MAJORANA NEUTRINO MASSES BY D-BRANE INSTANTON EFFECTS AND MODULAR TRANSFORMATIONS

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Introduction

Standard Model is not Perfect!

It contains many free parameters (not including electric charges) to be determined.

It cannot include gravity (at least as a unified gauge theory).

It cannot explain the mechanism for the (left-handed) neutrino masses.

D-brane instanton effect and See-saw mechanism

Outlook

Majorana neutrino masses by D-brane instanton effects in magnetized orbifold models

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(arXiv:2103.07147)

How do neutrinos get their masses？

In this paper

D-brane instanton effects → RH Majorana neutrino masses

See-saw mechanism → LH neutrino masses

Our current study

See-saw mechanism

Introduce the right-handed neutrinos

Standard model gauge invariant
& renormalizable terms

$$-\mathcal{L} = M_{Dij}\bar{\nu}_{Ri}\nu_{Lj} + \frac{1}{2}M_{Nij}\bar{\nu}_{si}\nu^c_{Rj} + \text{h.c.}$$

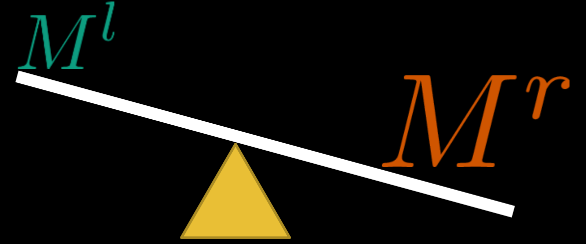
$$= \frac{1}{2}(\bar{\nu^c_L}, \bar{\nu_R})\begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{h.c.}$$

diagonalize

$$-\mathcal{L} = \frac{1}{2}\bar{\nu}_l M^l \nu_l + \frac{1}{2}\bar{N} M^r N$$

ν_L: LH neutrinos (active neutrinos)

ν_R: RH neutrinos (sterile neutrinos)



$$M^r \simeq V_r^T M_N V_r$$

$$M^l \simeq -V_l^T \boxed{M_D^T M_N^{-1} M_D} V_l$$

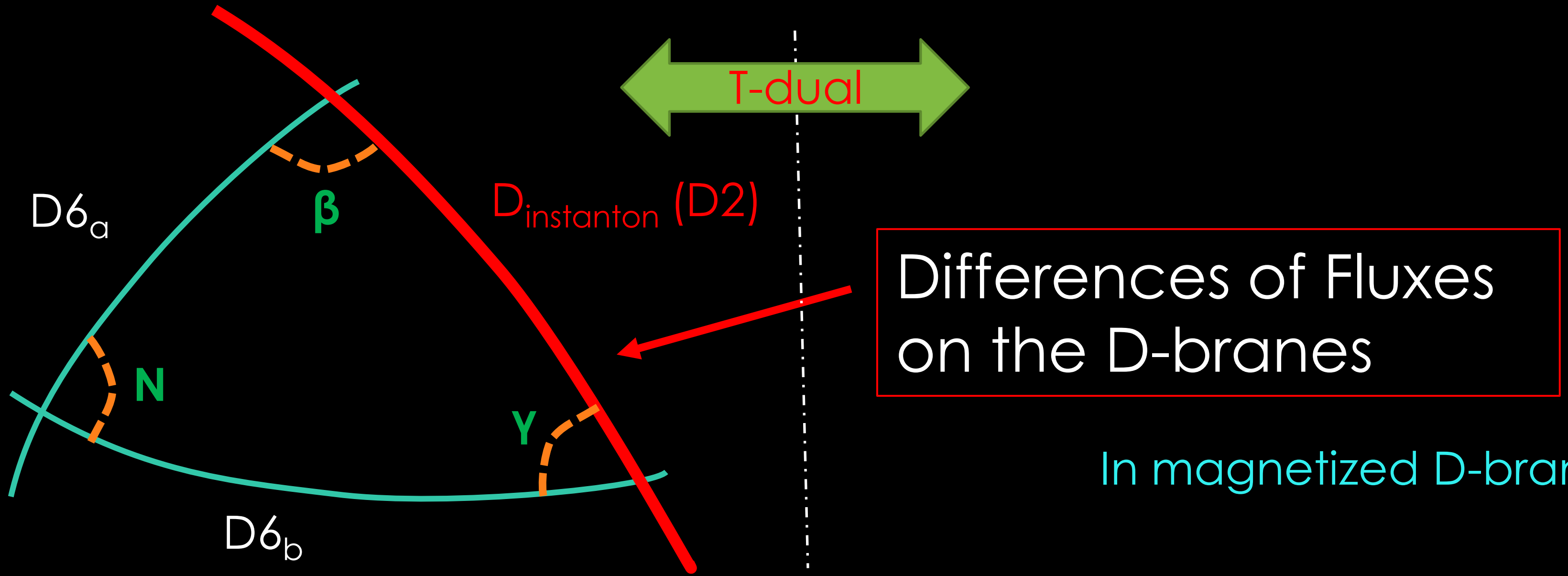
LH neutrino masses

Method

We will see some specific calculations for the right-handed Majorana neutrino masses deduced from the D-brane instanton effects.

In intersecting D-brane model

The intersecting angles correspond to the magnetic fluxes



In magnetized D-brane model

Procedure (1/4)

The Majorona mass terms due to the D-brane instanton effects

$$e^{-S_{cl}(D_{inst},M_{inst})}\int d^2\beta d^2\gamma e^{-\sum_{ija} d_a^{ij}\beta_i\gamma_j N_a},$$

β_i, γ_j : fermion zero-modes on D_{inst}
N_a : neutrinos

$$\int d^2\beta d^2\gamma e^{-\sum_{ija}\beta_i\gamma_j N_a} = N_a N_b(\varepsilon_{ij}\varepsilon_{k\ell}d_a^{ik}d_b^{j\ell}).$$

Grassmannian integral

$$M_{ab} = e^{-S_{cl}(D_{inst},M_{inst})}m_{ab}, \qquad m_{ab} = (\varepsilon_{ij}\varepsilon_{k\ell}d_a^{ik}d_b^{j\ell}).$$

Mass matrices

(arXiv:2103.07147)

Procedure (2/4)

$$e^{-S_{cl}(D_{inst},M_{inst})}\int d^2\beta d^2\gamma e^{-\sum_{ija} d_a^{ij}\beta_i\gamma_j N_a},$$

$$\int d^2\beta d^2\gamma e^{-\sum_{ija}\beta_i\gamma_j N_a} = N_a N_b(\varepsilon_{ij}\varepsilon_{k\ell}d_a^{ik}d_b^{j\ell}).$$

$$M_{ab} = e^{-S_{cl}(D_{inst},M_{inst})}m_{ab}, \qquad m_{ab} = (\varepsilon_{ij}\varepsilon_{k\ell}d_a^{ik}d_b^{j\ell}).$$

The formula we used to calculate each d-matrices

$$d_{ij}{}^{k} = \sigma_{abc}g\int \mathrm{d}z\mathrm{d}\bar{z}\, \psi_{M_1}^{(i+\alpha_1,\alpha_\tau)}(z)\psi_{M_2}^{(j+\alpha_1',\alpha_\tau')}(z)(\psi_{M_3}^{(k+\alpha_1'',\alpha_\tau'')}(z))^*$$

$$= c\sum_{m=0}^{|M_3|-1}\vartheta\begin{bmatrix}\frac{M_2(i+\alpha_1)-M_1(j+\alpha_1')+M_1M_2m}{M_1M_2M_3}\\ 0\end{bmatrix}(M_1\alpha_\tau' - M_2\alpha_\tau, \tau M_1M_2M_3)\delta_{(i+\alpha_1)+(j+\alpha_1')+M_1m,(k+\alpha_1'')+M_3l}$$

This has nonzero values only when

$$M_1 + M_2 = M_3,\ \alpha_1+\alpha_1' = \alpha_1'',\ \alpha_\tau+\alpha_\tau' = \alpha_\tau''$$

$$\eta_N^{(n)} = \vartheta\begin{bmatrix}\frac{N}{n}\\ 0\end{bmatrix}(0, n\tau)$$

(arXiv:2103.07147)

Procedure (3/4)

The wave functions on the T_2/Z_2-orbifold

$$\psi^{(j+\alpha_1,\alpha_2),|M|}_{T^2/\mathbb{Z}_2^m}(z) = \mathcal{N}\left(\psi^{(j+\alpha_1,\alpha_2),|M|}_{T^2}(z) + (-1)^m \psi^{(j+\alpha_1,\alpha_2),|M|}_{T^2}(-z)\right)$$
$$= \mathcal{N}\left(\psi^{(j+\alpha_1,\alpha_2),|M|}_{T^2}(z) + (-1)^{m-2\alpha_2} \psi^{(|M|-(j+\alpha_1),\alpha_2),|M|}_{T^2}(z)\right)$$

(m: parity)

where

$$\psi^{(j+\alpha_1,\alpha_\tau),|M_{12}|}_{T^2}(z) = \left(\frac{|M_{12}|}{\mathcal{A}^2}\right)^{1/4} e^{2\pi i \frac{(j+\alpha_1)\alpha_\tau}{|M_{12}|}} e^{\pi i |M_{12}| z \frac{\operatorname{Im} z}{\operatorname{Re}\tau}} \vartheta\begin{bmatrix} \frac{j+\alpha_1}{|M_{12}|} \\ -\alpha_\tau \end{bmatrix}(|M_{12}|\, z, |M_{12}|\, \tau)$$

$$\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(\nu,\tau) = \sum_{l\in\mathbb{Z}} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu+b)}.$$

$$\eta^{(n)}_N = \vartheta\begin{bmatrix} \frac{N}{n} \\ 0 \end{bmatrix}(0, n\tau)$$

$$\mathcal{N} = \begin{cases} 1/2 & (j+\alpha_1 = 0, |M|/2) \\ 1/\sqrt{2} & (\text{otherwise}) \end{cases}$$

(arXiv:2103.07147)

Procedure (4/4)

To have 3 generations for the RH neutrinos, M_N's must be
M_N = 4,8,5,7.

The possible values of the fluxes

M_N	β	γ
4	2	2
8	4	4
	2	6
	3	5
5	2	3
7	3	4
	2	5

We write as

2-2-4 model

4-4-8 model

etc…

⋮

(2-2-4 model)

Result with M_N=4 (1/3)

The mass matrix

$$m^{(2-2-4)} = c^2_{(2-2-4)} \begin{pmatrix} X_3 & 0 & X_1 \\ 0 & -\sqrt{2}X_2 & 0 \\ X_1 & 0 & X_3 \end{pmatrix}$$

where

$$\begin{aligned} X_1 &= (\eta_0+\eta_8)^2 + (\eta_4+\eta_{12})^2, \\ X_2 &= 2\sqrt{2}(\eta_2+\eta_{10})^2, \\ X_3 &= 2(\eta_0+\eta_8)(\eta_4+\eta_{12}). \end{aligned}$$

$$\eta_N = \vartheta\begin{bmatrix} \frac{N}{16} \\ 0 \end{bmatrix}(0, 16\tau).$$

The diagonalization matrix

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

The rotation by 45°

(bi-maximal)

The eigenvalues

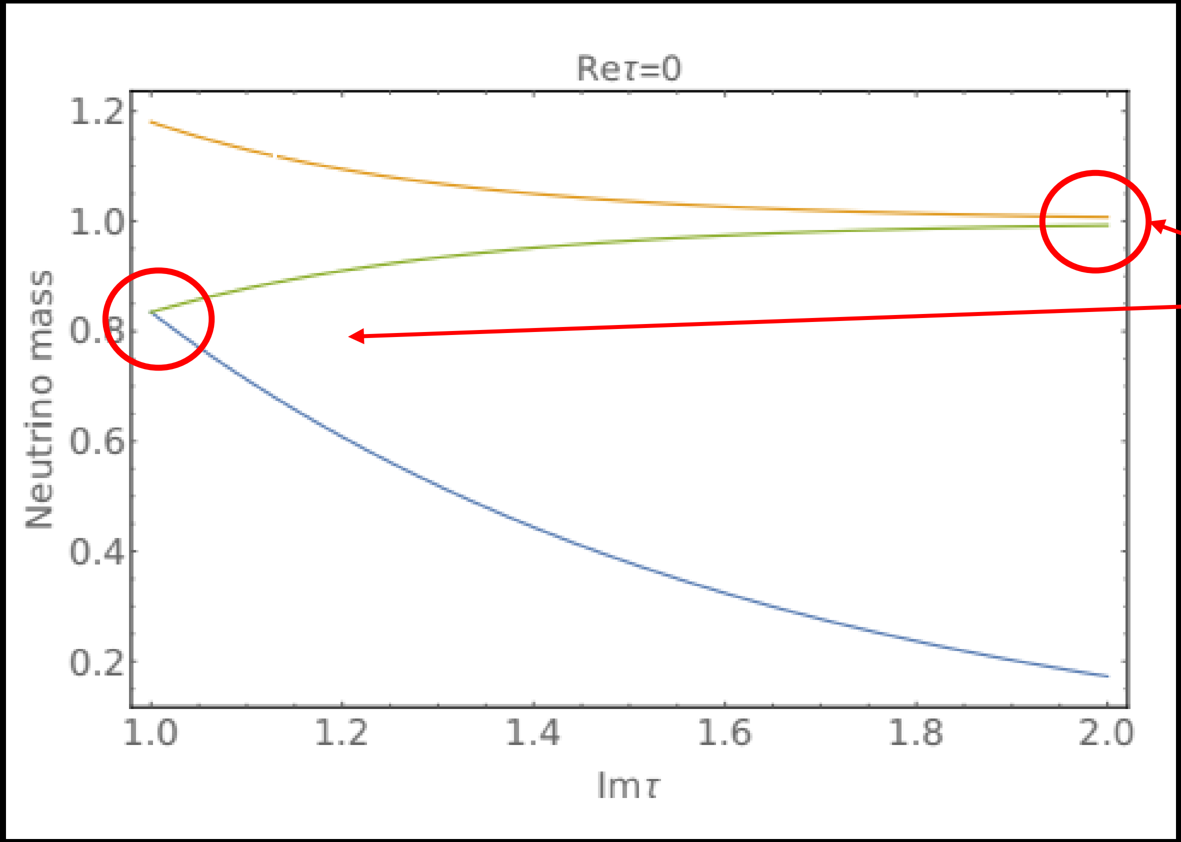
$$\begin{aligned} \lambda_1 &= c^2_{(2-2-4)}(X_1+X_3), \\ \lambda_2 &= -c^2_{(2-2-4)}\sqrt{2}X_2, \\ \lambda_3 &= c^2_{(2-2-4)}(X_3-X_1) \end{aligned}$$

(arXiv:2103.07147)

Result with M_N=4 (2/3)

(2-2-4 model)

The plots of the eigenvalues of the masses as the functions of Im τ.



These degeneracies can be explained by the Modular Transformations!

(arXiv:2103.07147)

Modular transformations (1/2)

Modular transformations send all lattice points on T^2 to lattice points.

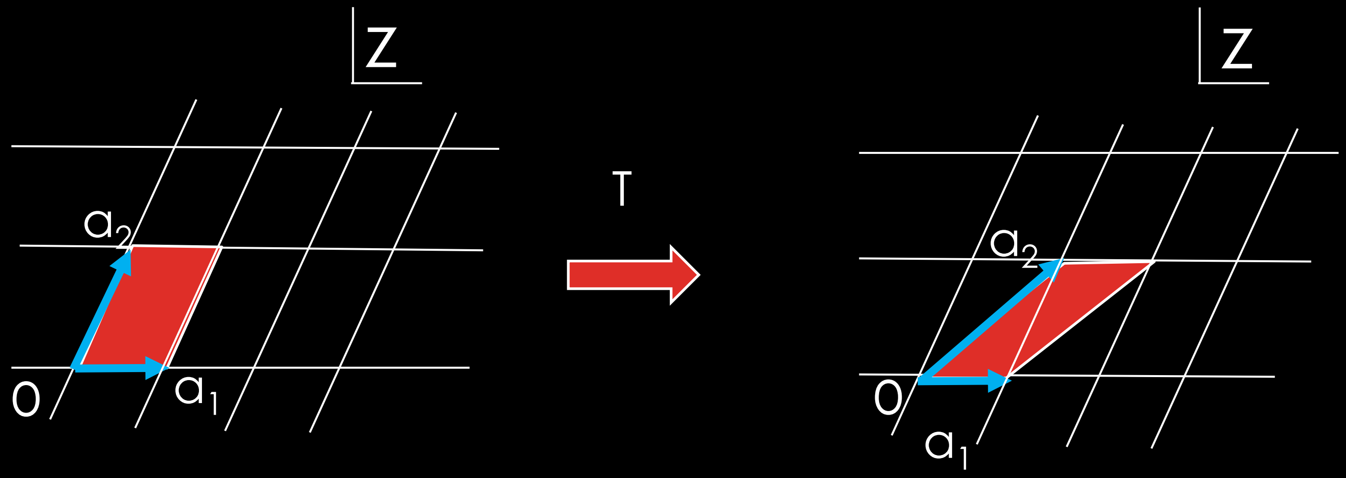
$$\begin{pmatrix} \alpha_2' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix} \quad (a, b, c, d \in \mathbb{Z}\ ,\ ad - bc = 1)$$

Modulus $\quad \tau := \dfrac{\alpha_2}{\alpha_1}$

The generators (T, S)

T-transformations $\quad \tau \to \tau + 1$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



Modular transformations (1/2)

Modular transformations send all lattice points on T^2 to

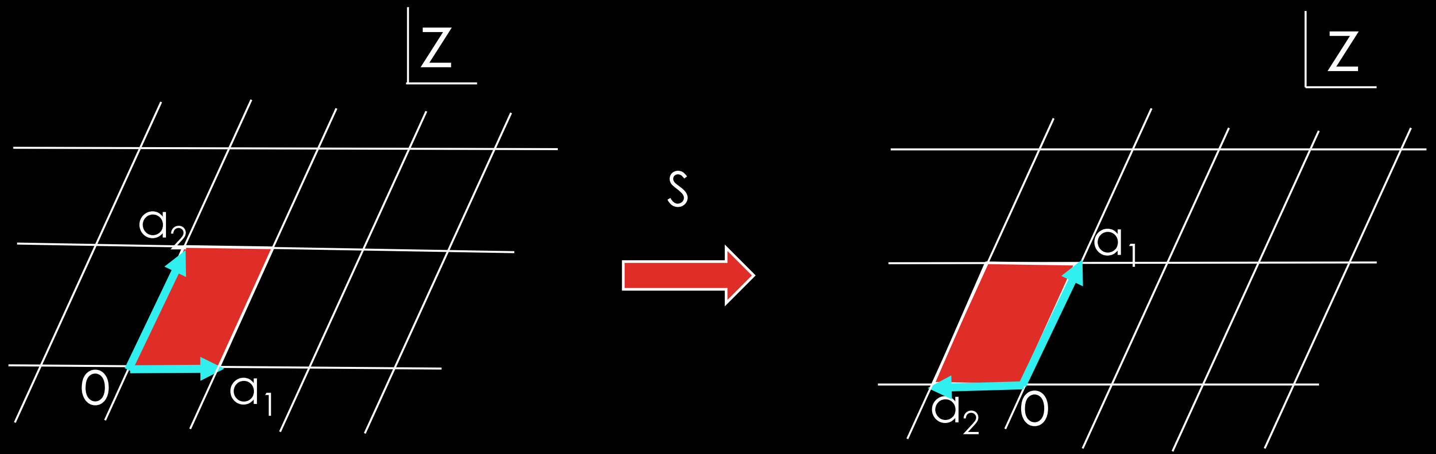
$$\begin{pmatrix} \alpha_2' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix} \quad (a,b,c,d \in \mathbb{Z}\ ,\ ad-bc=1)$$

Modulus $\tau := \dfrac{\alpha_2}{\alpha_1}$

The generators (T, S)

S-transformations $\tau \to -\dfrac{1}{\tau}$

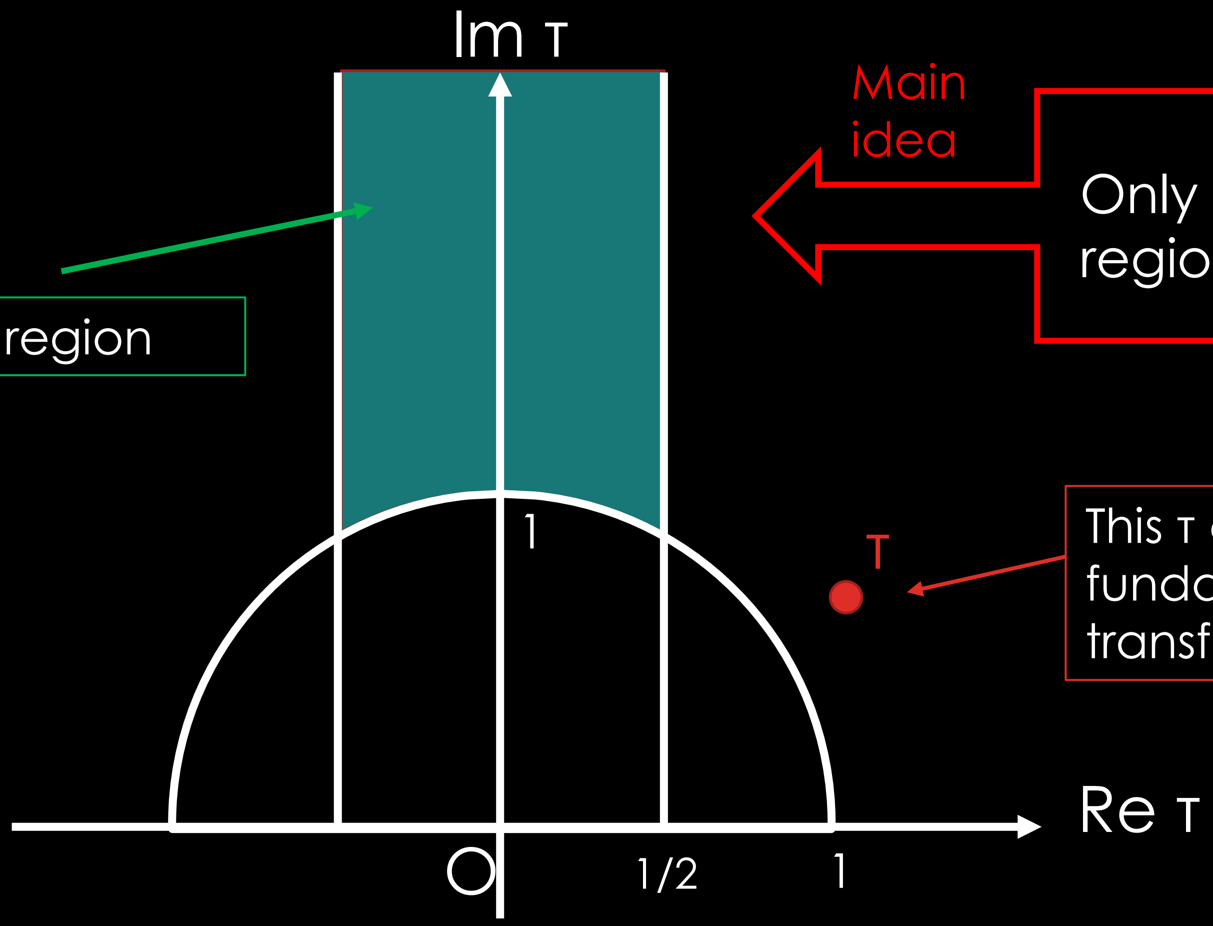
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



Modular transformations (2/2)

The fundamental region F defined as

$$F := \left\{ \tau \in \mathbb{C} \middle| \mathrm{Im}\tau > 0,\ |\mathrm{Re}| \leq \frac{1}{2},\ |\tau| \geq 1 \right\}$$



Only τ's which come from this region construct different tori.

This τ can be transformed into the fundamental region by a modular transformation.

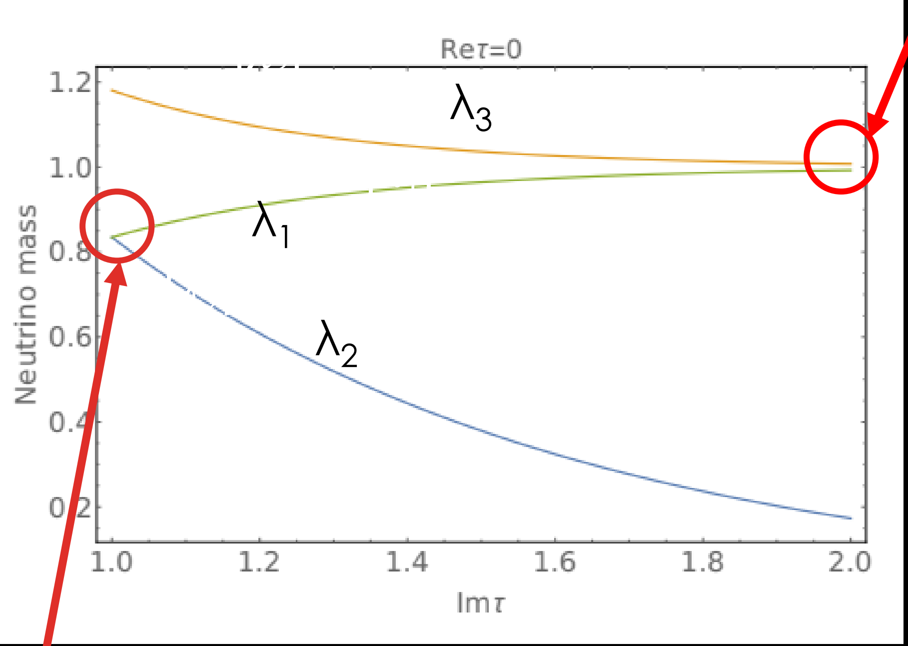
(2-2-4 model)

Result with M_N=4 (3/3)

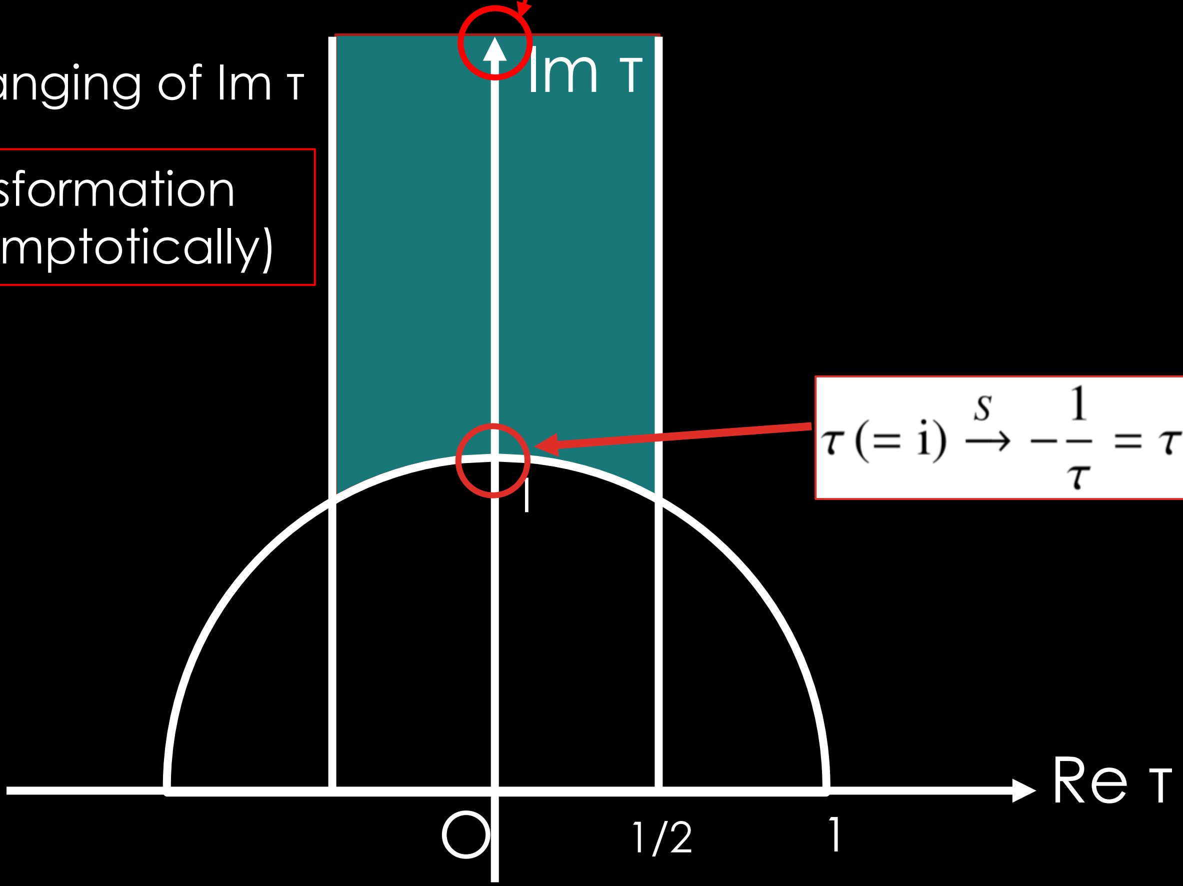
Assume that the contribution from τ→τ+1 can be ignored because Im τ= ∞

The behavior of the mass eigenvalues as the changing of Im τ

From the T-transformation invariance (asymptotically)



From the S-transformation invariance



$$\tau\,(=\mathrm{i}) \xrightarrow{S} -\frac{1}{\tau} = \tau$$

(arXiv:2103.07147)

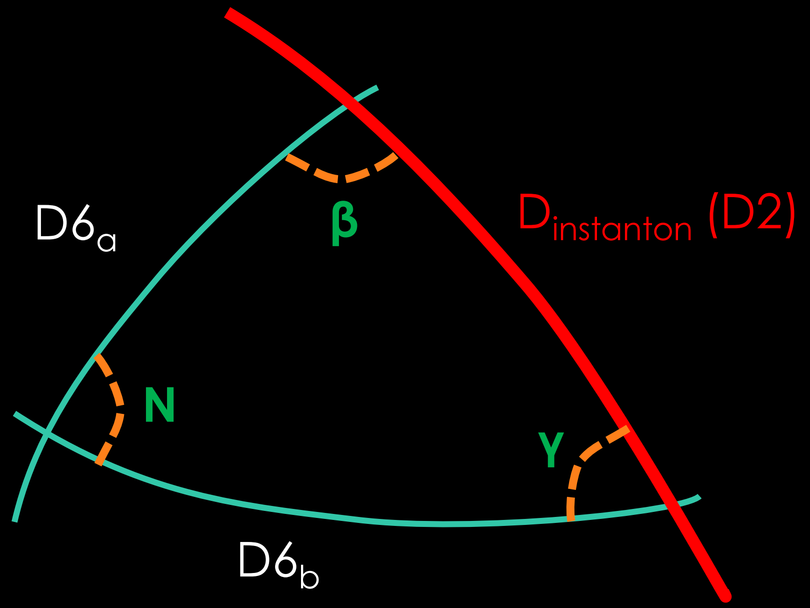
$$\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{pmatrix} \xrightarrow{S} \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1\end{pmatrix}\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{pmatrix}$$

$$\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{pmatrix} \xrightarrow{T} \begin{pmatrix} & & -1\\ & \mathrm{i} & \\ -1 & & \end{pmatrix}\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{pmatrix}$$

Summary

One of the problems in Standard Model is that the model does not give us the masses of neutrinos.

We have explored one possibility which exploits the "D-brane instanton effects".



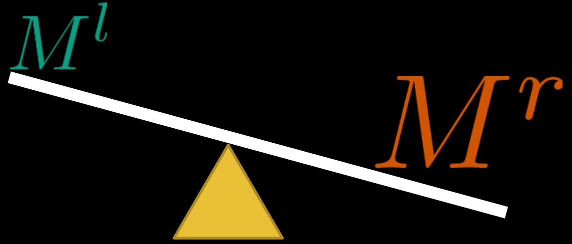
We have also seen that the RH Majorana neutrino masses of our calculation is consistent with the modular symmetric discussion.

Future Work

Unfortunately, the Majorana neutrino masses, which we have studied, are not observable.

We need to investigate observable quantities, e.x. PMNS matrix, the mixing angles of the leptons etc.…

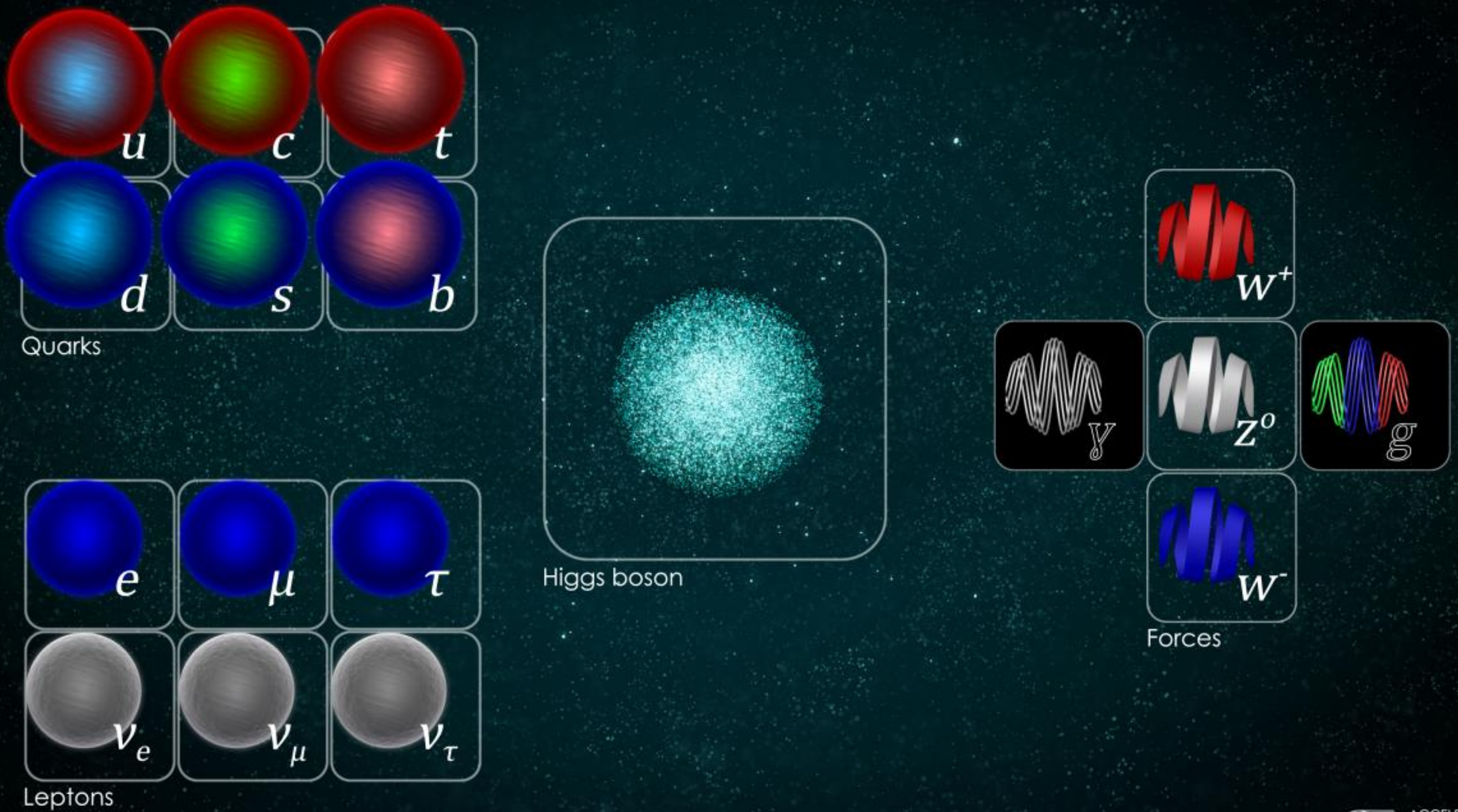
$$M^r$$



PMNS matrix

$$\begin{pmatrix} 0.80-0.84 & 0.52-0.58 & 0.14-0.16 \\ 0.24-0.49 & 0.47-0.68 & 0.64-0.77 \\ 0.28-0.52 & 0.49-0.70 & 0.62-0.75 \end{pmatrix}$$

Thank you for your attention!



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neutrino generation number	3	4	5	6	7
combination number of β and γ	81	108	54	12	1

Table 7: The combination numbers of zero-modes β_i and γ_j for the neutrino generation numbers. The number of all the possible combinations β_i and γ_j is equal to 256.

neutrino generation number	3	4	5	6	7
combination number of β and γ	27	18	12	6	1

Table 8: The combination numbers of zero-modes β_i and γ_j for the neutrino generation numbers such that the neutrino sector has vanishing SS phases. The number of all the possible combinations β_i and γ_j is equal to 64.

(arXiv:2103.07147)