

2021 Meeting of the Division of Particles of the American Physical Society – DPF21

# Some reflexions on hidden features of SM extensions with scalar triplets

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13<sup>th</sup> July 2021

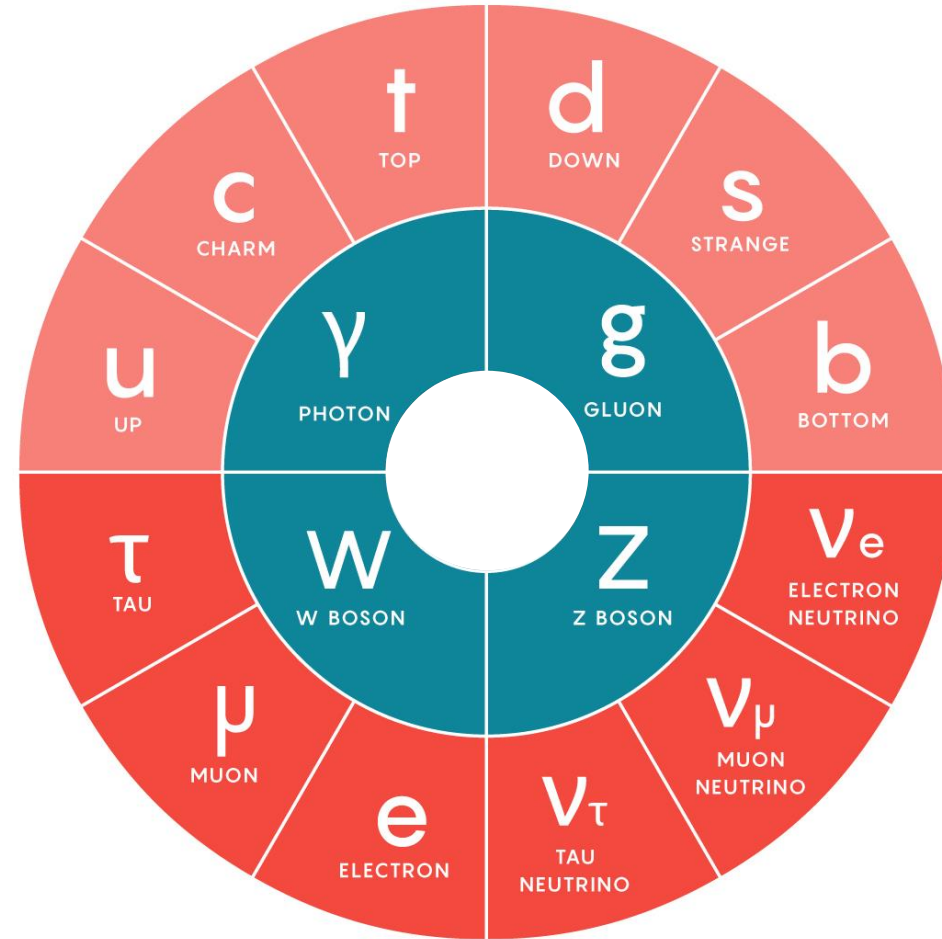
# The Standard Model of Particle Physics

On the 4<sup>th</sup> July of 2012...

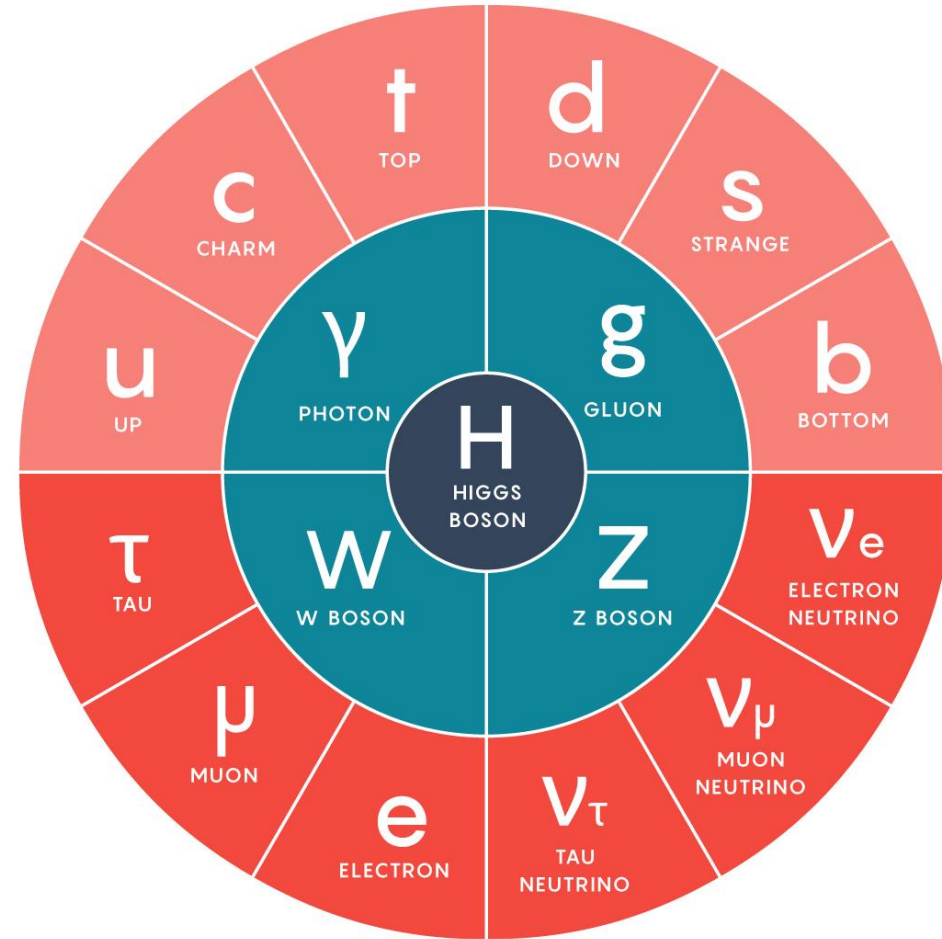


The discovery of a Higgs-like particle is announced at CERN

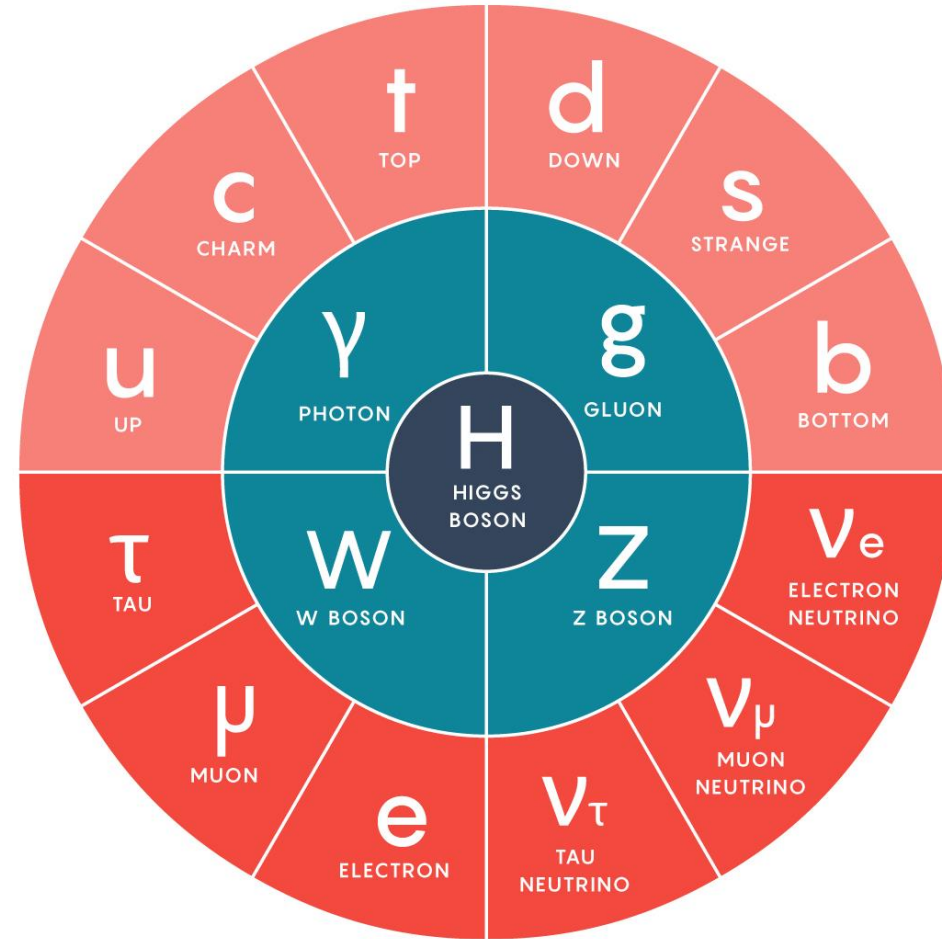
# The Standard Model of Particle Physics



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# The Standard Model of Particle Physics



However, this is not the whole story...

# Beyond the Standard Model

## Scalar triplet extensions of the Standard Model

**Multi-Higgs  
scenario**

Higgs-triplet model  
(HTM)

Two-scalar-triplet model  
(2STM)

**Motivation**

Neutrino masses  
in type-II seesaw  
mechanism

**Problem**

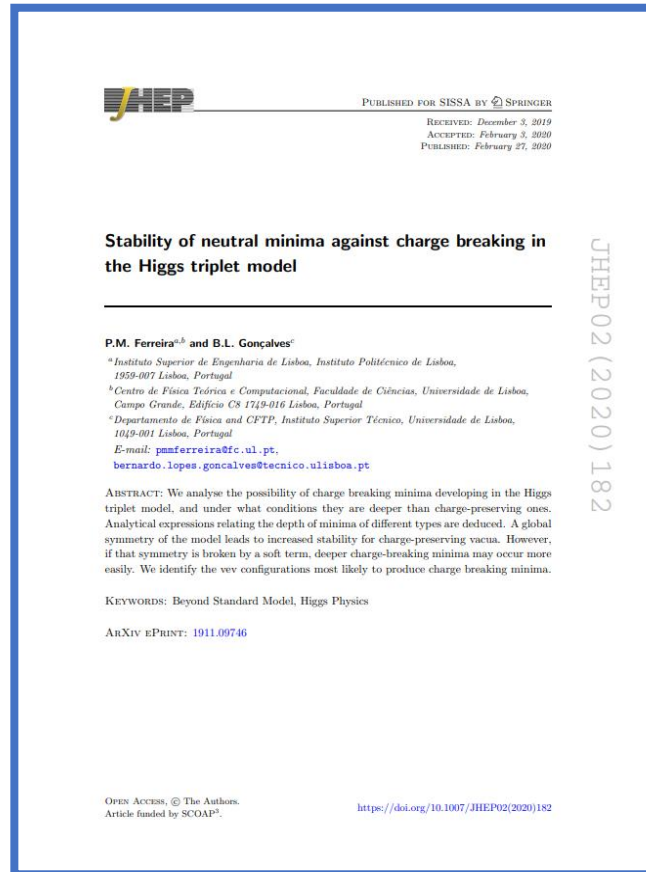
Are neutral minima  
stable against charge  
breaking?

# Beyond the Standard Model

## Scalar triplet extensions of the Standard Model

Multi-Higgs  
scenario

Two-scalar-triplet model  
(2STM)



[arXiv:1911.09746v3](https://arxiv.org/abs/1911.09746v3) [hep-ph]

Motivation

Problem



# The Higgs-Triplet Model

All SM fields, with the **addition of an SU(2) scalar triplet**

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

## **RICHER PARTICLE SPECTRUM:**

- Two CP-even scalars,  $h$  and  $H$
- One pseudoscalar,  $A$
- One charged scalar,  $H^+$
- One doubly charged scalar,  $H^{++}$



# The Higgs-Triplet Model

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## MODEL'S MOTIVATION:

- Smallness of neutrino masses (type-II seesaw)
- Dark matter candidates
- Rich phenomenology

# The Higgs-Triplet Model

All SM fields, with the **addition of an SU(2) scalar triplet**

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

## **DIFFERENT VACUUM POSSIBILITIES:**

- CP-breaking vacua
- Charge-breaking (CB) vacua
- Normal (N) vacua

# The Higgs-Triplet Model

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## DIFFERENT VACUUM POSSIBILITIES:

- ~~CP-breaking vacua~~ **NOT POSSIBLE!**
- Charge-breaking (CB) vacua
- Normal (N) vacua

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All SM fields, with the **addition of an SU(2) scalar triplet**

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## DIFFERENT VACUUM POSSIBILITIES:

- ~~CP-breaking vacua~~ **NOT POSSIBLE!**
- Charge-breaking (CB) vacua
- Normal (N) vacua

Are neutral minima stable against charge breaking in the Higgs triplet model?

# The Higgs-Triplet Model

Most general gauge invariant scalar potential

$$V = m^2 \Phi^\dagger \Phi + M^2 \text{Tr}(\Delta^\dagger \Delta) + \mu \left( \Phi^T i\tau_2 \Delta^\dagger \Phi + \text{h.c.} \right) + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 \left[ \text{Tr}(\Delta^\dagger \Delta) \right]^2 + \lambda_3 \text{Tr} \left[ (\Delta^\dagger \Delta)^2 \right] + \lambda_4 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_5 \Phi^\dagger \Delta \Delta^\dagger \Phi$$

Soft-breaking term

$$\Phi \rightarrow e^{i\theta} \Phi$$

- Potential **without** soft-breaking term  $\mu = 0$  Allows for dark matter particles
- Potential **with** soft-breaking term  $\mu \neq 0$  Helps generate neutrino masses

# The Higgs-Triplet Model

Three possibilities for neutral vacua

$$\langle \Phi \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}, \quad \langle \Delta \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

- Can occur whether the soft breaking term is present or not
- If such term is not present, we get a massless axion

$$\langle \Phi \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Delta \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

- Only occurs when the soft breaking term is not present
- Good dark matter candidates

$$\langle \Phi \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

- Can occur whether the soft breaking term is present or not
- Unphysical vacuum type (massless quarks)

Three possibilities for neutral vacua

$$\langle \Phi \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}, \quad \langle \Delta \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Delta \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

Six different possibilities for CB vacua

$$\langle \Phi \rangle_{CB1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Delta \rangle_{CB1} = \frac{1}{\sqrt{2}} \begin{pmatrix} -c_3/\sqrt{2} & 0 \\ c_2 & c_3/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Delta \rangle_{CB2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & c_3 \\ c_2 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{CB3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Delta \rangle_{CB3} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} & c_4 \\ c_2 & -c_3/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB4} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Delta \rangle_{CB4} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2/\sqrt{2} & 0 \\ 0 & -c_2/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB5} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Delta \rangle_{CB5} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2/\sqrt{2} & c_3 \\ 0 & -c_2/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB6} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Delta \rangle_{CB6} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & c_2 \\ 0 & 0 \end{pmatrix}$$



Using a bilinear formalism similar to the one developed for the 2HDM, it is possible to find analytical formulae relating the depth of the potential at different extrema of the potential

Stability of minima of type N2  
against charge breaking

Stability of minima of type N1  
against charge breaking

Stability of minima of type N2  
against charge breaking

$$\begin{aligned}V_{CB1} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_+^2) \\V_{CB2} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_{++}^2) \\V_{CB3} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_+^2 + c_4^2 m_{++}^2) \\V_{CB4} - V_{N2} &= \frac{1}{4} c_2^2 m_+^2 \\V_{CB5} - V_{N2} &= \frac{1}{4} (c_2^2 m_+^2 + c_3^2 m_{++}^2) \\V_{CB6} - V_{N2} &= \frac{1}{4} c_2^2 m_{++}^2\end{aligned}$$

Stability of minima of type N1  
against charge breaking

## Stability of minima of type N2 against charge breaking

$$\begin{aligned}
 V_{CB1} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_+^2) \\
 V_{CB2} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_{++}^2) \\
 V_{CB3} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_+^2 + c_4^2 m_{++}^2) \\
 V_{CB4} - V_{N2} &= \frac{1}{4} c_2^2 m_+^2 \\
 V_{CB5} - V_{N2} &= \frac{1}{4} (c_2^2 m_+^2 + c_3^2 m_{++}^2) \\
 V_{CB6} - V_{N2} &= \frac{1}{4} c_2^2 m_{++}^2
 \end{aligned}$$

## Stability of minima of type N1 against charge breaking

$$\begin{aligned}
 V_{CB1} - V_{N1} &= \frac{c_3^2 m_+^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \\
 V_{CB2} - V_{N1} &= \frac{1}{4} c_3^2 m_{++}^2 \\
 V_{CB3} - V_{N1} &= \frac{m_+^2 c_3^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} + \frac{1}{4} c_4^2 m_{++}^2 - \frac{1}{8} \lambda_3 v_\Delta^2 \frac{c_3^2 c_4}{c_2} \\
 V_{CB4} - V_{N1} &= \frac{c_1^2 m_+^2}{4 \left(2 + \frac{v_\Phi^2}{v_\Delta^2}\right)} + \frac{1}{8} c_2^2 m_{++}^2 \\
 V_{CB5} - V_{N1} &= \frac{c_1^2 m_+^2}{4 \left(2 + \frac{v_\Phi^2}{v_\Delta^2}\right)} + \frac{1}{8} c_2^2 m_{++}^2 + \frac{c_3^2 m_+^2}{2 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \\
 V_{CB6} - V_{N1} &= \frac{c_1^2 m_+^2}{2 \left(2 + \frac{v_\Phi^2}{v_\Delta^2}\right)} + \frac{c_2^2 m_+^2}{2 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)}
 \end{aligned}$$

Stability of minima of type N2  
against charge breaking

$$\begin{aligned}
 V_{CB1} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_+^2) \\
 V_{CB2} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_{++}^2) \\
 V_{CB3} - V_{N2} &= \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_+^2 + c_4^2 m_{++}^2) \\
 V_{CB4} - V_{N2} &= \frac{1}{4} c_2^2 m_+^2 \\
 V_{CB5} - V_{N2} &= \frac{1}{4} (c_2^2 m_+^2 + c_3^2 m_{++}^2) \\
 V_{CB6} - V_{N2} &= \frac{1}{4} c_2^2 m_{++}^2
 \end{aligned}$$

Stability of minima of type N1  
against charge breaking

$$\begin{aligned}
 V_{CB1} - V_{N1} &= \frac{c_3^2 m_+^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \\
 V_{CB2} - V_{N1} &= \frac{1}{4} c_3^2 m_{++}^2 \\
 V_{CB3} - V_{N1} &= \frac{m_+^2 c_3^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} + \frac{1}{4} c_4^2 m_{++}^2 - \frac{1}{8} \lambda_3 v_\Delta^2 \frac{c_3^2 c_4}{c_2} \\
 V_{CB4} - V_{N1} &= \frac{c_1^2 m_+^2}{4 \left(2 + \frac{v_\Phi^2}{v_\Delta^2}\right)} + \frac{1}{8} c_2^2 m_{++}^2 \\
 V_{CB5} - V_{N1} &= \frac{c_1^2 m_+^2}{4 \left(2 + \frac{v_\Phi^2}{v_\Delta^2}\right)} + \frac{1}{8} c_2^2 m_{++}^2 + \frac{c_3^2 m_+^2}{2 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \\
 V_{CB6} - V_{N1} &= \frac{c_1^2 m_+^2}{2 \left(2 + \frac{v_\Phi^2}{v_\Delta^2}\right)} + \frac{c_2^2 m_+^2}{2 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)}
 \end{aligned}$$

Stability of minima of type N2  
against charge breaking

$$V_{CB1} - V_{N2} = \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_+^2)$$

$$V_{CB2} - V_{N2} = \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_{++}^2)$$

$$V_{CB3} - V_{N2} = \frac{1}{4} (c_2^2 m_{H,A}^2 + c_3^2 m_+^2 + c_4^2 m_{++}^2)$$

$$V_{CB4} - V_{N2} = \frac{1}{4} c_2^2 m_+^2$$

$$V_{CB5} - V_{N2} = \frac{1}{4} (c_2^2 m_+^2 + c_3^2 m_{++}^2)$$

$$V_{CB6} - V_{N2} = \frac{1}{4} c_2^2 m_{++}^2$$

Stability of minima of type N1  
against charge breaking

$$V_{CB1} - V_{N1} = \frac{c_3^2 m_+^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)}$$

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$$V_{CB3} - V_{N1} = \frac{m_+^2 c_3^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} + \frac{1}{4} c_4^2 m_{++}^2 - \frac{1}{8} \lambda_3 v_\Delta^2 \frac{c_3^2 c_4}{c_2}$$

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$$V_{CB6} - V_{N1} = \frac{c_1^2 m_+^2}{2 \left(2 + \frac{v_\Phi^2}{v_\Delta^2}\right)} + \frac{c_2^2 m_+^2}{2 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)}$$

**STABILITY  
GUARANTEED**

Stability of minima of type N1 against charge breaking



## Stability of minima of type N1 against charge breaking

$$V_{CB1} - V_{N1} = \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta}{c_2} \frac{c_1^2}{v_\Phi^2}\right) + \frac{m_+^2 c_3^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)}$$

$$V_{CB2} - V_{N1} = \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta}{c_2} \frac{c_1^2}{v_\Phi^2}\right) + \frac{1}{4} c_3^2 m_{++}^2$$

$$V_{CB3} - V_{N1} = \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta}{c_2} \frac{c_1^2}{v_\Phi^2}\right) + \frac{m_+^2 c_3^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} + \frac{1}{4} c_4^2 m_{++}^2 - \frac{1}{8} \lambda_3 v_\Delta^2 \frac{c_3^2 c_4}{c_2}$$

$$V_{CB4} - V_{N1} = \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} \left(\frac{c_2^2}{2} + v_\Delta^2 + c_1^2 \frac{v_\Delta^2}{v_\Phi^2}\right) + \frac{1}{8} c_2^2 m_{++}^2 + \frac{v_\Delta^2}{v_\Phi^2} \frac{c_1^2 m_+^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)}$$

$$V_{CB5} - V_{N1} = \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} \left(\frac{c_2^2}{2} + v_\Delta^2 + c_1^2 \frac{v_\Delta^2}{v_\Phi^2} - c_3^2\right) + \frac{1}{8} c_2^2 m_{++}^2 + \frac{m_+^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \left(c_1^2 \frac{v_\Delta^2}{v_\Phi^2} + 2c_3^2\right)$$

$$V_{CB6} - V_{N1} = \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (v_\Delta^2 - c_2^2) + \frac{m_+^2}{2 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \left(c_1^2 \frac{v_\Delta^2}{v_\Phi^2} + c_2^2\right)$$

## Stability of minima of type N1 against charge breaking

$$\begin{aligned}
 V_{CB1} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta c_1^2}{c_2 v_\Phi^2}\right) + \frac{m_+^2 c_3^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \\
 V_{CB2} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta c_1^2}{c_2 v_\Phi^2}\right) + \frac{1}{4} c_3^2 m_{++}^2 \\
 V_{CB3} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta c_1^2}{c_2 v_\Phi^2}\right) + \frac{m_+^2 c_3^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} + \frac{1}{4} c_4^2 m_{++}^2 - \frac{1}{8} \lambda_3 v_\Delta^2 \frac{c_3^2 c_4}{c_2} \\
 V_{CB4} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} \left(\frac{c_2^2}{2} + v_\Delta^2 + c_1^2 \frac{v_\Delta^2}{v_\Phi^2}\right) + \frac{1}{8} c_2^2 m_{++}^2 + \frac{v_\Delta^2}{v_\Phi^2} \frac{c_1^2 m_+^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \\
 V_{CB5} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} \left(\frac{c_2^2}{2} + v_\Delta^2 + c_1^2 \frac{v_\Delta^2}{v_\Phi^2} - c_3^2\right) + \frac{1}{8} c_2^2 m_{++}^2 + \frac{m_+^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \left(c_1^2 \frac{v_\Delta^2}{v_\Phi^2} + 2c_3^2\right) \\
 V_{CB6} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (v_\Delta^2 - c_2^2) + \frac{m_+^2}{2 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \left(c_1^2 \frac{v_\Delta^2}{v_\Phi^2} + c_2^2\right)
 \end{aligned}$$

## Stability of minima of type N1 against charge breaking


$$\begin{aligned}
 V_{CB1} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta c_1^2}{c_2 v_\Phi^2}\right) + \frac{m_+^2 c_3^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \\
 V_{CB2} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta c_1^2}{c_2 v_\Phi^2}\right) + \frac{1}{4} c_3^2 m_{++}^2 \\
 V_{CB3} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta c_1^2}{c_2 v_\Phi^2}\right) + \frac{1}{8} \lambda_3 v_\Delta^2 \frac{c_3^2 c_4}{c_2} \\
 V_{CB4} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (c_2 - v_\Delta)^2 \left(1 - \frac{v_\Delta c_1^2}{c_2 v_\Phi^2}\right) + \frac{1}{8} \lambda_3 v_\Delta^2 \frac{c_3^2 c_4}{c_2} \\
 V_{CB5} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} \left(\frac{c_2^2}{2} + v_\Delta^2 + c_1^2 \frac{v_\Delta^2}{v_\Phi^2} - c_3^2\right) + \frac{1}{8} c_2^2 m_{++}^2 + \frac{m_+^2}{4 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \left(c_1^2 \frac{v_\Delta^2}{v_\Phi^2} + 2c_3^2\right) \\
 V_{CB6} - V_{N1} &= \frac{m_A^2}{4 \left(1 + \frac{4v_\Delta^2}{v_\Phi^2}\right)} (v_\Delta^2 - c_2^2) + \frac{m_+^2}{2 \left(1 + \frac{2v_\Delta^2}{v_\Phi^2}\right)} \left(c_1^2 \frac{v_\Delta^2}{v_\Phi^2} + c_2^2\right)
 \end{aligned}$$

**STABILITY NOT GUARANTEED**

CB minima  $c_1 \neq 0$

	$\mu = 0$	$\mu \neq 0$
N2 minima	STABILITY GUARANTEED	DOES NOT OCCUR
N1 minima	STABILITY GUARANTEED	STABILITY NOT GUARANTEED

$$\frac{\partial V}{\partial c_1} = c_1 \left[ m^2 + \lambda_1 c_1^2 + \frac{\lambda_4}{2} (c_2^2 + c_3^2 + c_4^2) + \frac{\lambda_5}{2} (2c_2^2 + c_3^2) \right] = 0$$

  $c_1 = 0$  disconnected solution from  $c_1 \neq 0$

Three possibilities for neutral vacua

$$\langle \Phi \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}, \quad \langle \Delta \rangle_{N1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Delta \rangle_{N2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{N3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$$

Six new possibilities for CB vacua

$$\langle \Phi \rangle_{CB7} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB7} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} & c_2 \\ c_2 & -c_3/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB8} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB8} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & c_2 \\ c_2 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{CB9} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB9} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -c_2 \\ c_2 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{CB10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB10} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} & -c_3^2/2c_2 \\ c_2 & -c_3/\sqrt{2} \end{pmatrix}$$

$$\langle \Phi \rangle_{CB11} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB11} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & c_4 \\ 0 & 0 \end{pmatrix}$$

$$\langle \Phi \rangle_{CB12} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Delta \rangle_{CB12} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3/\sqrt{2} & 0 \\ 0 & -c_3/\sqrt{2} \end{pmatrix}$$

## Stability of minima of type N1 against charge breaking

$$V_{CB7} - V_{N1} = \frac{v_\Phi^2}{v_\Delta^2} \frac{m_h^2 m_H^2}{16(\lambda_2 + \lambda_3)} \left( -\lambda_3 \right) \frac{[2(\lambda_2 + \lambda_3)v_\Delta^2 + (\lambda_4 + \lambda_5)v_\Phi^2]^2}{16(\lambda_2 + \lambda_3)(2\lambda_2 + \lambda_3)}$$

$$V_{CB10} - V_{N1} = \frac{v_\Phi^2}{v_\Delta^2} \frac{m_h^2 m_H^2}{16(\lambda_2 + \lambda_3)}$$

## Stability of minima of type N2 against charge breaking

$$V_{CB7} - V_{N2} = \frac{1}{4} \left( \frac{m^4}{\lambda_1} \left( - \right) \frac{M^4}{\lambda_2 + \frac{1}{2}\lambda_3} \right)$$

$$V_{CB10} - V_{N2} = \frac{1}{4} \left( \frac{m^4}{\lambda_1} \left( - \right) \frac{M^4}{\lambda_2 + \lambda_3} \right)$$

The expressions for  $V_{CB7} - V_{N1}$  hold for  $CB8$ ,  $CB9$  and  $CB12$ , while the second one also holds for  $V_{CB11} - V_{N1}$



## Stability of minima of type N1 against charge breaking

$$\begin{aligned}
 V_{CB7} - V_{N1} &= \frac{v_\Phi^2}{v_\Delta^2} \frac{m_h^2 m_H^2}{16(\lambda_2 + \lambda_3)} \ominus \frac{\lambda_1}{8(\lambda_2 + \lambda_3)} \frac{m_A^2}{1 + \frac{4v_\Delta^2}{v_\Phi^2}} \frac{v_\Phi^4}{v_\Delta^2} \ominus \lambda_3 \frac{[2(\lambda_2 + \lambda_3)v_\Delta^2 + (\lambda_4 + \lambda_5)v_\Phi^2]^2}{16(\lambda_2 + \lambda_3)(2\lambda_2 + \lambda_3)} \\
 &\quad + \frac{\lambda_3}{2(2\lambda_2 + \lambda_3)} \frac{m_A^2}{1 + \frac{4v_\Delta^2}{v_\Phi^2}} \left[ v_\Delta^2 + \frac{\lambda_4 + \lambda_5}{2(\lambda_2 + \lambda_3)} v_\Phi^2 \ominus \frac{1}{2(\lambda_2 + \lambda_3)} \frac{m_A^2}{1 + \frac{4v_\Delta^2}{v_\Phi^2}} \right] \\
 V_{CB10} - V_{N1} &= \frac{v_\Phi^2}{v_\Delta^2} \frac{m_h^2 m_H^2}{16(\lambda_2 + \lambda_3)} \ominus \frac{\lambda_1}{8(\lambda_2 + \lambda_3)} \frac{m_A^2}{1 + \frac{4v_\Delta^2}{v_\Phi^2}} \frac{v_\Phi^4}{v_\Delta^2}
 \end{aligned}$$

The expressions for  $V_{CB7} - V_{N1}$  hold for  $CB8$ ,  $CB9$  and  $CB12$ , while the second one also holds for  $V_{CB11} - V_{N1}$

# Full analytical conclusions

CB minima  $c_1 \neq 0$

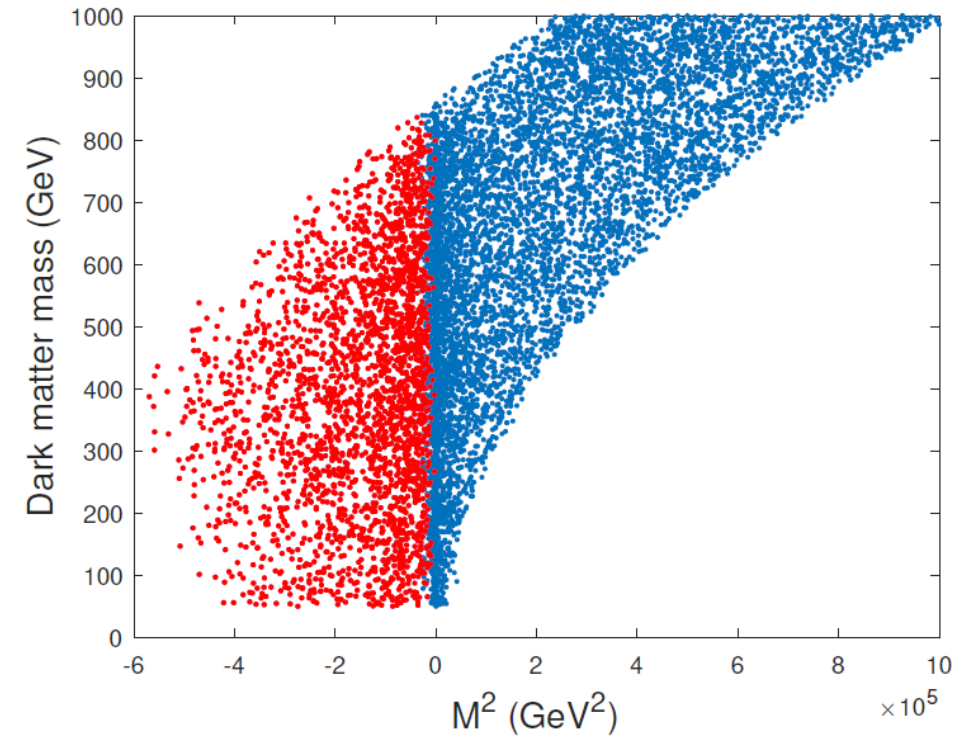
	$\mu = 0$	$\mu \neq 0$
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CB minima  $c_1 = 0$

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# Numerical scan without soft-breaking

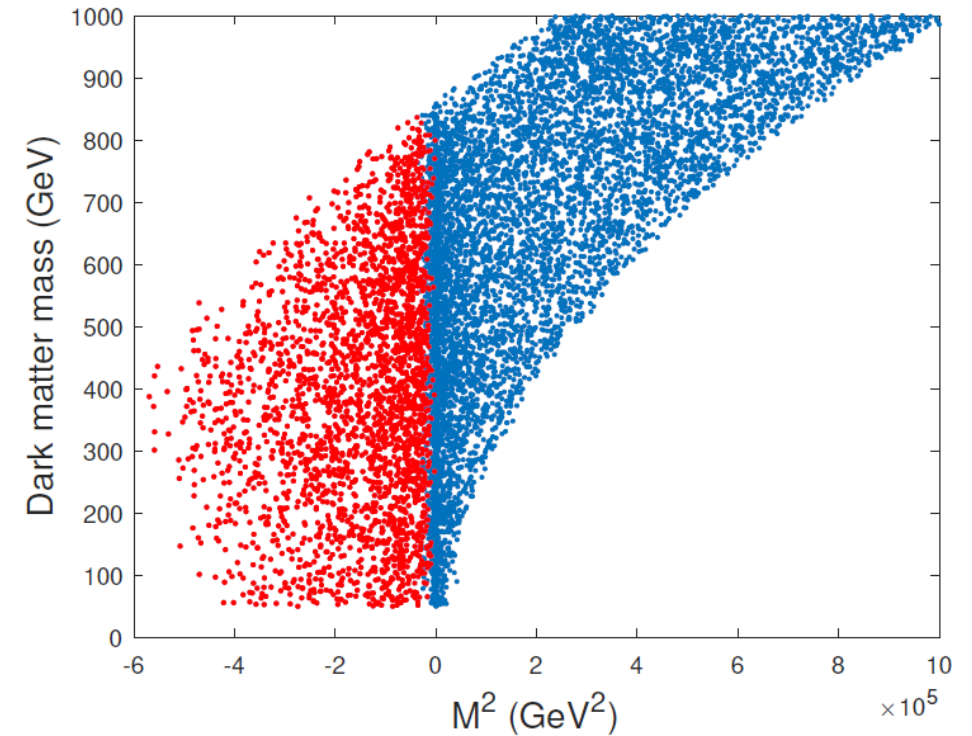
- N2 minima is stable against N1 minima type
- **N2 not necessarily stable against CB with vevless doublet**
- We have identified the most likely CB vacua as the vev combinations we dubbed CB7 and CB10



In blue, all scanned points for a minimum of type N2; in red, those points for which there exists a CB vacuum (CB7 or CB10) lower than N2

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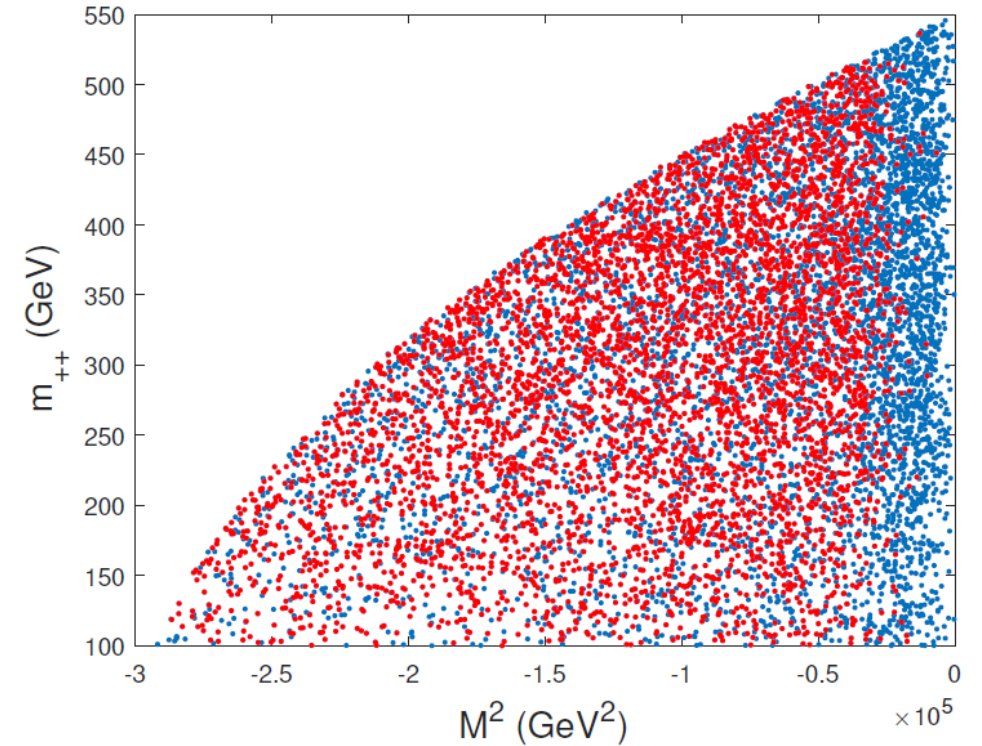


It can be shown analytically that

An  $N2$  minimum is stable against charge breaking iff  $M^2 > -\sqrt{\min\left(\lambda_2 + \frac{1}{2}\lambda_3, \lambda_2 + \lambda_3\right)} \frac{m_h v}{\sqrt{2}}$

# Numerical scan with soft-breaking

- N2 minima is not possible to occur
- **With a minimum of type N1, there are several possible deeper CB vacua**
- Large percentage of potentially-unstable neutral minima



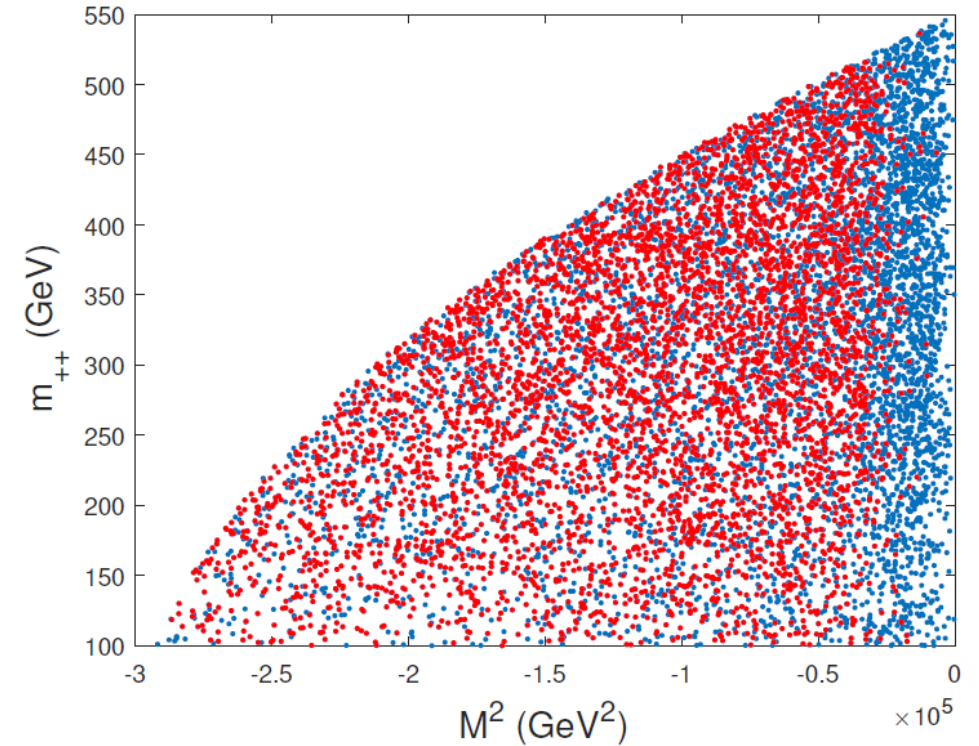
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# Numerical scan with soft-breaking

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## SOME REMARKS

- We found that for roughly 26% (48%) of the parameter space found for the globally symmetric (softly broken) potential neutral minima had deeper charge breaking ones
- CB global minima can indeed coexist, in some cases fairly frequently, with neutral minima

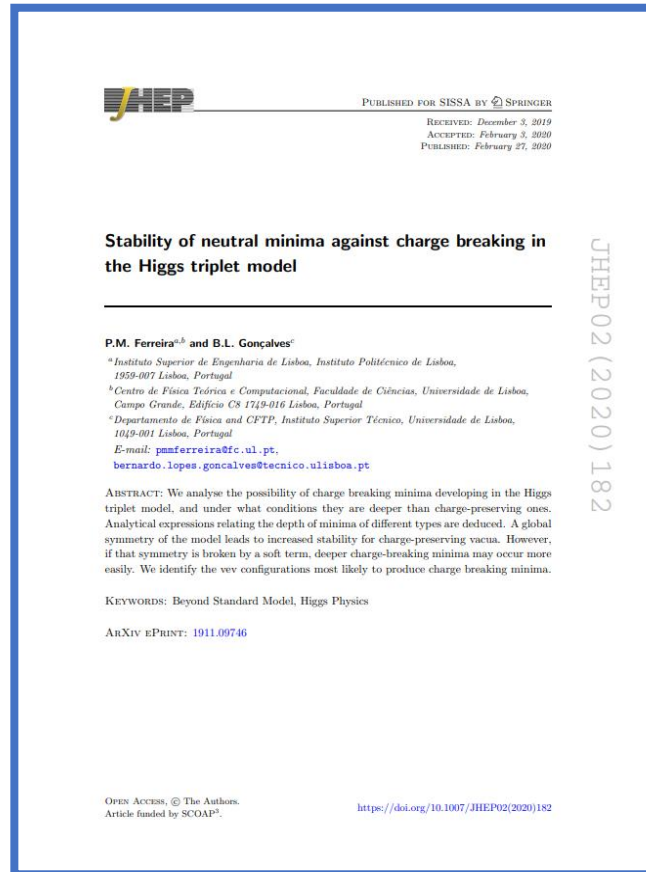


# Beyond the Standard Model

## Scalar triplet extensions of the Standard Model

Multi-Higgs  
scenario

Two-scalar-triplet model  
(2STM)



[arXiv:1911.09746v3](https://arxiv.org/abs/1911.09746v3) [hep-ph]

Motivation

Problem



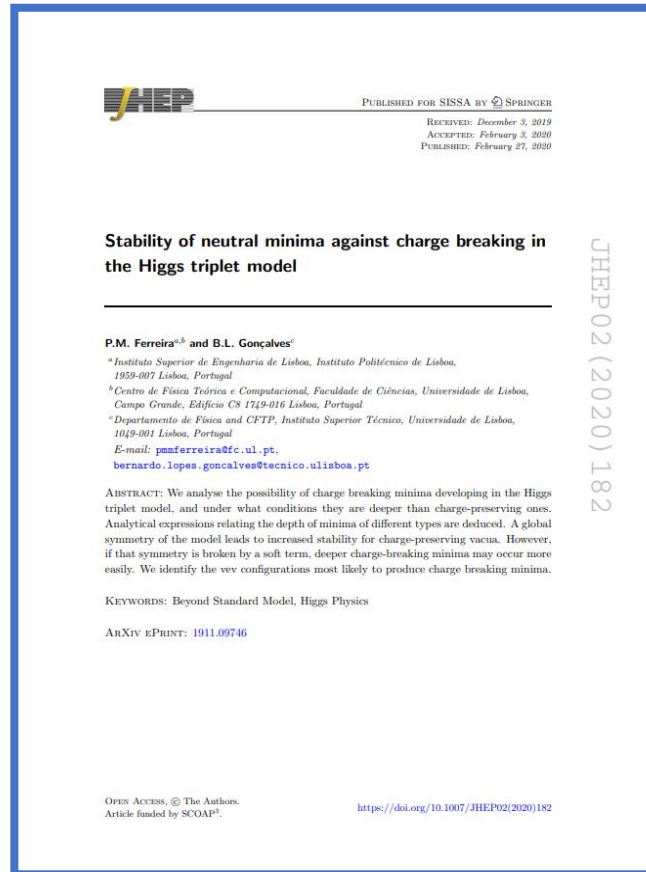
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Minimal triplet extension  
in which spontaneous CP  
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Do we have decoupling  
in the scalar mass  
spectrum?



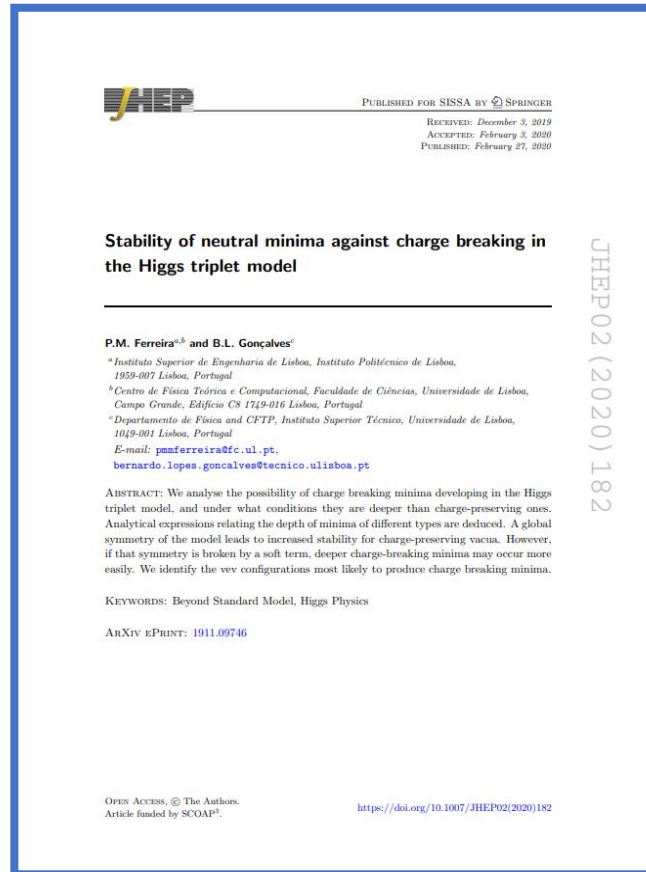
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**OUT  
SOON**

Do we have decoupling  
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# Spontaneous CP violation in scalar-triplet models

In the Higgs-triplet model:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

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$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}, \quad \langle \delta^0 \rangle = \frac{ue^{i\theta}}{\sqrt{2}} \longrightarrow \boxed{\mu v u \sin \theta = 0}$$

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# Spontaneous CP violation in scalar-triplet models

In the two-scalar-triplet model:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \Delta_{1,2} = \begin{pmatrix} \delta_{1,2}^+/\sqrt{2} & \delta_{1,2}^{++} \\ \delta_{1,2}^0 & -\delta_{1,2}^+/\sqrt{2} \end{pmatrix}$$

$$\begin{aligned} V_{U(1)} = & m^2 \Phi^\dagger \Phi + M_{11}^2 \text{Tr}(\Delta_1^\dagger \Delta_1) + M_{22}^2 \text{Tr}(\Delta_2^\dagger \Delta_2) + \lambda_0 (\Phi^\dagger \Phi)^2 \\ & + \lambda_1 [\text{Tr}(\Delta_1^\dagger \Delta_1)]^2 + \lambda_2 [\text{Tr}(\Delta_2^\dagger \Delta_2)]^2 + \lambda_{21} \text{Tr}(\Delta_2^\dagger \Delta_2) \text{Tr}(\Delta_1^\dagger \Delta_1) + \lambda_{12} \text{Tr}(\Delta_1^\dagger \Delta_2) \text{Tr}(\Delta_2^\dagger \Delta_1) \\ & + \tilde{\lambda}_1 \text{Tr}[(\Delta_1^\dagger \Delta_1)^2] + \tilde{\lambda}_2 \text{Tr}[(\Delta_2^\dagger \Delta_2)^2] + \tilde{\lambda}_{21} \text{Tr}(\Delta_2^\dagger \Delta_2 \Delta_1^\dagger \Delta_1) + \tilde{\lambda}_{12} \text{Tr}(\Delta_1^\dagger \Delta_2 \Delta_2^\dagger \Delta_1) \\ & + \lambda'_1 \text{Tr}(\Delta_1^\dagger \Delta_1) \Phi^\dagger \Phi + \lambda'_2 \text{Tr}(\Delta_2^\dagger \Delta_2) \Phi^\dagger \Phi + \hat{\lambda}_1 \Phi^\dagger \Delta_1 \Delta_1^\dagger \Phi + \hat{\lambda}_2 \Phi^\dagger \Delta_2 \Delta_2^\dagger \Phi \\ & + \end{aligned}$$

$$V_{\text{SB}} = M_{12}^2 [\text{Tr}(\Delta_1^\dagger \Delta_2) + \text{Tr}(\Delta_2^\dagger \Delta_1)] + (\mu_1 \Phi^T i\tau_2 \Delta_1^\dagger \Phi + \mu_2 \Phi^T i\tau_2 \Delta_2^\dagger \Phi + \text{H.c.})$$

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$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Delta_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ u_1 e^{i\theta_1} & 0 \end{pmatrix}, \quad \langle \Delta_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ u_2 e^{i\theta_2} & 0 \end{pmatrix} \quad \theta_1 \neq \theta_2$$

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$$u_1 = u c_\beta$$

$$u_2 = u s_\beta$$

$$\tan \beta = u_2/u_1$$


$$uv (\mu_1 c_\beta s_{\theta_1} + \mu_2 s_\beta s_{\theta_2}) = 0$$

# Spontaneous CP violation in scalar-triplet models

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↓

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**SCPV IS POSSIBLE**



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$$uv (\mu_1 c_\beta s \theta_1 + \mu_2 s_\beta s \theta_2) = 0$$

**SCPV IS  
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CP violation can be communicated to the fermion sector

# Spontaneous CP violation in scalar-triplet models

In the two-scalar-triplet model:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \Delta_{1,2} = \begin{pmatrix} \delta_{1,2}^+/\sqrt{2} \\ s_{1,2}^+ \end{pmatrix}$$

**BUT WHAT ABOUT THE SCALAR MASS SPECTRUM?**

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Delta_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u_1 e^{i\theta_1} \end{pmatrix}, \quad \langle \Delta_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u_2 e^{i\theta_2} \end{pmatrix}$$

$\theta_1 \neq \theta_2$

$$uv (\mu_1 c_\beta s_{\theta_1} + \mu_2 s_\beta s_{\theta_2}) = 0$$

**SCPV IS POSSIBLE**

CP violation can be communicated to the fermion sector

# Scalar mass spectrum

Using results from matrix theory, it is possible to find analytical results, exact or up to a good approximation, regarding the eigenvalues of the mass matrices, thus the scalar masses

- Six neutral scalars
- Three charged scalars
- Two doubly-charged scalars

# CP-conserving case

$$m_{h_2^0}^2 \simeq 2\lambda_0 v^2 - 2\sqrt{2}u\bar{\mu}, \quad \bar{\mu} = \mu_1 c_\beta + \mu_2 s_\beta$$

$$m_{h_{3,4}^0}^2 \simeq m_{h_{5,6}^0}^2 \simeq -\frac{M_{12}^2}{s_{2\beta}} + \frac{\sqrt{2}}{2} \frac{v^2 \bar{\mu}}{u s_{2\beta}} \pm \sqrt{\frac{v^2}{u^2 s_{2\beta}} \left( -\mu_1 \mu_2 v^2 + \sqrt{2} M_{12}^2 u \bar{\mu} \right) + \left( -\frac{M_{12}^2}{s_{2\beta}} + \frac{\sqrt{2}}{2} \frac{v^2 \bar{\mu}}{u s_{2\beta}} \right)^2}$$

$$m_{h_3^0}^2 \approx m_{h_4^0}^2 \approx m_{H_2^+}^2 \approx m_{H_1^{++}}^2, \quad m_{h_5^0}^2 \approx m_{h_6^0}^2 \approx m_{H_3^+}^2 \approx m_{H_2^{++}}^2$$

- Two Goldstone bosons
- One Higgs-like particle
- All the remaining particles decouple

# CP-violating case

This result is exact!

$$m_{h_{2,3}^0}^2 \leq \frac{u^2}{2} \left[ \Lambda_3 + \Lambda_5 + (\Lambda_3 - \Lambda_5)c_{2\beta} \pm \sqrt{[\Lambda_3 + \Lambda_5 + (\Lambda_3 - \Lambda_5)c_{2\beta}]^2 + (\Lambda_4^2 - 4\Lambda_3\Lambda_5) s_{2\beta}^2} \right]$$

$\Lambda_i \longrightarrow$  combinations of quartic couplings

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$\Lambda_i \longrightarrow$  combinations of quartic couplings

Two light neutral scalars!

# CP-violating case

$$M_{\Delta}^2 \equiv \frac{\mu_1}{\sqrt{2}u} v^2$$

$$m_{h_4^0}^2 \simeq 2\lambda_1 v^2 + 2\sqrt{2} \mu_1 u c_{\beta} \frac{s_{\theta_1 - \theta_2}}{s_{\theta_2}}$$

$$m_{h_5^0}^2 \simeq m_{h_6^0}^2 \simeq -\frac{s_{\theta_2}}{s_{\theta_1 - \theta_2}} [f_1(\beta, \theta_1, \theta_2) + f_2(\beta)] M_{\Delta}^2$$

$$m_{H_2^+}^2 \simeq \frac{m_{H_1^{++}}^2}{2} \simeq -\frac{1}{4} \frac{\hat{\lambda}_1 f_1(\beta, \theta_1, \theta_2) + \hat{\lambda}_2 f_2(\beta)}{f_1(\beta, \theta_1, \theta_2) + f_2(\beta)} v^2$$

$$m_{H_3^+}^2 \simeq m_{H_2^{++}}^2 \simeq -\frac{s_{\theta_2}}{s_{\theta_1 - \theta_2}} [f_1(\beta, \theta_1, \theta_2) + f_2(\beta)] M_{\Delta}^2$$



# CP-violating case

$$m_{h_4^0}^2 \simeq 2\lambda_1 v^2 + 2\sqrt{2} \mu_1 u c_\beta \frac{s_{\theta_1 - \theta_2}}{s_{\theta_2}}$$

Higgs-like

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Electroweak!

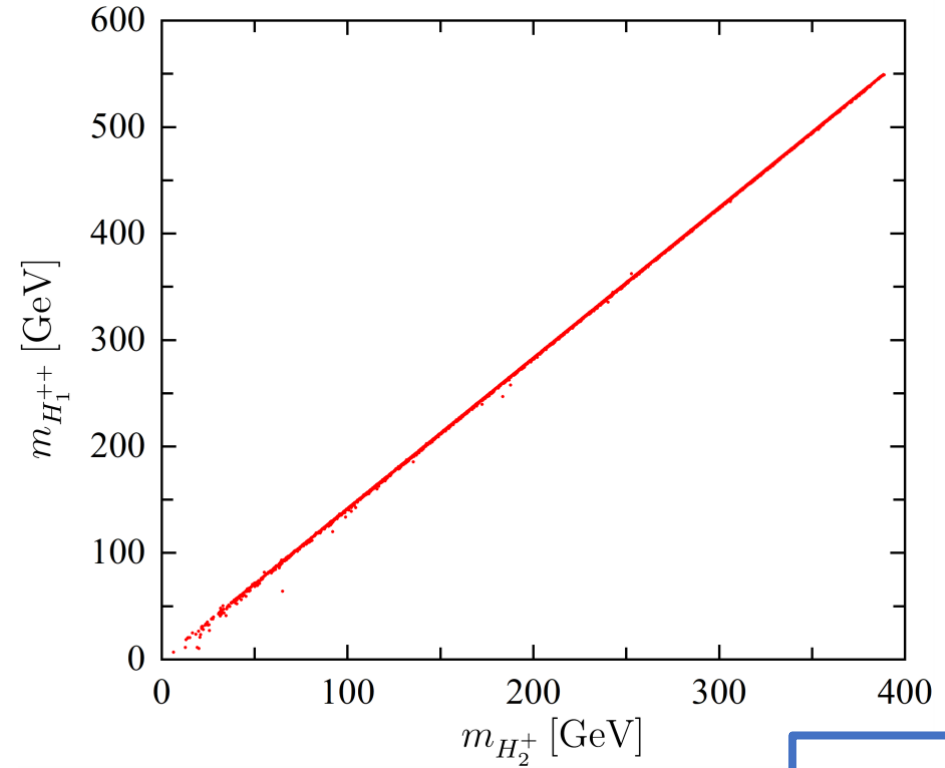
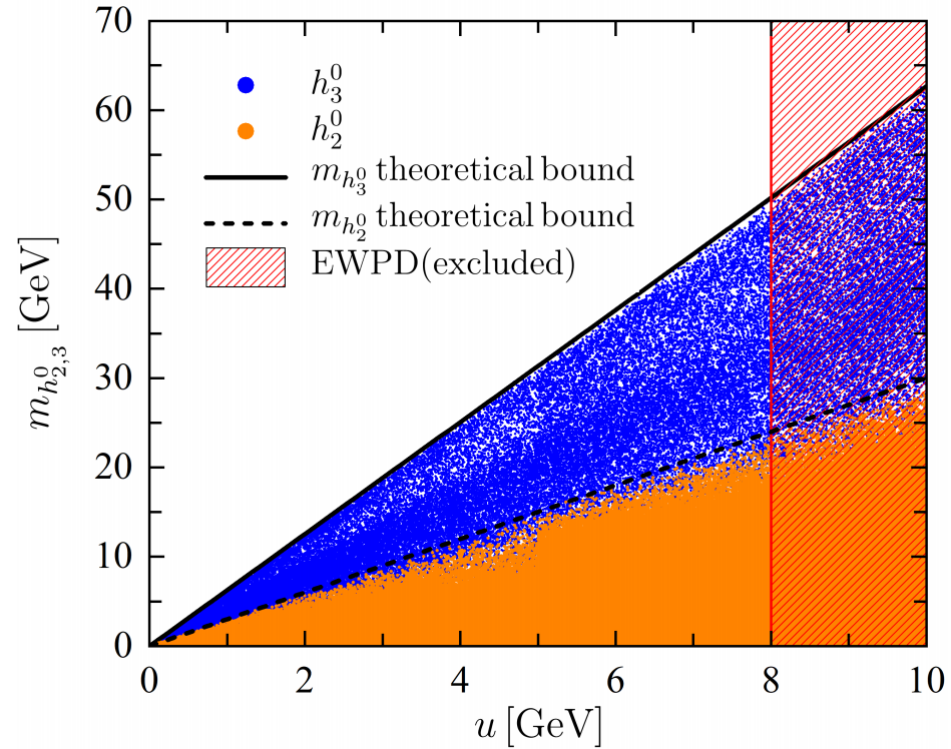
$$m_{H_3^+}^2 \simeq m_{H_2^{++}}^2 \simeq -\frac{s_{\theta_2}}{s_{\theta_1 - \theta_2}} [f_1(\beta, \theta_1, \theta_2) + f_2(\beta)] M_{\Delta}^2$$

Decoupled

# Full analytical conclusions

Mass spectrum		CP-Conserving	CP-Violating
Neutral	$h_1^0$	Massless - Goldstone boson	
	$h_2^0$	SM Higgs-like	Light
	$h_3^0$	Decoupled	
	$h_4^0$	Decoupled	SM Higgs-like
	$h_5^0$		Decoupled
	$h_6^0$		Decoupled
Singly-charged	$H_1^+$	Massless - Goldstone boson	
	$H_2^+$	Decoupled	Electroweak
	$H_3^+$	Decoupled	Decoupled
Doubly-charged	$H_1^{++}$	Decoupled	Electroweak
	$H_2^{++}$	Decoupled	Decoupled

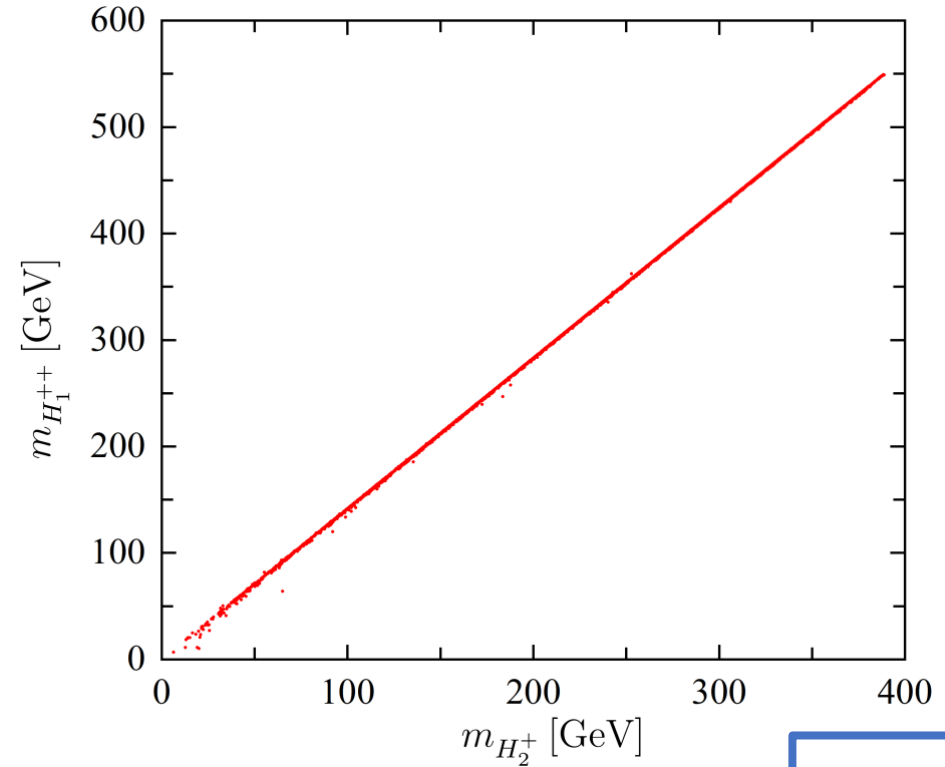
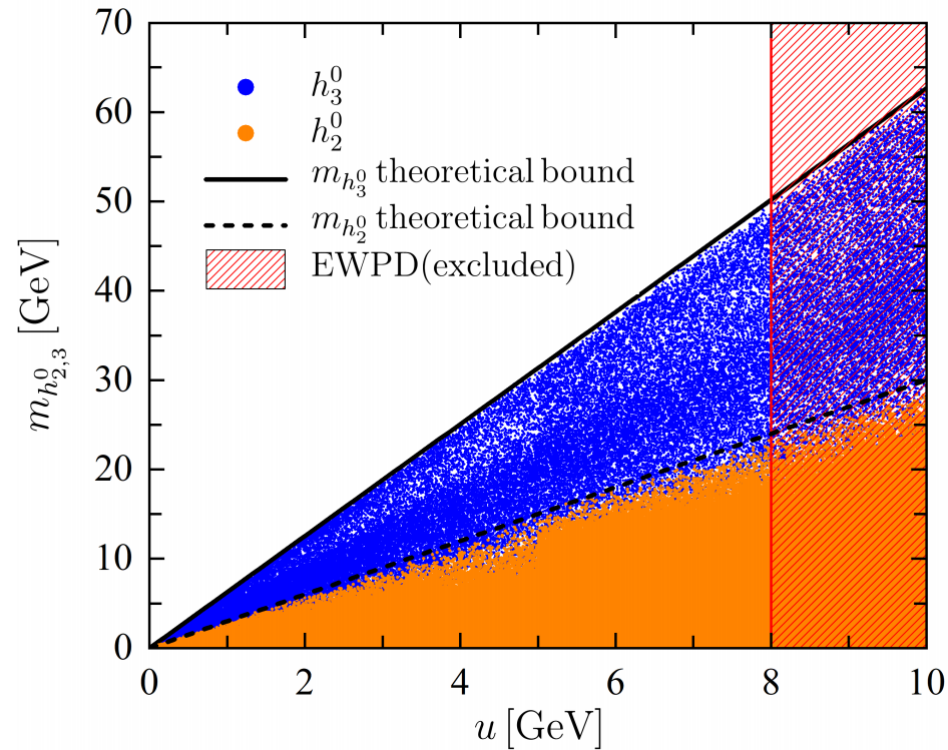
# Numerical scan



$$m_{h_{2,3}^0}^2 \leq \frac{u^2}{2} f(\Lambda, \beta)$$

$$m_{H_2^+}^2 \simeq \frac{m_{H_1^{++}}^2}{2}$$

# Numerical scan



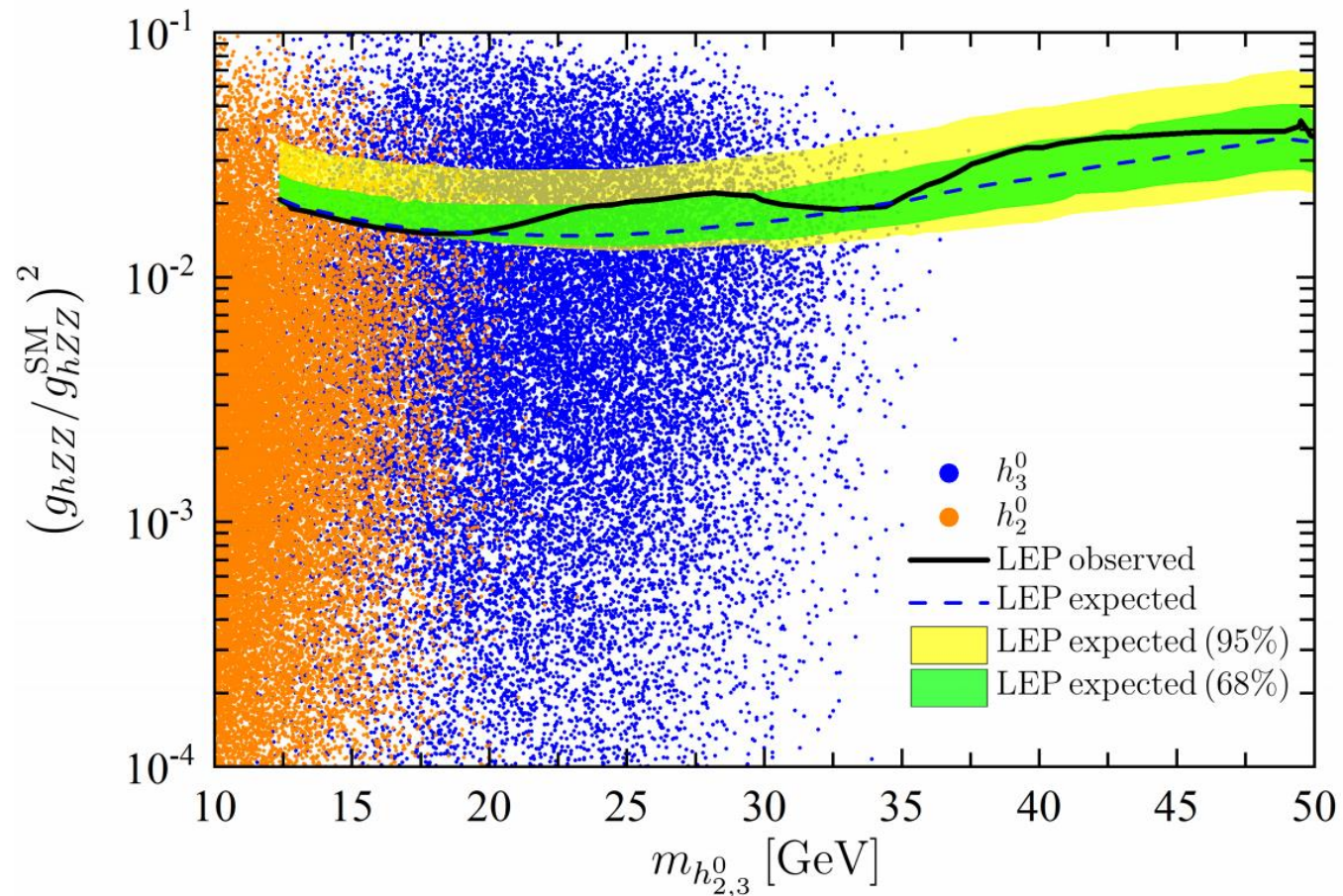
$$m_{h_{2,3}^0}^2 \leq \frac{u^2}{2} f(\Lambda, \beta)$$

Analytical results confirmed!

$$m_{H_2^+}^2 \simeq \frac{m_{H_1^{++}}^2}{2}$$

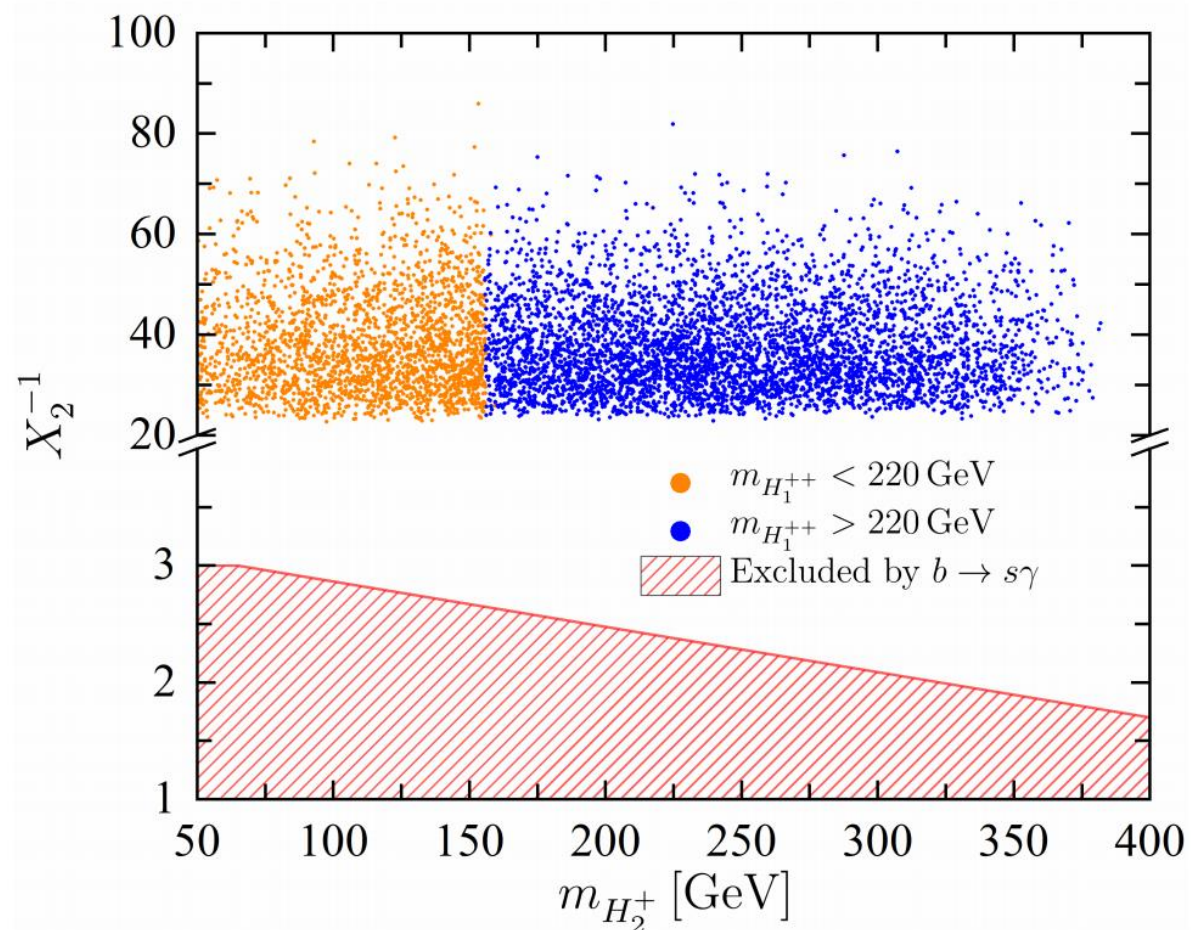
Could such scalars have evaded detection?

Could such scalars have evaded detection?





Could such scalars have evaded detection?



$$\phi^+ = X_1 H_1^+ + X_2 H_2^+ + X_3 H_3^+$$

BUT...

- Lepton-flavour-violation processes can impose a lower bound on the triplet's vev  $u$
- Can one simultaneously explain neutrino masses and leptonic CP violation via SCPV in the 2STM?
- Is this true for any number of triplets?