Unveiling triangular correlation of angular deviation in muon scattering tomography by means of GEANT4 simulations

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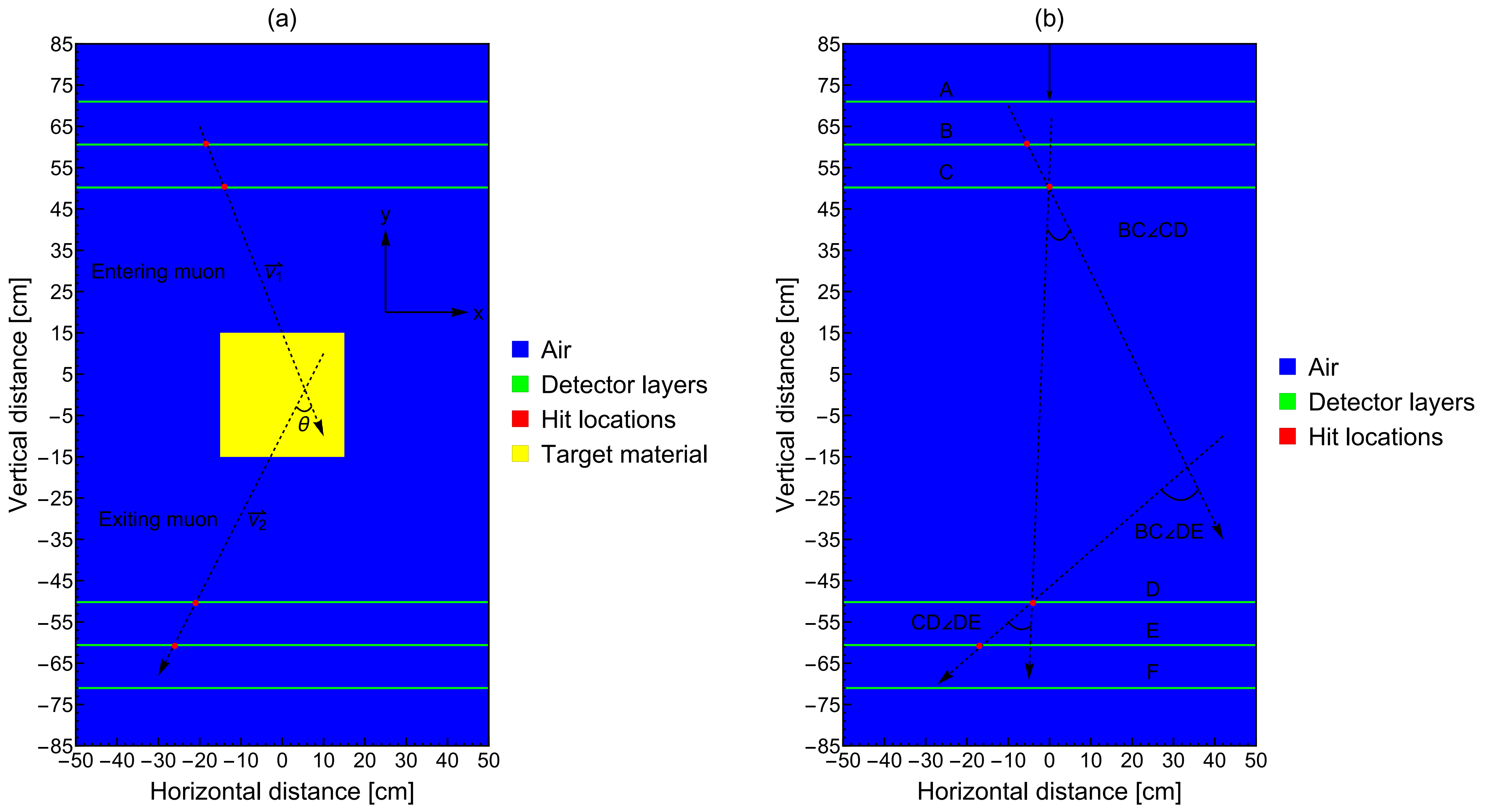
Objective

The angular deviation commonly represented by the scattering angle generally serves to provide the characteristic discrimination in the muon scattering tomography. The regular procedure to determine the scattering angle comprises the collection of exactly four hit locations in four detector layers among which two top detector layers are utilized to construct the first vector, whereas the second vector is built by using two bottom detector layers. Although this procedure acts to classify the target volumes in the tomographic systems based on the muon scattering, the scattering angle obtained through the usual methodology founded on four detector layers is dubious for not yielding any information about the position of target volume. Nonetheless, the same set of four detector layers also imparts the possibility of splitting the scattering angle into two separate angles by creating a triangular correlation in such a way that the scattering angle is referred to an exterior angle, whereas the separate angles are considered the interior opposite angles that are not neighboring this exterior angle. In this study, we first show that a combination of three detector layers out of four fulfills the calculation of the interior opposite angles. Then, by employing the GEANT4 simulations over our tomographic configuration composed of three plastic scintillators in either section, we demonstrate that the interior opposite angles differ towards the vertical spatial variation, while the exterior angle approximately remains constant, thereby implying a beneficial feature to be used for the image reconstruction purposes.

Triangular correlation



* The conventional scattering angle denoted by θ that also refers to the exterior angle is commonly defined as written in

$$\theta = \mathrm{BC}\angle\mathrm{DE} = \mathrm{BC}\angle\mathrm{CD} + \mathrm{CD}\angle\mathrm{DE} = \arccos\left(\frac{\overrightarrow{BC}\cdot\overrightarrow{DE}}{|\mathrm{BC}|\,|\mathrm{DE}|}\right) \qquad (1)$$

* The same set of four hit locations also gives access to compute two opposite interior angles as expressed in

$$\mathrm{BC}\angle\mathrm{CD} = \arccos\left(\frac{\overrightarrow{BC}\cdot\overrightarrow{CD}}{|\mathrm{BC}|\,|\mathrm{CD}|}\right) \qquad (2)$$

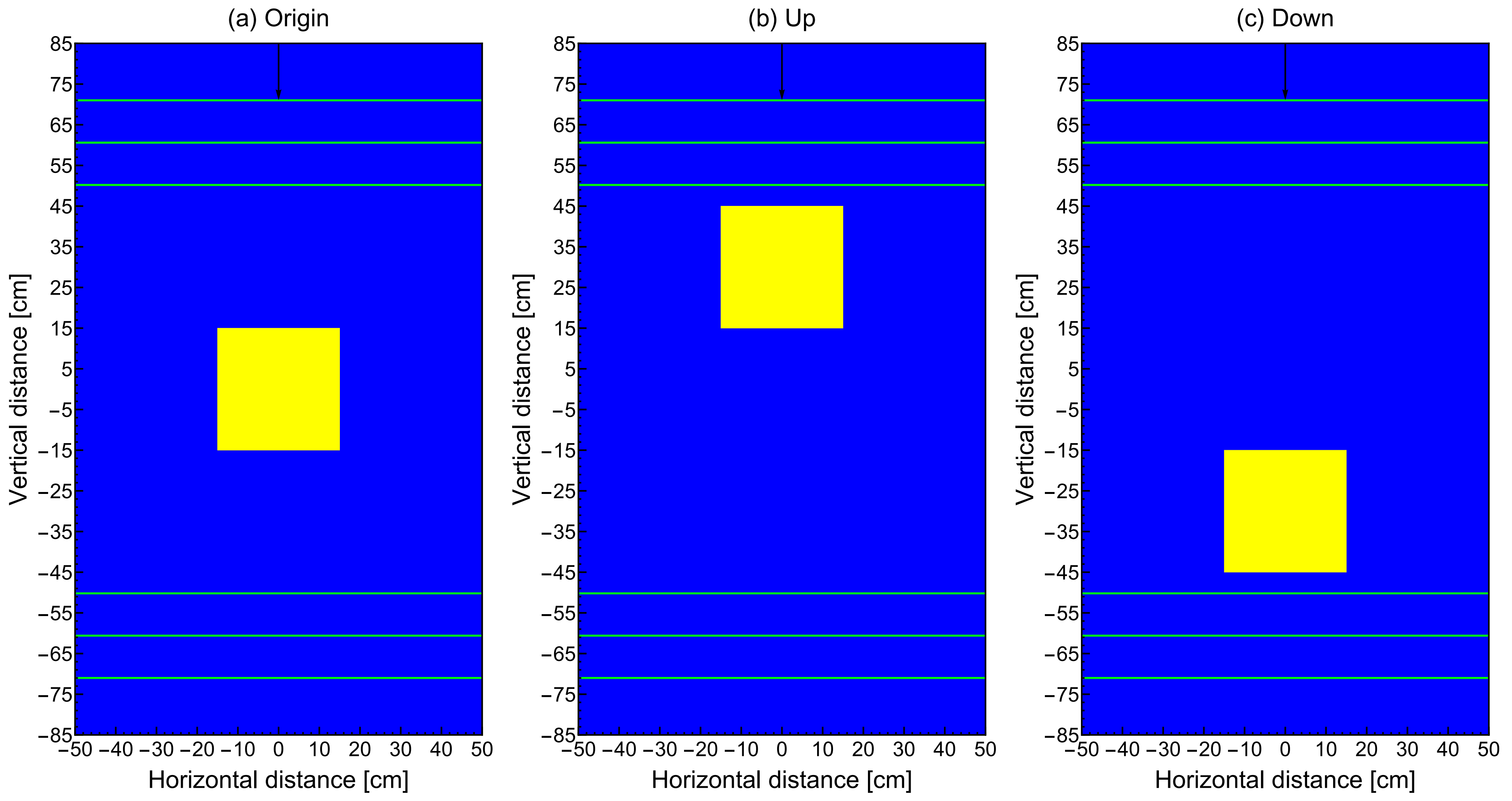
and

$$\mathrm{CD}\angle\mathrm{DE} = \arccos\left(\frac{\overrightarrow{CD}\cdot\overrightarrow{DE}}{|\mathrm{CD}|\,|\mathrm{DE}|}\right) \qquad (3)$$

* The average angular deviation of any combination, i.e $\overline{x\angle y}$, at a given energy value is determined by averaging over N number of the non-absorbed/non-decayed muons as defined in

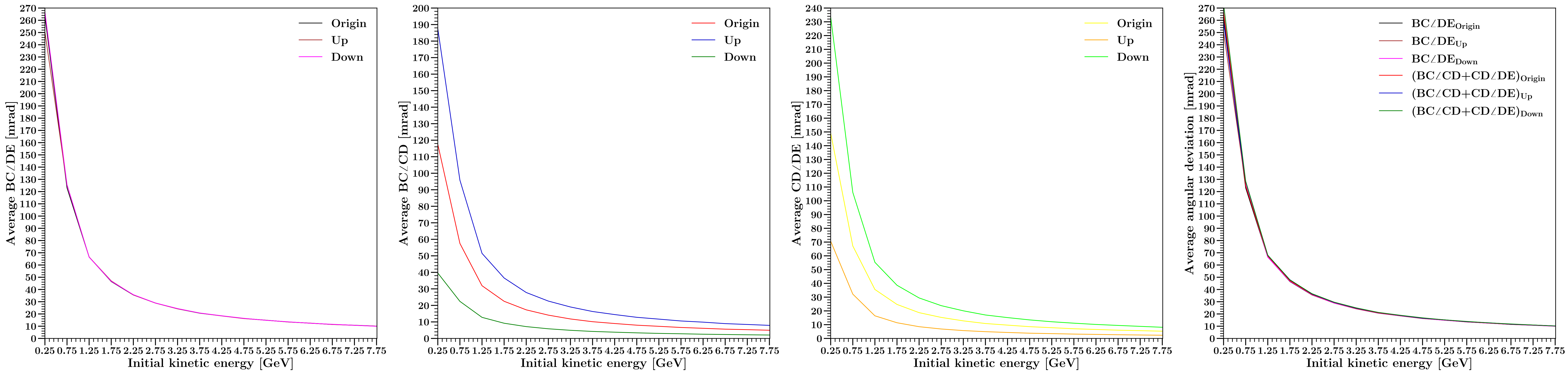
$$\overline{x\angle y} = \frac{1}{N}\sum_{i=1}^{N}(x\angle y)_i \qquad (4)$$

Simulation scheme for position sensitivity



Particle	μ^-
Beam direction	Vertical
Momentum direction	(0, -1, 0)
Source geometry	Planar
Initial position (cm)	([-0.5, 0.5], 85, [-0.5, 0.5])
Number of particles	10^5
Energy distribution	Uniform
Energy interval (GeV)	[0, 8]
Bin step length (GeV)	0.5
Energy cut-off (GeV)	0.1
Target material	Stainless steel
Target geometry	Cube
Target size (cm)	30
Material database	G4/NIST
Reference physics list	FTFP_BERT

Simulation outcomes



Concluding remarks

In the present study, we explore the triangular correlation of angular deviation by means of our GEANT4 simulations. We explicitly observe that the exterior angle remains constant towards the position change of target material, whereas the opposite interior angles exhibit differences due to this spatial variation. Finally, we analytically validate our post-processing framework by fulfilling the equality between the exterior angle and the superposition of two opposite interior angles.