Electroweak Symmetry Non-restoration in UV-complete Models with New Fermions

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INTRODUCTION

Electroweak symmetry is restored at high temperature in Standard Model and most of the BSM theories. For examples,

1. SM:
$$\frac{\partial^2 V_1^{th}}{\partial h^2}\Big|_{h=0} = T^2\left(\frac{3}{16}g^2 + \frac{1}{16}g'^2 + \frac{1}{4}\lambda_t^2 + \frac{1}{2}\lambda\right)$$
2. SM + real singlet scalar ($\mathbb{Z}_2$):
$$\frac{\partial^2 V_1^{th}}{\partial h^2}\Big|_{h=0} = T^2\left(\frac{3}{16}g^2 + \frac{1}{16}g'^2 + \frac{1}{4}\lambda_t^2 + \frac{1}{2}\lambda + \frac{1}{12}\lambda_{hs}\right)$$

EWSNR VIA NEW SCALARS

In some models, Electroweak symmetry was always broken (SNR) or only temporary restored (TR).

1. SM + singlet scalar s_i with $O(N_s)$ global symmetry (Meade & Ramani,1807.07578):

$$V = V_{SM} + \frac{1}{2}\mu_s^2(s_i s_i) + \frac{1}{4}\lambda_s(s_i s_i)^2 + \frac{1}{2}\lambda_{hs}h^2(s_i s_i)$$

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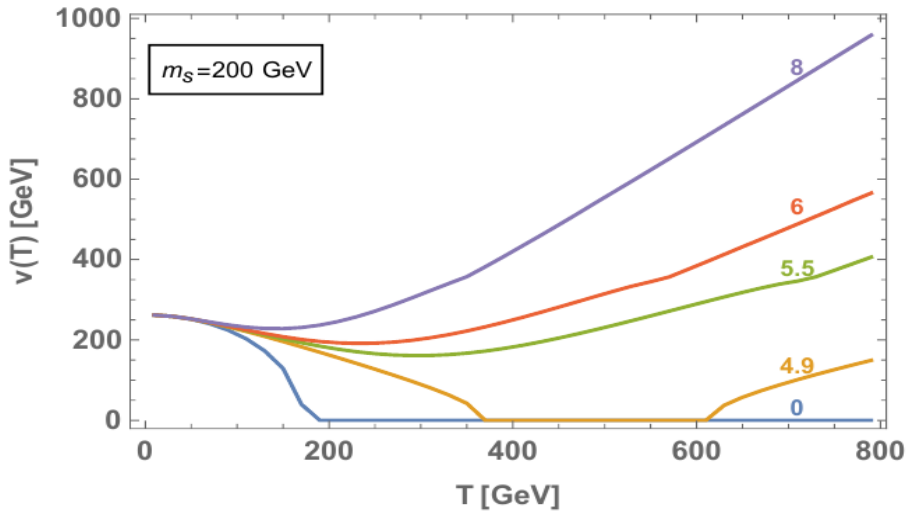

Temperature dependent vev for different values of $N_s|\lambda_{hs}|$.

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2. SM + Inert Higgs Doublet + singlet scalar($O(N_s)$) (Carena et al.,2104.00638)
3. 2HDM + real singlet scalar (Heinemeyer et at.,2103.12707)

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Is it possbile to achive SNR or TR for EW symmetry by adding new fermions to SM?

EWSNR VIA NEW FERMIONS

In high temperature limit (i.e. when $m_i^2 \ll T^2$) , (Matsedonskyi,Servant, 2002.05174)

$$\frac{\partial^2 V_{1,F}^{th}}{\partial h^2}\Big|_{h=0} = \sum_i T^2 \frac{n_F}{48} \frac{\partial^2 m_i^2}{\partial h^2} = T^2 \frac{n_F}{48} \frac{\partial^2}{\partial h^2} \sum_i m_i^2$$

$$= T^2 \frac{n_F}{48} \frac{\partial^2}{\partial h^2} \mathrm{Tr}\left(M_f^\dagger M_f\right) = T^2 \frac{n_F}{48} \frac{\partial^2}{\partial h^2} \sum_{i,j} |M_{ij}|^2$$

In renormalizable models, $M_{ij} = a_0 + a_1 h$, hence $\frac{\partial^2 V_{1,F}^{th}}{\partial h^2}\Big|_{h=0} \geq 0$. Thus, it is impossible to achieve EWSNR by adding new fermions!

SINGLET-DOUBLET MODEL

$$L_{L,R} = \begin{bmatrix} N \\ E \end{bmatrix}_{L,R} \sim (1,2)_{-\frac{1}{2}}, \; N'_{L,R} \sim (1,1)_0$$

$$\begin{aligned} \mathcal{L}^i_{mass} &= -y_1 \overline{L_L} \widetilde{\phi} N'_R - y_2 \overline{N'_L} \widetilde{\phi}^\dagger L_R - m_L \overline{L_L} L_R - m_{N'} \overline{N'_L} N'_R + h.c. \\ &= -\overline{F_L} M F_R \end{aligned}$$

$$F_{L,R} = \begin{bmatrix} N' \\ N \\ E \end{bmatrix}, \quad M = \begin{bmatrix} m_{N'} & m_{NN'2} & 0 \\ m_{NN'1} & m_L & 0 \\ 0 & 0 & m_L \end{bmatrix}, \quad m_{NN'i} = y_i h/\sqrt{2},$$

$$\phi = \begin{bmatrix} G^+ \\ (h + iG^0)/\sqrt{2} \end{bmatrix}, \quad \widetilde{\phi} = i\sigma_2 \phi^*.$$

The mass matrix can be diagonalized by biunitary transformation. The physical masses are m_L, m_{N1}, m_{N2}.

$$m_{N1}^2 = \frac{1}{2}\left(A_N^2 - \sqrt{\left(A_N^2\right)^2 - 4\left(\Delta_N^2\right)^2}\right)$$
$$m_{N2}^2 = \frac{1}{2}\left(A_N^2 + \sqrt{\left(A_N^2\right)^2 - 4\left(\Delta_N^2\right)^2}\right)$$

where

$$A_N^2 = \left|m_{NN'1}\right|^2 + \left|m_{NN'2}\right|^2 + \left|m_{N'}\right|^2 + \left|m_L\right|^2$$
$$\Delta_N^2 = \left|m_L m_{N'} - m_{NN'1} m_{NN'2}\right|$$

$$\left.\frac{\partial^2 V_{1,F}^{th}}{\partial h^2}\right|_{h=0} = \frac{T^2}{12}\frac{\partial^2}{\partial h^2}\sum_{N1,N2,L} m_i^2 > 0$$

What if the heavier fermion (N_2) is decoupled from the thermal bath?

When $m_L^2 \gg m_{N'}^2,\ \frac{1}{2}|y_1 y_2|h^2,\ T^2$,

$$m_{N1}^2 \approx m_{N'}^2 - \frac{m_{N'}\,\mathrm{Re}(y_1 y_2)}{m_L}h^2, \quad m_{N2}^2 \approx m_L^2 + \frac{m_{N'}\,\mathrm{Re}(y_1 y_2)}{m_L}h^2$$

$$\left.\frac{\partial^2 V_{1,F}^{th}}{\partial h^2}\right|_{h=0} = \frac{T^2}{12}\frac{\partial^2}{\partial h^2}m_{N1}^2 = -\frac{T^2}{6}\frac{m_{N'}\,\mathrm{Re}(y_1 y_2)}{m_L}$$

Thus, $\left.\frac{\partial^2 V_{1,F}^{th}}{\partial h^2}\right|_{h=0} < 0$ is possible by choosing $\mathrm{Re}(y_1 y_2) > 0$.
$m_L^2 >> T^2$ is possible if new fermions gain their masses through some dynamical mechanism.

Model I: Singlet-doublet fermions

+ real scalar singlet + scalar singlet with $O(N_\rho)$

$$L^i_{L,R} = \begin{bmatrix} N^i \\ E^i \end{bmatrix}_{L,R} \sim (1,2)_{-\frac{1}{2}},\ N'^i_{L,R} \sim (1,1)_0,\ E'^i_{L,R} \sim (1,1)_{-1}$$

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{i=1}^{N_F} \left(\mathcal{L}^i_{kin}(L, N', E') + \mathcal{L}^i_{yuk}\right) + \mathcal{L}_{kin}(\sigma, \rho) - V_0(\sigma, \rho)$$

$$\begin{aligned}\mathcal{L}^i_{yuk} = &- y^i_{NN'1}\overline{L^i_L}\widetilde{\phi}N'^i_R - y^i_{NN'2}\overline{N'^i_L}\widetilde{\phi}^\dagger L^i_R - y^i_{EE'1}\overline{L^i_L}\phi E'^i_R - y^i_{EE'2}\overline{E'^i_L}\phi^\dagger L^i_R \\ &- y^i_L\sigma\overline{L^i_L}L^i_R - y^i_{N'}\sigma\overline{N'^i_L}N'^i_R - y^i_{E'}\sigma\overline{E'^i_L}E'^i_R + h.c.\end{aligned}$$

$$\begin{aligned}V_0(\sigma, \rho) = &- \frac{1}{2}\mu^2_\sigma\sigma^2 + \frac{1}{4}\lambda_\sigma\sigma^4 \\ &- \frac{1}{2}\mu^2_\rho\rho_i\rho_i + \frac{1}{4}\lambda_\rho\left(\rho_i\rho_i\right)^2 + \frac{1}{4}\lambda_{\sigma\rho}\sigma^2\rho_i\rho_i\end{aligned}$$

The physical masses (for each copy of new fermions) are:

$$m_{N1}^2 = \frac{1}{2}\left(A_N^2 - \sqrt{\left(A_N^2\right)^2 - 4\left(\Delta_N^2\right)^2}\right)$$
$$m_{N2}^2 = \frac{1}{2}\left(A_N^2 + \sqrt{\left(A_N^2\right)^2 - 4\left(\Delta_N^2\right)^2}\right)$$

where

$$A_N^2 = \left|m_{NN'1}\right|^2 + \left|m_{NN'2}\right|^2 + \left|m_{N'}\right|^2 + \left|m_L\right|^2$$
$$\Delta_N^2 = \left|m_L m_{N'} - m_{NN'1} m_{NN'2}\right|$$
$$m_{NN'i} = \frac{1}{\sqrt{2}} y_{NN'i} h, \ \ (i = 1, 2)$$
$$m_x = y_x \sigma, \ \ (x = L, N', E')$$

m_{E1}, m_{E2} are same as equations above, except $N \to E$.

At high T, when $(y_L v_\sigma)^2 \gg T^2$,

$$\partial_h^2 V_1^{th}(h=0, v_\sigma) = -a_h T^2 \,,$$
$$a_h = \frac{N_F}{6y_L}\left(y_{N'} y_{NN'1} y_{NN'2} + y_{E'} y_{EE'1} y_{EE'2}\right)$$
$$- \left(\frac{3}{16}g^2 + \frac{1}{16}g'^2 + \frac{1}{4}y_t^2 + \frac{1}{2}\lambda_h\right) .$$

- When $\frac{N_F}{6y_L}\left(y_{N'} y_{NN'1} y_{NN'2} + y_{E'} y_{EE'1} y_{EE'2}\right)$ is large enough, the negative contribution outweigh the usual positive contributions from SM particles, thus $\partial_h^2 V_{eff}(h=0, v_\sigma) < 0$, and $v_h \neq 0$ at high T.

$$\partial_\sigma^2 V_1^{th}(h=0,\sigma=0) = -a_\sigma T^2 \,,$$
$$a_\sigma = -\frac{N_\rho}{24}\lambda_{\sigma\rho} - \frac{1}{4}\lambda_\sigma - \frac{N_F}{6}\left(2y_L^2 + y_{N'}^2 + y_{E'}^2\right) \,.$$

- To satisfy $(y_L v_\sigma)^2 \gg T^2$, we need large enough v_σ at high T too.
- $a_\sigma > 0$ guarantees that $v_\sigma \neq 0$ at high T. Although this condition is neither sufficient nor necessary, it helps us to obtain the correct benchmarks.
- $\lambda_{\sigma\rho} < 0$ and sufficiently large $N_\rho|\lambda_{\sigma\rho}|$ are required to satisfy $a_\sigma > 0$.

MODEL II: SINGLET-DOUBLET FERMIONS

+ REAL SCALAR SINGLET + SCALAR SINGLET WITH $O(N_\rho)$ + INERT HIGGS DOUBLET

$$V_0 = V_0(\Phi_1, \Phi_2) + V_0(\sigma, \rho) + \widetilde{V}_0$$

$$V_0(\Phi_1, \Phi_2) = -\mu_1^2 \Phi_1^\dagger \Phi_1 + \lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 - \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2$$

$$\widetilde{V}_0 = \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \sum_{i=1}^{2} \left(\frac{1}{2} \lambda_{\sigma \Phi_i} \sigma^2 (\Phi_i^\dagger \Phi_i) + \frac{1}{2} \lambda_{\rho \Phi_i} (\rho_j \rho_j)(\Phi_i^\dagger \Phi_i) \right)$$

$$\begin{aligned} \mathcal{L}^i_{yuk} = & - y^i_{NN'1} \overline{L^i_L} \widetilde{\Phi_2} N'^i_R - y^i_{NN'2} \overline{N'^i_L} \widetilde{\Phi_2}^\dagger L^i_R - y^i_{EE'1} \overline{L^i_L} \Phi_2 E'^i_R - y^i_{EE'2} \overline{E'^i_L} \Phi_2^\dagger L^i_R \\ & - y^i_L \sigma \overline{L^i_L} L^i_R - y^i_{N'} \sigma \overline{N'^i_L} N'^i_R - y^i_{E'} \sigma \overline{E'^i_L} E'^i_R + h.c. \end{aligned}$$

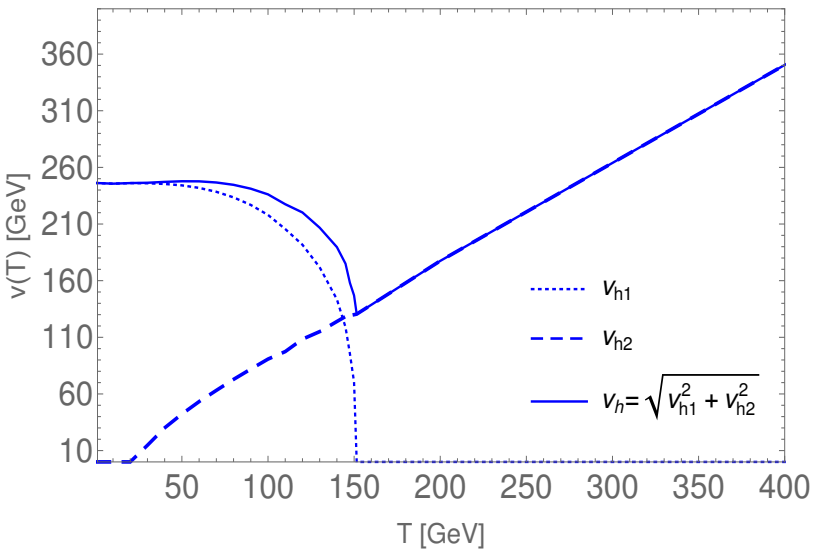
$$\begin{aligned} a_h = & \frac{N_F}{6 y_L} \left(y_{N'} y_{NN'1} y_{NN'2} + y_{E'} y_{EE'1} y_{EE'2} \right) \\ & - \left(\frac{3}{16} g^2 + \frac{1}{16} g'^2 + \frac{1}{2} \lambda_2 \right) \end{aligned}$$

$$\Phi_j = \begin{bmatrix} \varphi_j^+ \\ (h_j + i\varphi_j^0)/\sqrt{2} \end{bmatrix}$$

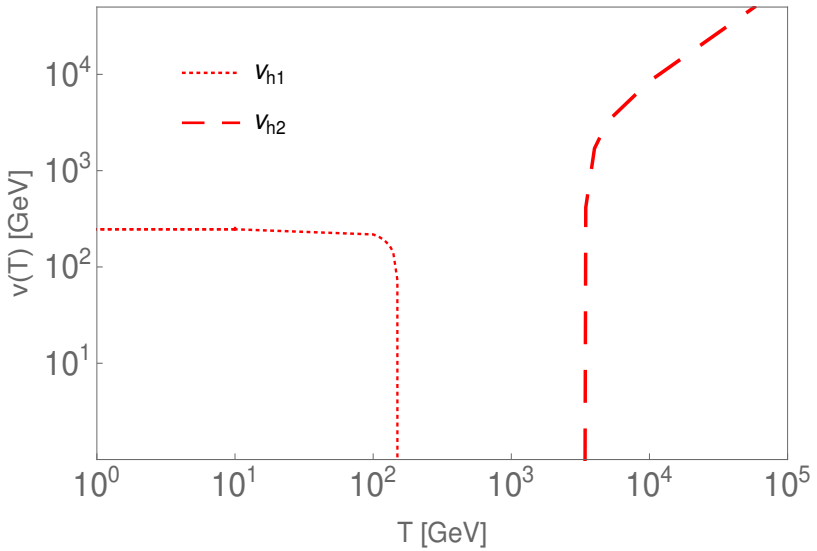
At $T = 0$,

$$\langle \Phi_2 \rangle = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

THERMAL HISTORIES

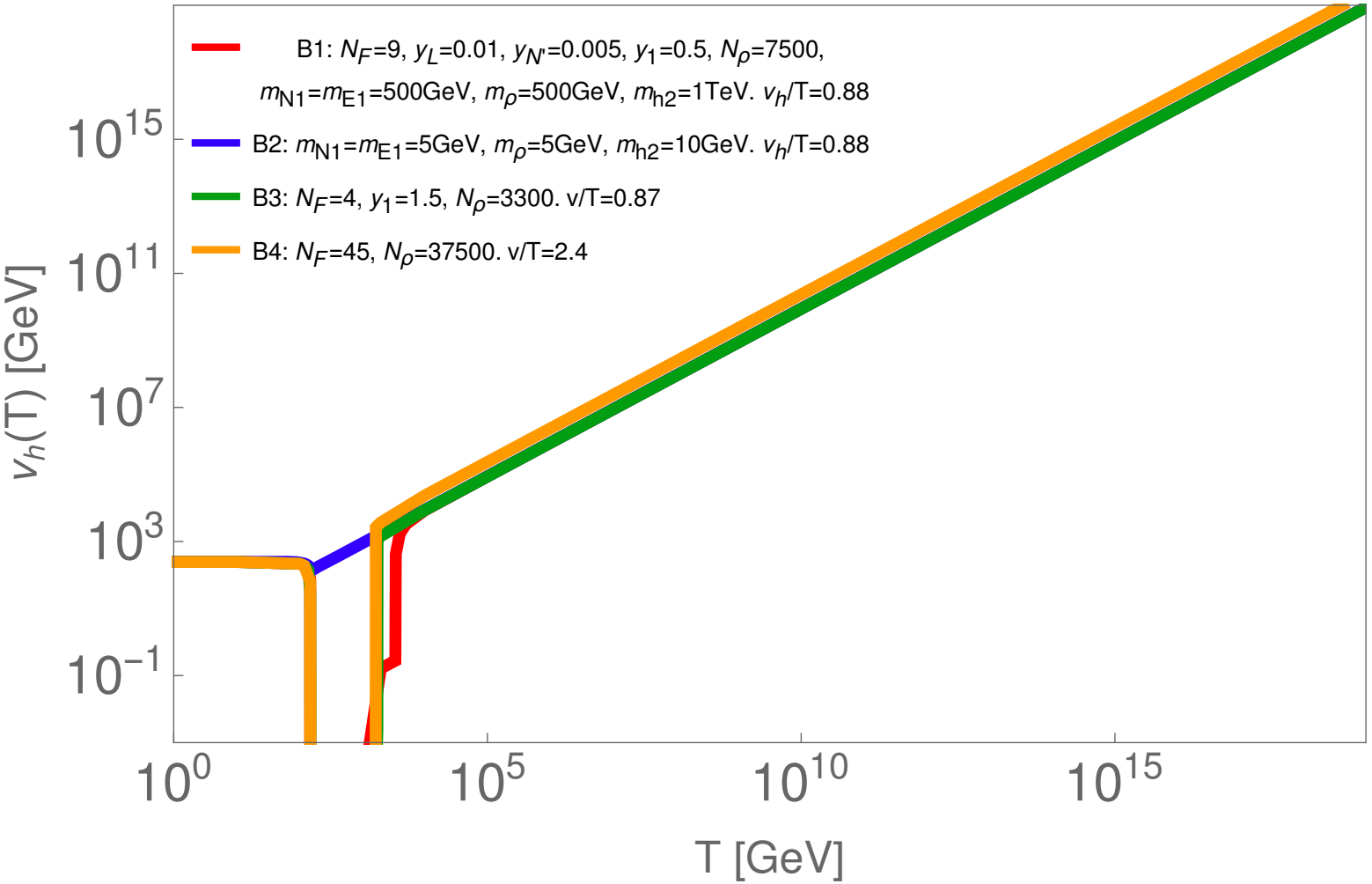


Thermal history in which the electroweak symmetry is always broken.



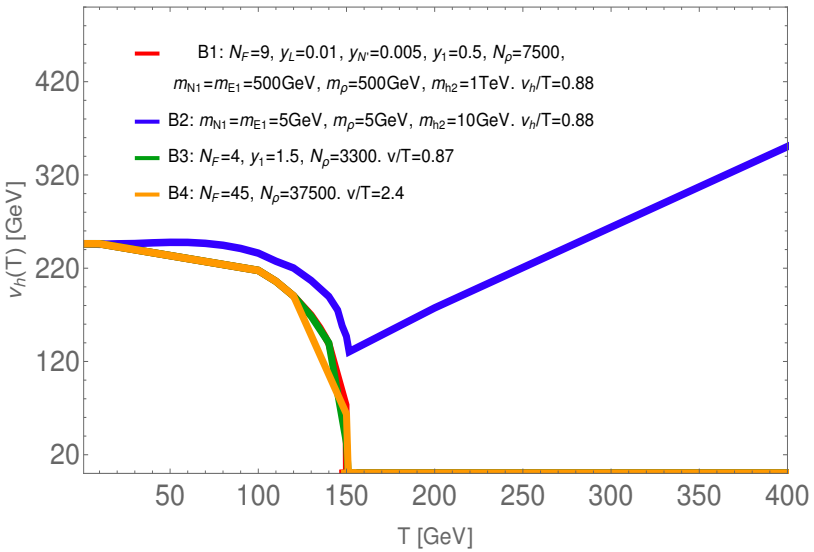
Thermal history in which the electroweak symmetry is only temporarily restored.

THERMAL HISTORIES



Temperature-dependent vev for several benchmarks.

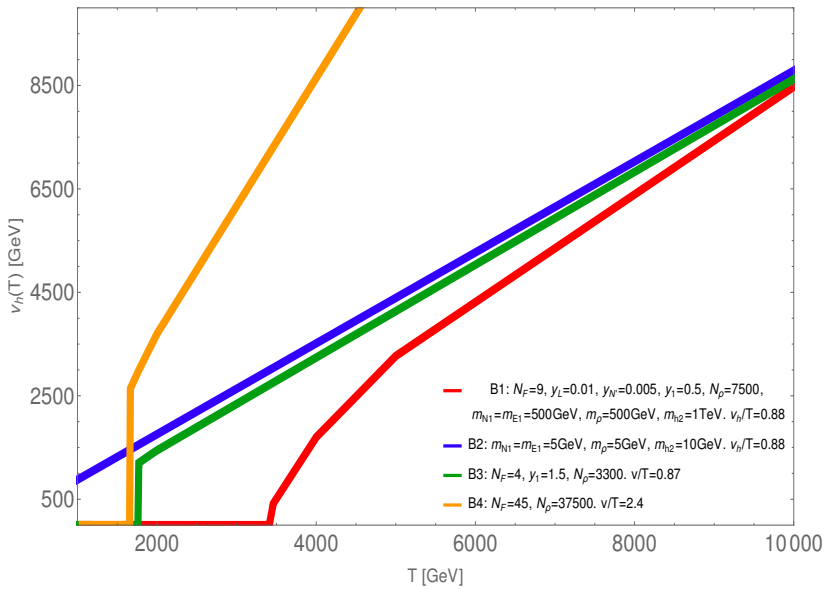
THERMAL HISTORIES



Lower temperature limits of temporary-restored phases.

- B1,B3,B4 have the same lower temperature limits of temporary-restored phases because SM-like Higgs does not couple with new fermions and and new scalars at tree-level.

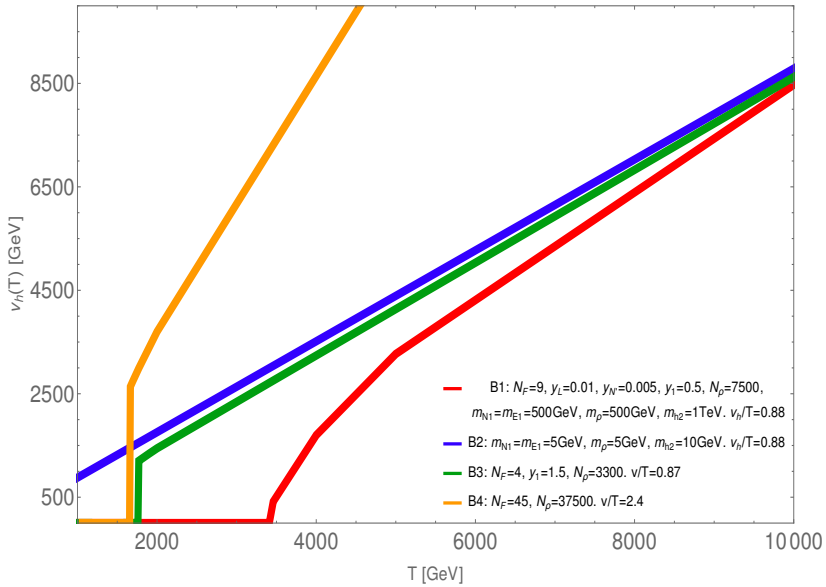
THERMAL HISTORIES



Upper temperature limits of temporary-restored phases.

- The value v_h/T of at high T depends on a_h.

THERMAL HISTORIES



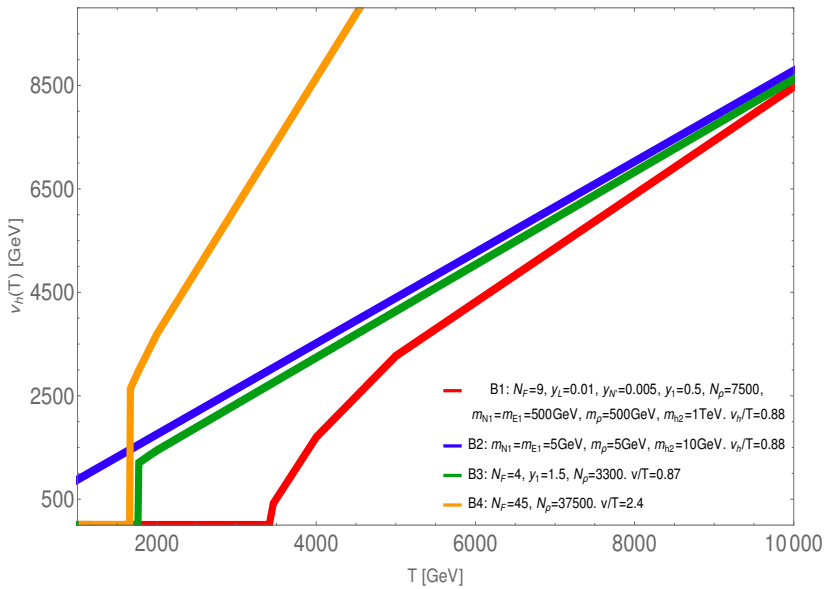
Upper temperature limits of temporary-restored phases.

- The value v_h/T of at high T depends on a_h.

$$\partial_h^2 V_{eff}(h=0, v_\sigma) \approx -a_h T^2 \,,$$

$$a_h = \frac{N_F}{6y_L}\left(y_{N'} y_{NN'1} y_{NN'2} + y_{E'} y_{EE'1} y_{EE'2}\right) - \left(\frac{3}{16}g^2 + \frac{1}{16}g'^2 + \frac{1}{2}\lambda_2\right) \,.$$

THERMAL HISTORIES



Upper temperature limits of temporary-restored phases.

- The value v_h/T of at high T depends on a_h.
- The length of temporary-restored phase depends on the masses of new fermions and new scalars (at $T = 0$), $N_\rho \lambda_\rho$, and a_h.

RGEs AND STABILITY OF V_{eff}

$$
\begin{aligned}
(4\pi)^2 \frac{d\lambda_2}{dt} &= 24\lambda_2^2 + \frac{3}{8}\left(2g_2^4 + \left(g_2^2 + g'^2\right)^2\right) - \left(9g_2^2 + 3g'^2\right)\lambda_2 \\
&\quad + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \frac{N_\rho}{2}\lambda_{\rho\Phi_2}^2 + \frac{1}{2}\lambda_{\sigma\Phi_2}^2 \\
&\quad + 2N_F\left(2\left(y_{NN'1}^2 + y_{NN'2}^2 + y_{EE'1}^2 + y_{EE'2}^2\right)\lambda_2 - \left(y_{NN'1}^4 + y_{NN'2}^4 + y_{EE'1}^4 + y_{EE'2}^4\right)\right) \\
(4\pi)^2 \frac{dg'}{dt} &= (7 + 2N_F)\, g'^3 \\
(4\pi)^2 \frac{dg_2}{dt} &= \left(-3 + \frac{2}{3}N_F\right) g_2^3 \\
(4\pi)^2 \frac{dy_{NN'1}}{dt} &= 2y_{N'}y_L y_{NN'2} + y_{NN'1}\left[\frac{3}{2}y_{NN'1}^2 - \frac{3}{2}y_{EE'1}^2 + \frac{1}{2}y_{N'}^2 + \frac{1}{2}y_L^2\right. \\
&\quad \left. + N_F(y_{NN'1}^2 + y_{NN'2}^2 + y_{EE'1}^2 + y_{EE'2}^2) - \frac{9}{4}g_2^2 - \frac{3}{4}g'^2\right] \\
(4\pi)^2 \frac{d\lambda_\rho}{dt} &= 2(N_\rho + 8)\lambda_\rho^2 + \frac{1}{2}\lambda_{\sigma\rho}^2 + 2\left(\lambda_{\rho\Phi_1}^2 + \lambda_{\rho\Phi_2}^2\right)
\end{aligned}
$$

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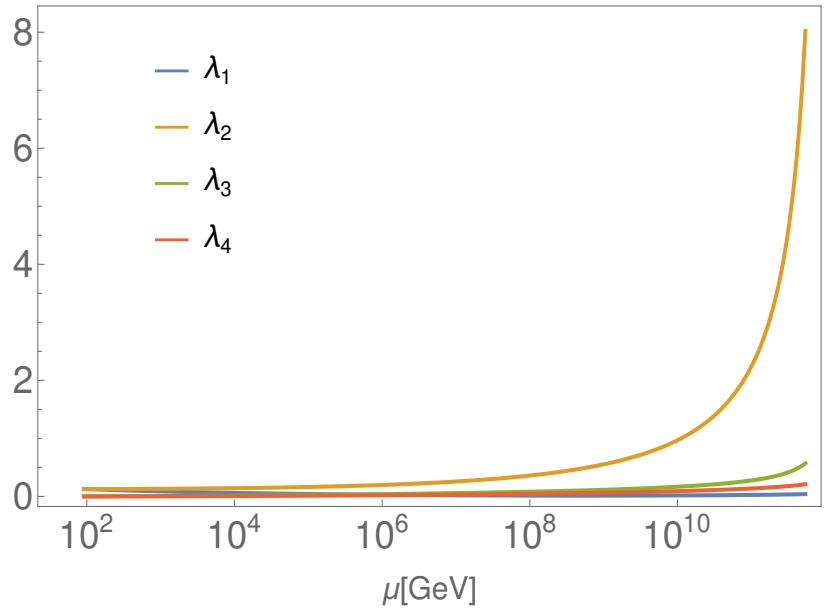
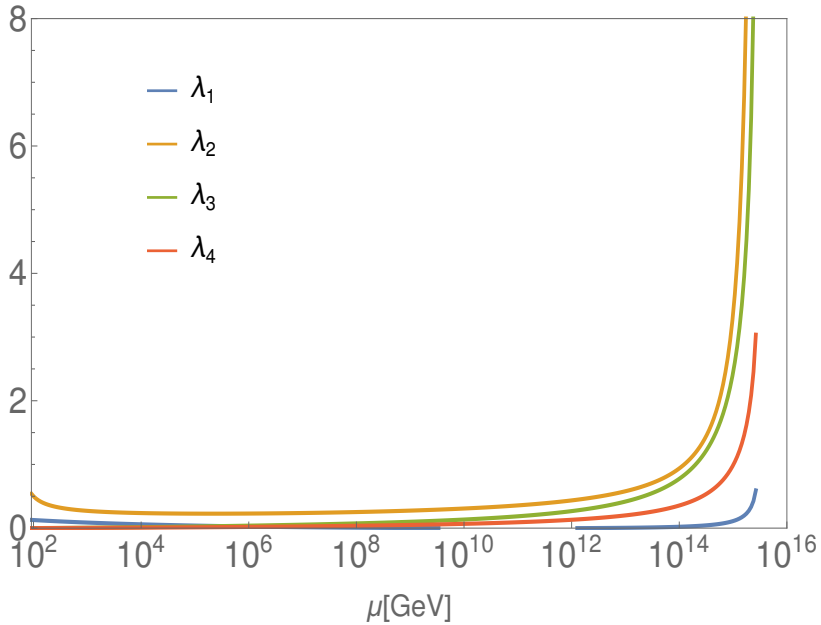
&\quad + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \frac{N_\rho}{2}\lambda_{\rho\Phi_2}^2 + \frac{1}{2}\lambda_{\sigma\Phi_2}^2 \\
&\quad + 2N_F\left(2\left(y_{NN'1}^2 + y_{NN'2}^2 + y_{EE'1}^2 + y_{EE'2}^2\right)\lambda_2 - \left(y_{NN'1}^4 + y_{NN'2}^4 + y_{EE'1}^4 + y_{EE'2}^4\right)\right) \\
(4\pi)^2 \frac{dg'}{dt} &= (7 + 2N_F)\, g'^3, \quad (4\pi)^2 \frac{dg_X}{dt} = -\frac{1}{3}\left(11N_X - 8n_F\right) g_X^3 \\
(4\pi)^2 \frac{dg_2}{dt} &= \left(-3 + \frac{2}{3}N_F\right) g_2^3 \\
(4\pi)^2 \frac{dy_{NN'1}}{dt} &= 2y_{N'}y_L y_{NN'2} + y_{NN'1}\left[\frac{3}{2}y_{NN'1}^2 - \frac{3}{2}y_{EE'1}^2 + \frac{1}{2}y_{N'}^2 + \frac{1}{2}y_L^2\right. \\
&\quad \left. + N_F(y_{NN'1}^2 + y_{NN'2}^2 + y_{EE'1}^2 + y_{EE'2}^2) - \frac{9}{4}g_2^2 - \frac{3}{4}g'^2 - 3\left(N_X - \frac{1}{N_X}\right)g_X^2\right] \\
(4\pi)^2 \frac{d\lambda_\rho}{dt} &= 2(N_\rho + 8)\lambda_\rho^2 + \frac{1}{2}\lambda_{\sigma\rho}^2 + 2\left(\lambda_{\rho\Phi_1}^2 + \lambda_{\rho\Phi_2}^2\right)
\end{aligned}
$$

(N, N', E, E' ARE CHARGED UNDER $SU(N_X)$. $N_F = N_x\, n_F$.)



B1: $n_F = 3, \quad N_x = 3.$



B5: $n_F = 1, \quad N_x = 4.$

The effective potential is stable, and the quartic couplings λ_i stays perturbative over a large range of energy scales.

Requiring all couplings stays perturbative up to 10^{10} GeV implies:

- $$g' \quad \text{stays perturbative} \quad \Rightarrow N_F \leq 12$$
- $$\lambda_\rho \quad \text{stays perturbative} \quad \Rightarrow N_\rho \lambda_\rho \lesssim 1.5$$
- $$\frac{y_L v_\sigma}{T} \gtrsim 4, \quad a_\sigma \gtrsim 0, \quad \lambda_\sigma \geq \frac{1}{\lambda_\rho}\left(\frac{\lambda_{\sigma\rho}}{2}\right)^2 \Rightarrow N_\rho \gtrsim \frac{64}{|x_{min}|} N_F$$

  x_{min} is the minimum of $f(x) = \frac{\pi^2}{2}x^2 + N_\rho \lambda_\rho J_B(x)$. E.g. choosing $N_\rho \lambda_\rho = 1.5$ implies $N_\rho \gtrsim 552 N_F$.
- $$\lambda_2 > 0 \Rightarrow \lambda_2 \gtrsim y^2_{NN'1}/2$$

  (assume $y_{NN'1} = y_{NN'2} = y_{EE'1} = y_{EE'2}$ at $T = 0$)
- $$y_{NN'i}, y_{EE'i} \quad \text{stays perturbative} \quad \Rightarrow \left(N_X - \frac{1}{N_X}\right) g_X^2 \gtrsim \frac{4}{3} N_F y^2_{NN'1}$$

BENCHMARKS

n_F	4	3	3	1
N_X	3	3	2	4
$y_{NN'i} = y_{EE'i}$	0.4	0.5	0.75	1.5
$y_{N'}$	0.005	0.005	0.005	0.005
$y_{L'}$	0.01	0.01	0.01	0.01
m_{N1}(GeV)	500	500	500	500
N_ρ	8500	7500	8500	8500
$\lambda_{\sigma\rho}$	-1.2×10^{-6}	-1.2×10^{-6}	-1.2×10^{-6}	-1.2×10^{-6}
λ_2	0.081	0.126	0.28	0.54
m_{h2}(GeV)	1000	1000	1000	1000
m_ρ(GeV)	500	500	500	500
g_X	1	1.1	1.64	$\sqrt{4\pi}$

$$N_\rho \lambda_\rho = 1.5, \quad \lambda_\sigma = \frac{1}{\lambda_\rho}\left(\frac{\lambda_{\sigma\rho}}{2}\right)^2$$

THERMAL EQUILIBRIUM CONDITIONS

- The effective potential at finite temperature above is valid only when N_1, σ, ρ_i are in thermal equilibrium.
- $$\Gamma(\rho_i \rho_i \to \sigma\sigma) \gtrsim H \quad \Rightarrow \quad T \lesssim \frac{\lambda_{\sigma\rho}^2}{\sqrt{N_\rho}} M_{Pl}$$
- $$\Gamma(N_1 N_1 \to \sigma\sigma) \gtrsim H \quad \Rightarrow \quad T \lesssim \frac{D_{1\sigma}}{f(m_{N_1}/T)} \frac{y_{11\sigma}^4}{\sqrt{N_\rho}} M_{Pl}$$

$$D_{i\sigma} = \frac{2}{\pi^2}\kappa_i e^{-2\kappa_i} \int_0^\infty dy_1 dy_2 (y_1 + y_1^2/(2\kappa_i))^{1/2} (y_2 + y_2^2/(2\kappa_i))^{1/2} e^{-y_1} e^{-y_2} \int_{-1}^{1} d\cos\theta \, F(\tilde{s})$$

$$F(s) = \left(3s - \frac{25}{16}(s - 1/4)^{-1} - \frac{9}{4}\right)/(8\pi)$$

$$\tilde{s} = s/(4m_i^2) = 1 + (y_1 + y_2)/(2\kappa_i) + y_1 y_2/(2\kappa_i^2) - (y_1 + y_1^2/(2\kappa_i))^{1/2} (y_2 + y_2^2/(2\kappa_i))^{1/2} \cos\theta/\kappa_i$$

$$\kappa_i = m_{N_i}/T$$

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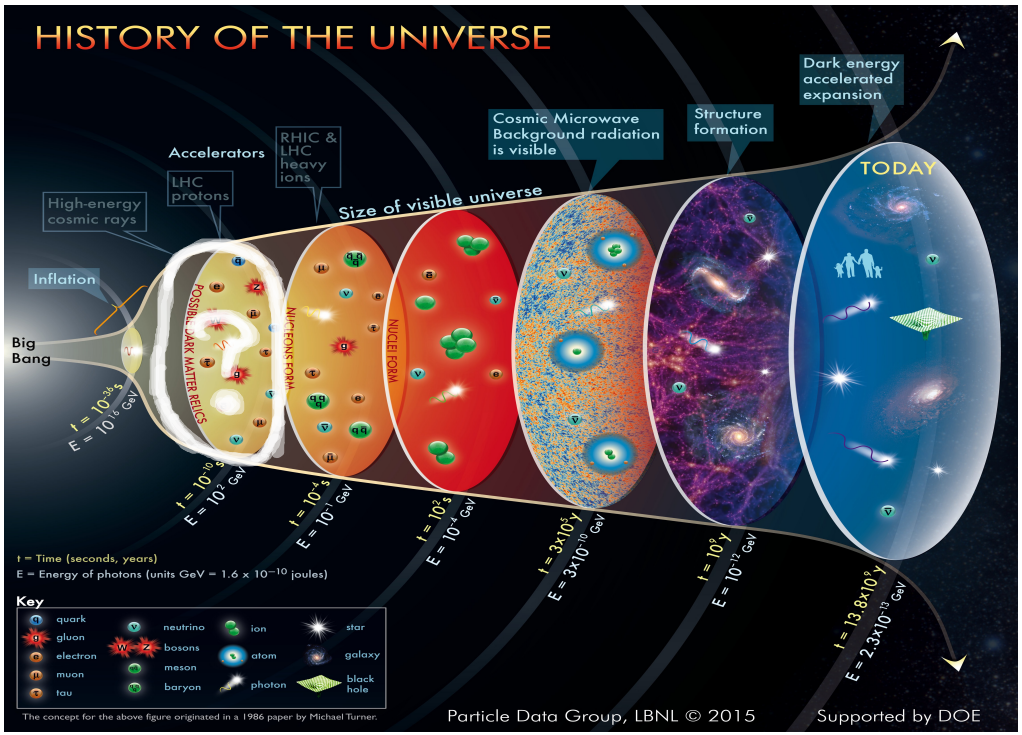
SUMMARY AND OUTLOOK

- EWSNR can be induced by new fermions from renormalizable models.
- The parameter spaces of these models are tightly constrained by theoretical constraints.
- Intriguing cosmological implications: origin of matter-antimatter asymmetry (e.g. high-temperature EWBG), unique gravitational wave signatures, early matter dominated era (N_ρ is large)



EFFECTIVE POTENTIAL

- Effective potential

$$V_{eff} = V_0 + \sum_i \left(V_i^{CW} + V_{1,i}^{th} + V_{ring,i}^{th} \right)$$

- Tree-level potential:

$$V_0 = V_0(\Phi_1, \Phi_2) + V_0(\sigma, \rho) + \widetilde{V}_0$$

$$V_0(\Phi_1, \Phi_2) = -\mu_1^2 \Phi_1^\dagger \Phi_1 + \lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 - \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2$$

$$\widetilde{V}_0 = \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \sum_{i=1}^{2} \left(\frac{1}{2} \lambda_{\sigma \Phi_i} \sigma^2 (\Phi_i^\dagger \Phi_i) + \frac{1}{2} \lambda_{\rho \Phi_i} (\rho_j \rho_j)(\Phi_i^\dagger \Phi_i) \right)$$

$$V_0(\sigma, \rho) = -\frac{1}{2} \mu_\sigma^2 \sigma^2 + \frac{1}{4} \lambda_\sigma \sigma^4 - \frac{1}{2} \mu_\rho^2 \rho_i \rho_i + \frac{1}{4} \lambda_\rho \left(\rho_i \rho_i \right)^2 + \frac{1}{4} \lambda_{\sigma\rho} \sigma^2 \rho_i \rho_i$$

(*Please see hep-ph/9901312 for more details.*)

- Coleman-Weinberg potential (for i-th particle)

$$V_i^{CW} = (-1)^{a_i} n_i \frac{m_i^4}{64\pi^2}\left[\log\left(\frac{m_i^2}{\mu^2}\right) - c_i\right]$$

- One-loop thermal potential (for i-th particle)

$$V_{1,i}^{th} = (-1)^{a_i} n_i \frac{T^4}{2\pi^2} J_{B/F}\left(\frac{m_i^2}{T^2}\right)$$

$$J_{B/F}(y^2) = \int_0^\infty dx\, x^2 \log\left[1 \mp \exp\left(-\sqrt{x^2+y^2}\right)\right]$$

- Daisy contribution (for i-th particle)

$$V_{ring,i}^{th} = \overline{n_i}\frac{T^4}{12\pi}\left[\left(\frac{m_i^2}{T^2}\right)^{3/2} - \left(\frac{\mathcal{M}_i^2}{T^2}\right)^{3/2}\right]$$

THERMAL CORRECTIONS BEYOND ONE LOOP

$$V_1^{th}(\phi,T) = \sum_i \frac{n_i T}{2} \sum_{-\infty}^{\infty} \int \frac{d^3\vec{k^2}}{(2\pi)^3} \log\left[\vec{k}^2 + \omega_n^2 + m_i^2(\phi)\right]$$

$$= \sum_i (-1)^{a_i} n_i \frac{T^4}{2\pi^2} J_{B/F}\left(\frac{m_i^2}{T^2}\right)$$

$$V_{ring}^{th}(\phi,T) = \sum_i \frac{\bar{n}_i T}{12\pi}\left[m_i^3(\phi) - \mathcal{M}_i^3(\phi,T)\right]$$

$$\omega_n = 2n\pi T \ (bosons), (2n+1)\pi T \ (fermions)$$

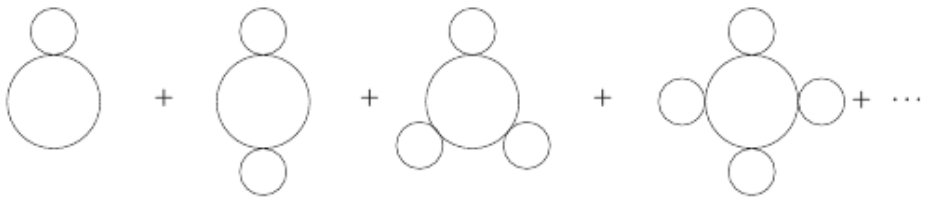


Figure 2: Some generic examples of ring diagrams where each solid line may represent either a scalar, a fermion or a gauge field. The small loops correspond to thermal loops in the IR limit. They are all separately IR divergent, but their sum is IR finite.

$\mathcal{M}_i^2(\phi,T) = m_i^2(\phi) + \Pi_i(\phi,T)$ (except $i = Z_L, \gamma_L$).

Truncated Full Dressing Method:

$$
\begin{aligned}
\Pi_h(\phi,T) &= \left(\frac{3g^2+g'^2}{16} + \frac{\lambda}{2} + \frac{y_t^2}{4}\right) T^2 = \Pi_\chi(\phi,T)\ , \\
\Pi_{W_L}(\phi,T) &= \frac{11}{6} g^2 T^2\ , \\
\Pi_{W_T}(\phi,T) &= \Pi_{Z_T}(\phi,T) = \Pi_{\gamma_T}(\phi,T) = 0\ , \qquad \text{(A14)} \\
\mathcal{M}^2_{Z_L}(\phi) &= \frac{1}{2}\left[m_Z^2(t) + \frac{11}{6}\frac{g^2}{\cos^2\theta_W} T^2 + \Delta(\phi,T)\right]\ , \\
\mathcal{M}^2_{\gamma_L}(\phi) &= \frac{1}{2}\left[m_Z^2(t) + \frac{11}{6}\frac{g^2}{\cos^2\theta_W} T^2 - \Delta(\phi,T)\right]\ , \\
& \qquad \text{(A15)}
\end{aligned}
$$

Optimized Partial Dressing Method:

$$\delta m^2_{\phi_j}(h,T) = \sum_i \frac{\partial}{\partial \phi_j}\left[\frac{\partial V^i_{\rm CW}}{\partial \phi_j}\Big(m^2_i(h)+\delta m^2_i(h,T)\Big) + \frac{\partial V^i_{\rm th}}{\partial \phi_j}\Big(m^2_i(h)+\delta m^2_i(h,T),T\Big)\right] \quad \text{(A.1)}$$

$$\delta m^2_j(h,T) \approx \delta m^2_{j(a)} + (h-h_a)\frac{\partial \delta m^2_{j(a)}}{\partial h} \quad \text{(A.2)}$$

$$V^{\rm pd}_{\rm eff}(h,T) = V_0 + \sum_i \int dh \left[\frac{\partial V^i_{\rm CW}}{\partial h}\Big(m^2_i(h)+\delta m^2_i(h,T)\Big) + \frac{\partial V^i_{\rm th}}{\partial h}\Big(m^2_i(h)+\delta m^2_i(h,T),T\Big)\right], \quad \text{(A.4)}$$

DAISY AND SUPER-DAISY CONTRIBUTIONS TO THERMAL MASS

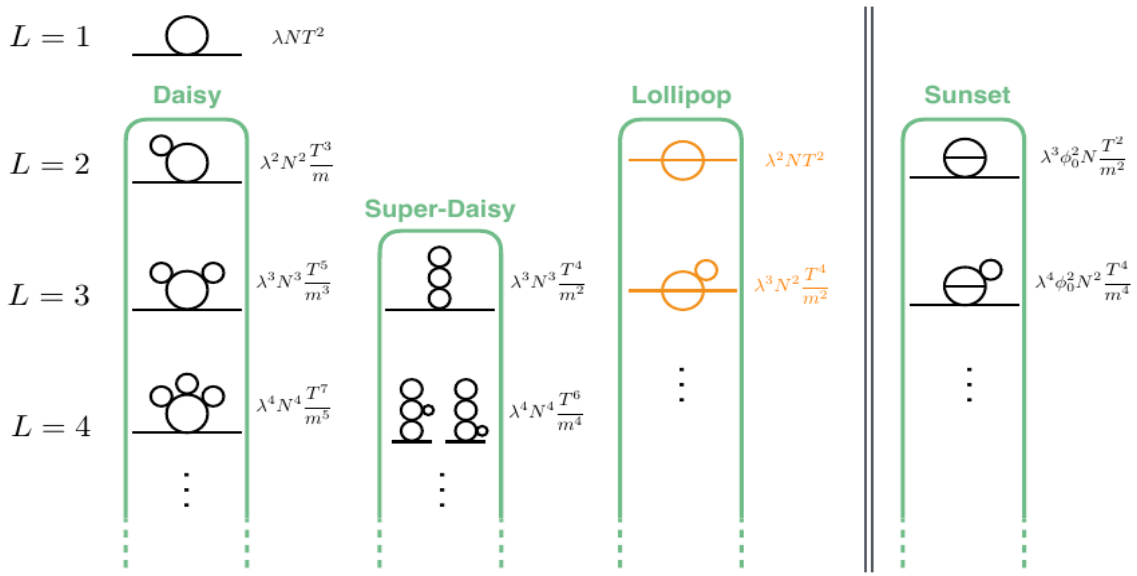


Figure 2. Complete set of 1- and 2- loop contributions to the scalar mass, as well as the most important higher loop contributions, in ϕ^4 theory. The scaling of each diagram in the high-temperature approximation is indicated, omitting symmetry- and loop-factors. Diagrams to the right of the vertical double-lines only contribute away from the origin when $\langle\phi\rangle = \phi_0 > 0$. We do not show contributions which trivially descend from e.g. loop-corrected quartic couplings. Lollipop diagrams (in orange) are not automatically included in the resummed one-loop potential.

Curtin, Meade & Ramani, arXiv:1612.00466

V_{eff} IN SM

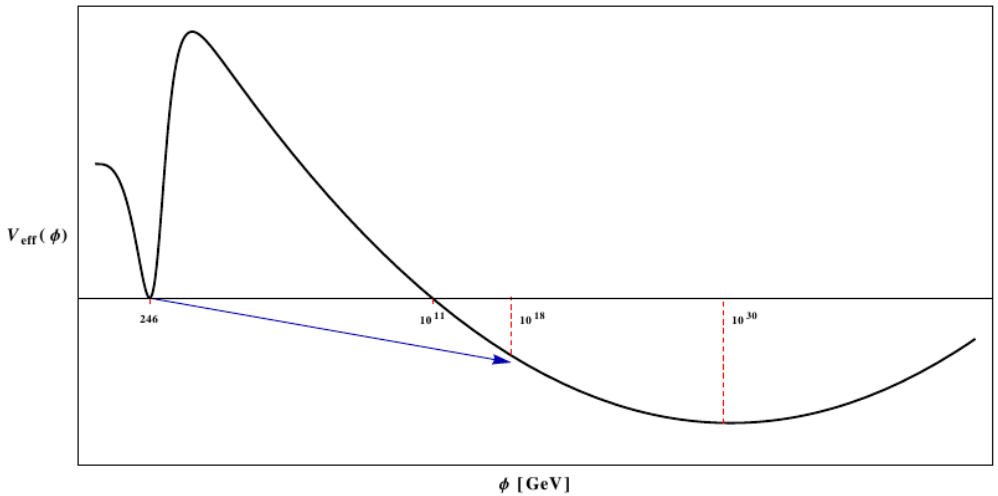
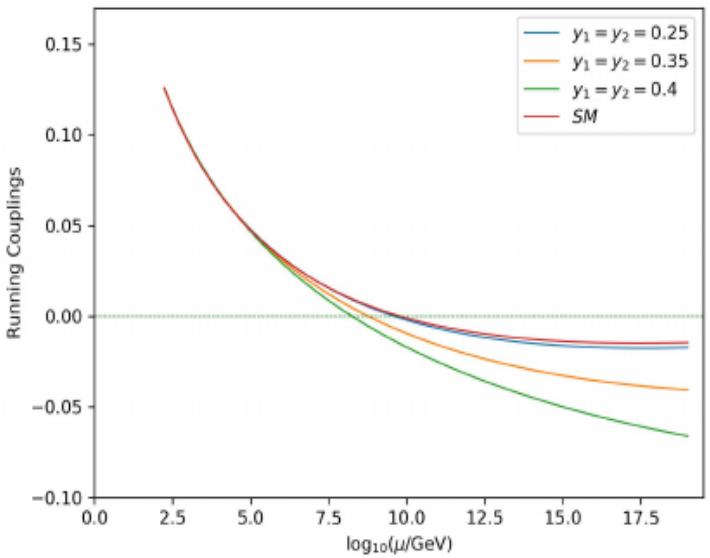


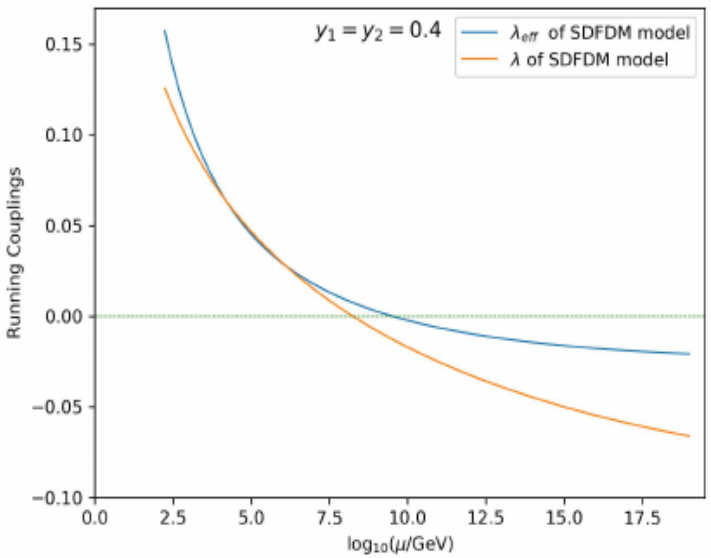
FIG. 2: The potential in the Standard Model, for $M_H = 125.7$ GeV and $M_t = 173.34$ GeV, is sketched (figure not to scale). The potential goes negative at a scale of 10^{11} GeV and reaches a new minimum at roughly 10^{30} GeV. The tunneling through the barrier goes from the base of the arrow ($\phi(r = \infty)$) to the tip ($\phi(0)$), which turns out to be close to or above the Planck scale.

Branchina, Messina & Sher, arXiv:1408.5302

STABILITY OF V_{eff} IN SM AND SINGLET-DOUBLET MODEL



(a)

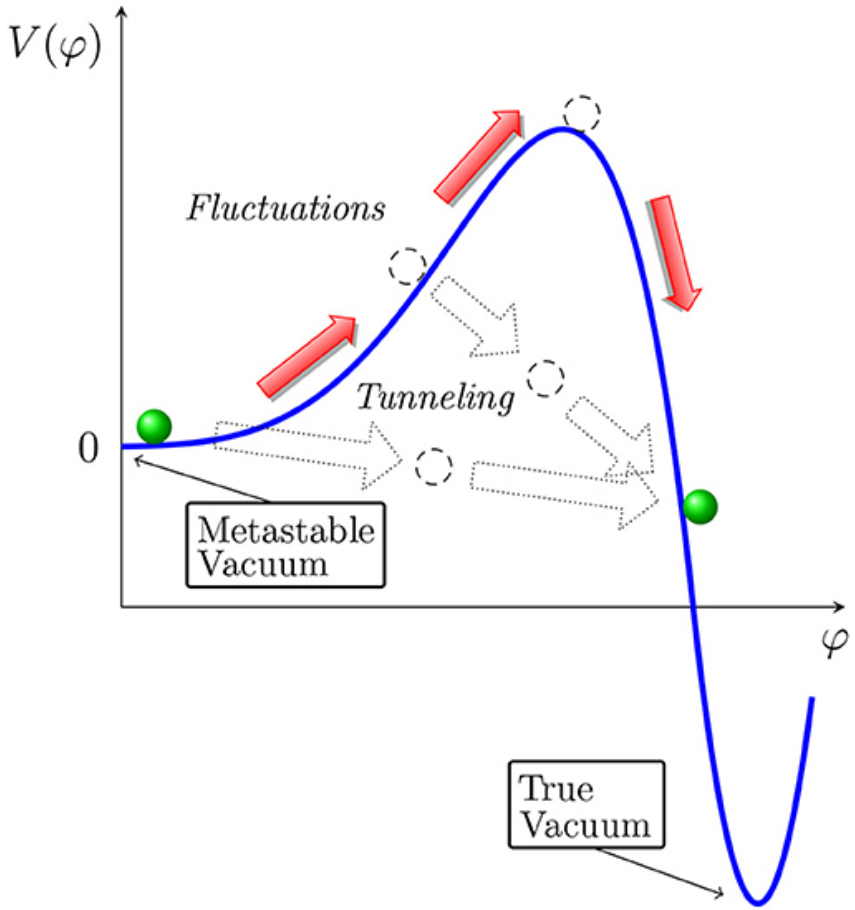


(b)

FIG. 1. (a) $\lambda(t)$ up to M_{Pl} for the SM and for various Yukawa couplings in the SDFDM model; (b) Running $\lambda(t)$ and $\lambda_{eff}(t)$ up to M_{Pl} scale for the SDFDM model.

Cheng & Liao, arXiv:1909.11941

METASTABLE VACUUM VS TRUE VACUUM



Markkanen, Rajantie & Stopyra, arXiv:1809.06923

In SM:

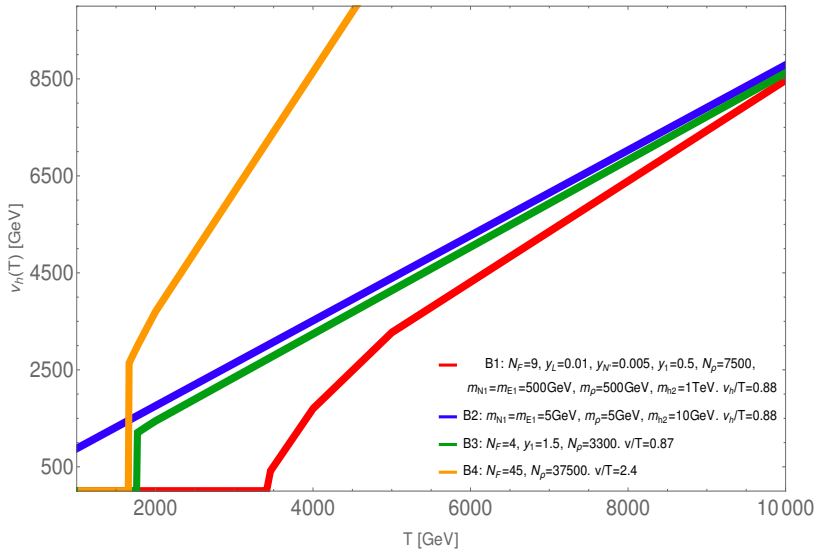
$$\tau = \left[\frac{R_M^4}{T_U^4}\, e^{\frac{8\pi^2}{3|\lambda(\mu)|}}\right] \times \left[e^{\Delta S}\right] \times T_U$$

$$R_M \sim 1.87 \cdot 10^{-17}\, GeV^{-1} = 224.5\, M_P^{-1}$$

$$\lambda(1/R_M) = -0.01345\,,$$

$$\tau_{tree} \sim 10^{613}\, T_U$$

$$(4\pi)^2 \frac{d\lambda_1}{dt} = 24\lambda_1^2 - 6y_t^4 + \frac{3}{8}\left(2g_2^4 + \left(g_2^2 + g'^2\right)^2\right) + \left(12y_t^2 - 9g_2^2 - 3g'^2\right)\lambda_1$$
$$+ 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \frac{N_\rho}{2}\lambda_{\rho\Phi_1}^2 + \frac{1}{2}\lambda_{\sigma\Phi_1}^2$$
$$(4\pi)^2 \frac{dg_3}{dt} = -7g_3^3$$



Upper temperature limits of temporary-restored phases.

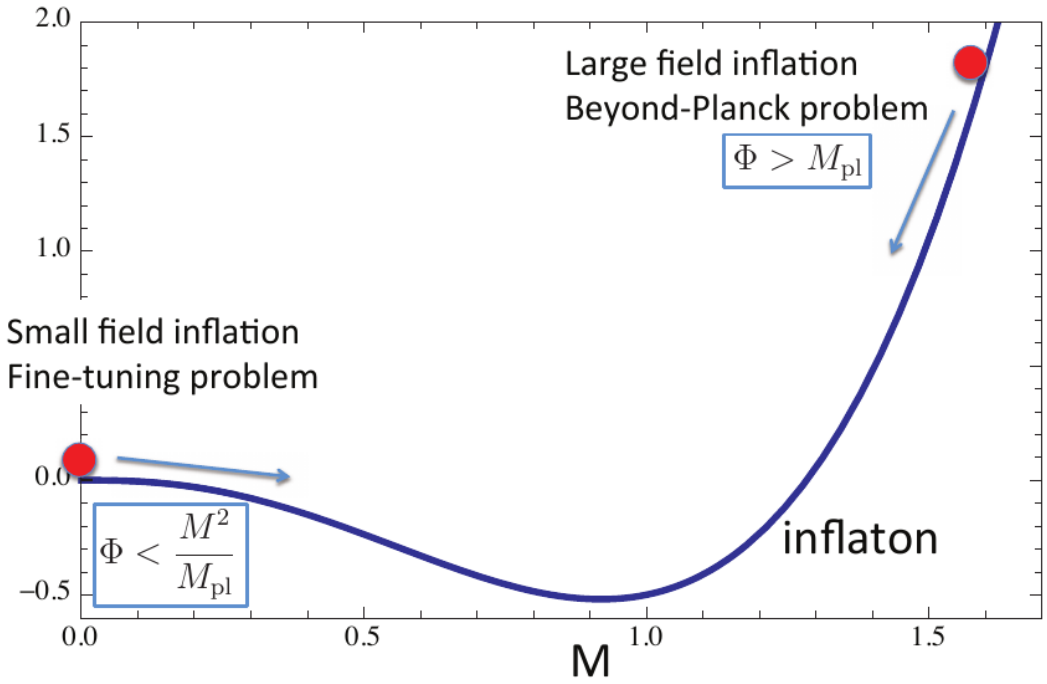
Table 1. Frequency classification of gravitational waves and their detection method [4-6]

Frequency band	Detection method
Ultra high frequency band: above 1 THz	Terahertz resonators, optical resonators, and magnetic conversion detectors
Very high frequency band: 100 kHz–1 THz	Microwave resonator/wave guide detectors, laser interferometers and Gaussian beam detectors
High frequency band (audio band)*: 10 Hz–100 kHz	Low-temperature resonators and ground-based laser-interferometric detectors
Middle frequency band: 0.1 Hz–10 Hz	Space laser-interferometric detectors of arm length 100 km − 60,000 km, atom and molecule interferometry, optical clock detectors
Low frequency band (milli-Hz band)†: 100 nHz–0.1 Hz	Radio Doppler tracking of spacecraft, space laser-interferometric detectors of arm length longer than 60,000 km, optical clock detectors
Very low frequency band (nano-Hz band): 300 pHz – 100 nHz	Pulsar timing arrays (PTAs)
Ultralow frequency band: 10 fHz–300 pHz	Astrometry of quasars and their proper motions
Extremely low (Hubble) frequency band (cosmological band): 1 aHz–10 fHz	Cosmic microwave background experiments
Beyond Hubble-frequency band: below 1 aHz	Through the verifications of inflationary/primordial cosmological models

*The range of audio band (also called LIGO band) normally goes only to 10 kHz.
†The range of milli-Hz band is 0.1 mHz to 100 mHz.

arXiv:1709.05659

SLOWLY-ROLLING SCALARS



Iso, Kohri & Shimada, arXiv:1511.05923