Dynamics of binary black hole mergers

From quantum scattering amplitudes to classical dynamics

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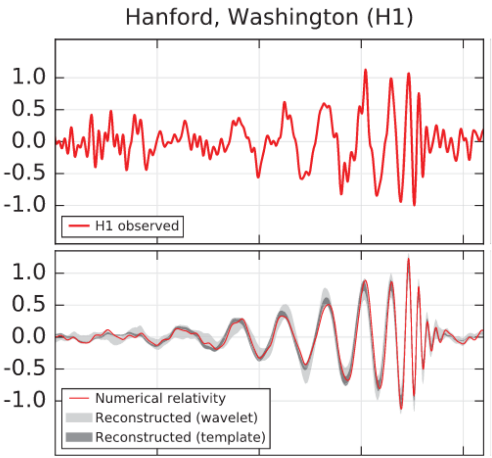
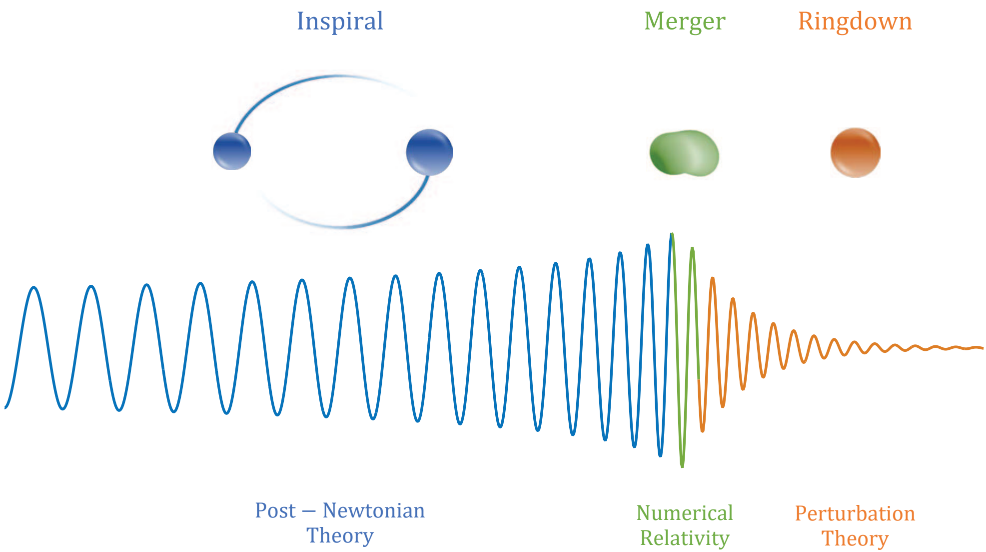
Gravitational wave astronomy



[LIGO & Virgo, arXiv:1602:03837]

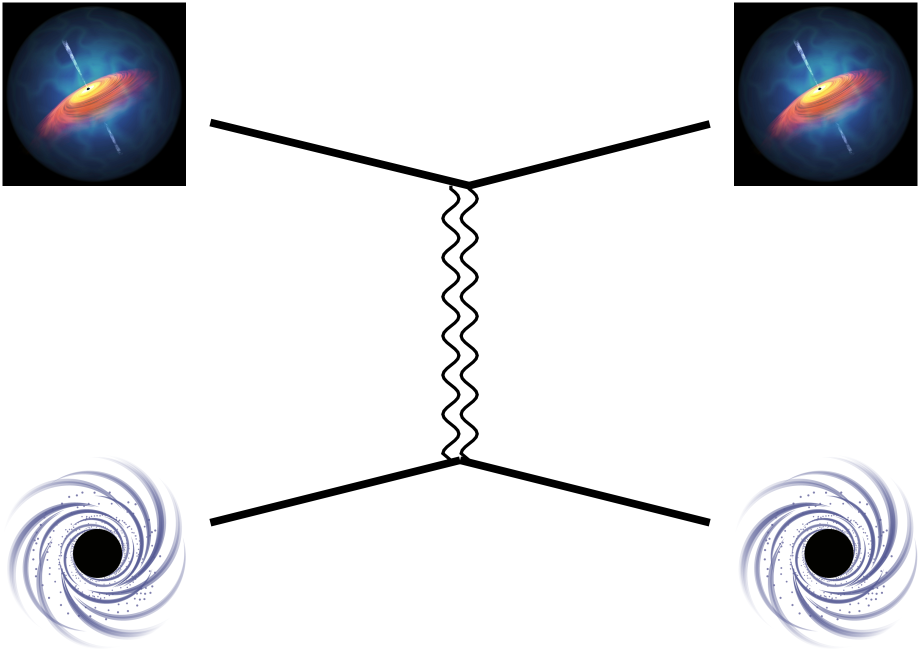
- The era of gravitational wave astronomy has just begun ...
- ... and can benefit from analytical high-precision calculations!

Different stages of binary black hole merger



[Antelis,Moreno arXiv:1610.03567]

How do we want to compute it?



Using particle physics methods!

- Consider Einstein-Hilbert gravity coupled to massive scalars

$$S = \int d^4x\sqrt{-g}\left(-\frac{2}{\kappa^2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m_\phi^2\phi^2\right)\,, \qquad \kappa = \sqrt{32\pi G}$$

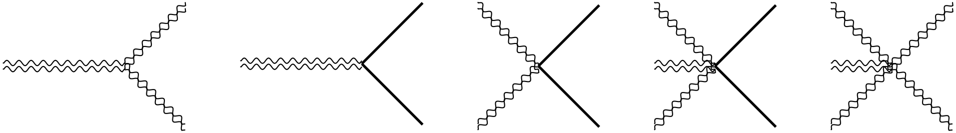
- Expand spacetime metric around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

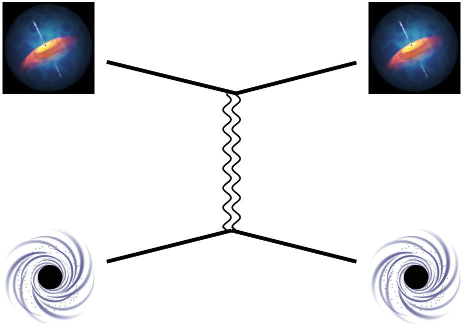
- General relativity is non-linear

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda}h^{\nu}{}_{\lambda} + \mathcal{O}(\kappa^3)$$
$$\sqrt{-g} = 1 + \frac{\kappa}{2}h + \frac{\kappa^2}{8}\left(h^2 - 2h_{\mu\nu}h^{\mu\nu}\right) + \mathcal{O}(\kappa^3)$$

- Leads to quantum EFT of gravity with **infinite** many interaction terms



Does this work?



$$M = \frac{4\pi\, G}{E_1 E_2} \frac{m_1^2 m_2^2 (1-2\sigma^2)}{q^2} + \mathcal{O}\left((q^2)^0\right)$$

$$\sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

The classical potential is then given by

$$V_{\mathrm{cl}} = \int \frac{d^3 q}{(2\pi)^3}\, M e^{i\vec{q}\cdot\vec{r}} = -\frac{G m_1 m_2}{r}\left(\frac{m_1 m_2}{E_1 E_2}(2\sigma^2 - 1)\right) = -\frac{G m_1 m_2}{r} + \mathcal{O}(\mathbf{v}^2)$$

- Scattering amplitude approach provides all-order $\mathbf{v}^2$ corrections!
- Loop amplitudes provide higher-order $\mathcal{O}(G^n)$ corrections!

Post-Newtonian vs. Post-Minkowskian expansion

Post-Newtonian expansion (PN) - virial theorem

$$\frac{v^2}{c^2} \sim \frac{GM}{r} \ll 1$$

0PN (1687)	1PN (1938)	2PN (1973)	3PN (2000)	4PN (2014)	5PN (2020)	6PN (2020)
Newton	Einstein,Infeld Hoffmann	Kimura et al	Damour et al Blanchet, Faye	Damour et al	Mastrolia et al Blümlein et al	Blümlein et al

$$
\begin{aligned}
&G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \dots)\\
&G^2(1 + v^2 + v^4 + v^6 + v^8 + \dots)\\
&G^3(1 + v^2 + v^4 + v^6 + \dots)\\
&G^4(1 + v^2 + v^4 + \dots)\\
&G^5(1 + v^2 + \dots)\\
&G^6(1 + \dots)
\end{aligned}
$$

Post-Minkowskian expansion (PM) - non-virial theorem

$$\frac{GM}{r} \ll 1$$

0PN (1687)	1PN (1938)	2PN (1973)	3PN (2000)	4PN (2014)	5PN (2020)	6PN (2020)
Newton	Einstein,Infeld Hoffmann	Kimura et al	Damour et al Blanchet, Faye	Damour et al	Mastrolia et al Blümlein et al	Blümlein et al

$$\boxed{G(\,1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \ldots)}$$

1PM

$$\boxed{G^2(\,1 + v^2 + v^4 + v^6 + v^8 + \ldots)}$$

2PM (1985) Westphal

$$\boxed{G^3(\,1 + v^2 + v^4 + v^6 + \ldots)}$$

3PM (2019) Bern, Cheung, Roiban Shen, Solon, Zeng

$$\boxed{G^4(\,1 + v^2 + v^4 + \ldots)}$$

4PM (2021) Bern,Parra-Martinez Roiban,Ruf,Shen Solon, Zeng

$$G^5(\,1 + v^2 + \ldots)$$

$$G^6(\,1 + \ldots)$$

Explore numerical unitarity for black hole mergers at $\mathcal{O}(G^3)$

Classical pieces of quantum loops

- multi-loop diagrams contain classical contributions

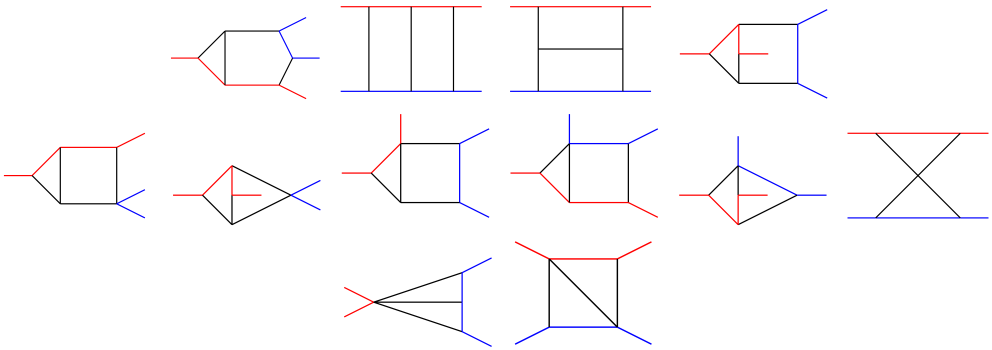
  [Iwasaki'1971, Holstein,Donoghue'2004, Neill,Rothstein'2013]

- The classical limit of gravity loop amplitude corresponds to

$$|\ell_1| \sim |\ell_2| \sim |q| \ll m_1, m_2, \sqrt{s}$$

- 1-loop diagrams (topologically inequivalent)

- 2-loop diagrams (topologically inequivalent)



Matching to non-relativistic EFT

How to connect multi-loop amplitudes with the classical potential?

[Cheung,Rothstein,Solon arXiv:1808.02489]

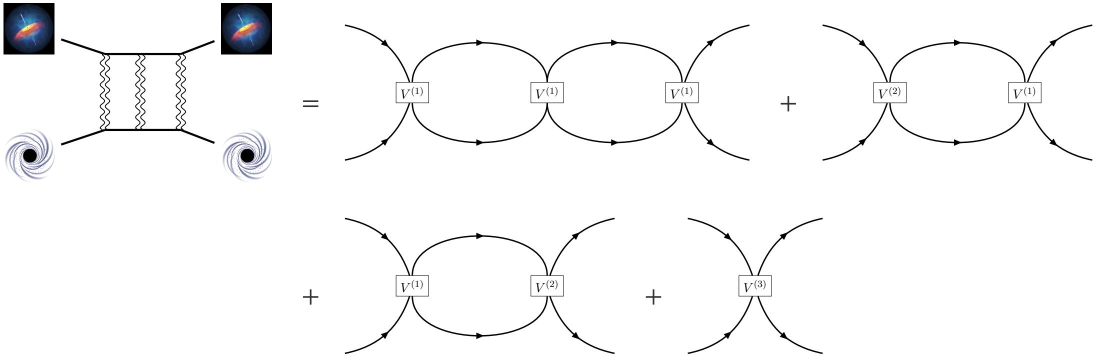
- Effective field theory (no anti-particles)

$$\mathscr{L} = \sum_{i=1}^{2} \int_{\mathbf{k}} \phi_i^\dagger(-\mathbf{k}) \left(i\partial_t - \sqrt{\mathbf{k}^2 + m_i^2} \right) \phi_i(\mathbf{k}) - \int_{\mathbf{k},\mathbf{p}} V(\mathbf{k},\mathbf{p}) \phi_1^\dagger(\mathbf{p}) \phi_1(\mathbf{k}) \phi_2^\dagger(-\mathbf{p}) \phi_2(-\mathbf{p})$$

with

$$V(\mathbf{k},\mathbf{p}) = \sum_{n=1}^{\infty} G^n \, V^{(n)}(\mathbf{k},\mathbf{p})$$

- Match gravity amplitude to EFT amplitude



Computational framework - I

Multi-loop Numerical unitarity method

- Integrand of amplitude factorizes for on-shell loop momenta ℓ_l^Γ

$$\lim_{\ell_l \to \ell_l^\Gamma} \left(M^{(2)}(\ell_l) \prod_{j \in P_\Gamma} \rho_j \right) = \sum_{\text{states}} \prod_{i \in T_\Gamma} M_i^{(0)}(\ell_l^\Gamma)$$

Caravel

- Tree-level Gravity amplitudes from numerical off-shell recursions
- Integrands are parametrized by tensor integral insertions

$$\lim_{\ell_l \to \ell_l^\Gamma} \left(M^{(2)}(\ell_l) \prod_{j \in P_\Gamma} \rho_j \right) = \sum_i c_{\Gamma,i}\, m_{\Gamma,i} + \sum_{\Gamma' > \Gamma} \sum_i c_{\Gamma',i} \frac{m_{\Gamma',i}}{\prod_{j \in P_{\Gamma'} \setminus P_\Gamma} \rho_j}$$

- Solve numerically for coefficients $c_{\Gamma,i}$ using finite field arithmetic
- Expressions are regularized in dimensional regularization: $d = 4 - 2\epsilon$

[Abreu,Dormans,Febres Cordero,Ita,MK,Page,Pascual,Ruf,Sotnikov arXiv:2009.11957]

Computational framework - II

Reconstruct analytics from numerical samples

[von Manteuffel, Schabinger'2014, Peraro'2016]

1. Multiple functional reconstructions in q^2 and ϵ for fixed $\vec{x} = \{s, m_1, m_2\}$
$$M^{(2)}(\vec{x}_0, q^2, \epsilon) = \sum_i f_i(\vec{x}_0, q^2, \epsilon)\, I_i(\vec{x}_0, q^2, \epsilon)$$
2. Reduce tensor integrals via *integration-by-part* identities to masters
(allows to make use of power counting)
$$I_i(\vec{x}_0, q^2, \epsilon) = \sum_j c_{ij}(\vec{x}_0, q^2, \epsilon)\, m_i(\vec{x}_0, q^2, \epsilon)$$
3. Combine with master integrals m_i from literature

[Parra-Martinez,Ruf,Zeng'2020]

4. Expand in ϵ and take the classical limit in $q^2 \to$ only non-analytic terms
5. Reconstuct the multi-variate functions in $\vec{x}$ of the remaining coefficients.
$\Rightarrow$ functional dependence tremendously simplified due to the classical limit
6. Match to non-relativistic EFT
$\Rightarrow$ Two-body Hamiltonian at 3rd post-Minkowskian order

Summary:

- Particle Physics methods allow to compute efficiently gravitational wave dynamics for the inspiral stage
- An extendable framework to compute gravity integrands
- First application of multiloop numerical unitarity with massive particles
- Final cross checks are still on the way

Outlook:

- finite size effects (higher-dim. operators)
- spin effects (fermions, vector bosons)
- radiation (external gravitons)
- higher-order corrections (more loops)