From Freezeout to Squeezeout for Heavy Thermal Dark Matter

Tracy Slatyer

MIT

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with Pouya Asadi, Eric Kramer, Eric Kuflik,
Gregory Ridgway, & Juri Smirnov

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Outline

- Introduction/review of thermal freezeout, the thermal mass window, and the unitarity bound
- Freezeout cosmology of ultraheavy strongly interacting dark sectors
    - estimating the effects of a dark phase transition
    - how a phase transition after initial freezeout leads to a second “squeezeout” phase
    - results for the dark matter depletion and the accidentally asymmetric limit

What do we know about dark matter?

We know it:

- Is roughly 84% of the matter in the universe.
- Has mass (and hence gravity).
- Doesn't scatter/emit/absorb light (really "transparent matter"!)
- Interacts with other particles weakly or not at all (except by gravity).
- Is distributed through galaxies and the universe in a way that we can predict and map.

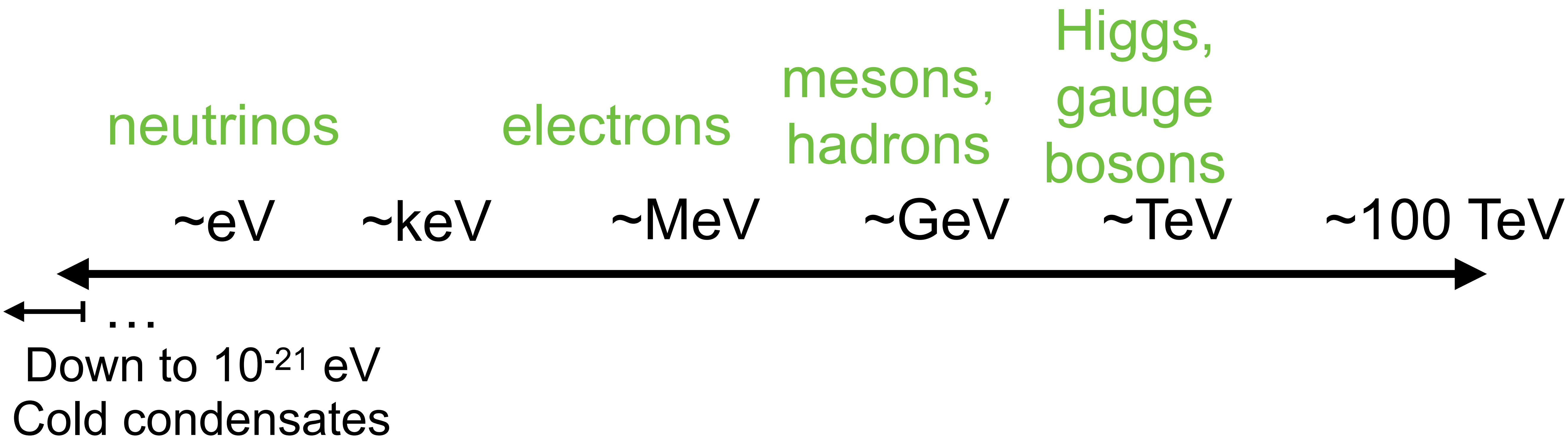
We don't know:

- What it's made from.
- Is it one particle, or more than one?
- How it interacts with other particles.
- Whether it's absolutely stable, or decays slowly over time.
- Why its abundance is what it is.
- If/how it's connected to other deep problems in particle physics.

A spectrum of DM scenarios

neutrinos		electrons	mesons, hadrons	Higgs, gauge bosons	
~eV	~keV	~MeV	~GeV	~TeV	~100 TeV

A spectrum of DM scenarios



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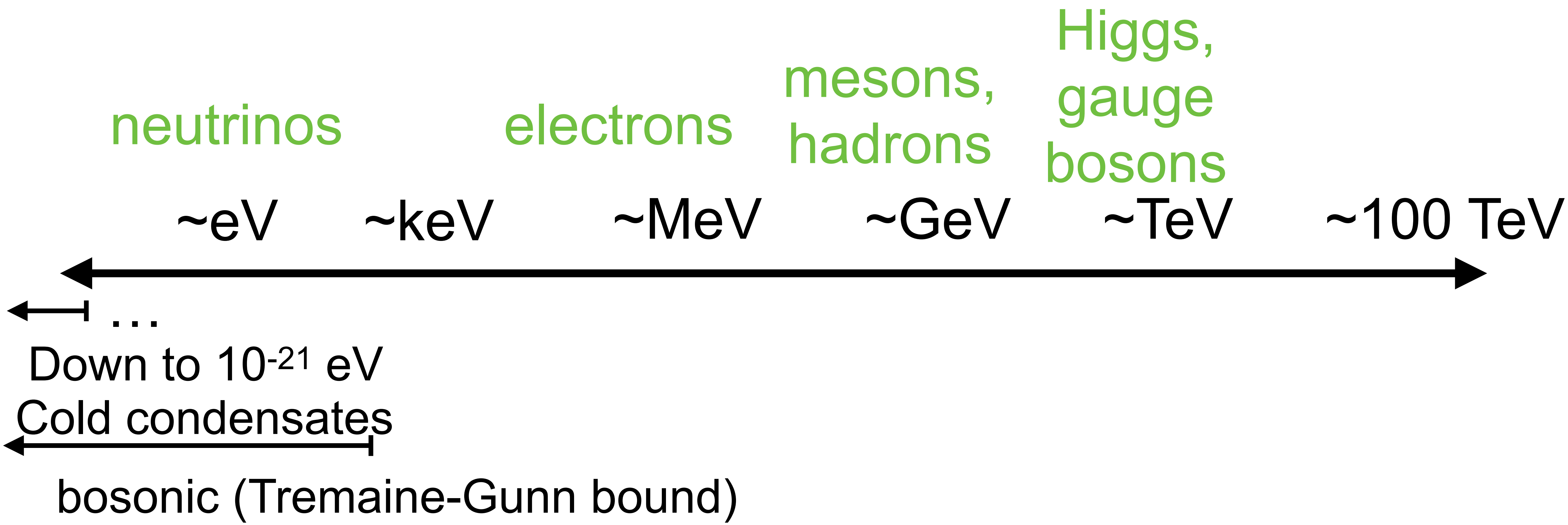

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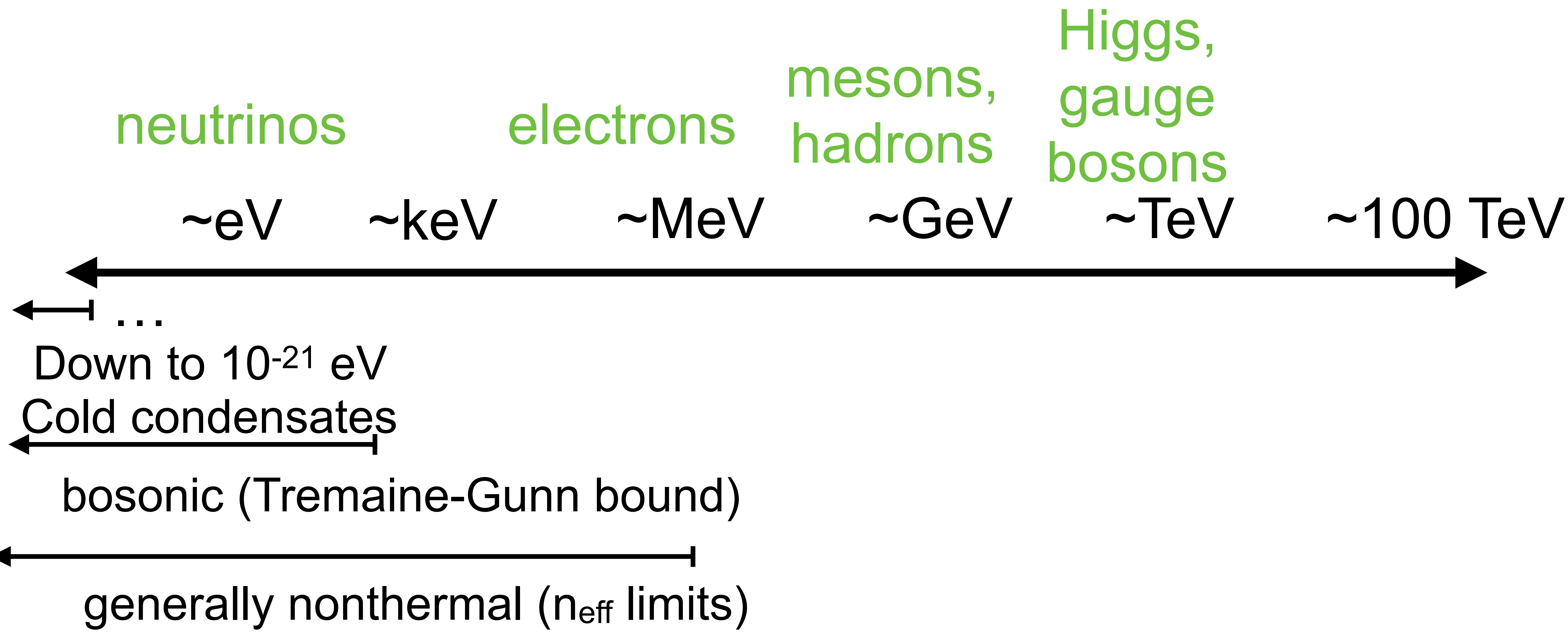

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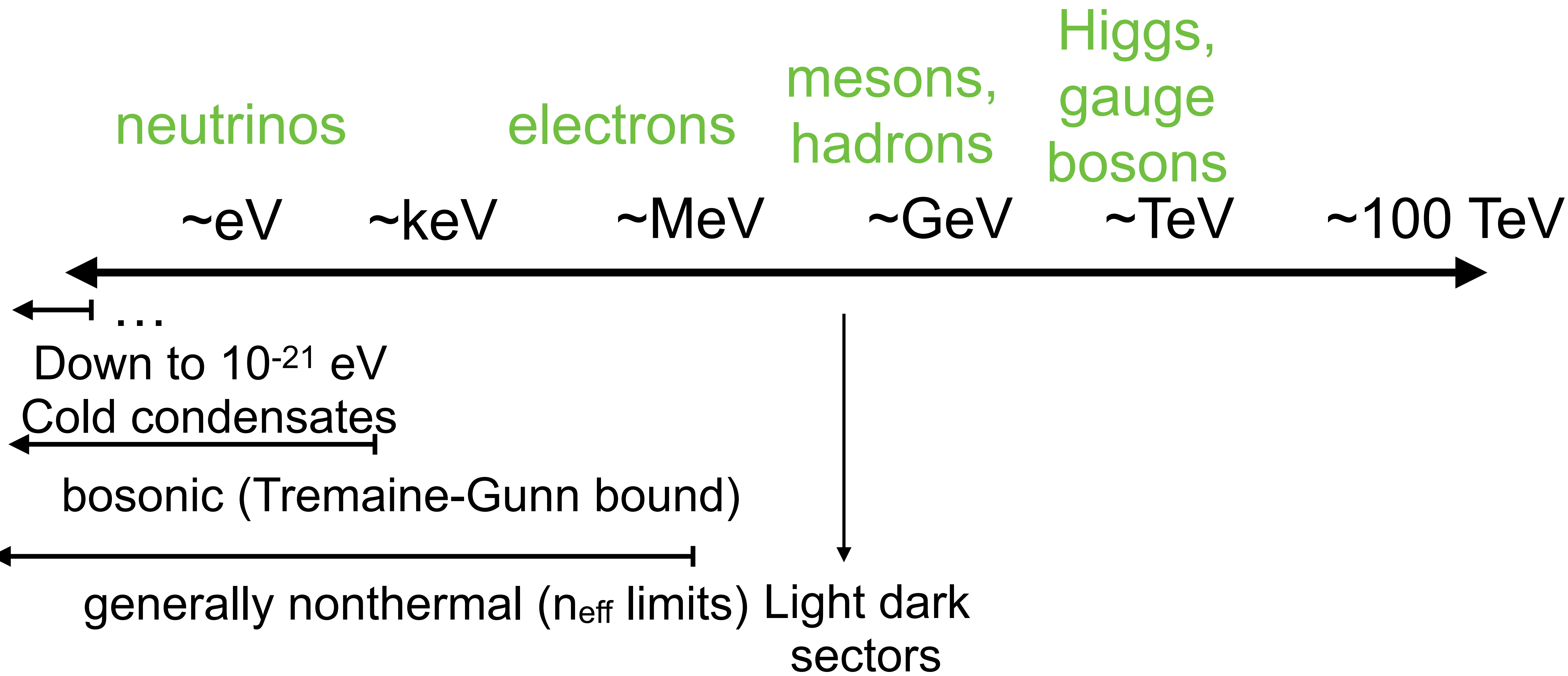

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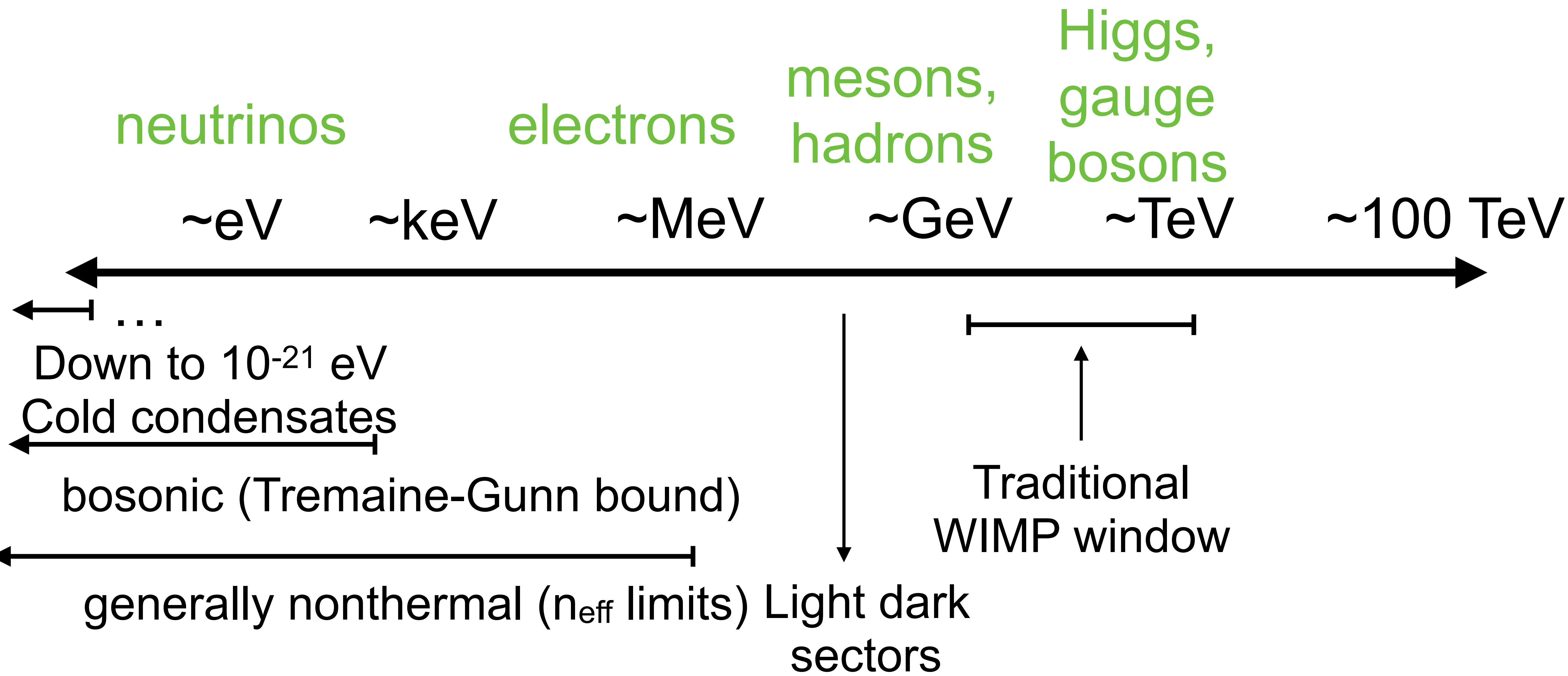

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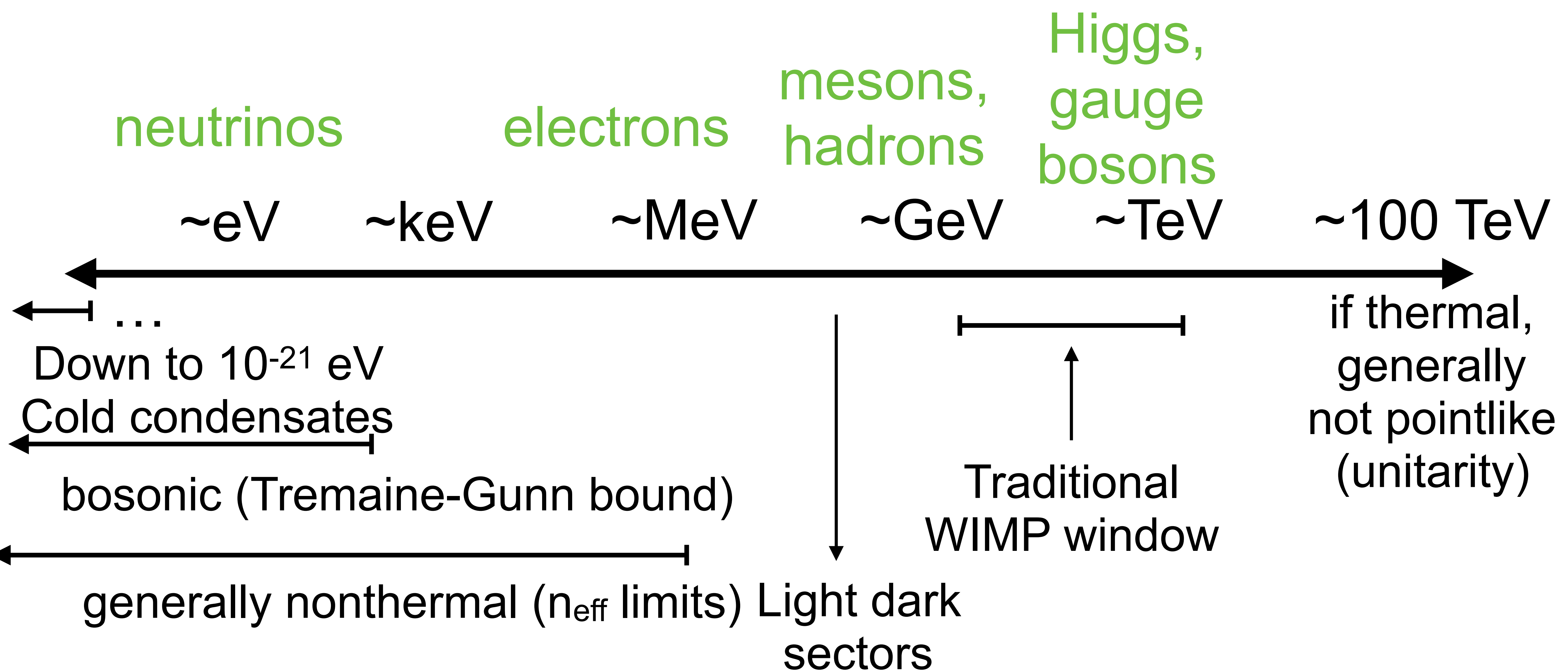

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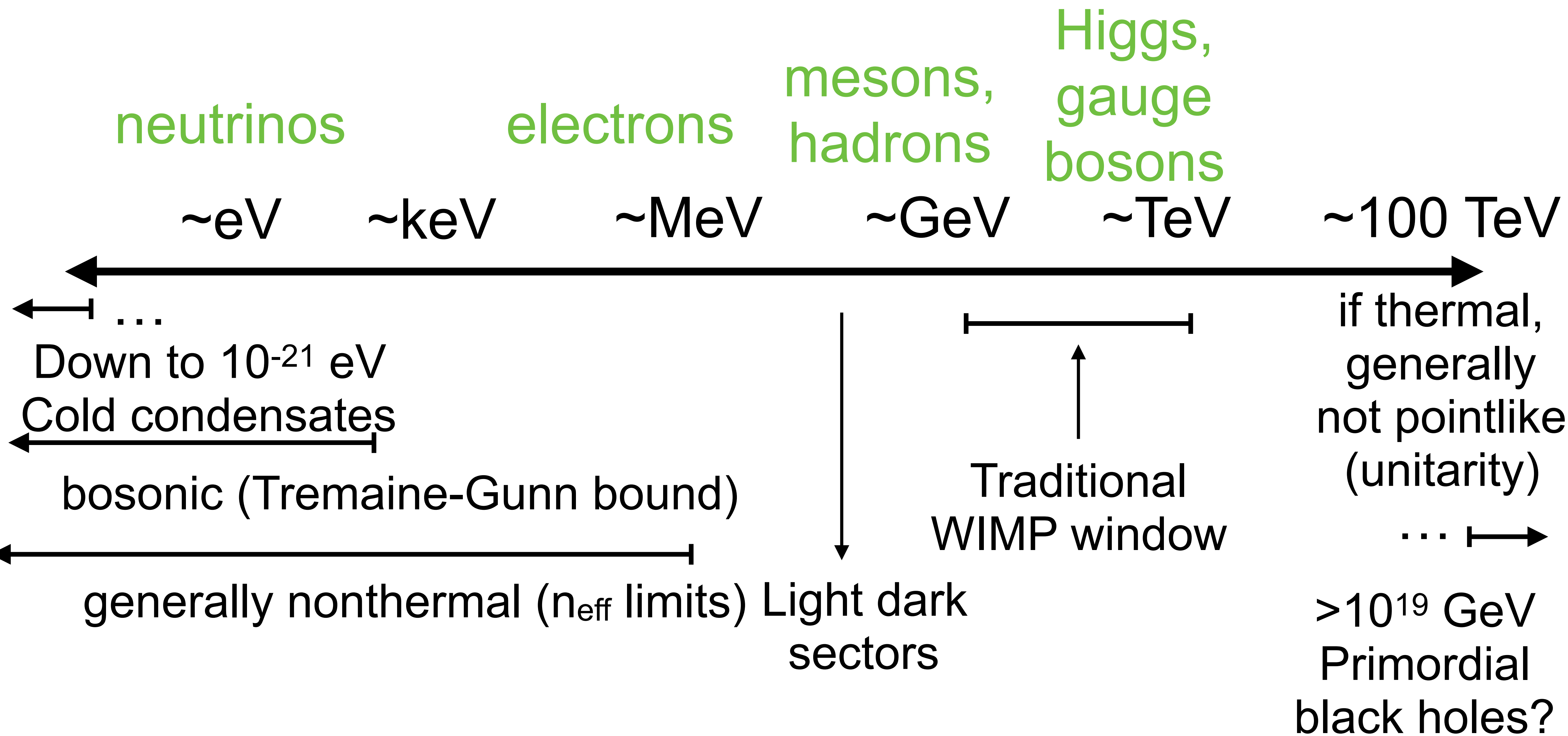

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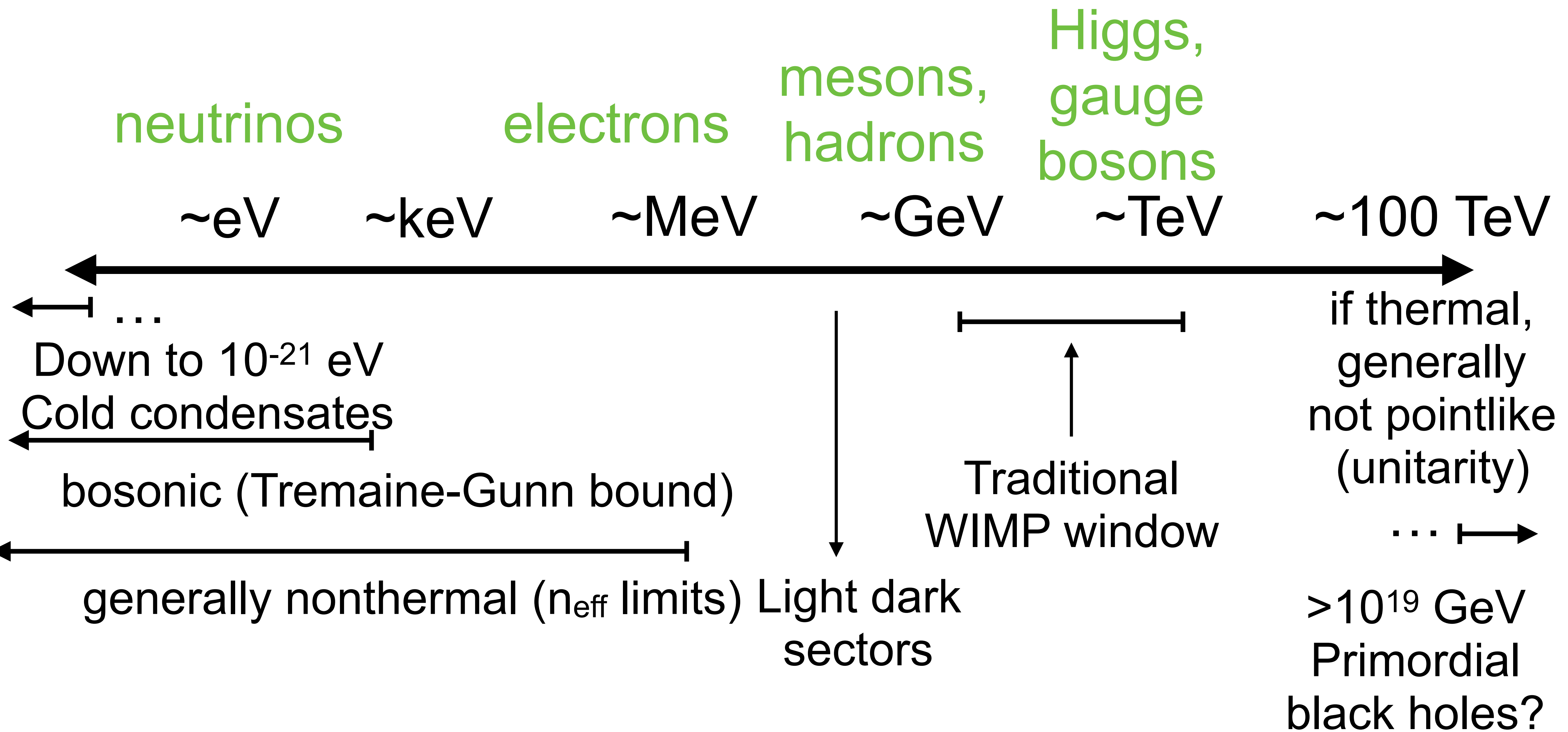

- The possibilities for dark matter span an enormous range of models and masses

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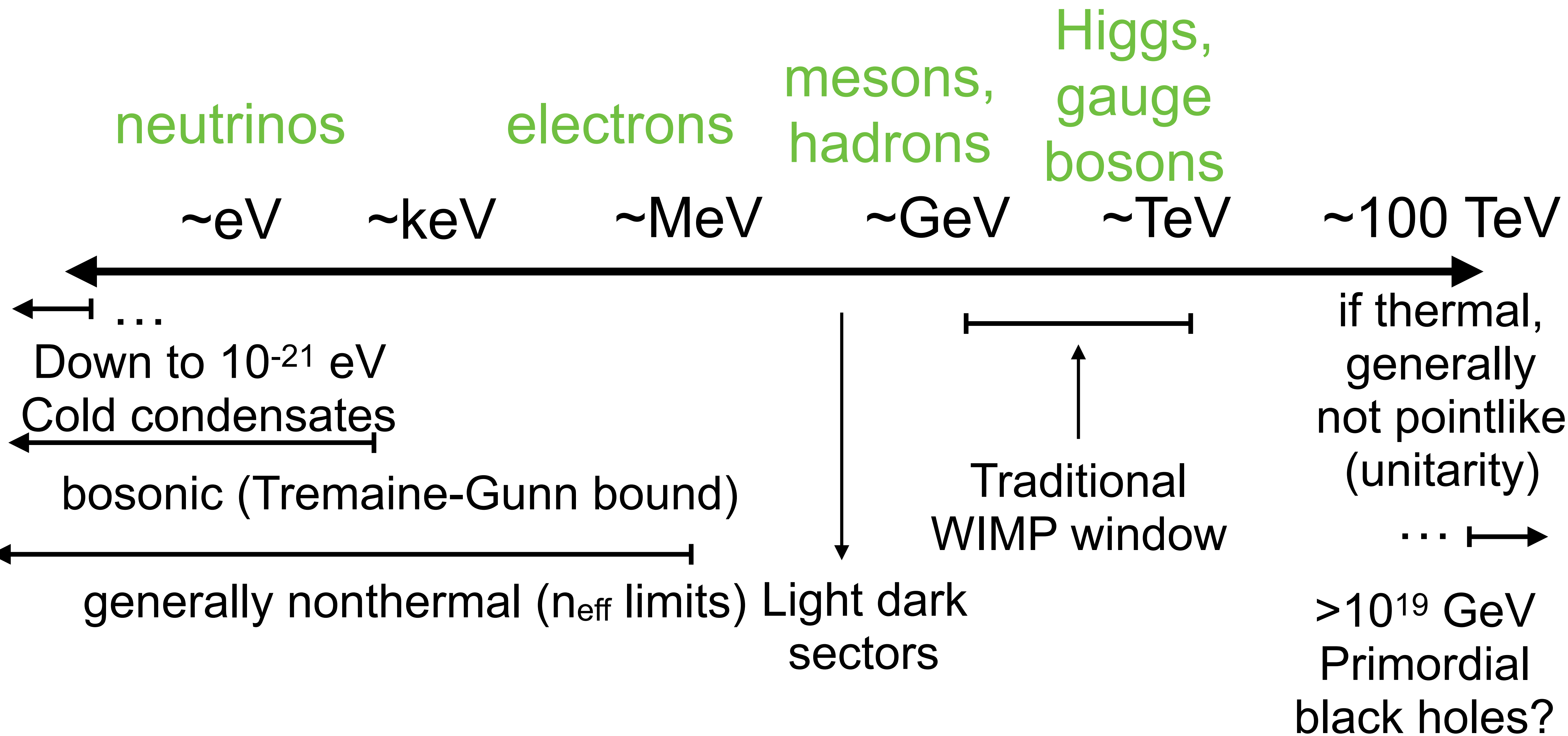

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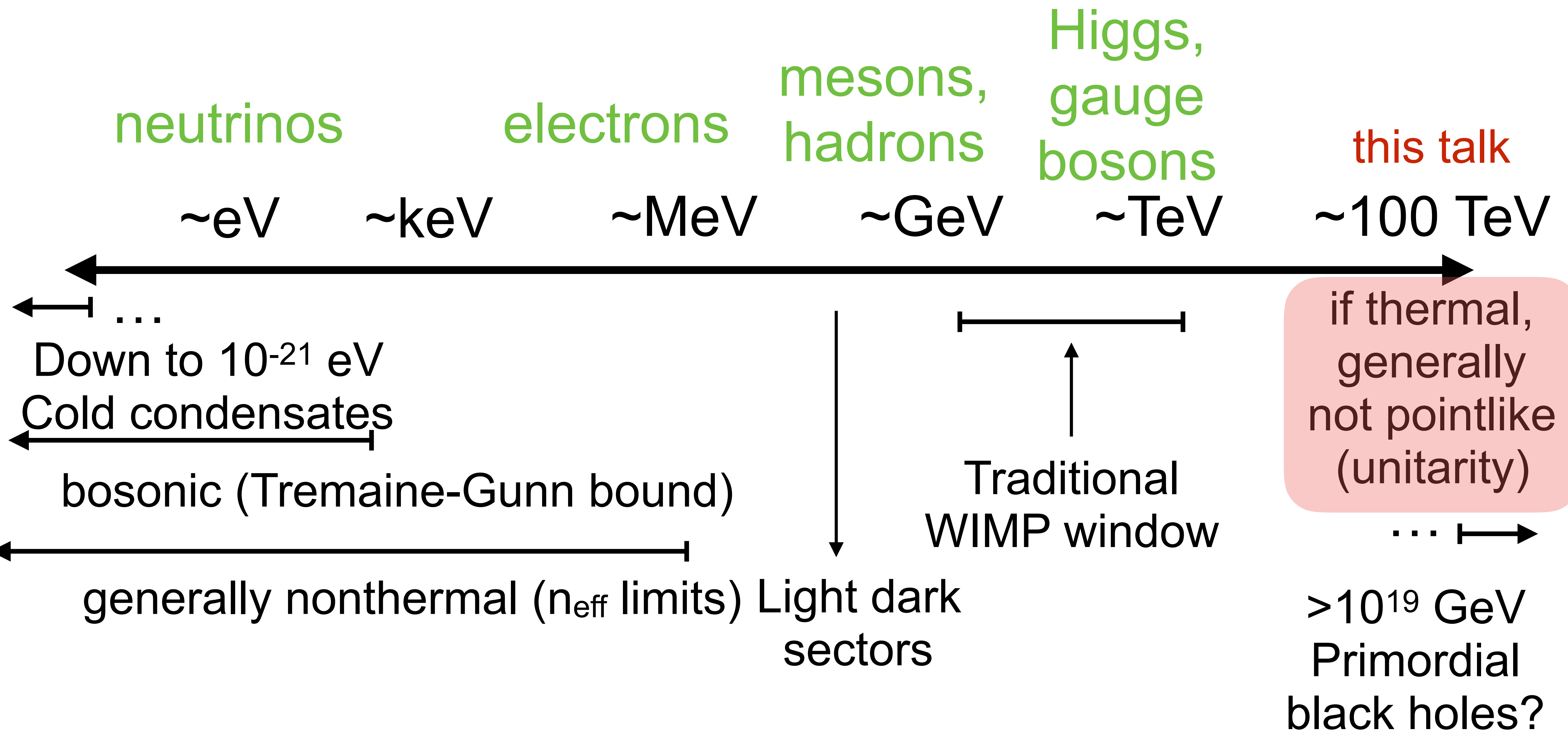

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The “thermal window”

- It is appealing to think of mechanisms that dynamically generate the observed DM abundance from its interactions with visible matter - classic examples are freezeout, freeze-in

- In the “freezeout” scenario, DM is initially in thermal equilibrium with the SM radiation bath, and is depleted through number-changing interactions with the SM. Conventional wisdom is that this defines a “thermal window” in DM mass:

  - Lower mass limit ~1-10 MeV: lighter DM is abundant during Big Bang nucleosynthesis and modifies n_{eff} (effective # of neutrino species for cosmological expansion) during that epoch [e.g. Sabti et al ’20]

  - Upper mass limit O(100) TeV: heavier DM cannot annihilate efficiently enough to deplete its abundance to that observed today [e.g. Smirnov & Beacom ’19]

Classic thermal freezeout

- Suppose there is some interaction that interconverts between dark matter and SM particles and is efficient in the early universe

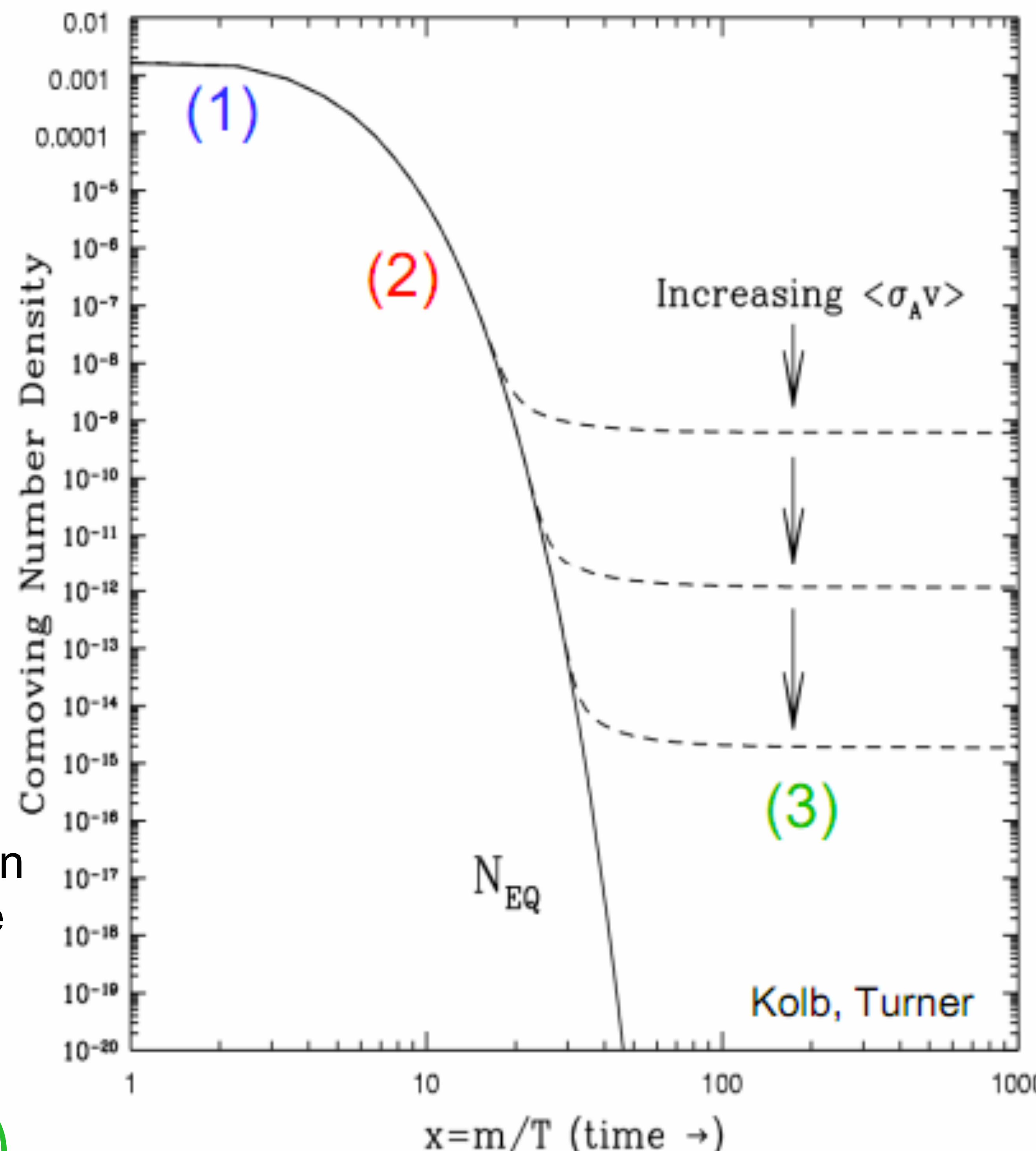
$$(1) \quad \chi\chi \leftrightarrow \mathrm{SM\,SM}$$

- As the universe expands, it cools down; eventually its temperature drops below the dark matter mass.
- At this stage, dark matter particles can efficiently annihilate to visible particles, but not the reverse:

$$(2) \quad \chi\chi \to \mathrm{SM\,SM} \not\to \chi\chi$$

- Dark matter abundance falls exponentially - eventually cuts off when the timescale for collision becomes comparable to the expansion timescale
- At this point we say the annihilation has <u>frozen out</u> and the late-time dark matter abundance is fixed

(3)



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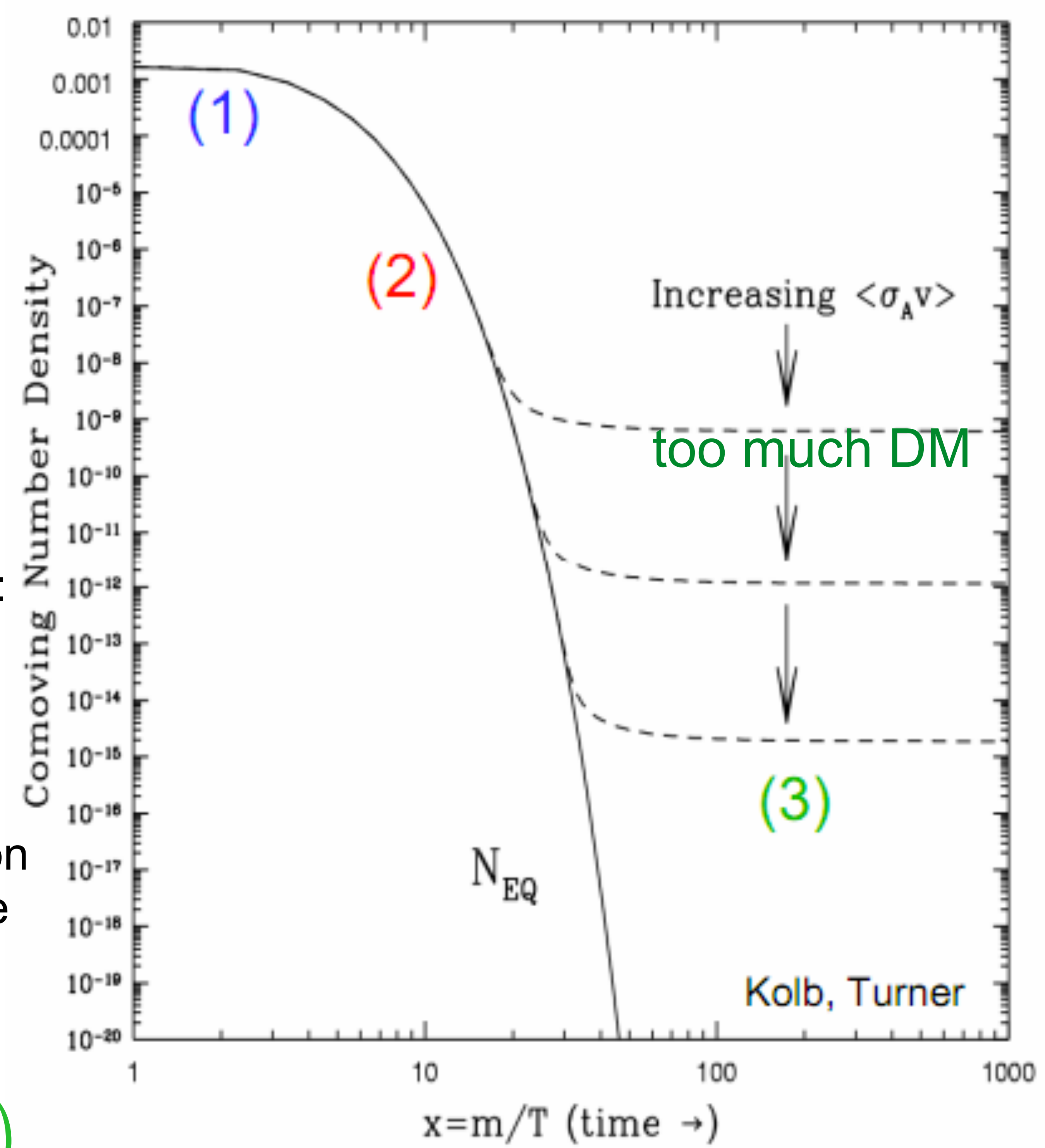

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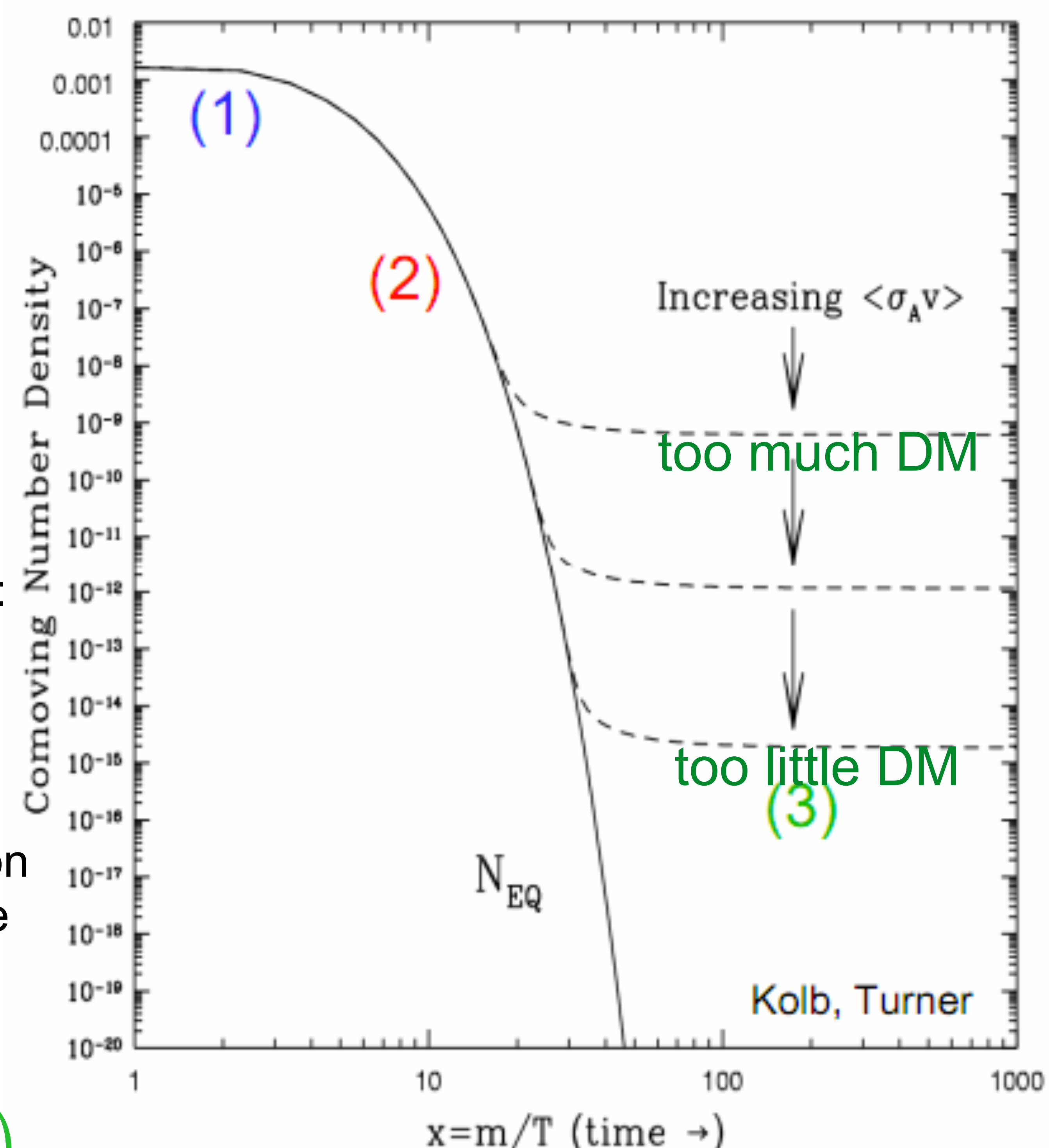

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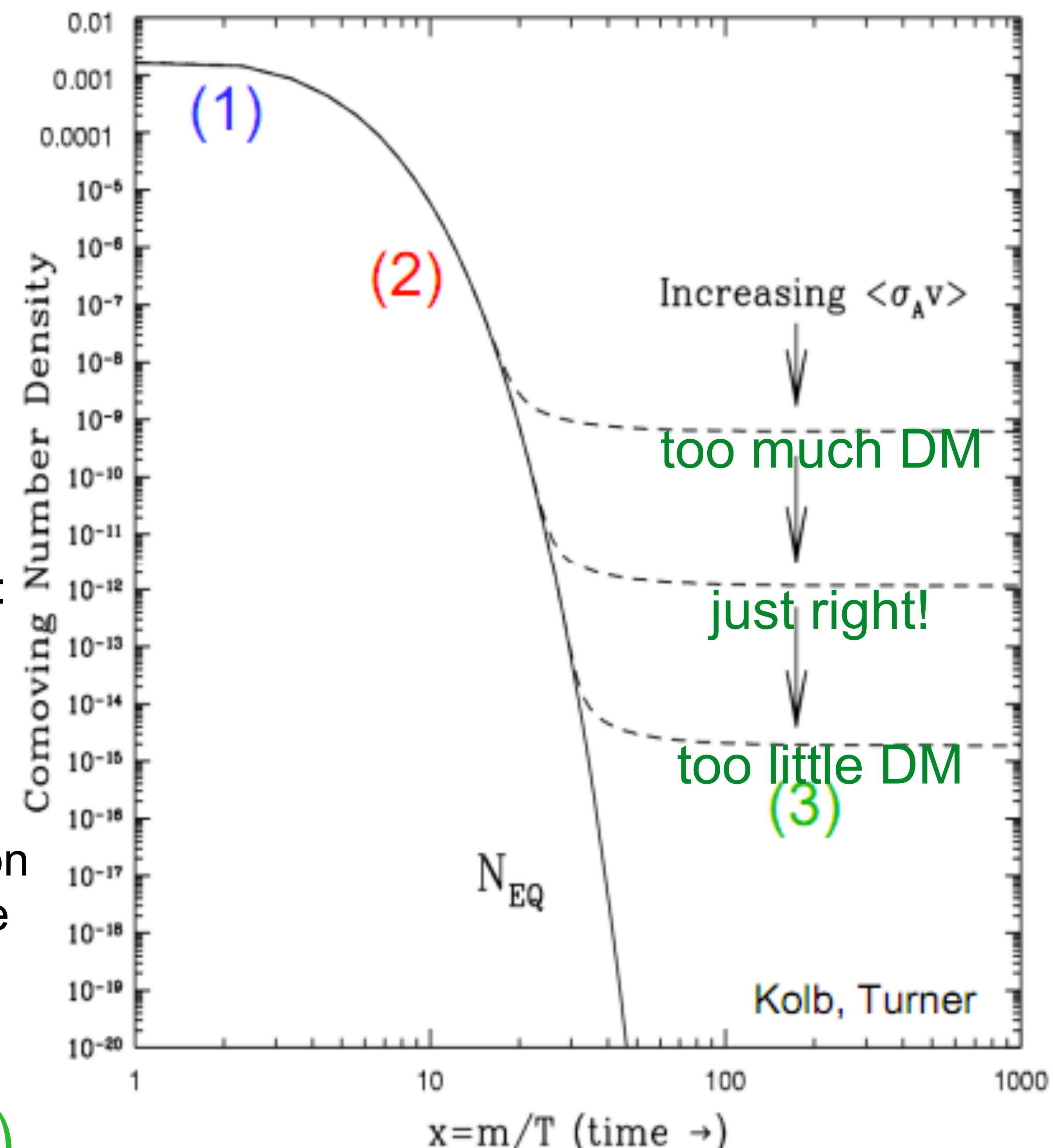


The unitarity bound

- In this scenario, the interaction strength controls the freezeout and hence the late-time ("relic") abundance of dark matter: stronger interactions = longer exponential decrease = lower abundance

- From measuring the relic abundance we can predict the annihilation rate:

$$\langle \sigma v \rangle \approx 2 \times 10^{-26} \mathrm{cm}^3/s \approx \frac{1}{(25\mathrm{TeV})^2} \sim \frac{1}{m_{\mathrm{Pl}} T_{\mathrm{eq}}}$$

- In the limit of weak interactions, this suggests a characteristic mass scale around $M \sim \alpha_D \times 25\mathrm{TeV}$, if α_D is the relevant coupling

- In the limit of strong interactions, partial-wave unitarity still sets a mass-dependent upper bound on the cross section, which implies a maximum mass scale around 100 TeV:

$$\sigma = \sum_{l=0}^{\infty} \sigma_l, \quad \sigma_l = \frac{4\pi}{k^2}(2l+1)\sin^2\delta_l \leq (2l+1)\frac{4\pi}{k^2}$$

Exceptions to the unitarity bound

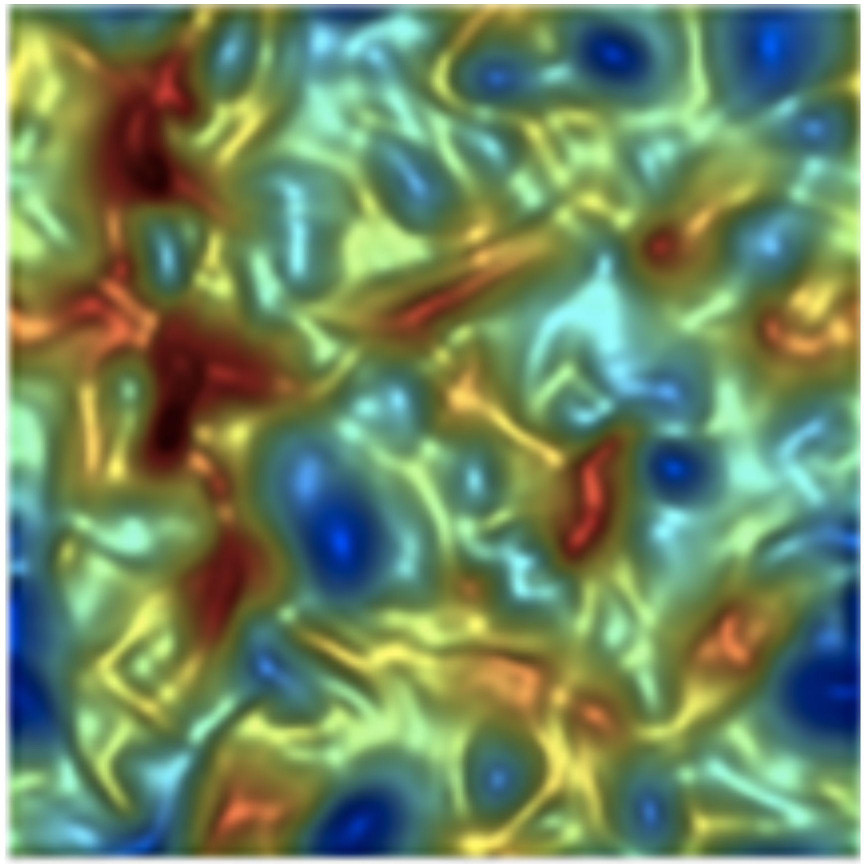
- One can evade this bound by modifying the cosmology
  - as one example, suppose the DM freezes out early, but then subsequently many other (currently unknown) degrees of freedom decay in a way that heats the SM and not the DM - increases the amount of energy in SM radiation bath relative to DM, effectively dilutes the DM
- The presence of bound states and other enhancements to annihilation can help saturate the unitarity bound, but generally not exceed it
- Often said that composite DM is an exception (since many partial waves can contribute for extended objects) but Smirnov & Beacom '19 argue that unitarity + BBN limits forces the maximum mass scale to ~1 PeV, due to plasma effects
- Obviously the bound does not apply to DM that was never in thermal equilibrium with the SM in the first place & does not need to be depleted

A confining dark sector

- Suppose we are thinking about strongly-interacting DM in the sense of a "dark sector" (DM + particles it interacts with) that is confining, like QCD.

- Then a modification to early-universe cosmology is already built in: the confinement phase transition.

- Today the dark matter would be comprised of stable dark baryons, but in the sufficiently-early universe there could/would have been a dark quark-gluon plasma.



- If the dark quarks are sufficiently heavy we expect (based on lattice studies) a first-order phase transition.

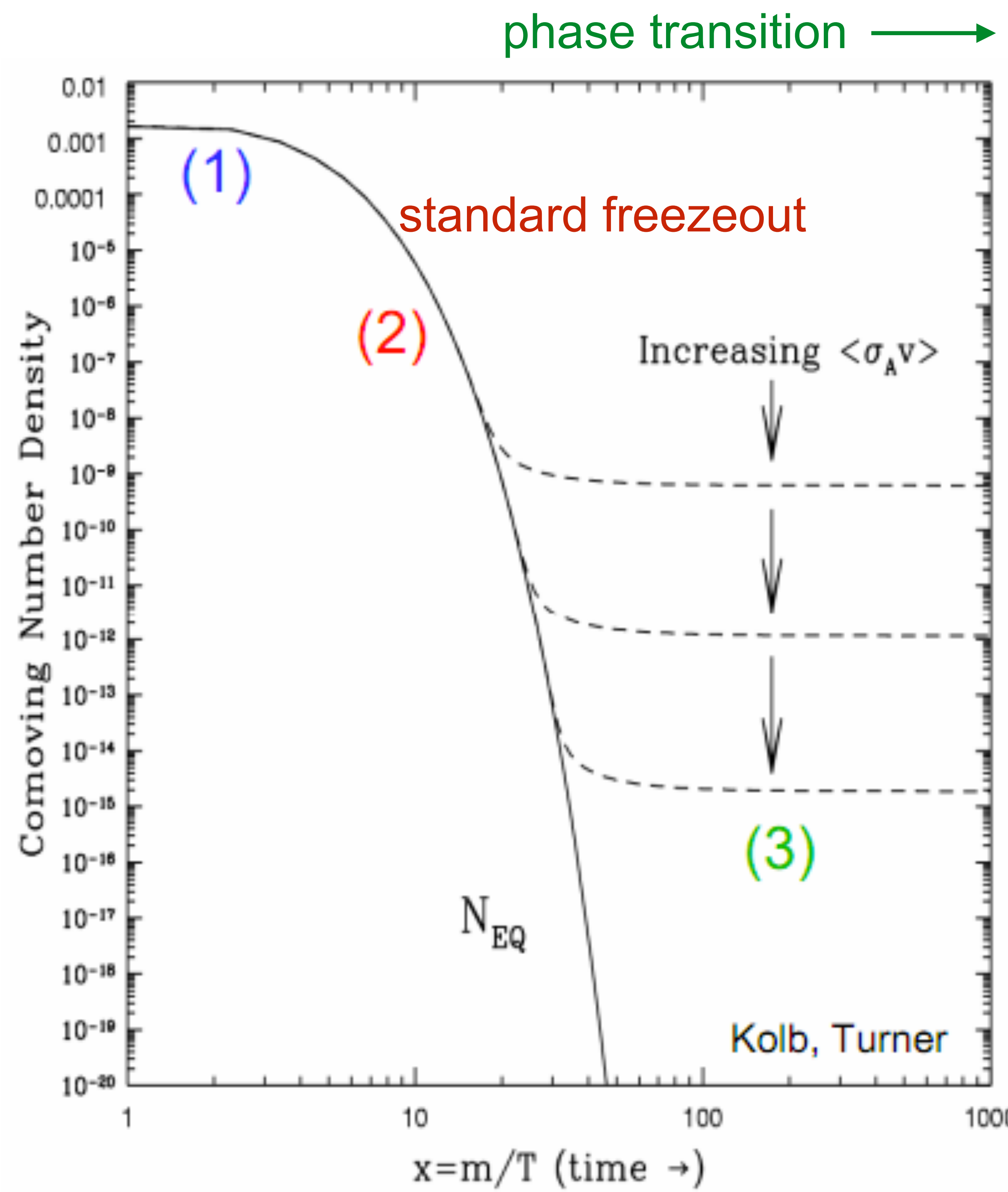
- The rest of this talk: a first-order phase transition in a strongly-interacting dark sector naturally strongly dilutes heavy thermal DM and points to a PeV-EeV mass scale.

A multi-stage history

- There are two relevant mass scales in the problem:
  - the confinement scale Λ - determines the phase transition temperature and the binding energies post-confinement
  - the quark mass m_q - determines the quark freezeout temperature
- If freezeout happens after confinement, similar to previous cases, with dark matter = dark baryons: annihilations keep the dark baryons/glueballs/other states in equilibrium with the SM, the relic abundance is fixed when the annihilation freezes out
- We will assume $m_q >> \Lambda$ so freezeout happens BEFORE the confinement phase transition

Stage I: freezeout

- Freezeout occurs as usual in the deconfined phase (we assume there is a portal to the SM that keeps DM & SM at same temperature)
- Sets initial conditions for the phase transition - stable comoving density of dark quarks + antiquarks
- If dark quarks are heavier than the unitarity bound, this density will be too high to match the relic abundance



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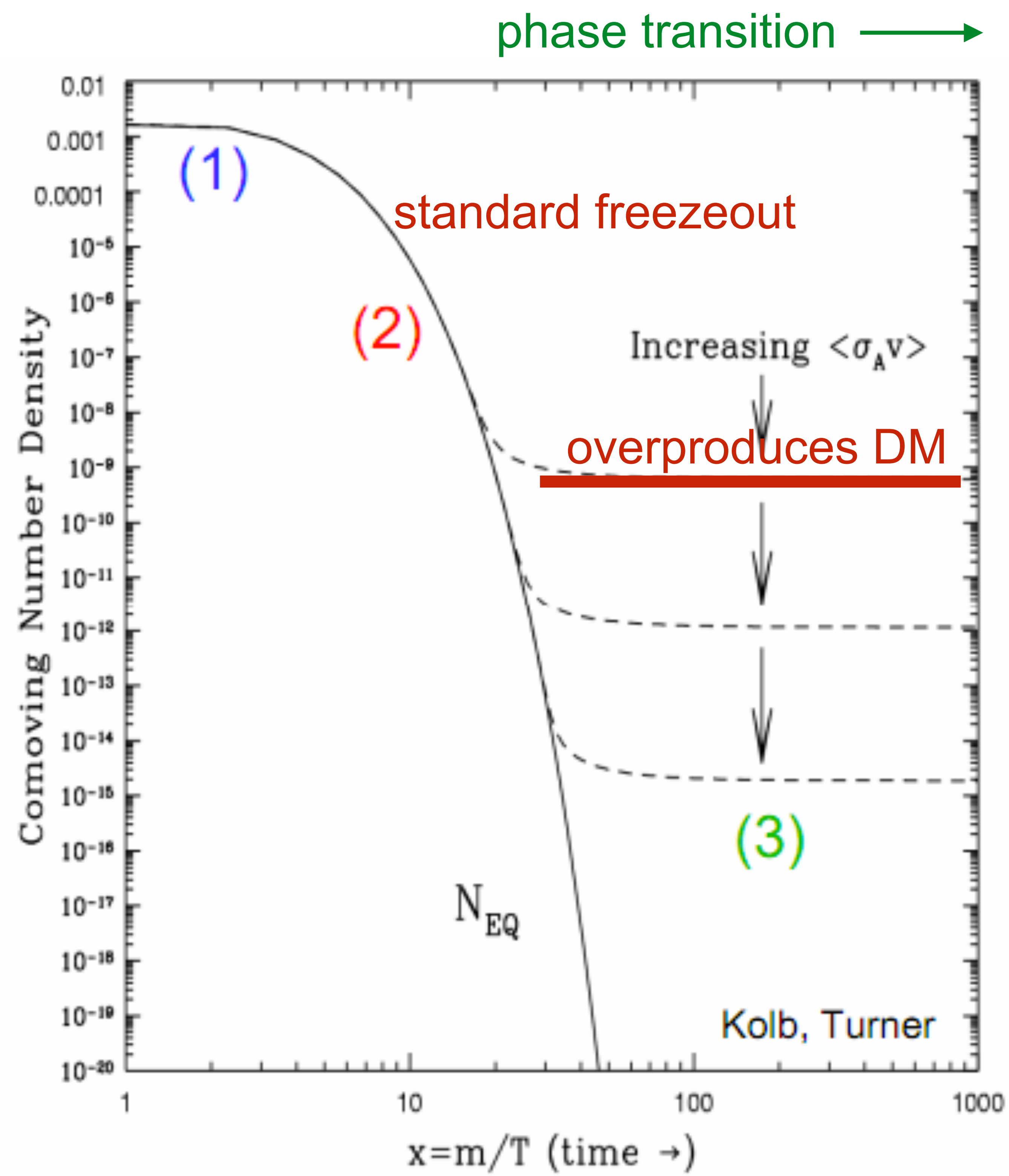


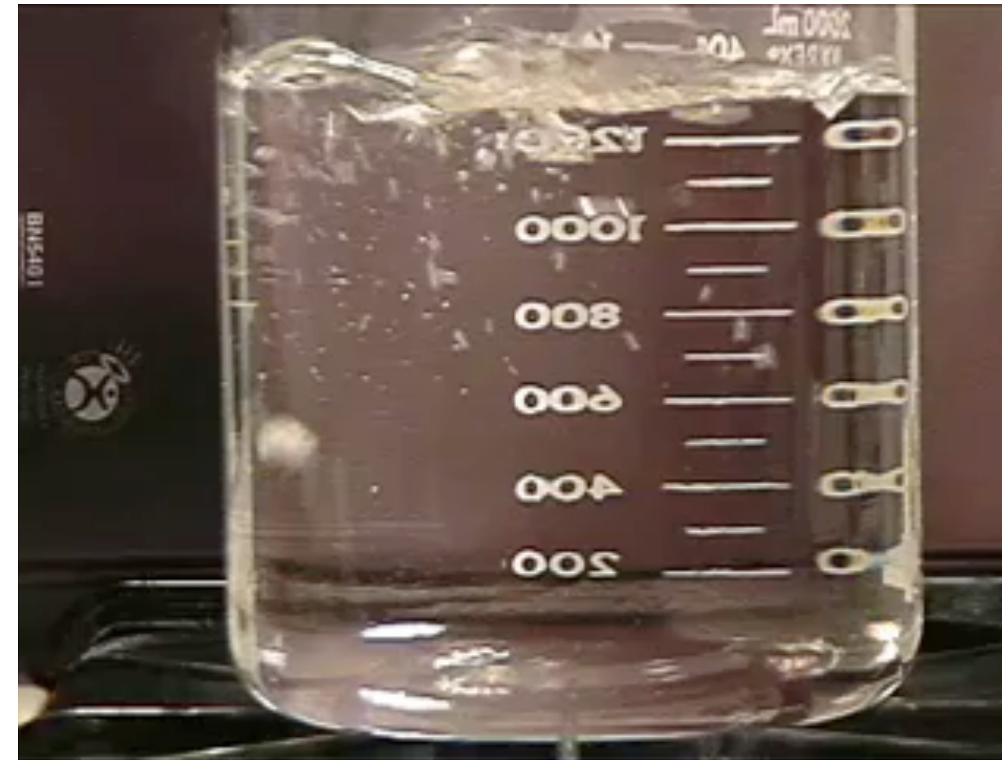
Stage 2: bubble growth

- After freezeout, once the temperature of the universe drops to Λ, bubbles of the confined phase begin to form and grow.

- These bubbles cannot form with free quarks inside, as free quarks cannot exist in the confined phase (requiring too much energy).

- Quarks (& antiquarks) must either quickly form hadrons or be shunted to the outside of the bubbles.

- Note: see also Hong, Jung & Xie, arXiv: 2008.04430, which uses similar “herding” of dark matter in a first-order phase transition to generate macroscopic “Fermi-balls” (or even primordial black holes, Kawana & Xie ‘21).



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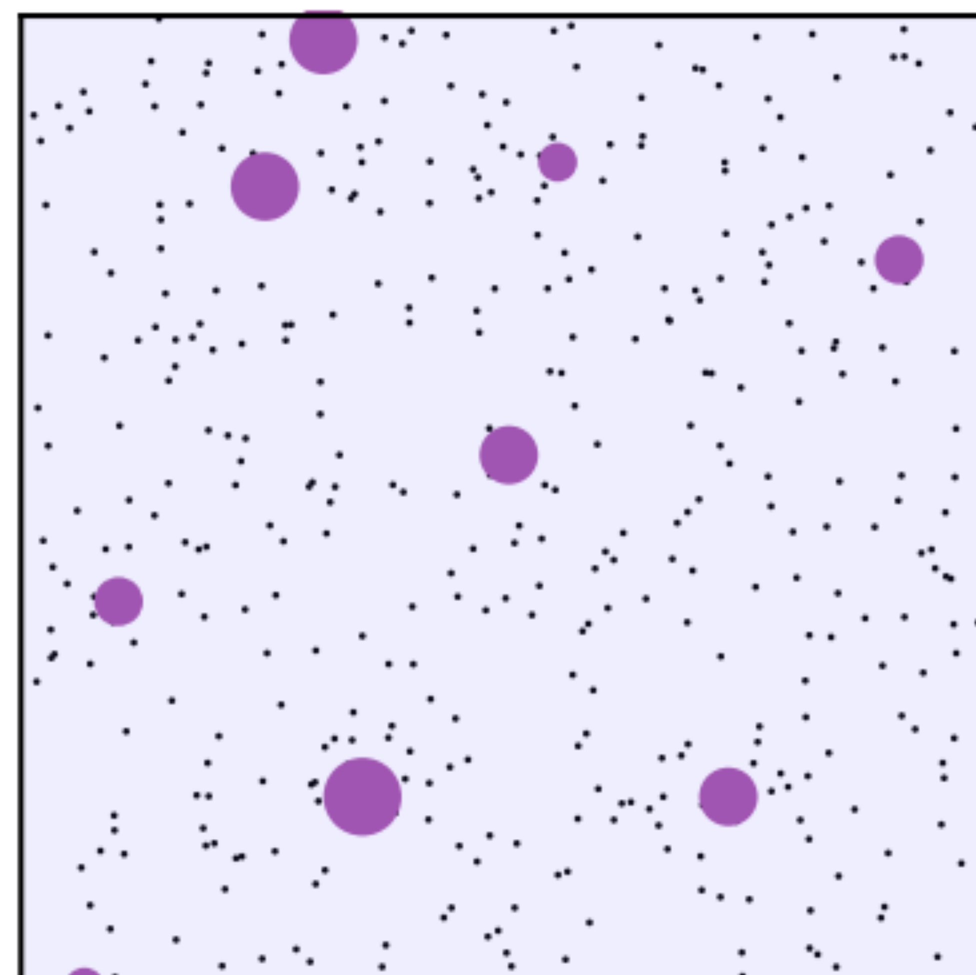

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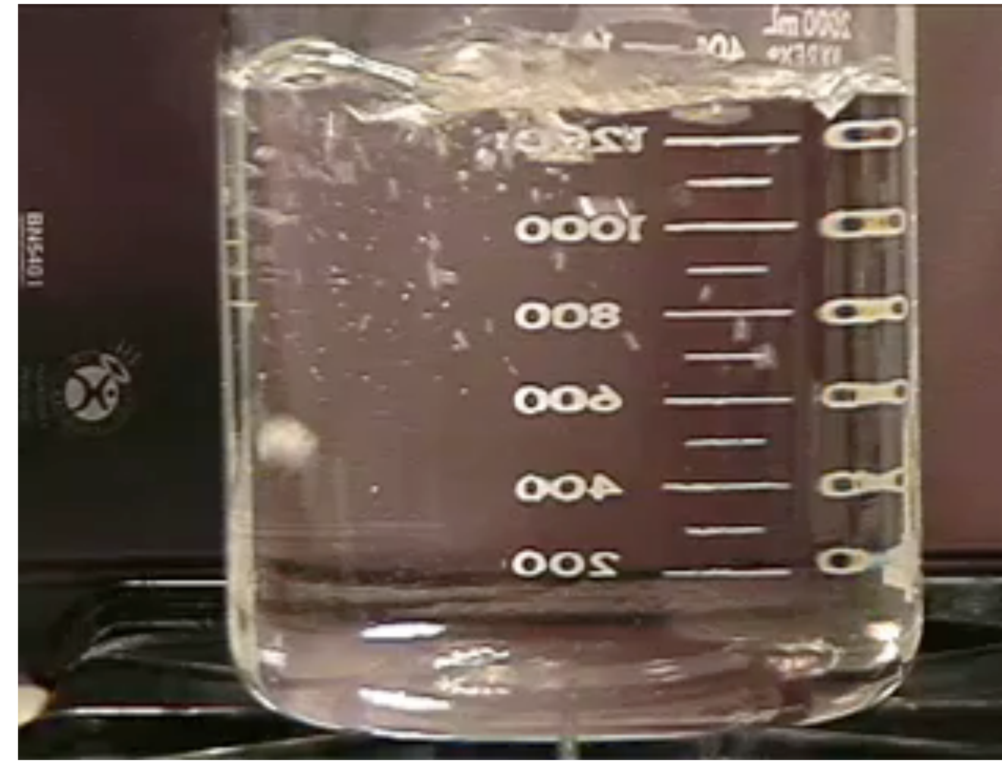

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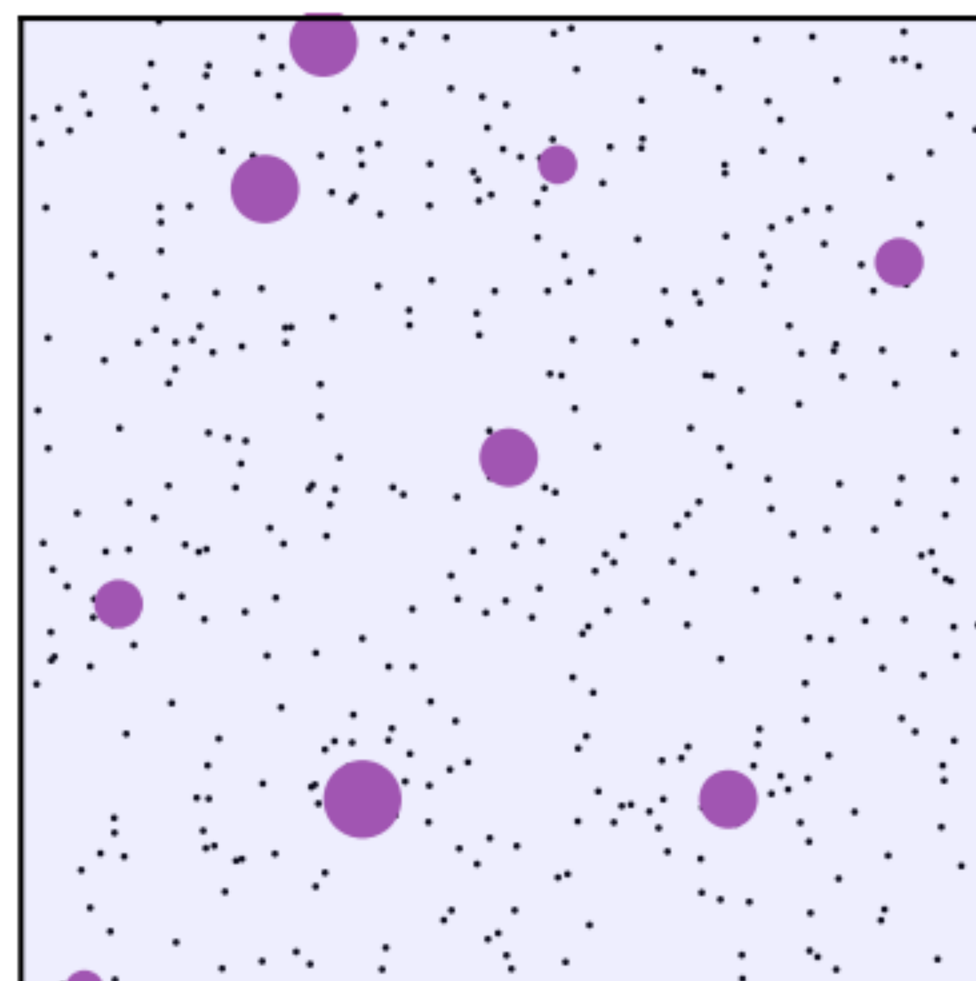

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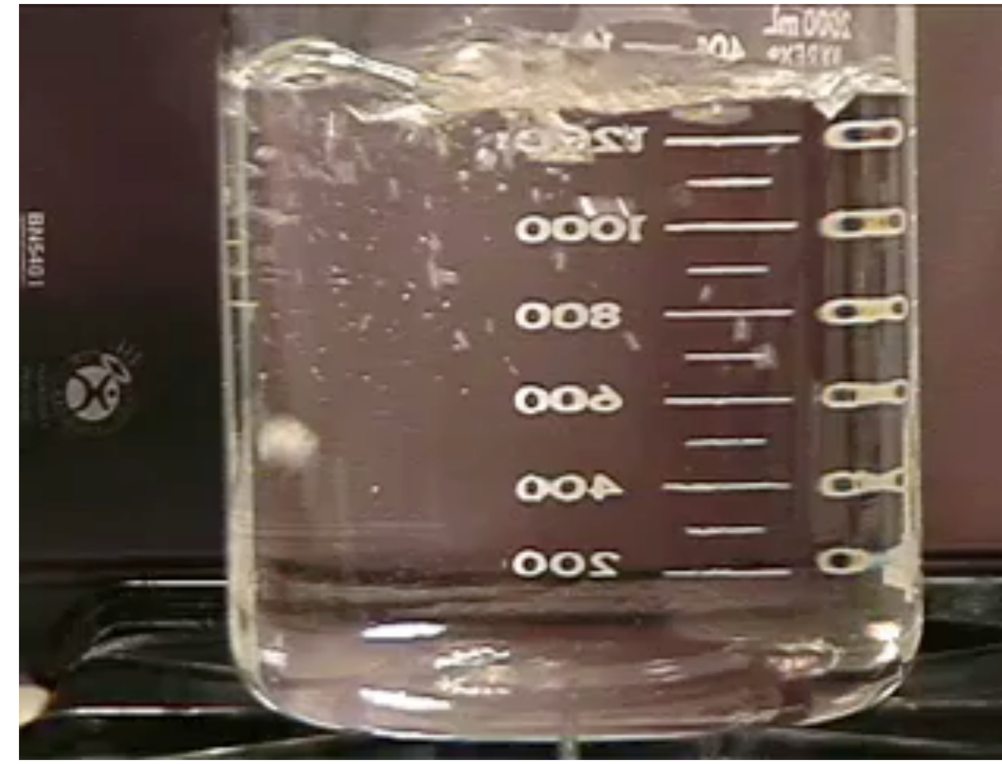

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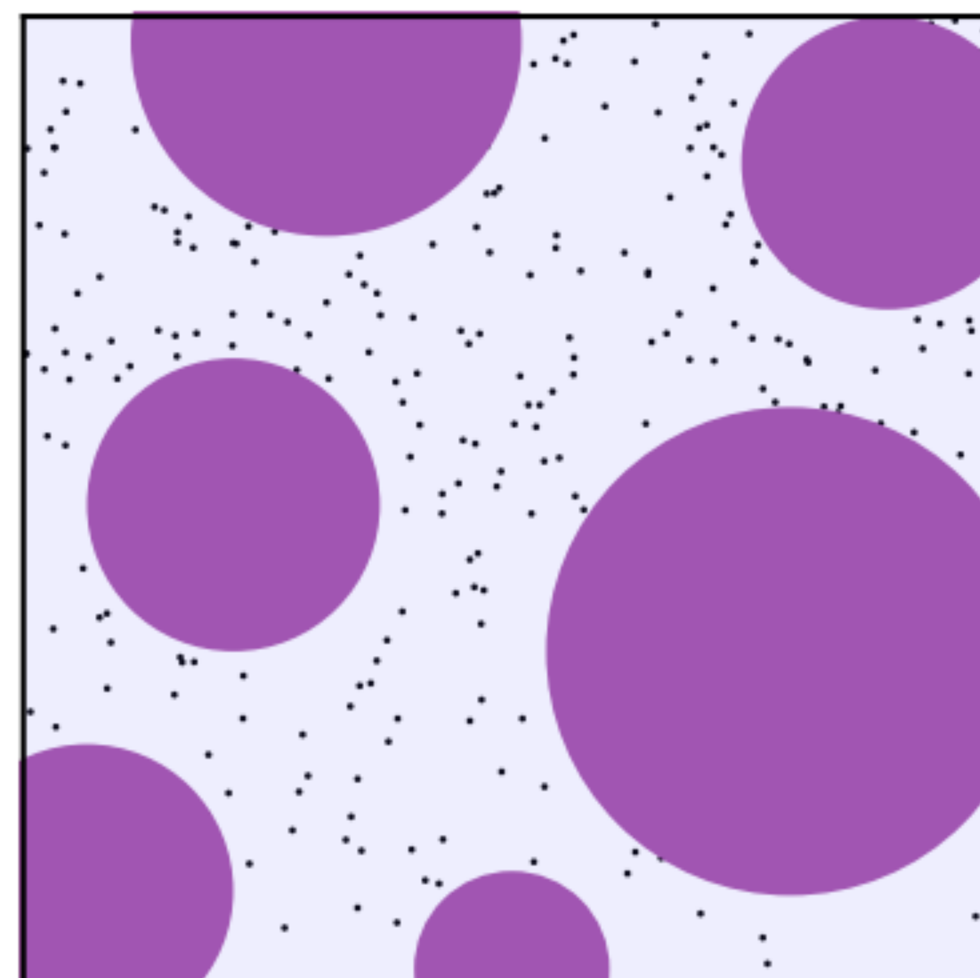

Quarks and bubbles

- What happens when a heavy (dark) quark encounters a bubble wall?
- Like a bowling ball hitting a trampoline (see https://www.youtube.com/watch?v=5GRyr0noXrw) - initially pushes into the bubble, putting a dent in its surface, but then recoils and bounces off

- We estimated this bounce time and found it to be very fast
- Alternatively, the energy from deforming the bubble wall could allow creation of more quarks/antiquarks from the vacuum, so the original quark can form a hadron and enter the bubble - but this process is very slow/rare if the dark quarks are sufficiently heavy

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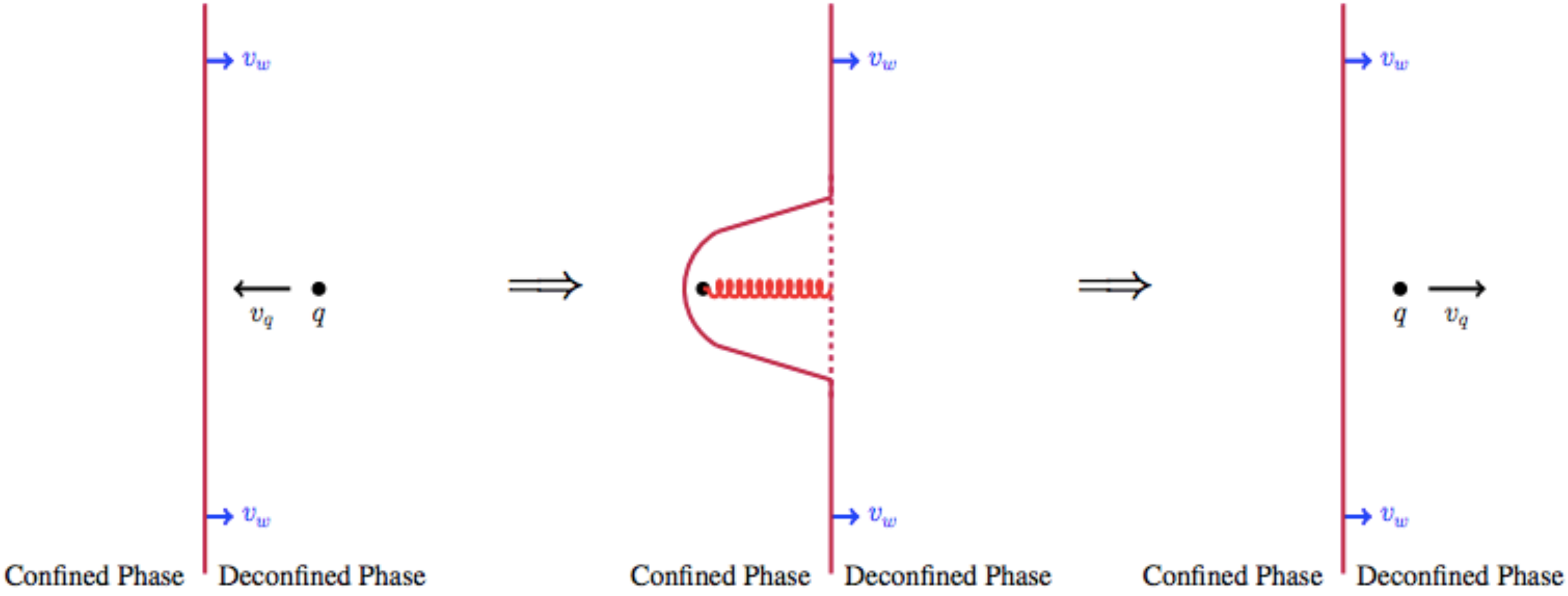

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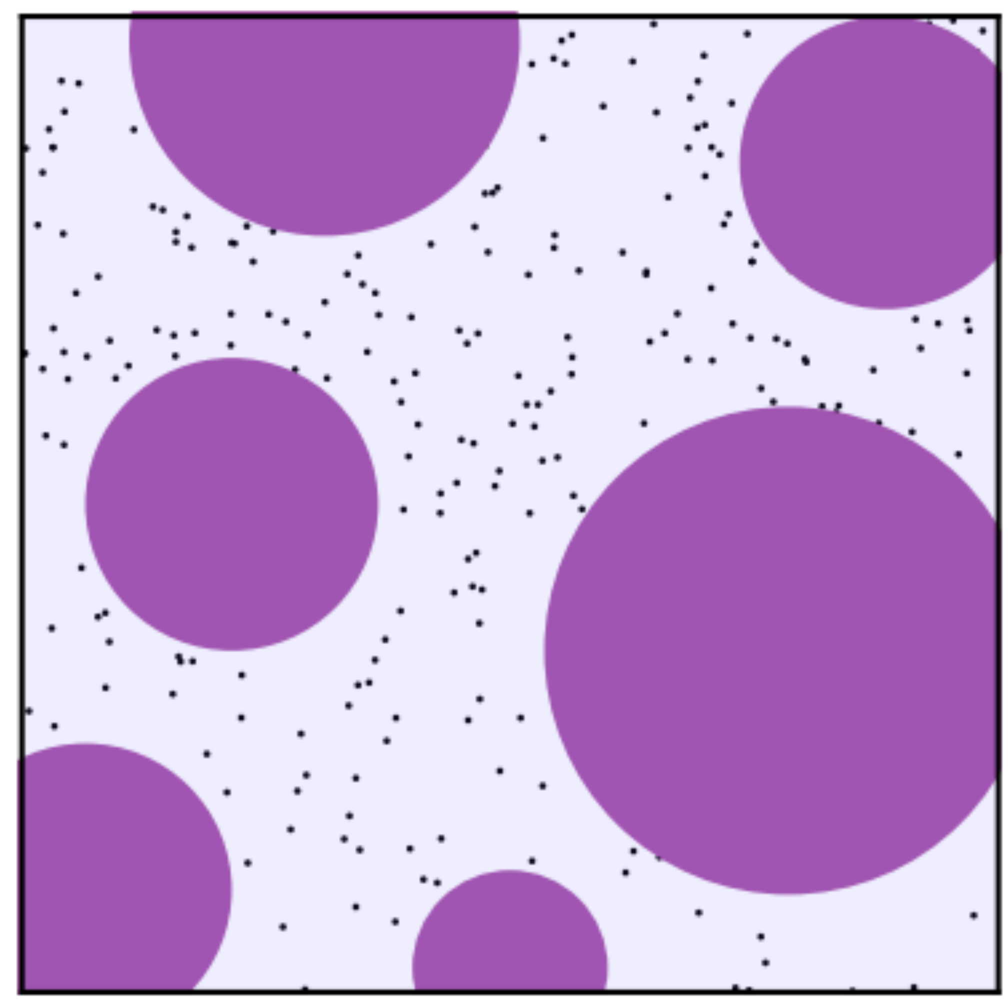
Stage 3: percolation

- As the bubbles continue to grow, eventually they will fill most of the universe - the remaining deconfined phase (gluon "sea" + heavy quarks) will occur only in isolated "pockets"
- All the heavy quarks will have been herded into these pockets by bouncing off the bubble walls
- As these pockets continue to shrink, they compress the heavy quarks to high density



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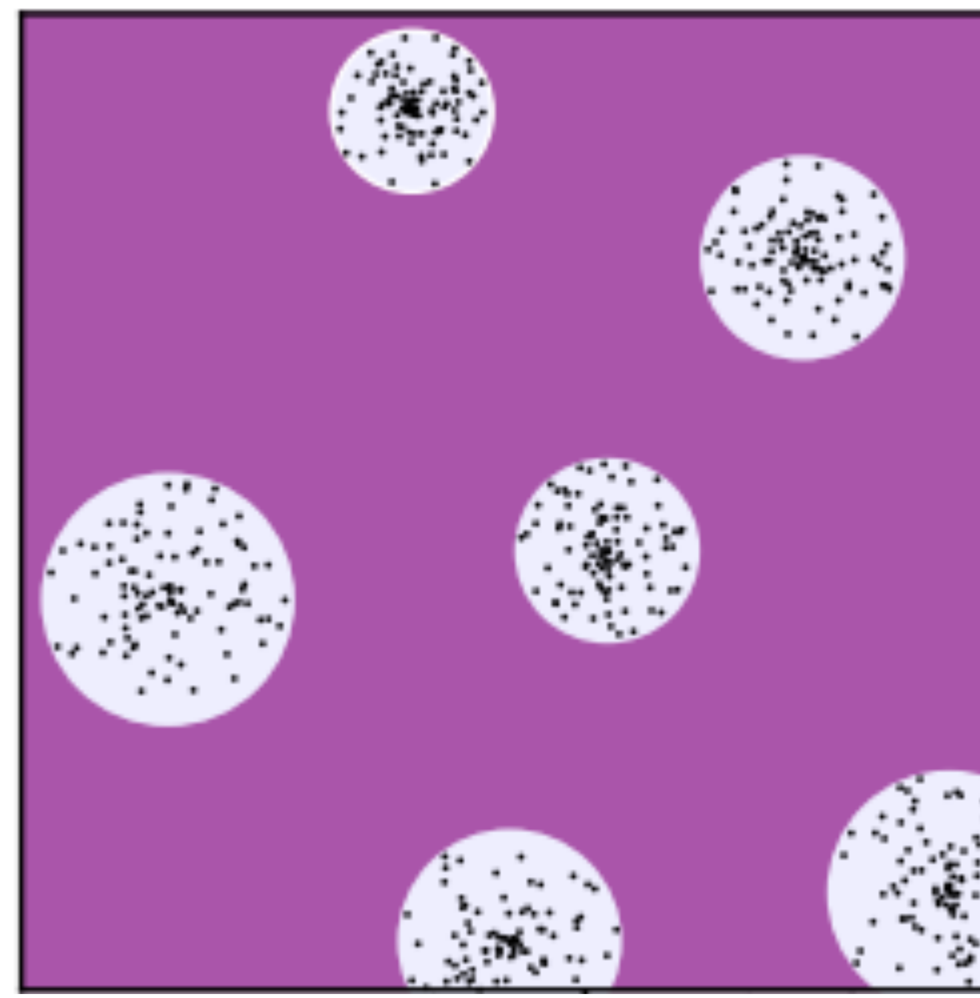
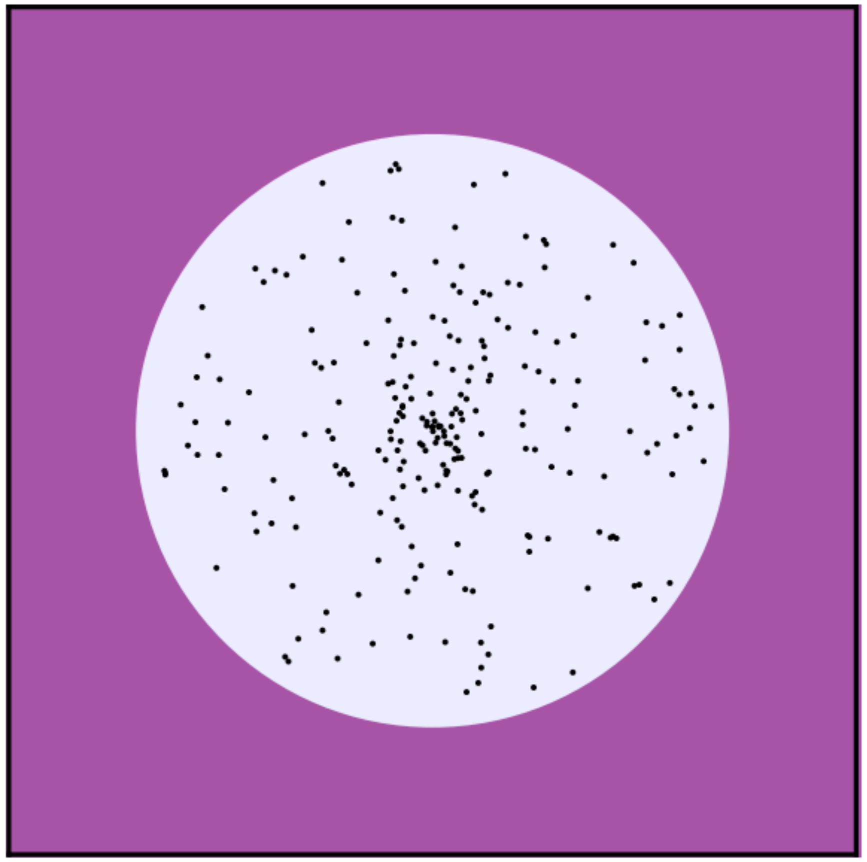


Stage 4: squeezeout

- Previously annihilation had frozen out
- But now the dark quarks are compressed into a much smaller volume, the density is high enough for it to re-start!
- At the same time, at these high densities the dark quarks can bind into dark hadrons
- Dark hadrons can leak through the shrinking pocket walls into the bulk of the universe that is now in the confined phase
- These hadrons form the dark matter at late times - DM is squeezed out of the pockets as they shrink down to zero size



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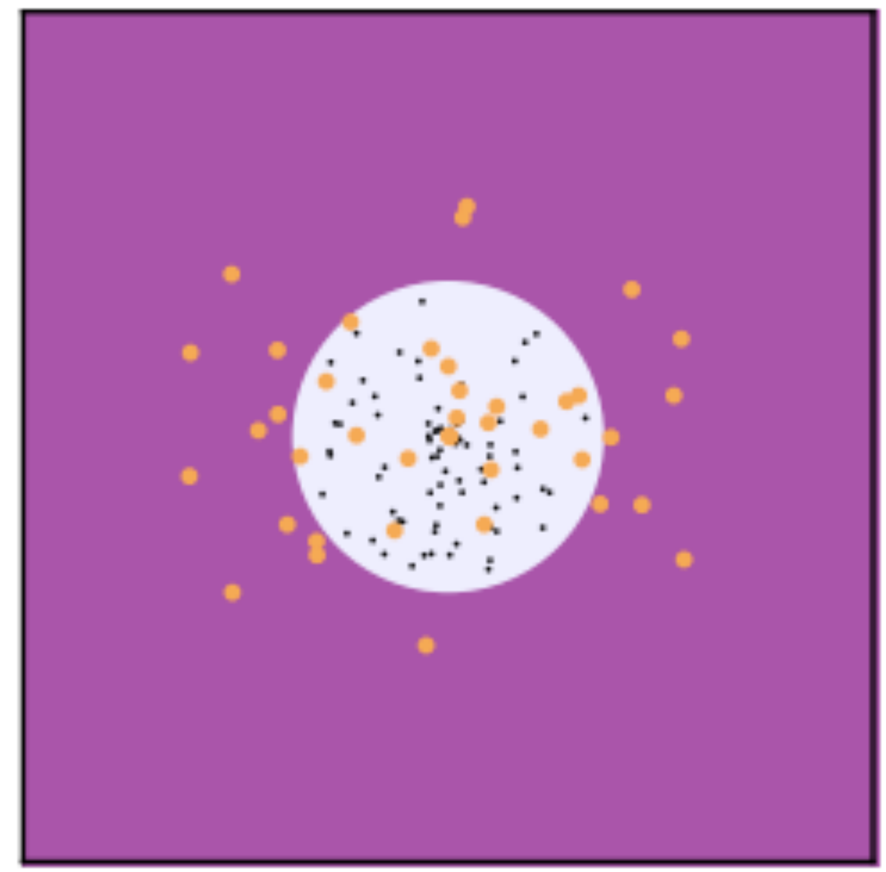

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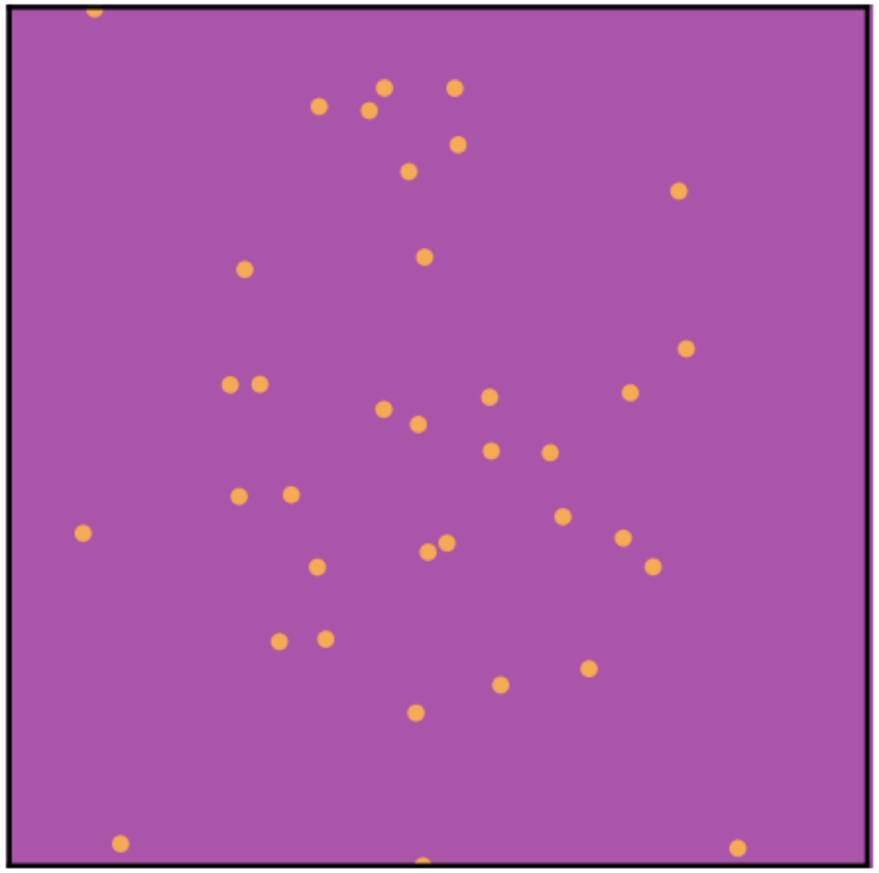

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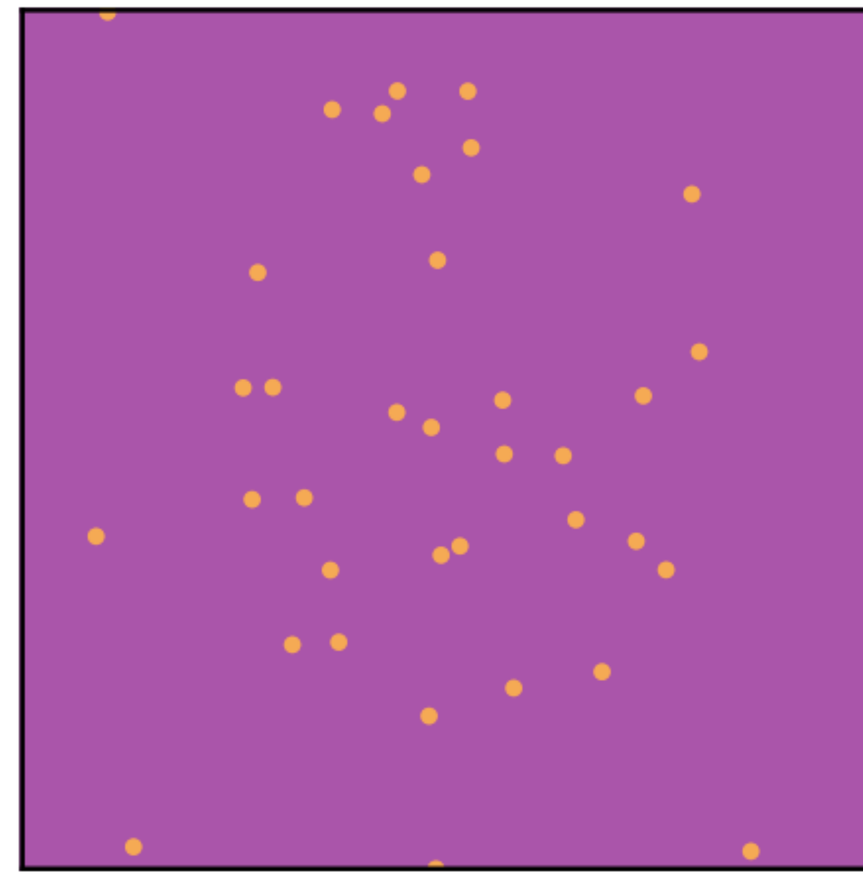
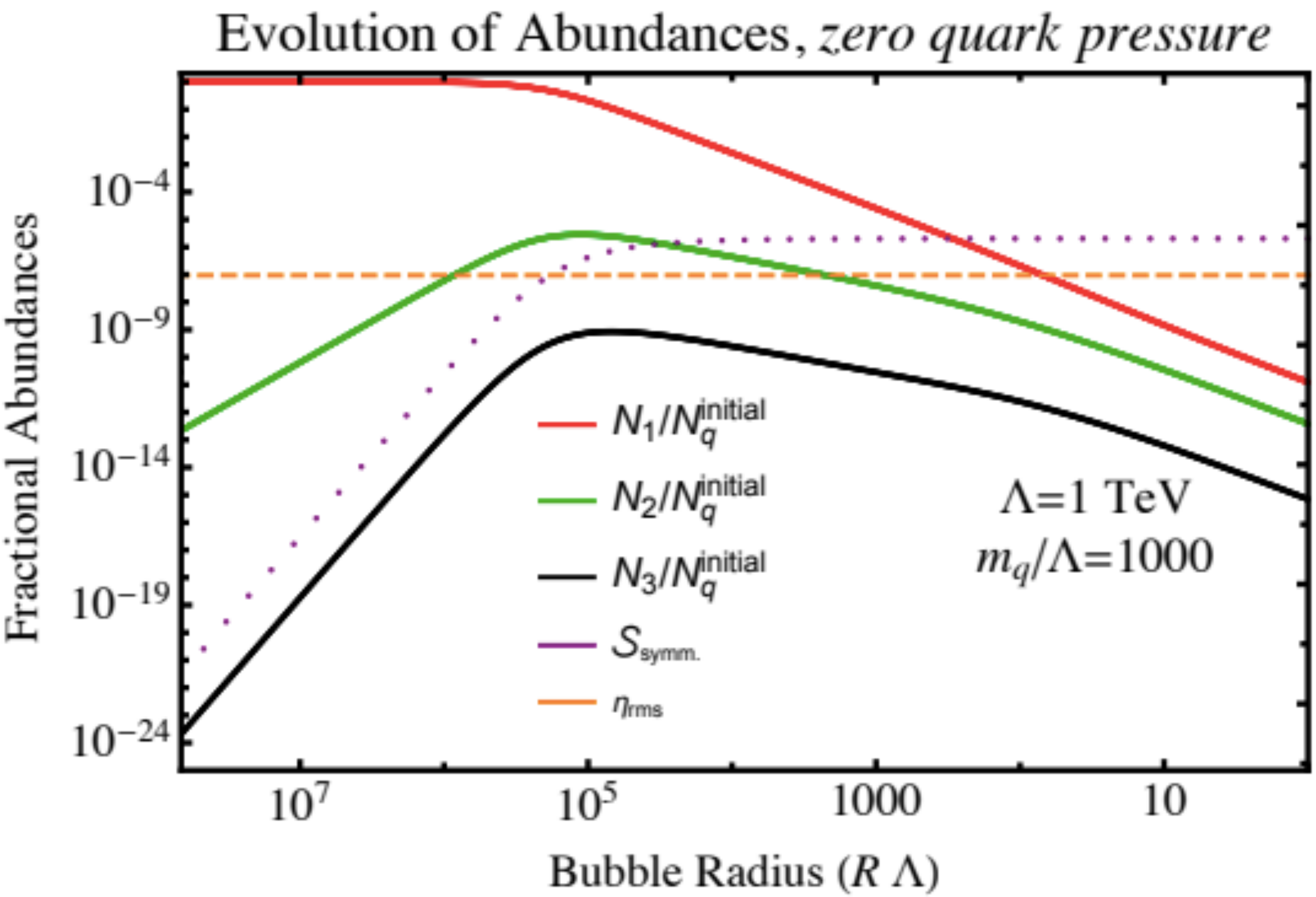


Hadronization vs annihilation?

- In this squeezeout phase, there is a competition between annihilation (destroys dark quarks) and hadronization (makes dark baryons).
- The baryon formation requires multiple steps (quarks → diquarks → baryons).
- Bound states do not necessarily survive to leave the pocket; they can be broken up before escaping.
- The shrinking of the pocket drives the quark density to continually higher values, increasing rates for all processes. Slower shrinkage = more time for annihilation to occur before hadronization+escape becomes efficient = less dark matter survives to be squeezed out.
- Other relevant parameters: initial quark density (set by freezeout), initial pocket size (set by phase transition dynamics, parametric estimate).
- We write down Boltzmann equations for all the processes and solve them numerically, using parametric estimates for the dark-strong-interaction cross sections.

An example simulation



- The <u>survival factor S</u> (purple dotted line) is the fraction of dark quarks that have escaped the pocket as baryons, compared to the initial post-freezeout dark quark abundance

The accidentally asymmetric limit

- So far we have assumed every pocket has equal amounts of dark quarks and dark antiquarks
- But even if overall the universe is symmetric, this is clearly not true in detail!
- A pocket with (initially) roughly N_{+q} quarks and N_{-q} antiquarks, summing to $N = N_{+q} + N_{-q}$, will be expected to have an asymmetry due to statistical fluctuations of order $|N_{+q} - N_{-q}| \sim \sqrt{N}$
- This "accidental asymmetry" can cut off the annihilations in the pockets - once all the quarks <u>or</u> antiquarks are eliminated, no further annihilations can occur, and all remaining quarks/antiquarks must hadronize and escape
- In turn this places a lower bound on the average survival factor S, $S \gtrsim 1/\sqrt{N}$

Quark pressure

- The simulation I showed previously made an extra approximation - it ignored the effects of quark pressure

- As the pockets shrink, the quark-gluon plasma within will exert a pressure on the pocket walls

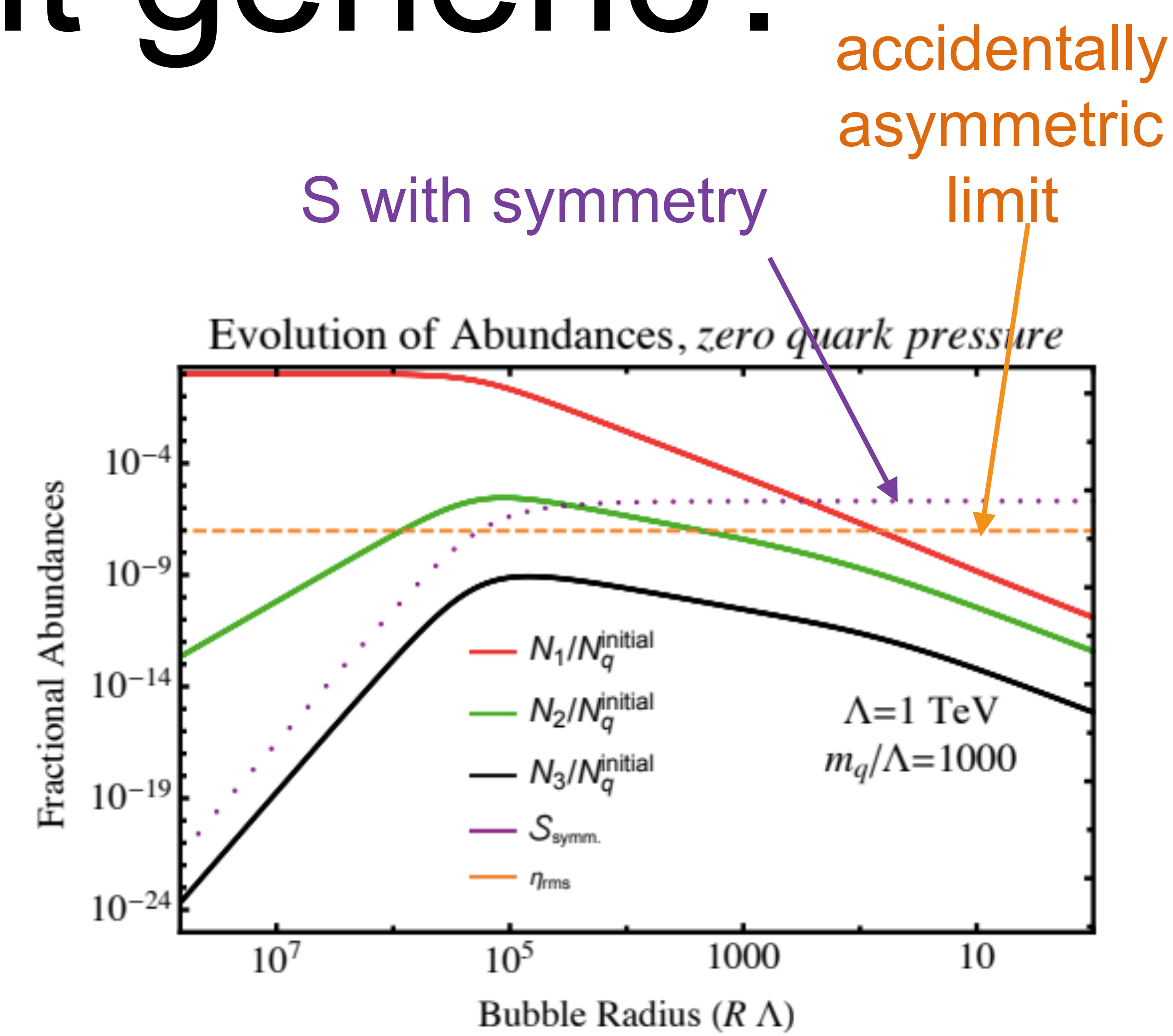
- This is a strong-interaction effect and we do not have an accurate model for it; however, parametric estimates indicate it is likely to be quite large

- The effect will be to slow down the pocket shrinkage velocity (possibly by a lot), which <u>decreases</u> the survival fraction

Is the accidentally-asymmetric limit generic?

- We scanned a wide range of input parameters and found that even when we <u>ignore</u> quark pressure, S generically either saturates the accidentally-asymmetric lower bound or comes close to it.

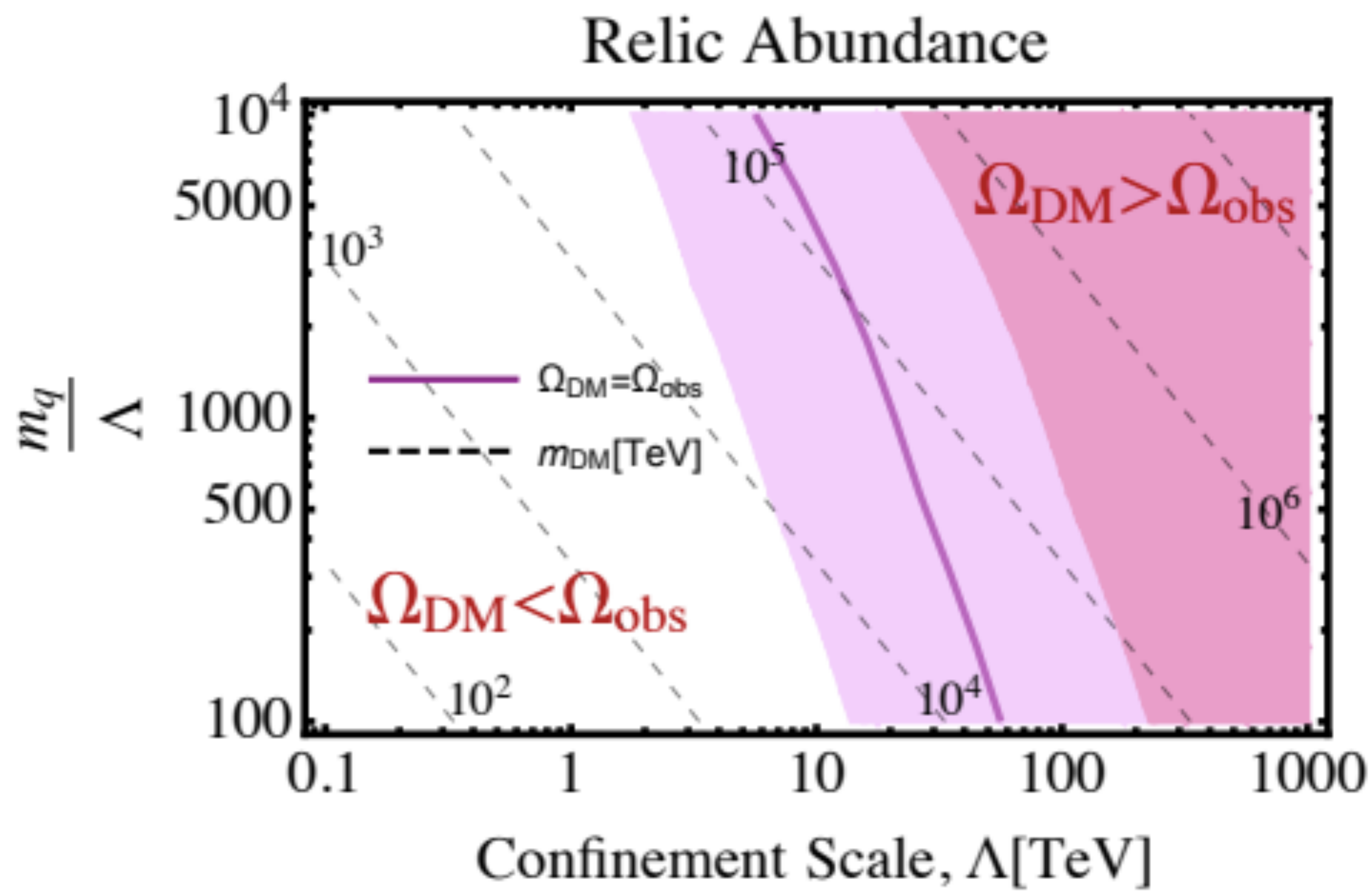
- Including quark pressure will generically decrease S - under simple estimates, causes saturation of the bound (easily) everywhere.



- Consequently, we argue that the accidentally-asymmetric limit is generically a good approximation.

The relic density

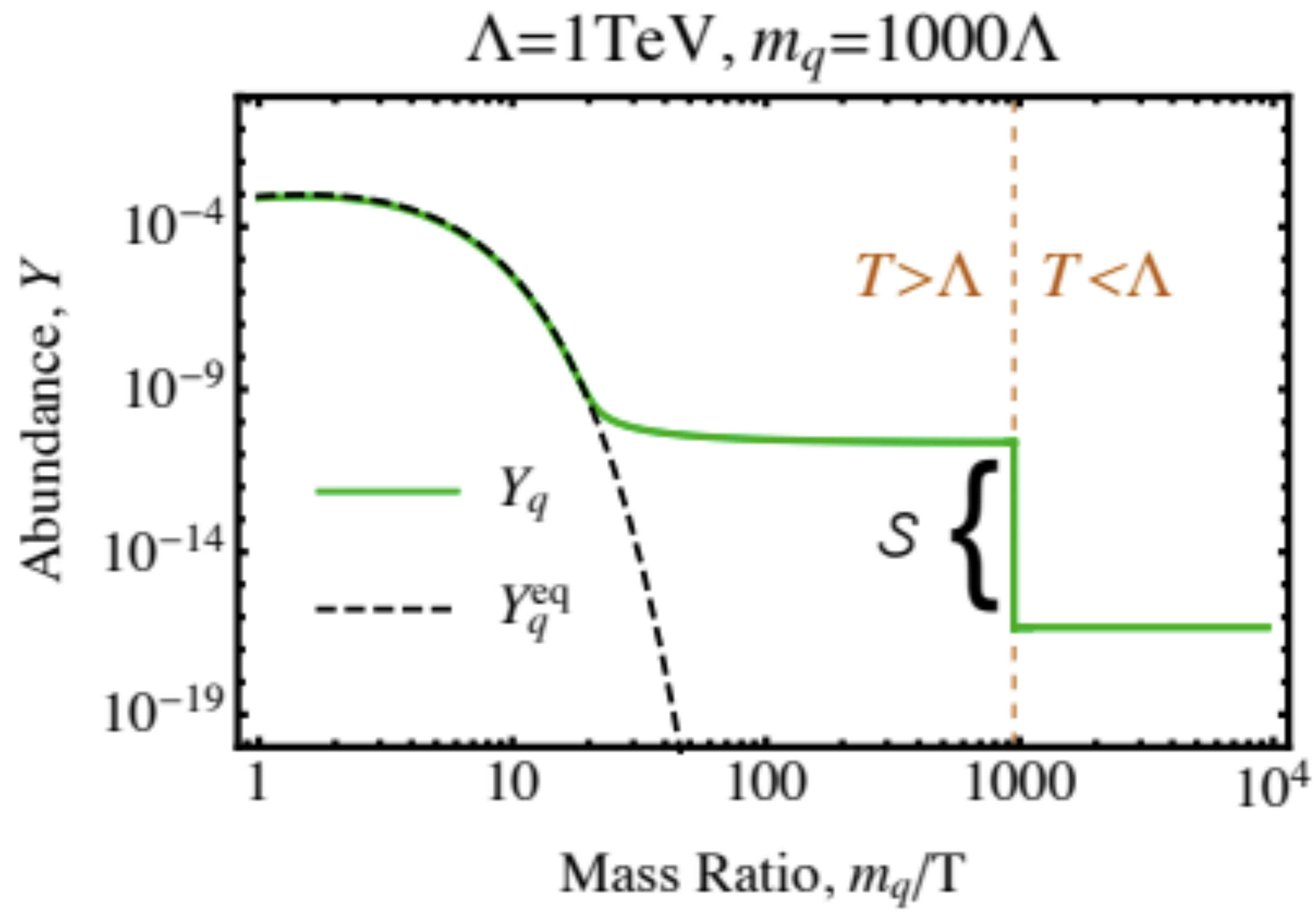
- In the accidentally asymmetric limit, the survival factor S is determined entirely by the initial number of quarks per pocket
- Fixed by:
  - post-freezeout number density (depends on quark mass + high-energy couplings, set by Λ)
  - radius of pockets at percolation (estimated as $R_1 \approx \left(\frac{M_{\mathrm{Pl}}}{10^4 \Lambda}\right)^{2/3} \frac{1}{\Lambda}$, from Witten 1984)



- We can calculate the relic density as a function of m_q and Λ, allowing for an order-of-magnitude variation in the pocket radius around our estimate
- We find preferred DM masses around 1-1000 PeV (also if we assume zero quark pressure)

Summary of cosmic history for this scenario

- Freezeout: the dark quark abundance is depleted through annihilation as normal.
- Squeezeout: the phase transition triggers a further sharp drop in the abundance, potentially by several orders of magnitude, as the dark quarks are compressed in contracting pockets and many of them annihilate before forming hadrons.
- Dark hadrons escape the pockets as the pocket size shrinks to zero, leading to the observed relic abundance for PeV+ DM.



Observational signatures?

- What I have shown you so far depends almost exclusively on the dark-sector physics - most signatures would depend on the details of the portal to the Standard Model, which is required to allow the glueballs to decay away

- Any first-order dark sector phase transition could generate a stochastic gravitational wave background that could be seen in future experiments [e.g. Geller et al, PRL 2018]

- This scenario predicts heavy unstable states (mesons + glueballs) lighter than the DM - glueballs might possibly be within reach for future colliders

- Indirect searches are limited by the unitarity upper bound on the annihilation cross-section

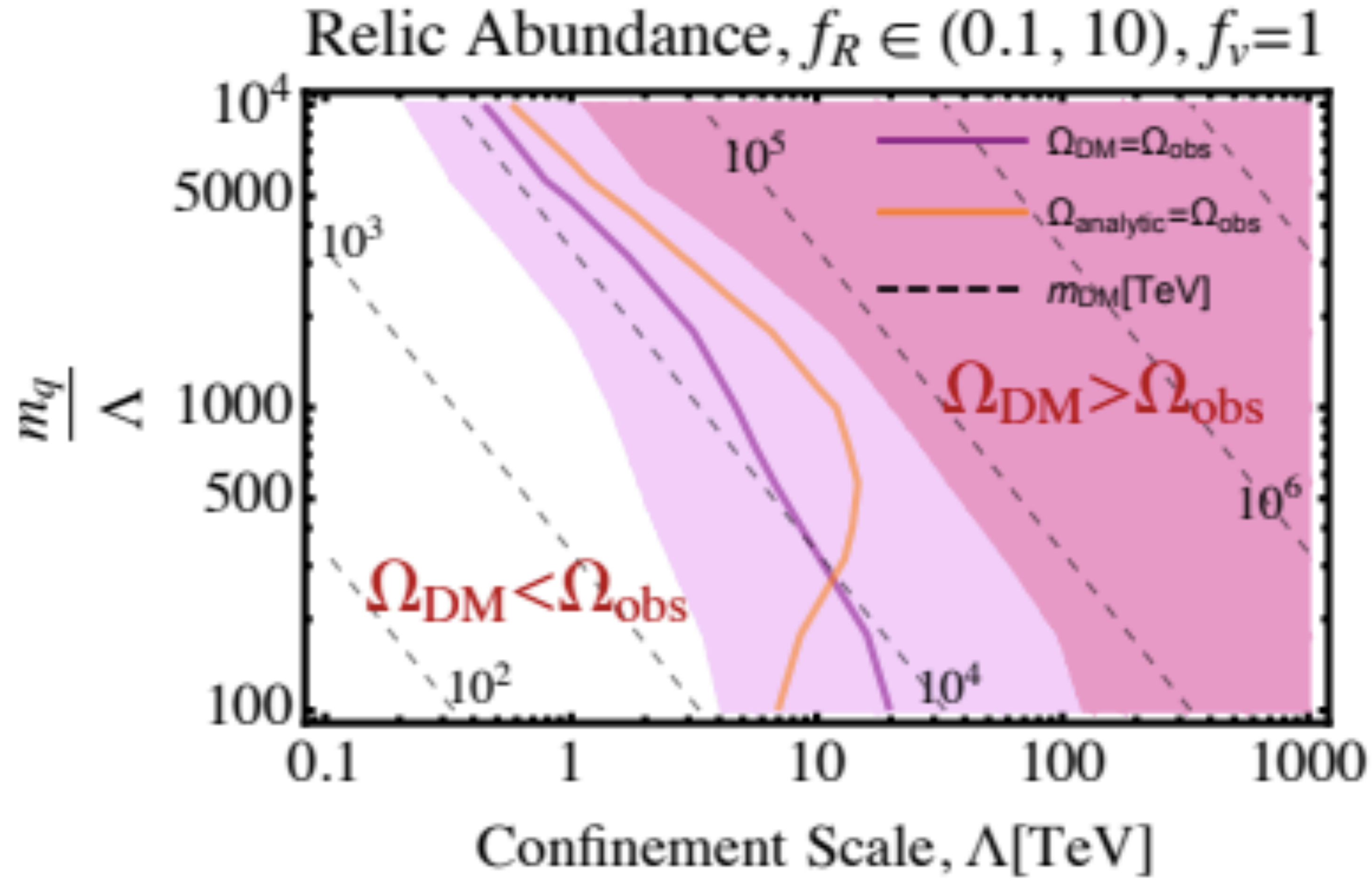
- Because the mass scale is so high, rather large interactions with the SM may be viable - interesting for direct detection? [e.g. Cappiello et al, PRD 2021]

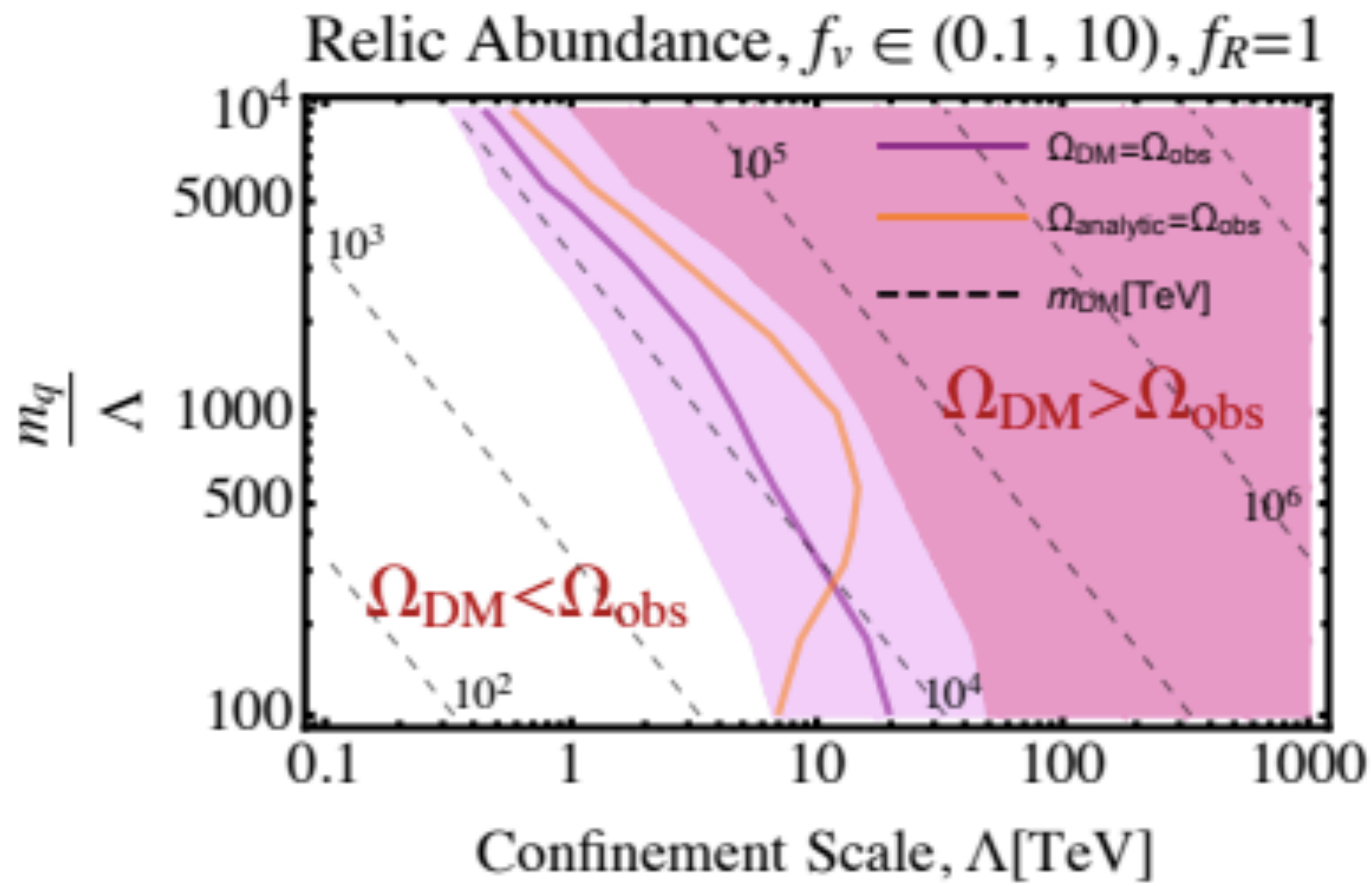
Summary

- A natural possibility for heavy thermal dark matter, beyond the standard unitarity bound at O(100) TeV, is a strongly-interacting dark sector.
- If the quark mass is much heavier than the confinement scale, the confinement phase transition is expected to be first-order.
- The interplay between thermal freezeout and a dark phase transition naturally leads to the correct abundance for PeV-EeV DM due to a second period of rapid annihilation during the phase transition.
- Heavy dark quarks and antiquarks are forced into shrinking pockets of the high-temperature phase, and annihilate away until only a residual *accidentally asymmetric* component (i.e. pure quarks or pure antiquarks) remains.
- This residual component forms dark hadrons which are squeezed out of the pockets as they shrink and vanish, in a process we call *squeezeout*.

BONUS SLIDES

Relic density assuming zero quark pressure





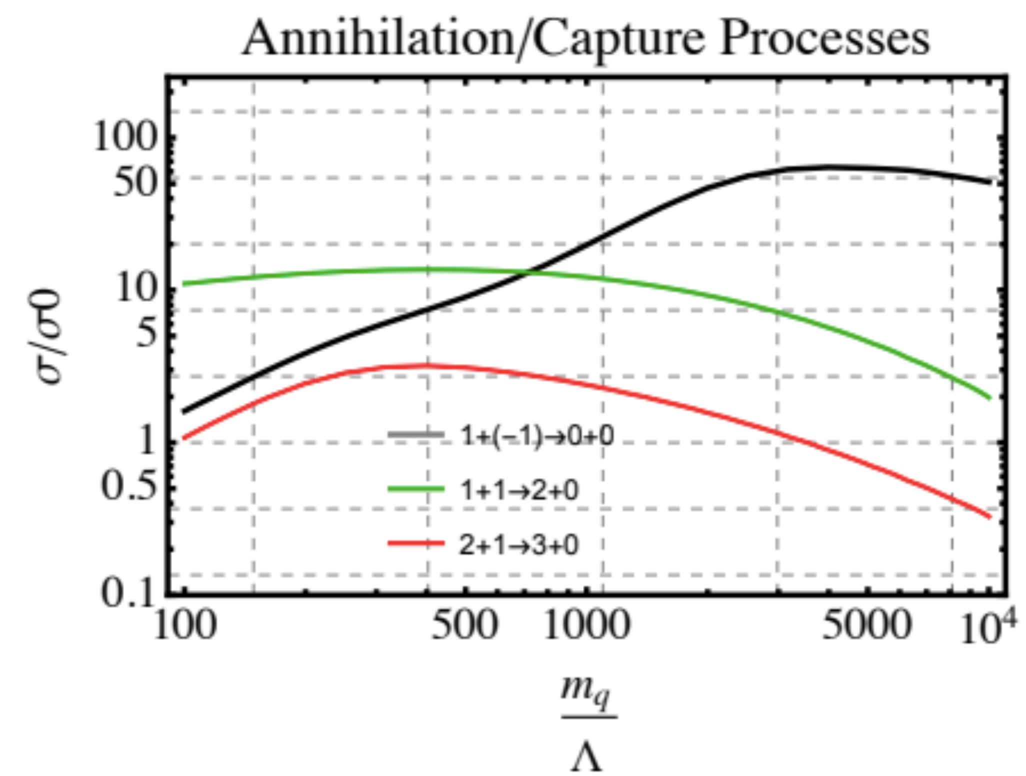
- These plots show the effect of varying initial pocket radius and wall velocity
- Preferred parameter space is similar to accidental-asymmetry case, 1-100 PeV DM

Rates for bound-state formation

- We need the rates to form diquark bound states, and to go from diquarks to baryons
- Simplifying approximations:
  - for mesons, which are expected to decay on a timescale fast relative to annihilations/hadronization, assume they are in equilibrium (so abundance is very small)
  - ignore heavy tetraquark/pentaquark states for the same reason
  - include only $2 \rightarrow 2$ processes as $3 \rightarrow 2$ and $2 \rightarrow 3$ are suppressed
  - treat gluons as a radiation species in equilibrium in deconfined phase
  - couplings can be evaluated at $m_q >> \Lambda$
- Notation: label each species by its quark number (gluons = 0, quarks = +1, anti-quarks = -1, diquarks = +2, etc)

Relevant processes

- Annihilation: particles and antiparticles annihilate directly (and completely) into gluons, e.g. 1 + -1 → 0 + 0



- Capture (and dissociation): quark number is conserved but a dark gluon is emitted to conserve momentum, e.g. 1 + 1 → 2 + 0

$$\langle \sigma_{\mathrm{ann./cap.}} v \rangle = \zeta \, \frac{\pi \alpha^2}{m_q^2} \equiv \zeta \, \sigma_0$$

- Rearrangement: quark number is conserved and no dark gluon is emitted, e.g. 2 + 2 → 3 + 1

$$\langle \sigma_{\mathrm{RA}} v \rangle = \frac{1}{C_N \alpha} \frac{\pi}{m_q^2} = \frac{\sigma_0}{C_N \alpha^3} \, ,$$

enhancement from finite size of colliding bound states

Boltzmann equations

$$
\begin{aligned}
L[i] &= C[i], \quad i = 1,2,3. \\
C[1] &= -\Big\langle(-3,1)\to(-1,-1)\Big\rangle - \Big\langle(-3,1)\to(-2,0)\Big\rangle + 2\Big\langle(3,-1)\to(1,1)\Big\rangle \\
&+ \Big\langle(3,-2)\to(1,0)\Big\rangle - \Big\langle(1,-1)\to(0,0)\Big\rangle + \Big\langle(2,2)\to(3,1)\Big\rangle - 2\Big\langle(1,1)\to(2,0)\Big\rangle \\
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&- \Big\langle(2,1)\to(3,0)\Big\rangle - \Big\langle(-2,1)\to(-1,0)\Big\rangle + \Big\langle(3,-3)\to(1,-1)\Big\rangle \quad , \\
C[2] &= \Big\langle(1,1)\to(2,0)\Big\rangle - \Big\langle(-3,2)\to(-1,0)\Big\rangle + \Big\langle(3,-1)\to(2,0)\Big\rangle \\
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L[i] &= -\frac{v_w}{V}N_i', \quad i = 1,2, \\
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\Big\langle(a,b)\to(\alpha,\beta)\Big\rangle &= \langle\sigma v\rangle_{ab\to\alpha\beta}\left(n_a n_b - n_\alpha n_\beta \frac{n_a^{eq} n_b^{eq}}{n_\alpha^{eq} n_\beta^{eq}}\right) \\
&= \frac{\langle\sigma v\rangle_{ab\to\alpha\beta}}{V^2}\left(N_a N_b - N_\alpha N_\beta f_{ab,\alpha\beta}\right) ,
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describes change in particle number with respect to pocket radius

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\end{aligned}
$$

describes escape of baryons from pocket