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SM Extension with Anomaly Free Flavor U(1)

Based on works: arXiv: 2107.????

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Outline

- Intro: Shortcomings, Problems & Puzzles of SM → *New Physics*
- New U(1)$_{\text{Flavor}}$ model proposed:

  - Non-anomalous flavor sym. with economical setup→ texture zeros ;

  - several successful charged fermion mass patterns emerged

  - Interesting pattern for neutrino masses & mixings - predictive neutrino sector– inverted hierarchical
- Summary

Some shortcomings / puzzles of SM:

Within the SM

- **Hierarchies of Ch. fermion masses / mixings**
- **Neutrino masses / mixings**
- **Higgs vacuum stability** →

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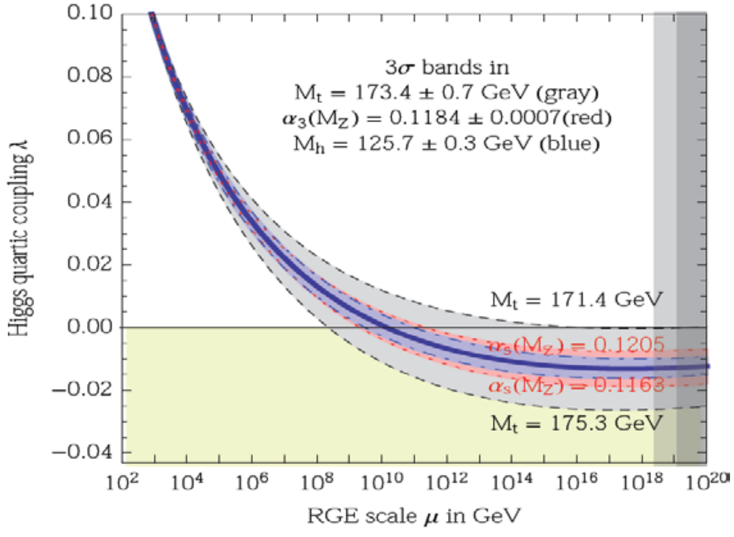

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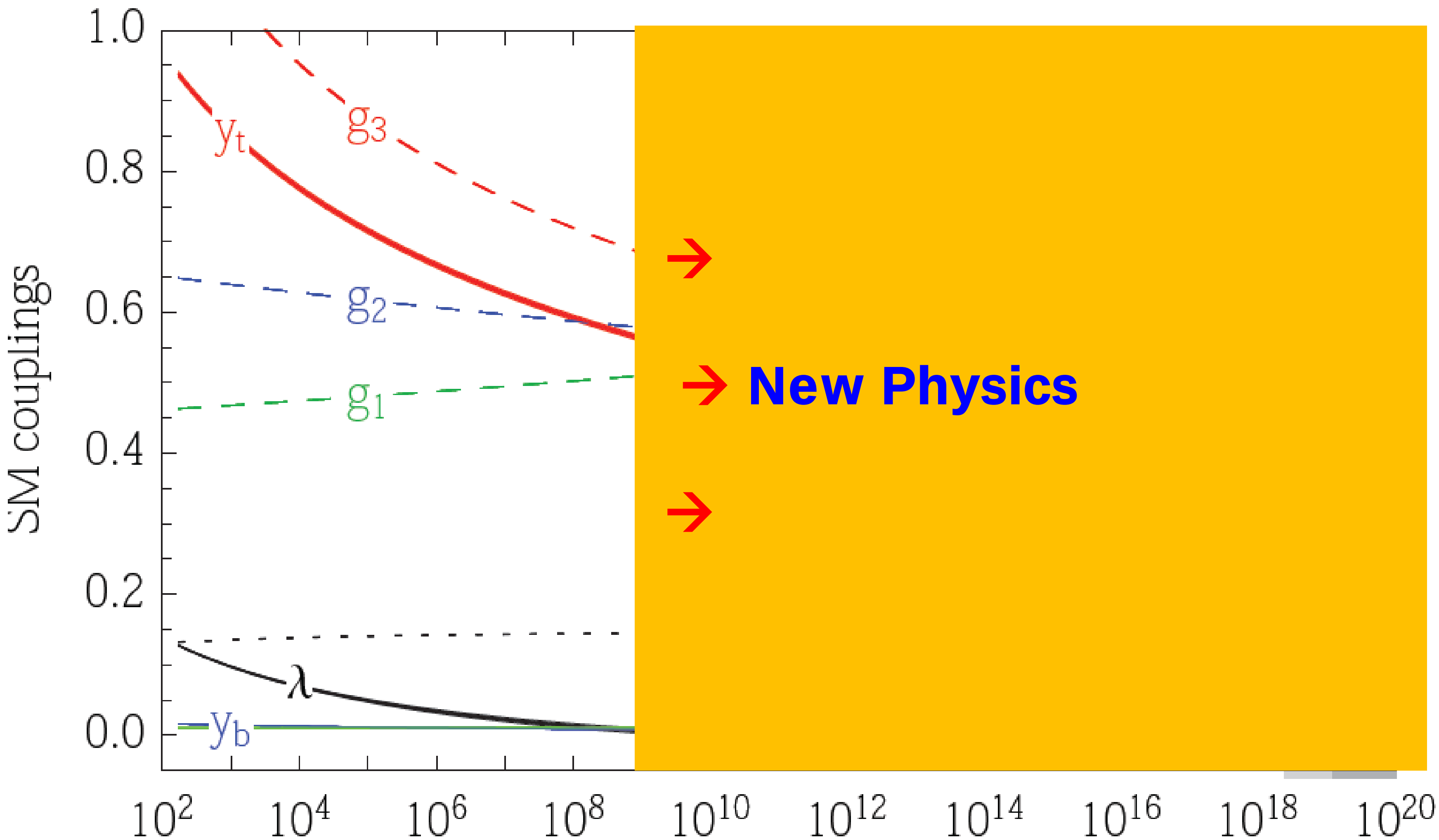


SM couplings
1.0
0.8
0.6
0.4
0.2
0.0
y_t
g_3
g_2
g_1
λ
y_b
New Physics
10^2
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
10^{18}
10^{20}

Extension With Flavor Symmetry

Flavor symmetry G_F distinguishing families can explain hierarchies

Simplest possibility: $G_F = U(1)_F$ (Froggatt, Nielsen'79)

$$U(1)_F \ : \qquad \phi_i \to e^{iQ(\phi_i)} \phi_i$$

$$Q(F_i) = n_i \ , \qquad Q(F_i^c) = \bar{n}_i \ , \qquad Q(H) = 0 \ , \qquad Q(X) = -1$$

'flavon'

With $n_i + \bar{n}_j \neq 0$: **coupling** $F_i F_j^c H$ (crossed out) **forbidden!**

$$\left(\frac{X}{M_*} \right)^{n_i + \bar{n}_j} F_i F_j^c H \longrightarrow \epsilon^{n_i + \bar{n}_j} F_i F_j^c H$$

→Suppressed couplings emerge

$\frac{\langle X \rangle}{M_*} \equiv \epsilon \ll 1$ $\quad M_*$ **- cut off scale (simplest possibility** $M_* \sim M_{\rm Pl}$**)**

Several/multiple flavons also can be considered

Possible candidates for flavor $U(1)_F$

- Global $U(1)_F$ is unattractive:

  -- Spont. breaking → pseudo-Goldstones (phen. difficulties)

  --Explicit breaking → against the ‘rules’ (selection criteria?)

Do gravity, non-perturbative effects respect global symmetries?
Trustful setting?

- Local $U(1)_F$:

  Models with gauged $U(1)_F$ are highly constrained
  due to anomaly cancellation condition

SM is anomaly free; But extra flavor $U(1)_F$ requires additional care

-- Anomalous U(1) (of stringy origin)

(Dine, Seiberg, Witten'87)

GS mechanism for anomaly cancellation.

Conditions:

$$\frac{A_{YY1}}{2k_Y} = \frac{A_{221}}{k_2} = \frac{A_{331}}{k_3} = \frac{A_{111}}{3k_1} = \frac{A_{GG1}}{24}$$

Anomaly coefficients:

$$(\text{Gravity})^2 \cdot U(1)_F : A_{GG1} = \text{Tr}[Q_{U(1)_F}]$$

$$U(1)_Y^2 \cdot U(1)_F : A_{YY1} = \sum_i Q_Y^2(i) Q_{U(1)_F}(i)$$

$$SU(1)_L^2 \cdot U(1)_F : A_{221} = \sum_i T_2(i) Q_{U(1)_F}(i) \ , \cdots$$

String Unification conds:

$$k_i g_i^2 = k_1 g_A^2 = 2 g_{st}^2$$

• • Anomalous U(1)$_F$ as flavor symmetry → successful fermion hierarchies

(Ibanez, Ross'94; Binetruy, Ramond'95; Jain, Shrock'95 ...)

-- Anomaly free $U(1)_F$

- Within MSSM, some anom. free $U(1)_F$'s with successful $Y_{U,D,E}$

(Dudas, Pokorski, Savoy, hp/9504292)

- Within MSSM & SU(5) GUT, some examples/models of anom. free U(1)F 's

(Mu-Chun Chen, et al, ph/0612017, 0801.0248)

Within SU(5) GUT: Z.T. PRD 87, 075026 ; PLB 706, 398-405

Within GUTs become more non-trivial [multiplet charges related]

Challenge to find simple anom. free $U(1)_F$ x G_{GUT}

Let's start $U(1)_F \times G_{SM}$ ...

Anomaly Constrain

Anomalies (direct and mixed) must vanish:

$$(U(1)_F)^3 : \qquad A_{111} = \sum_i Q_i^3$$

$$U(1)_Y \times (U(1)_F)^2 : \qquad A_{Y11} = \sum_i Y_i Q_i^2$$

$$(U(1)_Y)^2 \times U(1)_F : \qquad A_{YY1} = \sum_i Y_i^2 Q_i$$

$$(SU(2)_L)^2 \times U(1)_F : \qquad A_{221} = \sum_i [Q_i(l_i) + 3Q_i(q_i)]$$

$$(SU(3)_c)^2 \times U(1)_F : \qquad A_{331} = \sum_i [2Q_i(q_i) + Q_i(u_i^c) + Q_i(d_i^c)]$$

$$(\text{Gravity})^2 \times U(1)_F : \qquad A_{GG1} = \sum_i Q_i$$

Model: SM Extension with $U(1)_F$

$U(1)_F$ - gauge symmetry

X- scalar (SM singlet), for $U(1)_F$ breaking

$N_{1,2,3,4}$ - SM singlet fermions – RHN's

Table 1: $U(1)_F$ charge (Q) assignment for the model's states. $Q(X) = 1$, $Q(H) = -7$

	$\{q_1, q_2, q_3\}$	$\{u_1^c, u_2^c, u_3^c\}$	$\{d_1^c, d_2^c, d_3^c\}$	$\{l_1, l_2, l_3\}$	$\{e_1^c, e_2^c, e_3^c\}$	$\{N_1, N_2, N_3, N_4\}$
Q	$\{-11, -2, 0\}$	$\{26, 13, 7\}$	$\{-10, -1, -9\}$	$\{48, 6, -15\}$	$\{-61, -17, 6\}$	$\{-32, 10, 11, 5\}$

1) All anomalies vanish

2) This Q selection gives nice textures → Natural understanding of hierarchies

Yukawa couplings are fixed by $U(1)_F$ charges:

$$\begin{array}{c|ccc|c} Q & u^c_1(26) & u^c_2(13) & u^c_3(7) & \\ \hline q_1(-11) & \bar\epsilon^{8} & \epsilon^{5} & \epsilon^{11} & \\ q_2(-2) & \bar\epsilon^{17} & \bar\epsilon^{4} & \epsilon^{2} & H(-7) \\ q_3(0) & \bar\epsilon^{19} & \bar\epsilon^{6} & 1 & \end{array}$$

$$\begin{array}{c|ccc|c} Q & d^c_1(-10) & d^c_2(-1) & d^c_3(-9) & \\ \hline q_1(-11) & \epsilon^{14} & \epsilon^{5} & \epsilon^{13} & \\ q_2(-2) & \epsilon^{5} & \bar\epsilon^{4} & \epsilon^{4} & \tilde{H}(7) \\ q_3(0) & \epsilon^{3} & \bar\epsilon^{6} & \epsilon^{2} & \end{array}$$

$$\begin{array}{c|ccc|c} Q & e^c_1(-61) & e^c_2(-17) & e^c_3(6) & \\ \hline \ell_1(48) & \epsilon^{6} & \bar\epsilon^{38} & \bar\epsilon^{61} & \\ \ell_2(6) & \epsilon^{48} & \epsilon^{4} & \bar\epsilon^{19} & \tilde{H}(7) \\ \ell_3(-15) & \epsilon^{69} & \epsilon^{25} & \epsilon^{2} & \end{array}$$

$$\frac{X}{M_{\rm Pl}} \equiv \epsilon\,, \qquad \frac{X^*}{M_{\rm Pl}} \equiv \bar\epsilon$$

Hierarhical, good fit with: $|\epsilon| = |\bar\epsilon| \approx \mathbf{0.2}$

Some elements ≈ 0 → Texture zeros:

Neutrino Dirac & Majorana Couplings

Q

$$\begin{array}{c|ccc} & N_1(-32) & N_2(10) & N_3(11) \\ \hline \ell_1(48) & \bar{\epsilon}^{9} & \bar{\epsilon}^{51} & \bar{\epsilon}^{52} \\ \ell_2(6) & \epsilon^{33} & \bar{\epsilon}^{9} & \bar{\epsilon}^{10} \\ \ell_3(-15) & \epsilon^{54} & \epsilon^{12} & \epsilon^{11} \end{array} \; H(-7)$$

$$\begin{array}{c|ccc} & N_1 & N_2 & N_3 \\ \hline N_1 & \epsilon^{64} & \epsilon^{22} & \epsilon^{21} \\ N_2 & \epsilon^{22} & \bar{\epsilon}^{20} & \bar{\epsilon}^{21} \\ N_3 & \epsilon^{21} & \bar{\epsilon}^{21} & \bar{\epsilon}^{22} \end{array} \; M_{pl}$$

$$M_{pl} N_4 N_4 \bar{\epsilon}^{10}$$

$$\rightarrow Q = 5$$

No Yukawas & mixing with N_4

Possible to forbid:

$$N_4 \rightarrow -N_4$$

By reflection symm.

Quark Sector

Basis: $q^T Y_U u^c h_u$ $\qquad$ $q^T Y_D d^c h_d$

Parameterization:

$$Y_U \simeq \begin{pmatrix} a_1' \epsilon^8 & a_1 \epsilon^5 & 0 \\ 0 & a_2 \epsilon^4 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \lambda_t^0 \ ,$$

$$Y_D \simeq \begin{pmatrix} e^{-i\eta_1} & 0 & 0 \\ 0 & e^{-i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & b_1 \epsilon^3 & 0 \\ b_1' \epsilon^3 & b_2 \epsilon^2 & b_2' \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \kappa_b \epsilon^2$$

$\eta_{1,2}$ do not contribute to masses. Relevant for CP

Fit – Quark sector

input:

$$m_t(m_t) = 163.68 \text{ GeV} \qquad m_b(m_b) = 4.18 \text{ GeV}$$

$$\epsilon = 0.21, \quad \{a_1, a_1', a_2\} = \{0.8402,\ 1.7381,\ 1.9819\}, \quad \{\eta_1, \eta_2\} = \{-2.65608,\ -1.08617\}$$

$$\{b_1, b_1', b_2, b_2'\} = \{0.33636,\ 0.3619,\ 0.31151,\ 1.04025\}.$$

output:

$$(m_u,\ m_d,\ m_s,\ m_c)\,(2 \text{ GeV}) = (2.16,\ 4.67,\ 93.4,\ 1274)\,\text{MeV}$$

$$\text{at} \quad \mu = M_Z : \quad |V_{us}| = 0.2265, \quad |V_{cb}| = 0.04053$$

$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \qquad |V_{ub}| = 0.00361, \quad \overline{\rho} = 0.141, \quad \overline{\eta} = 0.357$$

Lepton Sector

$$Y_E \simeq \begin{pmatrix} c_1\epsilon^4 & 0 & 0 \\ 0 & c_2\epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \kappa_\tau \epsilon^2$$

input: $m_\tau(m_\tau) = 1.777 \text{ GeV}$

$$\text{at} \quad \mu = M_G \ , \qquad \{c_1, c_2\} \simeq \{0.143, 1.33\}$$

output:

$$m_e(m_e) = 0.511 \text{ MeV}, \qquad m_\mu(m_\mu) = 105.66 \text{ MeV}$$

Neutrino Sector

No important contribution from Y_E

Y_E^{diag} basis → Lepton mixing matrix U

$$\bar{M}_\nu = U^* M_\nu^{\mathrm{Diag}} U^\dagger$$

$$U = P_1 \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P_2$$

$$P_1 = \mathrm{Diag}\left(e^{-i\omega_1}\ ,\ e^{-i\omega_2}\ ,\ e^{-i\omega_3}\right)\ ,\qquad P_2 = \mathrm{Diag}\left(1\ ,\ e^{-i\rho_1/2}\ ,\ e^{-i\rho_2/2}\right)$$

Neutrino Dirac & Majorana Matrices

$$m_D \simeq \begin{pmatrix} A\epsilon^9 & 0 & 0 \\ 0 & B_1\epsilon^9 & C_1\epsilon^{10} \\ 0 & B_2\epsilon^{12} & C_2\epsilon^{11} \end{pmatrix} v \ , \quad M_R \simeq \begin{pmatrix} 0 & a\epsilon^2 & d\epsilon \\ a\epsilon^2 & b & c\epsilon \\ d\epsilon & c\epsilon & \epsilon^2 \end{pmatrix} \bar{c} M_{Pl} \epsilon^{20}$$

See-saw→

$$\bar{M}_\nu \simeq -m_D M_R^{-1} m_D^T \simeq \begin{pmatrix} \beta & \gamma & \gamma' \\ \gamma & \alpha^2 & \alpha \\ \gamma' & \alpha & 1 \end{pmatrix} \bar{m}$$

$$\bar{M}_\nu^{(2,2)} \bar{M}_\nu^{(3,3)} - (\bar{M}_\nu^{(2,3)})^2 \simeq 0$$

Relations→

$$\tan^2\theta_{13} = \frac{m_3}{m_2}\left| s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2 \right|$$

$$2\delta = \pi - \rho_2 + \mathrm{Arg}\left(s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2 \right)$$

Predict inverted hierarchical neutrinos!

(Z.T. PRD 87, 075026)

With input: $\{\alpha, \beta, \gamma, \gamma'\} = \{0.67, 1.38e^{-2.71i}, 0.97, 0.88e^{0.35i}\}$

$\bar{m} = 0.0259\ \mathrm{MeV}$

→ Perfect Fit: $\{\sin^2\theta_{12}, \sin^2\theta_{23}, \sin^2\theta_{13}\} = \{0.306, 0.587, 0.02179\}$

$$\Delta m^2_{\rm sol} = m_2^2 - m_1^2 = 7.5 \cdot 10^{-5}\mathrm{eV}^2, \quad \Delta m^2_{\rm atm} = m_2^2 - m_3^2 = 2.515 \cdot 10^{-3}\mathrm{eV}^2$$

All hierarchies, needed values Realized by original parameters' natural values:

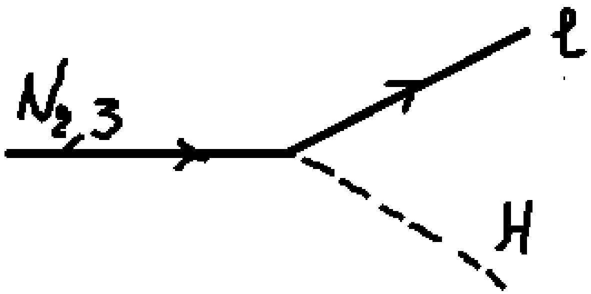
$$\{A, B_1, B_2, C_1, C_2\} = \{2.5, 2.5, 2, 3, -1\}$$

$$\{a, b, c, d, \bar{c}\} = \{3.05e^{-0.97i}, 0.61, 0.78, 0.17e^{-0.97i}, 1.1e^{1.42i}\}$$

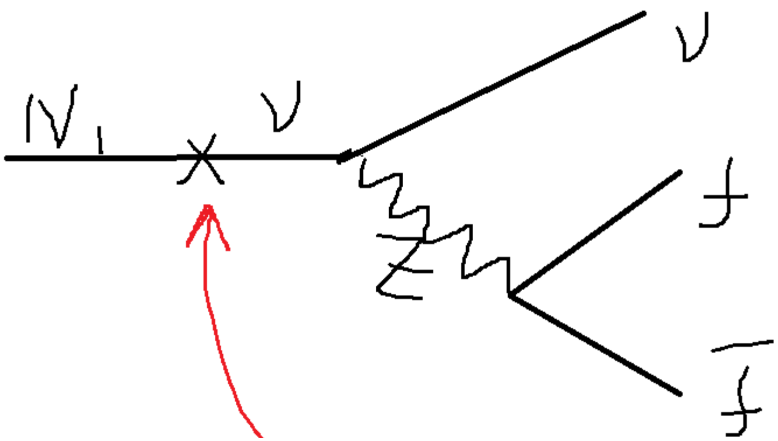
Consistency (with BBN)

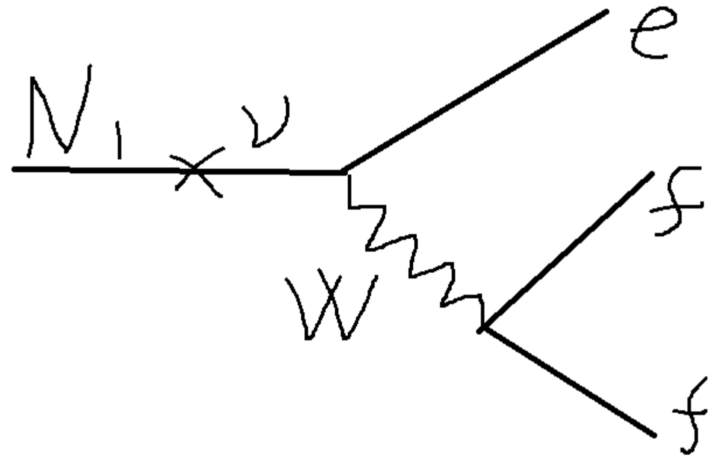
$M_{N_{1,2,3,4}} = \{1.6,\ 2\cdot 10^{3},\ 5\cdot 10^{4},\ 4\cdot 10^{11}\}$ **GeV**

$N_{2,\,3}$ **Decay quickly**



N_1 **Decays – mixing with ν's**





$\text{mixing} \sim 6\times 10^{-6}$

$$\Gamma(N_1) \simeq 1/(0.027\ \text{sec.})$$

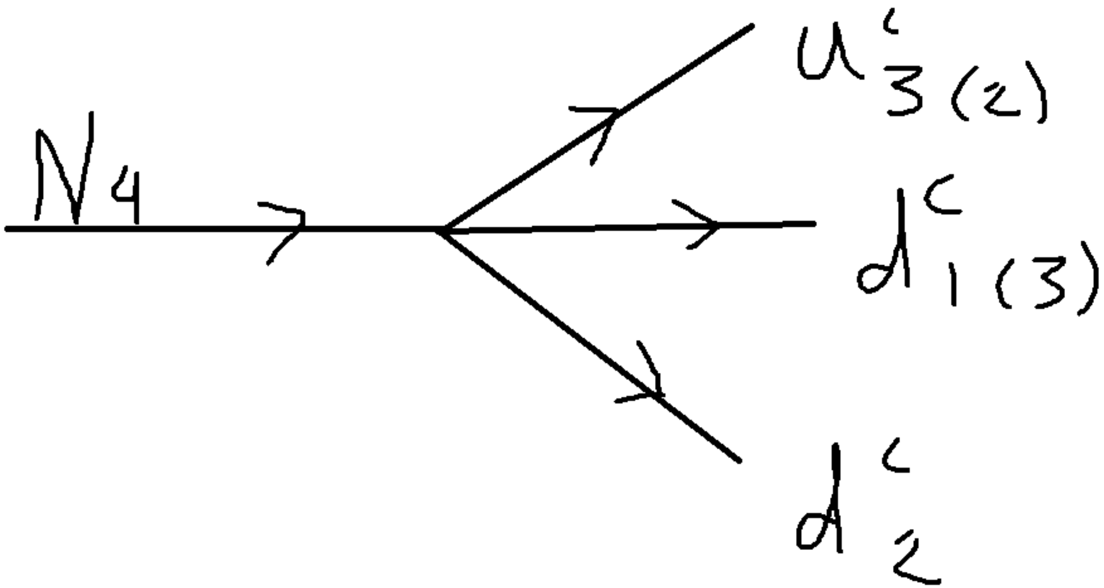
Consistency (with BBN)

N_4 Decays – due to d=6 operator couplings

$$\frac{1}{M_{Pl}^2}\left(\bar{\epsilon}(N_4 u_3^c)(d_1^c d_2^c) + \epsilon(N_4 u_2^c)(d_2^c d_3^c)\right)$$

Consistent with symmetry

$$N_4 \to -N_4$$
$$(q, u^c, d^c) \to -(q, u^c, d^c)$$



$$\Gamma(N_4) \simeq 1/(10^{-4}\text{sec.})$$

Higgs Vacuum Stability - 'λ_h -problem'

Within SM (1-loop):

$$16\pi^2 \frac{d}{dt}\lambda_h = 12\lambda_h^2 + 12\lambda_h\lambda_t^2 - 12\lambda_t^4 - \frac{3}{2}\lambda_h(3g_2^2 + g_1^2) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)$$

$$16\pi^2 \frac{d}{dt}\lambda_t = \frac{9}{2}\lambda_t^3 - \lambda_t\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)$$

Higgs Vacuum Stability - 'λ_h -problem'

New quartic interactions involving X:

$$V^{(4)} = \frac{\lambda_h}{2}(H^\dagger H)^2 + \lambda'(H^\dagger H)(X^\dagger X) + \frac{\lambda_X}{2}(X^\dagger X)^2$$

$$16\pi^2 \frac{d}{dt}\lambda_h = \beta_{SM} + (\lambda')^2$$

Can easily solve the problem – with $M_\chi < 10^{10}\ \mathrm{GeV}$

(see e.g. Lebedev 1203.0156)

$$X = (V_X + \frac{\chi}{\sqrt{2}})e^{i\theta}$$

SUMMARY

- SM extension with $U(1)_{\text{Flavor}}$ model proposed:
- Non-anomalous flavor sym. → texture zeros;
- Successful ch. fermion mass hierarchies /mixings;
- Desirable Neutrino (inverted hierarchical) oscillations
- Higgs vac. Stability can be easily achieved

-- Interesting be GUT embedding [SU(5), SO(10)] – more predictive?

Thank You

Backup Slides

• Charged fermion masses & mixings

Observed Noticeable Hierarchies:

$$\lambda_t \sim 1 \ , \qquad \lambda_u : \lambda_c : \lambda_t \sim \lambda^8 : \lambda^4 : 1$$

$$\lambda_b \sim \lambda_\tau \sim \frac{m_b}{m_t} \tan\beta \ , \qquad \lambda_d : \lambda_s : \lambda_b \sim \lambda^4 : \lambda^2$$

With λ=0.2

$$\lambda_e : \lambda_\mu : \lambda_\tau \sim \lambda^5 : \lambda^2 : 1$$

$$V_{us} \approx \lambda \ , \qquad V_{cb} \approx \lambda^2 \ , \qquad V_{ub} = \lambda^4 - \lambda^3$$

What is origin of these hierarchies?
Is there any relation or sum rule?
Why three families?

Within SM no answer to these questions...

Neutrino Data

← **hep-ph: 1611.01514**

global 3ν oscillation analysis

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 0.83$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.271 \to 0.345$
$\theta_{12}/°$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$31.38 \to 35.99$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.385 \to 0.635$	$0.587^{+0.020}_{-0.024}$	$0.393 \to 0.640$	$0.385 \to 0.638$
$\theta_{23}/°$	$41.6^{+1.5}_{-1.2}$	$38.4 \to 52.8$	$50.0^{+1.1}_{-1.4}$	$38.8 \to 53.1$	$38.4 \to 53.0$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.01934 \to 0.02392$	$0.02179^{+0.00076}_{-0.00076}$	$0.01953 \to 0.02408$	$0.01934 \to 0.02397$
$\theta_{13}/°$	$8.46^{+0.15}_{-0.15}$	$7.99 \to 8.90$	$8.49^{+0.15}_{-0.15}$	$8.03 \to 8.93$	$7.99 \to 8.91$
$\delta_{CP}/°$	261^{+51}_{-59}	$0 \to 360$	277^{+40}_{-46}	$145 \to 391$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.03 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.524^{+0.039}_{-0.040}$	$+2.407 \to +2.643$	$-2.514^{+0.038}_{-0.041}$	$-2.635 \to -2.399$	$\begin{bmatrix} +2.407 \to +2.643 \\ -2.629 \to -2.405 \end{bmatrix}$

Ivan Esteban,[a] M. C. Gonzalez-Garcia,[a,b,c] Michele Maltoni,[d] Ivan Martinez-Soler,[d] Thomas Schwetz[e]

Evidences for New Physics: Neutrino Data

- **Origin of these scales and mixings?**

Unexplained in SM $\leftarrow \; \mathbf{m}_\nu \lesssim \mathbf{10^{-4}}\ \mathrm{eV}$

$$m_\nu \sim \frac{M_{EW}^2}{M_{Pl}}$$

Without New Physics

Neutrino masses via see-saw

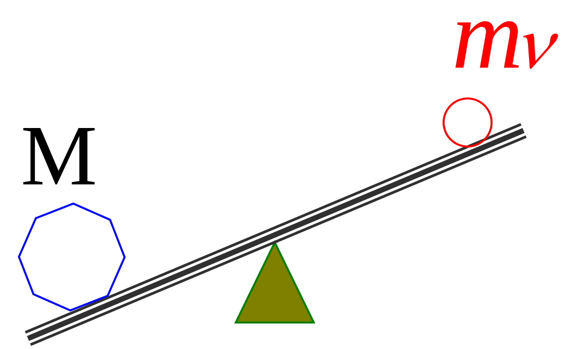
+ RHN - ν_R

→ Oscillations

→ Leptogenesis

$\nu^c \equiv N \sim (1,1,0)$ ← **SM singlet**

$l\, N \langle H \rangle$ $\quad M\, N\, N$ → **Δ L=2 Lepton number viol.**



$$\begin{pmatrix} 0 & \langle H \rangle \\ \langle H \rangle & M \end{pmatrix}$$

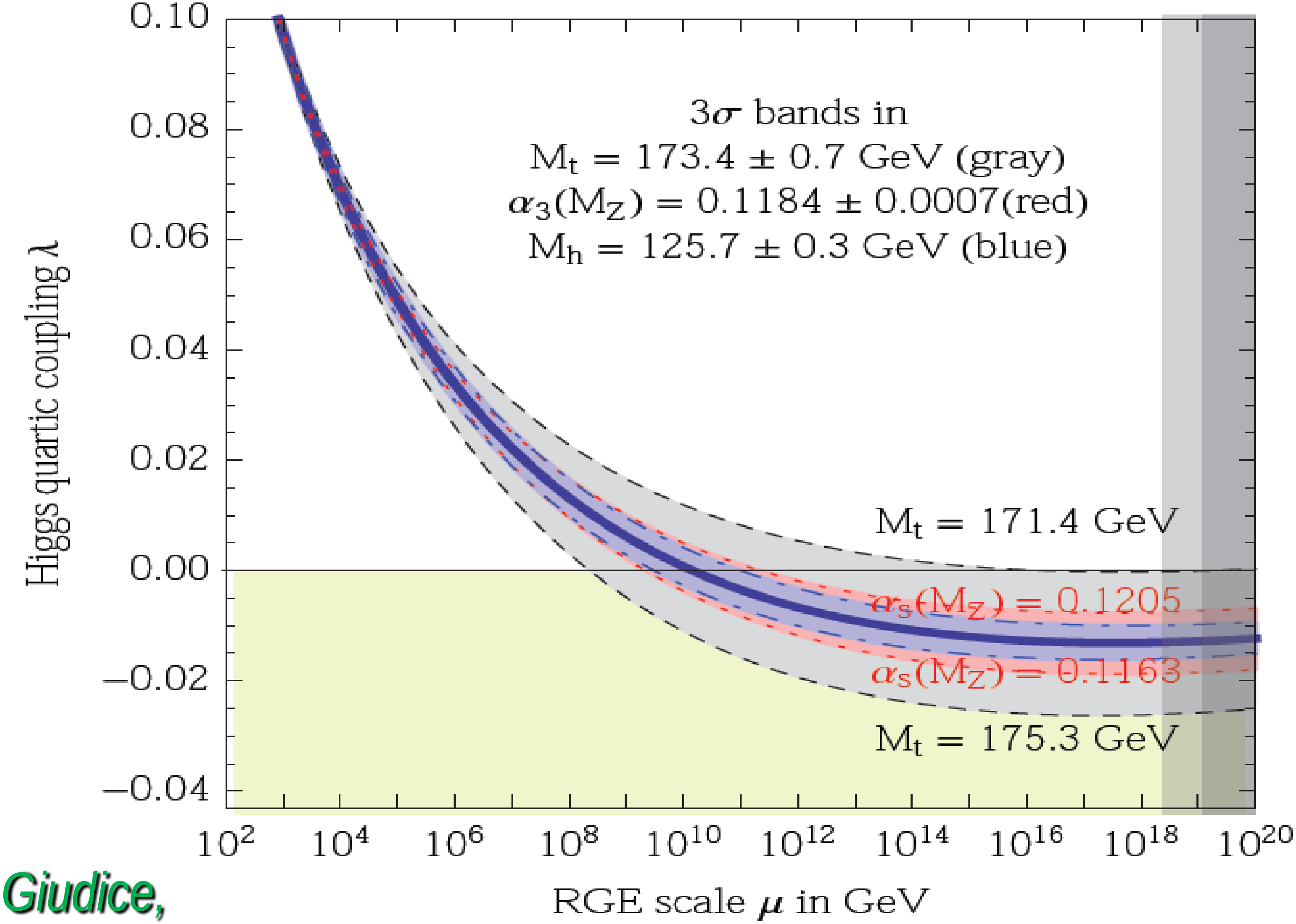
$$m_\nu \sim \frac{\langle H \rangle^2}{M}$$

$$M_N \simeq M$$

$$M \sim 10^{14}\ \mathrm{GeV} \ \rightarrow \ m_\nu \sim \mathrm{few} \cdot 0.01\ \mathrm{eV}$$

- **Higgs vacuum stability**

$$V = -\frac{1}{2}m^2|H|^2 + \lambda|H|^4$$



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, arX: 1307.3536

Some related works:

-- Within MSSM, anom. free $U(1)_F$'s with successful $Y_{U,D,E}$

Dudas, Pokorski, Savoy, hp/9504292;

-- Within MSSM & SU(5) GUT, some examples/models of anom. free $U(1)_F$'s : *Mu-Chun Chen, et al, ph/0612017, 0801.0248;*