Complete Unification with E_8

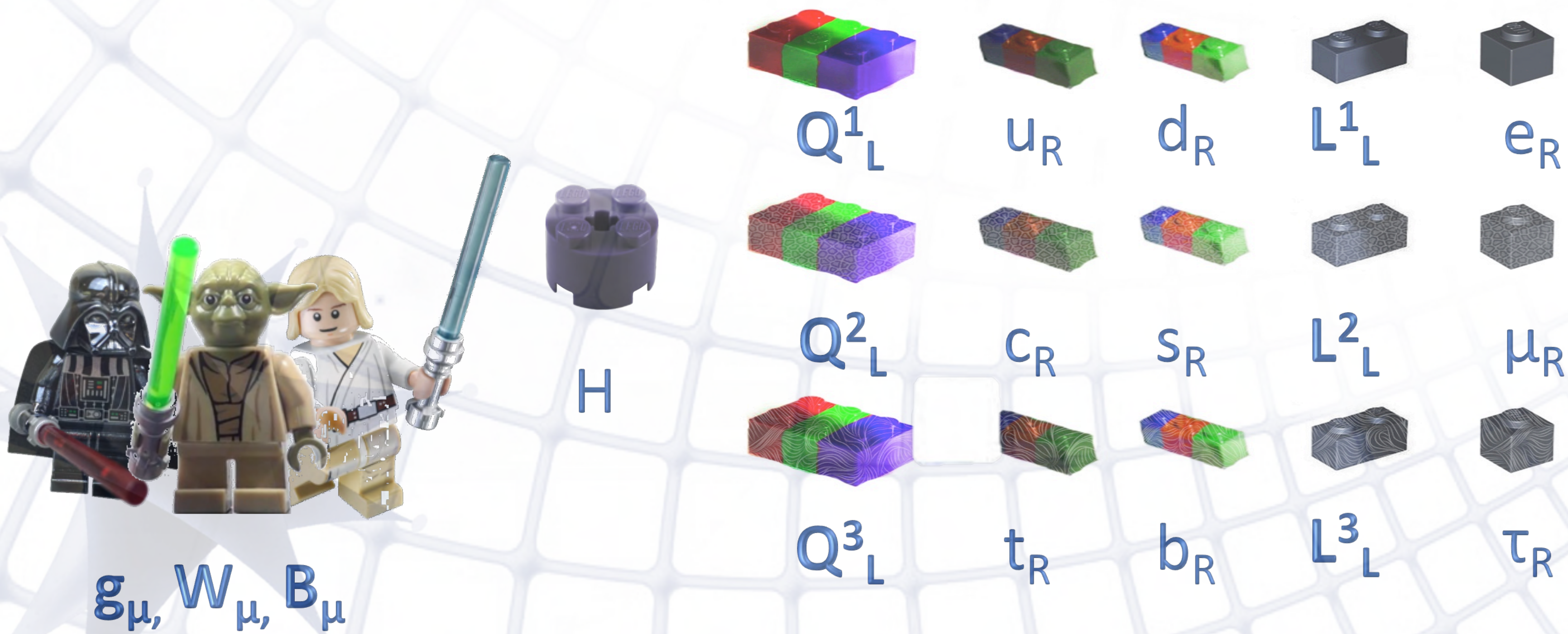
Francisco J. de Anda
Tepatitlán's Institute for Theoretical Studies

The Standard Model

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

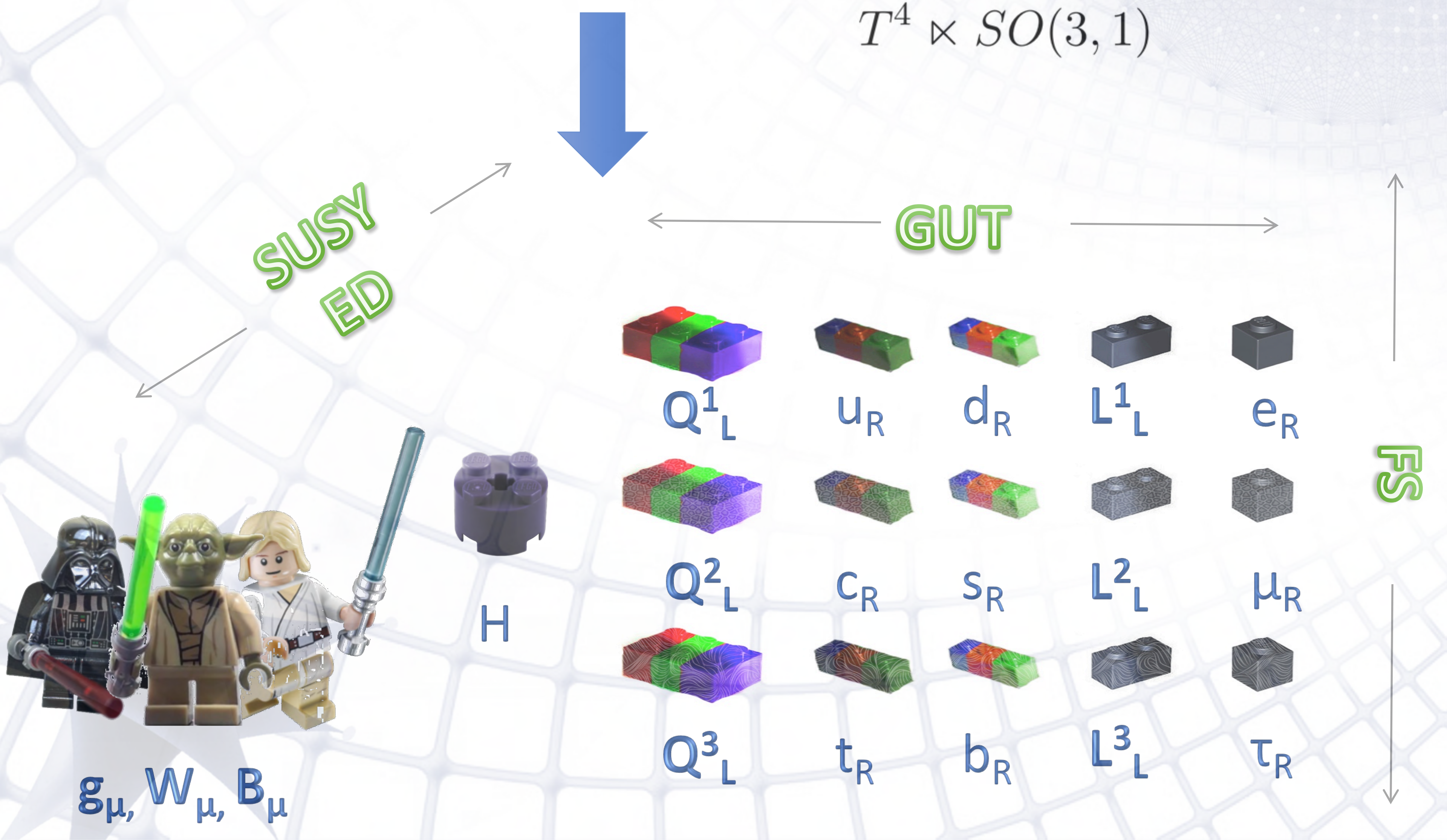
$$T^4 \ltimes SO(3,1)$$

26 Real Parameters

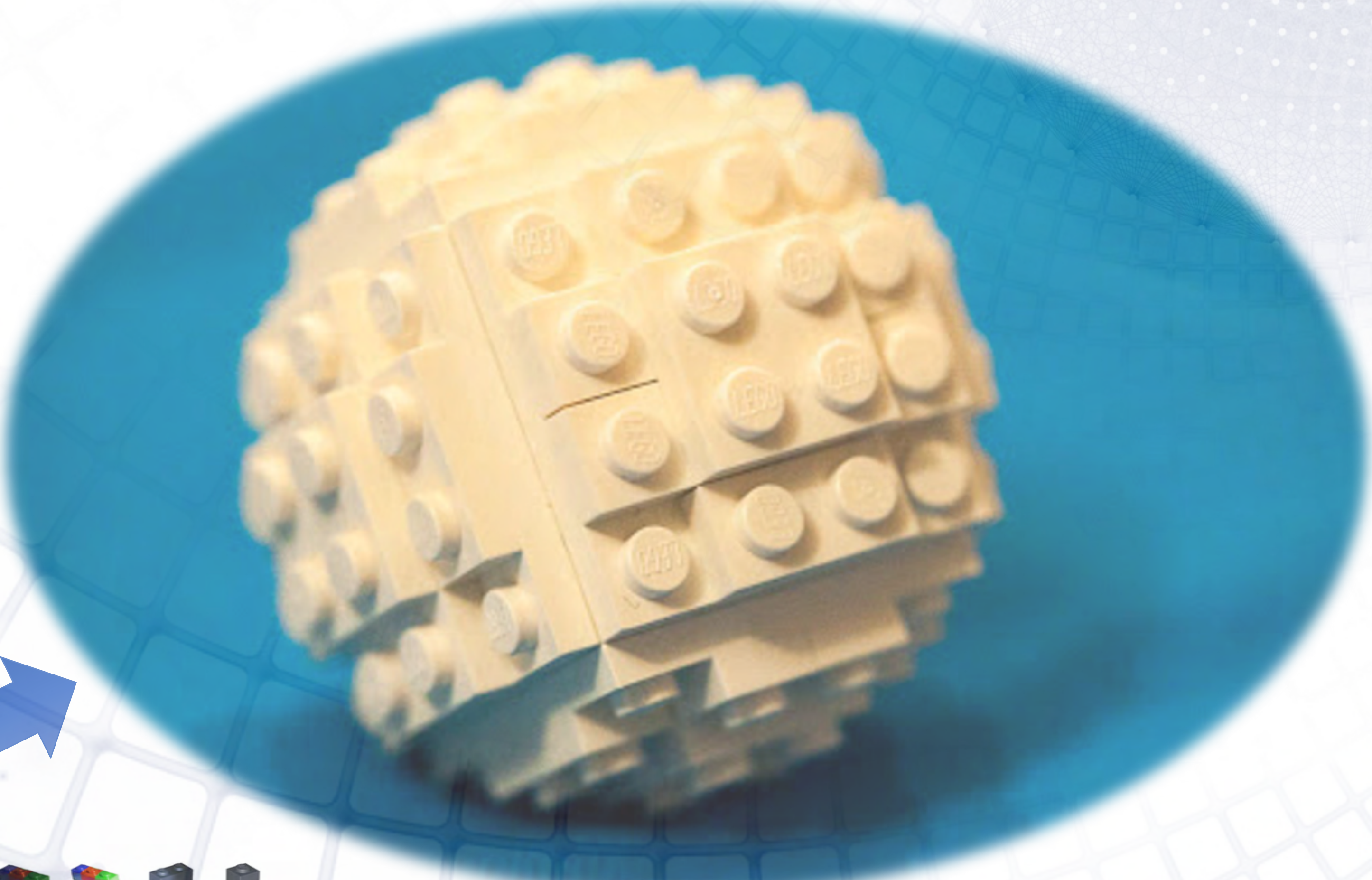


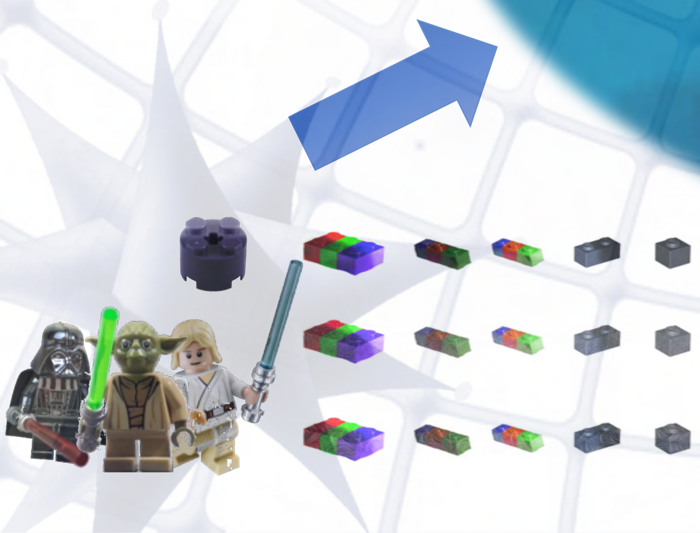
Beyond the Standard Model

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$T^4 \ltimes SO(3,1)$$



Full Unification






Exceptional Chain
1980s

SU(3) X SU(2) X U(1)

$T^4 \ltimes SO(3,1)$	$SU(3)_C \times SU(2)_L \times U(1)_Y$
$(\mathbf{1})$	$(\mathbf{1}, \mathbf{2}, 3)$
$(\mathbf{2})$	$3 \times (\mathbf{1}, \mathbf{2}, -3) + 3 \times (\mathbf{1}, \mathbf{1}, 6) + 3 \times (\bar{\mathbf{3}}, \mathbf{1}, -4) + 3 \times (\bar{\mathbf{3}}, \mathbf{1}, 2) + 3 \times (\mathbf{3}, \mathbf{2}, 1)$
$(\mathbf{4})$	$(\mathbf{8}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{3}, 0) + (\mathbf{1}, \mathbf{1}, 0)$

26 Real Parameters

The Standard Model

SU(5)

$T^4 \ltimes SO(3,1)$	$SU(5)$
$(\mathbf{1})$	$(\overline{\mathbf{5}})$
$(\mathbf{2})$	$3 \times (\overline{\mathbf{5}}) + 3 \times (\mathbf{10})$
$(\mathbf{4})$	$(\mathbf{24})$

+ Gauge Field Unification

SO(10)

$T^4 \ltimes SO(3,1)$	$SO(10)$
$(\mathbf{1})$	$(\mathbf{10})$
$(\mathbf{2})$	$3 \times (\mathbf{16})$
$(\mathbf{4})$	$(\mathbf{45})$

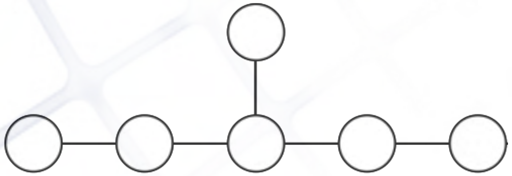
+ Fermion Unification

E$_6$

$T^4 \ltimes SO(3,1)$	E_6
$(\mathbf{1})$	$(\mathbf{27})$
$(\mathbf{2})$	$3 \times (\mathbf{27})$
$(\mathbf{4})$	$(\mathbf{78})$

Suggests SUSY

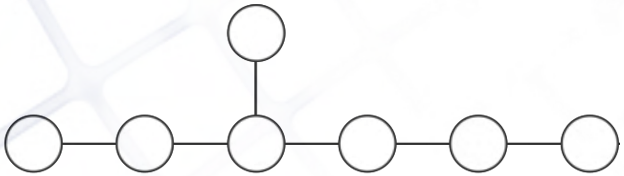
SUSY E_6



$S^1 \ltimes [T^4 \ltimes SO(3,1)]$	E_6
$(2 \times \mathbf{1} + \mathbf{2})$	$3 \times (\mathbf{27})$
$(\mathbf{4} + \mathbf{2})$	$(\mathbf{78})$

+ Fermion-Higgs Unification

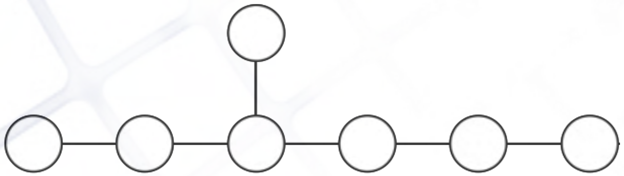
SUSY E_7



$S^1 \ltimes [T^4 \ltimes SO(3,1)]$	E_7
$(2 \times \mathbf{1} + \mathbf{2})$	$(\mathbf{912})$
$(\mathbf{4} + \mathbf{2})$	$(\mathbf{133})$

Only real representations
Suggests Orbifolding

SUSY 6d E_7



$S^1 \ltimes [T^6 \ltimes SO(5,1)]$	E_7
$(2 \times \mathbf{1} + \mathbf{4})$	$(\mathbf{912})$
$(\mathbf{6} + \mathbf{4})$	$(\mathbf{133})$

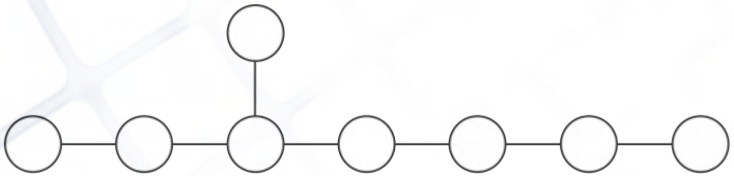
+ Family Unification

SUSY 6d E_8

$S^1 \ltimes [T^6 \ltimes SO(5,1)]$	E_8
$(2 \times \mathbf{1} + \mathbf{4})$	$(\mathbf{248})$
$(\mathbf{6} + \mathbf{4})$	$(\mathbf{248})$

Suggests
N=4 **SUSY in 4d or**
N=1 **SUSY in 10d**

SUSY 10d E_8



$S^1 \ltimes [T^{10} \ltimes SO(9,1)]$	E_8
$(\mathbf{10}+\mathbf{8})$	$(\mathbf{248})$

Full Unification


Exceptional Chain

String theorists

The Model

10d QFT

N=1 Super Yang-Mills based on E_8.

A 10d vector and a 10d Weyl/Majorana fermion
in the adjoint/fundamental representation (**248**).

SM

SUSY

10d E_8

SM

SUSY
10d E_8

The Full Model

10d QFT, *N=1* Super Yang-Mills based on E_8.

A 10d vector and a 10d Weyl/Majorana fermion in the adjoint/fundamental representation (**248**).

1 Complex Parameter

Extra dimensions as

$T^6 / (Z_3 \times Z_3)$

$$\mathbb{Z}_3 : \ (x, z_1, z_2, z_3) \sim (x, \omega^2 z_1, \omega^2 z_2, \omega^2 z_3), \quad \mathcal{V} \to e^{2i\pi q_8^F/3}\, \mathcal{V}$$

$$\mathbb{Z}_3 : \ (x, z_1, z_2, z_3) \sim (x, \omega^3 z_1, \omega z_2, \omega^2 z_3), \quad \mathcal{V} \to e^{2i\pi q_8^C/3}\, \mathcal{V}$$

$$z_i \sim z_i + 2\pi R_i, \quad z_i \sim z_i + 2\pi e^{i\pi/3} R_i$$

13 Complex Parameters

Wilson line aligned with

$$\nu_i^c, \quad \varphi_i$$

$$\omega = e^{2i\pi/3}$$

SUSY

10d E_8

SM

Extra Dimensions

$$S^1 \ltimes \left[T^{10} \ltimes SO(9,1)\right] \times E_8$$

Open EDs
Not observed

No 4d Chirality

Orbifolded Extra Dimensions

$$S^1 \ltimes \left[(T^4 \times T^6/\Gamma) \ltimes \{SO(9,1)/F\}\right] \times \ E_8$$

Lattice group Compactification

Orbifold group

Incomplete Poincarè Group

The Full Model

10d QFT, *N=1* Super Yang-Mills based on E_8.

A 10d vector and a 10d Weyl/Majorana fermion in the adjoint/fundamental representation (**248**).

1 Complex Parameter

Extra dimensions as

$T^6 / (Z_3 \times Z_3)$

$$\mathbb{Z}_3 : \ (x, z_1, z_2, z_3) \sim (x, \omega^2 z_1, \omega^2 z_2, \omega^2 z_3), \quad \mathcal{V} \to e^{2i\pi q_8^F/3}\, \mathcal{V}$$

$$\mathbb{Z}_3 : \ (x, z_1, z_2, z_3) \sim (x, \omega^3 z_1, \omega z_2, \omega^2 z_3), \quad \mathcal{V} \to e^{2i\pi q_8^C/3}\, \mathcal{V}$$

$$z_i \sim z_i + 2\pi R_i, \quad z_i \sim z_i + 2\pi e^{i\pi/3} R_i$$

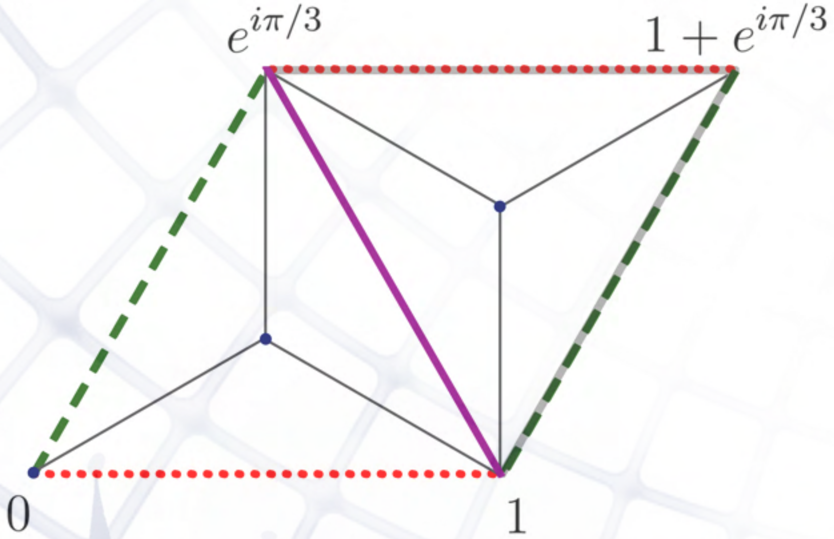
13 Complex Parameters

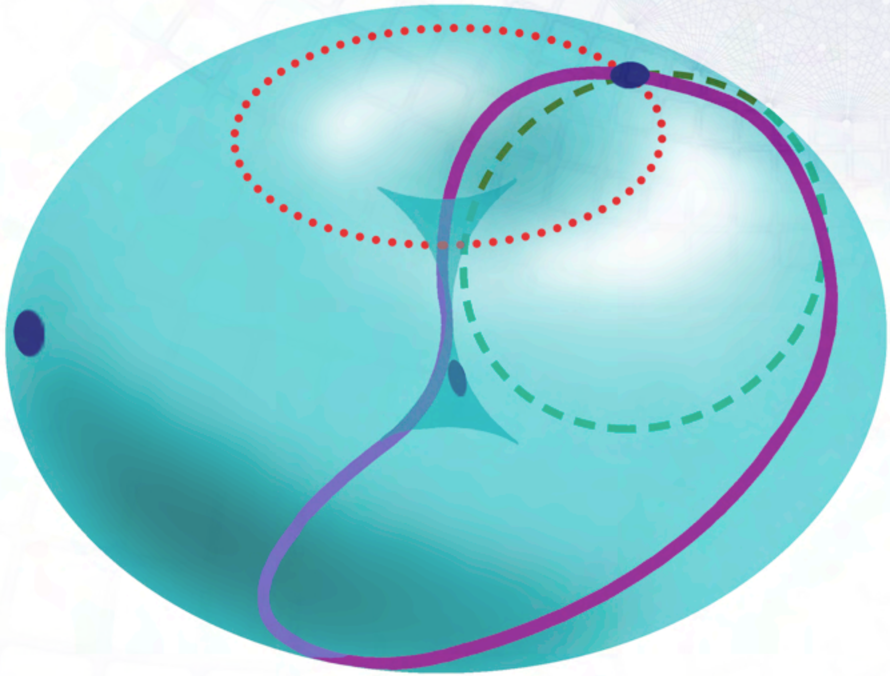
Wilson line aligned with

$$\nu_i^c, \quad \varphi_i$$

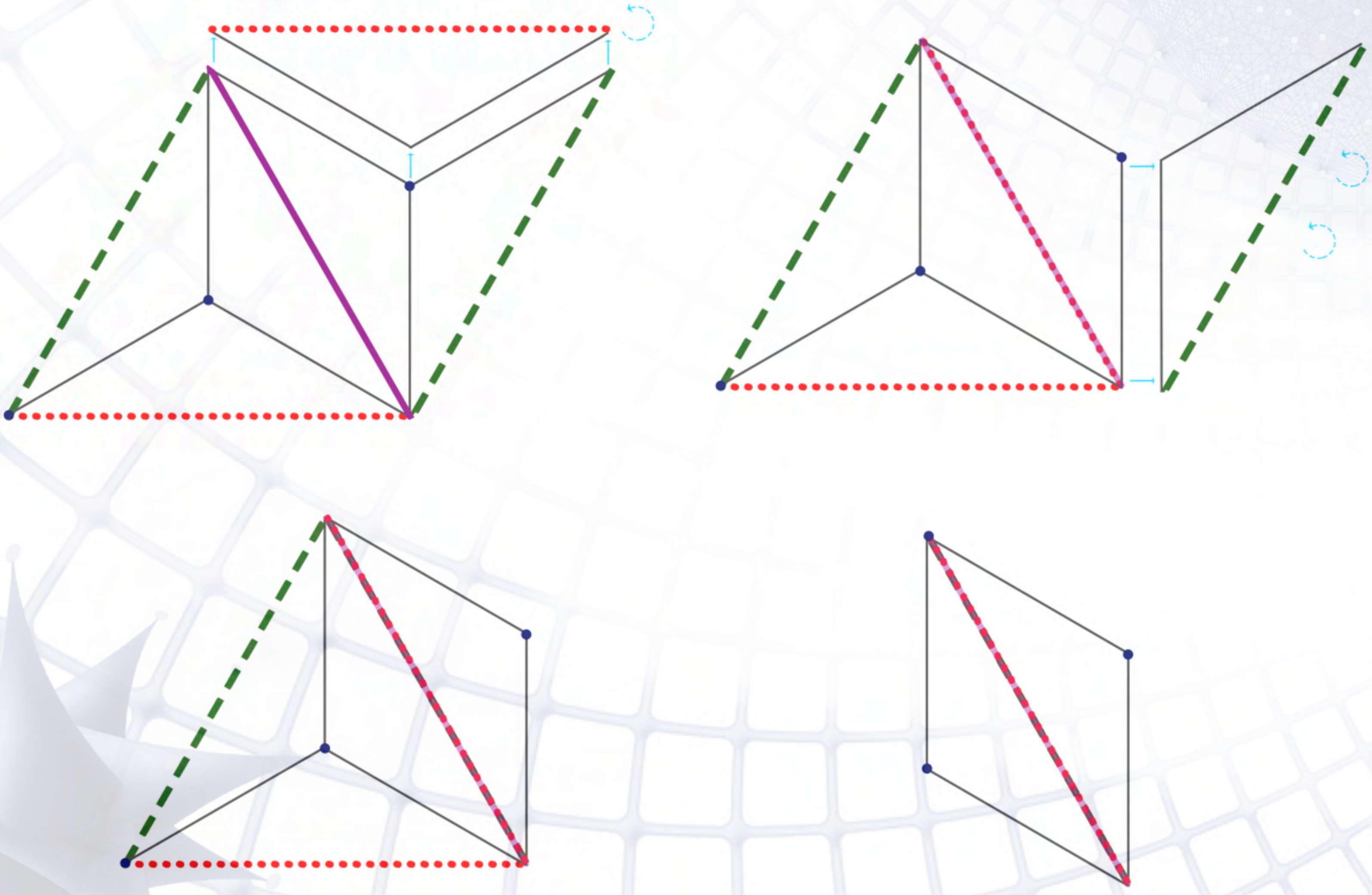
$\omega = e^{2i\pi/3}$

T^2 / Z_3

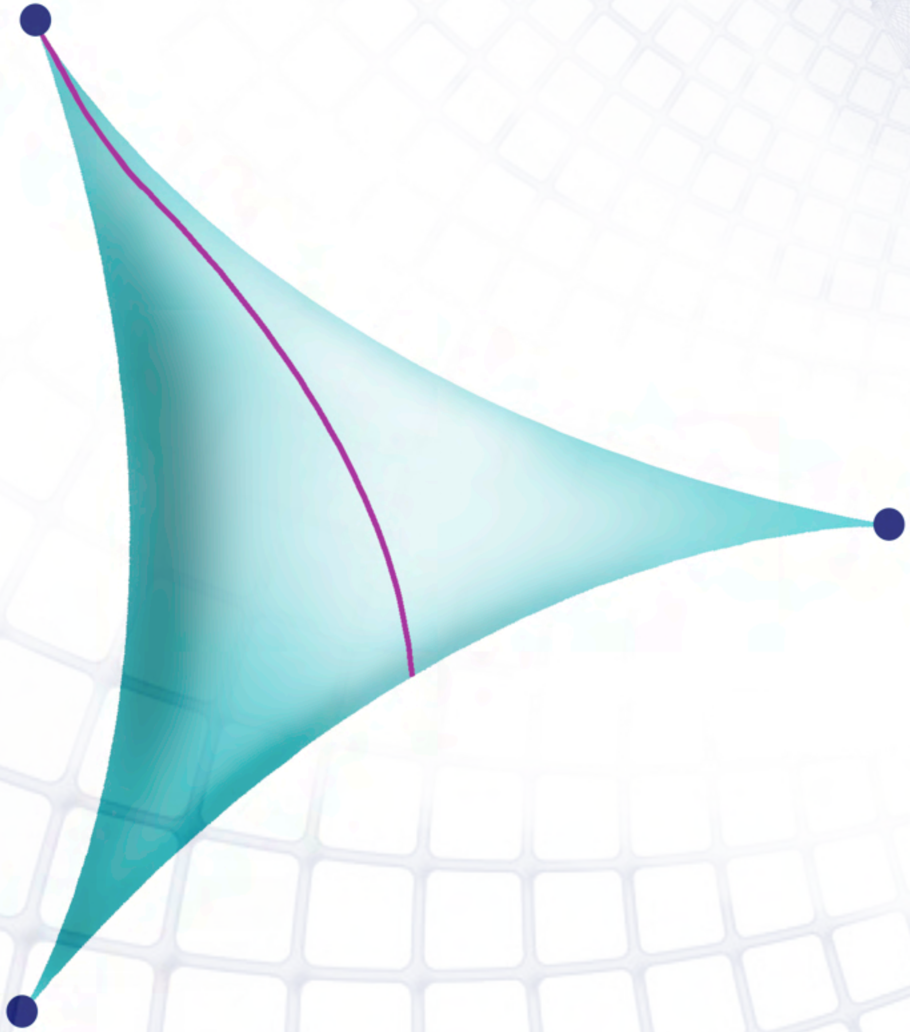




T^2 / Z_3



T^2 / Z_3



$T^6 / (Z_3 \times Z_3)$

Full Symmetry

$$S^1 \ltimes \left[(T^4 \times T^6/\mathbb{Z}^6) \ltimes \{SO(9,1)/(\mathbb{Z}_3 \times \mathbb{Z}_3)\} \right] \ltimes \; E_8$$

Orbifold Symmetry Breaking

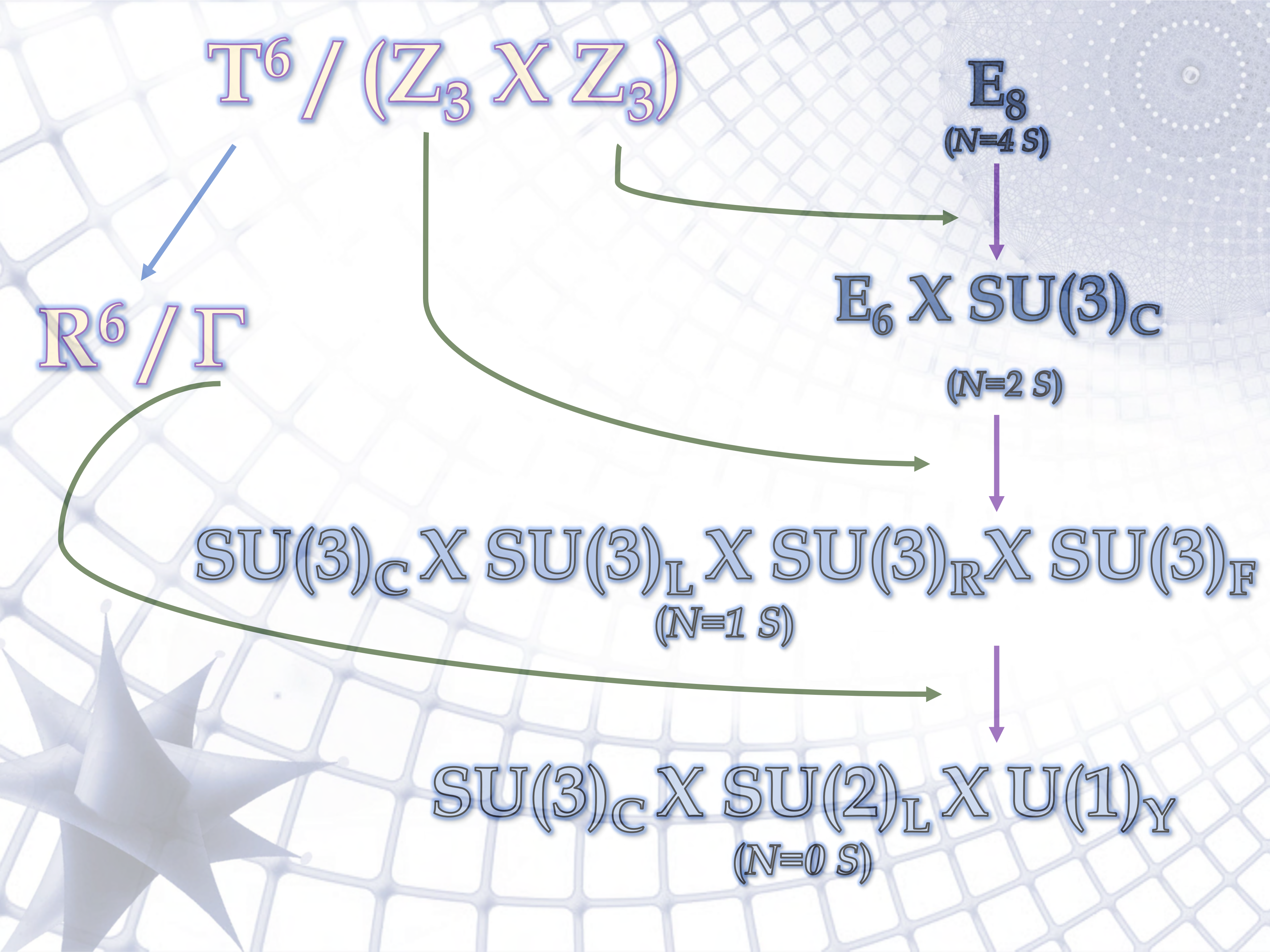
$$S^1 \ltimes \left[(T^4 \times T^6/\mathbb{Z}^6) \ltimes \{SO(9,1)/(\mathbb{Z}_3 \times \mathbb{Z}_3)\} \right] \ltimes E_8$$

Simultaneous breaking of ED SuperPoincarè and E_8 at compactification

$$\left[E_8,\ \mathbb{Z}^6 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3) \right] \neq 0$$

Wilson Line — **Orbifolding**

Nucl.Phys.B 960 (2020) 115209 2005.03048 [hep-ph]



$T^6 / (Z_3 \times Z_3)$
E_8
(N=4 S)
R^6 / Γ
$E_6 \times SU(3)_C$
(N=2 S)
$SU(3)_C \times SU(3)_L \times SU(3)_R \times SU(3)_F$
(N=1 S)
$SU(3)_C \times SU(2)_L \times U(1)_Y$
(N=0 S)

$T^6 / (Z_3 \times Z_3)$

E_8
($N=4$ S)

$SU(3)_C \times SU(2)_L \times U(1)_Y$
($N=0$ S)

Free of gauge anomalies in QFT

2007.13248 [hep-ph]

Field Content

$$\mathcal{V}(x_\mu, z_1, z_2, z_3)$$

Single 10d *N=1* gauge superfield

10d vector + 10d Majorana/Weyl fermion
in adjoint/fundamental representation (248)

Field Content

$$\mathcal{V}(x_\mu, z_1, z_2, z_3)$$

Compactification

$$V(x_\mu) \qquad \phi_1(x_\mu) \qquad \phi_2(x_\mu) \qquad \phi_3(x_\mu)$$

KK tower of

4d *N=1* SUSY: vector superfield and 3 chiral superfields

= 4d *N=4* SUSY gauge superfield

Orbifolded Field Content

$\phi_1(x_\mu)$ → $\mathbf{L} + \langle \mathbf{L} \rangle$ →

$$\begin{pmatrix} H_{di} & H_{ui} & L_i \\ \nu_i^c & e_i^c & \varphi_i \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ \langle \nu_i^c \rangle & 0 & \langle \varphi_i \rangle \end{pmatrix}$$

$\phi_2(x_\mu)$ → $\mathbf{Q}_R$ → $(d_i^c \; u_i^c \; D_i^c)$

Orbifolding

$\phi_3(x_\mu)$ → $\mathbf{Q}_L$ → $(Q_i \; D_i)$

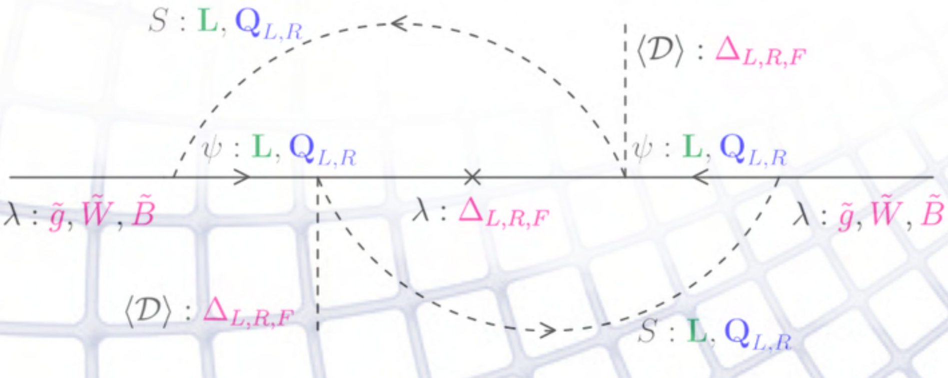
$V(x_\mu)$ → $\Delta_C + \Delta_L + \Delta_R + \Delta_F$

SUSY breaking

$$\langle \mathcal{D} \rangle \sim \langle \mathbf{L}^\dagger \mathbf{L} \rangle$$

$$m_0^2 \sim \langle \mathcal{D} \rangle$$

$$m_{1/2} \sim \frac{l^2}{\Lambda} \langle \mathcal{D} \rangle$$



SUSY breaking corrections

Quarks $\sim \mathbf{L}\mathbf{Q}_R\mathbf{Q}_L + \frac{1}{\Lambda^2}\mathbf{L}\mathbf{Q}_R\mathbf{Q}_L \left\langle \mathbf{L}^\dagger \mathbf{L} \right\rangle + \frac{1}{\Lambda^4}\mathbf{L}\mathbf{Q}_R\mathbf{Q}_L \left\langle \mathbf{L}^\dagger \mathbf{L} \right\rangle^2$

Leptons $\sim \frac{1}{\Lambda^4}\mathbf{L} \left\langle \mathbf{L}^\dagger \mathbf{L} \right\rangle \mathbf{L} \left\langle \mathbf{L}^\dagger \mathbf{L} \right\rangle \mathbf{L}$

RHN $\sim \frac{1}{\Lambda}\mathbf{L}\mathbf{L} \left\langle \mathbf{L}^\dagger \mathbf{L}^\dagger \right\rangle$

SUSY breaking corrections

Quarks $\sim \mathbf{L}\mathbf{Q}_R\mathbf{Q}_L + \frac{1}{\Lambda^2}\mathbf{L}\mathbf{Q}_R\mathbf{Q}_L \langle \mathbf{L}^\dagger \mathbf{L}\rangle + \frac{1}{\Lambda^4}\mathbf{L}\mathbf{Q}_R\mathbf{Q}_L \langle \mathbf{L}^\dagger \mathbf{L}\rangle^2$

↓ Hierarchy

Leptons $\sim \frac{1}{\Lambda^4}\mathbf{L}\langle \mathbf{L}^\dagger \mathbf{L}\rangle \mathbf{L} \langle \mathbf{L}^\dagger \mathbf{L}\rangle \mathbf{L}$

RHN $\sim \frac{1}{\Lambda}\mathbf{L}\mathbf{L}\langle \mathbf{L}^\dagger \mathbf{L}^\dagger\rangle$

SUSY breaking corrections

Quarks $\sim \mathbf{L}\mathbf{Q}_R\mathbf{Q}_L + \frac{1}{\Lambda^2}\mathbf{L}\mathbf{Q}_R\mathbf{Q}_L \langle \mathbf{L}^\dagger \mathbf{L} \rangle + \frac{1}{\Lambda^4}\mathbf{L}\mathbf{Q}_R\mathbf{Q}_L \langle \mathbf{L}^\dagger \mathbf{L} \rangle^2$

Hierarchical: Small mixing

Leptons $\sim \frac{1}{\Lambda^4}\mathbf{L} \langle \mathbf{L}^\dagger \mathbf{L} \rangle \mathbf{L} \langle \mathbf{L}^\dagger \mathbf{L} \rangle \mathbf{L}$

Not hierarchical: Large Mixing

RHN $\sim \frac{1}{\Lambda}\mathbf{L}\mathbf{L} \langle \mathbf{L}^\dagger \mathbf{L}^\dagger \rangle$

SUSY breaking corrections

Quarks $\sim \mathbf{L}\mathbf{Q}_R\mathbf{Q}_L + \frac{1}{\Lambda^2}\mathbf{L}\mathbf{Q}_R\mathbf{Q}_L\left\langle \mathbf{L}^\dagger\mathbf{L}\right\rangle + \frac{1}{\Lambda^4}\mathbf{L}\mathbf{Q}_R\mathbf{Q}_L\left\langle \mathbf{L}^\dagger\mathbf{L}\right\rangle^2$

Leptons $\sim \frac{1}{\Lambda^4}\mathbf{L}\left\langle \mathbf{L}^\dagger\mathbf{L}\right\rangle\mathbf{L}\left\langle \mathbf{L}^\dagger\mathbf{L}\right\rangle\mathbf{L}$

RHN $\sim \frac{1}{\Lambda}\mathbf{L}\mathbf{L}\left\langle \mathbf{L}^\dagger\mathbf{L}^\dagger\right\rangle$

Seesaw mechanism 6 RHN

Viable SM!!

Conceptually
Without Specifying Parameters

The Full Model

10d QFT, *N=1* Super Yang-Mills based on E_8.

A 10d vector and a 10d Weyl/Majorana fermion in the adjoint/fundamental representation (**248**).

1 Complex Parameter

Extra dimensions as

$T^6 / (Z_3 \times Z_3)$

$$\mathbb{Z}_3 : \ (x, z_1, z_2, z_3) \sim (x, \omega^2 z_1, \omega^2 z_2, \omega^2 z_3), \quad \mathcal{V} \to e^{2i\pi q_8^F/3}\, \mathcal{V}$$

$$\mathbb{Z}_3 : \ (x, z_1, z_2, z_3) \sim (x, \omega^3 z_1, \omega z_2, \omega^2 z_3), \quad \mathcal{V} \to e^{2i\pi q_8^C/3}\, \mathcal{V}$$

$$z_i \sim z_i + 2\pi R_i, \quad z_i \sim z_i + 2\pi e^{i\pi/3} R_i$$

13 Complex Parameters

Wilson line aligned with

$$\nu_i^c, \quad \varphi_i$$

$$\omega = e^{2i\pi/3}$$

The Full Model

$$\Re g + \Im g$$

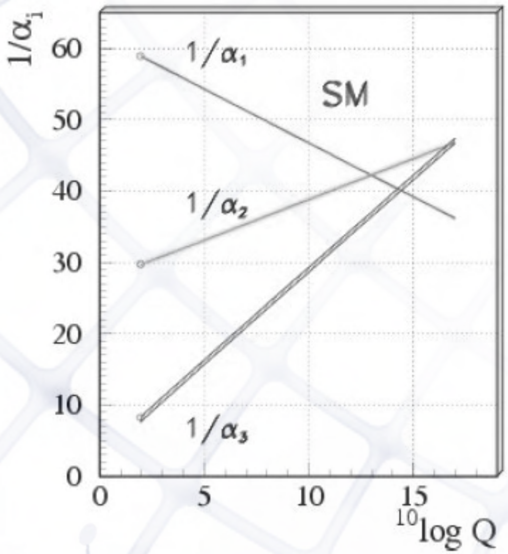
1 Complex Parameter

$T^6 / (Z_3 \times Z_3)$

$$R_1 + iR_2 \quad \begin{pmatrix} v_{di} & v_{ui} & 0 \\ \langle \nu_i^c \rangle & 0 & \langle \varphi_i \rangle \end{pmatrix}$$

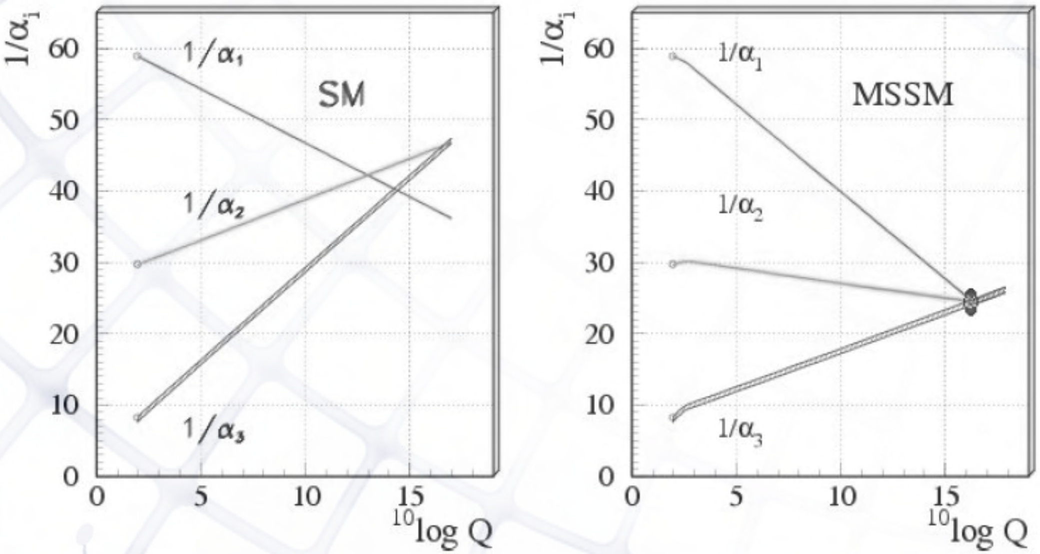
13 Complex Parameters

Gauge Coupling Unification



SM

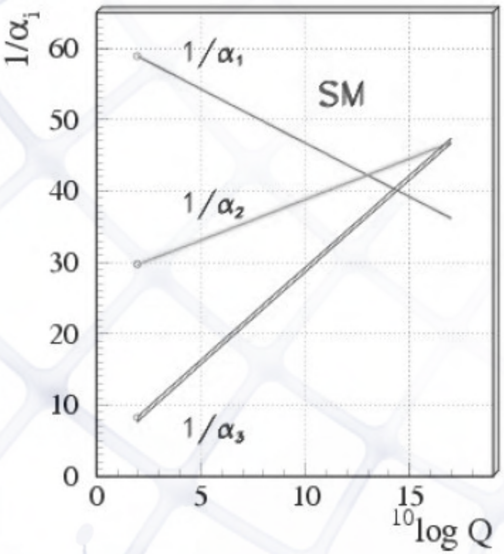
Gauge Coupling Unification

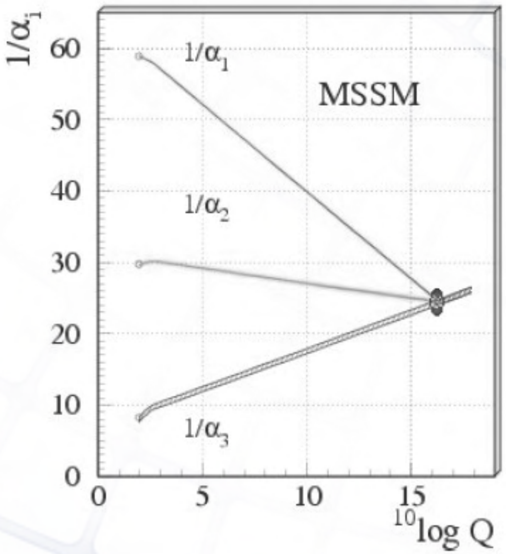


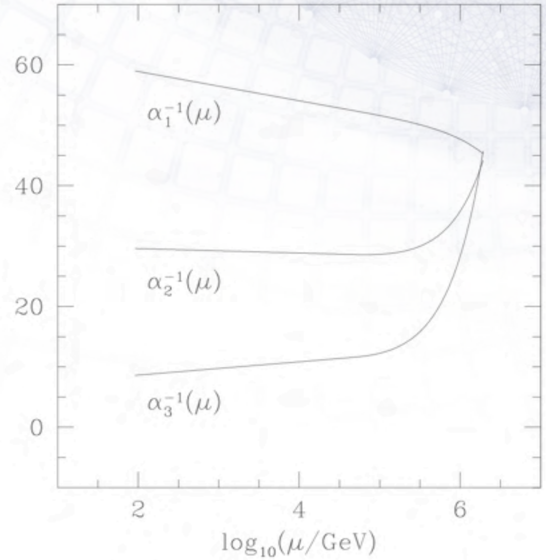
SM

+ SUSY fields

Gauge Coupling Unification





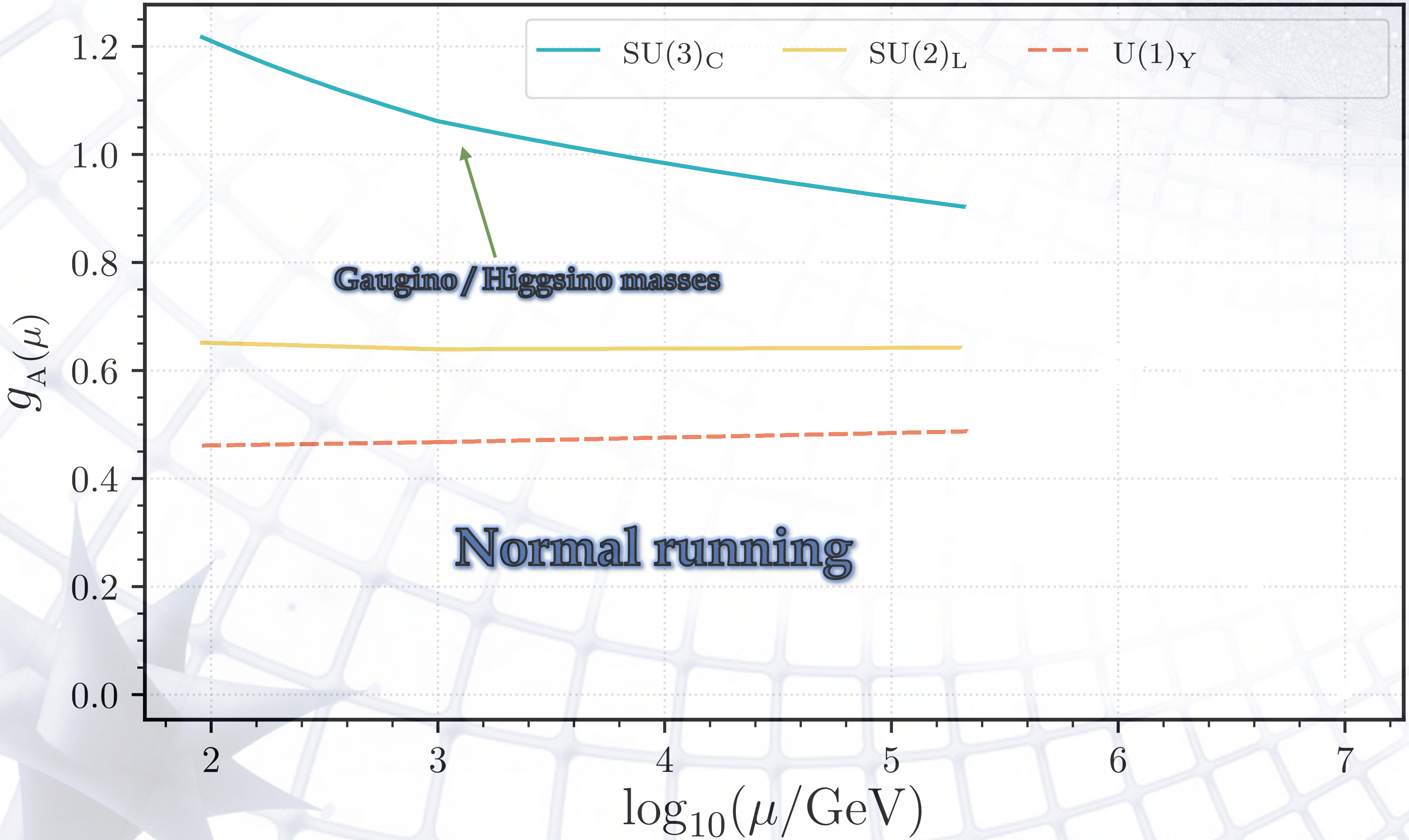


SM

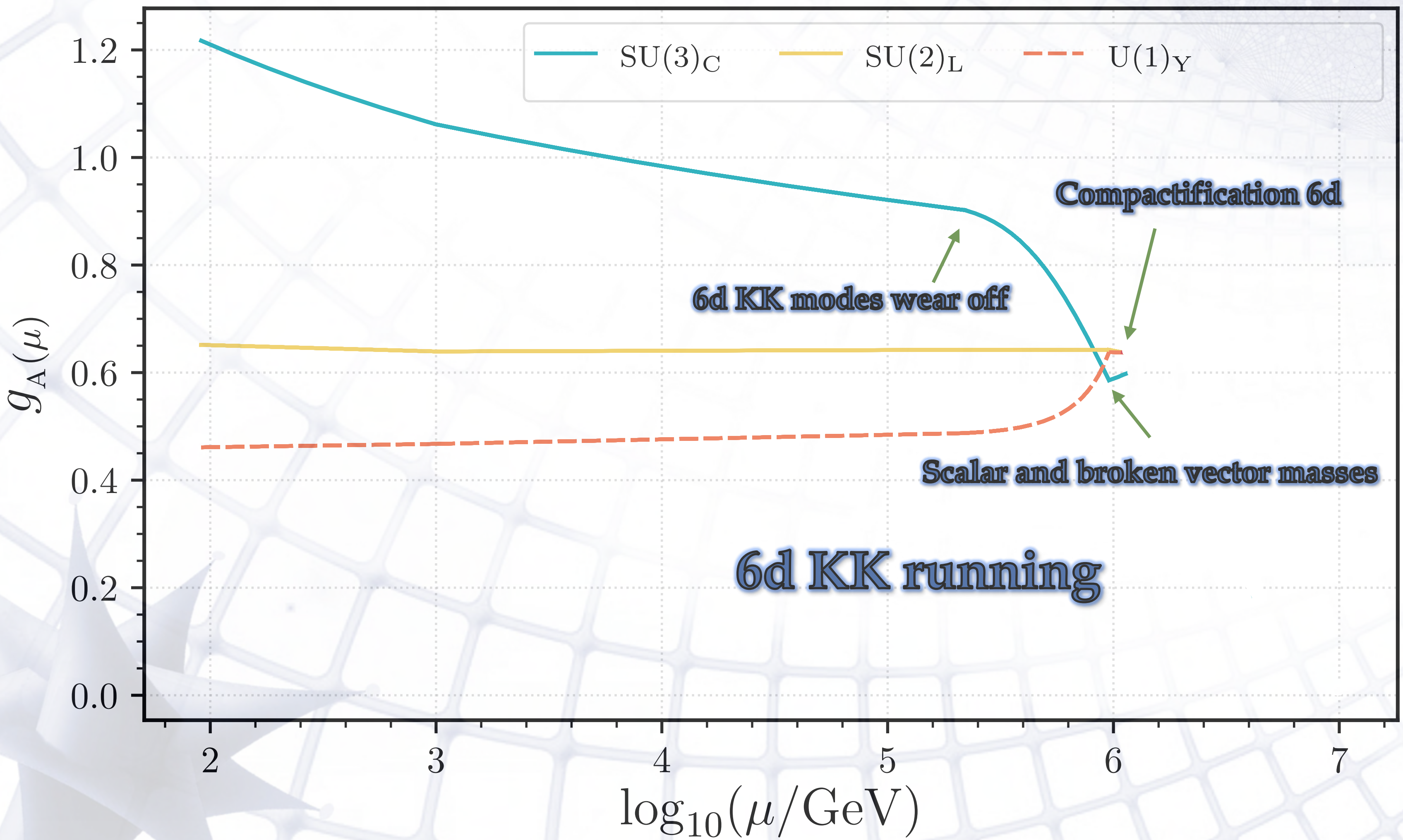
+ SUSY fields

+ KK modes

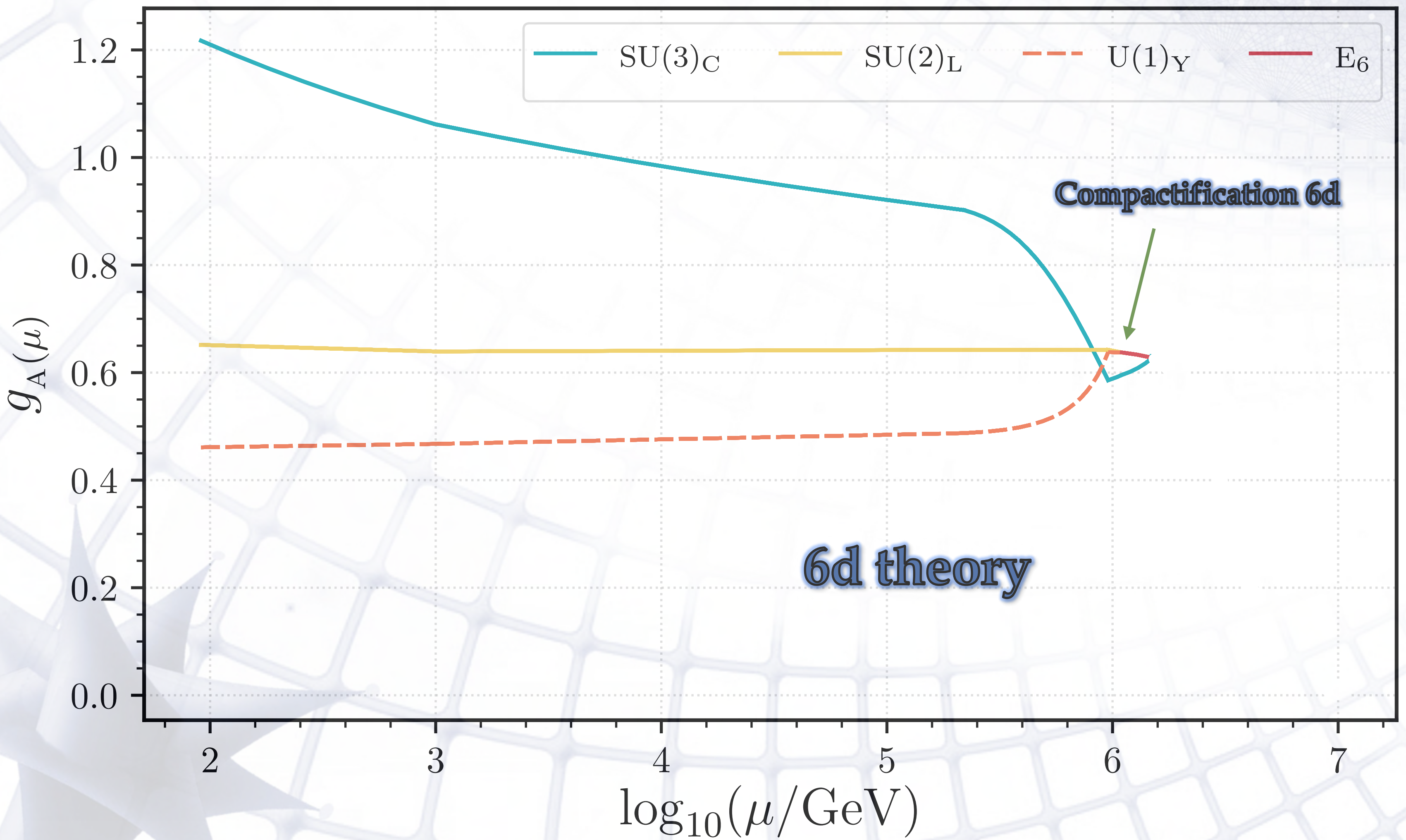
Gauge Coupling Unification



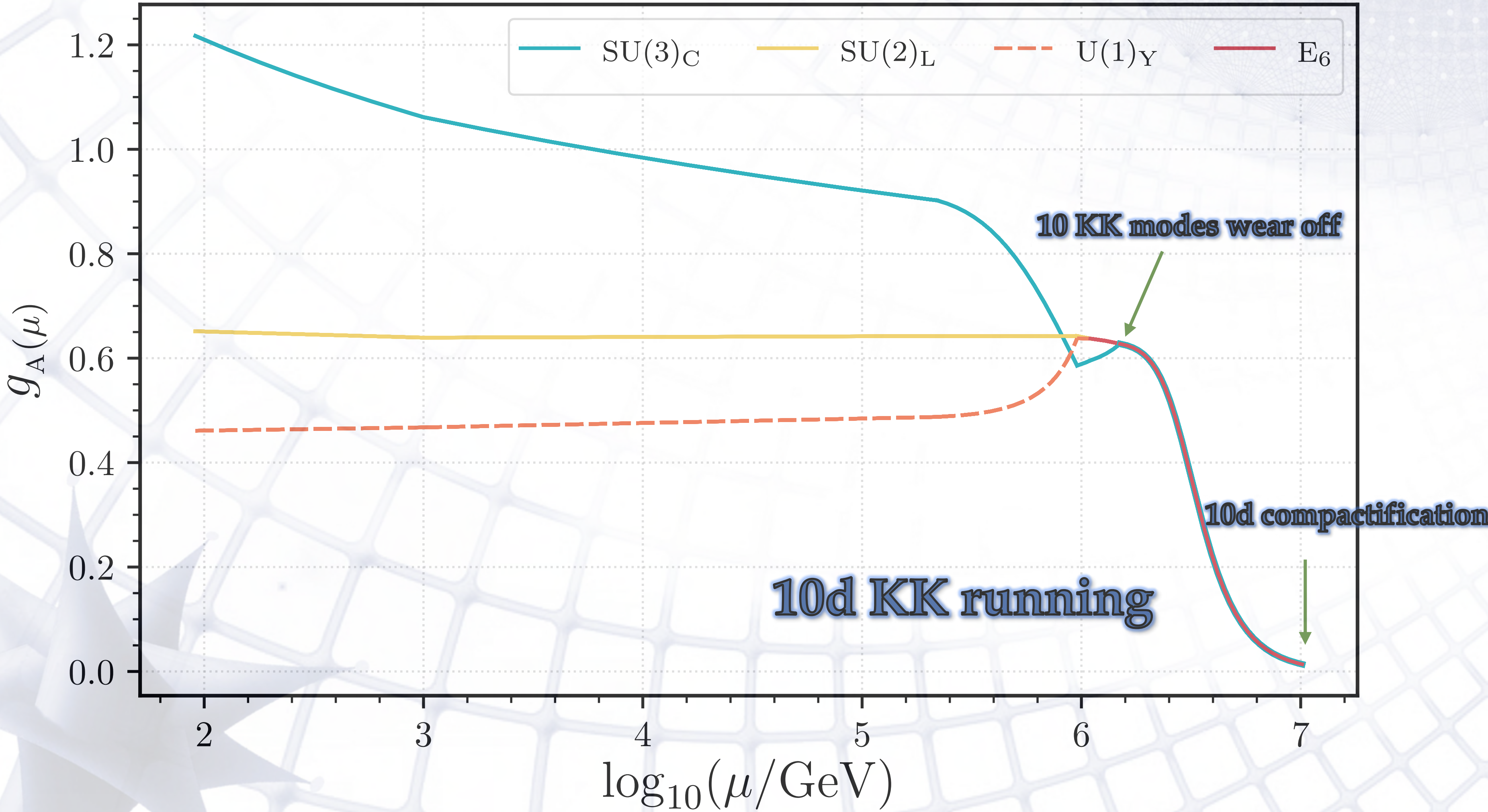
Gauge Coupling Unification



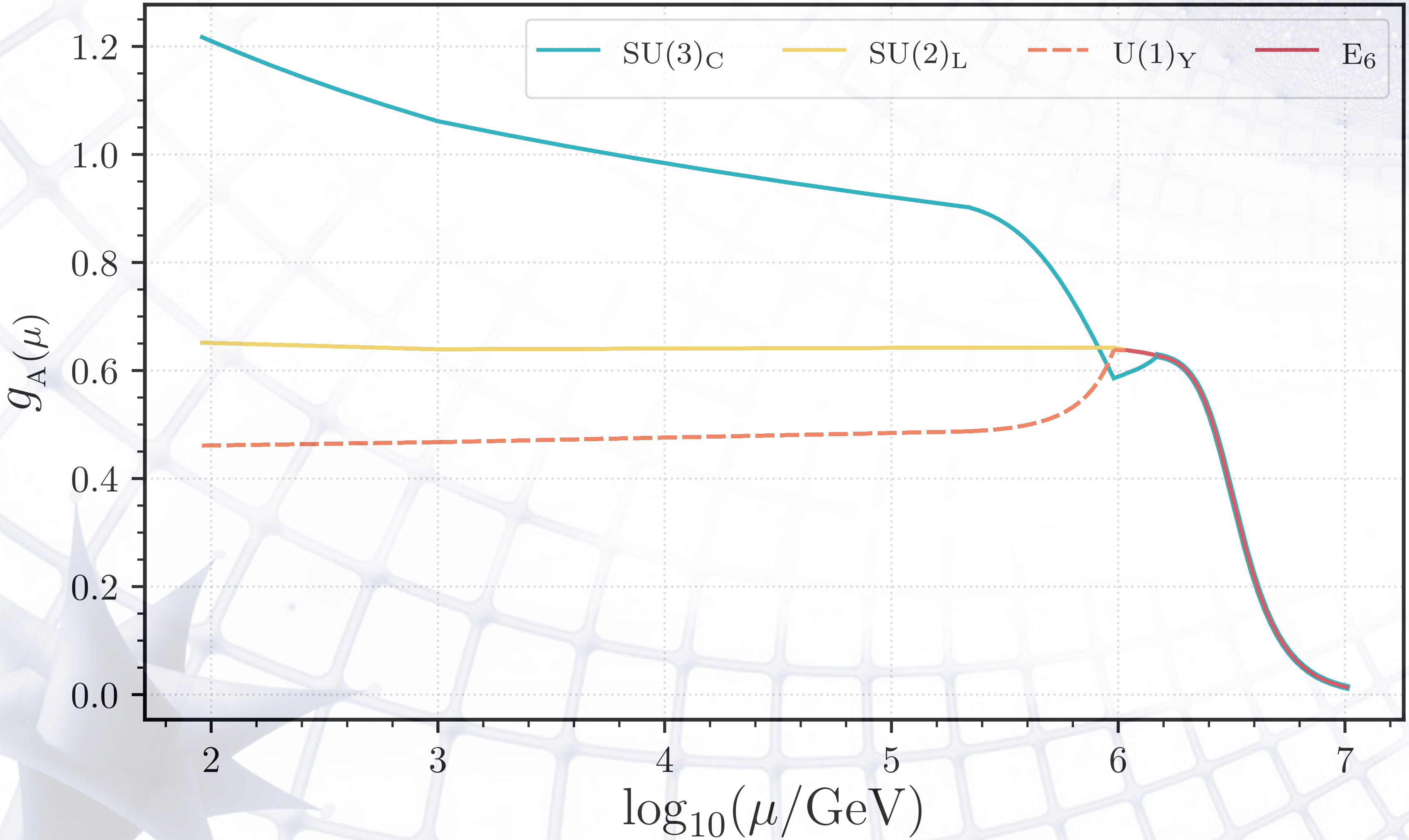
Gauge Coupling Unification



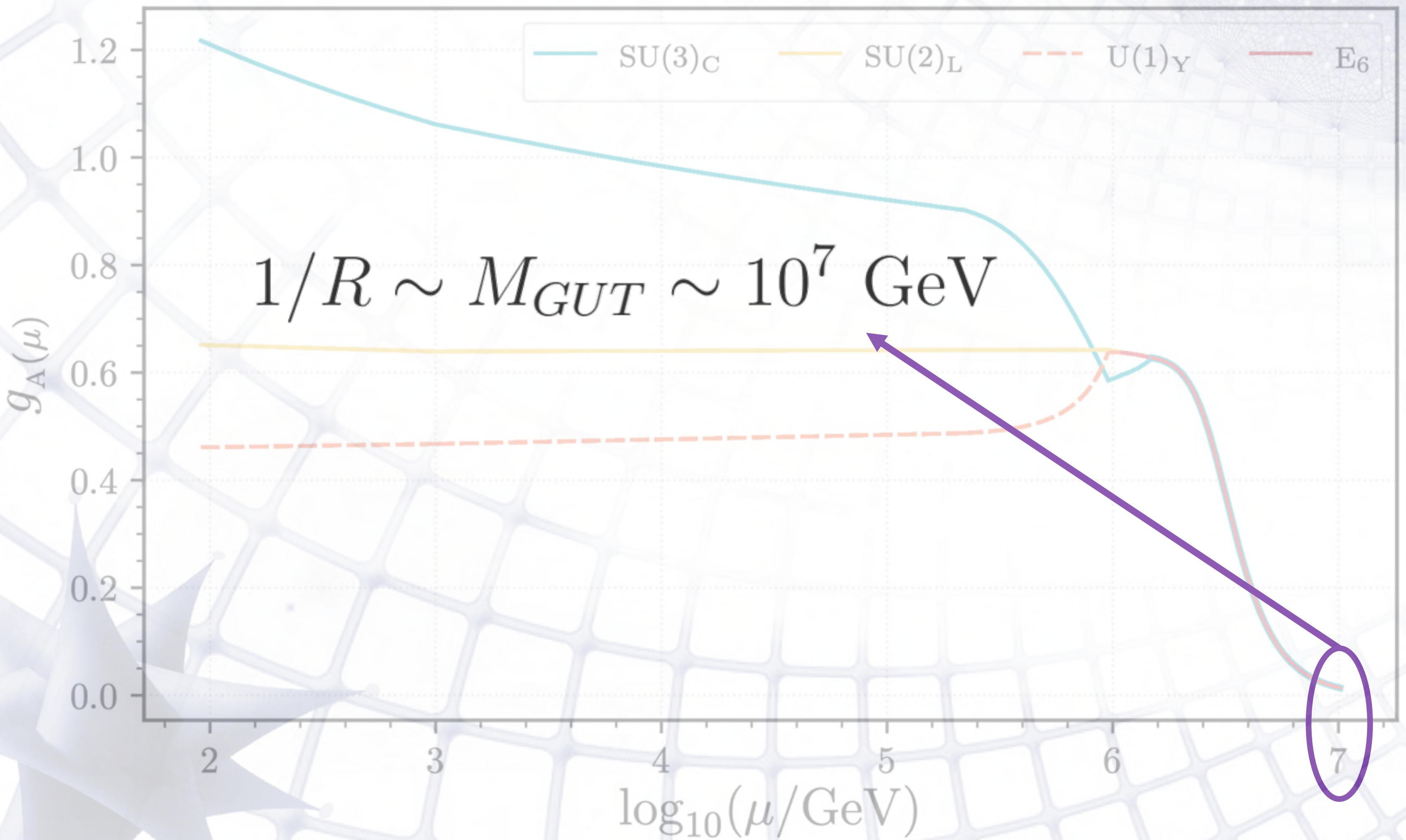
Gauge Coupling Unification



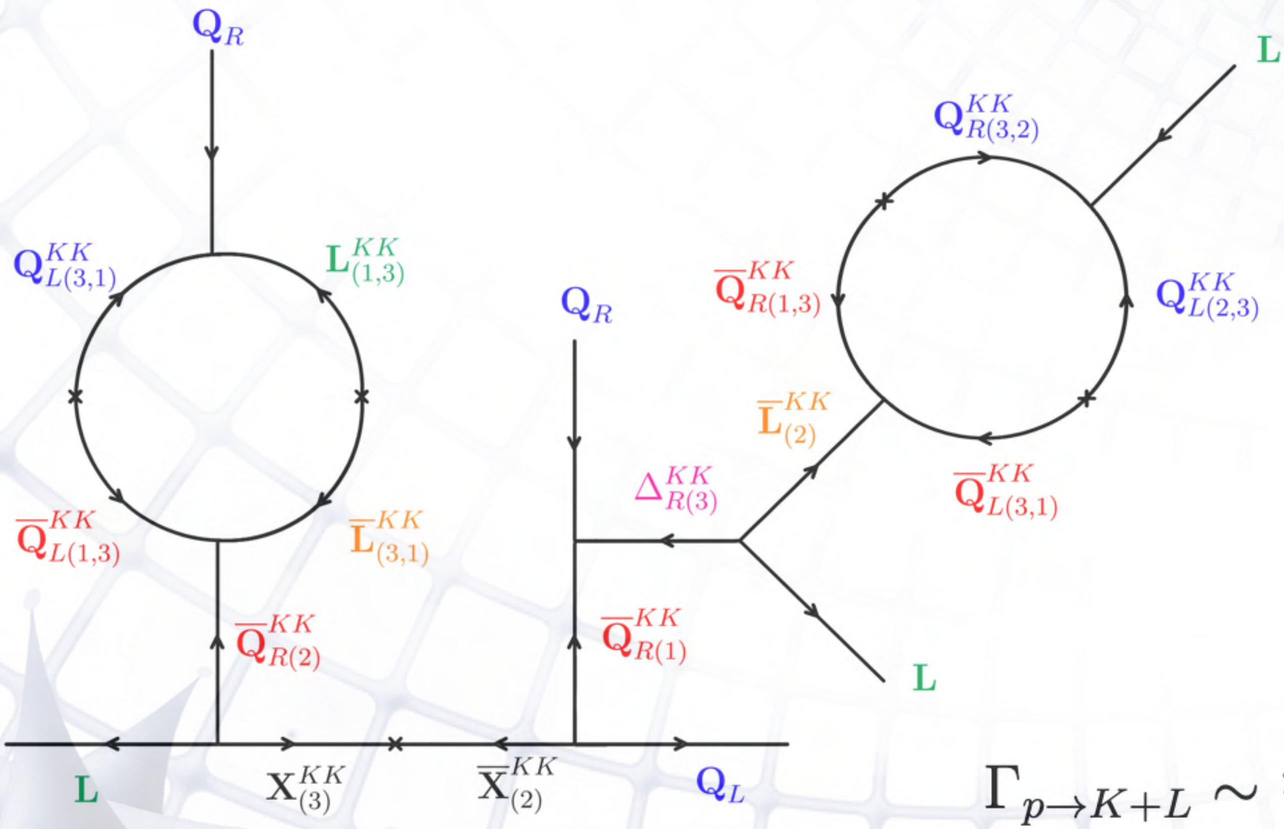
Gauge Coupling Unification



Gauge Coupling Unification



Proton Decay

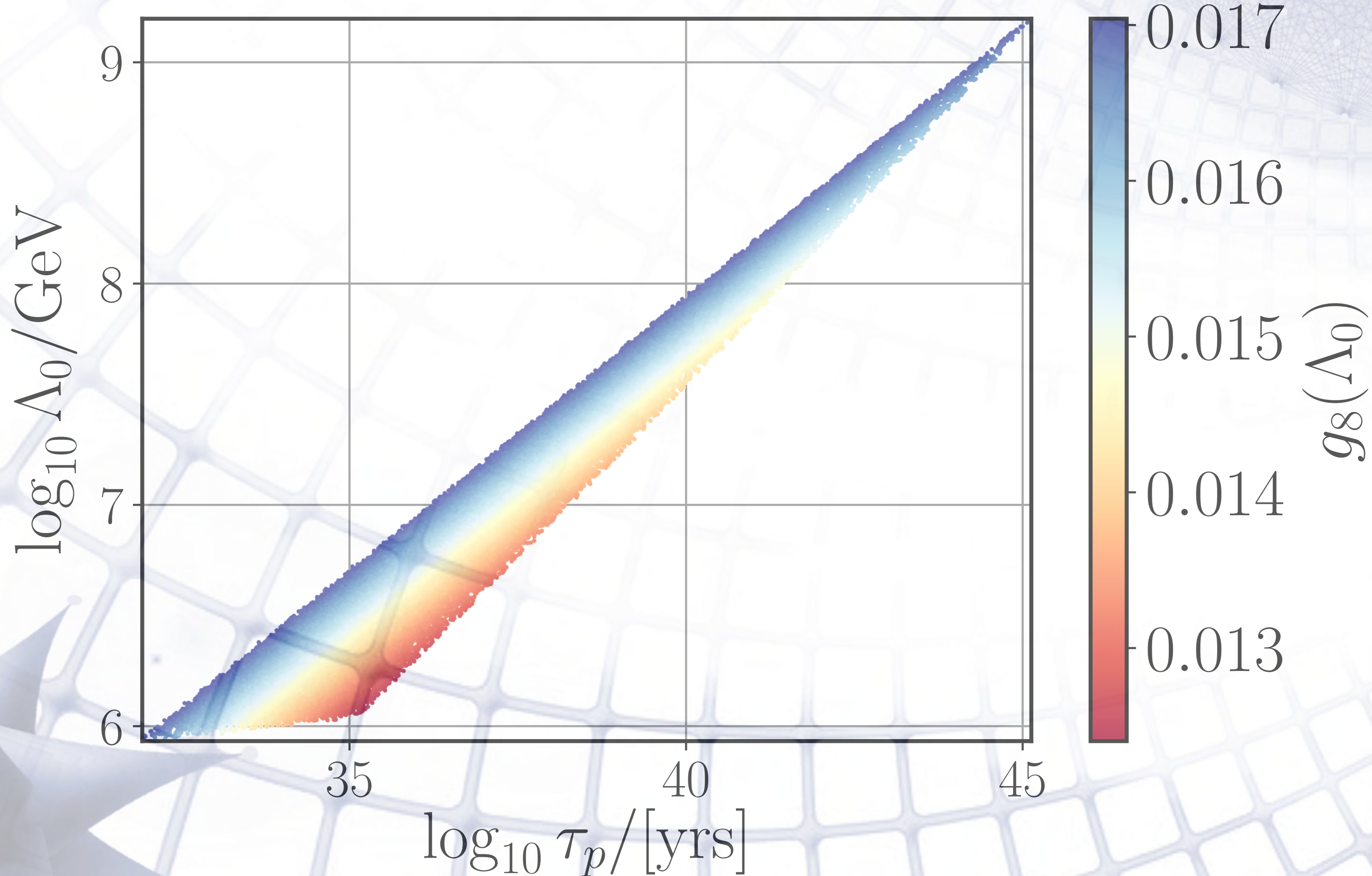


$$\Gamma_{p\to K+L} \sim \frac{g^{18} l^4}{\Lambda^4} \left\langle \tilde{\nu}^{c\dagger s} \tilde{\nu}^c_s \right\rangle^2 \frac{m_p^5}{M_X^4}$$

Suppressed by antimmetry, KK mass, loops and g powers.

Constraints

No unification



Proton decay and light gluino

$1/R \sim M_{GUT} \sim 10^6 - 10^9$ GeV

Viable PeV SUSY

Everything fits!

Within reach!!

And gravity??

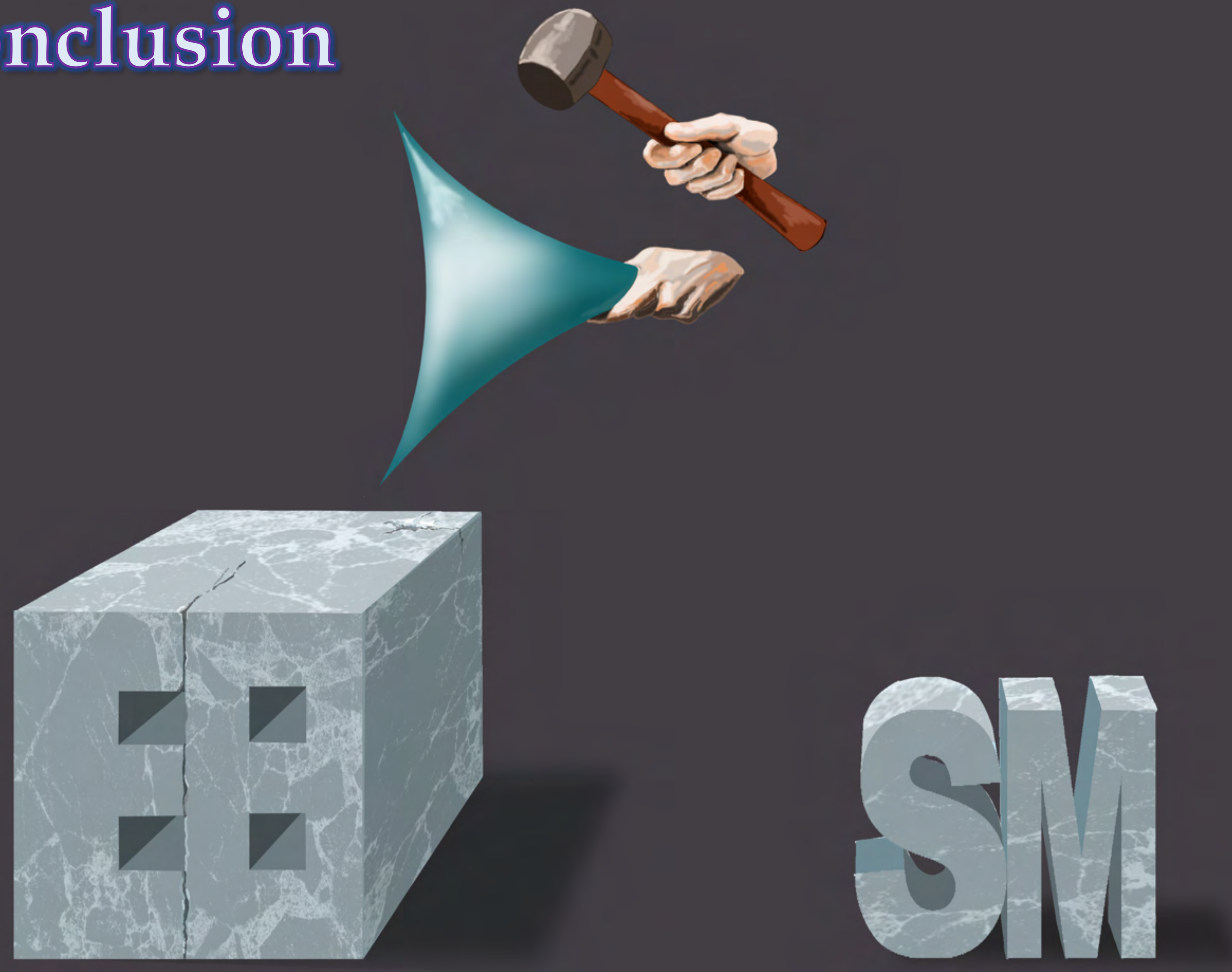
New degrees of freedom or

Emergent Gravity

Gravitational phenomena may arise from the same degrees of freedom of the original theory

Class.Quant.Grav. 37 (2020) 19, 195012 1910.03599 [hep-th]

Conclusion



A new take on E_8 Unification (short letter)

arXiv:2107.05421

Sculpting the SM from E_8 (full details)

arXiv:2107.05495

by Alfredo Aranda, Francisco J. de Anda, António P. Morais and Roman Pasechnik