Non-Radial Neutrino Emission upon Black Hole Formation in Core-Collapse Supernovae

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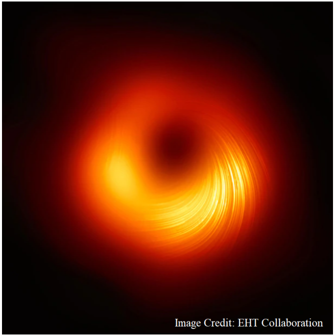
Black Hole Formation in Core-Collapse Supernovae

- Expected to happen when progenitor mass $\gtrsim 20M_{\odot}$
- Shock wave revival fails during accretion phase, and matter starts to fall back exceeding the neutron star mass limit
- When $M > 40M_{\odot}$, the core bounce might fail to form a shock and the star might collapse to a BH directly
- The BH formation is believed to lead a sharp cut-off in luminosity



Neutrino Luminosity for a Collapsing Shell

- *Podurets (1965)* discussed the case of a free falling mass shell emitting photons radially
- The resulting luminosity is then characterised by a sharp exponential $exp(-\frac{t}{3\sqrt{3}M})$, in the case of $M = 2.5M_{\odot}$ the decay time will be 0.06ms
- This result has been widely applied in later studies on neutrinos, such as *Beacom et al. (2001)*
- The need for non-radial ray-tracing, which is essential for full general relativistic treatment, has been pointed out in *Baumgarte et al. (1996)*

Time Delay caused by Non-Radial Null Geodesics

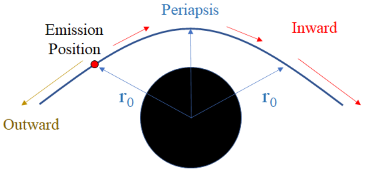
- Many work has been done in evaluating the time delays of non-radial geodesics for photons from a collapsing stellar surface, e.g. *Ames et al. (1968), Lake et al. (1979)*...etc.
- Based on those results, we investigate the neutrino time delays during the BH cut-off
- We will discuss two cases: Schwarzschild metric and Kerr metric (with planar emission, disc model)

Emission from a Collapsing Surface

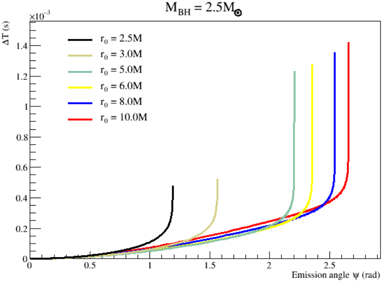
- We are basing our calculations on the case of a collapsing surface opaque to neutrinos (neutrinosphere), analogous to the case of a collapsing stellar surface
- In this scenario, the luminosity is effectively emitted from a collapsing shell
- We've carried out calculations at different radii in case those last neutrinos come from radii different from those we (or various models) expect

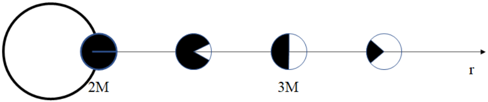
Evaluating Time Delays

- We denote the travel time of the null ray as $T(r_0, r_E, b)$, where r_0 is the emission radial position, r_E is the distance to Earth and b is the impact parameter which depends on the emission angle relative to the radial direction ($b = 0$ corresponds to radial)
- Depending on the travelling direction the ΔT expression can be different:
  - Outward Travelling: $\Delta T = T(r_0, r_E, b) - T(r_0, r_E, 0)$
  - Inward Travelling: an extra contribution from a Shapiro-like delay as they pass the periapsis, $\Delta T = 2T(r_p, r_0, b) + T(r_0, r_E, b) - T(r_0, r_E, 0)$, where r_p is the periapsis position

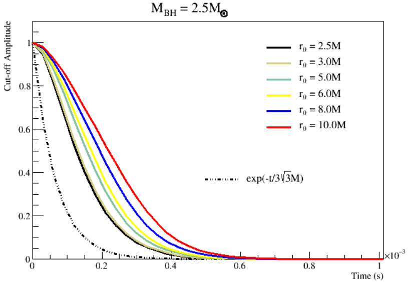


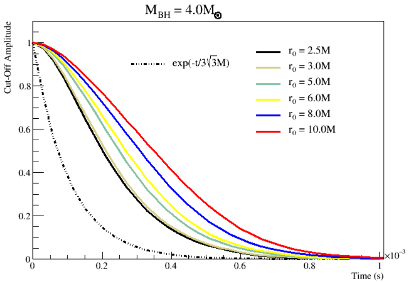
- The emission angle as observed in the local inertial frame: $\psi = \arcsin\left(\frac{b}{r_0}\left(1-\frac{2M}{r_0}\right)^{1/2}\right)$ where $b \equiv \frac{L}{E}$ and r_0 is the emission position
- The travel time in this case is: $T(r_0, r_E, b) = \int_{r_0}^{r_E} \frac{r^{5/2}\,\mathrm{d}r}{(r-2M)\sqrt{r^3 - b^2(r-2M)}}$
- A major fraction of the geodesics lead to time delays with fractions of a millisecond, and as path approaches the photon orbit the delay increases until they don't leave at all





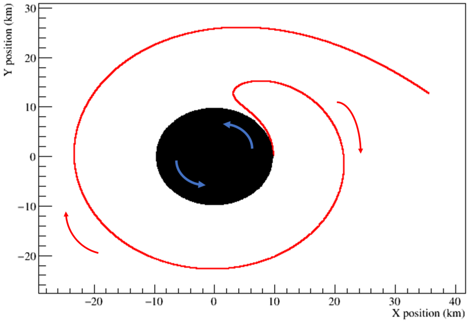
- The cutoff factor for a given geodesic is $exp(-\frac{t-\Delta T}{3\sqrt{3}M})$
- To add up the contributions we integrate through the escape cone surface S_E (the collection of emission directions which lead to an eventual escape to infinity): $\frac{\int_{S_E} e^{-\frac{t-\Delta T}{3\sqrt{3}M}} 2\pi \sin\psi d\psi}{\int_{S_E} 2\pi \sin\psi d\psi}$
  where ψ is the emission angle relative to the radial direction
- The cut-off can be extended by $0.1 \sim 0.4$ms





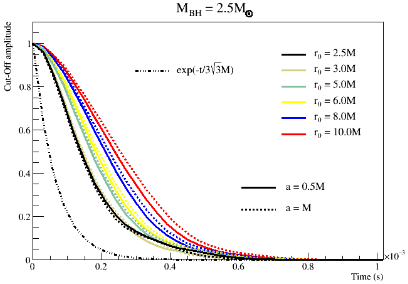
Formulation - Kerr Case

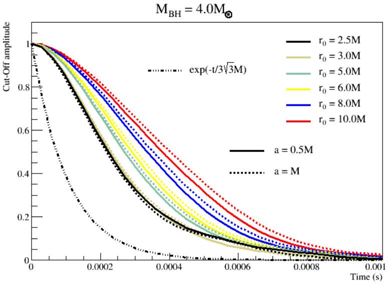
- The emission direction is now determined by two angles as the geodesics are no longer planar
- For the Kerr Case, we consider a disc model and the escape conditions are worked out in *Igata et al. (2021)*



Result - Kerr Case

- The cut-off profile integral has to be modified slightly:
  $$\frac{\int_{S_E} e^{-\frac{t-\Delta T}{3\sqrt{3}M}} \sin\psi \mathrm{d}\psi \mathrm{d}\eta}{\int_{S_E} \sin\psi \mathrm{d}\psi \mathrm{d}\eta}$$
  where ψ is the polar emission angle and η is the azimuth emission angle
- In this case, the cut-off profile can be further extended by roughly 10%



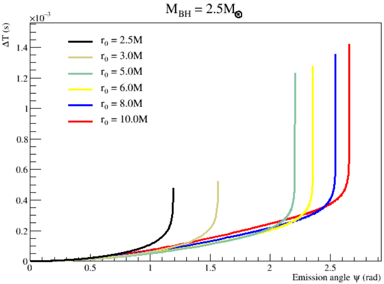


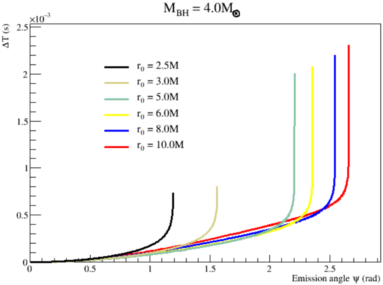
Summary

- In this study, we calculated the effects of the non-radial contributions on the BH cut-off with a toy model
- The results suggests that instead of the usual 0.1ms decay time, the decay time can potentially be extended by several 0.1ms depending on mass and position
- Rotation can enhance the effect slightly
- If an actual cut-off is observed, one could potentially gain information regarding the state of the PNS upon transition to a BH
- Should include the effect of the speed of the shell (thanks to Evan, Shuai and Samuel!)
- More can be done with detailed simulations

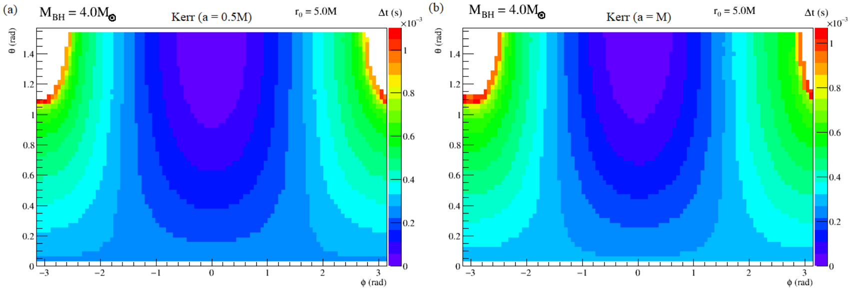
Backup

Time Delays - Schwarzschild Case





Time Delays - Kerr Case



Metrics

- Schwarzschild metric:
$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$
- Kerr metric: $\mathrm{d}s^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)\mathrm{d}t^2 - \frac{4aMr\sin^2\theta}{\Sigma}\mathrm{d}t\mathrm{d}\phi + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\mathrm{d}\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\Sigma}\right)\sin^2\theta\mathrm{d}\phi^2$ where $a \equiv J/M$,
$\Delta \equiv r^2 + a^2 - 2Mr$ and $\Sigma \equiv r^2 + a^2\cos^2\theta$