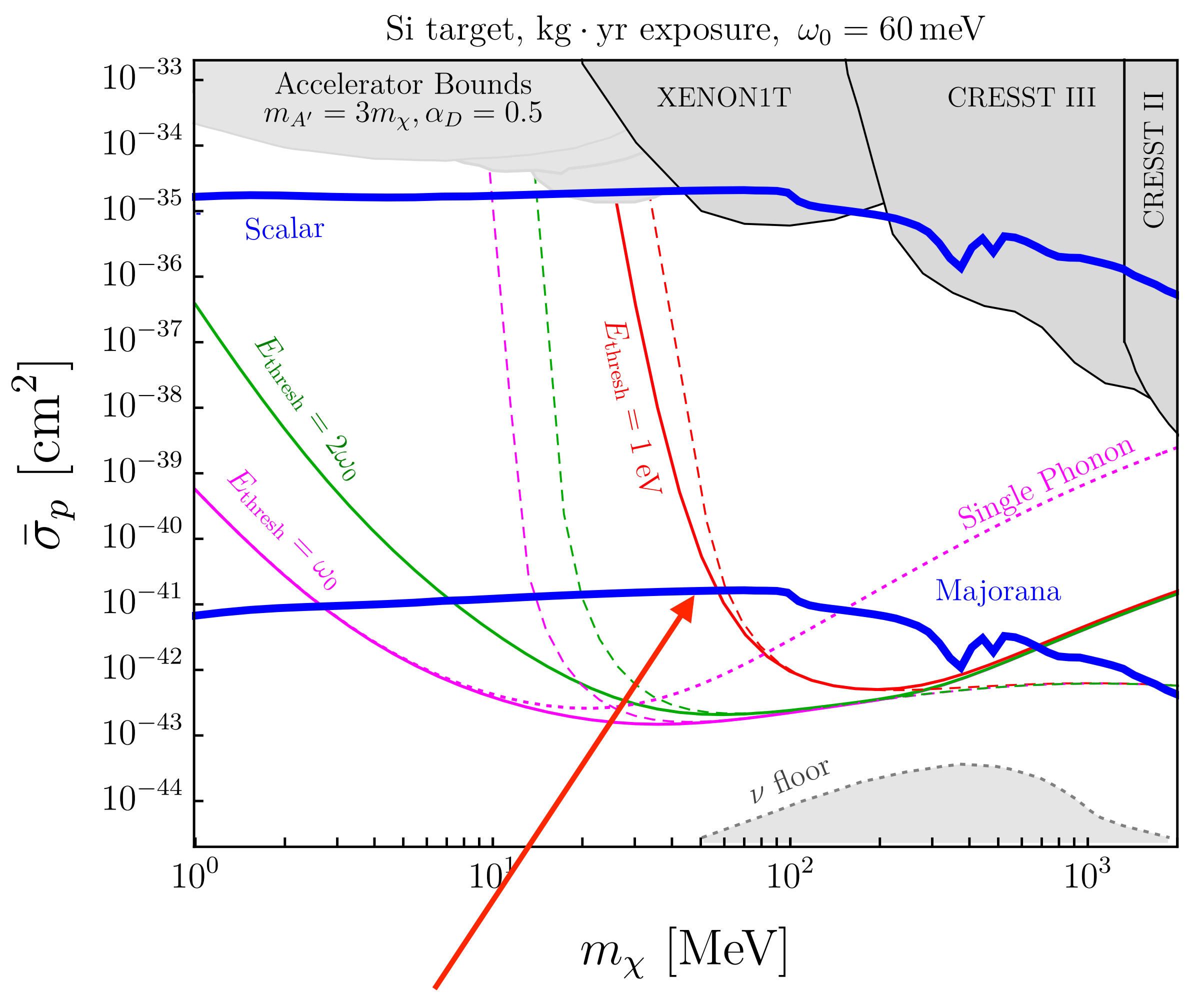
Poisson tail = more sensitivity

Integrate over DM velocities:



extra events above "elastic" energy

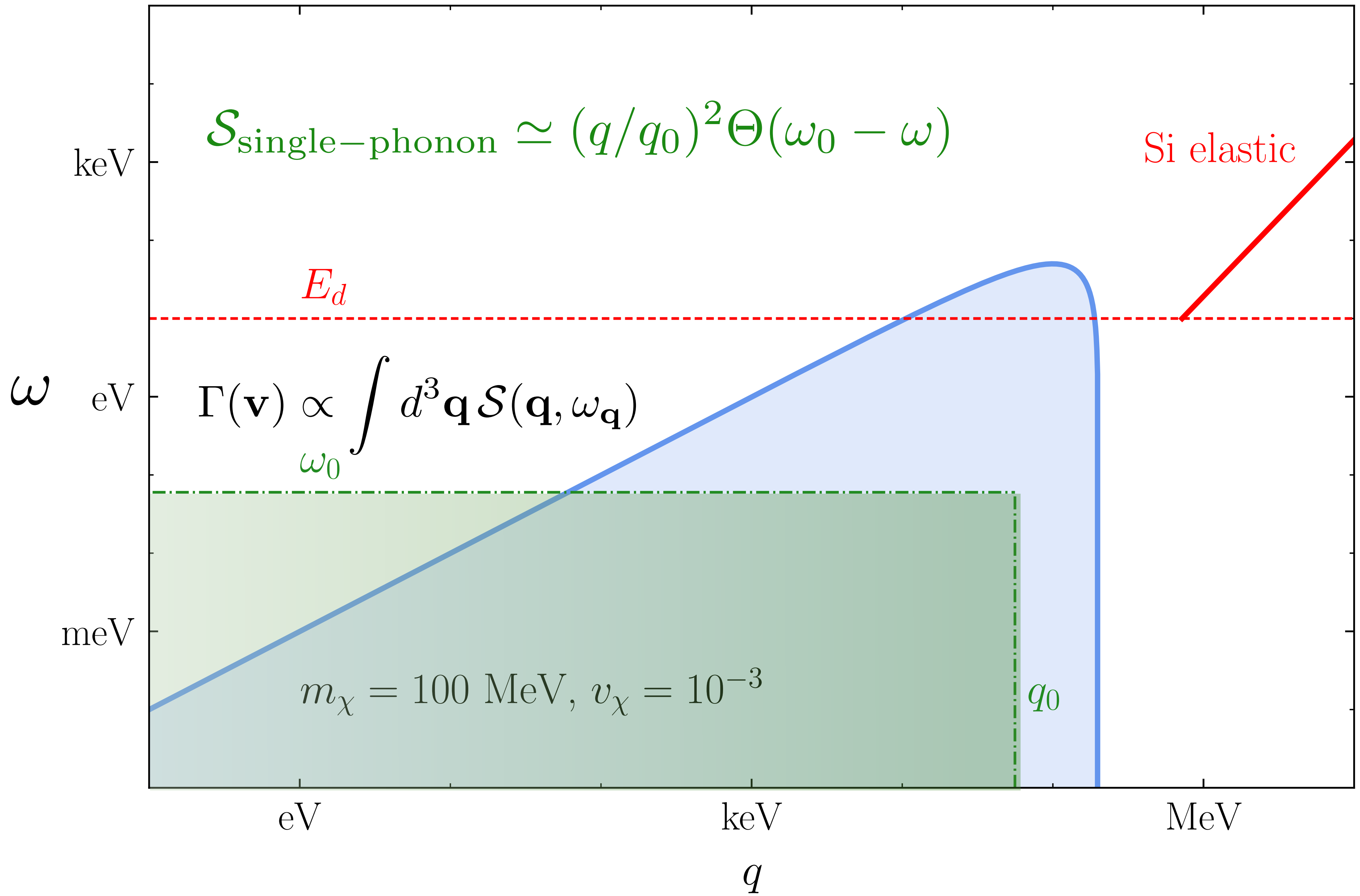
Integrate over all energy deposits:



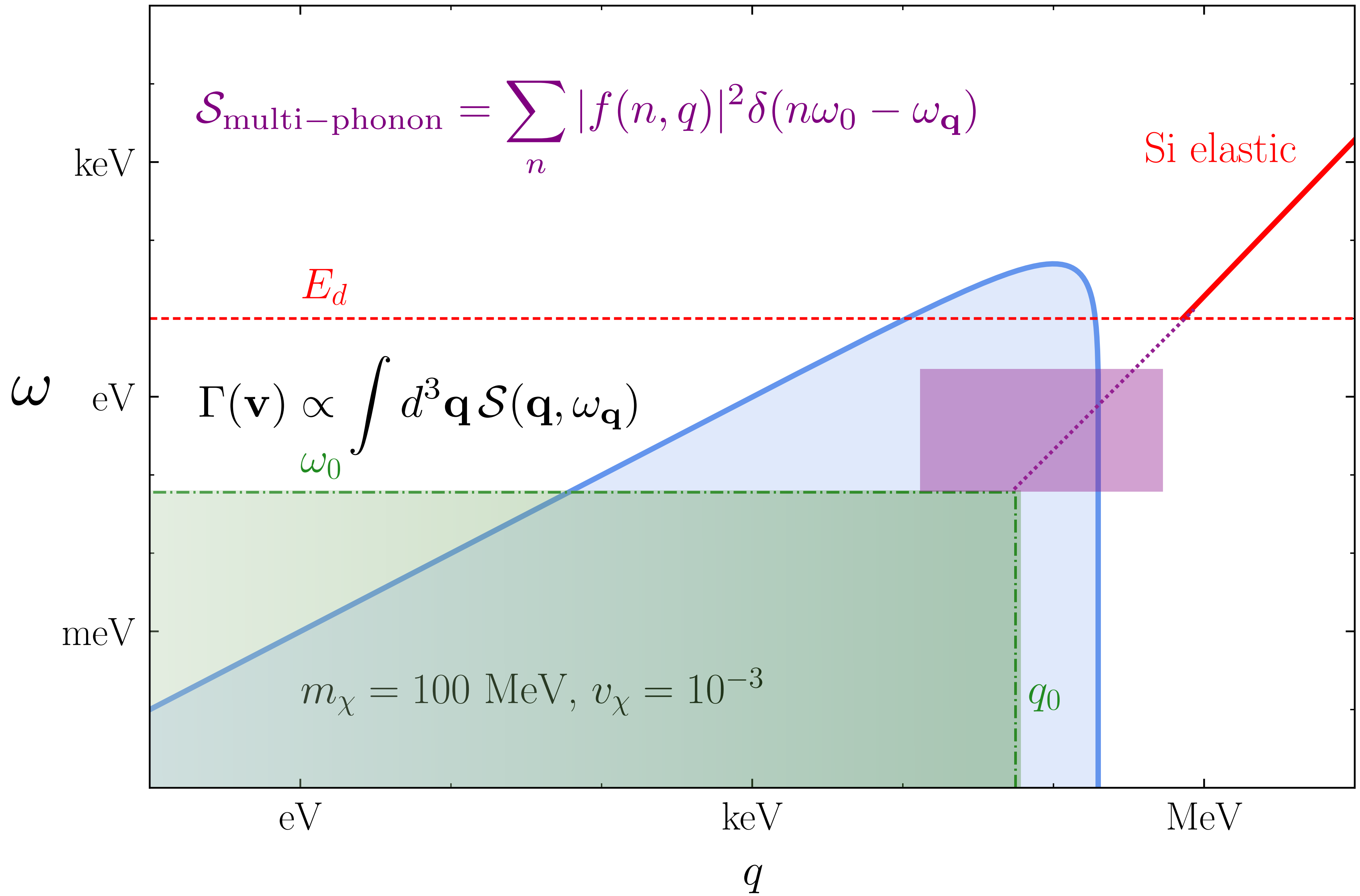
velocity-suppressed thermal target seems accessible!!

Single-phonon resolution is still several years off: can get important physics out of detectors much sooner

Phonon response functions

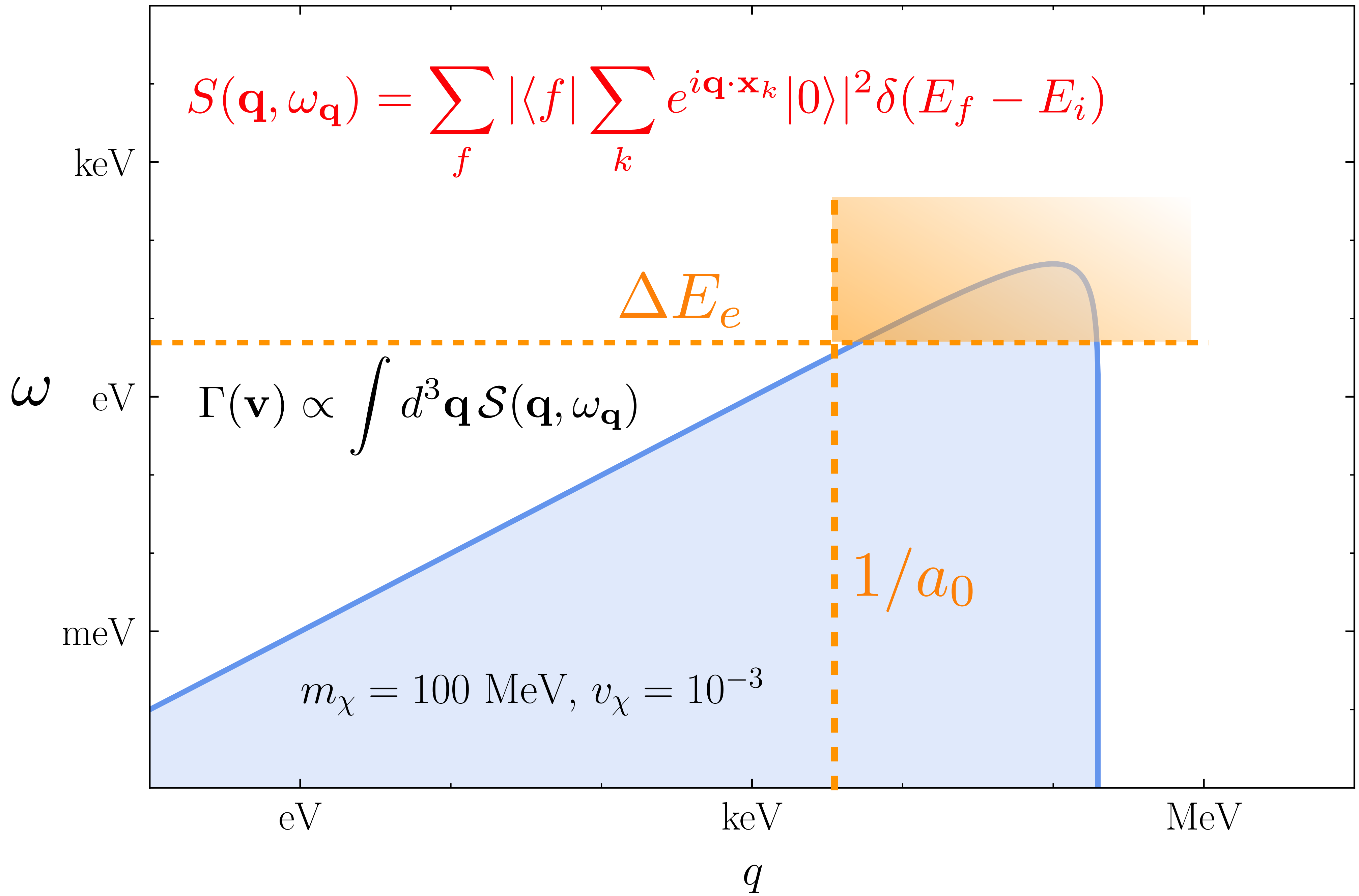


Phonon response functions



Bound nuclei still act as sort-of-free particles if you hit them hard enough, up to Poisson fluctuations in phonon number

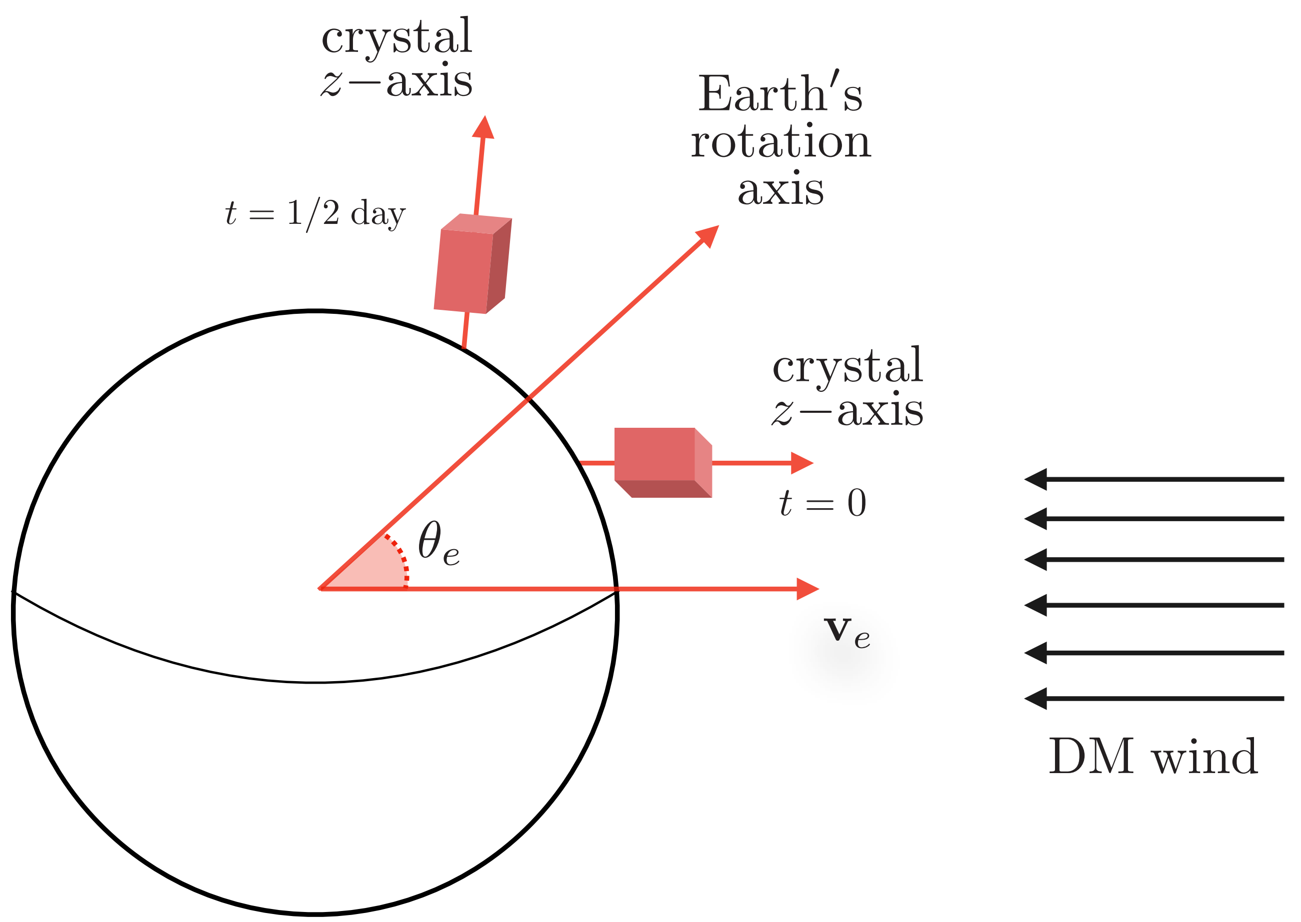
What about electrons?



Atomic/molecular scales set by Bohr radius are just right for sub-GeV DM

[Essig, Mardon, Volansky, PRD 2012; Essig, Manalaysay, Sorensen, Volansky, PRL 2012; Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu, JHEP 2016]

Daily modulation



$$R(t) \sim \int d^3v\, d^3q\, f_\chi(\mathbf{v}, t)\, \mathcal{S}(\mathbf{q}, \omega_\mathbf{q})$$

$$\omega_\mathbf{q} = \mathbf{q} \cdot \mathbf{v} - \frac{q^2}{2m_\chi}$$

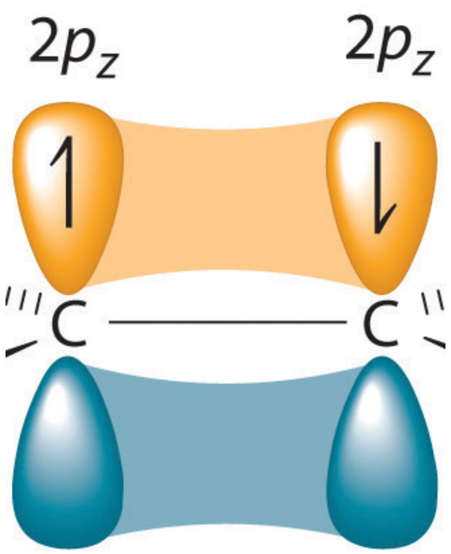
If S is peaked in particular directions of **q**, R will change periodically over 24 hours as $\langle \mathbf{v} \rangle$ rotates in lab frame

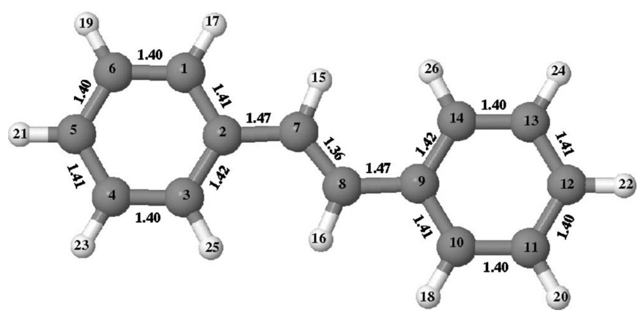
Smoking gun for DM signal!

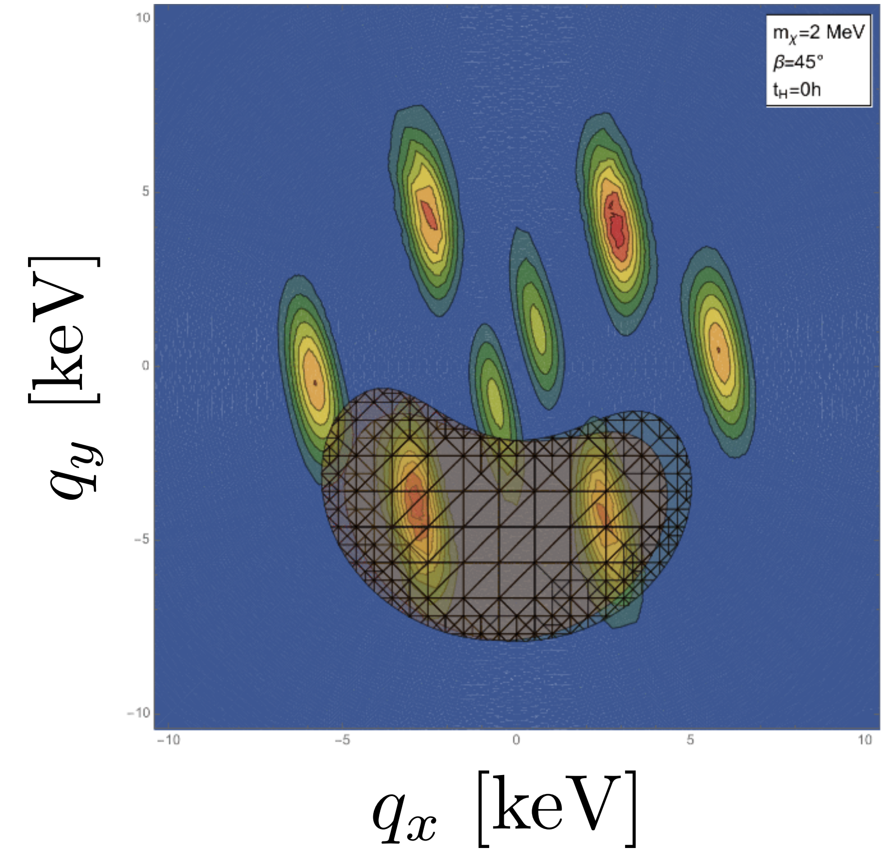
 [Spergel, PRD 1988; cartoon from Coskuner, Mitridate, Olivares, Zurek, 1909.09170]

Organic crystals

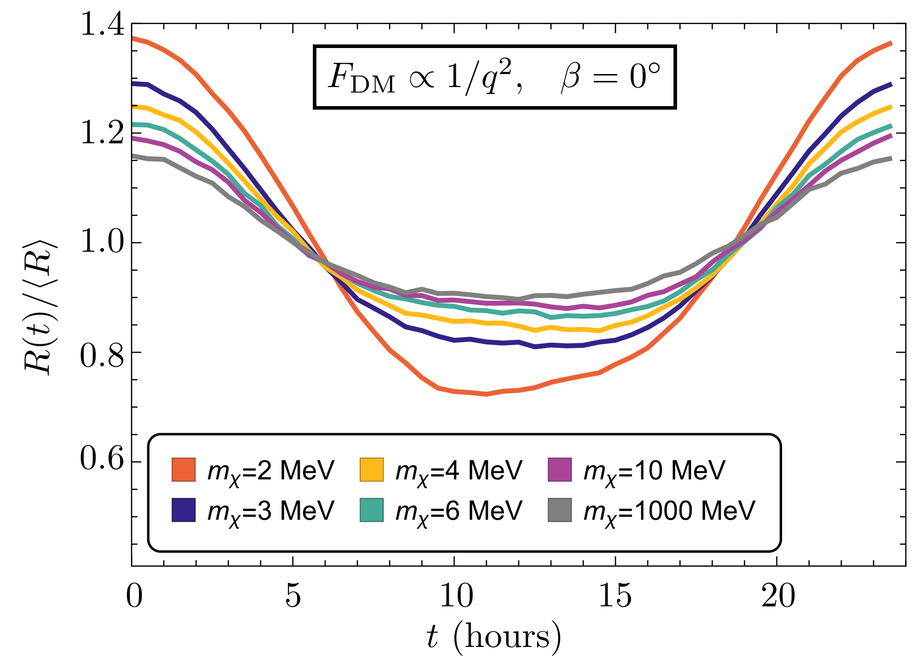
Carbon bonds give eV-scale energy gaps, anisotropic response







response function
(lowest transition)

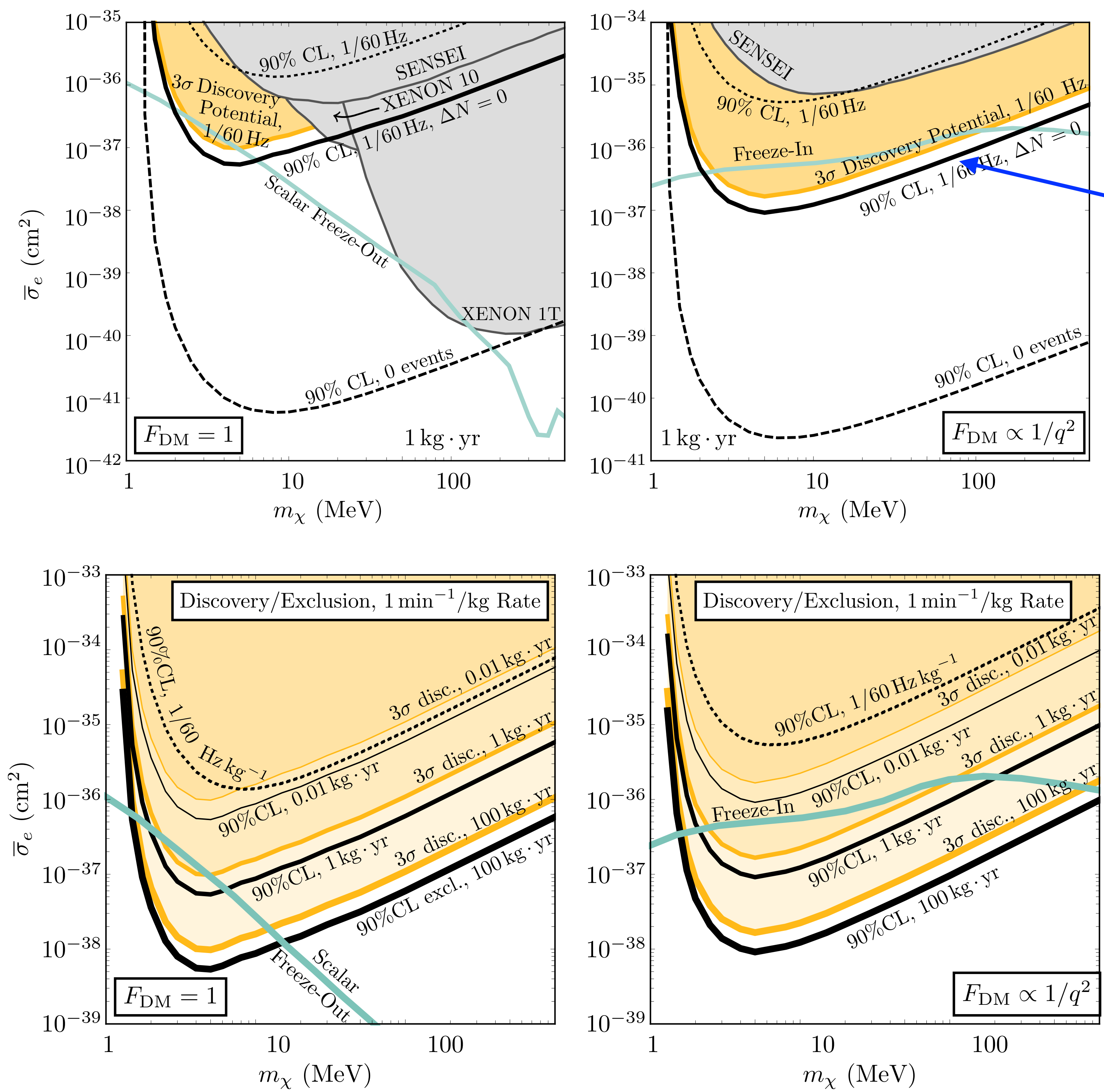


20% daily modulation!

Total rate comparable to Si for same target mass,
but modulation means discovery does not require zero background

[Blanco, **YK**, Lillard, McDermott, arXiv:2103.08601]

The power of daily modulation

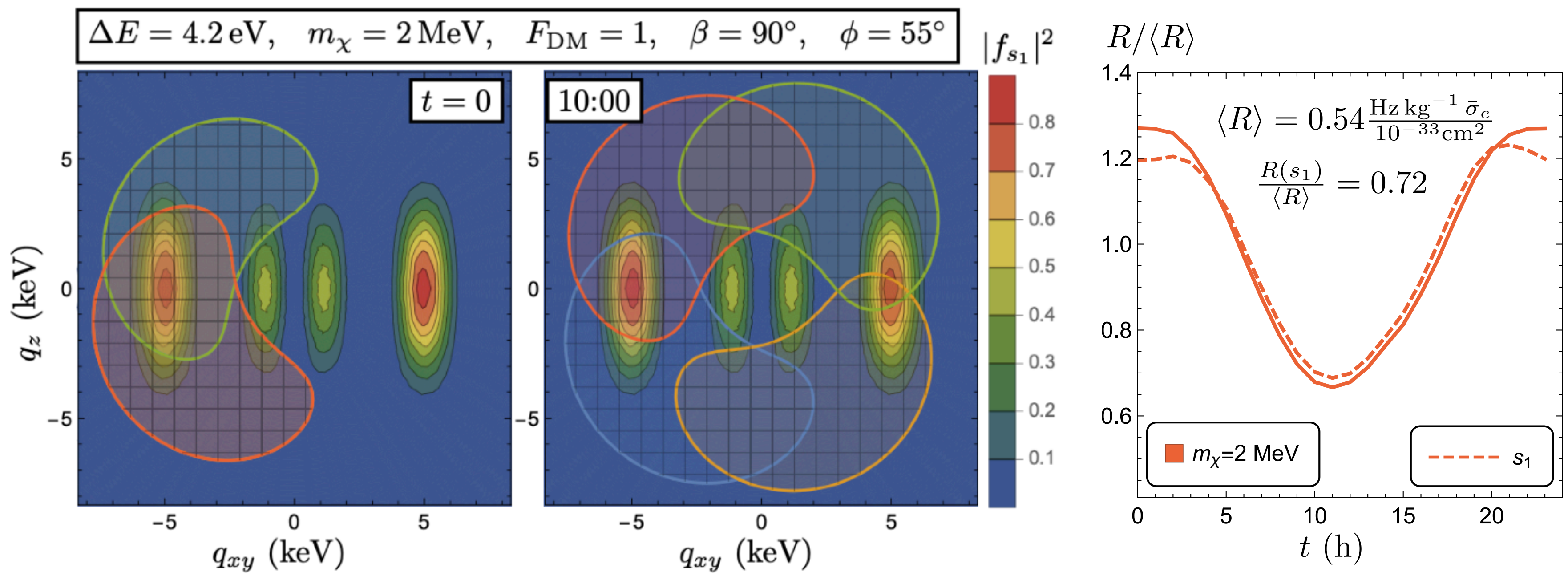


Freeze-in discoverable with 1/min background, but only in modulation analysis!

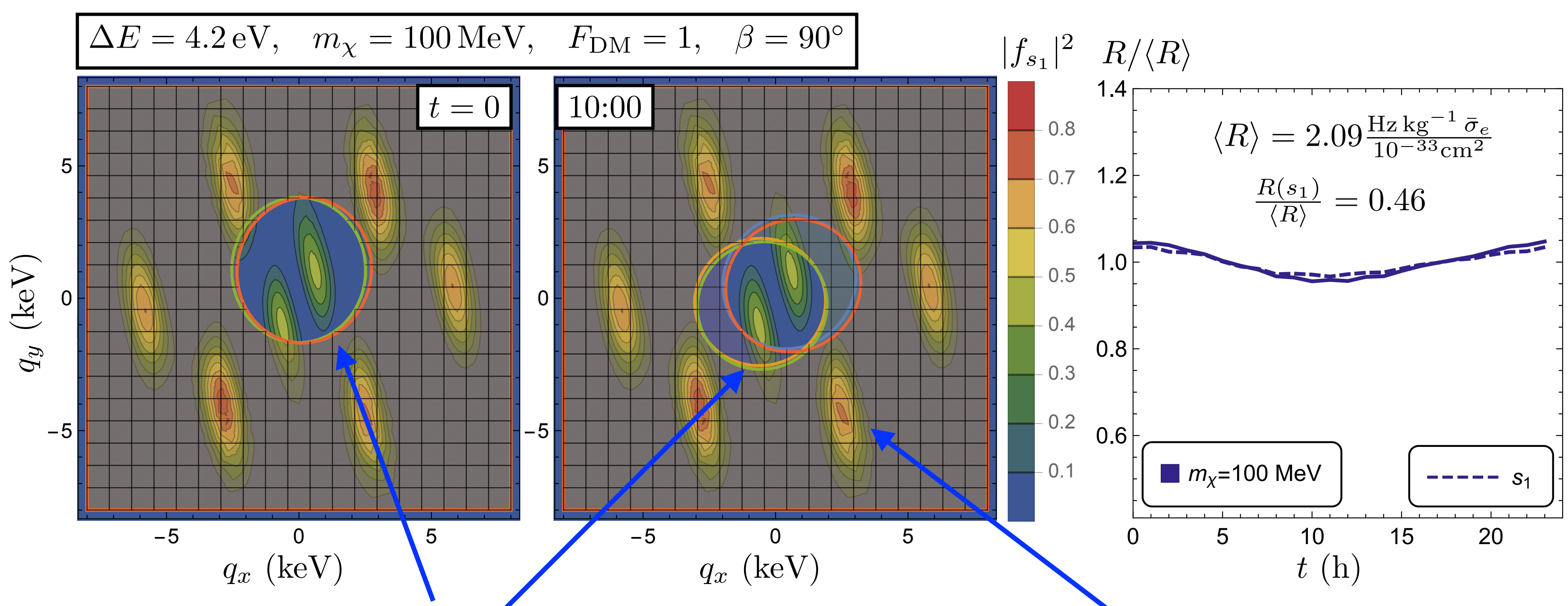
Reach keeps improving with exposure, even with large (constant) background

[Blanco, **YK**, Lillard, McDermott, arXiv:2103.08601]

Why can't we do better?



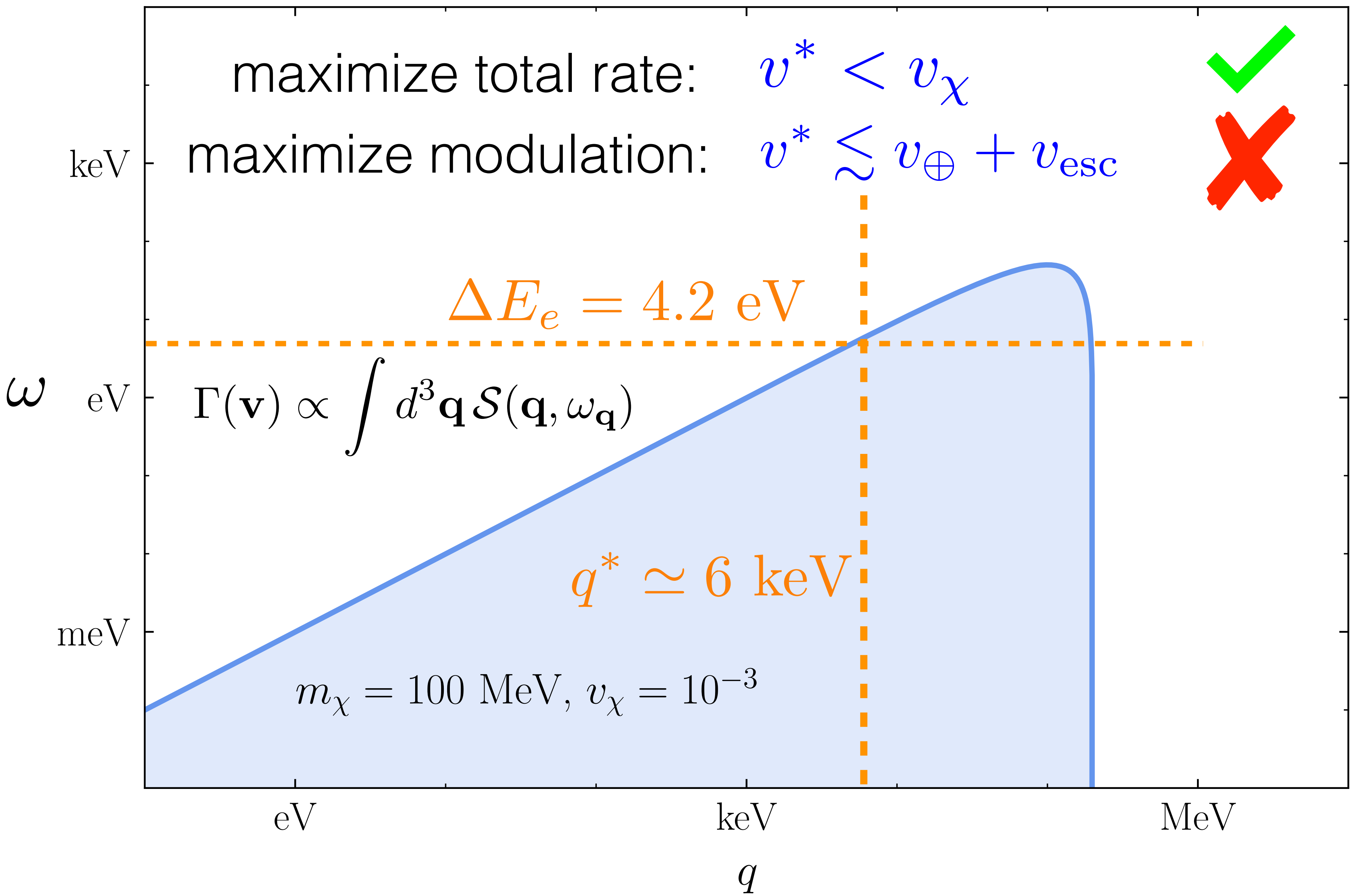
Low masses: kinematically-allowed "beans" traverse peaks of response function



High masses: peaks are always accessible, residual modulation driven by secondary peaks

$$q_{\rm min} = \Delta E / v_{\rm max} \qquad v^* \equiv \Delta E / q^* \simeq 200 \ \mathrm{km/s}$$

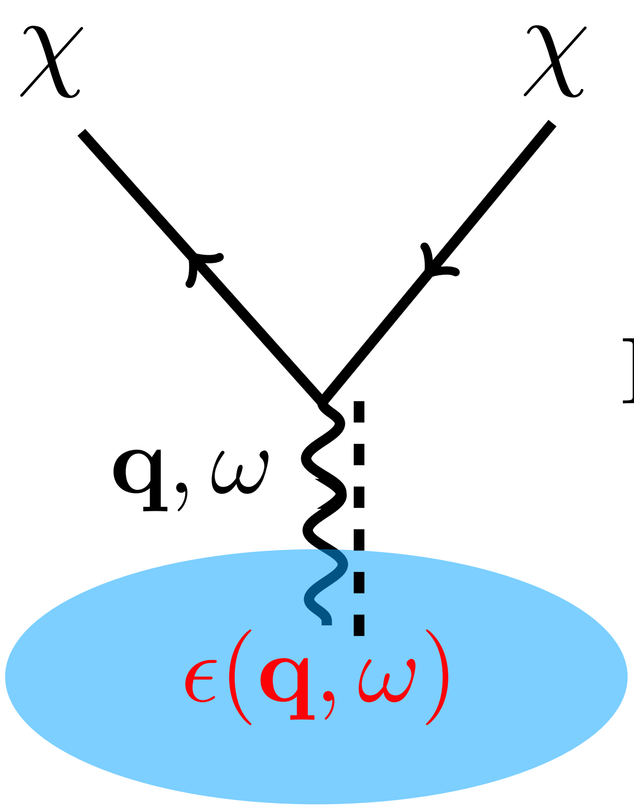
Why can't we do better?



Effective electron velocity controls both total rate and modulation!

From molecules to solids: many-body effects

$$S(\mathbf{q}, \omega_{\mathbf{q}}) = \sum_f |\langle f| \sum_k e^{i\mathbf{q}\cdot\mathbf{x}_k} |0\rangle|^2 \delta(E_f - E_i)$$



DM-electron interaction (assumed spin-independent)

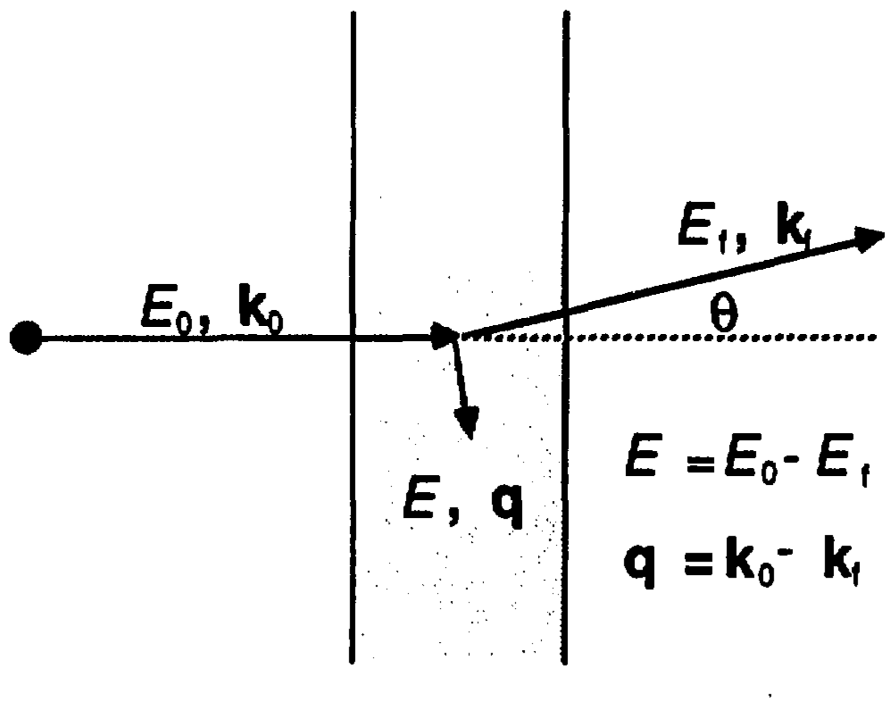
Response function (dielectric, for electrons)

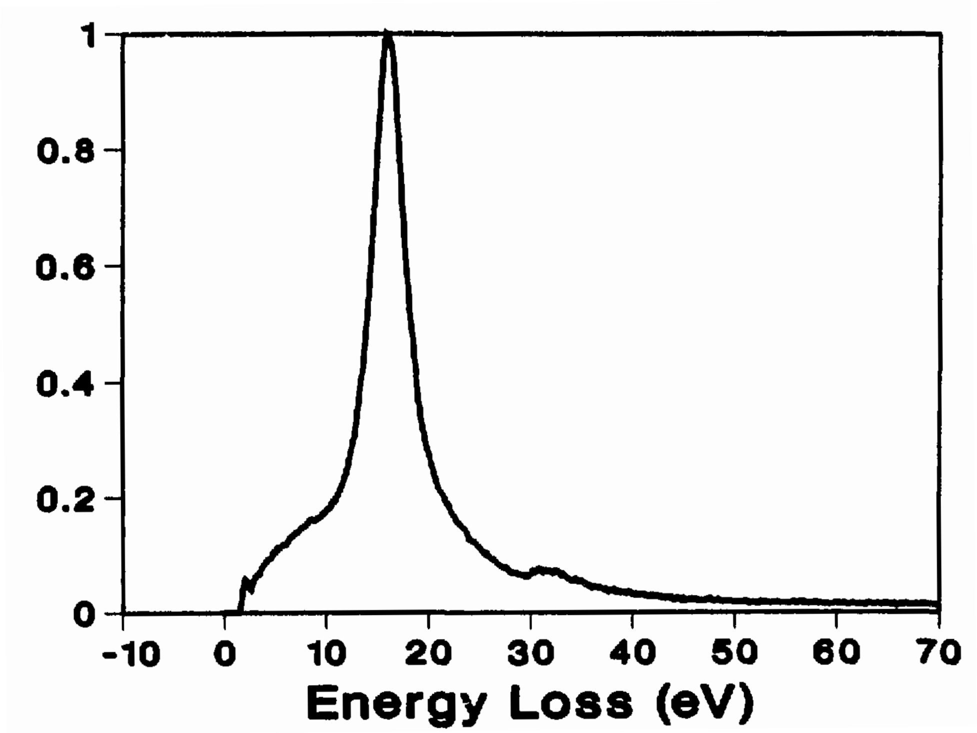
$$\Gamma(\mathbf{v}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} |V(\mathbf{q})|^2 \left[\frac{q^2}{e^2} 2\,\mathrm{Im}\left(-\frac{1}{\epsilon(\mathbf{q}, \omega)}\right)\right]$$

Response function has a huge resonance when $q < \omega_p / v_F$: plasmon

[Berggren, Hochberg, **YK**, Kurinsky, Lehmann, Yu, arXiv:2101.08263; Lin, Knapen, Kozaczuk, 2101.08275]

Plasmons





Semi-relativistic electron scattering dominated by a collective long-range charge wave (plasmon). Electron preferentially deposits ~15 eV of energy, regardless of initial KE

$$\omega_p = \sqrt{4\pi\alpha n_e / m_e}$$

$$\int_0^\infty d\omega \, \omega S(\mathbf{q}, \omega) = \frac{\pi}{2}\omega_p^2$$

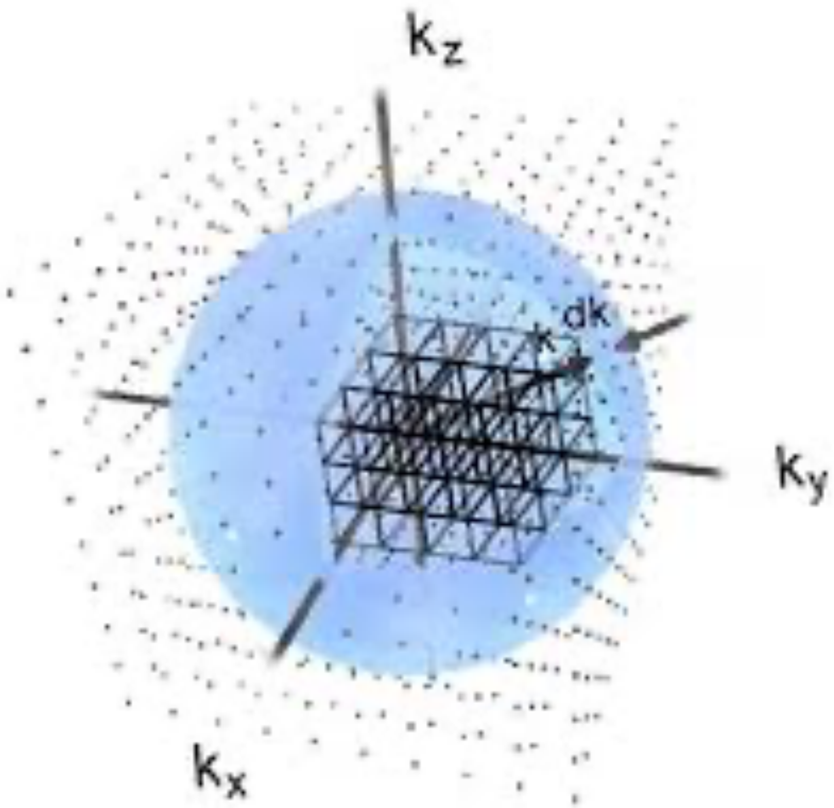
Peak damps out when

$$v_{\text{probe}} < v_F$$

[M. Kundmann, Ph.D. thesis 1988]

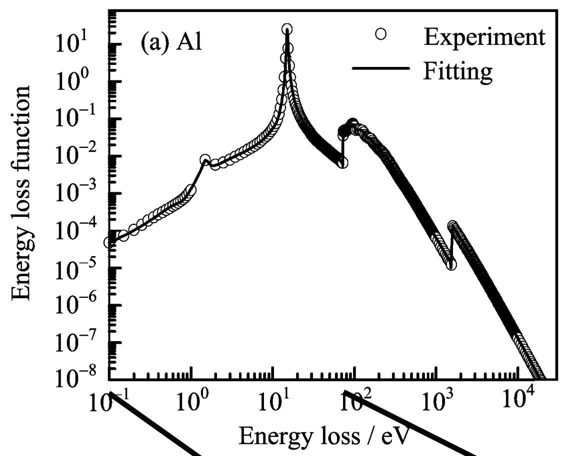
Newton vs. Fermi

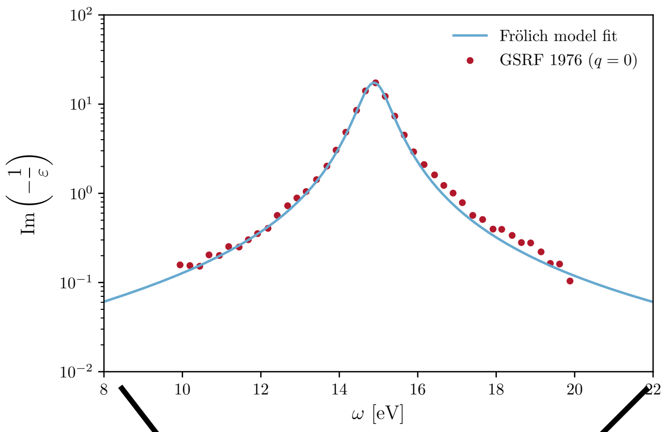
$v_{\rm DM} \sim 10^{-3} c$ $\ll$ $v_F \simeq 10^{-2} c$

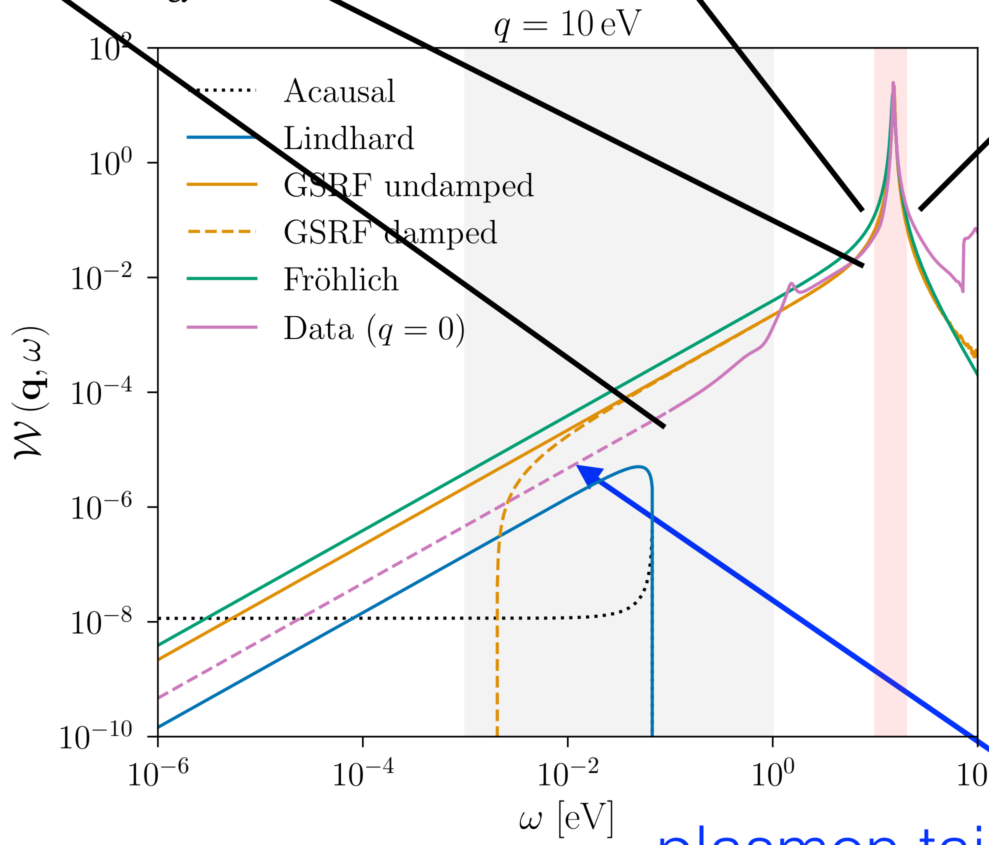


Peaks of loss function are kinematically inaccessible.
Unfortunate coincidence for DM direct detection!

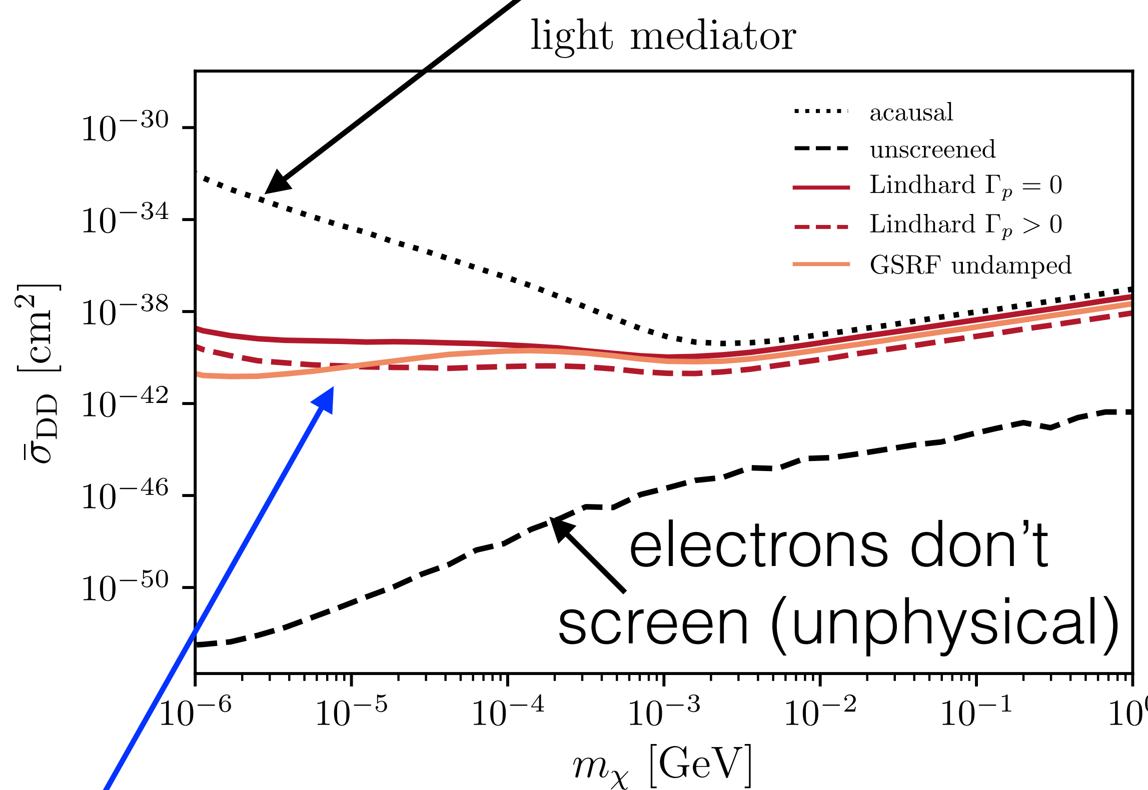
Peaks and long tails







S fails causality checks



electrons don't screen (unphysical)

plasmon tail buys you 2-3 orders of magnitude!

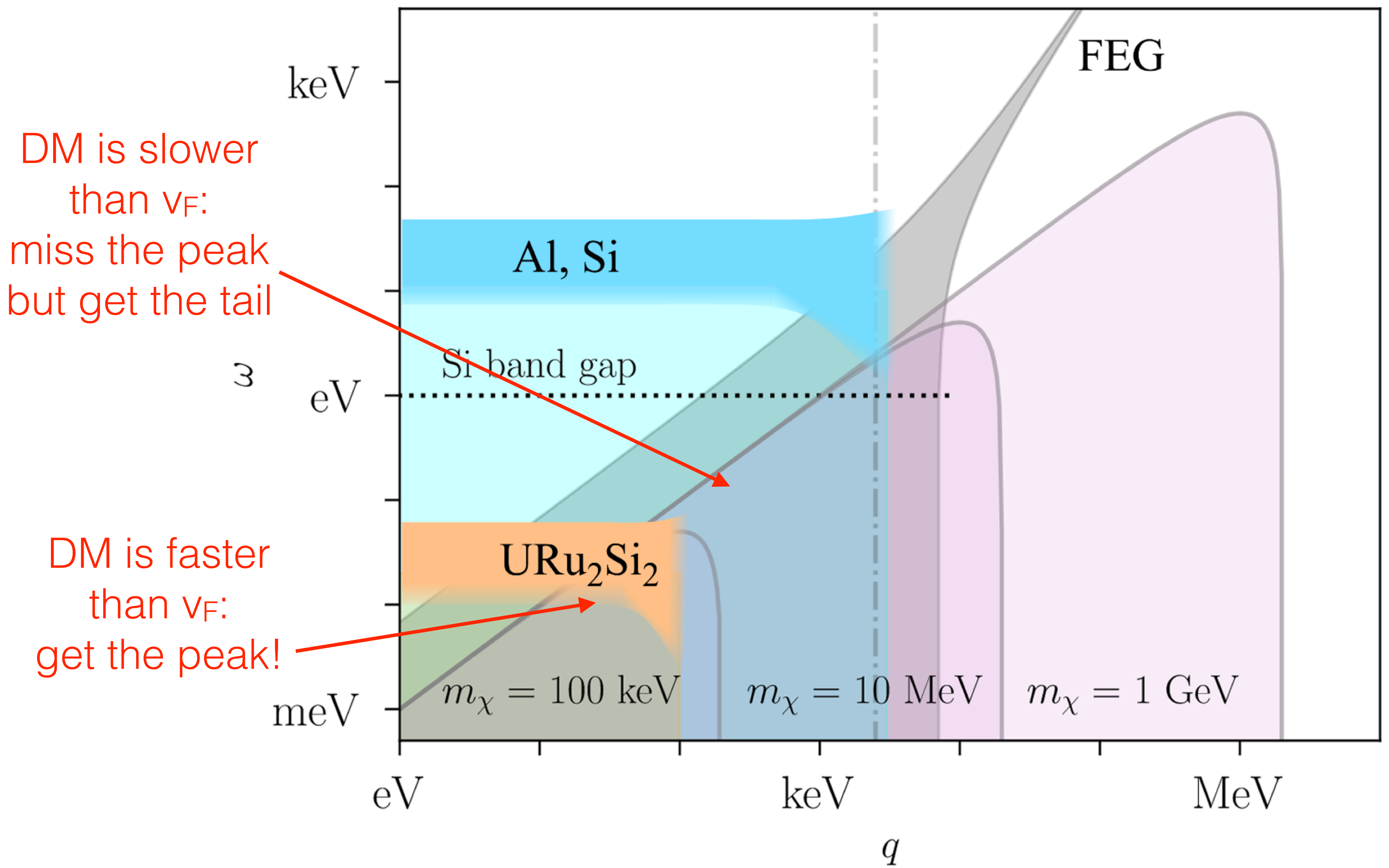
Invisible in single-particle formalism

What happens in a real-life superconductor? Need measurements!

[Gibbons, Schnatterly, Ritsko, Fields, PRB 1975; Sun et al., Chinese J. Chem. Phys. 2016, Berggren, Hochberg, **YK**, Kurinsky, Lehmann, Yu, arXiv:2101.08263]

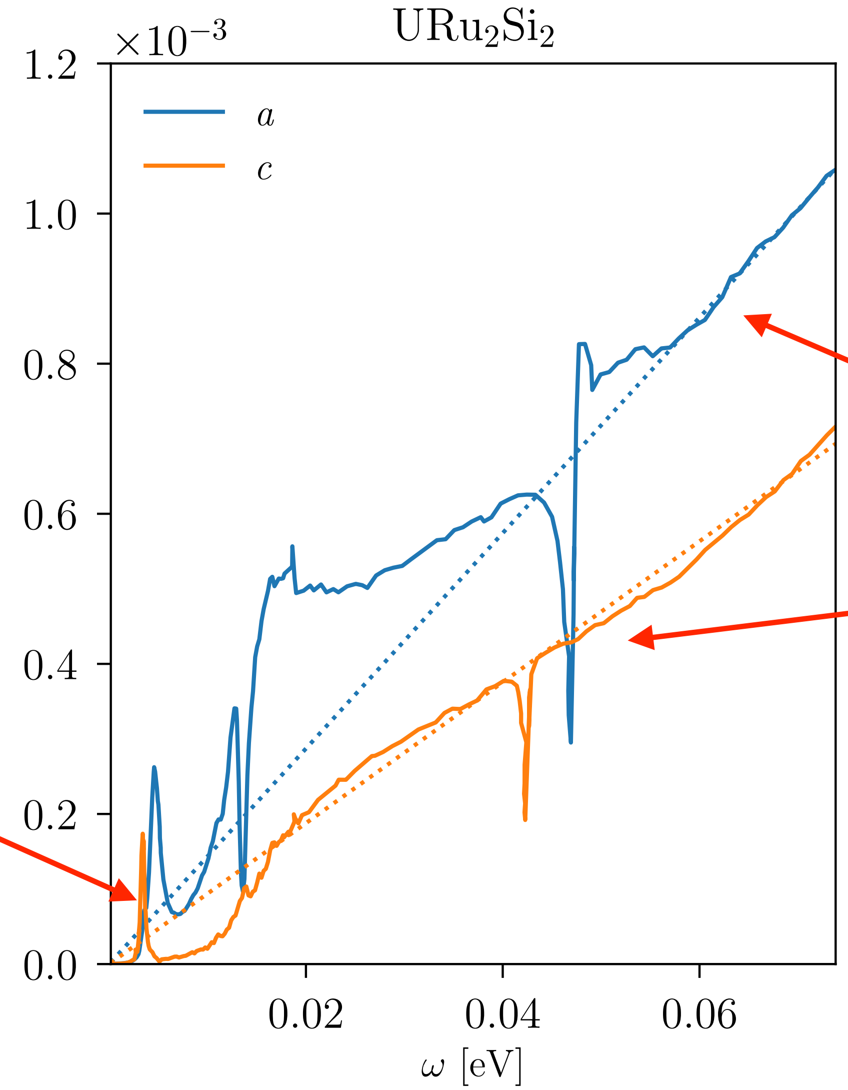
Optimizing direct detection

Best option: move to a heavier galaxy where DM is faster.
Next best option: find a material with slow electrons



[Berggren, Hochberg, **YK**, Kurinsky, Lehmann, Yu, arXiv:2101.08263]

Anisotropic plasmons!



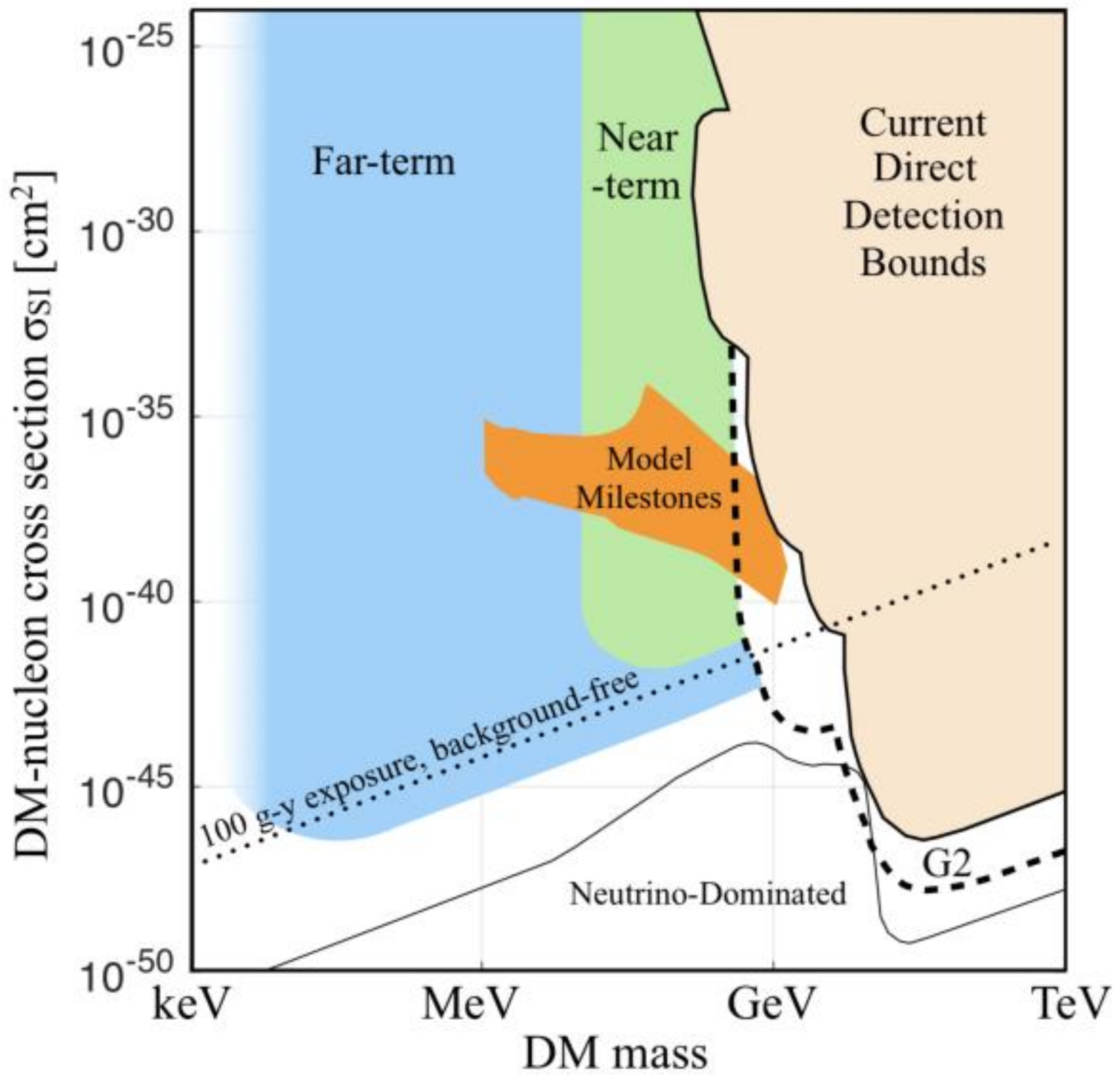
heavy-fermion plasmons (low v_F)

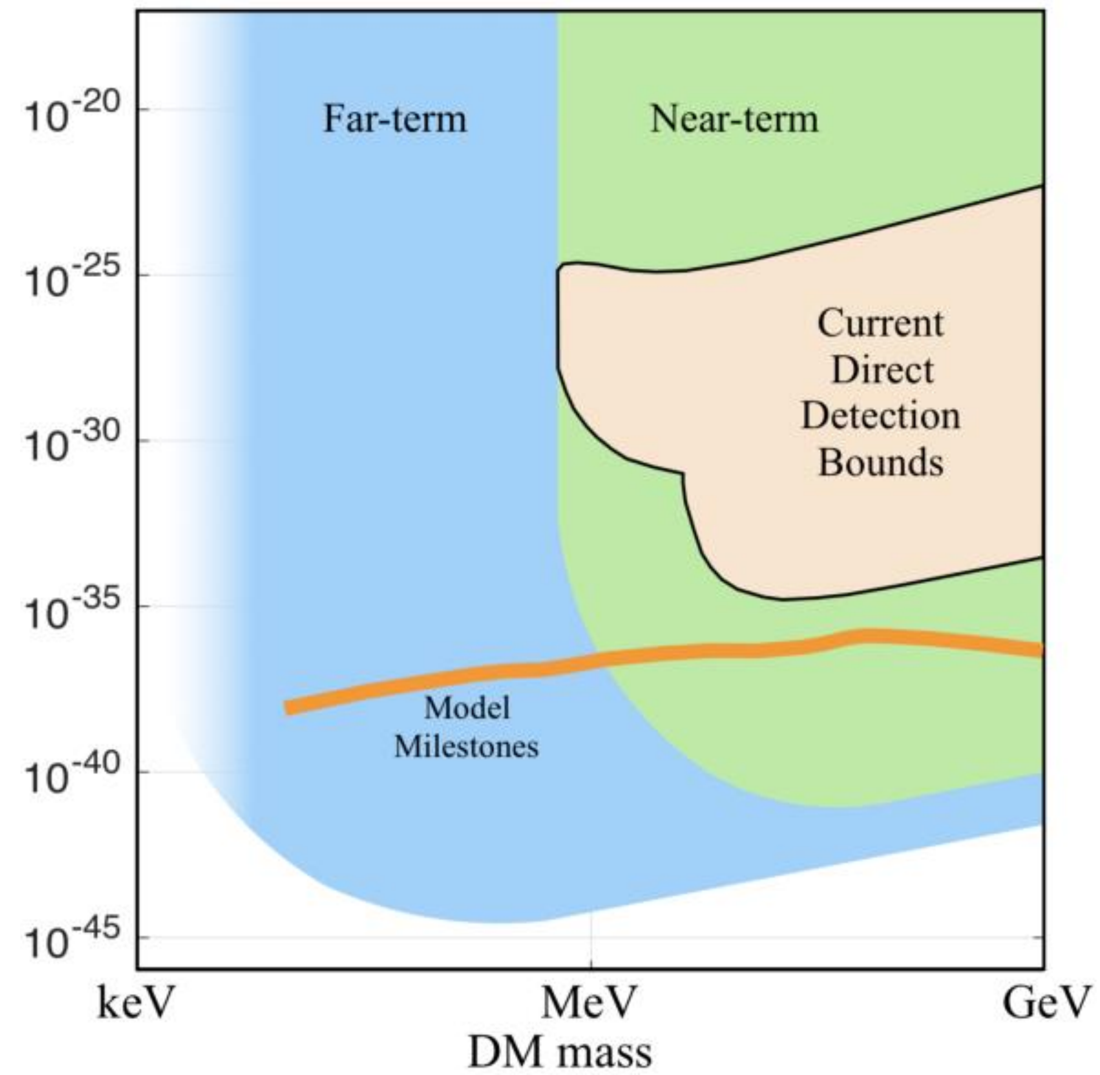
large anisotropy in tails of bulk plasmons

Best of both worlds! Peaks of loss functions accessible, and possibility of daily modulation — stay tuned

[Berggren, Hochberg, **YK**, Kurinsky, Lehmann, Yu, arXiv:2101.08263]

The future for sub-GeV DM





Detectors are uncovering new parameter space every day,
and more on the horizon. CM tools help determine the true sensitivity!

[DOE BRN report]

BACKUP

Avatars of the dielectric

Polarizability: measures linear response to E-fields

$$\epsilon(\mathbf{q}) = \frac{\mathbf{D}(\mathbf{q})}{\mathbf{E}(\mathbf{q})} = \frac{V_{\text{Coulomb}}(\mathbf{q})}{V_{\text{eff}}(\mathbf{q})}$$

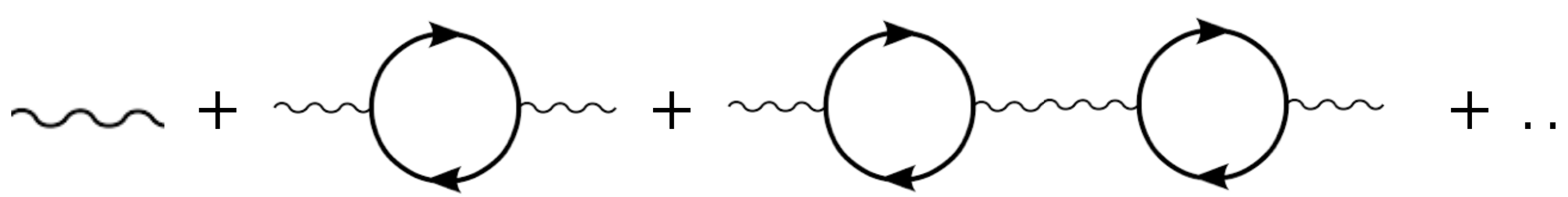
Many-body density operator: generalizes single-particle picture

Real-time correlator for electron density

$$\epsilon^{-1}(\mathbf{q},\omega) = 1 - \frac{e^2}{q^2}\frac{1}{V}\int d\tau e^{i\omega\tau}\langle\hat{\rho}_e(\mathbf{q},\tau)\hat{\rho}_e(-\mathbf{q},0)\rangle\Big|_{i\omega\to\omega}$$

Can compute with random phase approximation (RPA):

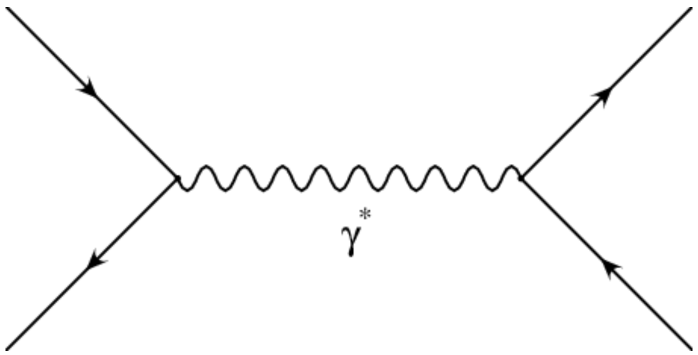
$$\epsilon(\mathbf{q}) = 1 - \frac{e^2}{q^2}\Pi_q$$



Exact dielectric function contains **all** screening and many-body effects: required to move beyond single-particle formalism

An allegory

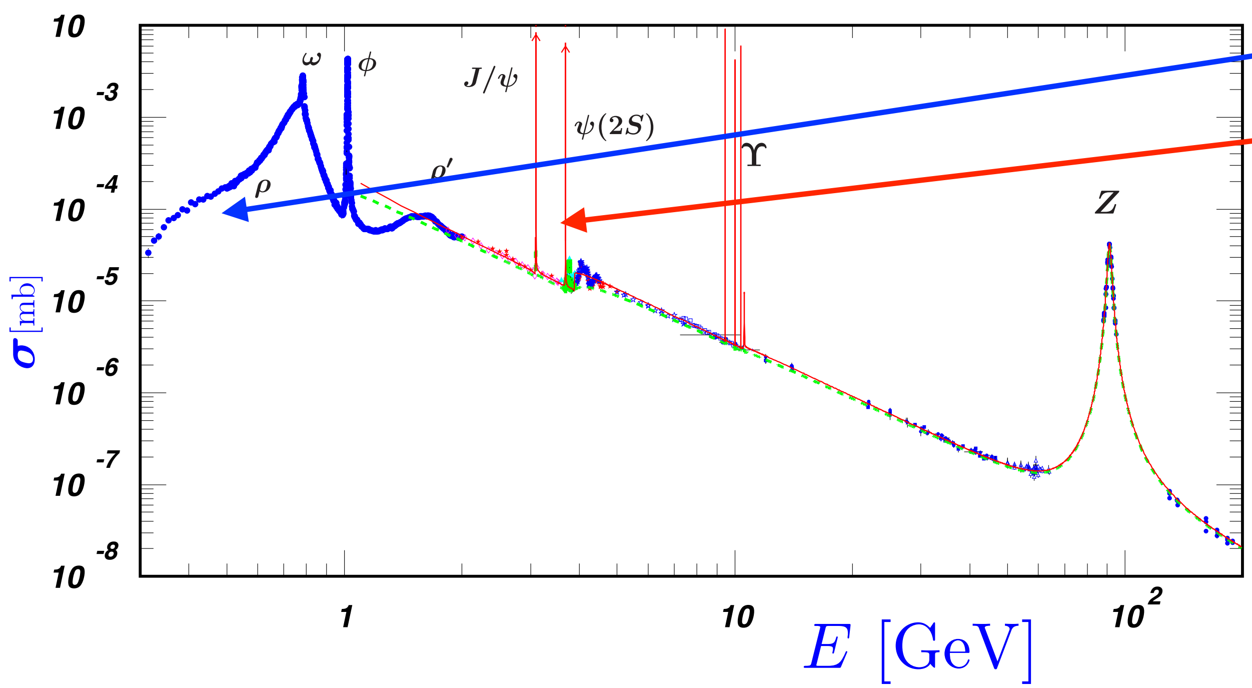
What is the e+e- annihilation cross section as a function of energy?



OK, my Lagrangian has quarks in it…
let me calculate $e^+e^- \to q\bar{q}$.

$$\sigma(E) = \frac{4\pi\alpha^2}{3E^2}$$

Looks pretty smooth as a function of E…



hey, why is it rising instead of falling?

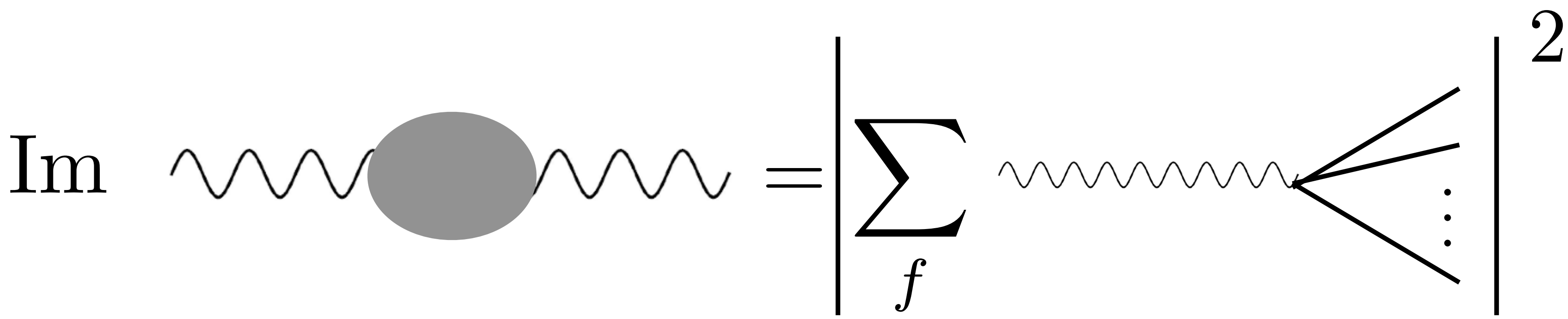
and what the heck are those?

Guess I have to include resonances. But how to fit them? What about final states with more than 2 particles?

An allegory

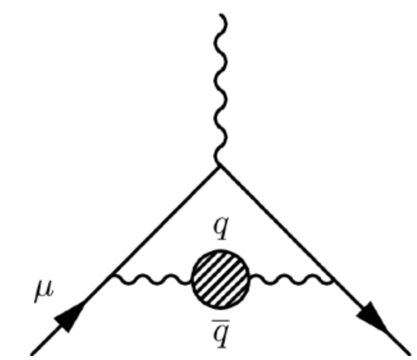
What is the e^+e^- annihilation cross section as a function of energy?

Optical theorem:



If someone gave you the exact photon propagator, you could not only compute the cross section, you could look for poles to find resonances, even if they weren't in your Lagrangian.

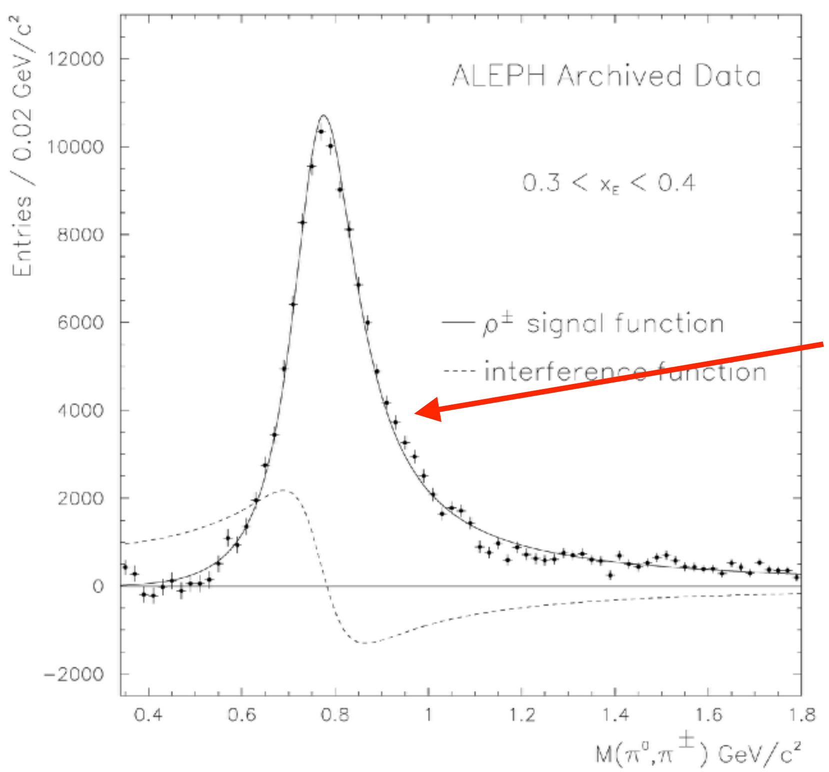
Even better: measure $\mathrm{Im}(\Pi)$ by doing a scattering experiment, then use that as input to another experiment, e.g. $(g-2)_\mu$



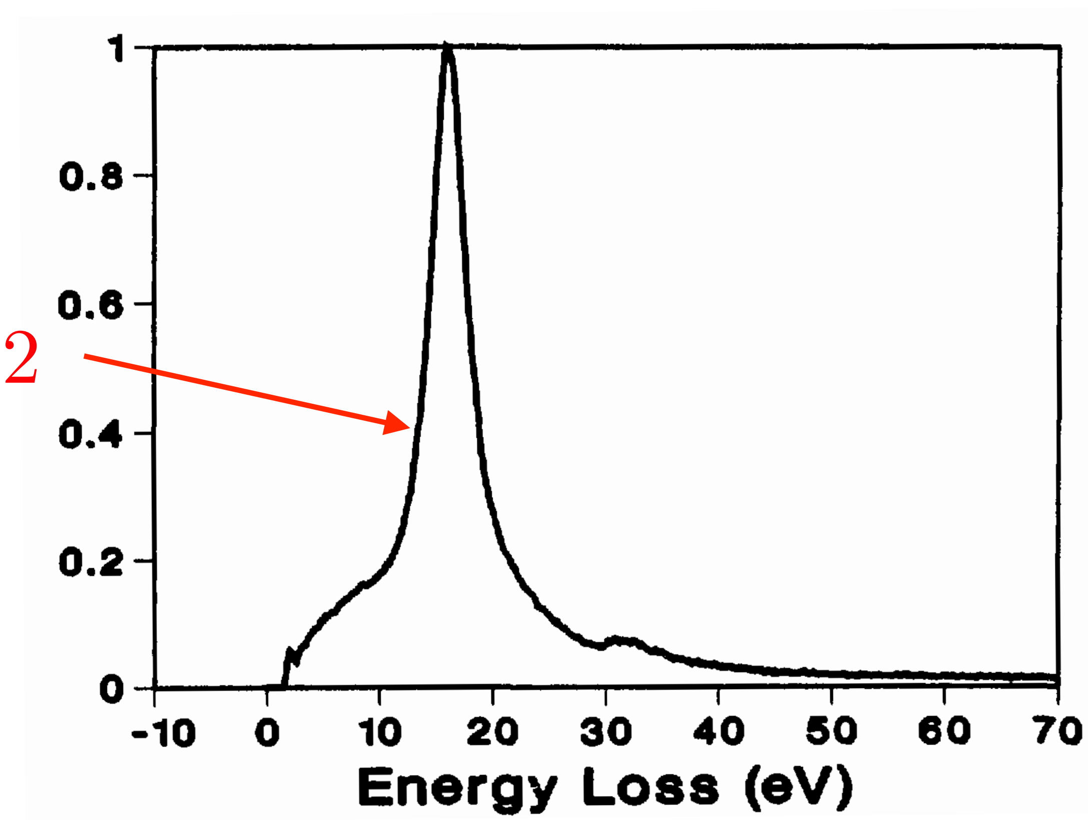
In condensed matter, this is often a better starting point.
"what is the response function?" vs. "what are the wavefunctions?"

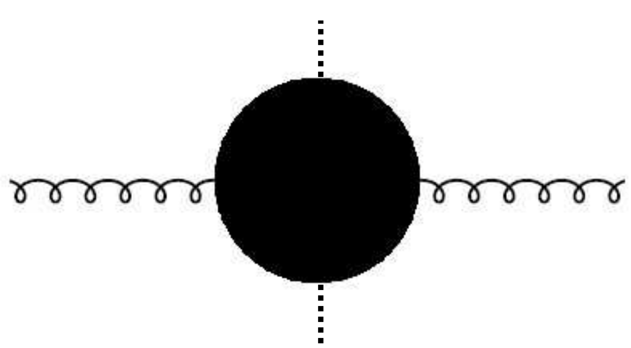
Some possibly useful analogies

The plasmon is like the ρ meson



$\Gamma/E \approx 0.2$





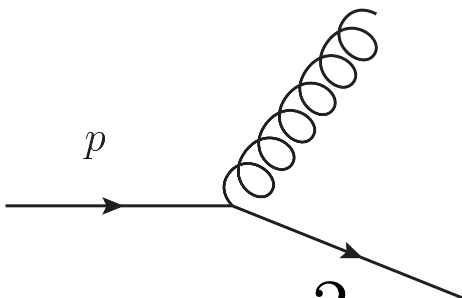
$$S(\omega) \equiv \mathrm{Im}\left\{\frac{-1}{\epsilon(\omega)}\right\}$$

Narrow width approximation fails miserably! Strong coupling at work

Some possibly useful analogies

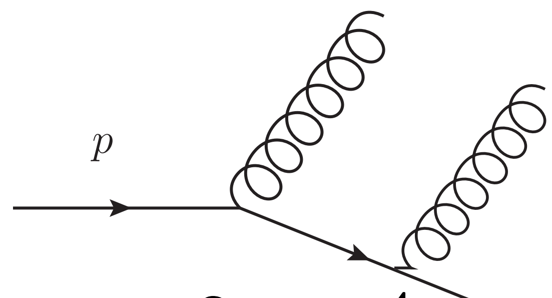
The plasmon is like the Sudakov factor

QCD:



$\alpha_s \log^2$

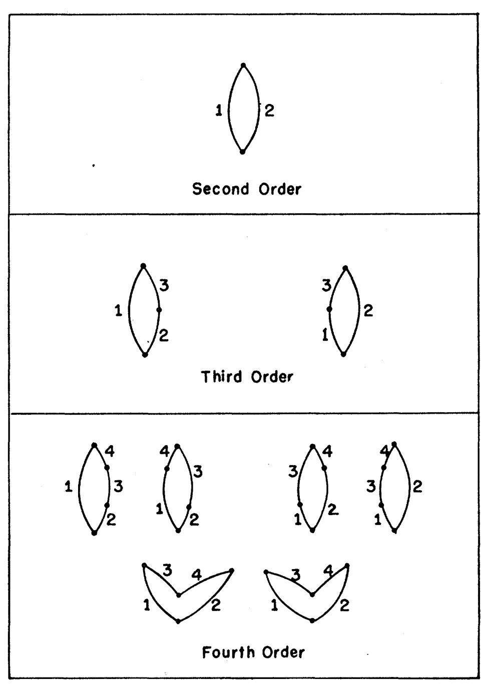
$+$



$\alpha_s^2 \log^4$

$$+ \ldots \quad = \exp\left[-\frac{\alpha_s}{\pi}\log^2\right]$$

Coulomb:



$e^2\Lambda^2$

$e^4\Lambda^4$

$e^6\Lambda^6$

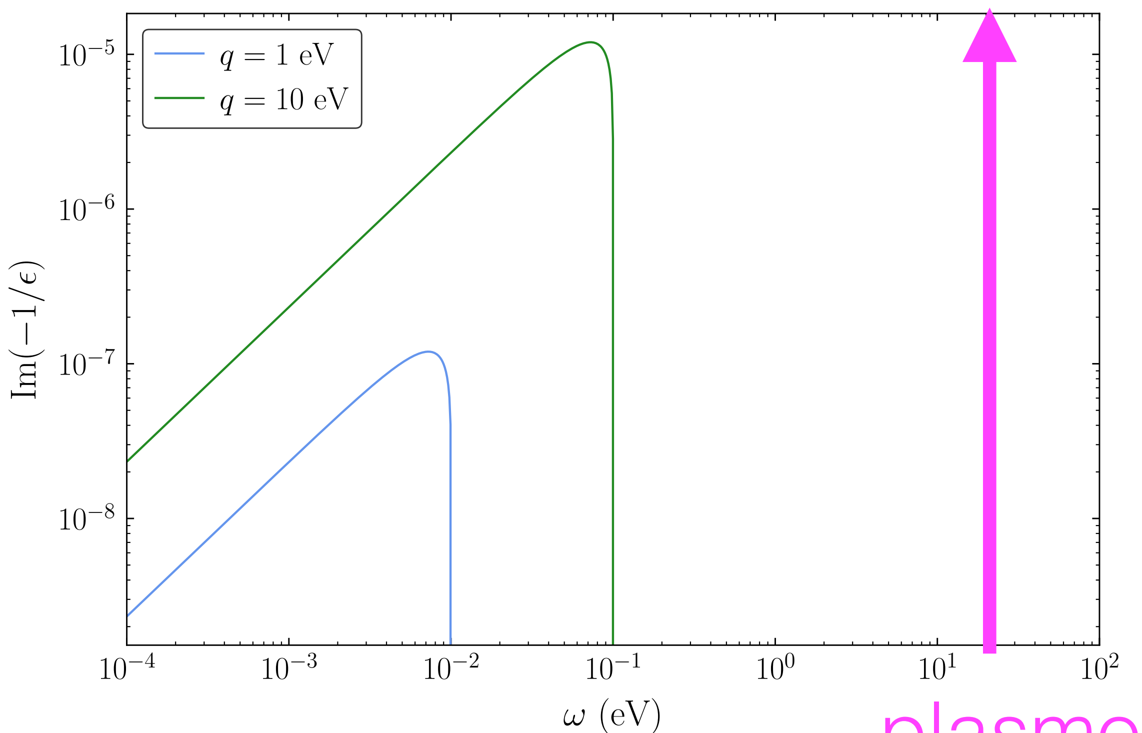
$$\Longrightarrow \quad C \log e^2$$

Invisible at finite order in perturbation theory!

A model: free electron gas

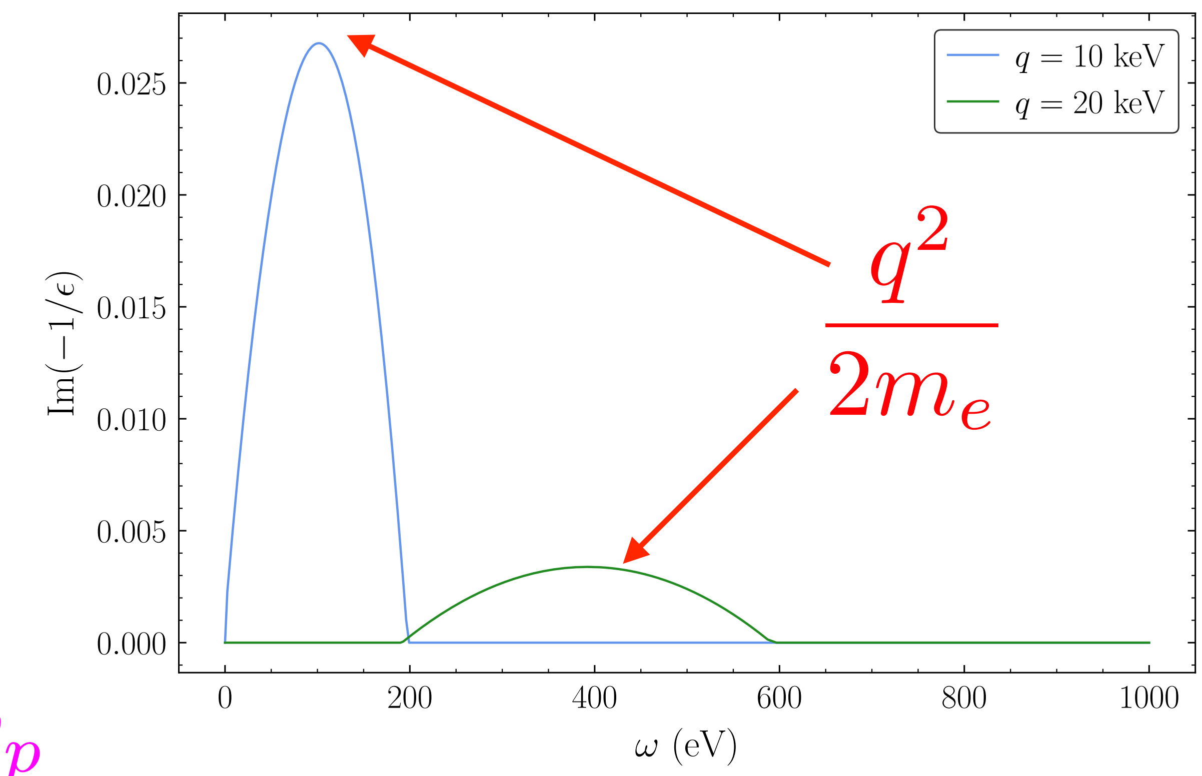
Characteristic momentum scale is $k_F \simeq \lambda_{\text{TF}} \simeq 2\pi/a \simeq 5 \text{ keV}$

$q \ll k_F$



plasmon ω_p

$q \gg k_F$



screening and collective modes

single-particle elastic scattering

All "ordinary" materials have $v_F \simeq 10^{-2}, \; \omega_p \simeq 15 \text{ eV}$

In real life, plasmon has finite width…

[Berggren, Hochberg, **YK**, Kurinsky, Lehmann, Yu, arXiv:2101.08263]