RF Transport

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HELMHOLTZ | ASSOCIATION

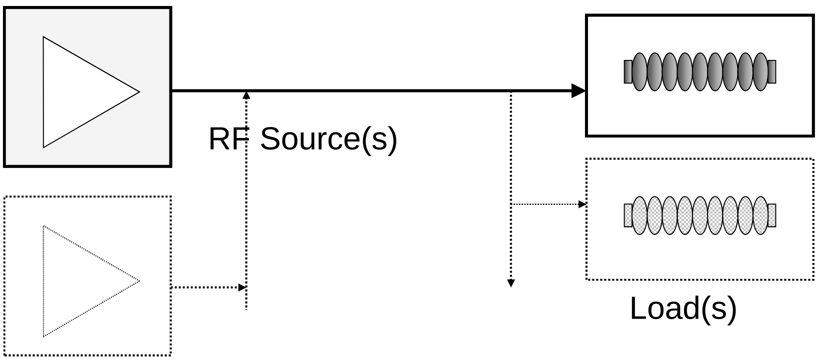
Overview

- Introduction
- Electromagnetic Waves in Waveguides
- TE_{10}-Mode
- Waveguide Elements

- Waveguide Distributions
- Limitations, Problems and Countermeasures

Introduction

RF Transport



•Task: Transmission of RF power of typical several kW up to several MW at frequencies from the MHz to GHz range. The RF power generated by an RF generator or a number of RF generators must be combined, transported and distributed to a load or cavity or a number of loads or cavities.

•Requirements: low loss, high efficieny, low reflections, high reliability, high stability, adjustment of phase and amplitude, ….

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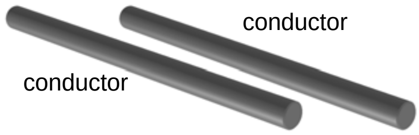
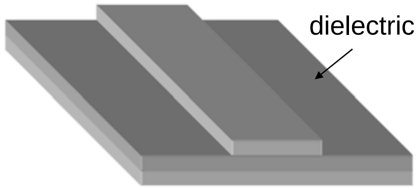
Transmission Lines for RF Transport

- Two-wire lines (Lecher Leitung)
  - often used for indoor antenna (e.g. radio or TV)
  - problem: radiation to the environment, can not be used for high power transportation

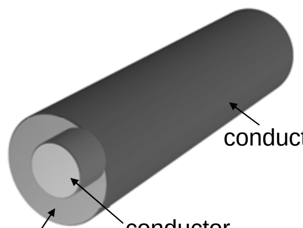


- Strip-lines
  - often used for microwave integrated circuits
  - problem: radiation to the environment and limited power capability, can not be used for high power transportation

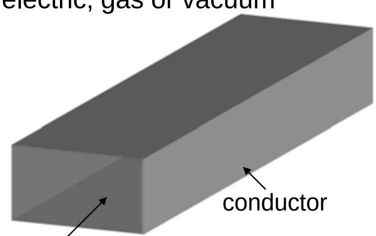


Transmission Lines for RF Transport (2)

- Coaxial transmission lines
  - often used for power RF transmission and connection of RF components
  - problem: high loss above a certain frequency due to heating of inner conductor and dielectric material and limited power capability at higher frequencies due to small dimensions



- Waveguides (rectangular,cylindrical or elliptical)
  - often used for high power RF transmission (mostly rectangular)
  - problem: waveguide plumbing, rigidity



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Electromagnetic Waves in Waveguides

Strategy for Calculation of Fields

- start with Maxwell equation
- derive wave equation
- Ansatz: separation into transversal and longitudinal field components
- wave equation for transversal and longitudinal components
- rewrite Maxwell equation in transversal and longitudinal components
- solve eigenvalue problem for three cases
  TEM (E_z=H_z =0), TE (E_z=0, $H_z \neq 0$), TM (H_z=0, $E_z \neq 0$)
- derive properties of the the solutions

Maxwell Equations

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D} + \mathbf{j} \qquad \text{Amperes law}$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \qquad \text{Faradays law}$$

$$\nabla \cdot \mathbf{D} = \rho \qquad \text{Gauss law}$$

$$\nabla \cdot \mathbf{B} = 0$$

$\mathbf{B}$ magnetic field, $\mathbf{H}$ magnetic intensity, $\mathbf{D}$ electric displacement,
$\mathbf{E}$ electric field

with $\rho = 0$ (no external charges), $\mathbf{j} = 0$ (no external current),
$\mathbf{B} = \mu\mathbf{H}$ (μ permeability) and $\mathbf{D} = \varepsilon\mathbf{E}$ (ε permittivity) one gets

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial}{\partial t}\mathbf{E}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}\mathbf{H}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

Wave Equation

The wave equation can be derived from the Maxwell equations.

start with $\nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}\mathbf{H}$ and apply curl

$$\Rightarrow \quad \nabla \times \nabla \times \mathbf{E} = -\mu \nabla \times \frac{\partial}{\partial t}\mathbf{H}$$

use of $\nabla \times \mathbf{H} = \varepsilon \frac{\partial}{\partial t}\mathbf{E}$ and $\nabla \times \nabla \times = \nabla(\nabla \cdot) - \nabla^2$

$$\Rightarrow \quad \nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E} = -\mu\varepsilon \frac{\partial^2}{\partial t^2}\mathbf{E}$$

use $\nabla \cdot \mathbf{E} = 0$

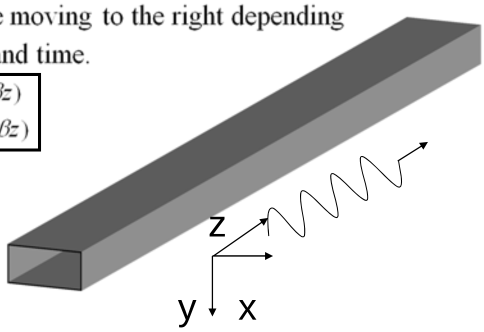
$$\Rightarrow \quad \nabla^2\mathbf{E} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\mathbf{E} = 0$$ wave equation for $\mathbf{E}$

in the same way for H

$$\nabla^2\mathbf{H} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\mathbf{H} = 0$$ wave equation for $\mathbf{H}$

Ansatz for Wave Equation

Separation of fields in a function depending on transversal coordinates only and a wave moving to the right depending on longitudinal coordinate and time.



$$\mathbf{E}(x,y,z,t) = \mathbf{E}(x,y)\exp i(\omega t - \beta z)$$
$$\mathbf{H}(x,y,z,t) = \mathbf{H}(x,y)\exp i(\omega t - \beta z)$$

results in:

$$\nabla^2 \mathbf{E} + \mu\varepsilon\omega^2 \mathbf{E} = 0$$

with: $k^2 = \mu\varepsilon\omega^2$ and $\gamma = i\beta$

$$\nabla_t^2 \mathbf{E} + (\gamma^2 + k^2)\mathbf{E} = 0$$

Note:

∇_t operates on transversal coordinates only, e.g. x, y (or r, Φ).

$\mathbf{E}(x,y), \mathbf{H}(x,y)$ are still vectors with longitudinal and transversal components but they depend only on transversal coordinates.

Derivation of Maxwell Equation for transversal and longitudinal Components

With $\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}$

$$\nabla \times \mathbf{E} = (\nabla_t - i\beta\mathbf{e}_z) \times (\mathbf{E}_t + \mathbf{E}_z) = -i\omega\mu(\mathbf{H}_t + \mathbf{H}_z)$$

$$\nabla_t \times \mathbf{E}_t - i\beta\mathbf{e}_z \times \mathbf{E}_t + \nabla_t \times \mathbf{E}_z - i\beta\mathbf{e}_z \times \mathbf{E}_z = -i\omega\mu\mathbf{H}_t - i\omega\mu\mathbf{H}_z$$

$$\nabla_t \times \mathbf{E}_z = \nabla_t \times \mathbf{e}_z E_z = -\mathbf{e}_z \times \nabla_t E_z$$

Now one can see

$\nabla_t \times \mathbf{E}_t \parallel \mathbf{e}_z$

$\mathbf{e}_z \times \mathbf{E}_t \perp \mathbf{e}_z$

$\nabla_t \times \mathbf{E}_z \perp \mathbf{e}_z$

$\mathbf{e}_z \times \mathbf{E}_z = 0$

Separation of longitudinal and transversal components

$$\nabla_t \times \mathbf{E}_t = -i\omega\mu\mathbf{H}_z$$

$$-i\beta\mathbf{e}_z \times \mathbf{E}_t + \nabla_t \times \mathbf{E}_z =$$

$$-i\beta\mathbf{e}_z \times \mathbf{E}_t - \mathbf{e}_z \times \nabla_t E_z = -i\omega\mu\mathbf{H}_t$$

Derivation Maxwell Equation for transversal and longitudinal Components (2)

With $\nabla \times \mathbf{H} = i\omega\varepsilon\mathbf{E}$ in the same manner

$$\nabla_t \times \mathbf{H}_t = i\omega\varepsilon\mathbf{E}_z$$
$$i\beta\mathbf{e}_z \times \mathbf{H}_t + \mathbf{e}_z \times \nabla_t H_z = -i\omega\varepsilon\mathbf{E}_t$$

With $\nabla \cdot \mathbf{H} = 0$

$$\nabla_t \cdot \mathbf{H}_t = i\beta H_z$$

With $\nabla \cdot \mathbf{E} = 0$

$$\nabla_t \cdot \mathbf{E}_t = i\beta E_z$$

Equations of transversal and longitudinal Components as Function of transversal Coordinates

$$\nabla_t^2\mathbf{E} + (\gamma^2 + k^2)\mathbf{E} = 0$$
$$\nabla_t^2\mathbf{H} + (\gamma^2 + k^2)\mathbf{H} = 0$$

Wave equation

$$\nabla_t \times \mathbf{E}_t = -i\omega\mu\mathbf{H}_z$$
$$-i\beta\mathbf{e}_z \times \mathbf{E}_t - \mathbf{e}_z \times \nabla_t E_z = -i\omega\mu\mathbf{H}_t$$
$$\nabla_t \times \mathbf{H}_t = i\omega\varepsilon\mathbf{E}_z$$
$$i\beta\mathbf{e}_z \times \mathbf{H}_t + \mathbf{e}_z \times \nabla_t H_z = -i\omega\varepsilon\mathbf{E}_t$$
$$\nabla_t \cdot \mathbf{H}_t = i\beta H_z$$
$$\nabla_t \cdot \mathbf{E}_t = i\beta E_z$$

Maxwell equation for transversal and longitudinal components

TEM-, TE-, TM- Waves

1. $E_z = H_z = 0$ TEM
2. $E_z = 0$ and $H_z \neq 0$ TE (or H-wave)
3. $H_z = 0$ and $E_z \neq 0$ TM (or E-wave)

On the next slides we will try to find solutions for TE-waves. The treatment for TM- modes is similar. For TEM modes the treatment is even easier, but TEM-modes do not exist in hollow transmission lines, because transversal E components require longitudinal H components and transversal H components require longitudinal E components. These are 0 in TEM fields. TEM-modes exist in coaxial lines since on the inner conductor we might have $j \neq 0$.

Derivation of TE-Wave Equations

With wave equation

$$\nabla_t^2 \mathbf{H} + (\gamma^2 + k^2)\mathbf{H} = 0 \text{ and } \gamma^2 + k^2 = k_c^2$$

$$\nabla_t^2 \mathbf{H} + k_c^2 \mathbf{H} = 0$$

or for transversal and longitudinal components

$$\nabla_t^2 \mathbf{H}_t + k_c^2 \mathbf{H}_t = 0$$

$$\nabla_t^2 H_z + k_c^2 H_z = 0$$

With $E_z = 0$ in Maxwell equations

$$\nabla_t \times \mathbf{E}_t = -i\omega\mu \mathbf{H}_z$$

$$-i\beta \mathbf{e}_z \times \mathbf{E}_t = -i\omega\mu \mathbf{H}_t$$

$$\nabla_t \times \mathbf{H}_t = 0$$

$$i\beta \mathbf{e}_z \times \mathbf{H}_t + \mathbf{e}_z \times \nabla_t H_z = -i\omega\varepsilon \mathbf{E}_t$$

$$\nabla_t \cdot \mathbf{H}_t = i\beta H_z$$

$$\nabla_t \cdot \mathbf{E}_t = 0$$

Derivation of TE-Wave Equations(2)

Application of curl on $\nabla_t \times \mathbf{H}_t = 0$ gives $\nabla_t \times \nabla_t \times \mathbf{H}_t = 0$

With $\nabla_t \times \nabla_t \times = \nabla_t(\nabla_t \cdot) - \nabla_t^2$ and $\nabla_t^2 \mathbf{H}_t + k_c^2 \mathbf{H}_t = 0$

$\Rightarrow \nabla_t(\nabla_t \cdot \mathbf{H}_t) + k_c^2 \mathbf{H}_t = 0$

and with $\nabla_t \cdot \mathbf{H}_t = i\beta H_z$

$\nabla_t(i\beta H_z) + k_c^2 \mathbf{H}_t = 0$

$$\mathbf{H}_t = -\frac{i\beta}{k_c^2} \nabla_t H_z$$

Now

$$\mathbf{e}_z \times (\beta \mathbf{e}_z \times \mathbf{E}_t) = \beta(\mathbf{e}_z(\mathbf{e}_z \cdot \mathbf{E}_t) - \mathbf{E}_t(\mathbf{e}_z \cdot \mathbf{e}_z)) = -\beta \mathbf{E}_t = \omega\mu \mathbf{e}_z \times \mathbf{H}_t$$

$$\mathbf{E}_t = -\frac{\omega\mu}{\beta} \mathbf{e}_z \times \mathbf{H}_t = -\sqrt{\frac{\mu}{\varepsilon}} \frac{\sqrt{\mu\varepsilon}\,\omega}{\beta} \mathbf{e}_z \times \mathbf{H}_t = -Z_F \frac{k}{\beta} \mathbf{e}_z \times \mathbf{H}_t$$

$\mathbf{H}_t$ can be calculated from H_z and $\mathbf{E}_t$ from $\mathbf{H}_t$ and therefore from H_z too.

Derivation of TE-Wave Equations(3) Impedance of a TE-Wave

$Z_F = \sqrt{\frac{\mu}{\varepsilon}}$ impedance of an electromagnetic wave in free space

$Z_{TE} = Z_F \frac{k}{\beta}$ impedance of a TE - wave (H - wave)

$Z_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$ for TE - wave moving to the right

TE-Wave Equation in rectangular Waveguides

$$\nabla_t^2 H_z + k_c^2 H_z = 0$$

$$\mathbf{H}_t = -\frac{i\beta}{k_c^2}\nabla_t H_z$$

$$\mathbf{E}_t = -Z_{TE}\mathbf{e}_z \times \mathbf{H}_t$$

TE wave equation

$$\partial^2/\partial x^2 \, H_z + \partial^2/\partial y^2 \, H_z + k_c^2 H_z = 0 \quad \text{Eigenvalue problem}$$

$$H_x = -\frac{i\beta}{k_c^2}\partial/\partial x \; H_z$$

$$H_y = -\frac{i\beta}{k_c^2}\partial/\partial y \; H_z$$

$$E_x = -Z_{TE}\frac{i\beta}{k_c^2}\partial/\partial y \; H_z$$

$$E_y = Z_{TE}\frac{i\beta}{k_c^2}\partial/\partial x \; H_z$$

TE wave equation
written in components

Solution of TE- Wave Equation

$$\partial^2/\partial x^2 \, H_z + \partial^2/\partial y^2 \, H_z + k_c^2 H_z = 0$$

Ansatz: Separation into x and y coordinates.

$H_z = f \cdot g$ with $f = f(x)$ and $g = g(y)$

$$\frac{1}{f} d^2/dx^2 \, f + \frac{1}{g} d^2/dy^2 \, g + k_c^2 = 0$$

$$\frac{1}{f} d^2/dx^2 \, f = -k_x^2$$

$$\frac{1}{g} d^2/dy^2 \, g = -k_y^2$$

$$k_x^2 + k_y^2 = k_c^2$$

Solution of TE-Wave Equation(2)

$\Rightarrow$

$f = A_1 \cos k_x x + A_2 \sin k_x x$

$g = B_1 \cos k_y y + B_2 \sin k_y y$

Boundary conditions :

Normal component of H : $H_\perp = 0$ on surface

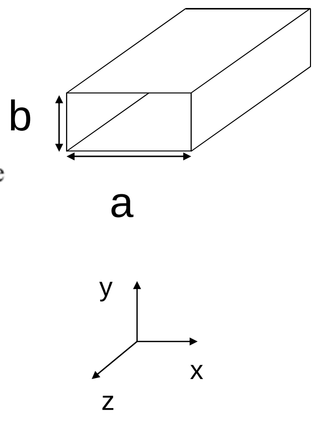
$H_x = 0$ for $x = 0$ and $x = a$

$H_y = 0$ for $y = 0$ and $y = b$

$\Rightarrow$

$\partial / \partial x \, H_z = 0$ for $x = 0$ and $x = a$

$\partial / \partial y \, H_z = 0$ for $y = 0$ and $y = b$



Solution of TE-Wave Equation(3)

$-A_1 k_x \sin k_x x + A_2 k_x \cos k_x x = 0$ for $x = 0$ and $x = a$

$\Rightarrow$

$A_2 = 0$ from $x = 0$

$k_x = \frac{n\pi}{a}$ from $x = a$

$n = 0,1,2,\ldots.$

in the same manner

$B_2 = 0$ from $y = 0$

$k_y = \frac{m\pi}{b}$ from $y = b$

$m = 0,1,2,\ldots.$

$H_z(x,y) = H_{nm} \cos \frac{n\pi x}{a} \cos \frac{m\pi y}{b}$ with n = 0,1,2.. m = 0,1,2,.. but n = m ≠ 0

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Solution of TE-Wave Equation(3)

$$k_{cnm} = \left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^{1/2} \text{ cut off wave number}$$

$$k_c^2 = \gamma^2 + k^2$$

$$k_{cnm}^2 = \gamma_{nm}^2 + k^2$$

with

$$\gamma = \mathrm{i}\beta$$

$$\exp i(\omega t - \beta z)$$

$$k_{cnm}^2 = k^2 - \beta_{nm}^2$$

$$\beta_{nm}^2 = k^2 - k_{cnm}^2$$

for $k > k_{cnm}$ β_{nm} is real $\Rightarrow$ propagation

for $k < k_{cnm}$ β_{nm} is imaginary $\Rightarrow$ exponential decay

TE_{nm} -Fields

$$E_z(x,y,z,t) = 0$$

$$E_x(x,y,z,t) = -\frac{\beta_{nm} m\pi}{bk_{cnm}^2} Z_{TE,nm} H_{nm} \cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sin(\omega t - \beta_{nm} z)$$

$$E_y(x,y,z,t) = -\frac{\beta_{nm} n\pi}{ak_{cnm}^2} Z_{TE,nm} H_{nm} \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\sin(\omega t - \beta_{nm} z)$$

$$H_z(x,y,z,t) = H_{nm} \cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\cos(\omega t - \beta_{nm} z)$$

$$H_x(x,y,z,t) = -\frac{\beta_{nm} n\pi}{ak_{cnm}^2} H_{nm} \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\sin(\omega t - \beta_{nm} z)$$

$$H_y(x,y,z,t) = -\frac{\beta_{nm} m\pi}{bk_{cnm}^2} H_{nm} \cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sin(\omega t - \beta_{nm} z)$$

Cut Off Frequency and Wavelength

for $\beta_{nm} = 0$ is $k = k_{cnm}$

since $k_{cnm} = \frac{2\pi}{\lambda_{cnm}} = \frac{2\pi}{c} f_{cnm}$

$$f_{cnm} = \frac{c}{2\pi}\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^{1/2} \quad \text{cut off frequency}$$

$$\lambda_{cnm} = \frac{2}{\left[\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]^{1/2}} \quad \text{cut off wavelength}$$

Waves with frequency lower than the cut off frequency ($f<f_{cnm}$) or wavelength longer than the cut off wavelength ($\lambda>\lambda_{cnm}$) can not propagte in nm-mode.

Guide Wavelength

$\beta_{nm}^2 = k^2 - k_{cnm}^2$ propagation constant of mode nm

$$\lambda_{gnm} = \frac{2\pi}{\beta_{nm}} = \frac{2\pi}{\sqrt{k^2 - k_{cnm}^2}} = \frac{1}{\sqrt{\frac{1}{\lambda^2} - \frac{1}{\lambda_{cnm}^2}}} = \frac{\lambda}{\sqrt{1 - \frac{\lambda^2}{\lambda_{cnm}^2}}}$$

λ_{gnm} is called guide wavelength of mode nm.

It gives the distance after which the mode pattern repeats in the waveguide.

$\lambda_{gnm} > \lambda$

TM-Waves

$H_z = 0$ and $E_z \neq 0$

in the same manner as for TE – Waves one calculates

$$E_z = E_{nm} \sin\frac{n\pi x}{a} \sin\frac{n\pi y}{b} \quad \text{with n} = 1,2,\ldots \text{and m} = 1,2,\ldots$$

and the other field components. One also obtains :

$$k_{cnm} = \left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^{1/2} \quad \text{cut off wave number}$$

$$f_{cnm} = \frac{c}{2\pi}\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^{1/2} \quad \text{cut off frequency}$$

$$\lambda_{cnm} = \frac{2}{\left[\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]^{1/2}} \quad \text{cut off wavelength}$$

$$Z_{\text{TMnm}} = \frac{\beta_{nm}}{k} Z_0 \quad \text{impedance of the TM wave}$$

TM_{nm} -Fields

$$E_z(x,y,z,t) = E_{nm} \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\cos(\omega t - \beta_{nm} z)$$

$$E_x(x,y,z,t) = \frac{\beta_{nm} n\pi}{a k_{cnm}^2} E_{nm} \cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sin(\omega t - \beta_{nm} z)$$

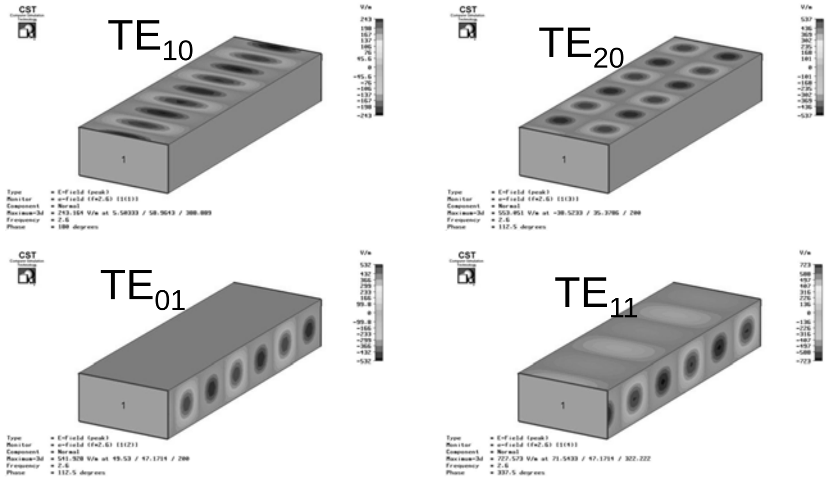
$$E_y(x,y,z,t) = \frac{\beta_{nm} n\pi}{b k_{cnm}^2} E_{nm} \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\sin(\omega t - \beta_{nm} z)$$

$$H_z(x,y,z,t) = 0$$

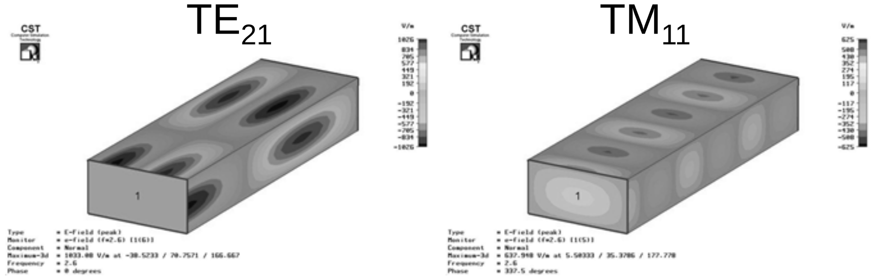
$$H_x(x,y,z,t) = -\frac{\beta_{nm} n\pi}{b k_{cnm}^2} \frac{1}{Z_{TMnm}} E_{nm} \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\sin(\omega t - \beta_{nm} z)$$

$$H_y(x,y,z,t) = \frac{\beta_{nm} n\pi}{a k_{cnm}^2} \frac{1}{Z_{TMnm}} E_{nm} \cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sin(\omega t - \beta_{nm} z)$$

Rectangular Waveguide Mode Pattern



Rectangular Waveguide Mode Pattern(2)



TE- and TM- Mode Pattern

Mode pattern images can be found for instance in

•N. Marcuvitz, Waveguide Handbook, MIT Radiation Laboratory Series, Vol. 10, McGraw Hill 1951
•H. J. Reich, P. F. Ordung, H. L.Krauss, J. G. Skalnik, Microwave Theory and Techniques, D. van Nostrand 1953

and probably in other books, too.

TE_{10} (H_{10})-Mode

Waveguide Size and Modes

It is common to use $a = 2b$ in a retangular waveguide system.

$$\Rightarrow \lambda_{cnm} = \frac{2a}{\left(n^2 + 4m^2\right)^{1/2}}$$

TE – modes cutoff wavelengths

$\lambda_{c10} = 2a,\ \lambda_{c01} = a,\ \lambda_{c11} = 2a/\sqrt{5},\ \lambda_{c20} = a,$

$\lambda_{c02} = a/2,\ \lambda_{c21} = 2a/\sqrt{8}, ...$

$\Rightarrow$ for $\lambda_{c01} = a < \lambda < \lambda_{c10} = 2a$ only TE_{10} can propagate.

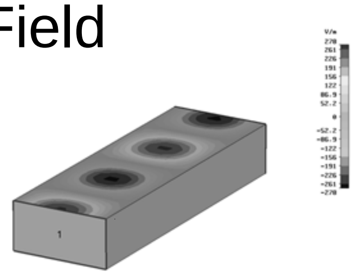
TM – modes cutoff wavelength

$\lambda_{c11} = 2a/\sqrt{5},\ \lambda_{c21} = 2a/\sqrt{8}, ...$

TM - mode with lowest frequency (longest wavegenth), which can propagate in a half height waveguide is TM_{11} with cut off wavelength $\lambda_{c11} = 2a/\sqrt{5}$

$\Rightarrow$ for $a < \lambda < 2a$ only the TE_{10} – mode can propagate

TE_{10} (H_{10})-Field



•The mode with lowest frequency propagating in the waveguide is the TE_{10} (H_{10}) mode. For $a<\lambda<2a$ only this mode can propagate.

$$E_z(x,y,z,t) = 0$$

$$E_x(x,y,z,t) = 0$$

$$E_y(x,y,z,t) = Z_{TE}\, H_{nm} \frac{\beta}{k_c} \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

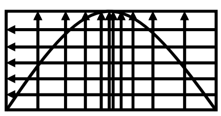
$$H_z(x,y,z,t) = H_{nm} \cos\left(\frac{\pi x}{a}\right) \cos(\omega t - \beta z)$$

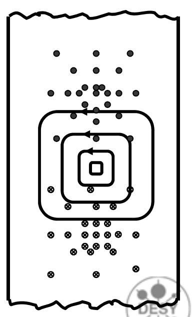
$$H_x(x,y,z,t) = H_{nm} \frac{\beta}{k_c} \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

$$H_y(x,y,z,t) = 0$$

E-Field

H-Field





Some TE_{10} Properties

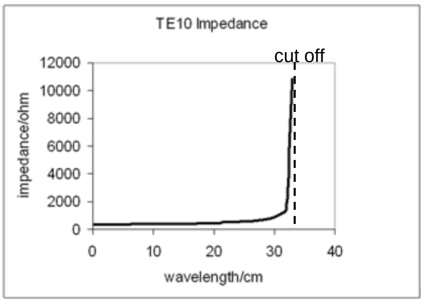
cut off frequency $f_c = \frac{c}{2a}$

cut off wavelength $\lambda_c = 2a$

guide wavelength $\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$

$\Rightarrow$ guide wavelength $\lambda_g > \lambda$ free space wavelength

example : WR650 at f = 1.3GHz
$\lambda = 23.1cm$ but $\lambda_g = 32.2cm$



impedance $Z = \frac{377\Omega}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$

Some TE_{10} Properties (2)

propagation constant $\beta_g = \frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2} = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{a}\right)^2}$

$\Rightarrow \beta_g$ depends on λ : dispersion

phase velocity $v_{ph} = \frac{\omega}{\beta_g} = \frac{\frac{2\pi c}{\lambda}}{\frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}} = \frac{c}{\sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{a}\right)^2}} > c$

but

group velocity $v_g = \left(\frac{d\beta_g}{d\omega}\right)^{-1} = \frac{c^2}{\omega}\beta_g = \frac{c^2}{v_{ph}} < c$

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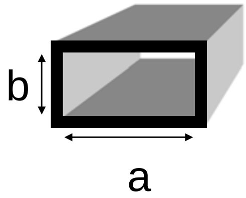
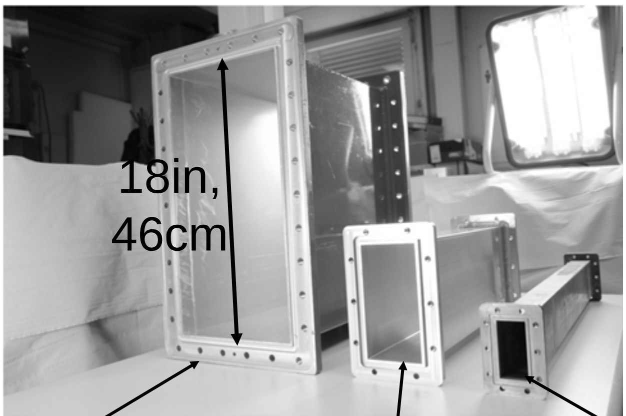
Some Standard Waveguide Sizes



Waveguide type	a (in)	b (in)	f_{c10} (GHz)	frequency range (GHz)
WR 2300	23.000	11.500	.256	.32–.49
WR 2100	21.000	10.500	.281	.35 –.53
WR 1800	18.000	9.000	.328	.41 –.62
WR975	9.750	4.875	.605	.75 – 1.12
WR770	7.700	3.850	.767	.96 – 1.45
WR650	6.500	3.250	.908	1.12 – 1.70
WR430	4.300	2.150	1.375	1.70 – 2.60
WR284	2.84	1.34	2.08	2.60 – 3.95
WR187	1.872	.872	3.16	3.95 – 5.85
WR137	1.372	.622	4.29	5.85 – 8.20
WR90	.900	.450	6.56	8.2 – 12.4
WR62	.622	.311	9.49	12.4 - 18

Some Waveguides of different Size



WR1800
e.g. for 500MHz
P-Band

WR650
e.g. for 1.3GHz
L-Band

WR284
e.g. for 3GHz
S-Band

Power in TE_{10}-Mode

Poynting Vector : $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Power transported in TE_{nm} $\quad P_{nm} = \frac{1}{T}\int_0^T\int_0^a\int_0^b \left(\mathbf{E}_{nm} \times \mathbf{H}_{nm}\right)\cdot \mathbf{e}_z\, dxdydt$

$\Rightarrow$

$$P_{nm} = \frac{\left|H_{nm}\right|^2 ab}{2\varepsilon_{0n}\varepsilon_{0m}}\left(\frac{\beta_{nm}}{k_{cnm}}\right)^2 Z_{TEnm}$$

with $\varepsilon_{0n} = 1$ for $n = 0$ and $\varepsilon_{0n} = 2$ for $n \neq 0$

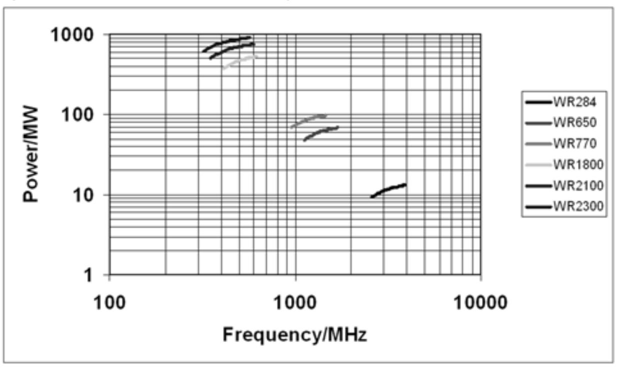
with $\varepsilon_{0m} = 1$ for $m = 0$ and $\varepsilon_{0m} = 2$ for $m \neq 0$

for TE_{10} one can caluclate :

$$P_{10} = 6.63\cdot 10^{-4} a[cm] b[cm]\left(\lambda / \lambda_g\right)\left(E[V/cm]\right)^2$$

Theoretical Power Limit in TE_{10}

The maximum power which can be transmitted theoretically in a waveguide of certain size a, b and frequency f is determined by the electrical breakdown limit E_{max}.
In air it is E_{max}=30kV/cm. From this one can find the maximum handling power in air filled waveguides.



Attenuation in TE_{10}

- The walls of the waveguides are not perfect conductors. They have finite conductivity σ, resulting in a skin depth of

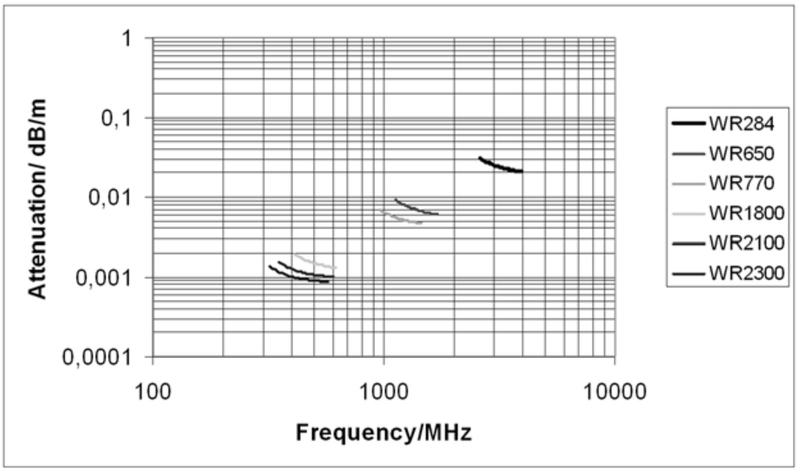
$$\delta_s = (\omega\mu\sigma/2)^{-1/2}$$

- Due to current in the walls of the waveguides loss appears and the waves are attenuated.
- The attenuation constant for the TE_{10} is:

$$\alpha[dB/m] = 0.2026 k_1 \frac{1}{b[cm]\sqrt{\lambda\,[cm]}} \frac{\frac{1}{2} + \frac{b}{a}\left(\frac{\lambda}{2a}\right)^2}{\left(1 - \left(\frac{\lambda}{2a}\right)^2\right)^{1/2}}$$

k_1= 1.00 Ag, 1.03 Cu, 1.17 Au, 1.37 Al, 2.2 Brass

Attenuation in Al-Waveguides in TE_{10}



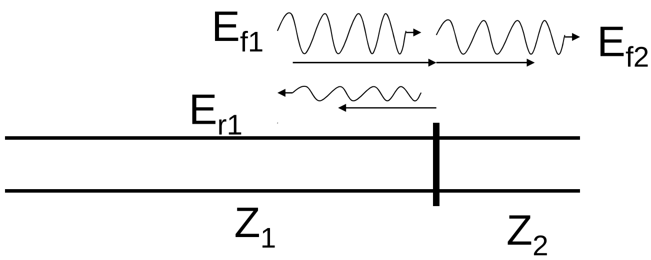
Reflection and Impedance

$\rho = \frac{E_r}{E_f}$ reflection coefficient,

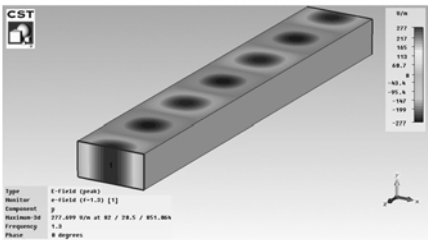
E_r and E_f amlitude of the reflected and incoming wave

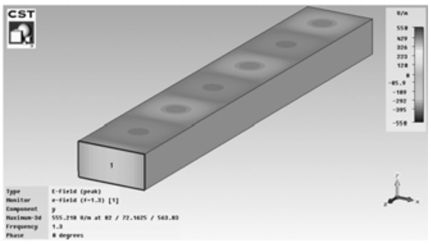
$$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$VSWR = \frac{|E_f| + |E_r|}{|E_f| - |E_r|} = \frac{1+\rho}{1-\rho}$$ voltage standing wave ratio



Travelling and Standing Wave





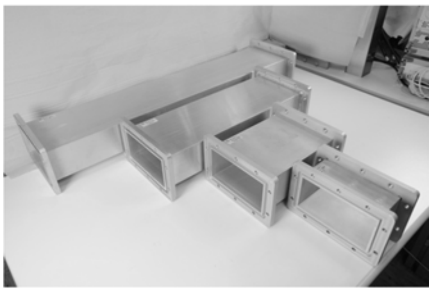
TE_{10} travelling wave

TE_{10} standing wave due to full reflection ρ=1.

The maximum electrical field strength in the standing wave is double the strength of the travelling wave. The same field strength can only be found in a travelling wave of 4-times power.

Waveguide Elements

Straight Waveguides



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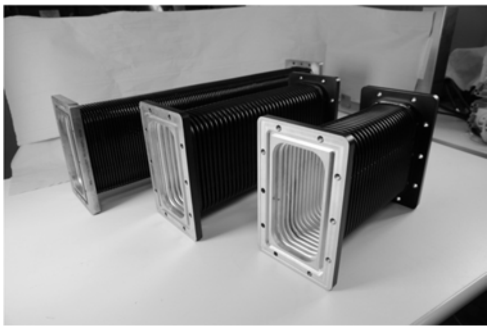
Bellows

Sometimes it is necessary to use flexible waveguides because a small misalignment exists or for compensation of displacements or expansion e.g. because of heating during operation. This can be done by bellows.

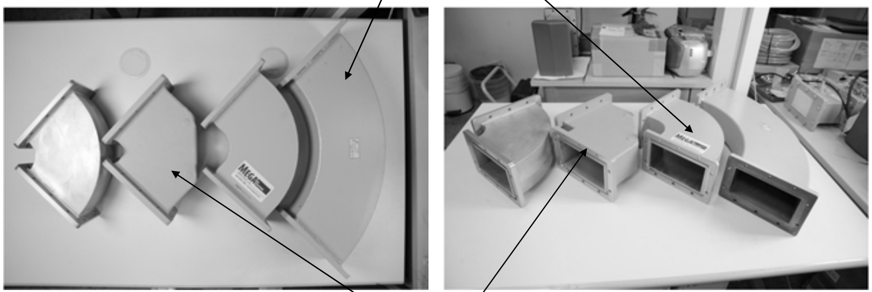


E- and H-Bends

E- and H-bends are used to change the direction of a waveguide. If the x-direction stays constant it is called E-bend (direction of E of the TE_{10} mode changes). If the y-direction stays constant it is called H-bend (direction of H changes). Both types come as mitred or swept bends. The VSWR of both types is typically 1.02.

H-Bends

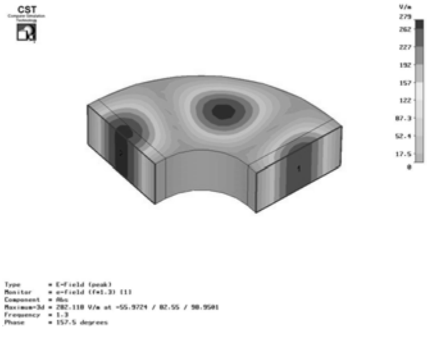
Swept bend



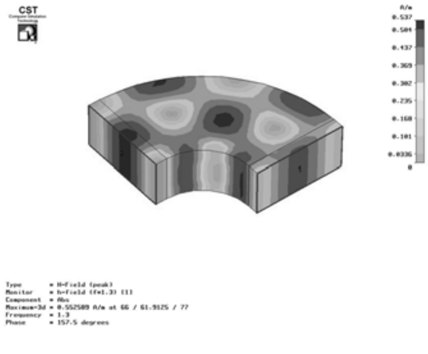
Mitred bend

E- and H- Field of TE_{10} in a H-Bend

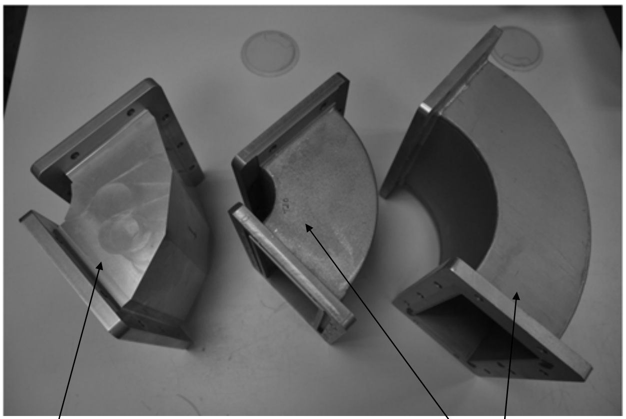
E-Field



H-Field



E-Bends

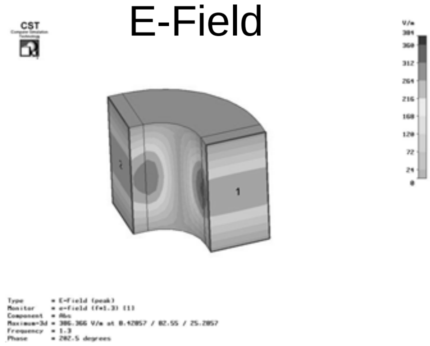


Swept-mitred bend

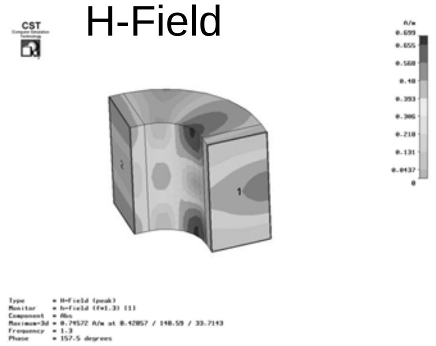
Swept bend

E- and H- Field of TE_{10} in a E-bend

E-Field

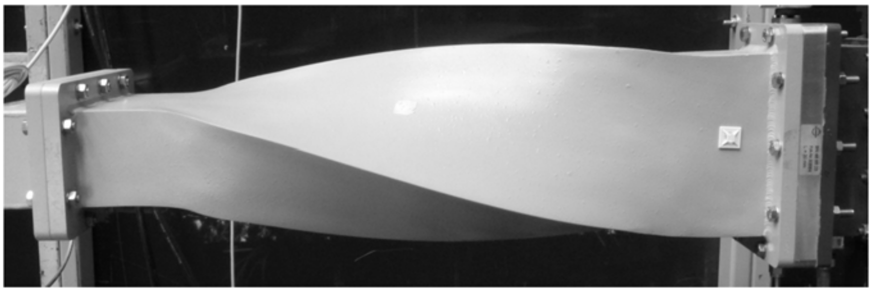


H-Field



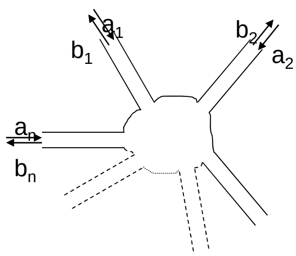
Twisted Waveguide

It is necessary to change the orientation of the of the electric field. This can be accomplished by twisted waveguides.



Combiner, Divider, Directional Coupler

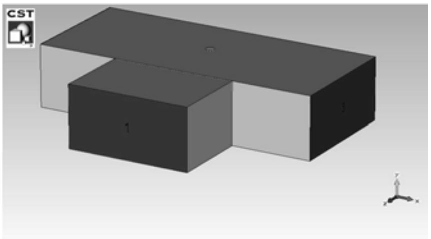
Combiners, dividers and directional couplers are waveguide elements which have several ports. They allow to combine, divide, split or couple RF power.

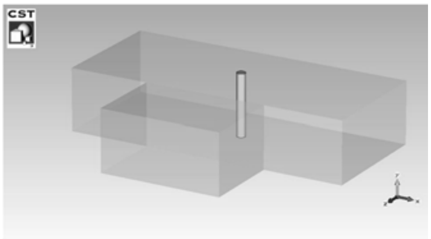


$$S = \begin{pmatrix} S_{11} & .. & .. & .. & S_{1n} \\ & .. & .. & .. & \\ & & .. & & \\ & .. & .. & .. & \\ S_{n1} & .. & .. & .. & S_{nn} \end{pmatrix}$$

Incoming electromagnetic waves with amplitude a_j entering at ports j are connected to the outgoing waves with amplitude b_i leaving at ports i by the S-matrix with matrix elements S_{ij}.
Due to time and space restrictions only some examples can be discussed on the next transparencies.

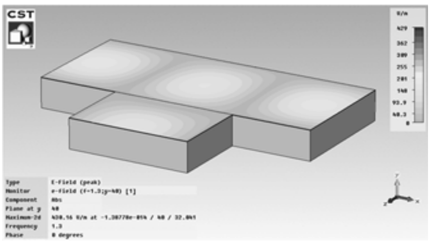
Shunt Tee

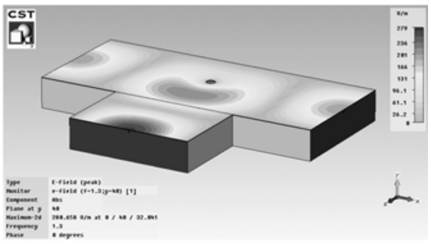




A 3-port shunt tee is a device which allows to divide or combine power. It is not matched. Therefore reflections occur. By using additional elements, e.g. inductive posts one can achieve matching to one port.

Shunt Tee as Divider

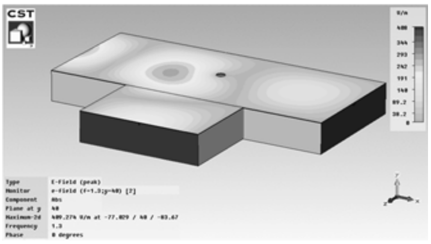


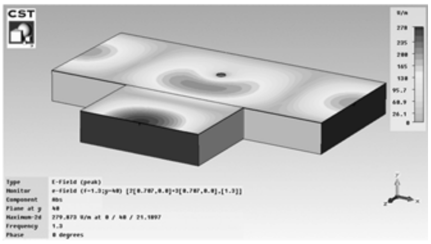


Shunt tee without matching post. Therefore reflections occur.

3-dB shunt tee with matching post. No reflections occur. The power is equally distributed.

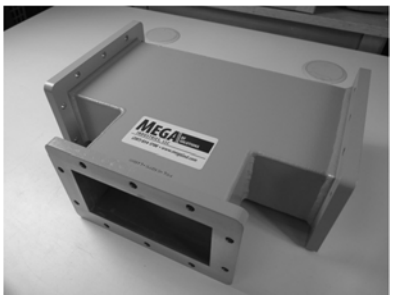
Shunt Tee as Combiner

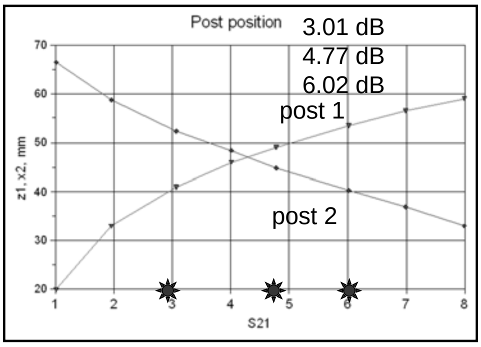


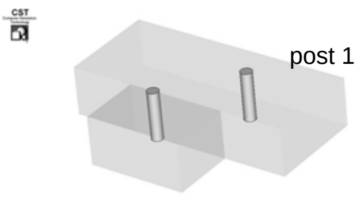


The shunt tee works only as combiner without reflections at the input port, if both input ports are used with the right amplitude.

WR650 Asymmetric Shunt Tee Adjustment

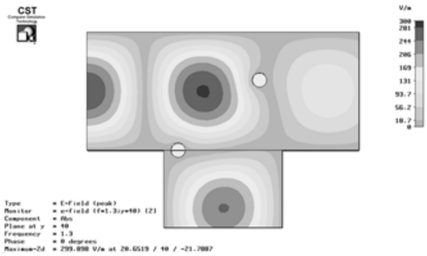


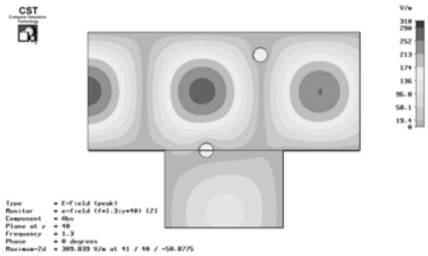




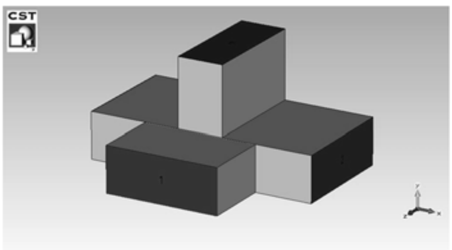
post 2

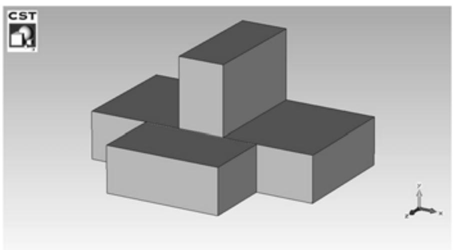
Shunt Tee with 1dB (left) and 8dB(right) Coupling Ratio





Magic Tee





The magic tee is a combination of an E-tee and and H-tee. It is usally used as power divider from port 1 to ports 2 and 3 or vice versa as combiner from ports 2 and 3 to port 1. It overcomes the short coming of shunt tees.

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

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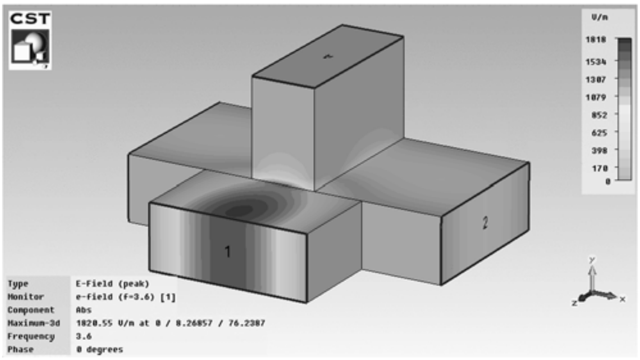
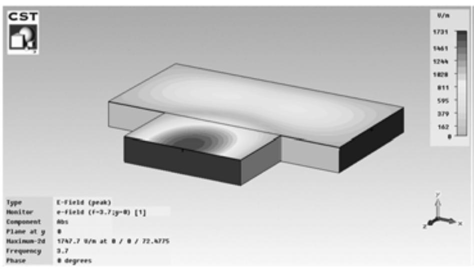
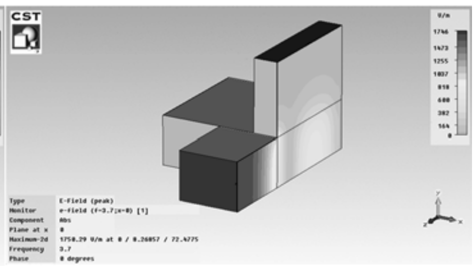
Magic Tee



Magic Tee



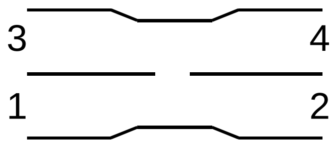
H part of a Magic Tee



E part of a Magic Tee

Hybrid (Riblet Coupler)

The hybrid is a 4-port device which works as divider or coupler. By proper choice of the dimensions of the hole between the two waveguides the S-parameter can be adjusted.

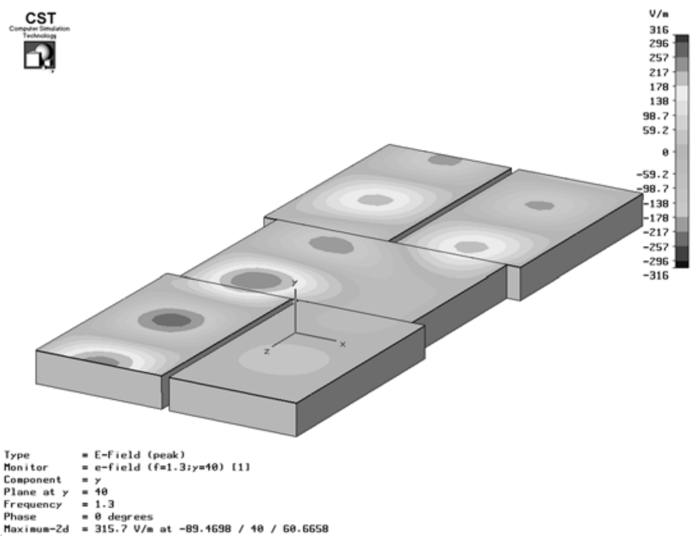


S-matrix of an ideal 3dB hybrid:

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 & i \\ 1 & 0 & i & 0 \\ 0 & i & 0 & 1 \\ i & 0 & 1 & 0 \end{pmatrix}$$

The power entering port 1 is equally divided between port 2 and 4. The phase between port 2 and 4 is 90degree.

3dB Hybrid



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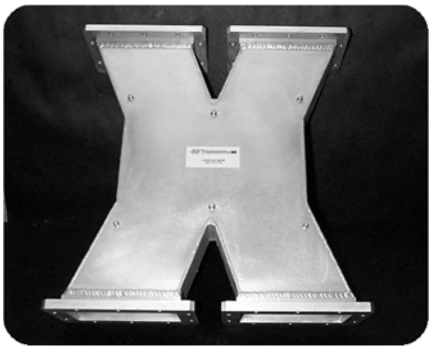
Two Examples of a Hybrid



Directional Coupler

Directional coupler can be used to measure signals of waves in waveguides. They make use of holes or loops in the waveguides. The coupled signal of the waves between the ports 1 and 2 can be measured at port 3 and 4 for the reflected and the forward wave, respectively. Good directivity can be accomplished by proper size, seperation or orientation of the holes or loops.
This makes use of constructive and deconstructive interference of the signals in the holes or loops.
The coupling is described by the coupling factor:

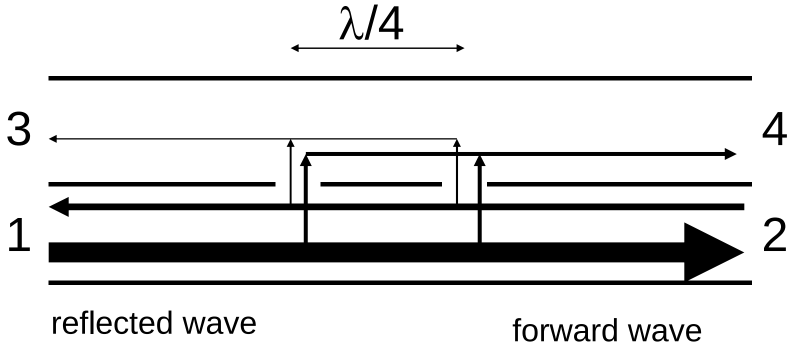
$$C = 20\log\left|\frac{E_{forw}}{E_{measured,forw}}\right| dB$$

The directivity is described by:

$$D = 20\log\left|\frac{E_{measured,forw}}{E_{measured,refl}}\right| dB$$

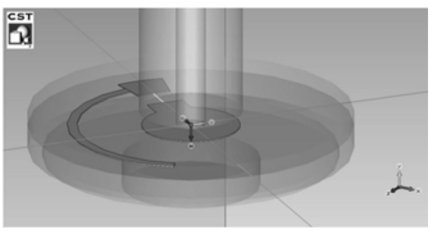
2-hole Directional Coupler

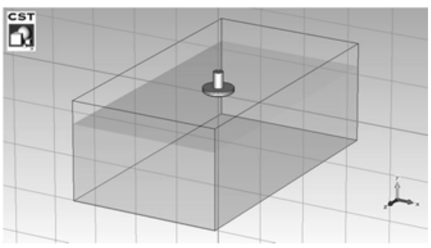
One example of a directional coupler is hole coupler which has two holes between one wall of two waveguides. The separation of the holes is $\lambda/4$. Therefore constructive interference occurs for the forward wave in the forward direction of the other waveguide at port 4 and deconstructive interference at port 3. For the reflected wave it is vice versa. By adding more holes the directivity can be improved.



Directional Loop Coupler

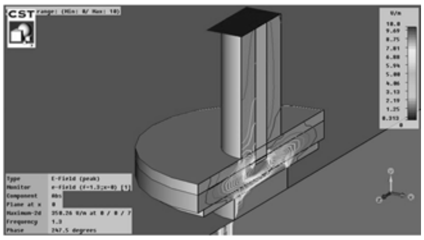
Another example of a directional coupler is a hole coupler with one hole and a loop in the hole. The coupling and the directivity is adjusted by adjusting the diameter of the couling, the distance to the loop and the alignment of the loop. Electrical and magnetic field components are launched in the hole. Due to orientation of the field components for forward and reflected wave and choice of the loop orientation one achieves cancellation or summation for forward and reflected waves.



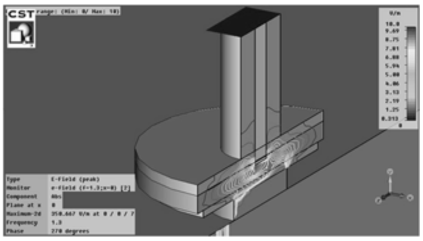


Fields in a Directional Coupler

YZ plane Forward Wave
Max Field in Coax

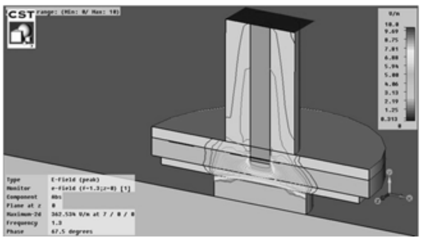


YZ plane Reflected Wave
~0 Field in Coax

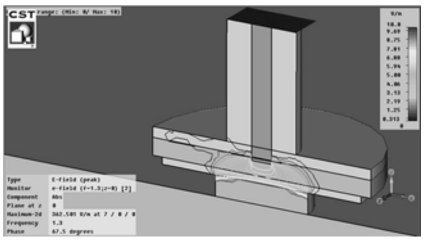


Fields in a Directional Coupler (2)

YX plane Forward Wave
Max Field in Coax

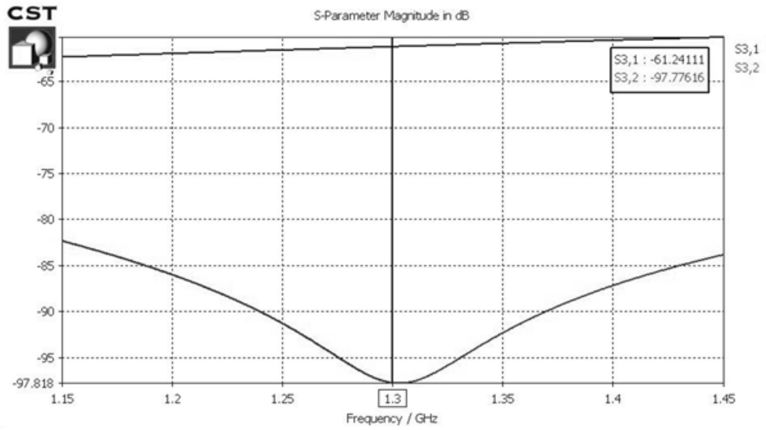


YX plane Reflected Wave
~0 Field in Coax

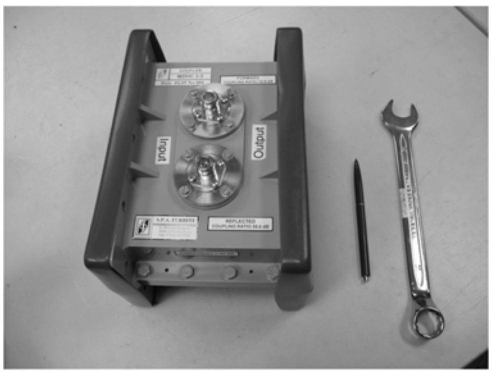


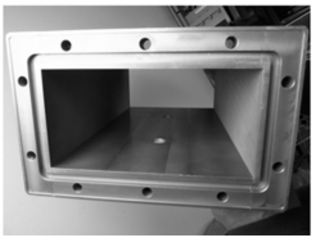
S-Parameter of a Directional Coupler

Result of simulation. Directivity is 97.7 – 61.2 = 36.5 dB



Two Examples of Directional Coupler

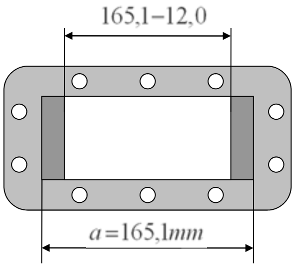


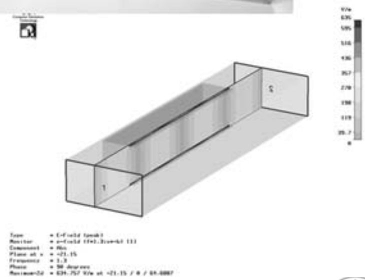


Phase Shifter

•By adjusting the dimensions of the waveguide e.g. the width a the phase constant changes.

$$\beta_g = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$





Waveguides using Ferrites

Ferrites have the chemical formula $XOFe_2O_3$ where XO is a metal oxide. These materials have low electrical conductivity and are anisotropic in magnetic fields. Therefore they can pass electromagnetic waves with only low loss and with different velocities, depending on propagation direction and polarisation of the electromagnetic wave relative to the external magnetic field. The last property results in different phase advance and different propagation direction of the wave in the ferrite component. By the use of ferrites devices with non reciprocal properties can be built.

Interaction of Electron Spin with B-Field

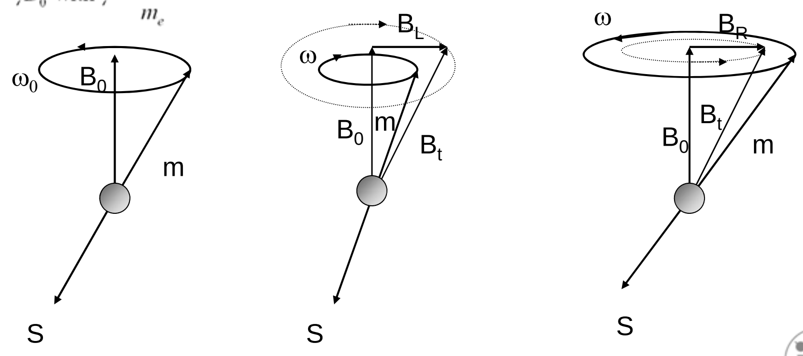
Static B-Field results in precession at ω_0

$\mathbf{T} = \mathbf{m} \times \mathbf{B}_0$

$\omega_0 = \gamma B_0$ with $\gamma = \frac{e}{m_e}$

Static B-Field plus LHCP of frequency $-\omega$ results in forced precession at $-\omega$

Static B-Field plus RHCP of frequency ω results in forced precession at ω



Tensor of Permeability

In a macroscopic material one uses the magnetization **M** of the material which is defined as:

$$\mathbf{M} = \frac{1}{V}\sum_i \mathbf{m}_i$$

$$\mathbf{B} = \mu_0(\mathbf{M} + \mathbf{H})$$

M behaves like **m**,

that means differently for LHCP and RHCP

Every LHCP and RHCP can be separated into x and y coordinates with ± 90degree phase advance between x and y.

Instead of $\mathbf{B} = \mu\mathbf{H}$

$$\mathbf{B} = \|\mu\|\mathbf{H}$$

Tensor of the Permeability (2)

$$\|\mu\| = \begin{pmatrix} \mu & i\kappa & 0 \\ -i\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{pmatrix}$$

Tensor of permeability for the case $\mathbf{B}_0 \parallel \mathbf{e}_z$

$$\mu = 1 + \frac{\omega_0 \gamma \mu_0 M_s}{\omega_0^2 - \omega^2} \quad \text{and} \quad \kappa = \frac{\omega \gamma \mu_0 M_s}{\omega_0^2 - \omega^2} \text{ without loss}$$

or

$$\mu = 1 + \frac{\omega_0 \omega_m \left(\omega_0^2 - \omega^2\right) + \omega_0 \omega_m \omega^2 \alpha^2}{\left(\omega_0^2 - \omega^2 (1+\alpha^2)\right)^2 + 4\omega_0^2 \omega^2 \alpha^2} - i \frac{\omega \omega_m \alpha \left(\omega_0^2 - \omega^2 (1+\alpha^2)\right)}{\left(\omega_0^2 - \omega^2 (1+\alpha^2)\right)^2 + 4\omega_0^2 \omega^2 \alpha^2}$$

$$\kappa = \frac{\omega \omega_m \left(\omega_0^2 - \omega^2 (1+\alpha^2)\right)}{\left(\omega_0^2 - \omega^2 (1+\alpha^2)\right)^2 + 4\omega_0^2 \omega^2 \alpha^2} - i \frac{2\omega^2 \omega_0 \omega_m \alpha}{\left(\omega_0^2 - \omega^2 (1+\alpha^2)\right)^2 + 4\omega_0^2 \omega^2 \alpha^2} \quad \text{with loss}$$

$\omega_m = \gamma \mu_0 M_s$ and α damping constant for precession

$\gamma = \frac{e}{m_e}$ and M_s saturation magnetization

Waves in Ferrite Waveguides

Now the Maxwell equation

$$\nabla \times \mathbf{H} = i\omega\varepsilon\mathbf{E}$$

$$\nabla \times \mathbf{E} = -i\omega \|\mu\| \partial/\partial t\, \mathbf{H}$$

$$\nabla \cdot \mathbf{E} = 0$$

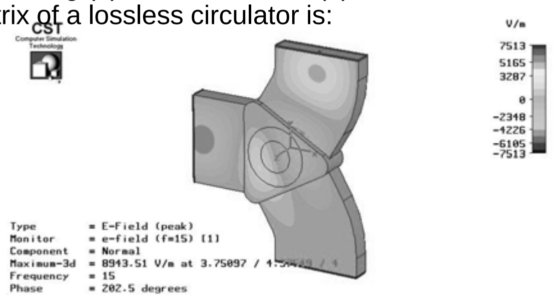
$$\nabla \cdot \mathbf{H} = 0$$

could be solved, what will be not done here.
One can distinguish two cases. Propagation of the wave parallel to the bias magnetic field and propagation perpendicular to the bias magnetic field. Within the last case one can distinguish polarisation of the waves H field parallel the bias field (ordinary wave) and perpendicular to the bias field (extraordinary wave).
A number of waveguide components make use of the anisotropic properties of ferrites.

3-Port Circulator

- A circulator is a device with ferrite material in the middle of 3 waveguide connections. The bias field is applied perpendicular to the propagation direction. The circulator has an input port (1), output port (3) and load port (2). If power is entering (1) it is transfered to port (3), but if power is entering (3) it is tranfered to (2) and than absorbed in a load. The S-matrix of a lossless circulator is:

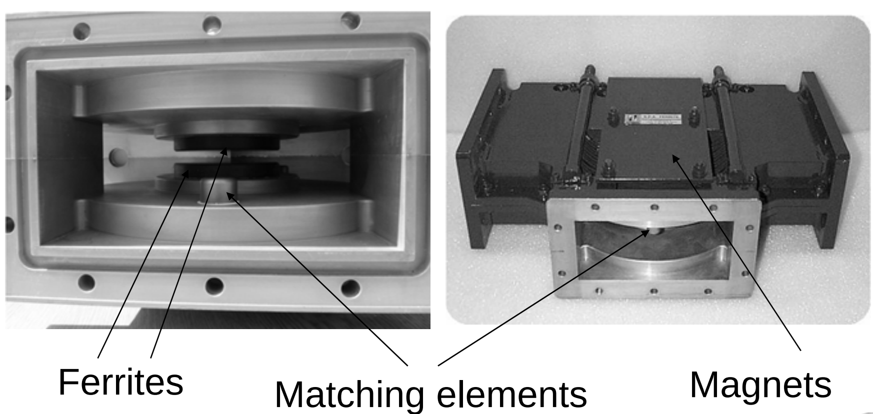
$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$



- The ciculator protects the RF source from reflected power. Usually the circulator is not ideal and lossless.The isolation is usually more than 25dB. The insertion loss can be less then 0.15dB, but is sometimes larger. A typical VSWR is 1.1.

Example of a 3-Port Circulator

WR650 400kW circulator



Loads

- Loads absorb the power generated by an RF source
- Absorbing material can be ferrite, SiC or water.
- The amount of power reflected by a load is described by the *VSWR* defined as

$$VSWR = \frac{|E_f| + |E_r|}{|E_f| - |E_r|} = \frac{1+\rho}{1-\rho} \quad \text{and}$$

$$\rho = \frac{Z_L - Z}{Z_L + Z}$$

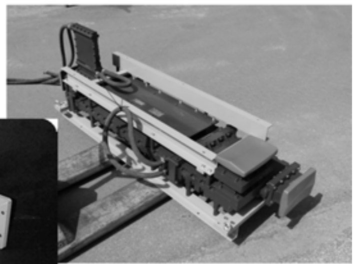
with Z impedance of the waveguide and Z_L load impedance

Three WR650 ferrite loads,
200W air cooled, 500kW water cooled and 5MW water cooled
(from left to right)

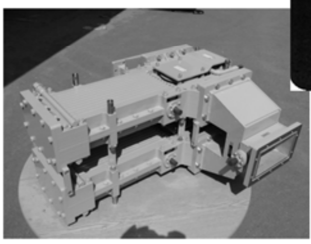
Some other Waveguide Elements

Adjustable short circuit

3-Stub tuner



WR650 4-port (phase shift) isolator weight ca 280kg



WR650 1MW isolator made of two 3-port circulators, two E-tees and two ferrite loads

5MW RF waveguide gas window

Acknowledgement: I would like to thank two persons who supported me during preparation of this lecture. Valery Katalev provided simulation results and Ingo Sandvoss took photographs.

Thank you very much for your your attention

Literature: Textbooks and School Proceedings

A large number of very good books on microwave and waveguide theory exists. Some of them are listed here. One should use according to personal preference.

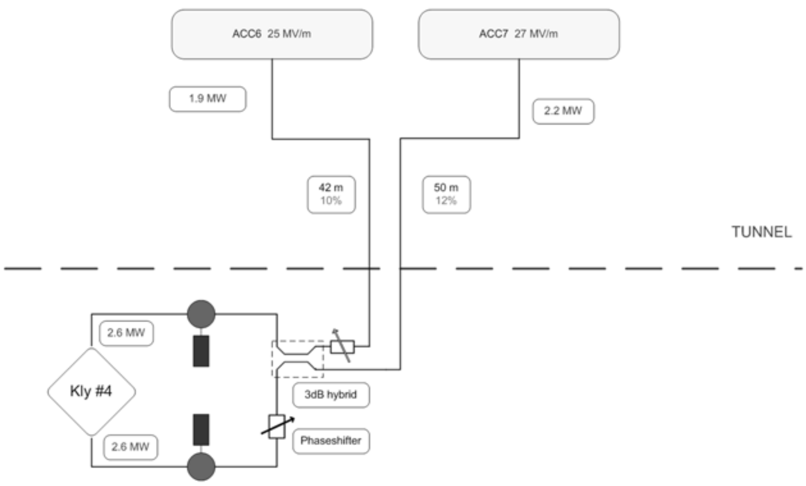
- R. E. Collin, Foundations For Microwave Engineering, McGraw Hill 1992
- D. M. Pozar, Microwave Engineering, Wiley 2004
- N. Marcuvitz, Waveguide Handbook, MIT Radiation Laboratory Series, Vol. 10, McGraw Hill 1951
- H. J. Reich, P. F. Ordung, H. L.Krauss, J. G. Skalnik, Microwave Theory and Techniques, D. van Nostrand 1953

- R. K. Cooper, R. G. Carter, High Power RF Transmission, in Proceedings of the CERN Accelerator School: Radio Frequency Engineering, 8-16 May 2000, Seeheim, Germany
- R. K. Cooper, High Power RF Transmission, in Proceedings of the CERN Accelerator School: RF Engineering for Particle Accelerators, 3-10 April 1991, Oxford, UK
- A. Nassiri, Microwave Physics and Techniques, USPAS, Santa Barbara, Summer 2003

- Meinke, Gundlach, Taschenbuch der Hochfrequenztechnik

Waveguide Distributions

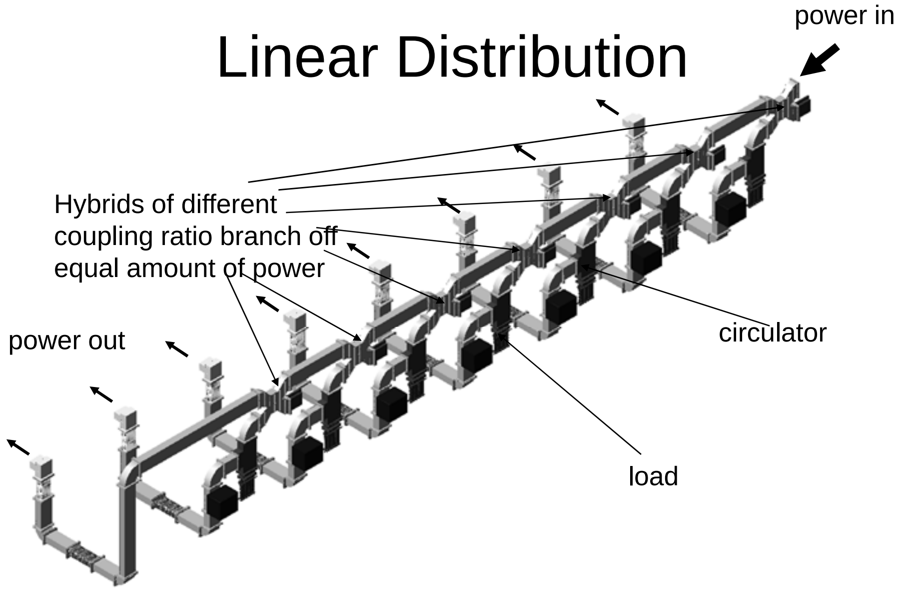
Task for a real distribution



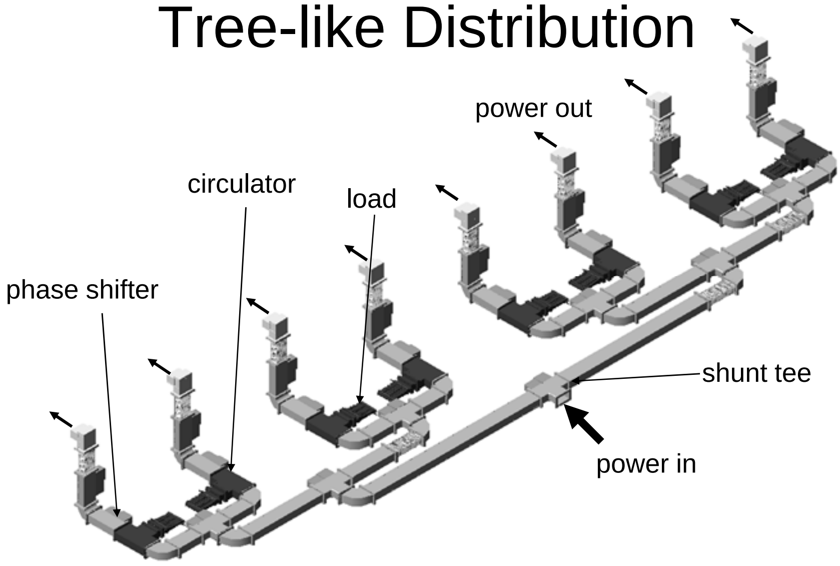
Waveguide Distribution Schemes

- Waveguide distributions are combinations of different waveguide elements
- One can distinguish two basic types :
  linear distributions and tree like distributions.
- Combinations of both are possible. The layout depends on a number of requirements: e.g. power capability, isolation between cavities, weight, space availability, ease of assembly, cost, etc.

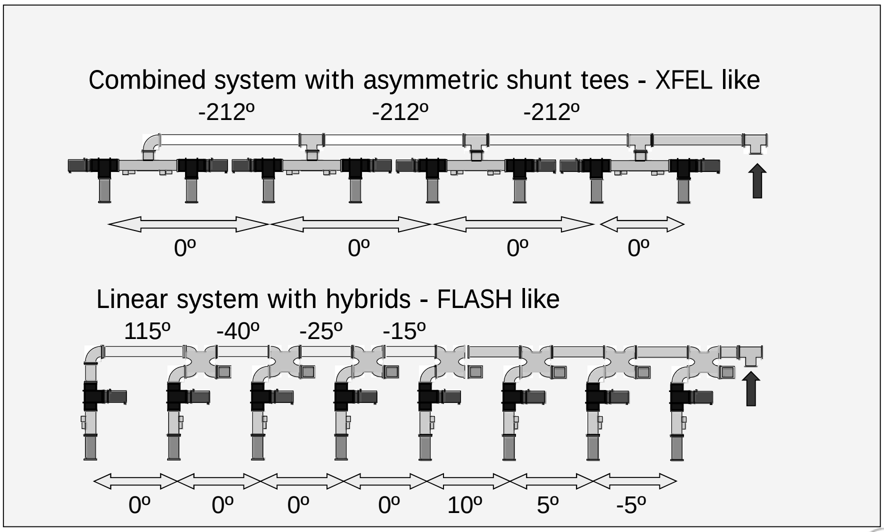
Linear Distribution



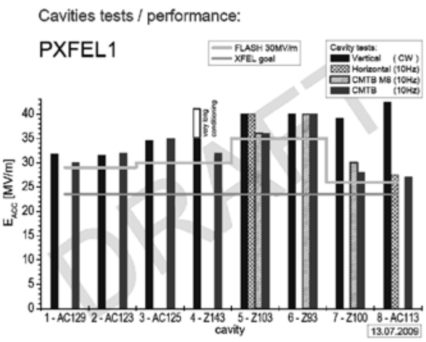
Tree-like Distribution



A linear and a combined Distribution

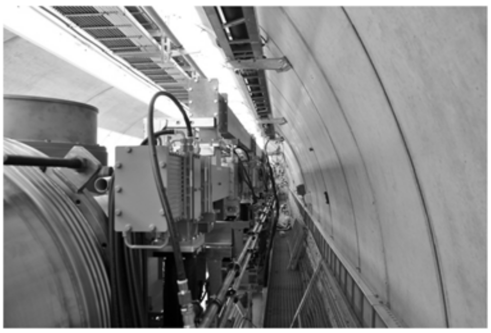


Combined XFEL-type Distribution for FLASH

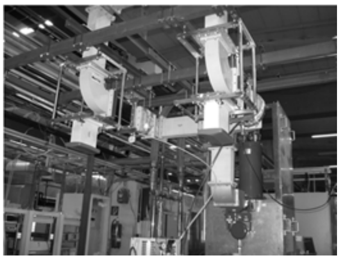


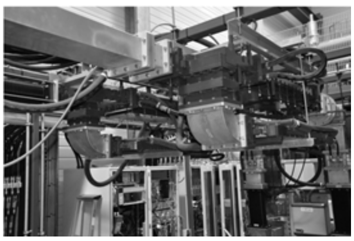
By choosing the coupling ratio of the shunt tees, operation of the cavities at maximum gradient can be achieved (green line). In case of the same coupling ratio, the cavities can be operated only at the gradient of the weakest cavity (red line).

Accelerator installation



Waveguides near the cavities





Waveguides near to klystron

Limitations, Problems and Countermeasures

Maximum Power in TE_{10}

$$P_{RF} = 6.63 \times 10^{-4} a[cm] b[cm] \left[\lambda / \lambda_g \right] E[V/cm]^2$$

•The maximum power which can be transmitted theoretically in a waveguide of certain size a, b and wavelength λ is determined by the breakdown limit E_{max}.

•In air it is E_{max}=30kV/cm. Therefore the theoretical limit is 58MW at 1.3GHz in WR650.

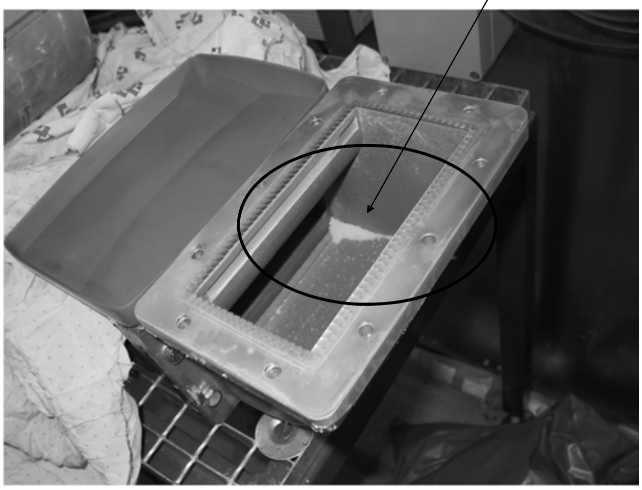
•But experience shows that in real distributions it is lower, typically 5-10 times lower. One could increase the gas pressure inside the waveguide, which due to Paschens law would increase the power capbility. But this requires enforced and gas tight waveguides. In addition the pressure vessel rules most be observed.

•By using SF6 instead of air, which has Emax=89kV/cm (at 1bar, 20°C), the power capability can be increased, too

Maximum Power in TE_{10} (2)

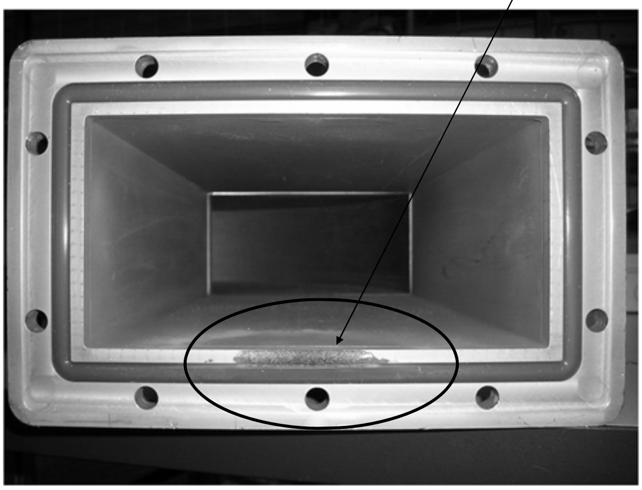
- The problem of SF6 is that although it is chemically very stable it is a green house gas and if cracked in sparcs products can form HF, which is a very aggressive acid. Other chemical poisonous chemicals e.g. S_2F_{10} are being produced too.
- The practical power limit is lower, because of a variety of different reasons: smaller size (e.g. within circulators), surface effects (roughness, steps at flanges etc.), dust in waveguides, humidity, reflections (VSWR) or because of higher order modes TE_{nm}/TM_{nm}. These HOMs are also generated by the power source. If these modes are not damped, they can be excited resonantly and reach very high field strength above the breakdown limit.

Fluorides inside a WR650 Waveguide



Staff for opening and cleaning SF_6 filled waveguide must use protection clothes

Damaged Waveguide due to bad Connection of two Waveguide Flanges



HOMs

HOMs can be sometimes damped by installing small antenna which are than connected to small loads. The exact mode pattern is proberbly not known, but if these antanna couple to HOMs, the HOMs are damped. The disadvantage of this solution is that one always couple out part of the fundamental mode.