SELF-INTERACTING (DARK) RADIATION AND THE HUBBLE TENSION

ArXiv:2012.11830

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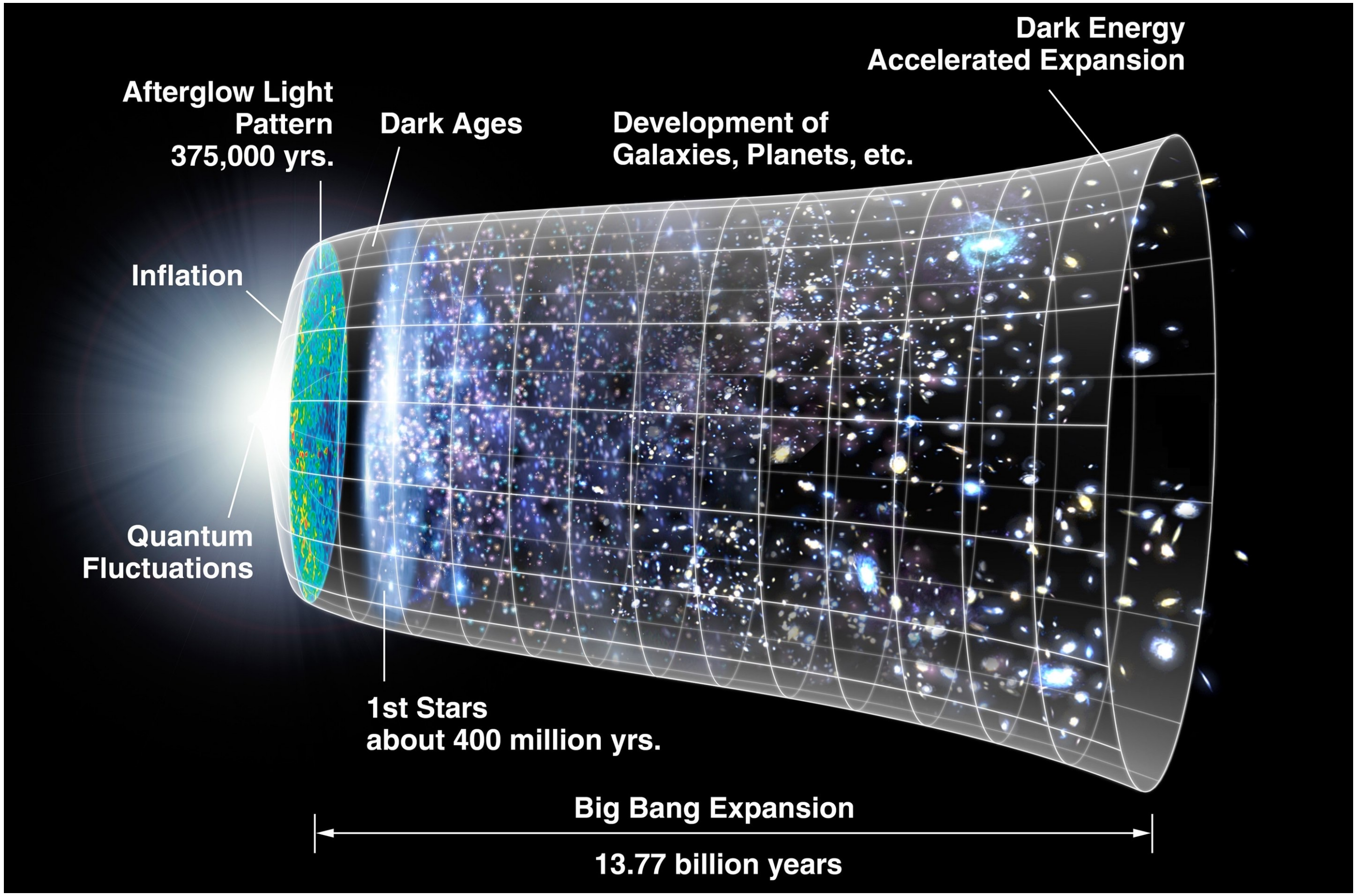
2021/06/08 The 2021 CERN-CKC Theory Workshop

THE HUBBLE PARAMETER TENSION

We live in an expanding Universe

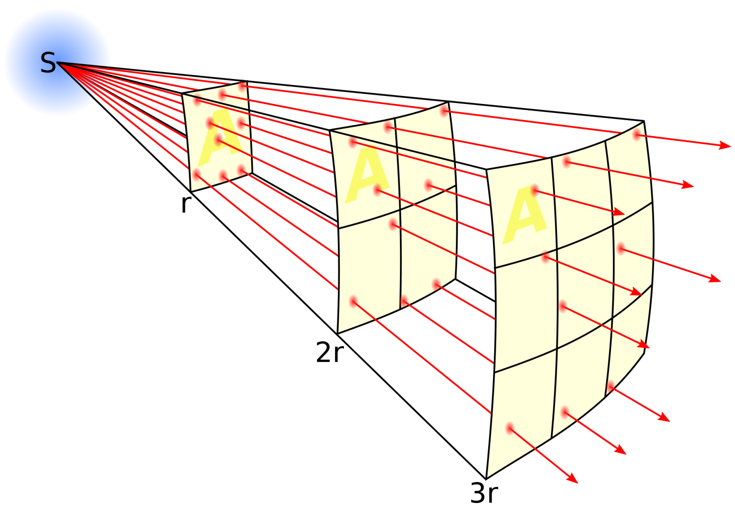


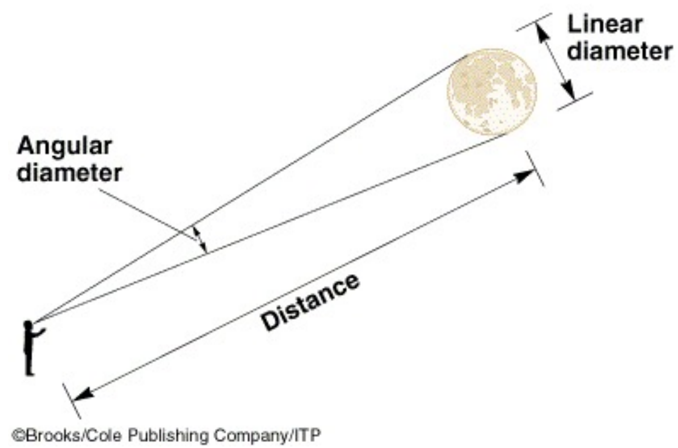
- What is the speed of the expansion?

Hubble Parameter and Hubble's Law

- $ds^2 = dt^2 - a(t)^2 \gamma_{ij} dx^i dx^j$
- Hubble Parameter : $H \equiv \frac{\dot{a}}{a}$
- Hubble's Law : $z = H_0 D \ \ (z \ll 1)$
  - z is the redshift
  - H_0 is the Hubble parameter today
  - D is the proper distance

Luminosity and Angular Distances





- Luminosity Distance

$$J_{obs} = \frac{L_s}{4\pi D_L^2}$$

$$D_L = D/a$$

Standard Candle

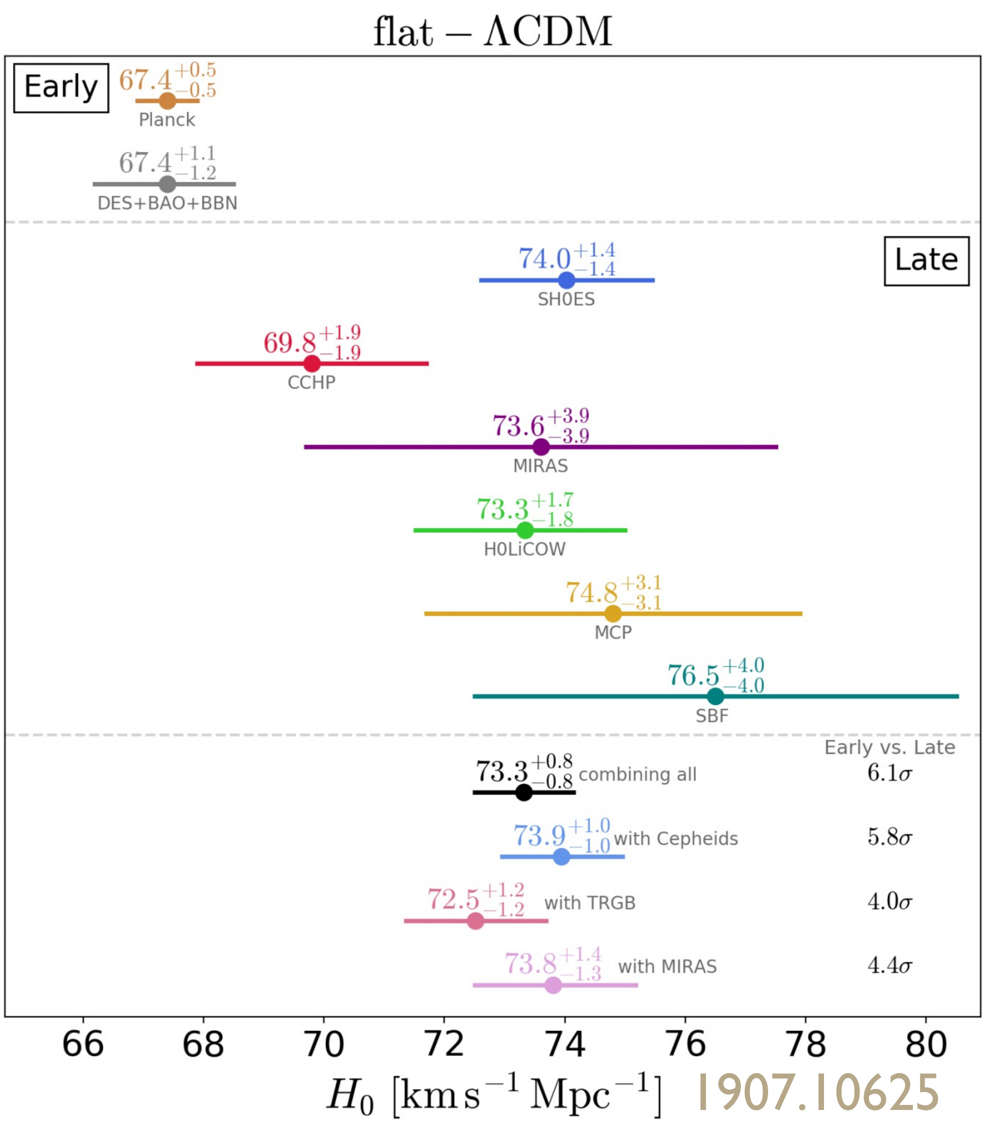
- Angular Distance

$$\sin\theta = \frac{d_s}{D_A}$$

$$D_A = aD$$

Standard Ruler

Hubble Parameter H_0 Measurements



1907.10625

- Large difference between early vs late H_0 measurement
- Is it just due to systematics?

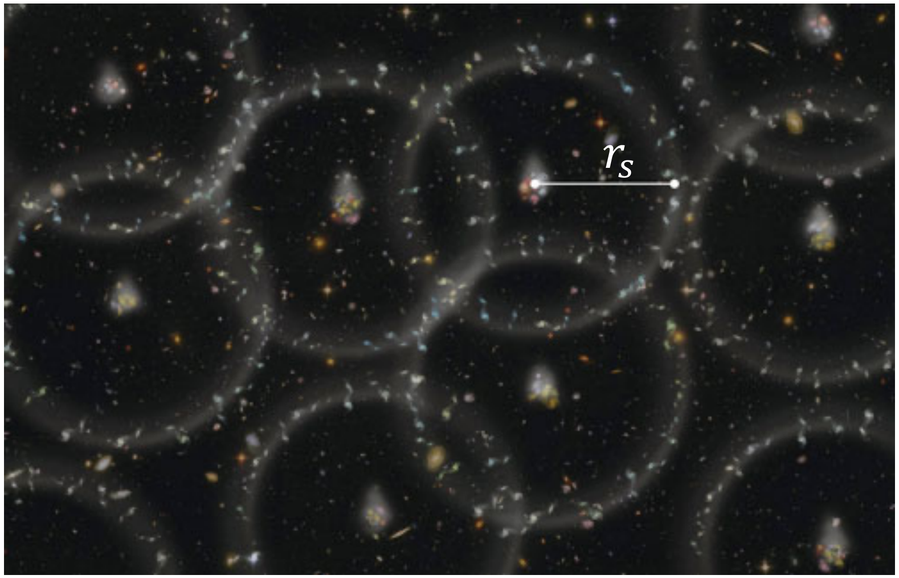
What if our knowledge is wrong?

- If the standard candle or the standard ruler is wrong, we predict different H_0

$$H_0 \propto \frac{1}{L_s}, \qquad H_0 \propto \frac{1}{d_s}$$

- In this talk, we focus on the standard ruler for early Universe : BAO scale

Baryon Acoustic Oscillation



http://www.astro.ucla.edu/~wright/BAO-cosmology.html

$$r_s = \int_0^{a_r} da \frac{c_s}{a^2 H}$$

$$H^2 = \frac{8\pi G}{3}\rho$$

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu = \rho_\gamma \left(1 + N_{\rm eff}\frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4\right)$$

- $H_0 \propto \frac{1}{r_s}$ from CMB
- Additional radiation predicts smaller r_s, hence large H_0

Additional Radiation

- Decreases r_s
  $\longrightarrow$ Helps to solve the Hubble tension

- Suppresses small scales
  $\longrightarrow$ Conflicts with the observations

- Solutions : Interacting Radiation Kreisch et al, 1902.00534
  $\longrightarrow$ Interacting radiation has smaller sound speed, so it can compensate the suppression

- The Kreisch paper finds larger H_0 with self-interacting neutrinos

SELF-INTERACTING NEUTRINOS

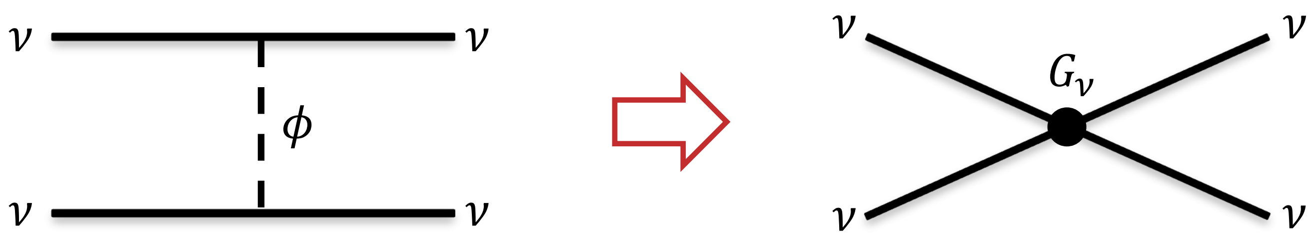
The Majoron Model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}g_{ij}\bar{\nu}_i\nu_j\phi$$

- We introduce a scalar coupled to neutrinos, called the Majoron

- For simplicity, we assume the diagonal and universal coupling : $g_{ij} = g_\phi\delta_{ij}$

Neutrino Self-Interactions



$$\mathcal{L} = \frac{1}{8} G_\nu \bar{\nu}_i \nu_i \bar{\nu}_j \nu_j$$

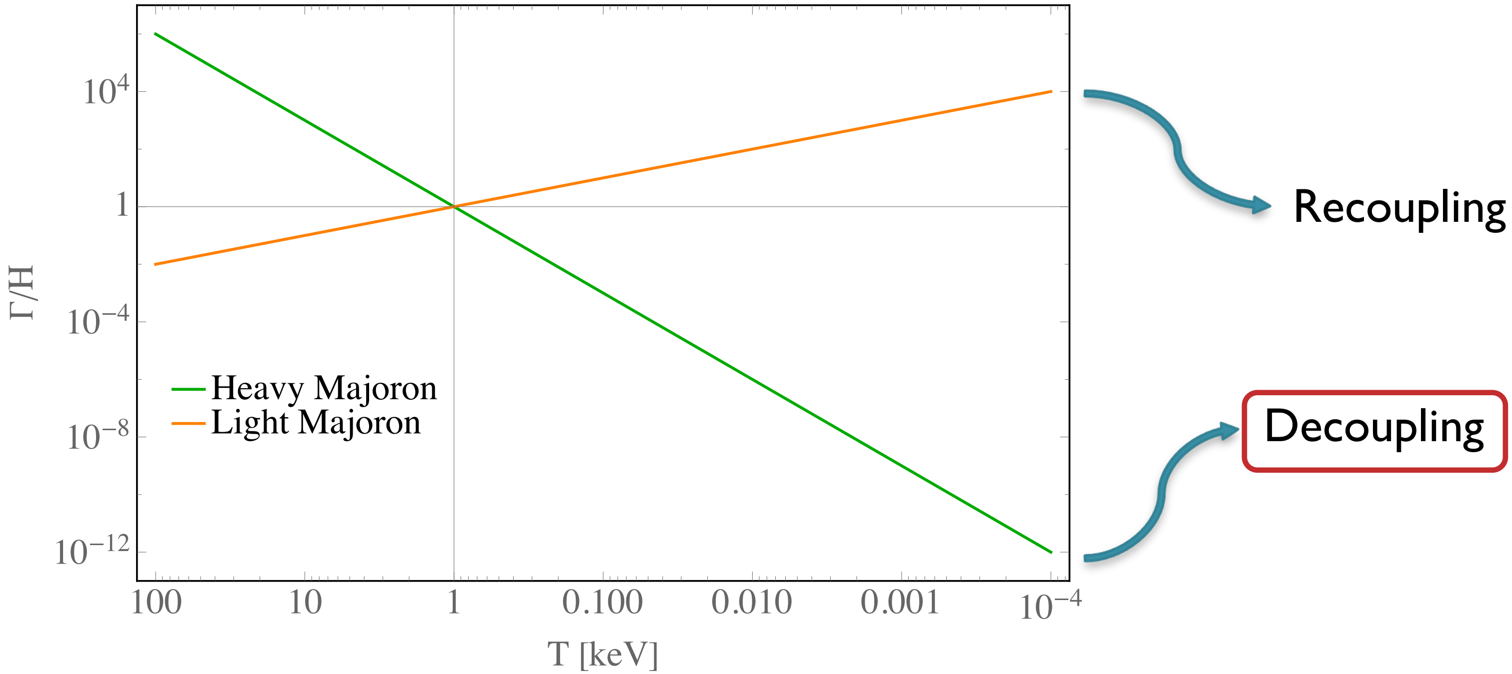
- Neutrinos interact with each other by exchanging a Majoron

- For light Majoron : $\Gamma \sim g_\phi^4 T$

- For heavy Majoron : $\Gamma \sim G_\nu^2 T^5, \; G_\nu \equiv \dfrac{g_\phi^2}{m_\phi^2}$

Decoupling and Recoupling

- Interaction is active if $\Gamma > H \sim \frac{T^2}{m_{pl}}$
- Γ/H increases or decreases depending on Majoron mass

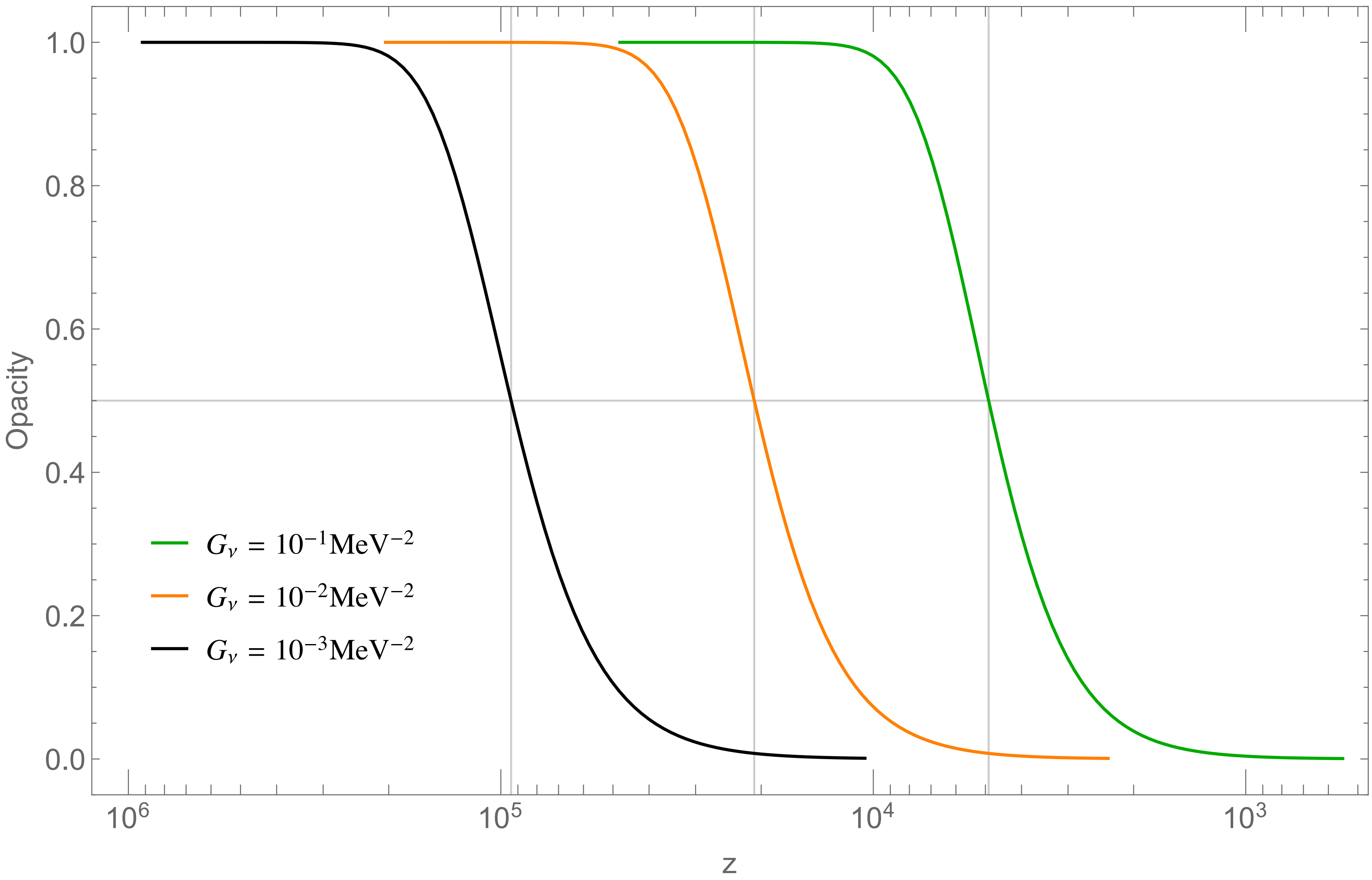


The Opacity Function

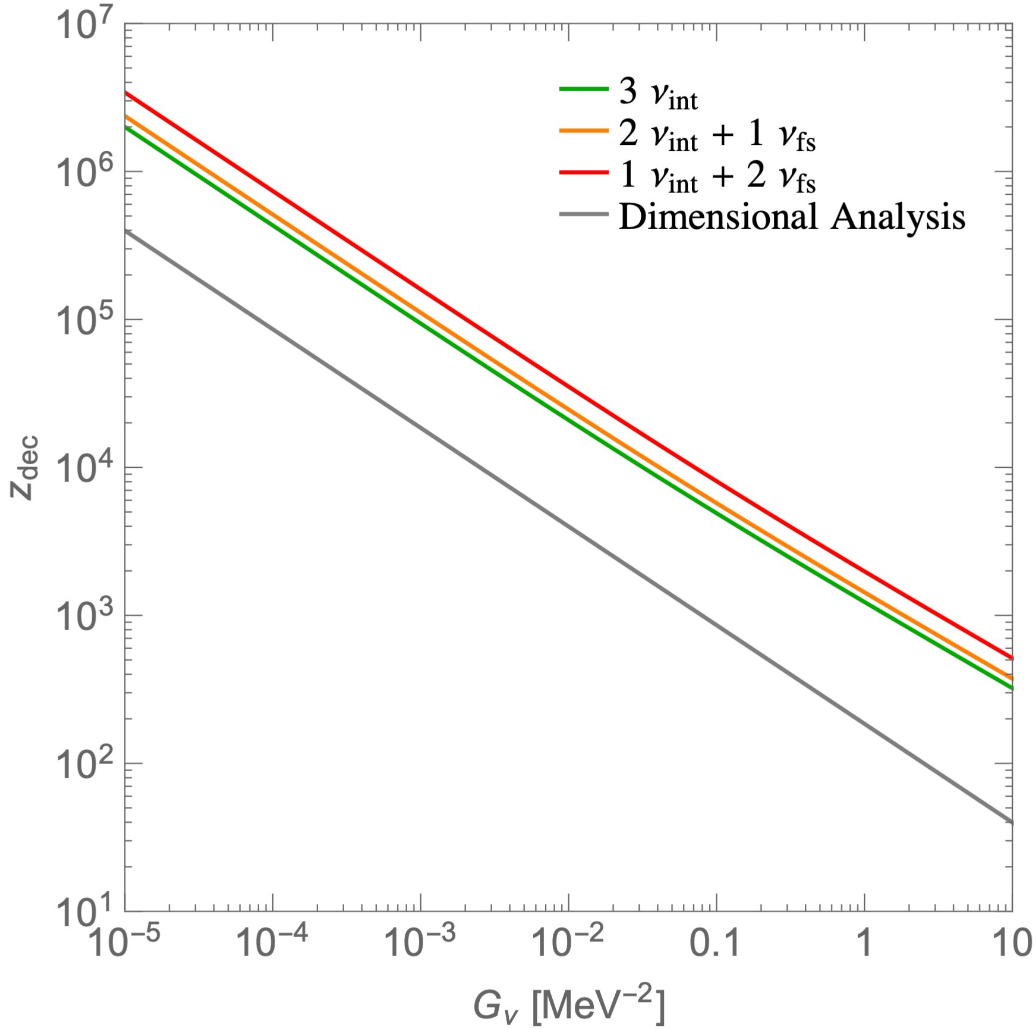
$$O(T, E_1) \ = \ 1 - \exp\left[-\int_{t(T)}^{t_0} \Gamma_\nu \mathrm{d}t\right]$$

- Opacity indicates the fraction of particles that have interacted at least once from the time t to today.
- We calculate thermal averaged opacity $\langle O(T) \rangle$
- Define T_d where $\langle O(T) \rangle = \frac{1}{2}$

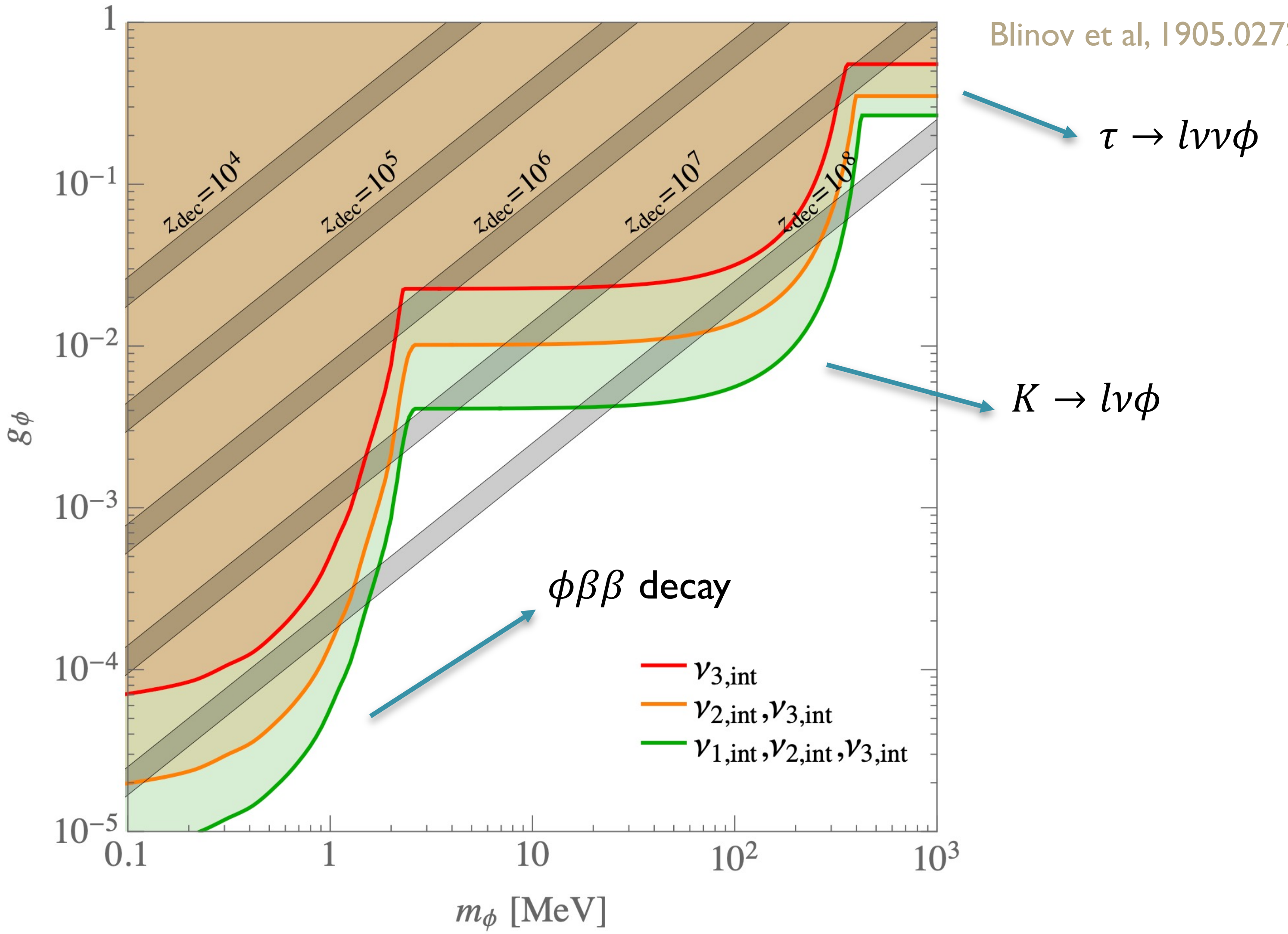
The Opacity Function



Decoupling Redshift



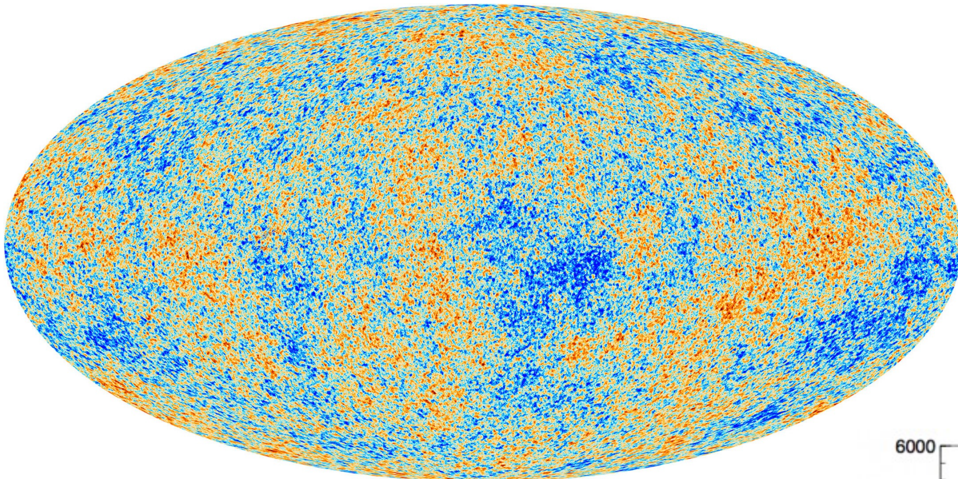
Experimental Constraints

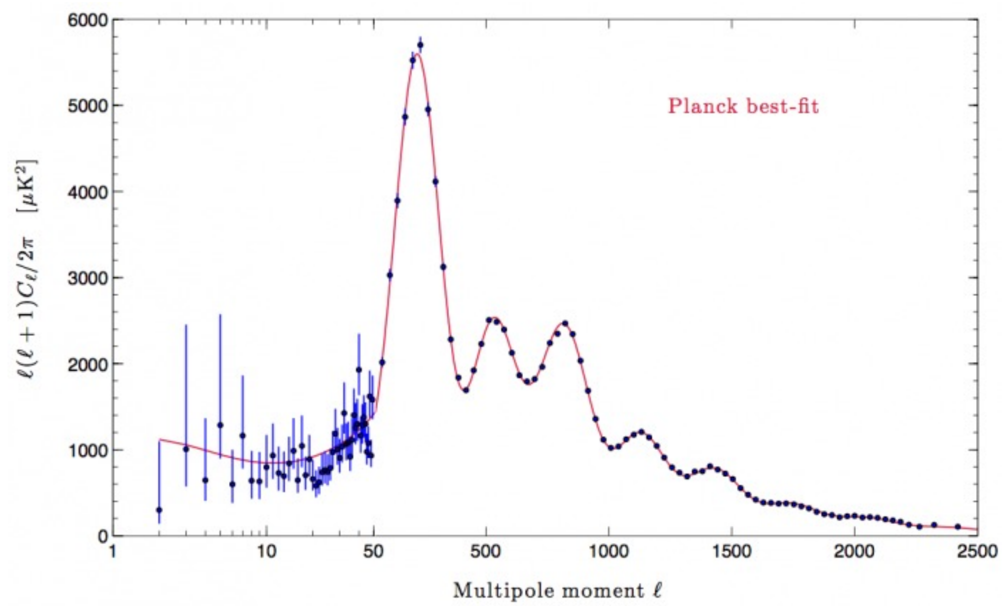


Blinov et al, 1905.02727

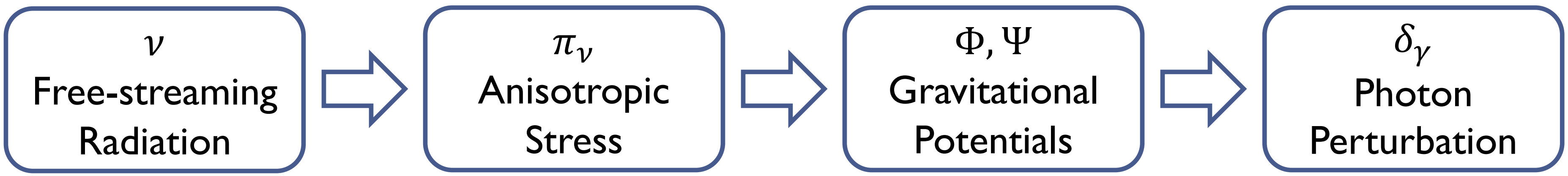
COSMIC MICROWAVE BACKGROUND

Cosmic Microwave Background





Effects of Neutrinos on CMB



Effects of Neutrinos on CMB

ν Free-streaming Radiation → π_ν Anisotropic Stress → Φ, Ψ Gravitational Potentials → δ_γ Photon Perturbation

$$F_\nu(\mathbf{n};\eta,\mathbf{k}) = \frac{4\pi}{\rho_\nu a^4}\int \kappa^3 d\kappa f_\nu^{(0)}(\kappa)\cdot \delta f_\nu(\boldsymbol{\kappa};\eta,\mathbf{x})\cdot \mathrm{e}^{i\mathbf{kx}} d^3\mathbf{x} = \sum_{l=0}^{\infty}(2l+1)(-i)^l \cdot F_{\nu,l}(\eta,\mathbf{k})\cdot P_l\left(\frac{\mathbf{kn}}{k}\right)$$

$F_\nu(\mathbf{n};\eta,\mathbf{k})$ → Neutrino Perturbations

$F_{\nu,l}(\eta,\mathbf{k})$ → Decompose with Legendre polynomial

Gorbunov and Rubakov, 2011

Effects of Neutrinos on CMB

ν Free-streaming Radiation ⇨ π_ν Anisotropic Stress ⇨ Φ, Ψ Gravitational Potentials ⇨ δ_γ Photon Perturbation

$$\pi_\nu = \frac{3}{16\pi}\int d\mathbf{n}\left(\left(\frac{\mathbf{kn}}{k}\right)^2 - \frac{1}{3}\right)F_\nu = \frac{6\pi}{16\pi}\int_{-1}^{1} dx\,\frac{2}{3}P_2\left(x\right)F_\nu = -\frac{1}{2}F_{\nu,2}$$

For neutrino

$$\pi_\nu' = \frac{4}{15}k^2 v_\nu + \frac{3}{10}kF_{\nu,3}$$

- π_ν evolves freely

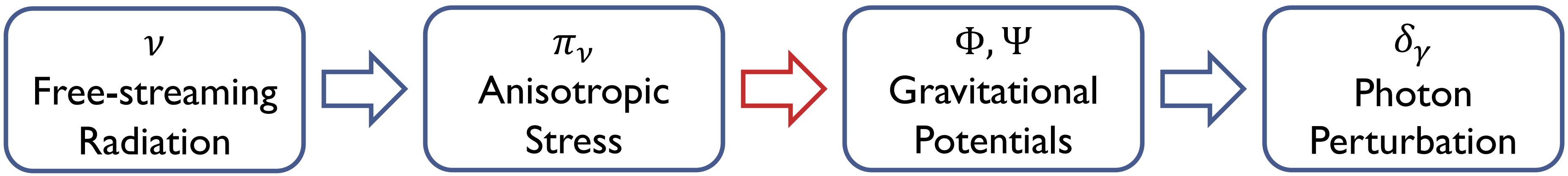
For photon

$$\pi_\gamma' - \frac{4}{15}k^2 v_\gamma + \frac{3}{10}kF_{\gamma,3} = \boxed{-\frac{9}{10}\tau'\pi_\gamma}$$

- τ' is the interaction rate
- The interaction suppresses π_γ

Gorbunov and Rubakov, 2011

Effects of Neutrinos on CMB



$$ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1+2\Psi)d\mathbf{x}^2]$$

$$\Delta(\Phi+\Psi) = -12\pi G a^2 \cdot [(\rho+p)\,\pi]_{tot}$$

- If $\pi = 0$, we have $\Psi = -\Phi$
- In Standard Model, $\Psi = -\left(1 + \frac{2}{5}\mathrm{R}_\nu \Phi\right) \approx -1.16\Phi$
  where $R_\nu = \frac{\rho_\nu}{\rho} \approx 0.41$ due to neutrinos

Gorbunov and Rubakov, 2011

Effects of Neutrinos on CMB

ν Free-streaming Radiation → π_ν Anisotropic Stress → Φ, Ψ Gravitational Potentials → δ_γ Photon Perturbation

$$\delta_\gamma' - \frac{4k^2}{3} v_\gamma + 4\Psi' = 0,$$

$$v_\gamma' + \frac{1}{4}\delta_\gamma + \pi_\gamma + \Phi = -\left(v_\gamma - v_B\right)\frac{\tau'}{k}$$

- Phase shift
- Amplitude suppression

Gorbunov and Rubakov, 2011

If Neutrinos have Self-Interaction

ν Free-streaming Radiation → π_ν Anisotropic Stress → Φ, Ψ Gravitational Potentials → δ_γ Photon Perturbation

For neutrino

$$\pi'_\nu = \frac{4}{15}k^2 v_\nu + \frac{3}{10}kF_{\nu,3} - \frac{9}{10}\tau'_\nu \pi_\nu$$

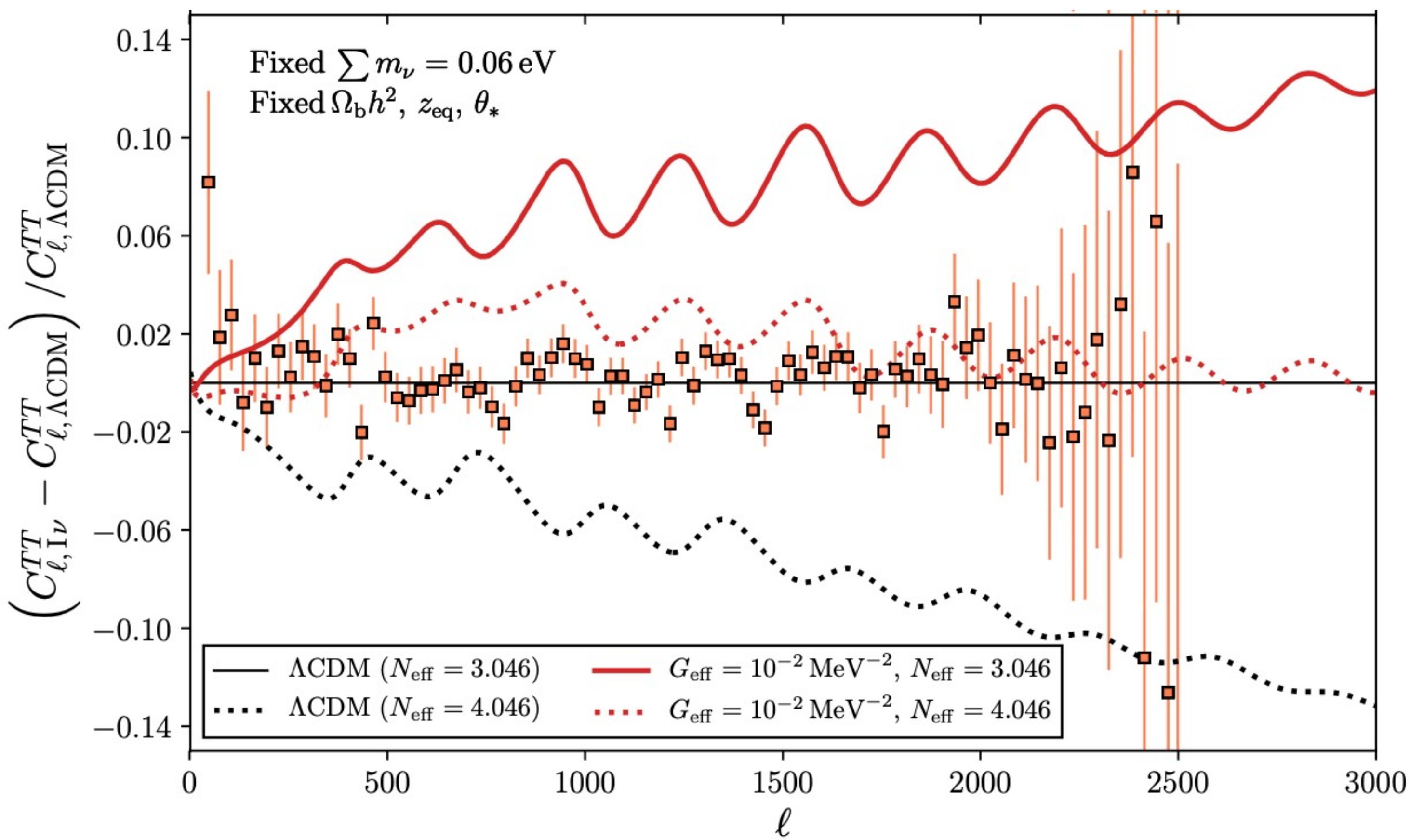
For photon

$$\pi'_\gamma - \frac{4}{15}k^2 v_\gamma + \frac{3}{10}kF_{\gamma,3} = -\frac{9}{10}\tau' \pi_\gamma$$

- π_ν suppressed at early time
- After decoupling, π_ν starts to evolve
- Can compensate the effect of additional radiation

Cyr-Racine and Sigurdson, 1306.1536

Power Spectrum Comparison



Kreisch et al, 1902.00534

RESULTS

Data Sets

- P18: Planck 2018 CMB temperature and polarization auto- and cross-correlation

- lens: Planck 2018 CMB lensing

- BAO: 6dFGS, SDSS DR7 MGS, and BOSS DR12

- R19: Prior on the Hubble parameter today, H_0, from Riess et al. 2019

Parameterization

We have 4 new parameters in addition to 6 basic cosmological parameters

- $N_{\mathrm{eff,fs}}$:The number of free-streaming species
- $N_{\mathrm{eff,int}}$:The number of interacting species
- $N_{\mathrm{eff}} = N_{\mathrm{eff,fs}} + N_{\mathrm{eff,int}}$
- z_{dec} :The decoupling redshift
- $\Sigma m = \sum_i N_{\mathrm{eff},i}\, m_i$

Implementation in CLASS

- We suppress $F_{\nu,l\geq 2}$ with a transition function

$$\mathcal{T}(z) = \frac{1}{2}\left(\tanh\left(\frac{z - z_{\rm dec}}{\Delta z_{\rm dec}}\right) + 1\right)$$

Choi et al, 1804.10180

- $\Delta z_{\rm dec} = 0.4 z_{\rm dec}$ to match the opacity function

- We've checked this approximation is valid for the decoupling model

Cases

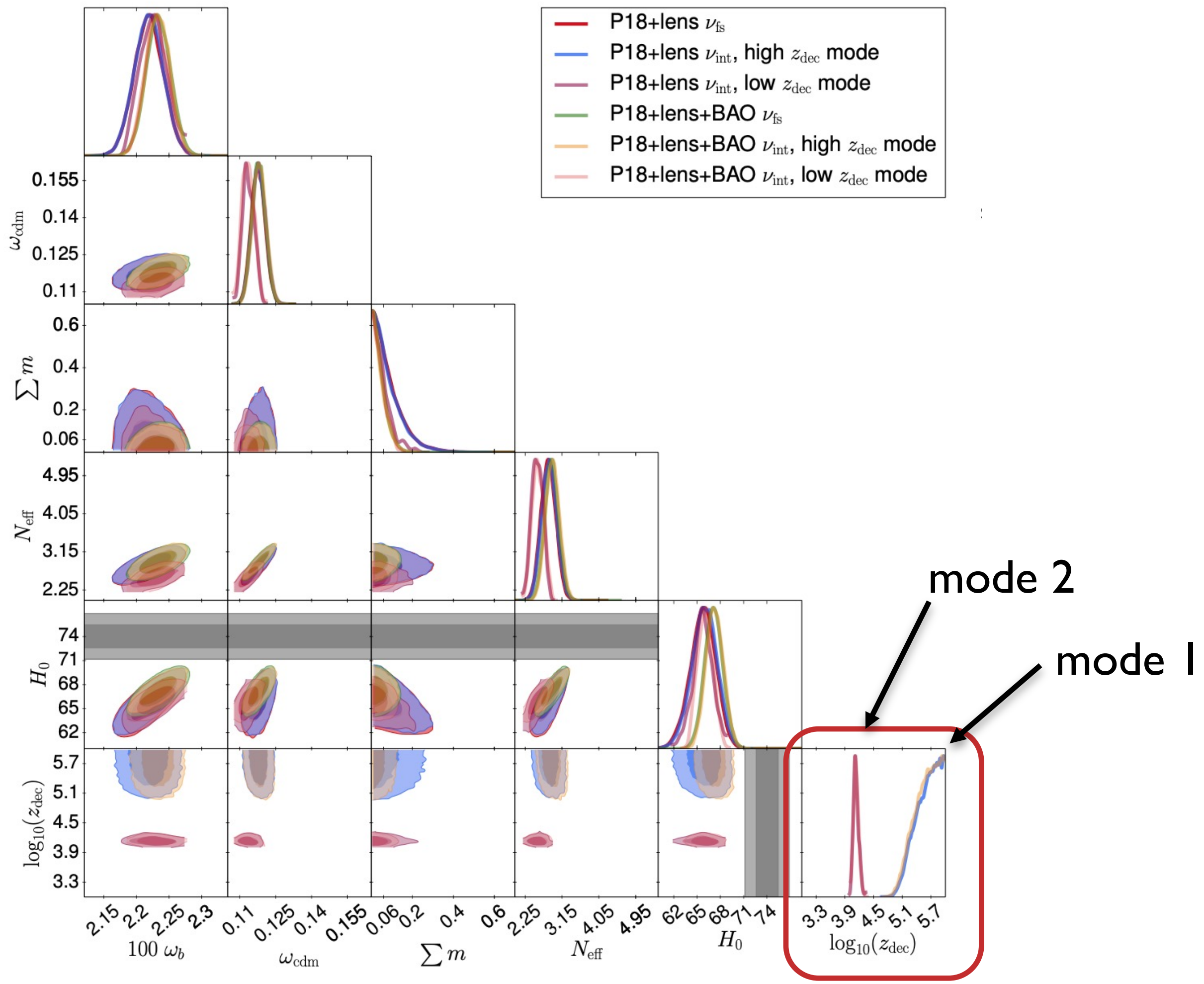
For computational efficiency, we fix one of the new parameters

- Case 1 : $N_{\mathrm{eff,fs}} = 0$
- Case 2 : $N_{\mathrm{eff,fs}} = 2.0328$
- Case 3 : $N_{\mathrm{eff}} = 3.046$
- Case 4 : $\Sigma m = 0.11\ \mathrm{eV}$

Case 1: All species interacting

- All neutrino species are interacting
- Corresponds to the case in the Kreisch paper
- Fixed parameters : $N_{\mathrm{eff,fs}} = 0$
- Varying parameters : $N_{\mathrm{eff,int}},\ z_{\mathrm{dec}},\ \Sigma m$

Case 1: Triangle Plot



Case 1: Summary Table

	Free-streaming		Self-interacting (Case 1)			
	P18 +lens	+BAO	P18 +lens		+BAO	
			mode 1	mode 2	mode 1	mode 2
ω_b	0.02219 ± 0.00022	0.02234 ± 0.00019	0.02219 ± 0.00022	0.02226 ± 0.00020	0.02233 ± 0.018	0.02230 ± 0.0017
ω_{cdm}	0.1177 ± 0.0029	0.1179 ± 0.0028	0.1180 ± 0.0029	0.1136 ± 0.0024	0.1182 ± 0.0029	0.1135 ± 0.0025
$100 \times \theta_s$	1.04226 ± 0.00051	1.04217 ± 0.00050	1.04225 ± 0.00051	1.04679 ± 0.00055	1.04217 ± 0.00049	1.04678 ± 0.00055
$\ln(10^{10} A_s)$	3.037 ± 0.017	3.042 ± 0.017	3.035 ± 0.017	2.967 ± 0.014	3.040 ± 0.016	2.967 ± 0.014
n_s	0.9573 ± 0.0085	0.9631 ± 0.0071	0.9560 ± 0.0085	0.9209 ± 0.0061	0.9613 ± 0.0071	0.9226 ± 0.0055
z_{reio}	7.57 ± 0.76	7.75 ± 0.73	7.56 ± 0.76	7.45 ± 0.67	7.74 ± 0.72	7.48 ± 0.65
$\log_{10}(z_{\rm dec})$	—	—	> 5.2 (95%CL)	4.14 ± 0.058	> 5.1 (95%CL)	4.14 ± 0.056
$N_{\rm eff}$	2.82 ± 0.18	2.90 ± 0.17	2.84 ± 0.18	2.55 ± 0.14	2.92 ± 0.17	2.57 ± 0.14
$\sum m$	< 0.227 (95%CL)	< 0.108 (95%CL)	< 0.225 (95%CL)	< 0.160 (95%CL)	< 0.107 (95%CL)	< 0.108 (95%CL)
$H_0 \left[\frac{(\rm km/s)}{\rm Mpc}\right]$	65.8 ± 1.6	67.2 ± 1.1	65.9 ± 1.7	65.7 ± 1.3	67.3 ± 1.1	66.1 ± 1.0
S_8	0.835 ± 0.013	0.828 ± 0.012	0.835 ± 0.013	0.825 ± 0.013	0.829 ± 0.011	0.821 ± 0.011
$\ln(E)$	-0.5282×10^3	-0.5320×10^3	-0.5333×10^3	-0.5388×10^3	-0.5370×10^3	-0.5418×10^3
$E_{\rm int}/E_{\rm fs}$	—	—	6.1×10^{-3}	2.5×10^{-5}	6.7×10^{-3}	5.5×10^{-5}
Best fit						
$N_{\rm eff}$	2.846	2.922	2.859	2.572	2.819	2.519
$\log_{10}(z_{\rm dec})$	—	—	5.953	4.119	5.997	4.126
$\chi^2_{\rm eff}$	1011.08	1016.72	1011.67	1018.35	1016.94	1023.39
$\Delta\chi^2_{\rm eff}$	—	—	+0.59	+7.27	+0.22	+6.67

ΛCDM

Self-interacting neutrinos

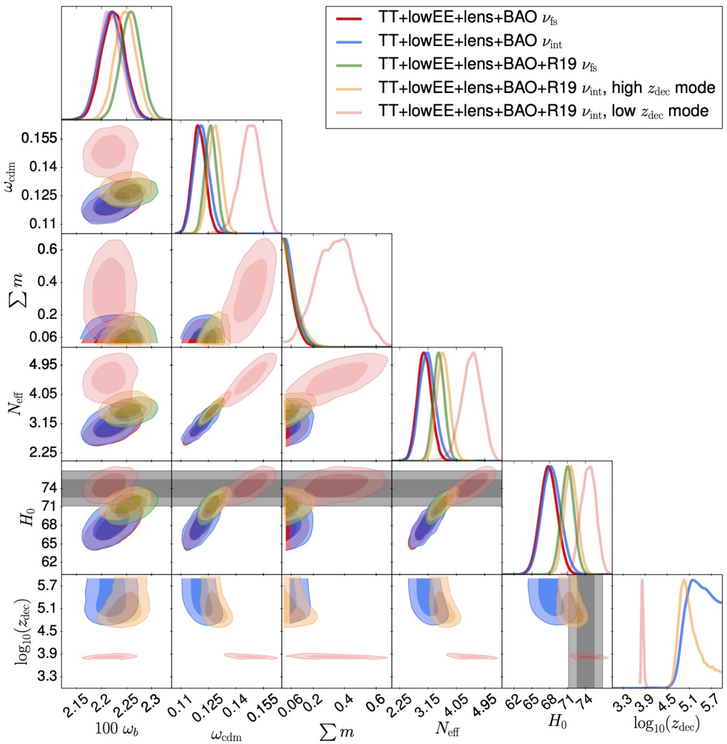
Case 1: Results

- Unlike the Kreisch paper, we don't find increased H_0

- The difference is the data set
  - We used Planck2018 data instead of Planck2015
  - Planck2015 data had large errors for high-l polarization

- What if we remove high-l polarization data?

Case 1-2: Removing high-l polarization



	Free-streaming		Self-interacting (Case 1)		
	TT +lowEE +lens +BAO	+R19	TT +lowEE +lens +BAO	+R19 mode 1	+R19 mode 2
ω_b	0.02223 ± 0.00023	0.02259 ± 0.00020	0.02219 ± 0.00024	0.02248 ± 0.021	0.02216 ± 0.0021
ω_{cdm}	0.1199 ± 0.0036	0.1263 ± 0.0031	0.1213 ± 0.0041	0.1286 ± 0.0036	0.1478 ± 0.0057
$100 \times \theta_s$	1.04189 ± 0.00063	1.04204 ± 0.00054	1.04103 ± 0.00056	1.04217 ± 0.00049	1.04617 ± 0.00046
$\ln(10^{10}A_s)$	3.045 ± 0.018	3.067 ± 0.017	3.041 ± 0.018	3.057 ± 0.018	2.984 ± 0.016
n_s	0.9668 ± 0.0085	0.9839 ± 0.0065	0.9639 ± 0.0092	0.9760 ± 0.0087	0.9411 ± 0.0067
z_{reio}	7.74 ± 0.76	8.10 ± 0.78	7.69 ± 0.77	7.95 ± 0.78	7.63 ± 0.87
$\log_{10}(z_{\rm dec})$	—	—	> 4.9 (95%CL)	$5.15^{+0.40}_{-0.16}$	3.83 ± 0.03
$N_{\rm eff}$	3.04 ± 0.22	3.50 ± 0.18	3.13 ± 0.25	3.63 ± 0.20	4.53 ± 0.32
$\sum m$	< 0.130 (95%CL)	< 0.144 (95%CL)	< 0.151 (95%CL)	< 0.173 (95%CL)	0.35 ± 0.13
$H_0\ \left[\frac{\rm (km/s)}{\rm Mpc}\right]$	67.8 ± 1.4	71.0 ± 1.0	68.3 ± 1.5	71.5 ± 1.1	74.5 ± 1.2
S_8	0.828 ± 0.014	0.829 ± 0.014	0.832 ± 0.015	0.836 ± 0.014	0.816 ± 0.015
$\ln(E)$	-0.3419×10^3	-0.3468×10^3	-0.3462×10^3	-0.3504×10^3	-0.3550×10^3
$E_{\rm int}/E_{\rm fs}$	—	—	1.4×10^{-2}	2.7×10^{-2}	2.7×10^{-4}
			Best fit		
$N_{\rm eff}$	2.971	3.494	3.123	3.591	4.653
$\log_{10}(z_{\rm dec})$	—	—	5.224	4.970	3.8208
P18 highTT	205.34	211.24	204.95	208.11	216.01
P18 lowTT	23.58	21.56	23.95	22.74	24.44
P18 highEE	—	—	—	—	—
P18 lowEE	395.77	396.28	395.75	396.15	395.87
P18 lensing	8.81	9.46	8.88	9.36	11.20
P18 total	633.5	638.5	633.5	636.4	647.5
BAO	5.40	6.54	5.25	6.53	4.96
R19	—	3.75	—	2.97	0.33
$\chi^2_{\rm eff}$	638.89	648.83	638.79	645.85	652.81
$\Delta\chi^2_{\rm eff}$	—	—	-0.10	-2.98	+3.98

- Now we find increased H_0
- Polarization data constrain $N_{\rm eff}$ tightly
- With polarization data, we find self-interacting neutrino is not a solution to the Hubble tension

Case 4: Fixed Σm

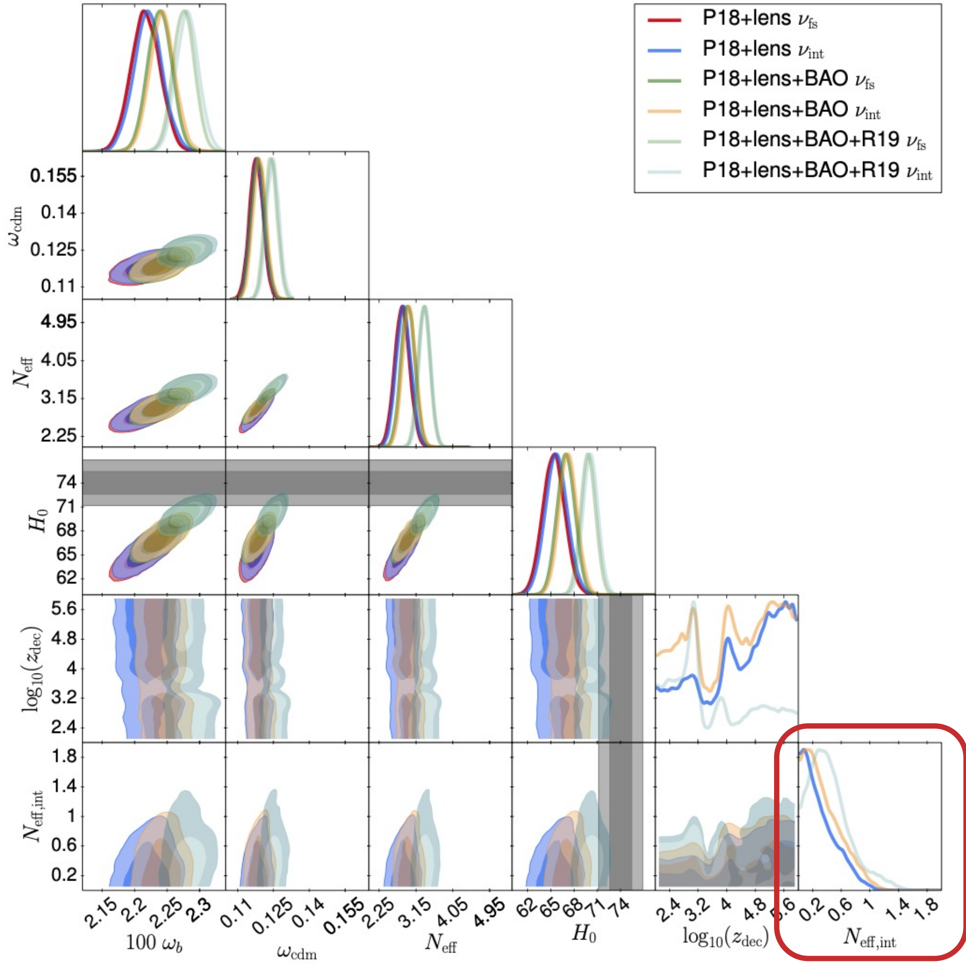
- We fix total Σm while varying both $N_{\mathrm{eff,int}}$ and $N_{\mathrm{eff,fs}}$

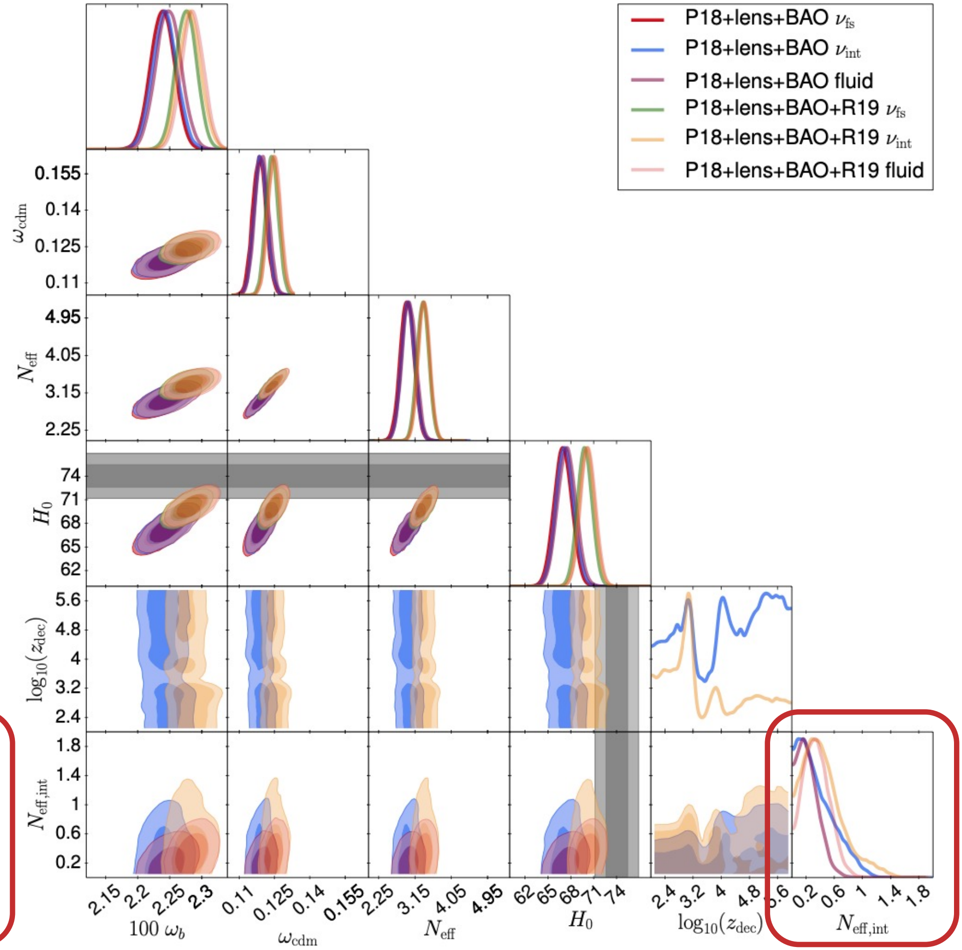
- Fixed parameters : $\Sigma m = 0.11\ \mathrm{eV}$

- Varying parameters :$N_{\mathrm{eff,int}}$, $N_{\mathrm{eff,fs}}$, z_{dec}

- We also consider fluid-like case ($z_{\mathrm{dec}} = 0$)

Case 4: Triangle Plot





Case 4: Summary Table

	Free-streaming			Self-interacting (Case 4)			Self-interacting (fluid-like)	
	P18 +lens	+BAO	+R19	P18 +lens	+BAO	+R19	P18 +lens +BAO	+R19
ω_b	0.02217 ± 0.00022	0.02239 ± 0.00019	0.02276 ± 0.00017	0.02221 ± 0.00022	$0.02242^{+0.00018}_{-0.00020}$	$0.02281^{+0.00017}_{-0.00018}$	0.02248 ± 0.00020	0.02286 ± 0.00017
ω_{cdm}	0.1178 ± 0.0028	0.1186 ± 0.0029	0.1240 ± 0.0026	$0.1183^{+0.0027}_{-0.0030}$	0.1191 ± 0.0028	$0.1247^{+0.0026}_{-0.0027}$	0.1196 ± 0.0029	0.1249 ± 0.0027
$100 \times \theta_s$	1.04226 ± 0.00054	1.04207 ± 0.00052	1.04127 ± 0.00046	$1.04244^{+0.00054}_{-0.00066}$	$1.04238^{+0.00055}_{-0.00075}$	$1.04198^{+0.00068}_{-0.00094}$	1.04269 ± 0.00066	$1.04226^{+0.00066}_{-0.00075}$
$\ln(10^{10} A_s)$	3.038 ± 0.017	3.050 ± 0.017	3.071 ± 0.017	$3.036^{+0.017}_{-0.018}$	$3.044^{+0.018}_{-0.017}$	$3.058^{+0.020}_{-0.019}$	3.040 ± 0.018	3.054 ± 0.019
n_s	0.9567 ± 0.0085	0.9654 ± 0.0070	0.9810 ± 0.0059	$0.9561^{+0.0086}_{-0.0082}$	$0.9633^{+0.0076}_{-0.0071}$	$0.9757^{+0.0073}_{-0.0070}$	0.9622 ± 0.0073	0.9745 ± 0.0068
z_{reio}	7.63 ± 0.75	8.06 ± 0.73	8.45 ± 0.76	$7.69^{+0.72}_{-0.74}$	$8.07^{+0.70}_{-0.72}$	8.46 ± 0.75	8.13 ± 0.75	8.53 ± 0.77
$N_{\rm eff}$	2.81 ± 0.18	2.95 ± 0.17	3.35 ± 0.15	$2.85^{+0.18}_{-0.19}$	$2.98^{+0.16}_{-0.17}$	$3.37^{+0.14}_{-0.15}$	3.00 ± 0.17	3.37 ± 0.15
$N_{\rm eff,int}$	—	—	—	< 0.74 (95%CL)	< 0.86 (95%CL)	$0.44^{+0.14}_{-0.37}$	< 0.51 (95%CL)	$0.35^{+0.15}_{-0.22}$
$H_0 \left[\frac{\rm (km/s)}{\rm Mpc}\right]$	65.4 ± 1.4	67.0 ± 1.2	69.8 ± 1.0	$65.7^{+1.4}_{-1.5}$	$67.2^{+1.1}_{-1.2}$	$70.1^{+0.9}_{-1.0}$	67.5 ± 1.2	70.3 ± 1.0
S_8	0.835 ± 0.014	0.823 ± 0.012	0.823 ± 0.012	0.833 ± 0.014	0.821 ± 0.012	0.818 ± 0.013	0.818 ± 0.013	0.815 ± 0.013
$\ln(E)$	-0.5275×10^3	-0.5322×10^3	-0.5399×10^3	-0.5273×10^3	-0.5317×10^3	-0.5385×10^3	-0.5325×10^3	-0.5391×10^3
$E_{\rm int}/E_{\rm fs}$	—	—	—	1.18	1.57	3.94	0.71	2.19
Best fit (corresponding to low $z_{\rm dec}$ mode)								
$N_{\rm eff}$	2.798	2.937	3.321	2.807	2.924	3.376	2.982	3.365
$N_{\rm eff,int}$	—	—	—	0.030	0.193	0.564	0.168	0.312
$\log_{10}(z_{\rm dec})$	—	—	—	3.038	3.077	3.078	—	—
$\chi^2_{\rm eff}$	1012.85	1021.61	1036.65	1012.79	1021.01	1032.73	1021.22	1034.32
$\Delta\chi^2_{\rm eff}$	—	—	—	-0.06	-0.60	-3.91	-0.39	-2.32
Intermediate $z_{\rm dec}$ mode best fit								
$N_{\rm eff}$	—	—	—	2.768	2.930	3.463	—	—
$N_{\rm eff,int}$	—	—	—	0.002	0.297	0.448	—	—
$\log_{10}(z_{\rm dec})$	—	—	—	3.849	4.004	3.773	—	—
$\chi^2_{\rm eff}$	—	—	—	1012.93	1021.46	1034.96	—	—
$\Delta\chi^2_{\rm eff}$	—	—	—	+0.07	-0.15	-1.68	—	—
High $z_{\rm dec}$ mode best fit								
$N_{\rm eff}$	—	—	—	2.954	2.924	3.321	—	—
$N_{\rm eff,int}$	—	—	—	0.012	0.028	0.305	—	—
$\log_{10}(z_{\rm dec})$	—	—	—	5.542	5.860	5.180	—	—
$\chi^2_{\rm eff}$	—	—	—	1013.07	1021.74	1037.36	—	—
$\Delta\chi^2_{\rm eff}$	—	—	—	+0.22	+0.13	+0.71	—	—

Case 4: Results

- We find an upper bound on $N_{\mathrm{eff,int}} < 0.86$ (95% C.L.) for the decoupling case
- For fluid-like case, we find $N_{\mathrm{eff,int}} < 0.51$ (95% C.L.)

CONCLUSIONS

Conclusions

- Self-interacting neutrinos do not help to solve the Hubble tension with new data
- We put constraints on self-interacting radiation (on $z_{\rm dec}, N_{\rm eff,int}$)

Future Works

- We are working on the recoupling case

- For the recoupling case, the Majoron can be produced in later Universe

- Recoupling width is much larger compared to the decoupling case

THANK YOU

BACK UP

Decoupling Time

- Decoupling happens at $\Gamma \sim H$

- With $\Gamma \sim G_\nu^2 T^5$ and $H \sim \frac{T^2}{m_{pl}}$,
  we get $T_d \sim \left(G_\nu^2 m_{pl}\right)^{-1/3}$

The Exact Interaction Rate

For $\nu_i(p_1) + \nu_j(p_2) \to \nu_k(p_3) + \nu_l(p_4)$

$$\Gamma_\nu(E_1) = \frac{1}{2E_1}\int d\Pi_2 d\Pi_3 d\Pi_4 f_\nu(E_2)(1-f_\nu(E_3))(1-f_\nu(E_4))\boxed{|\mathcal{M}|^2}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

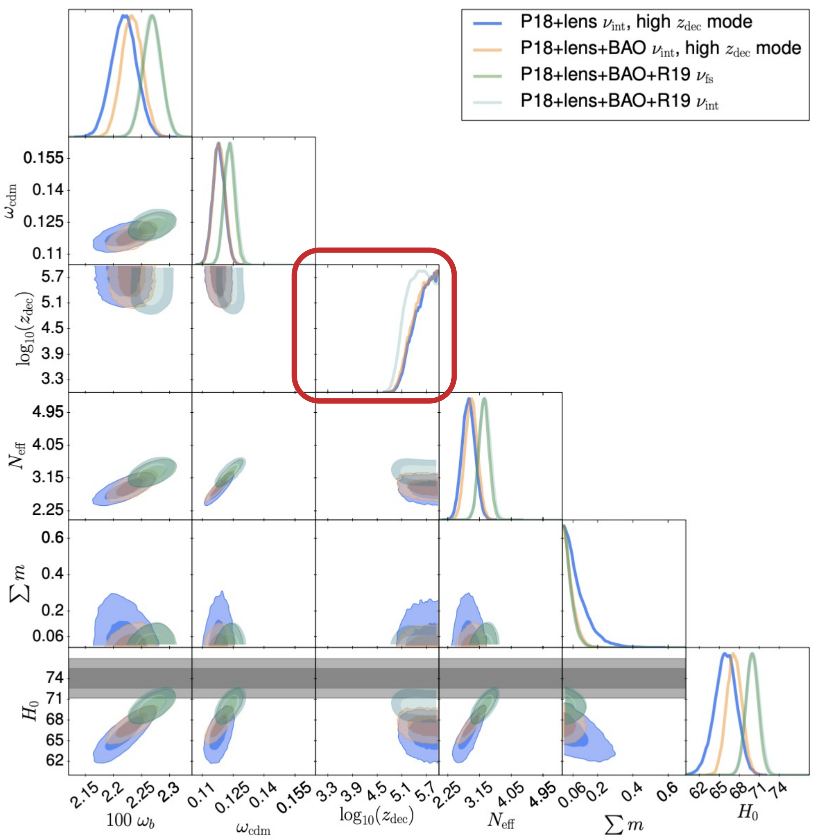
$$|\mathcal{M}_{\nu\nu\to\nu\nu}|^2 = \boxed{\frac{1}{2}}|\mathcal{M}_{\nu_i\nu_i\to\nu_i\nu_i}|^2 + \boxed{\frac{1}{2}}\times\boxed{2}|\mathcal{M}_{\nu_i\nu_i\to\nu_j\nu_j}|^2 + \boxed{2}|\mathcal{M}_{\nu_i\nu_j\to\nu_i\nu_j}|^2$$

For identical outgoing particles For different species

$$\begin{aligned}
|\mathcal{M}_{\nu_i\nu_i\to\nu_i\nu_i}|^2 &= G_\nu^2(s^2+st+t^2) \\
|\mathcal{M}_{\nu_i\nu_i\to\nu_j\nu_j}|^2 &= G_\nu^2 s^2 \qquad (i\neq j) \\
|\mathcal{M}_{\nu_i\nu_j\to\nu_i\nu_j}|^2 &= G_\nu^2 t^2 \qquad (i\neq j)
\end{aligned}$$

$$\begin{aligned}
\Gamma_\nu(E_1) &= \int \frac{d^3p_2}{(2\pi)^3} g_\nu f_\nu(E_2)\frac{s}{2E_1E_2}\sigma_{\nu\nu\to\nu\nu} \\
&= \frac{35\pi}{1728}G_\nu^2 E_1 T_\nu^4
\end{aligned}$$

Case 1: With R19



	Free-streaming		Self-interacting (Case 1)
	P18 +lens +BAO	P18 +lens +BAO +R19	P18 +lens +BAO + R19
ω_b	0.02234 ± 0.00019	0.02270 ± 0.00016	0.2268 ± 0.0016
ω_{cdm}	0.1179 ± 0.0028	0.1231 ± 0.0026	0.1235 ± 0.0027
$100 \times \theta_s$	1.04217 ± 0.00050	1.04139 ± 0.00043	1.04139 ± 0.00045
$\ln(10^{10} A_s)$	3.042 ± 0.017	3.062 ± 0.016	3.058 ± 0.016
n_s	0.9631 ± 0.0071	0.9780 ± 0.0058	0.9751 ± 0.0066
z_{reio}	7.75 ± 0.73	8.10 ± 0.74	8.07 ± 0.74
$\log_{10}(z_{\rm dec})$	—	—	> 5.3 (68%CL)
$N_{\rm eff}$	2.90 ± 0.17	3.28 ± 0.14	3.30 ± 0.15
$\sum m$	< 0.108 (95%CL)	< 0.0965 (95%CL)	< 0.102 (95%CL)
$H_0\ \left[\frac{\rm (km/s)}{\rm Mpc}\right]$	67.2 ± 1.1	69.93 ± 0.92	70.05 ± 0.94
S_8	0.828 ± 0.012	0.828 ± 0.012	0.829 ± 0.012
$\ln(E)$	-0.5320×10^3	-0.5393×10^3	-0.5439×10^3
$E_{\rm int}/E_{\rm fs}$	—	—	1.0×10^{-2}
	Best fit		
$N_{\rm eff}$	2.922	3.209	3.254
$\log_{10}(z_{\rm dec})$	—	—	5.571
P18 highTTTEEE	583.23	588.54	589.17
P18 lowTT	23.45	21.95	22.17
P18 lowEE	396.01	396.43	396.26
P18 lensing	8.73	9.08	9.08
P18 total	1011.4	1016.0	1016.7
BAO	5.30	5.69	5.78
R19	—	9.14	8.17
$\chi^2_{\rm eff}$	1016.72	1030.83	1030.64
$\Delta\chi^2_{\rm eff}$	—	—	-0.19

- mode 2 is ruled out
- We have the lower bound on $z_{\rm dec} > 10^{5.3}$ (68% C.L.)

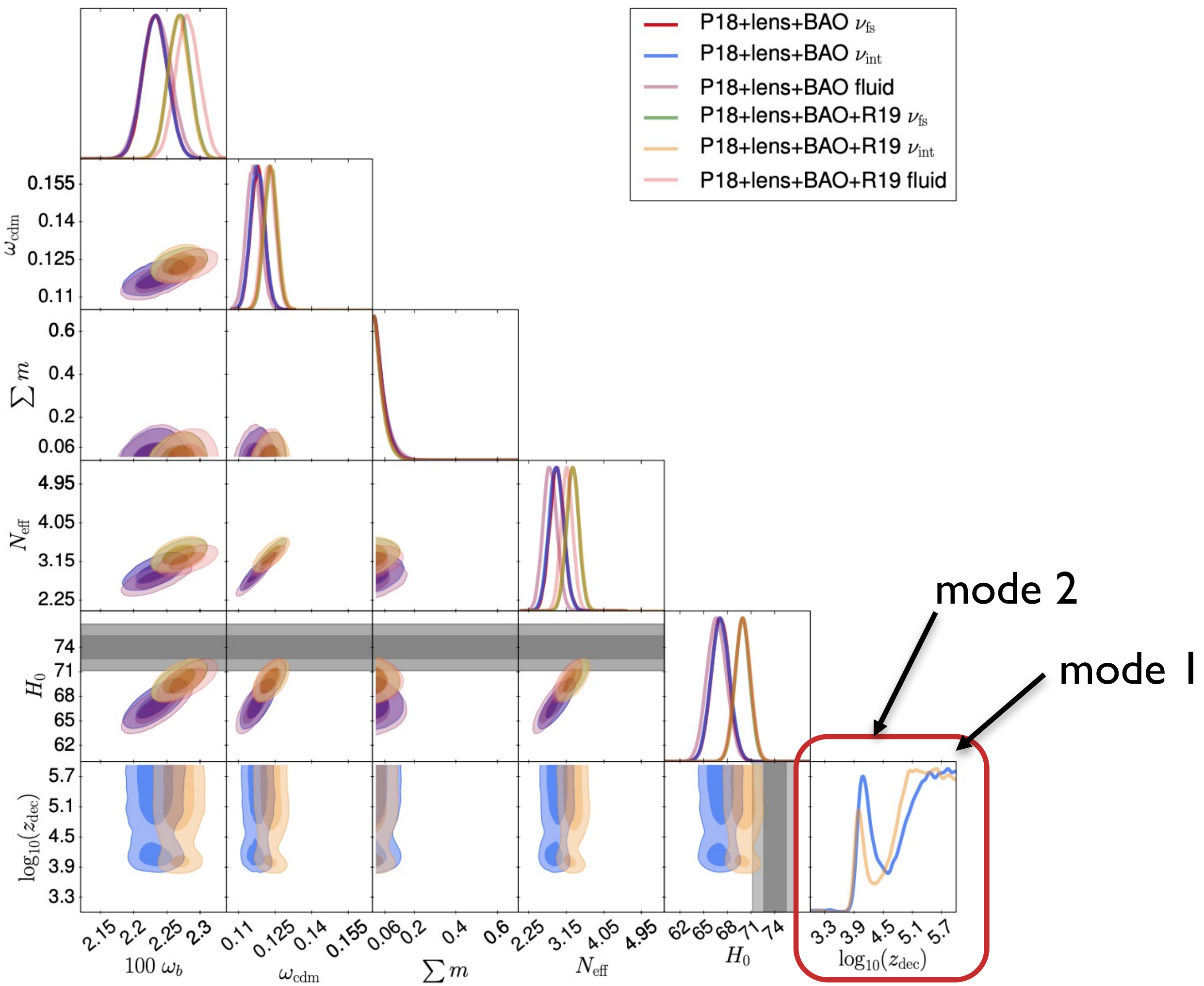
Case 2: $2\ \nu_{\mathrm{fs}} + 1\ \nu_{\mathrm{int}}$

- We leave two species of neutrino free-streaming, but one of the species is self-interacting

- Fixed parameters : $N_{\mathrm{eff,fs}} = 2.0328$

- Varying parameters :$N_{\mathrm{eff,int}},\ z_{\mathrm{dec}},\ \Sigma m$

Case 2: Triangle Plot



Case 2: Summary Table

	Free-streaming			Self-interacting (Case 2)			Self-interacting (fluid-like)	
	P18 +lens	+BAO	+R19	P18 +lens	+BAO	+R19	P18 +lens +BAO	+R19
ω_b	0.02216 ± 0.00023	0.02233 ± 0.00019	0.02269 ± 0.00016	0.02215 ± 0.00023	0.02232 ± 0.00019	0.02268 ± 0.00016	0.02235 ± 0.00021	0.02281 ± 0.00018
ω_{cdm}	0.1177 ± 0.0029	0.1178 ± 0.0029	0.1231 ± 0.0026	$0.1175^{+0.0029}_{-0.0031}$	0.1177 ± 0.0029	$0.1232^{+0.0028}_{-0.0027}$	0.1162 ± 0.0030	$0.1224^{+0.0025}_{-0.0029}$
$100 \times \theta_s$	1.04229 ± 0.00051	1.04219 ± 0.00050	1.04140 ± 0.00044	$1.04257^{+0.00056}_{-0.00082}$	$1.04254^{+0.00056}_{-0.00093}$	$1.04173^{+0.00032}_{-0.00088}$	1.04510 ± 0.00032	1.04537 ± 0.00030
$\ln(10^{10} A_s)$	3.036 ± 0.017	3.042 ± 0.017	3.062 ± 0.016	$3.031^{+0.019}_{-0.018}$	3.035 ± 0.018	$3.054^{+0.020}_{-0.018}$	2.992 ± 0.015	2.994 ± 0.030
n_s	0.9563 ± 0.0087	0.9629 ± 0.0071	0.9781 ± 0.0059	$0.9530^{+0.0092}_{-0.0095}$	$0.9588^{+0.0084}_{-0.0085}$	$0.9735^{+0.0091}_{-0.0067}$	0.9389 ± 0.0044	0.9465 ± 0.0040
z_{reio}	7.58 ± 0.77	7.78 ± 0.74	8.12 ± 0.74	7.56 ± 0.75	7.73 ± 0.72	$8.06^{+0.72}_{-0.74}$	7.81 ± 0.73	8.24 ± 0.75
$N_{\rm eff}$	2.83 ± 0.18	2.92 ± 0.17	3.30 ± 0.15	$2.83^{+0.18}_{-0.19}$	$2.91^{+0.18}_{-0.17}$	$3.30^{+0.15}_{-0.16}$	2.76 ± 0.17	3.16 ± 0.14
$\sum m$	< 0.301 (95%CL)	< 0.108 (95%CL)	< 0.095 (95%CL)	< 0.312 (95%CL)	< 0.110 (95%CL)	< 0.097 (95%CL)	< 0.122 (95%CL)	< 0.110 (95%CL)
$H_0 \left[\frac{(\rm km/s)}{\rm Mpc}\right]$	65.4 ± 1.7	67.1 ± 1.1	69.9 ± 0.9	$65.4^{+1.9}_{-1.7}$	67.1 ± 1.2	70.0 ± 1.0	66.7 ± 1.2	69.9 ± 1.0
S_8	0.833 ± 0.014	0.827 ± 0.012	0.828 ± 0.012	$0.832^{+0.015}_{-0.014}$	0.827 ± 0.012	0.828 ± 0.012	0.804 ± 0.012	0.796 ± 0.011
$\ln(E)$	-0.5280×10^3	-0.5322×10^3	-0.5394×10^3	-0.5324×10^3	-0.5323×10^3	-0.5436×10^3	-0.5365×10^3	-0.5444×10^3
$E_{\rm int}/E_{\rm fs}$	—	—	—	1.3×10^{-2}	0.86	1.6×10^{-2}	1.4×10^{-2}	6.8×10^{-3}
Best fit								
$N_{\rm eff,int}$	—	—	—	0.834	0.787	1.239	0.646	1.153
$\log_{10} z_{\rm dec}$	—	—	—	5.442	4.085	5.163	—	—
$\chi^2_{\rm eff}$	1011.10	1016.79	1030.98	1011.24	1016.62	1030.85	1025.76	1041.15
$\Delta\chi^2_{\rm eff}$	—	—	—	+0.14	-0.16	-0.12	+8.97	+10.17
Second mode best fit								
$N_{\rm eff,int}$	—	—	—	0.687	0.822	1.127	—	—
$\log_{10} z_{\rm dec}$	—	—	—	4.118	5.456	4.002	—	—
$\chi^2_{\rm eff}$	—	—	—	1011.43	1016.71	1031.39	—	—
$\Delta\chi^2_{\rm eff}$	—	—	—	+0.33	-0.08	+0.41	—	—

Case 2: Results

- We find a room for self-interacting species
- This does not change other cosmological variables such as H_0 much
- CMB does not prefer the interacting neutrino over free-streaming case

Case 3: Fixed N_{eff}

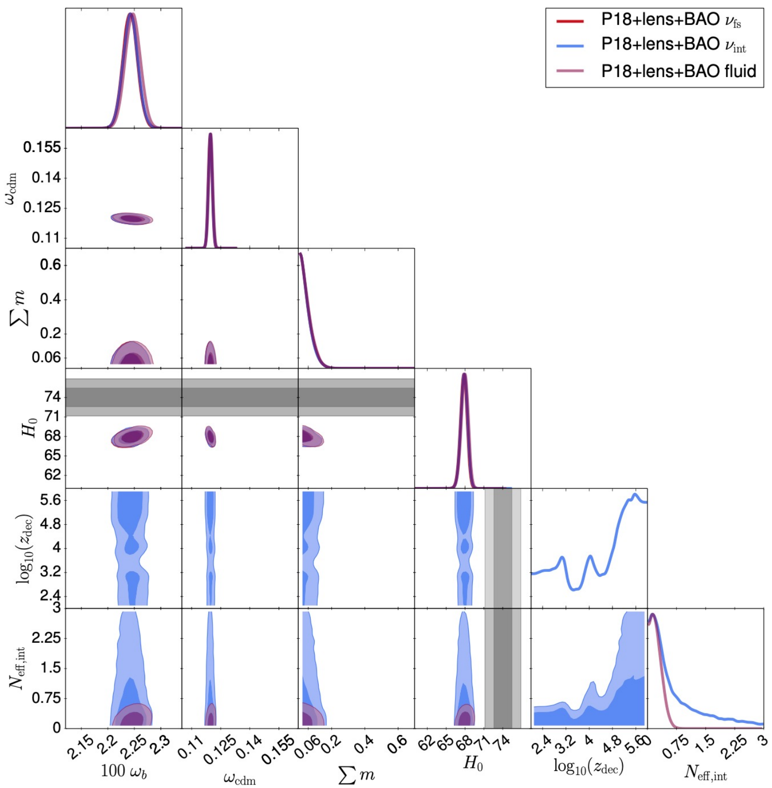
- We fix total N_{eff} while varying the fraction of interacting species

- Fixed parameters : $N_{\mathrm{eff}} = 3.046$

- Varying parameters :$N_{\mathrm{eff,int}}$, z_{dec}, Σm

- We also consider fluid-like case ($z_{\mathrm{dec}} = 0$)

Case 3: Triangle Plot and Summary Table



	Free-streaming P18 +lens +BAO	Self-interacting (Case 3) P18 +lens +BAO	Self-interacting (fluid-like) P18 +lens +BAO
ω_b	0.02242 ± 0.00014	0.02243 ± 0.00014	0.02247 ± 0.00015
ω_{cdm}	0.1196 ± 0.0010	0.1197 ± 0.0010	0.1199 ± 0.0010
$100 \times \theta_s$	1.04191 ± 0.00032	$1.04220^{+0.00033}_{-0.00066}$	$1.04261^{+0.00050}_{-0.0067}$
$\ln(10^{10} A_s)$	3.043 ± 0.015	3.042 ± 0.016	3.036 ± 0.017
n_s	0.9669 ± 0.0038	$0.9643^{+0.0055}_{-0.0041}$	0.9621 ± 0.0050
z_{reio}	7.82 ± 0.73	7.82 ± 0.72	7.87 ± 0.73
$N_{eff,int}$	—	< 0.79 (68%CL)	< 0.28 (68%CL)
$\sum m$	< 0.115 (95%CL)	< 0.115 (95%CL)	< 0.118 (95%CL)
$H_0 \left[\frac{(km/s)}{Mpc}\right]$	67.8 ± 0.5	$67.9^{+0.6}_{-0.5}$	68.0 ± 0.6
S_8	0.829 ± 0.012	0.829 ± 0.013	0.823 ± 0.013
$\ln(E)$	-0.5306×10^3	-0.5309×10^3	-0.5320×10^3
E_{int}/E_{fs}	—	0.74	0.24
Best fit			
$N_{eff,int}$	—	0.199	0.139
$\log_{10}(z_{dec})$	—	3.084	—
χ^2_{eff}	1017.32	1016.65	1017.05
$\Delta\chi^2_{eff}$	—	-0.67	-0.28
High z_{dec} mode best fit			
$N_{eff,int}$	—	0.020	—
$\log_{10}(z_{dec})$	—	5.077	—
χ^2_{eff}	—	1017.47	—
$\Delta\chi^2_{eff}$	—	+0.15	—

Case 3: Results

- We find an upper bound on $N_{\mathrm{eff,int}} < 0.79$ (68% C.L.) for the decoupling case

- For fluid-like case, we find $N_{\mathrm{eff,int}} < 0.28$ (68% C.L.)