NATIONAL SCIENCE CENTRE
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THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

Hot debate on spin polarization of Lambda hyperons

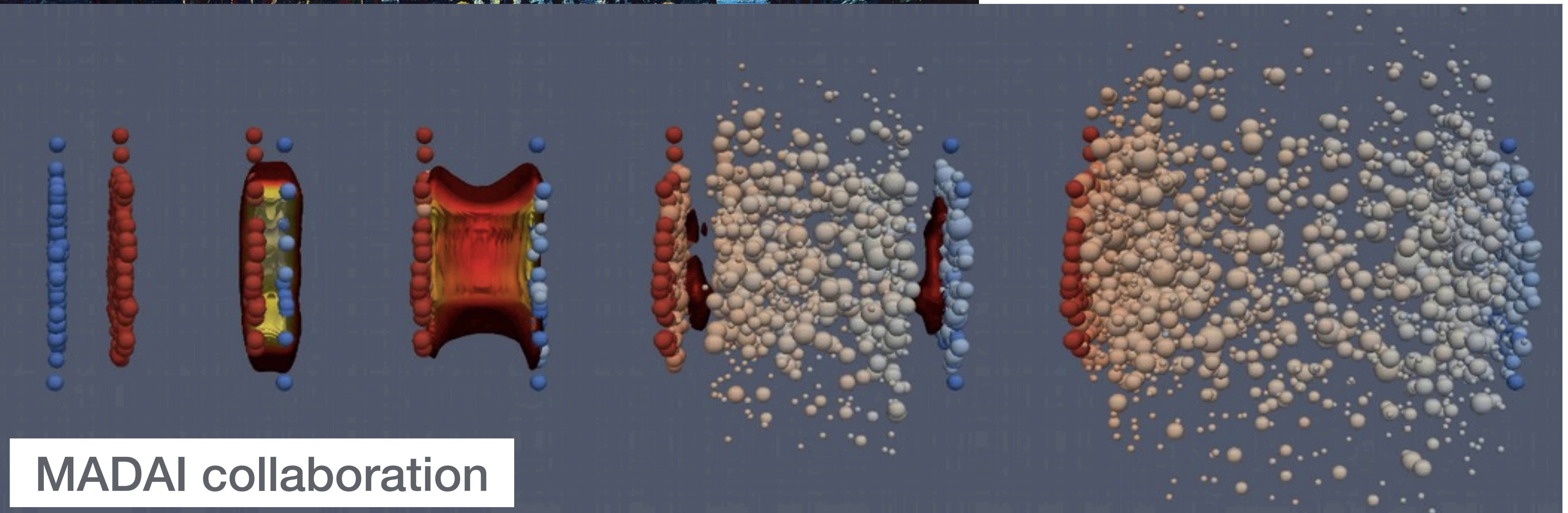
Rajeev Singh
The H. Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences
Kraków, Poland

Based on: arXiv:2103.02592, arXiv:2011.14907, and arXiv:1901.09655.

Meeting of the Division of Particles and Fields of the APS

14 July 2021

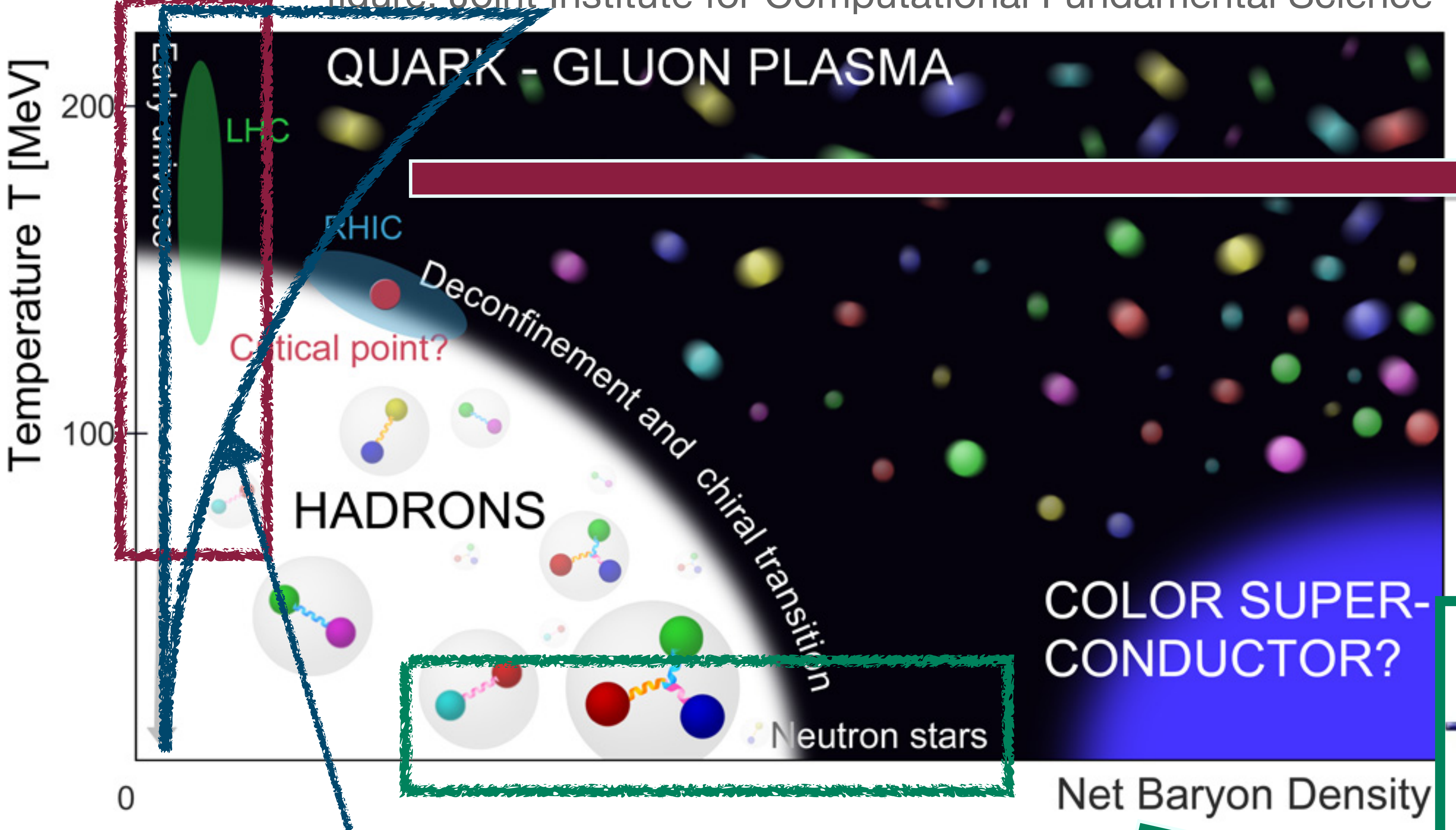

Vincent van Gogh

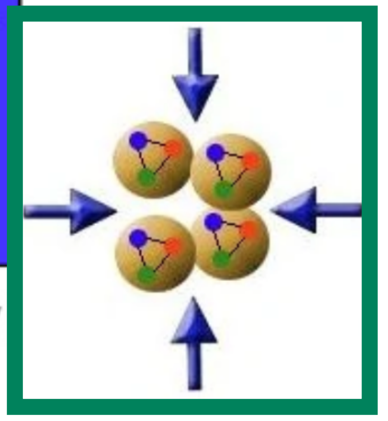

MADAI collaboration

Quantum Chromodynamics (QCD) pushed to extreme

QCD phase diagram

figure: Joint Institute for Computational Fundamental Science





Early Universe

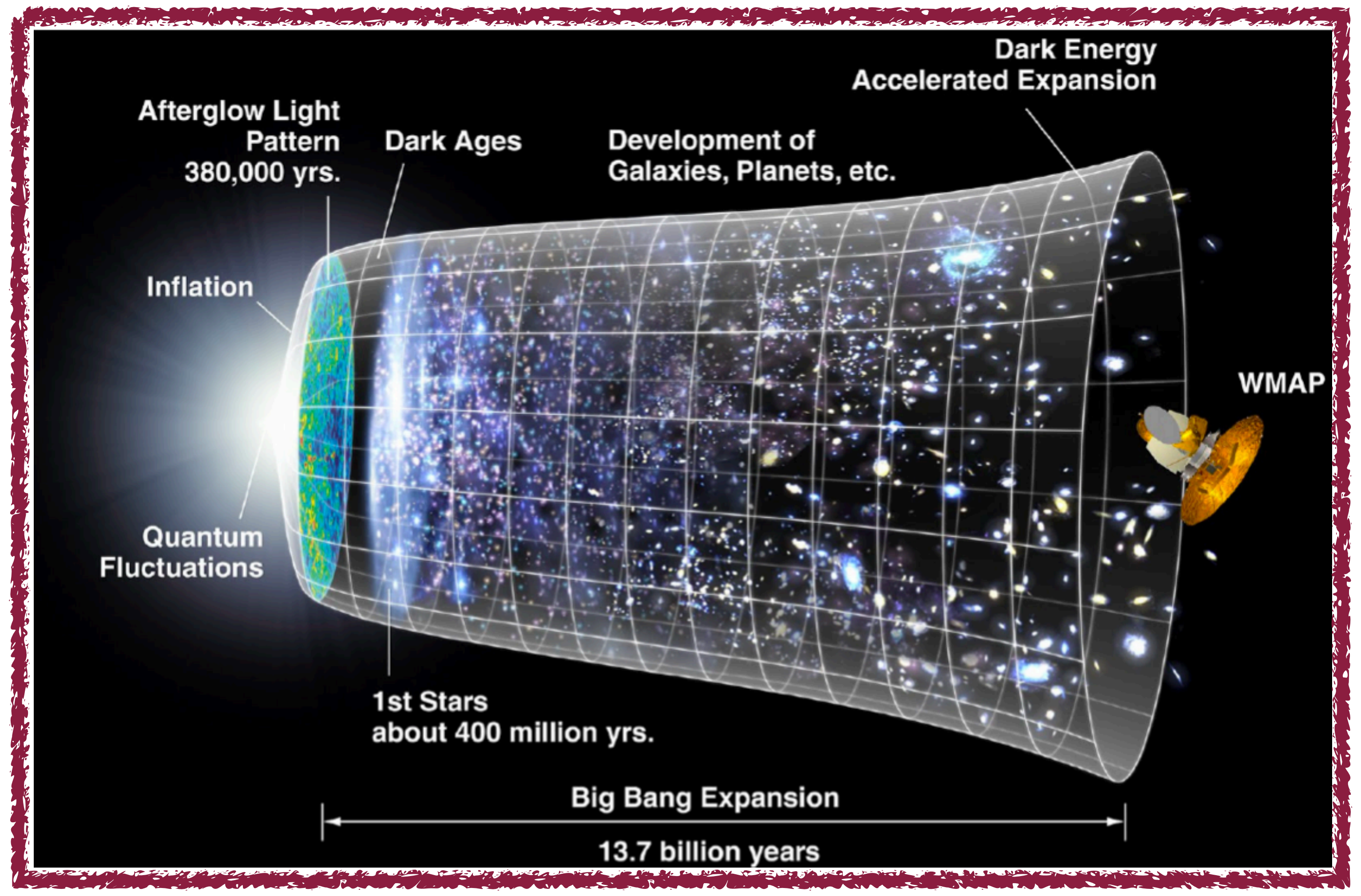


figure: NASA

Lattice-QCD simulations

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

$$\text{where } G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$$

$$\text{and } D_\mu \equiv \partial_\mu + i t^a A^a_\mu$$

That's it!

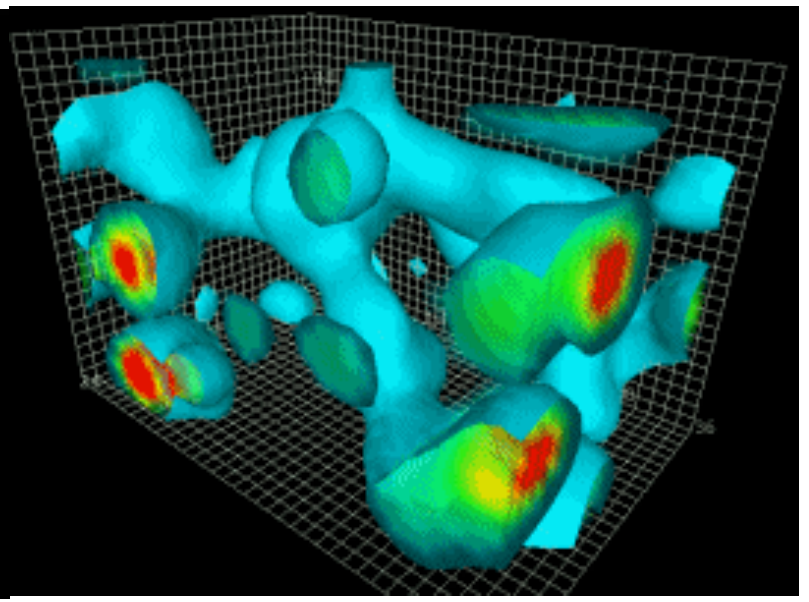


figure: D. Leinweber (www.physics.adelaide.edu.au)

Cores of neutron stars

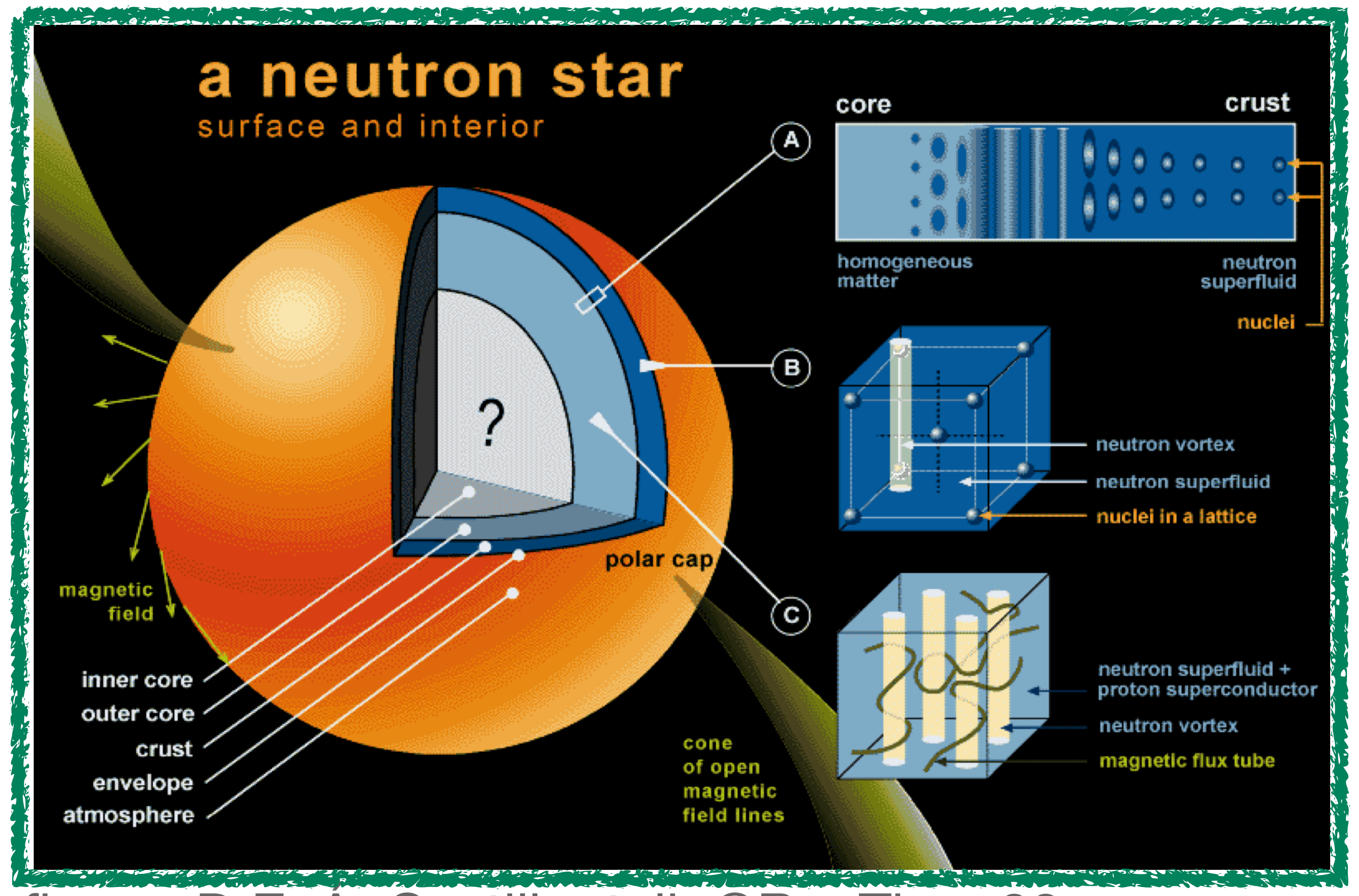
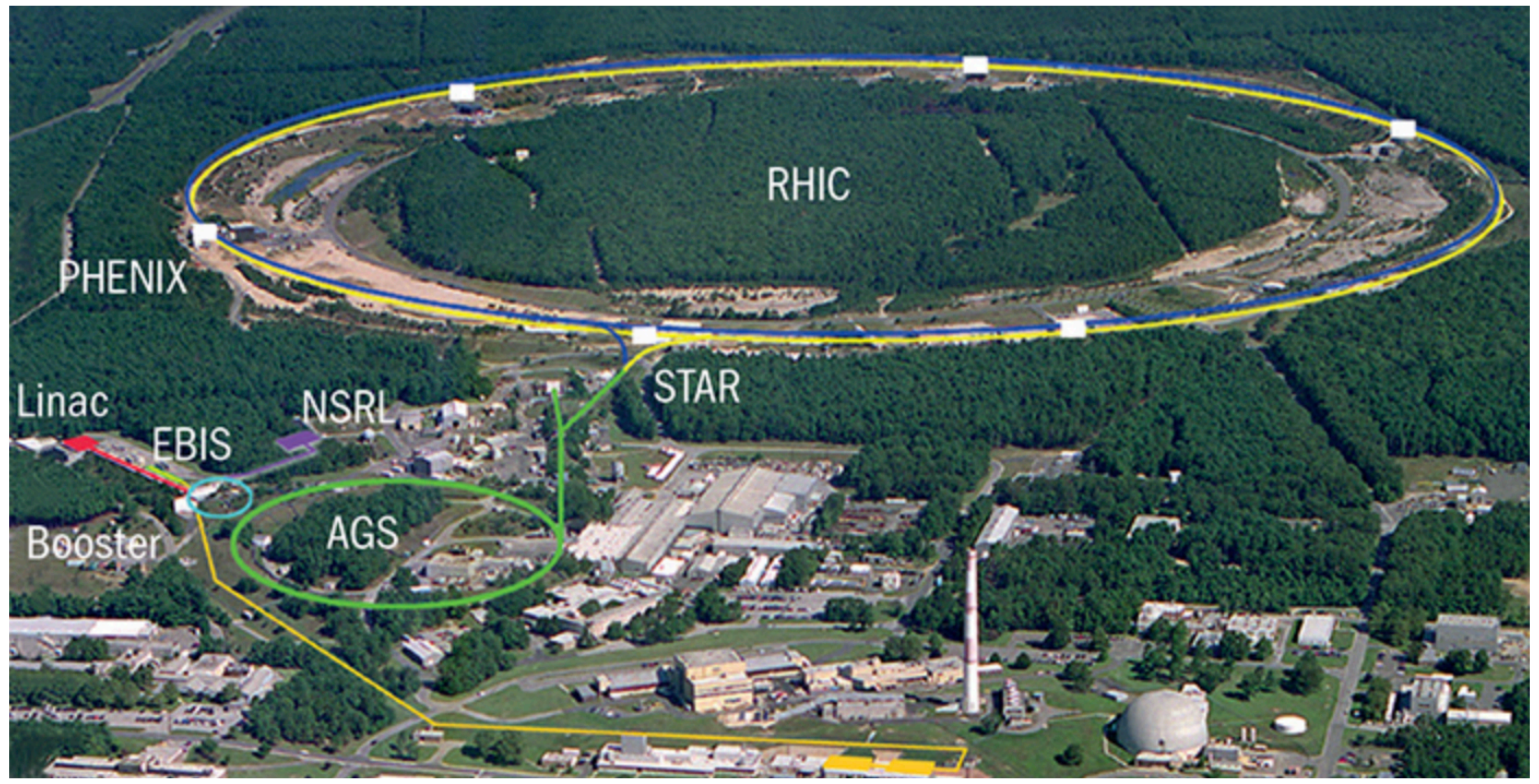


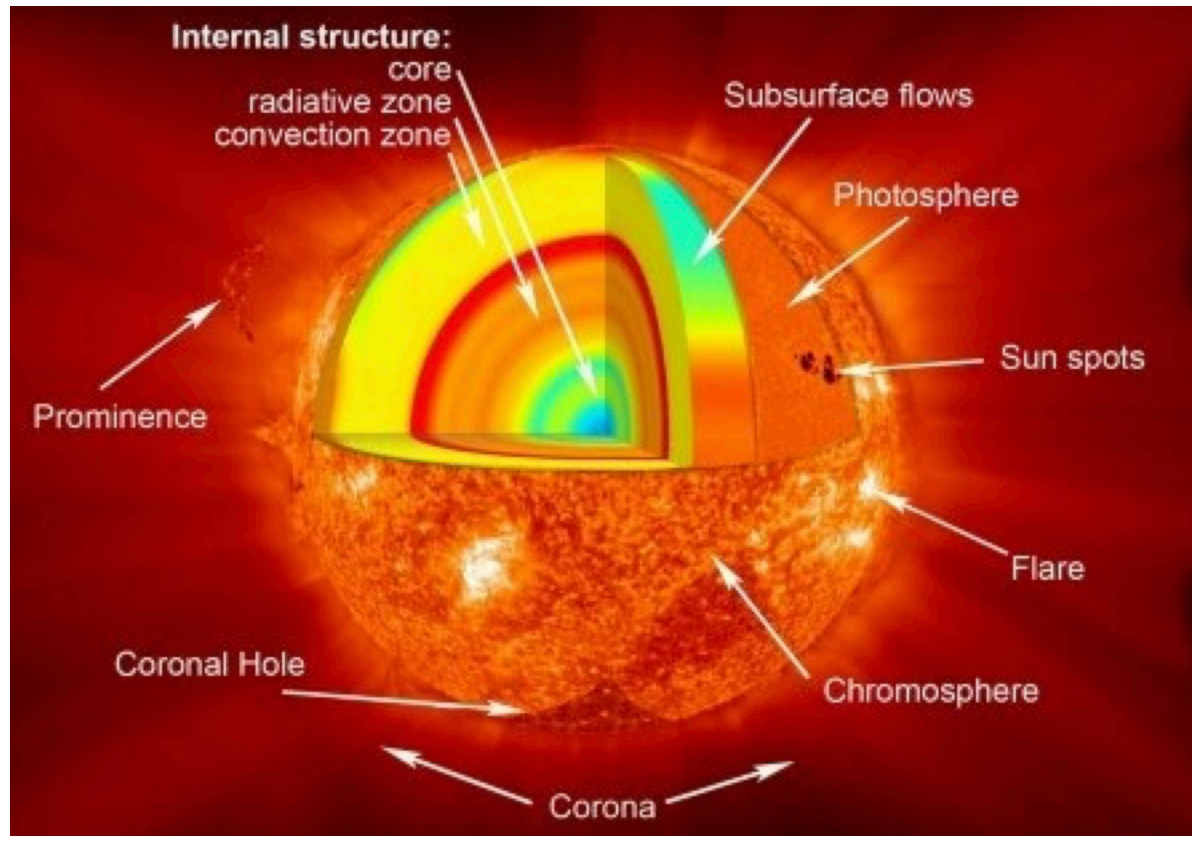
figure: D.E. A. Castillo, talk @RagTime 22

Relativistic heavy-ion collisions - a tool to study QGP

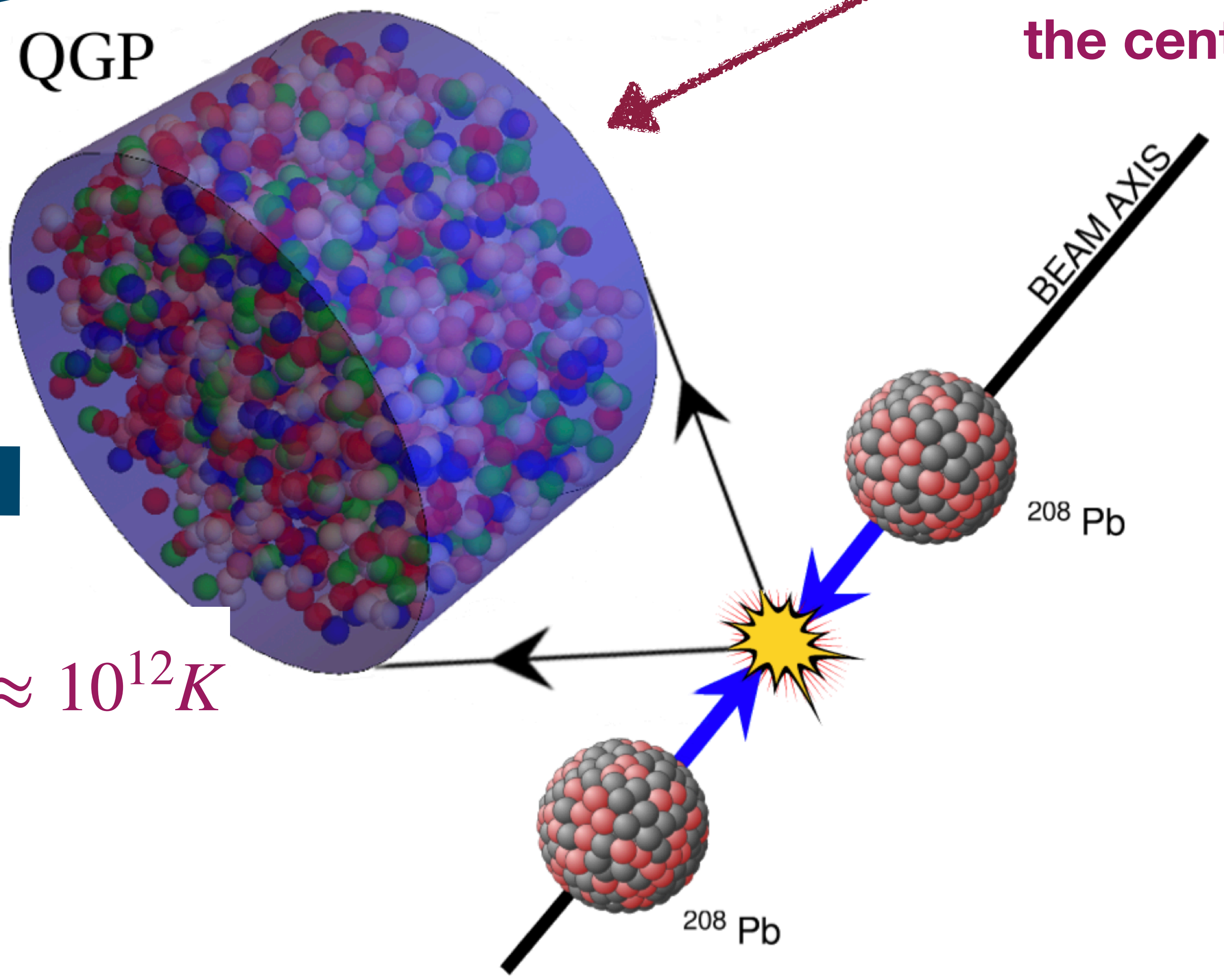


At high beam energies we observe many particles being produced

figure: NASA



100 000 times hotter than the center of the Sun



$T \approx 10^{12} K$

The study of QGP possible only indirectly through the energy and momenta of emitted particles

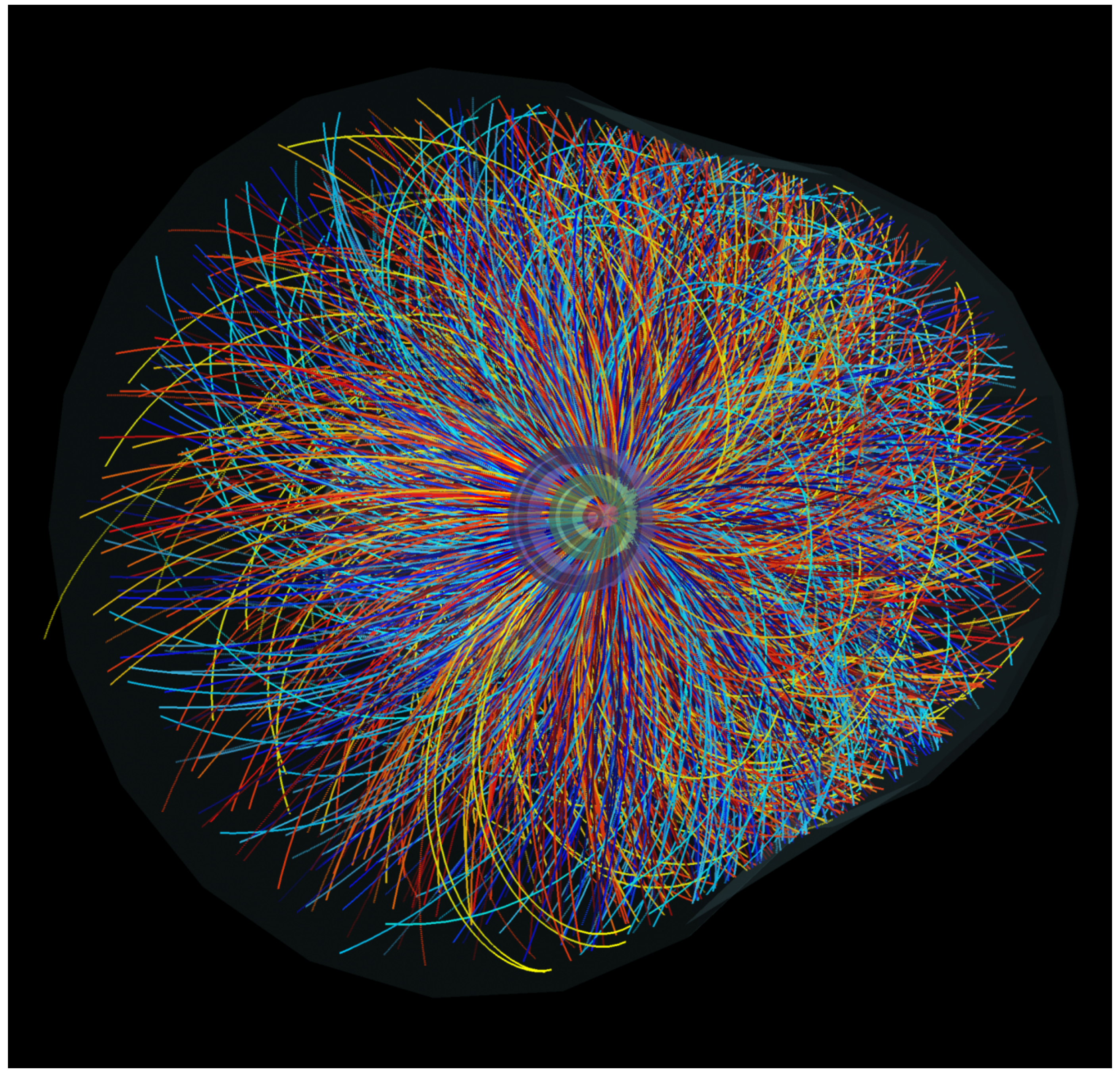
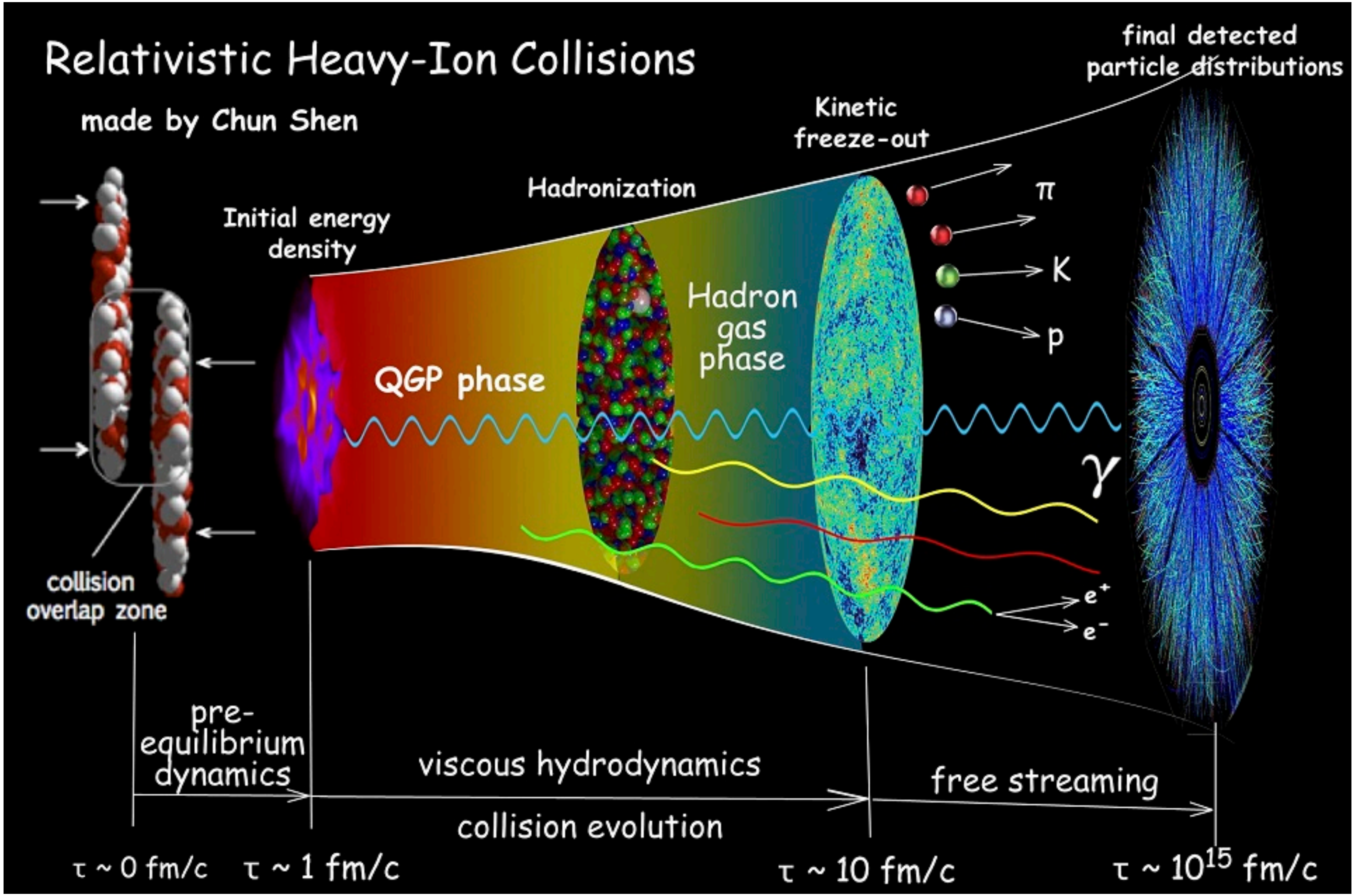


figure: Nature Physics 16, 615–619(2020)

Relativistic heavy-ion collisions - a tool to study QGP



Spin polarization in heavy-ion collisions - new sensitive probe!

Non-central heavy-ion collisions create fireballs with large global orbital angular momenta

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

$$\boldsymbol{L}_{\rm init} \sim 10^5 \hbar$$

Part of the angular momentum can be transferred from the orbital to the spin part

$$\boldsymbol{J}_{\rm init} = \boldsymbol{L}_{\rm init} = \boldsymbol{L}_{\rm final} + \boldsymbol{S}_{\rm final}$$

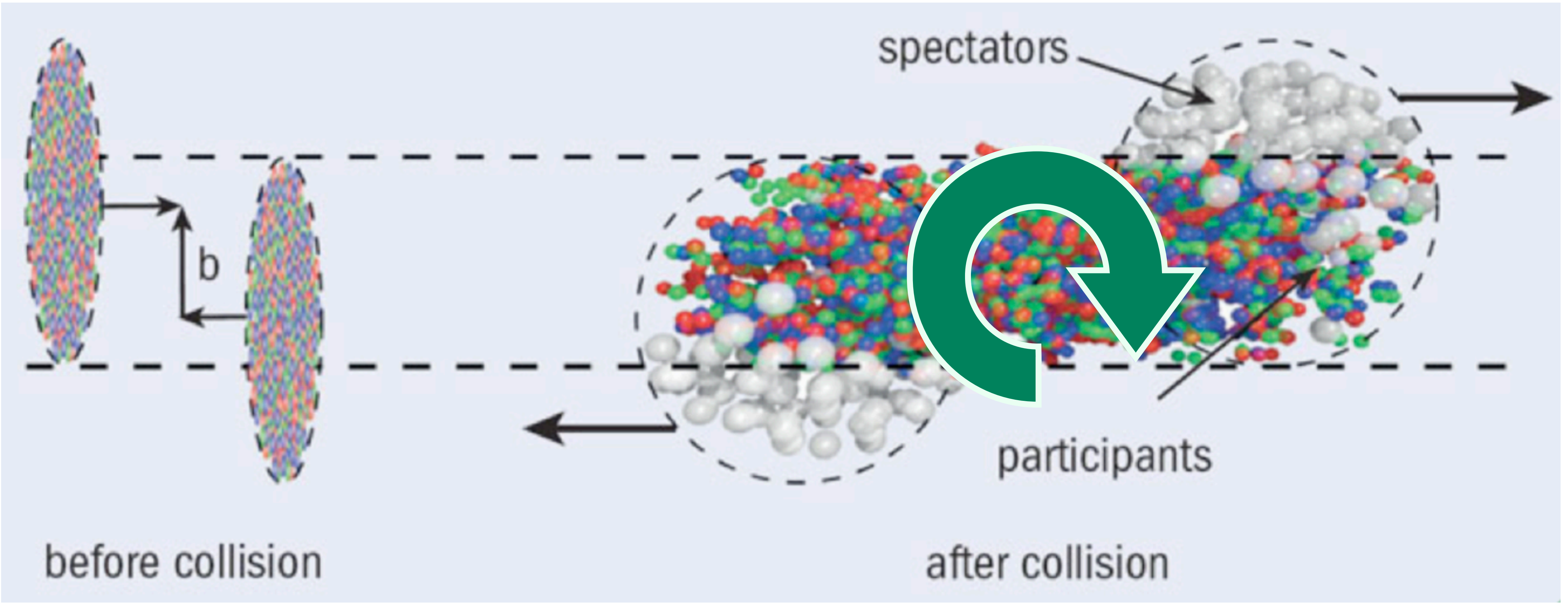
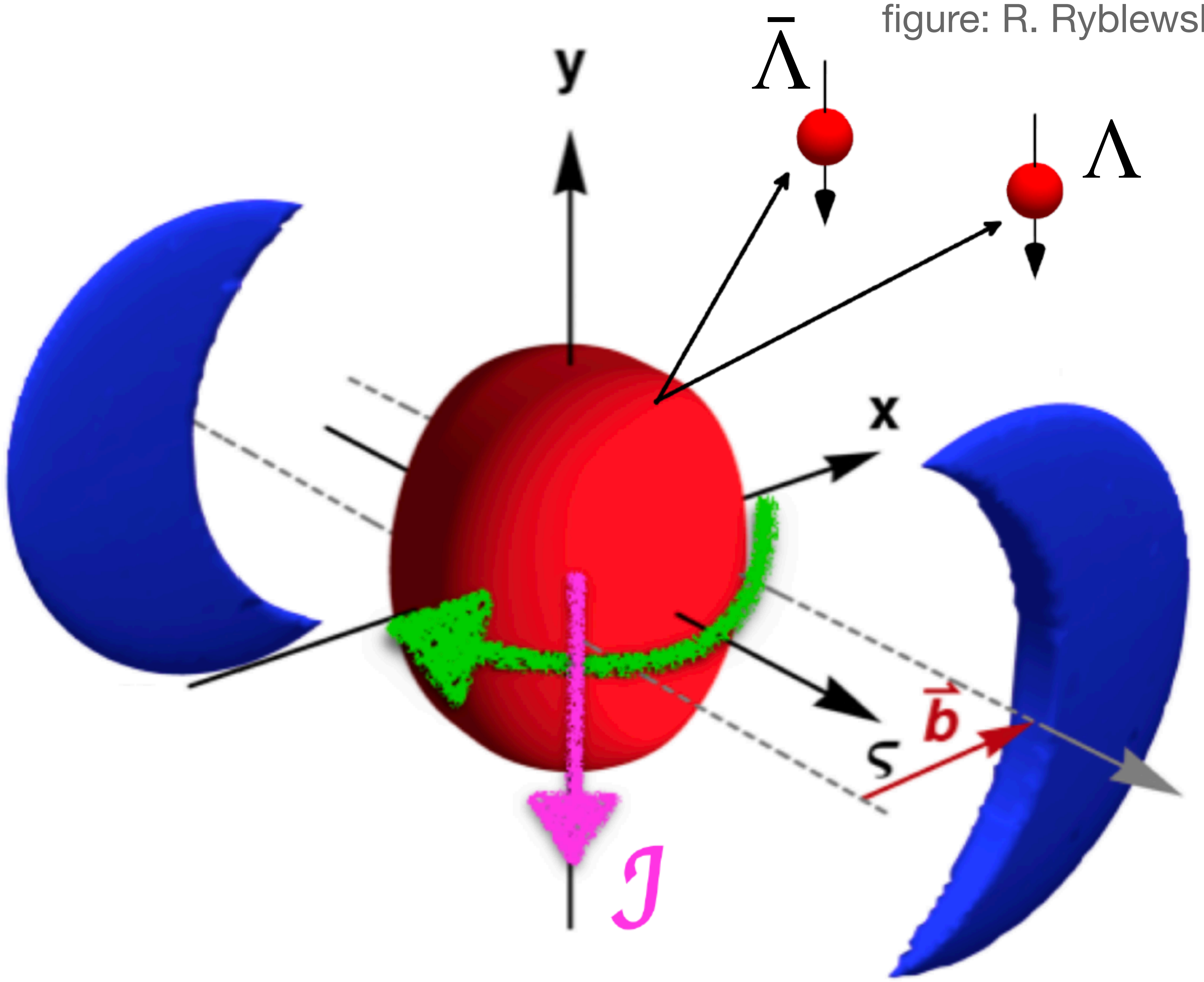


figure: M. Lisa, talk @ "Strangeness in Quark Matter 2016"

figure: R. Ryblewski

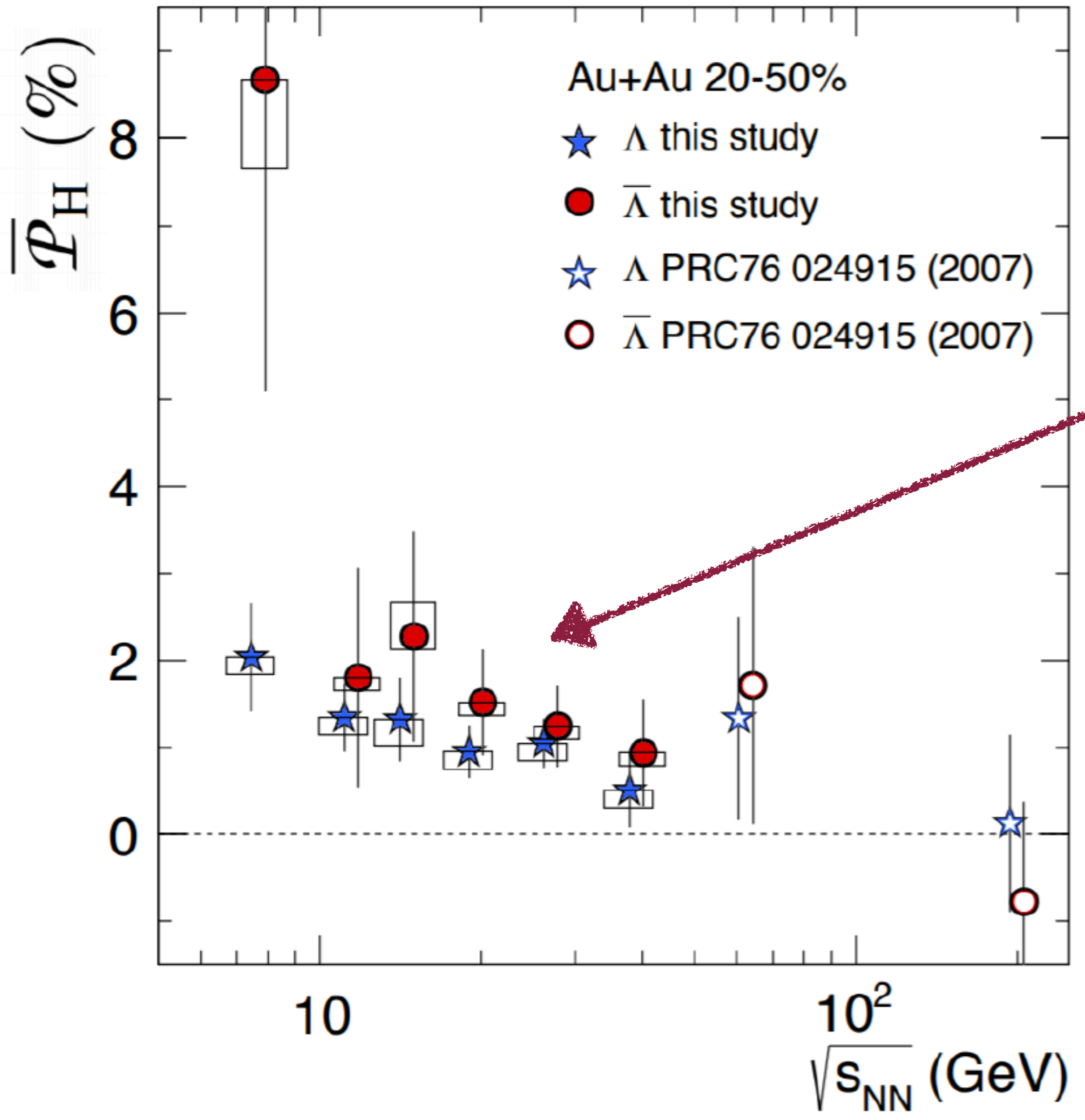


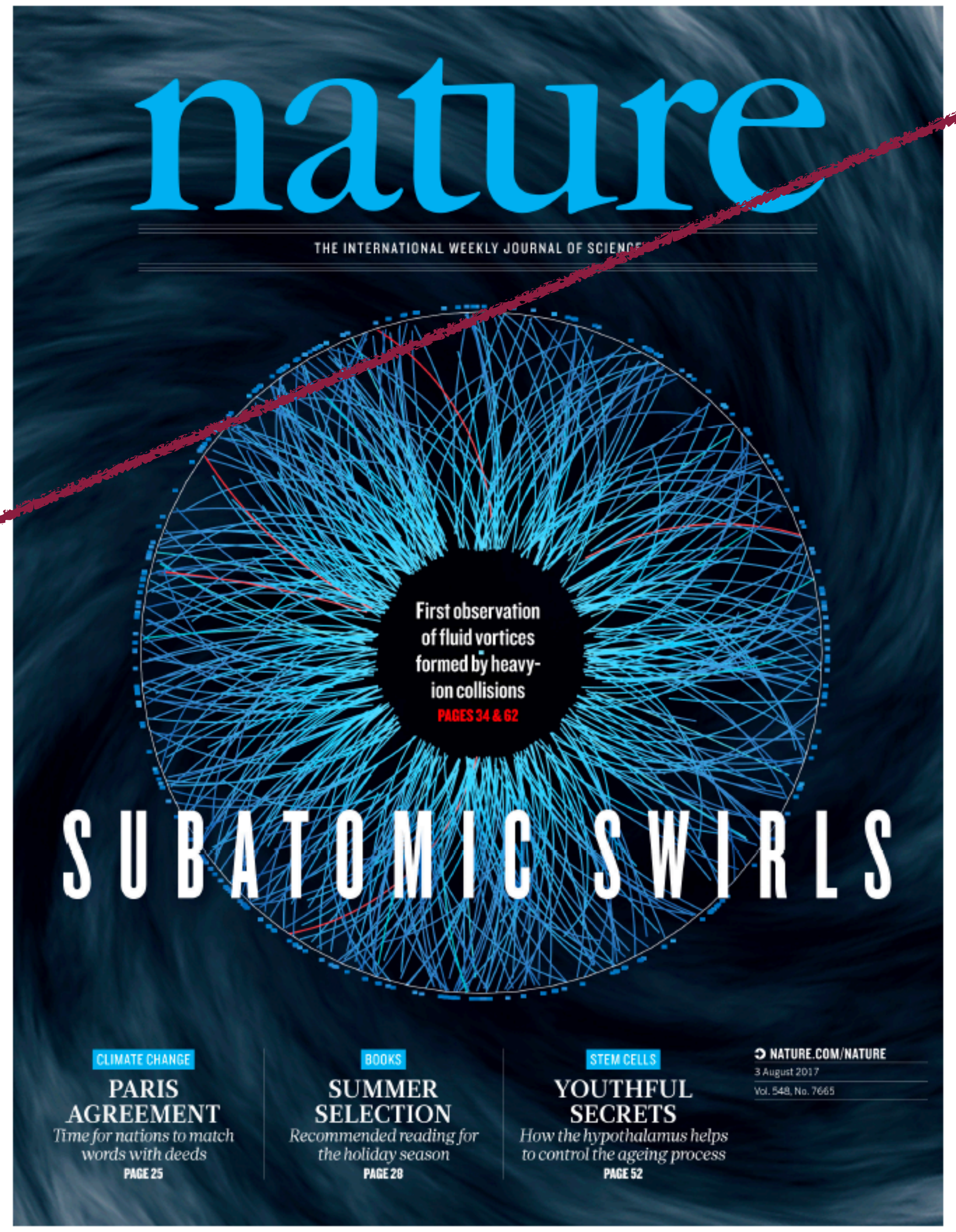
Emitted particles are expected to be globally polarized along the system's angular momentum

Measurement of Λ and $\bar{\Lambda}$ spin polarization in heavy-ion collisions

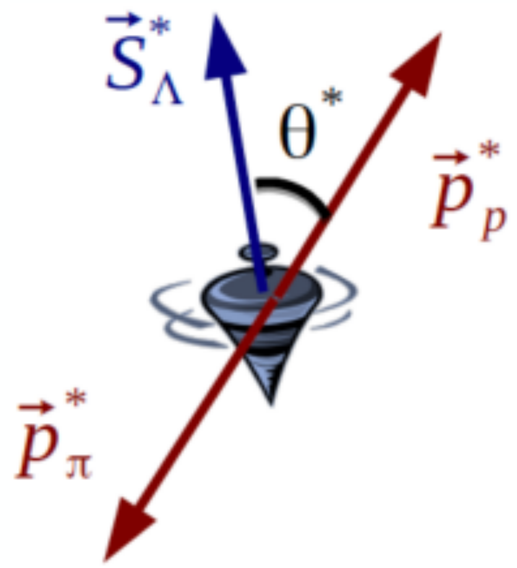
L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

~2% - small but measurable effect





Self-analysing parity-violating hyperon weak decay allows to measure polarization of Λ



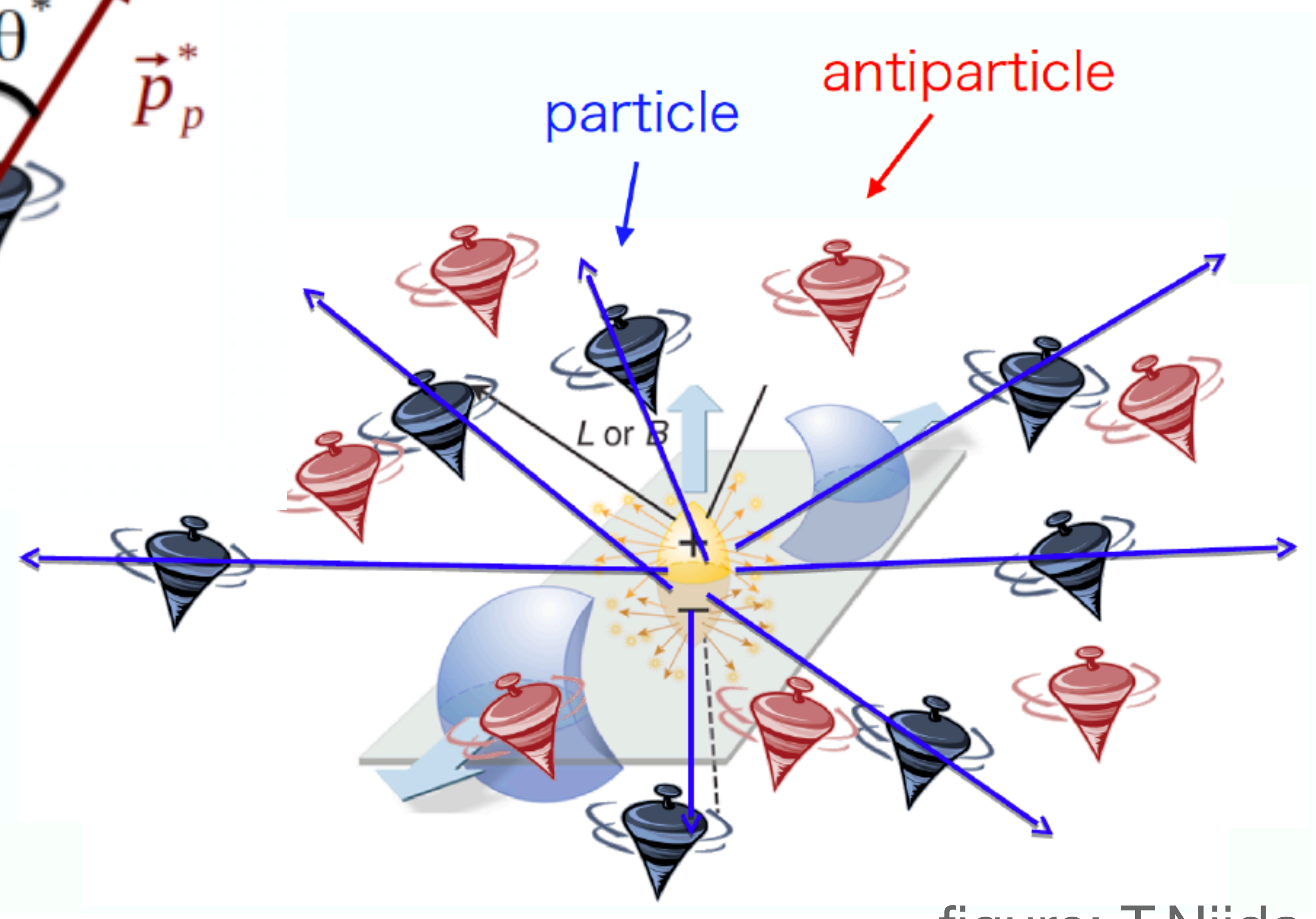


figure: T.Niida

. . . the hottest, least viscous – and now, most vortical – fluid produced in the laboratory . . .

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$

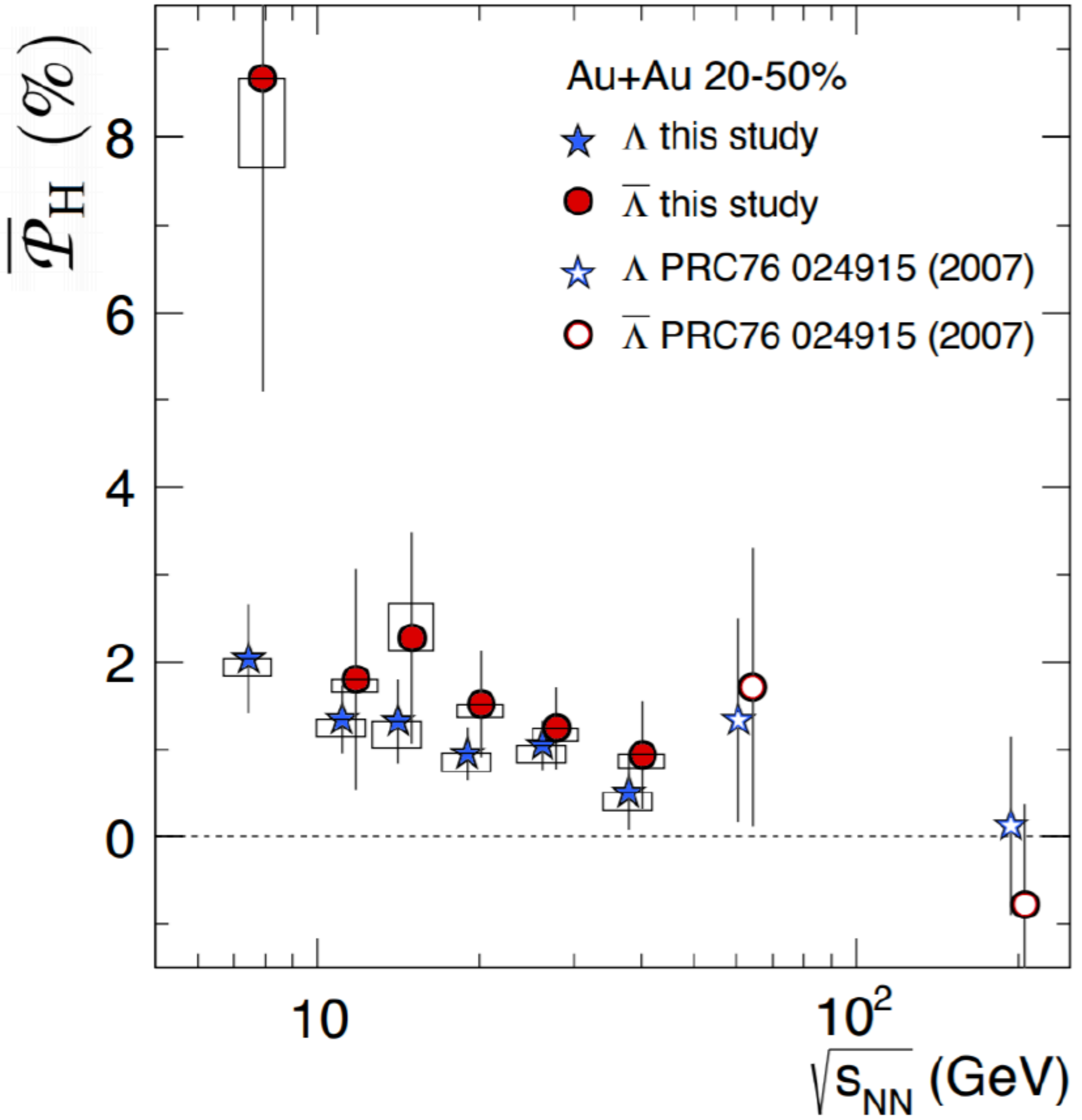
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\mathrm{H}} \mathbf{P}_{\mathrm{H}} \cdot \mathbf{p}^*_{\mathbf{p}})$$

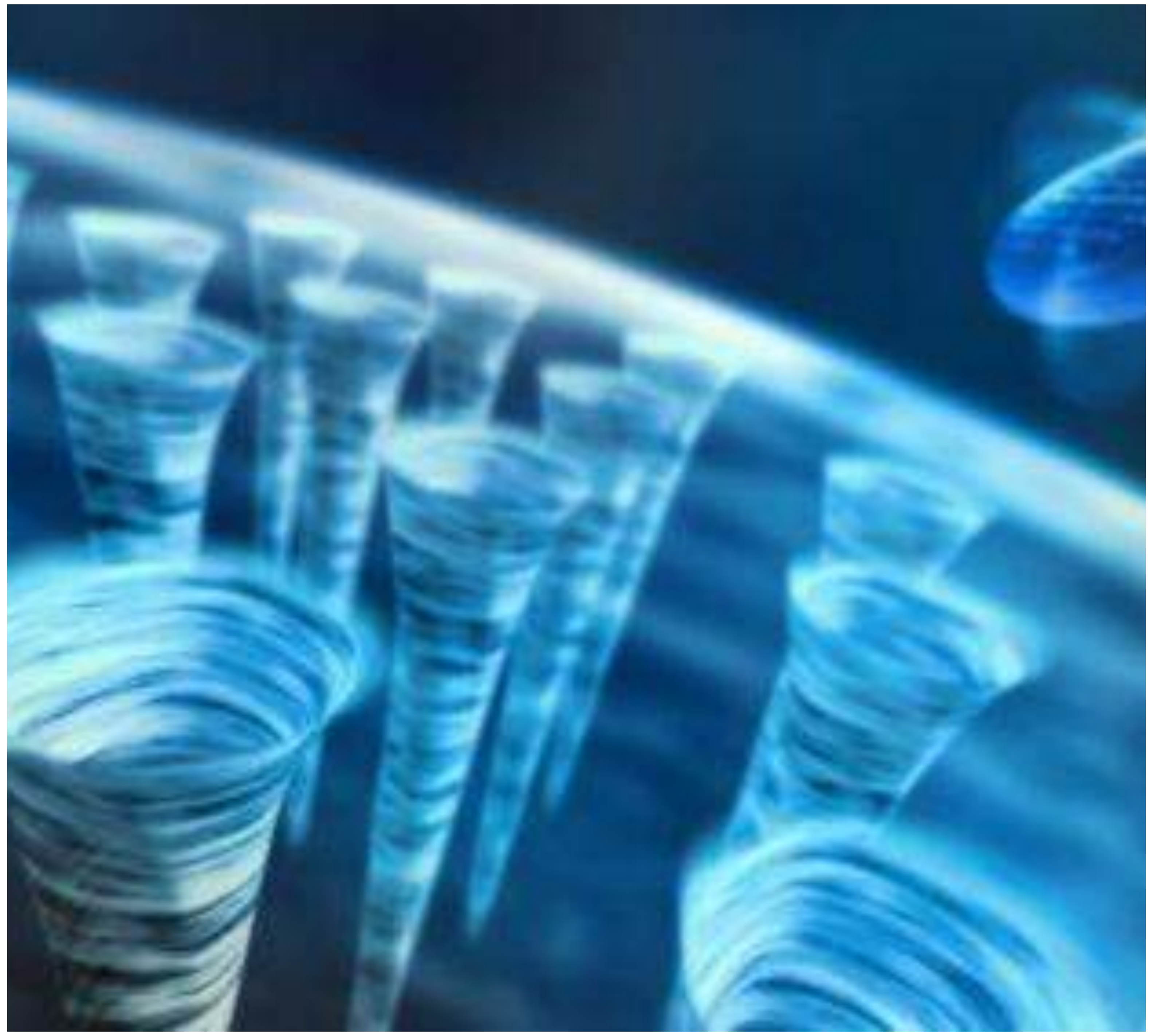
$$P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T} \qquad P_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

$P_\Lambda \approx P_{\bar{\Lambda}}$ ➡ **first direct observation of spin**

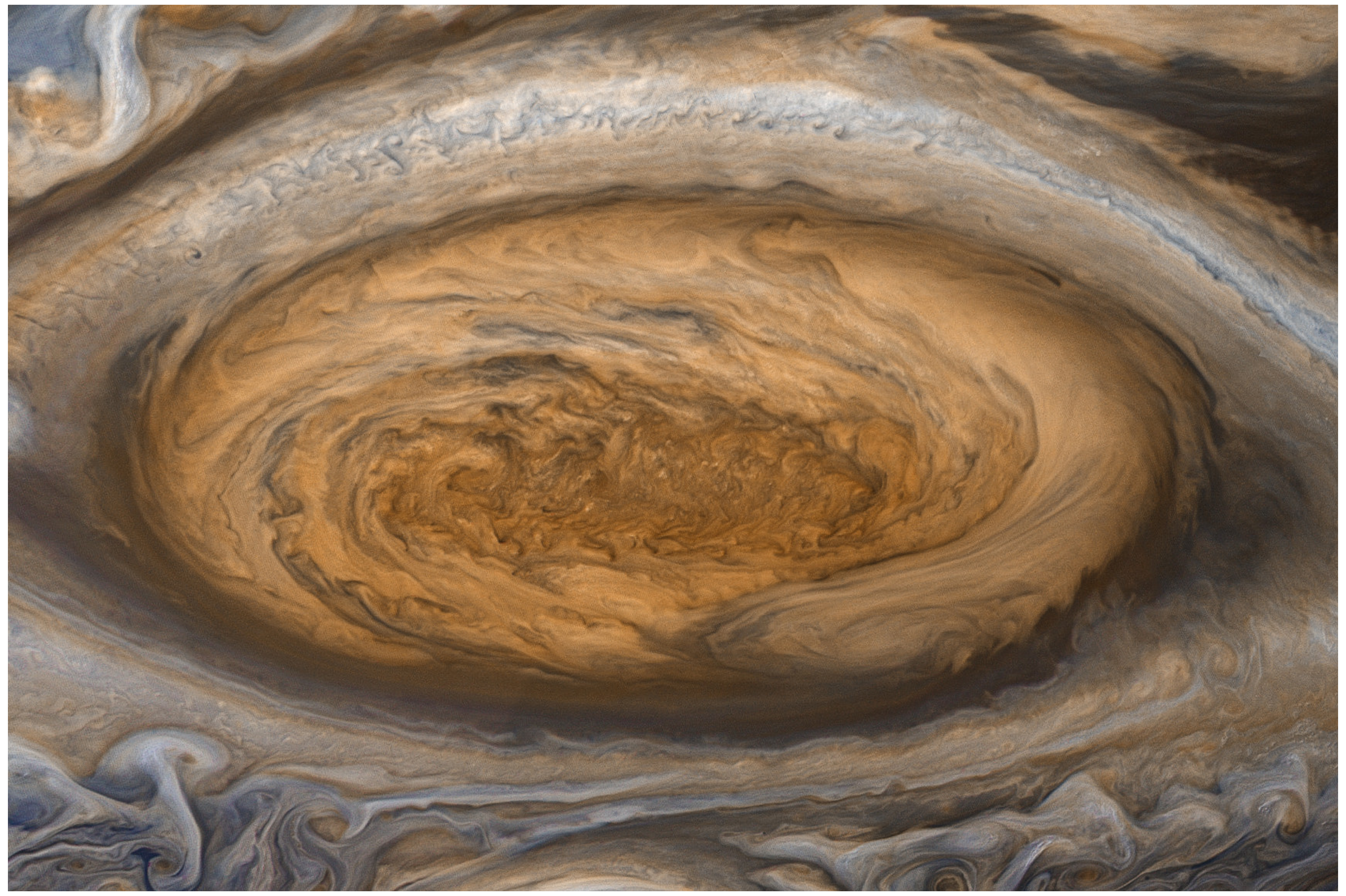
Measurement of Λ and $\bar{\Lambda}$ spin polarization in heavy-ion collisions

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

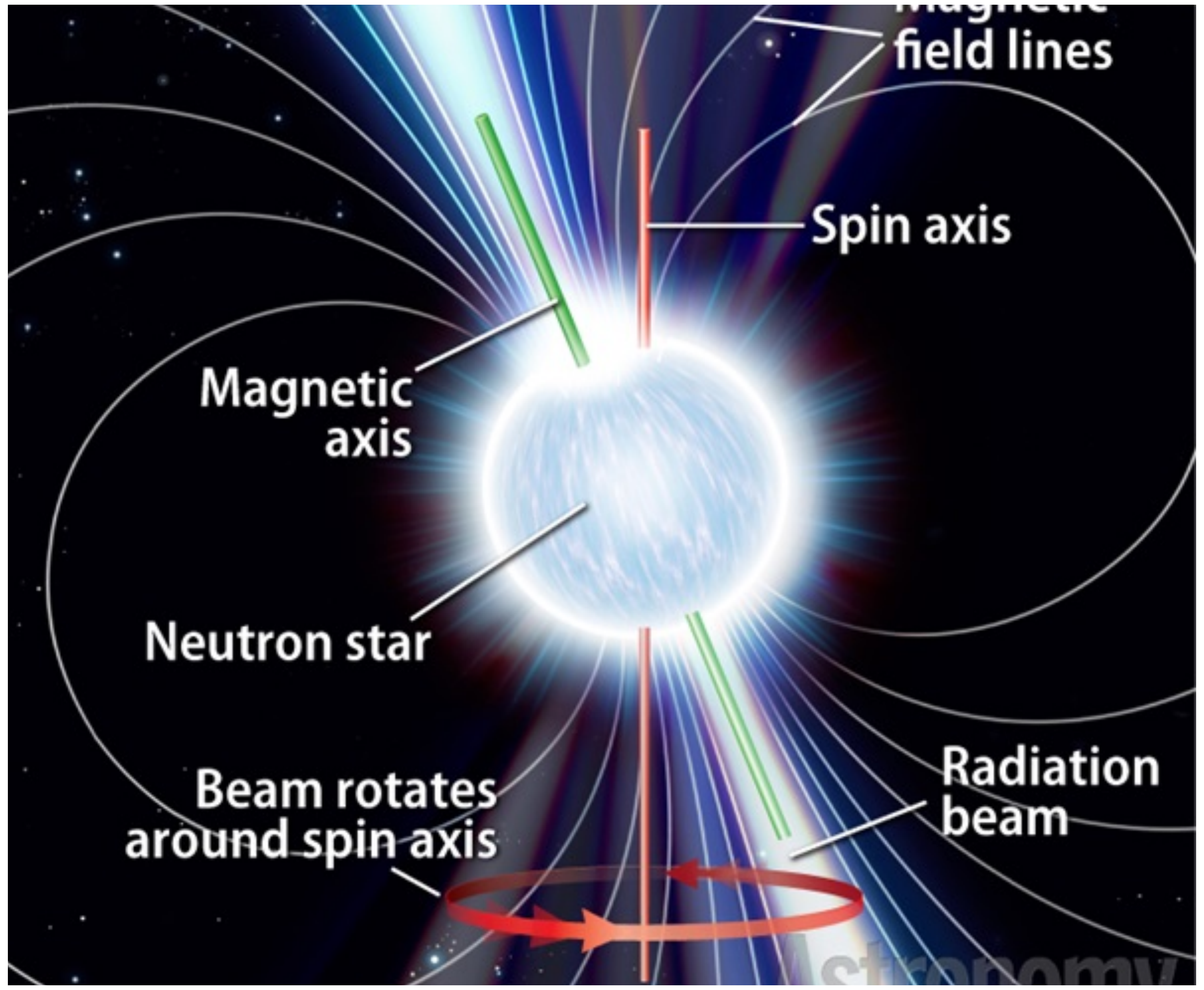




Science 345, 906–909 (2014)



arXiv:1301.6119



10.1126/science.1123430

. . . the hottest, least viscous – and now, most vortical – fluid produced in the laboratory . . .

$$\omega = (P_\Lambda + P_{\bar{\Lambda}}) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$

$$P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T} \qquad P_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

Spin polarization in equilibrated QGP - spin-thermal approach

In local thermodynamic equilibrium at $\mathcal{O}((\varpi^{\mu\nu})^2)$ one can establish a link between spin and thermal vorticity

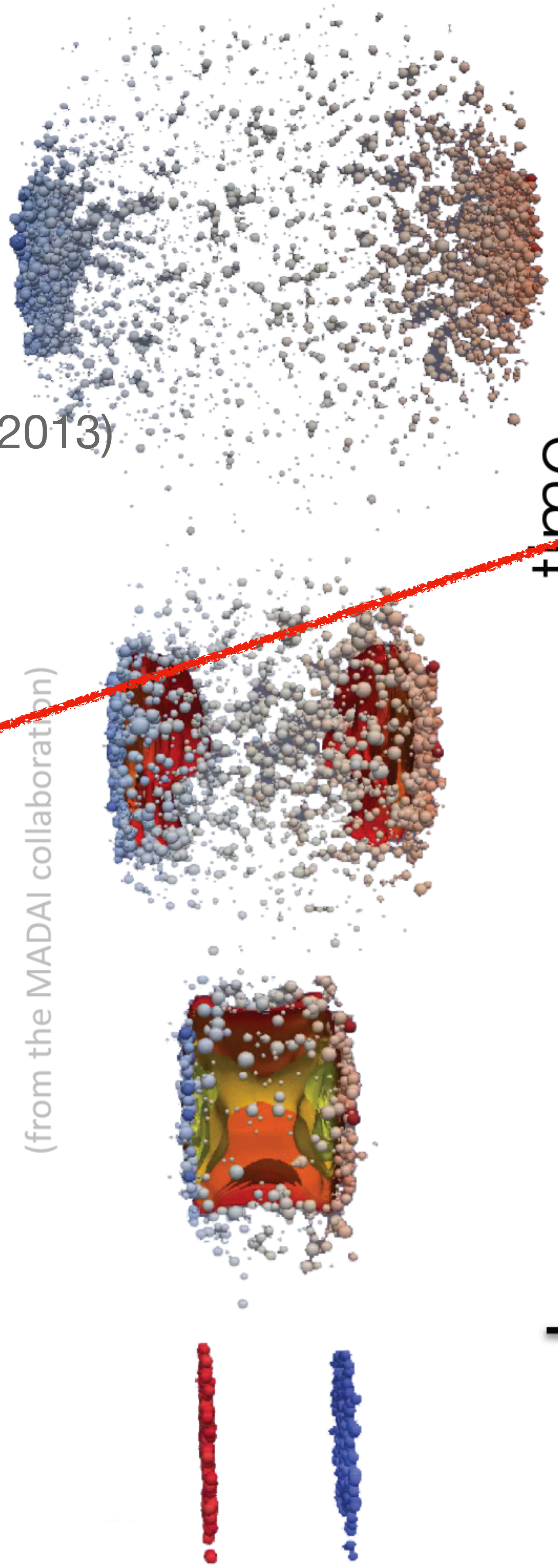
Becattini F, Piccinini F. Ann. Phys. 323:2452 (2008)
Becattini F, Chandra V, Del Zanna L, Grossi E. Ann. Phys. 338:32 (2013)
Fang R, Pang L,Wang Q,Wang X. Phys. Rev. C 94:024904 (2016)

$$\boxed{S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int d\Sigma_\lambda p^\lambda n_F\left(1-n_F\right)\boxed{\varpi_{\rho\sigma}}}{\int d\Sigma_\lambda p^\lambda n_F}}$$

$$\boxed{\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \beta^\mu = \frac{u^\mu}{T}}$$

$$n_F = (1+\exp[\beta\cdot p - \mu Q/T])^{-1}$$

Allows to extract polarisation at the freeze-out hypersurface in <u>any</u> model which provides u^μ, T and μ



relativistic heavy-ion collision

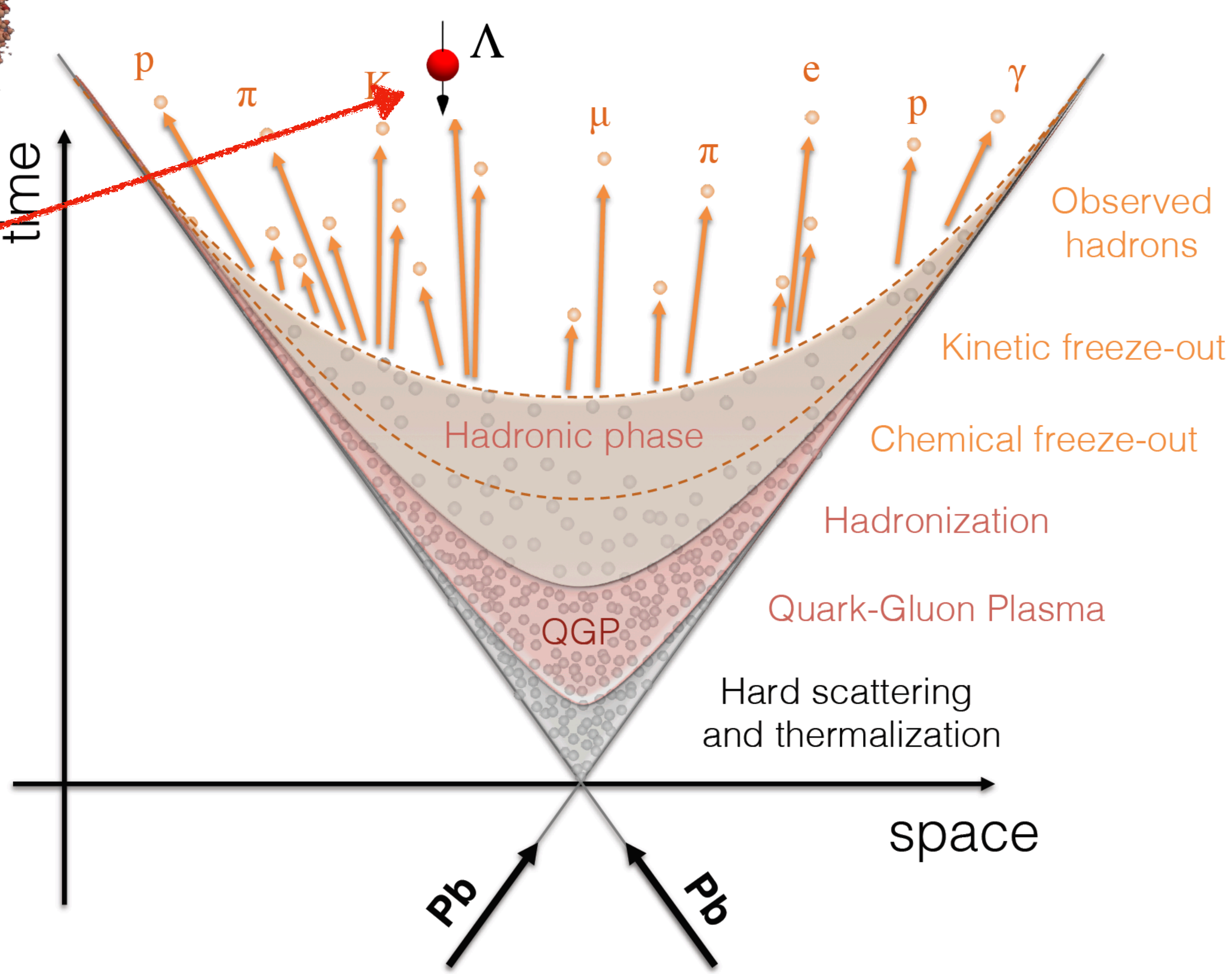


figure: D.D. Chinellato

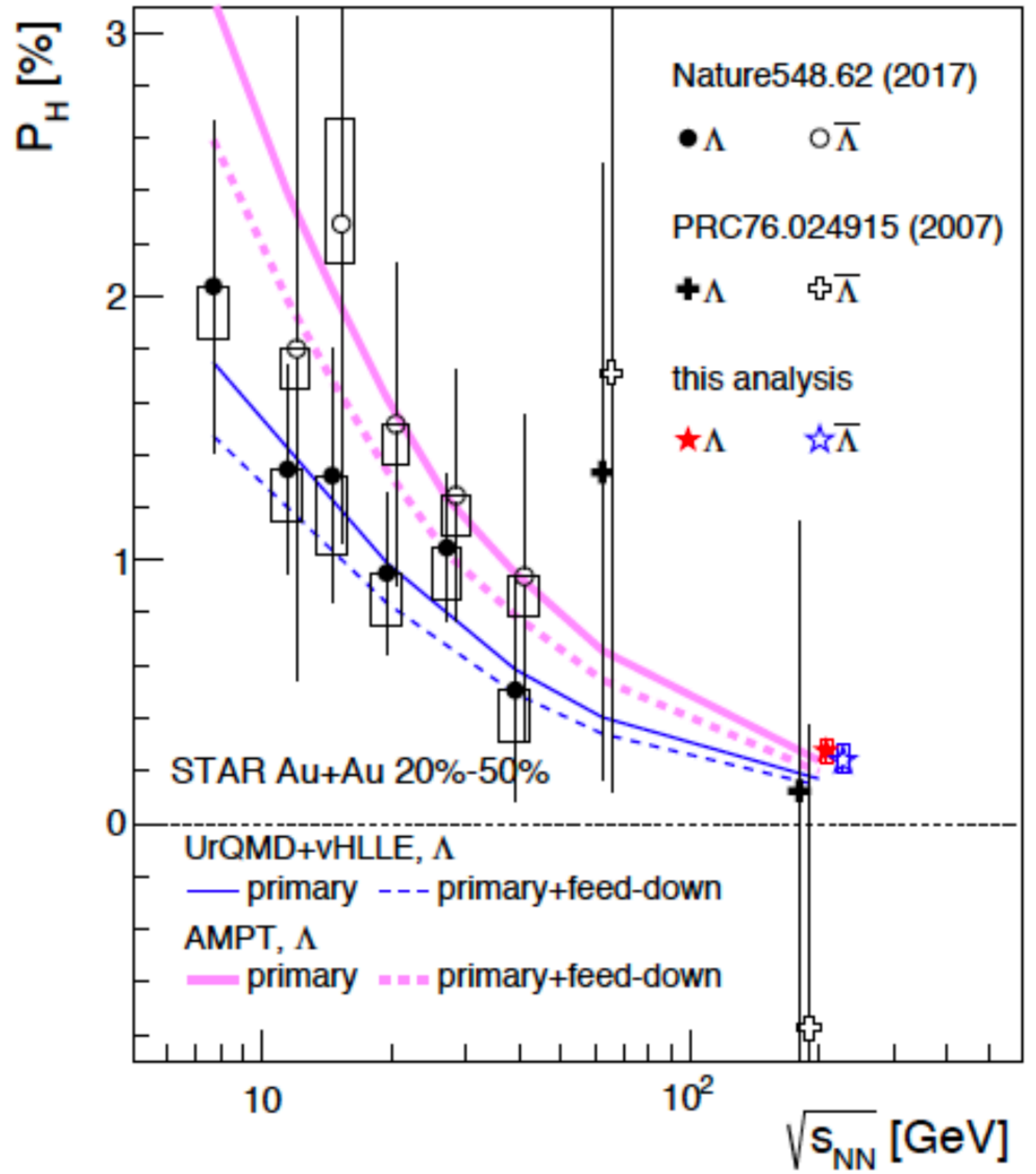
Global polarization

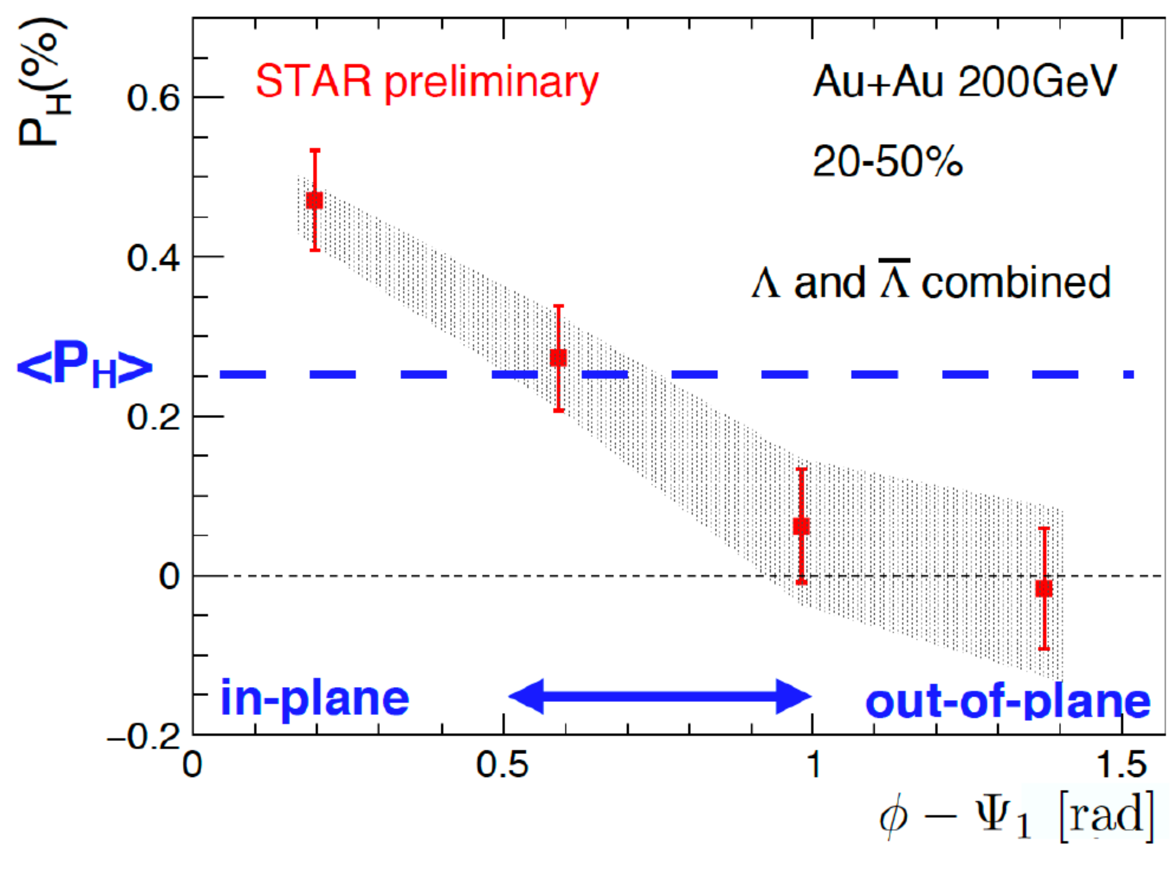
Global polarization data supports the spin-thermal approach

Signal is pretty robust and agrees for both multiphase transport model (AMPT) and viscous hydrodynamics (UrQMD+vHLLE)

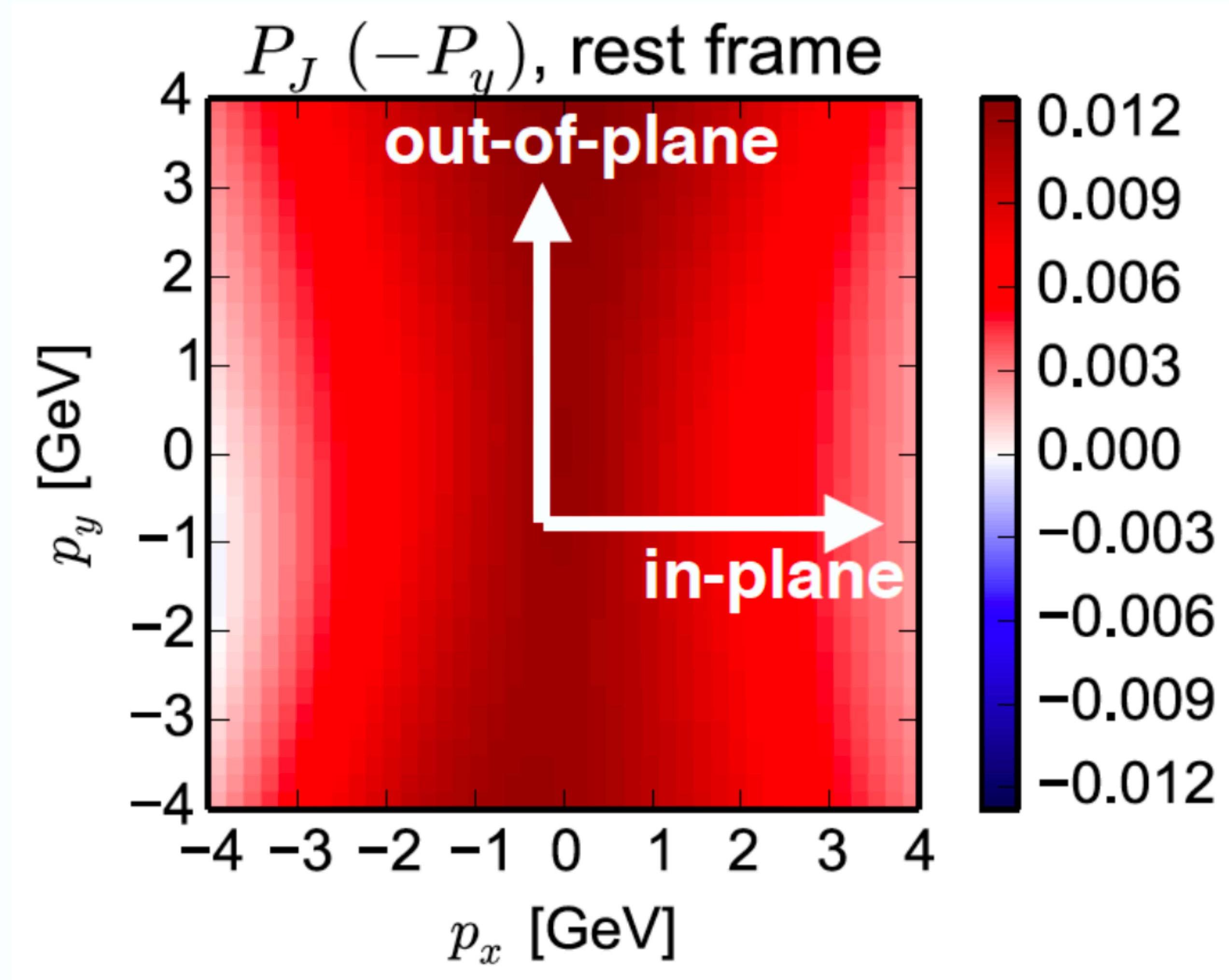
Azimuthal modulation is not captured

J. Adam, et al., Phys. Rev. C 98, 014910 (2018)





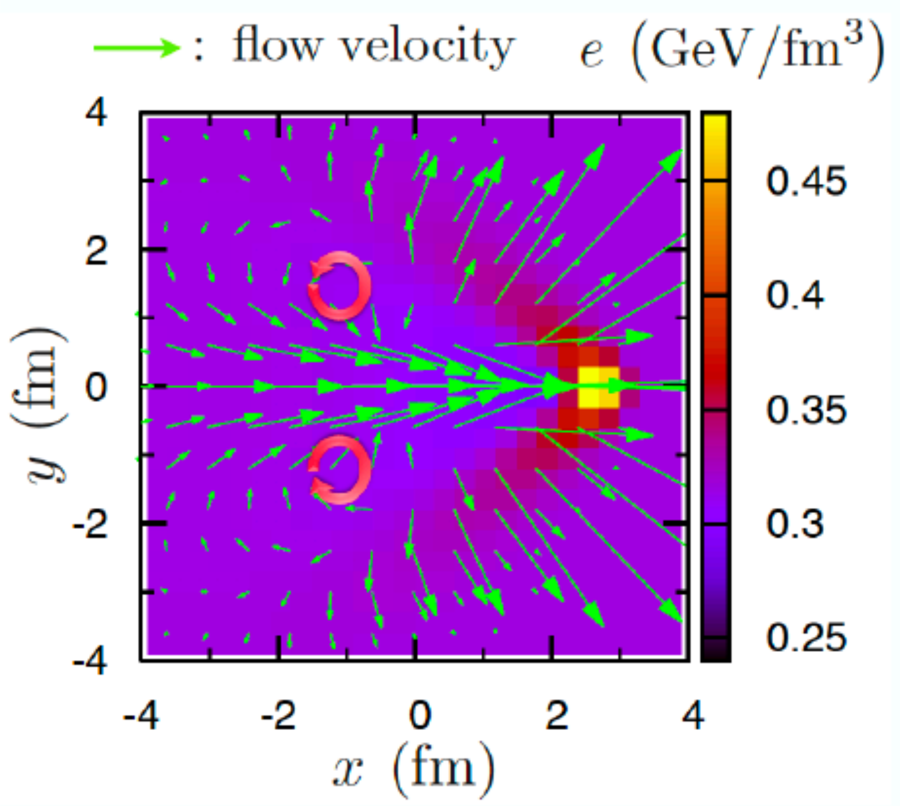
Credit: T.Niida, The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, 2019



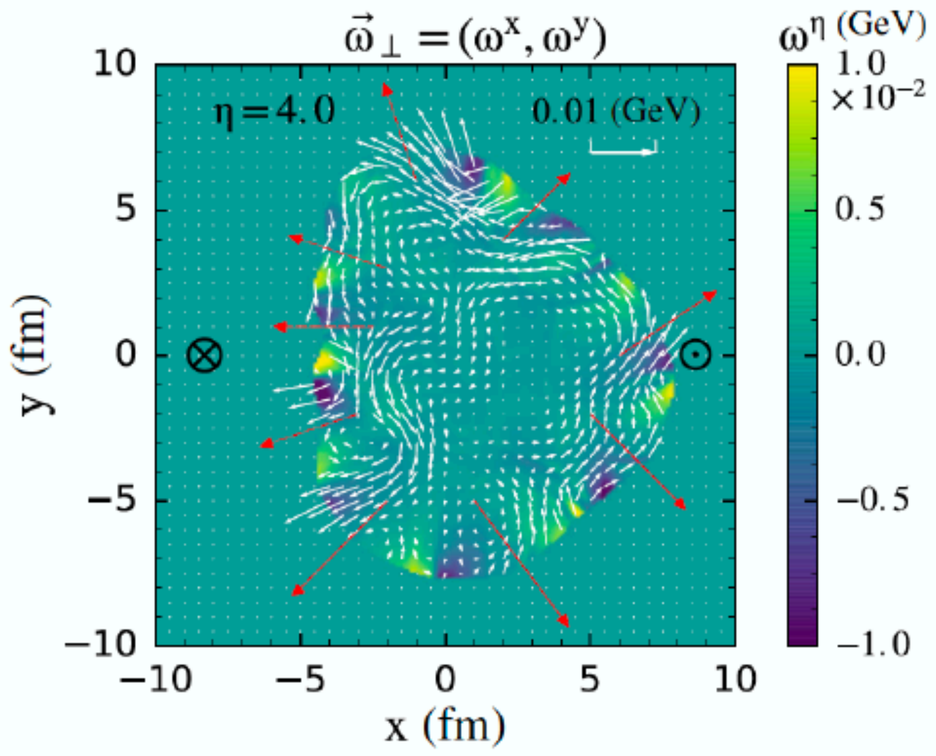
I. Karpenko, F. Becattini, EPJC 77, 213 (2017)

UrQMD+vHLLE: I. Karpenko, F. Becattini, EPJC 77, 213 (2017)
AMPT: H. Li, L. Pang, Q. Wang, and X. Xia, PRC 96, 054908 (2017)

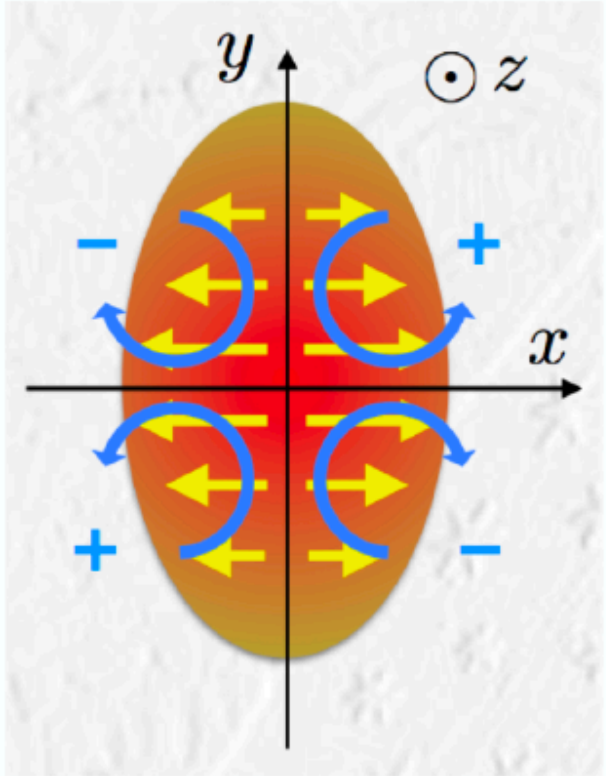
Local (momentum-differential) polarization



Y. Tachibana and T. Hirano, NPA904-905 (2013) 1023

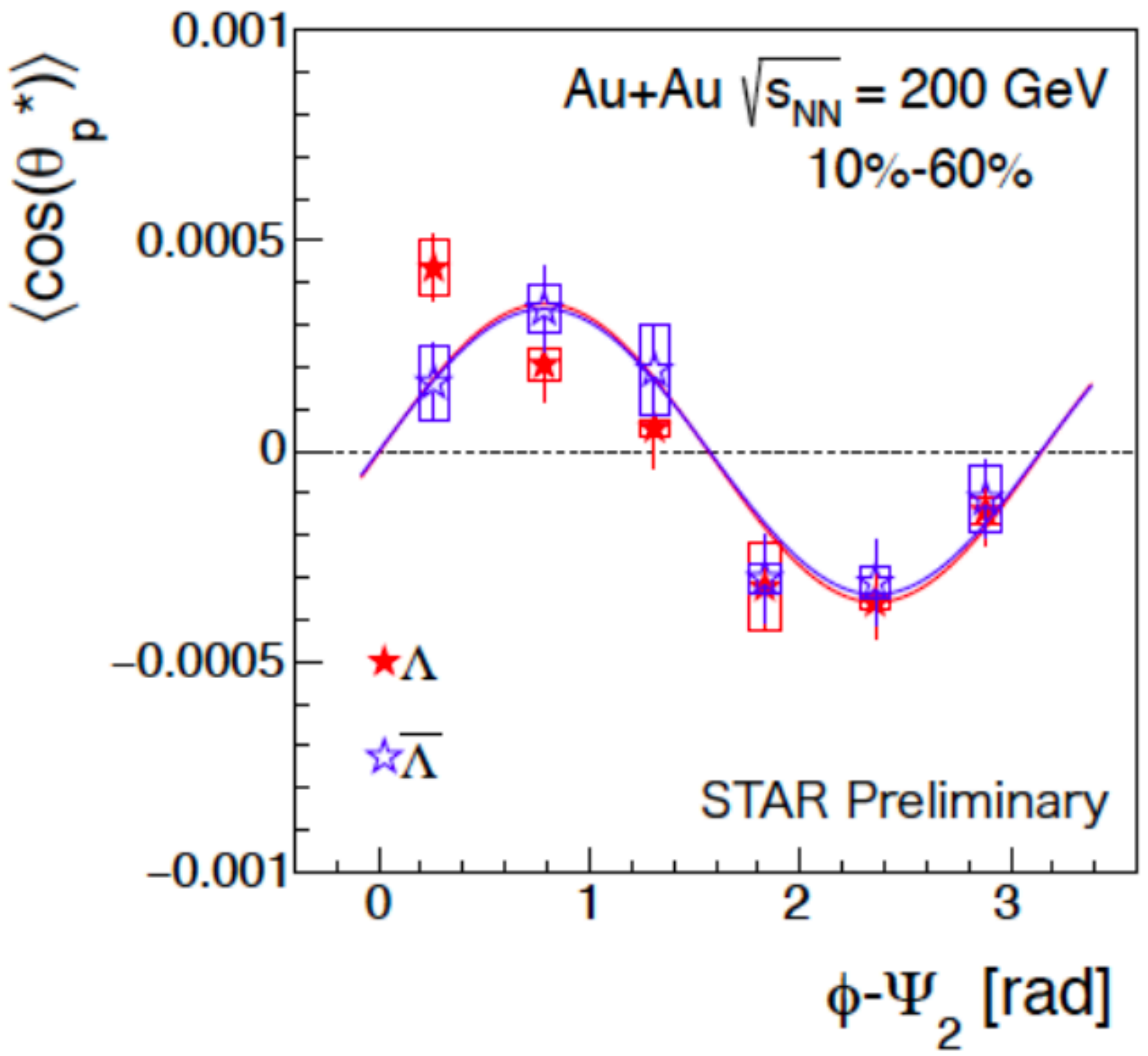


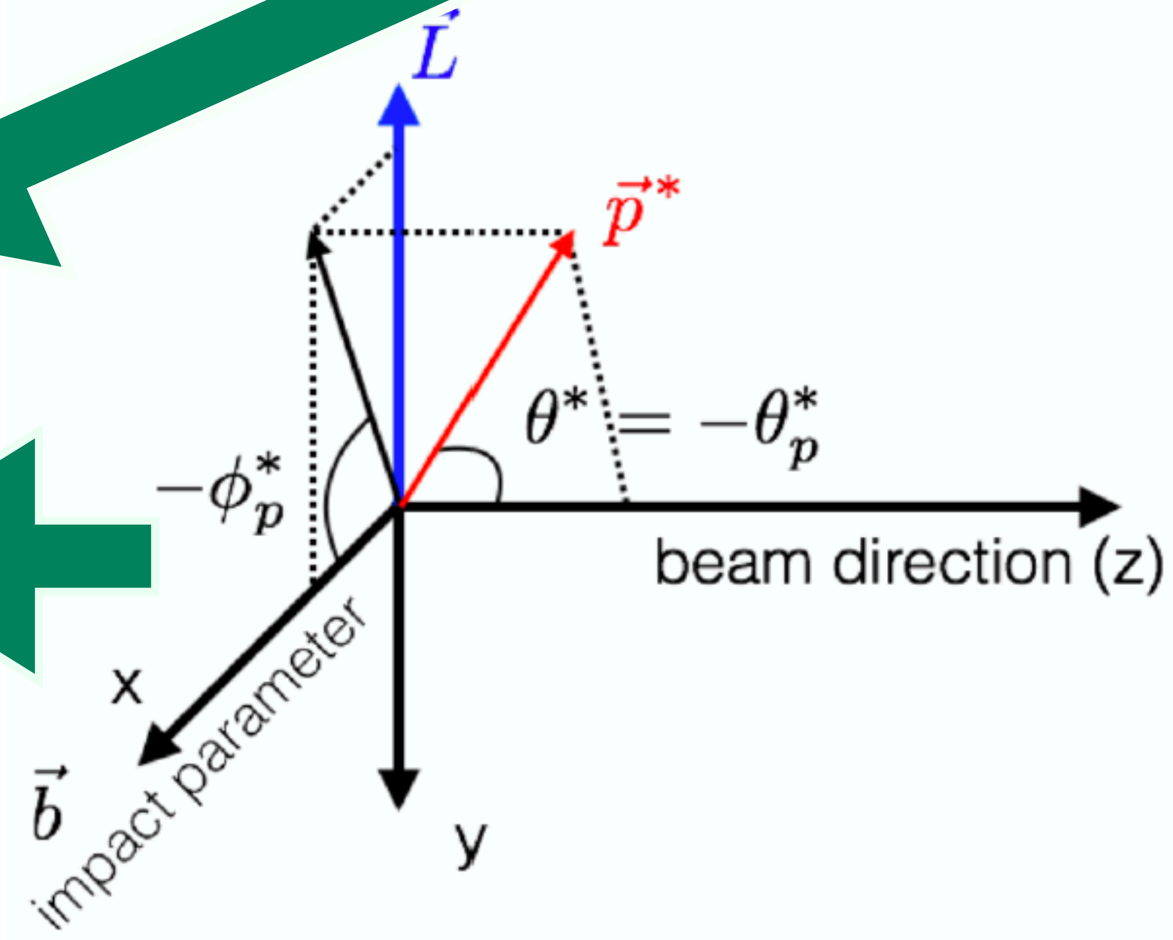
L.-G. Pang, H. Peterson, Q. Wang, and X.-N. Wang PRL117, 192301 (2016)



Becattini and I. Karpenko, PRL120.012302 (2018)
S. Voloshin, EPJ Web Conf.171, 07002 (2018)

Flow structure in the transverse plane (jet, ebe fluctuations etc.) may generate longitudinal polarization





$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\mathrm{H}} \mathbf{P_H} \cdot \mathbf{p}_p^*)$$

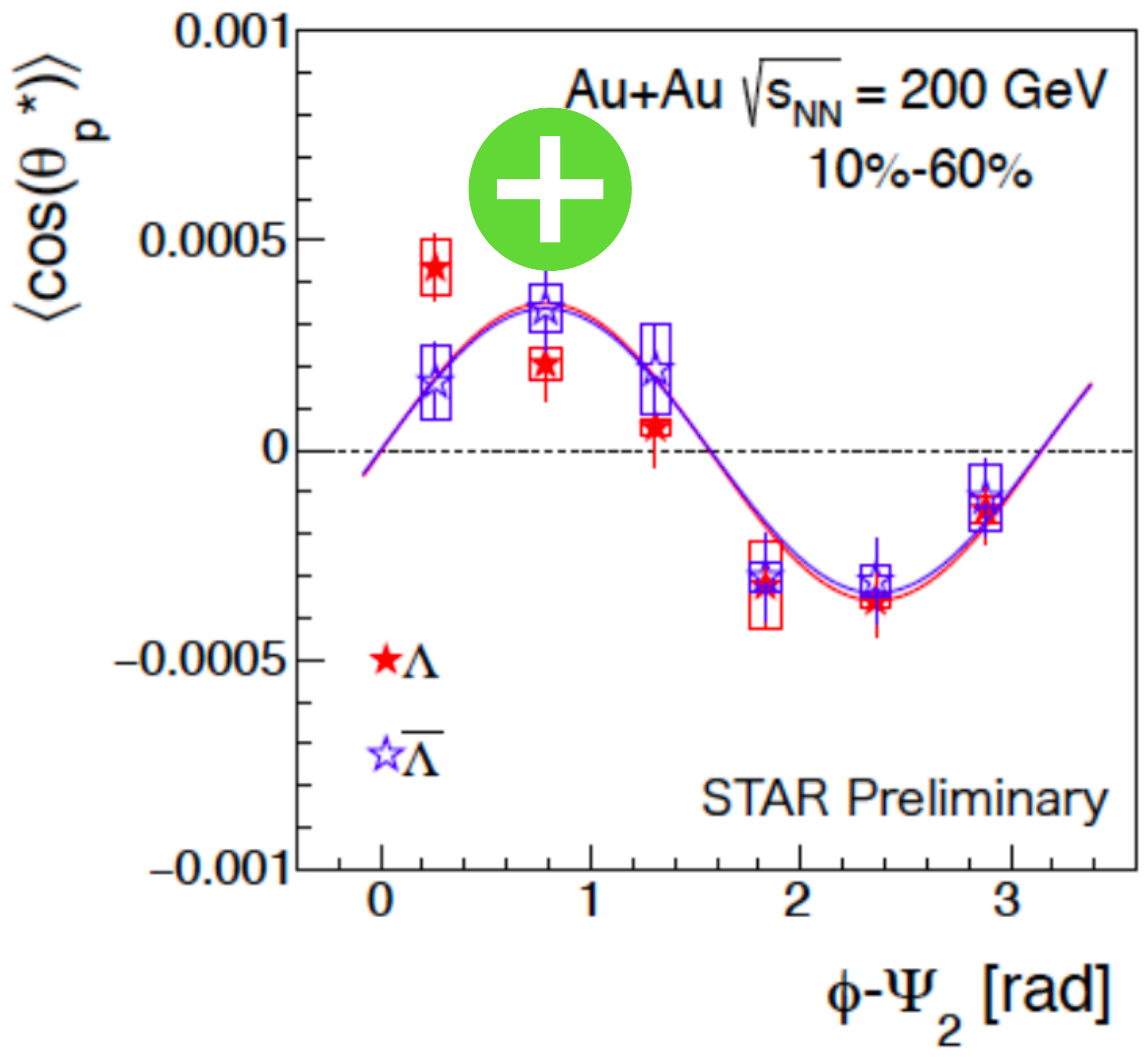
$$\langle \cos\theta_p^* \rangle = \int \frac{dN}{d\Omega^*} \cos\theta_p^* d\Omega^* = \alpha_{\mathrm{H}} P_z \langle (\cos\theta_p^*)^2 \rangle$$

$$\therefore \; P_z = \frac{\langle \cos\theta_p^* \rangle}{\alpha_{\mathrm{H}} \langle (\cos\theta_p^*)^2 \rangle} = \frac{3\langle \cos\theta_p^* \rangle}{\alpha_{\mathrm{H}}}$$ (if perfect detector)

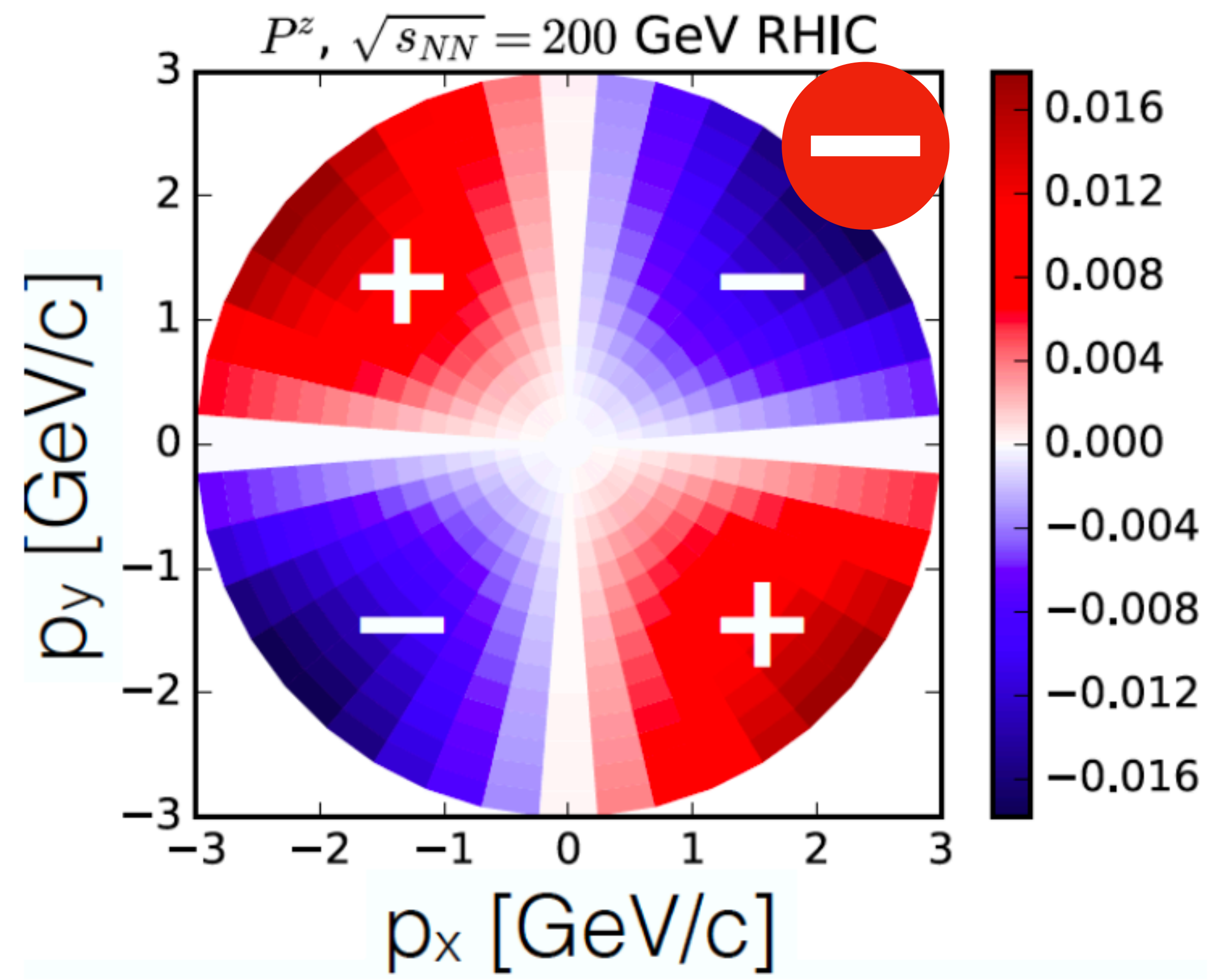
α_H: hyperon decay parameter
θ_p^*: θ of daughter proton in Λ rest frame

Credit: T.Niida, The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, 2019

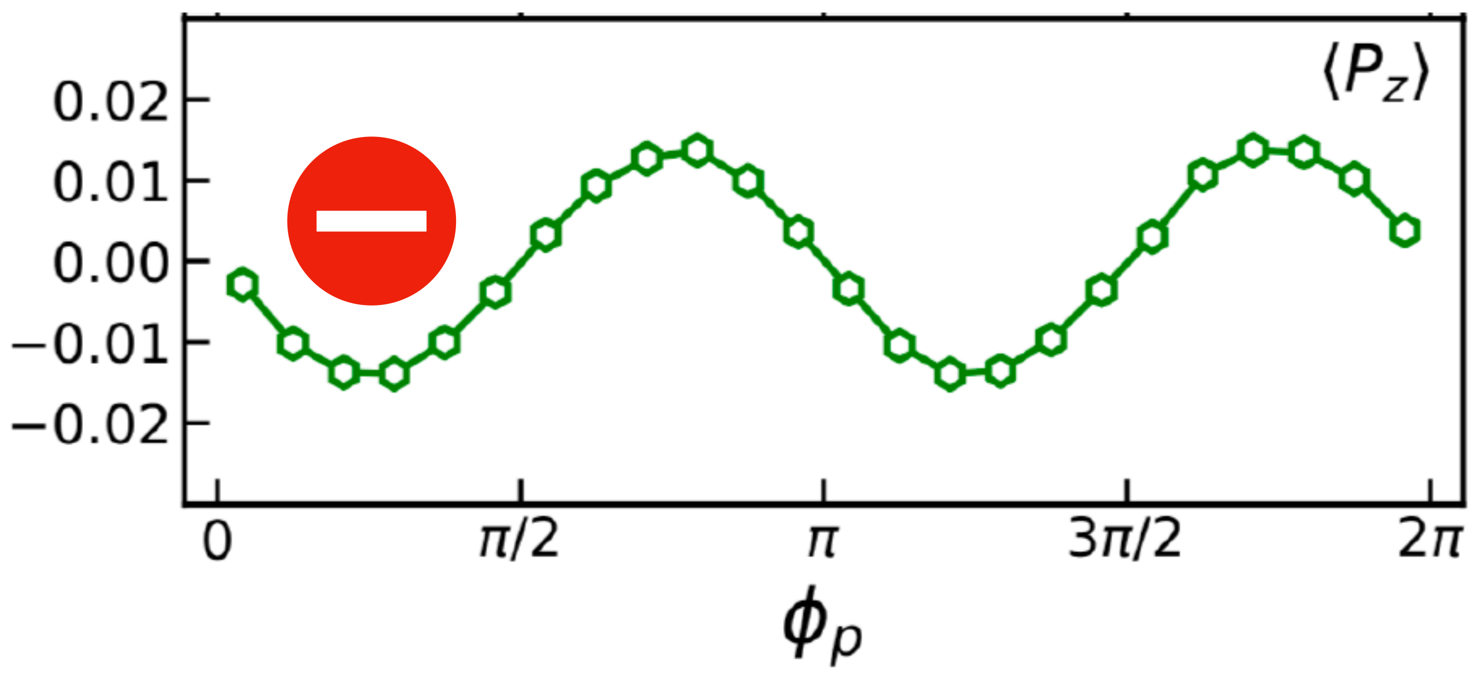
Local (momentum-differential) polarization



T. Niida, NPA 982 (2019) 511514



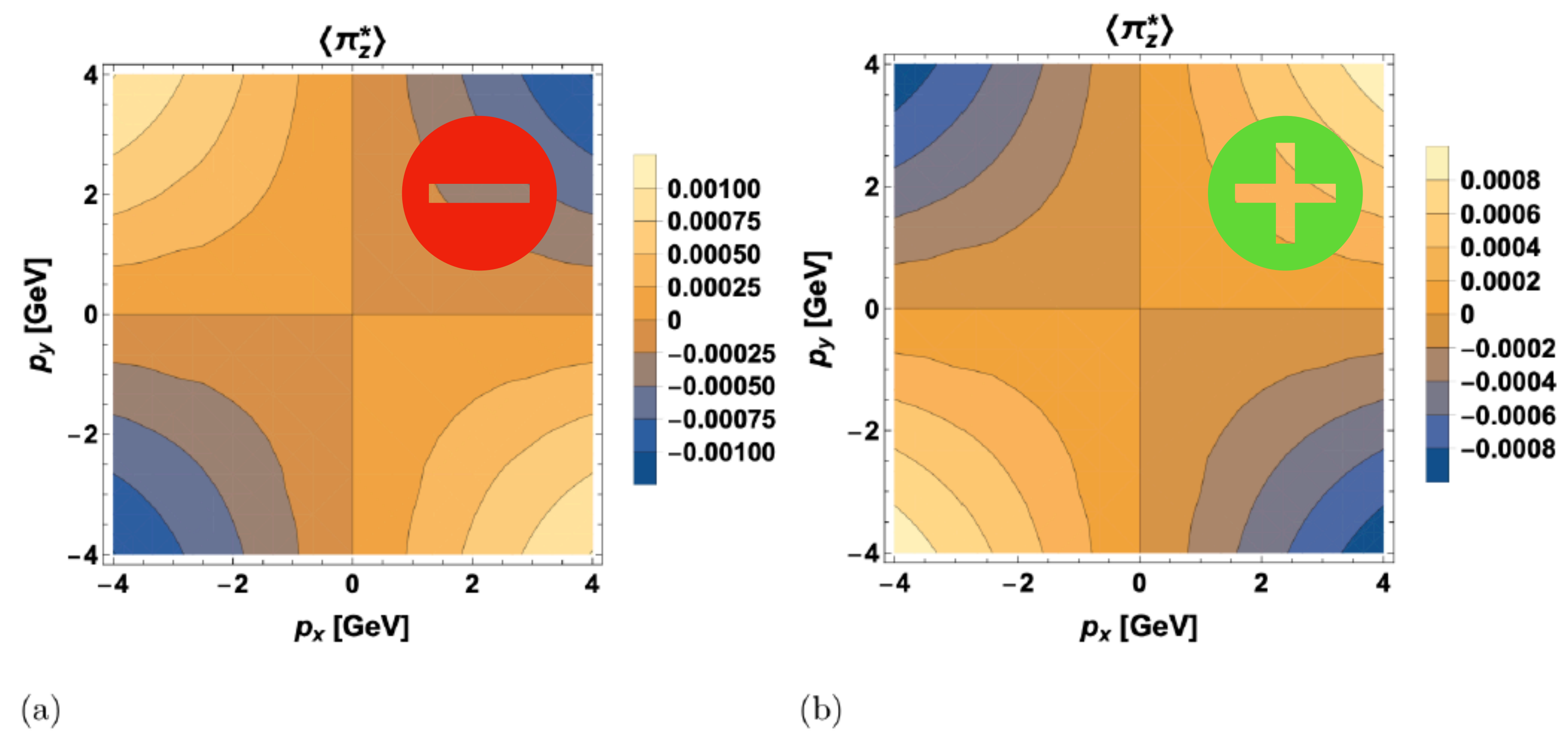
UrQMD+vHLLE: F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302,



AMPT: X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)

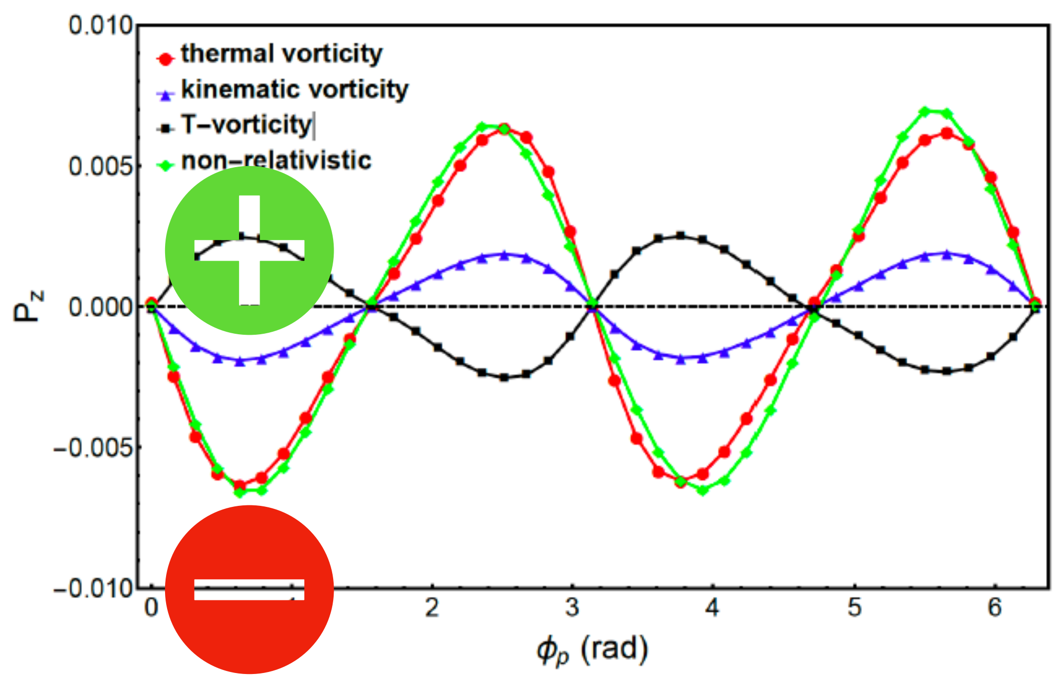
thermal model with projected vorticity $\omega_{\mu\nu} = \varpi_{\alpha\beta}\overline{\Delta}^{\alpha}_{\mu}\overline{\Delta}^{\beta}_{\nu}$

W.Florkowski, A. Kumar, A. Mazeliauskas, R.R., [1904.00002]



3D VH + AMPT IC with T-vorticity $\omega^{(T)}_{\mu\nu} = -\frac{1}{2}\left[\partial_\mu(Tu_\nu) - \partial_\nu\left(Tu_\mu\right)\right]$

H-Z Wu, L-G Pang, X-G Huang, Q. Wang [1906.09385]



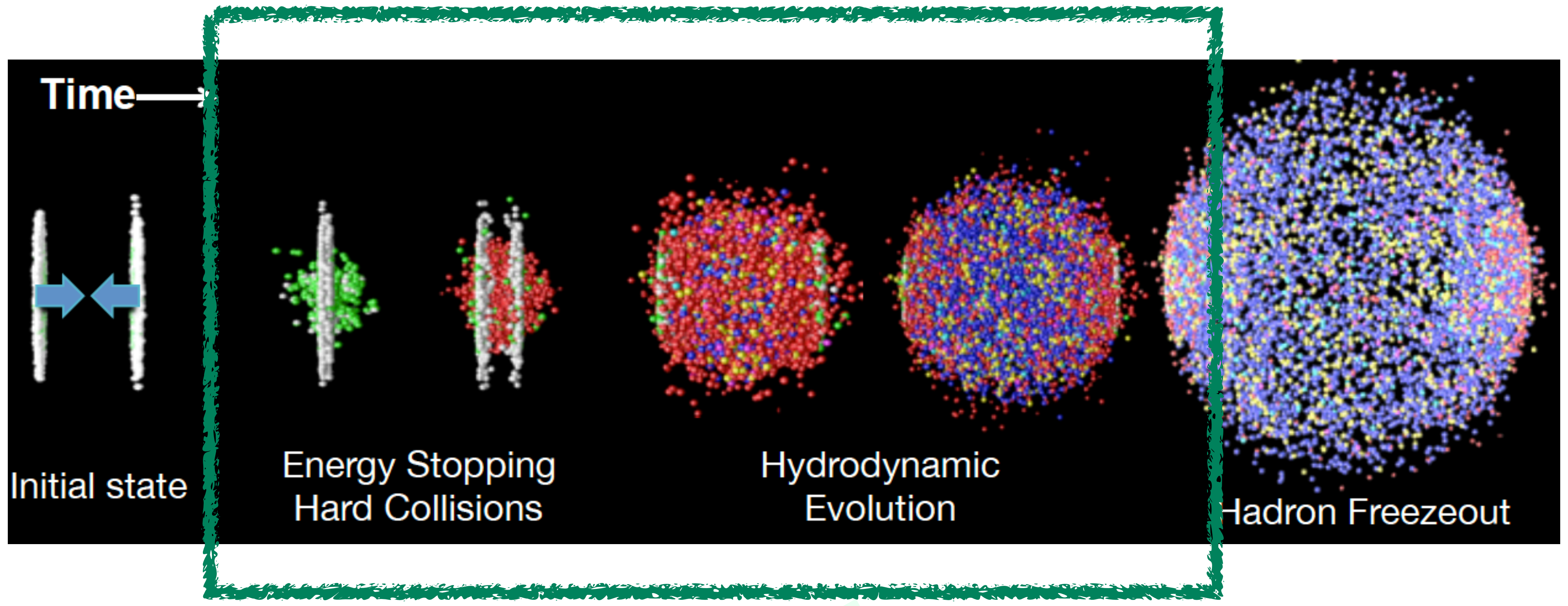
How to describe dynamics of spin?

Spin-thermal approach does not capture differential observables

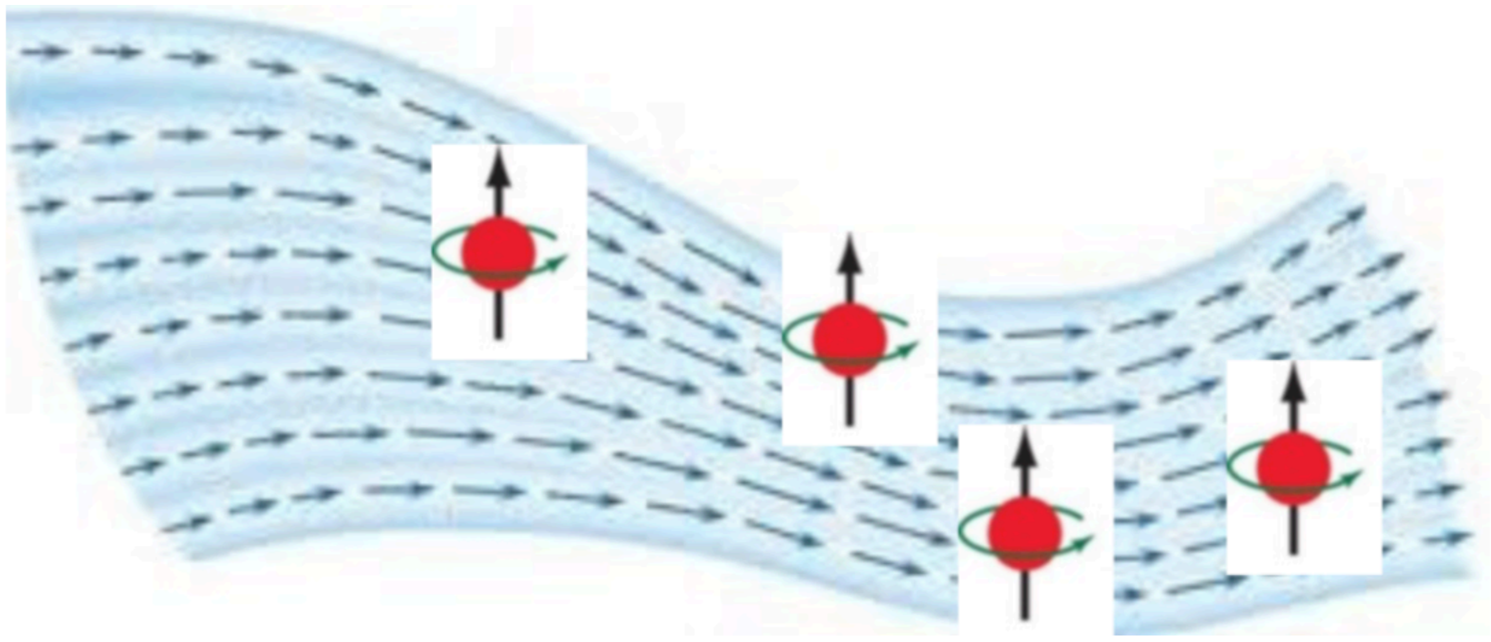
Is spin polarization always enslaved to thermal vorticity?

Non-trivial space-time dynamics of spin?

Relativistic fluid dynamics forms the basis of HIC models



Fluid dynamics with spin?



Most of the time close to equilibrium but the dissipation is also important

Ideal fluid dynamics with spin

If the energy-momentum tensor is symmetric the hydrodynamics with spin is given by

Prog. Part. Nucl. Phys. 108 (2019) 103709

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

What are the constitutive relations which enter equations of motion?

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \omega, \xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta, \omega, \xi], \quad N^\mu = N^\mu[\beta, \omega, \xi]$$

Summary

The spin polarization provides a new probe of the QGP properties

The disagreements between spin-thermal approach and data motivates developments of dynamical models

The fluid dynamics with spin is a natural framework one should seek for QGP

Present ideal spin hydro formulation is readily applicable

The theory is developing fast - future looks interesting!

Spinless relativistic fluid dynamics - basics

Ideal fluid dynamics = local equilibrium + conservation laws

energy-linear momentum conservation

baryon number conservation

Ideal	Dissipative
$T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu}$ $N^\mu = n u^\mu$ Unknowns: $\underbrace{\boxed{\epsilon,\ P,\ n,\ u^\mu}}_{=6}$	$T^{\mu\nu} = \epsilon u^\mu u^\nu - [P + \Pi]\Delta^{\mu\nu} + \pi^{\mu\nu}$ $N^\mu = n u^\mu + \nu^\mu$ Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu,\ \Pi,\ \pi^{\mu\nu},\ \nu^\mu}_{=15}$
Equations: $\underbrace{\partial_\mu T^{\mu\nu} = 0,\ \ \partial_\mu N^\mu = 0,\ \ EoS}_{4+1+1=6}$	
Closed set of equations	9 additional equations are needed

$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

L. Landau

Caution: Eckart-Landau theory is acausal!

For particles with spin the conservation of angular momentum implies introduction of new hydrodynamic (polarization) variables

Fluid dynamics with spin should tell how the polarisation variables evolve but not their origin!

Conservation of angular momentum and spin chemical potential

Noether's theorem:
for each continuous symmetry of the action there is a corresponding conserved (canonical) current

Conservation of charge (baryon number, electric charge, ...)

$$\partial_\mu \widehat{N}^\mu(x) = 0 \qquad \text{(1 equation/charge)}$$

$\rightarrow$ $\mu \equiv \xi T$

Conservation of energy and momentum

$$\widehat{J}_C^{\mu,\alpha\beta}(x) = \underbrace{x^\alpha \widehat{T}_C^{\mu\beta}(x) - x^\beta \widehat{T}_C^{\mu\alpha}(x)}_{\widehat{L}_C^{\mu,\alpha\beta}(x)} + \widehat{S}_C^{\mu,\alpha\beta}(x)$$

$$\partial_\mu \widehat{T}_C^{\mu\alpha}(x) = 0 \qquad \text{(4 equations)}$$

$\rightarrow$ T, u^ν

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, Phys. Rev. C97 (4) (2018) 041901
W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, PRD 97 (2018) 116017
F.Becattini, W. Florkowski, E. Speranza, PLB 789 (2019) 419-425

Conservation of total angular momentum

$$\partial_\mu \widehat{J}_C^{\mu,\alpha\beta}(x) = 0 \qquad \Rightarrow \qquad \partial_\mu \widehat{S}_C^{\mu,\alpha\beta}(x) = \widehat{T}_C^{\beta\alpha}(x) - \widehat{T}_C^{\alpha\beta}(x)$$

Spin chemical potential

$\rightarrow$ $\Omega_{\mu\nu} \equiv T\omega_{\mu\nu}$

Pseudogauges and the problem of energy and spin localization

Pseudo-gauge transformation

W. Hehl, Rept. Math. Phys. 9 (1976) 55–82;
F. Becattini, L. Tinti, PRD 84 (2011) 025013; PRD 87(2) (2013) 025029

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\lambda\left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu}\right)$$

$$\widehat{S}'^{\lambda,\mu\nu} = \widehat{S}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

- preserve $\widehat{P}^\mu = \int d^3\Sigma_\lambda\, \widehat{T}^{\lambda\mu}(x)$ $\quad \widehat{J}^{\mu\nu} = \int d^3\Sigma_\lambda\, \widehat{J}^{\lambda,\mu\nu}(x)$
- conservation laws unchanged

Belinfante-Rosenfeld pseudo-gauge (choosing superpotential $\widehat{\Phi} = \widehat{S}_C^{\lambda,\mu\nu}$)

Belinfante, F. J. (1939): Physica 6. 887-898, (1940); Rosenfeld, L. (1940): Mem. Acad. Roy. Belgique, cl. SC., tome 18, fasc. 6

$$\widehat{T}_B^{\mu\nu} = \widehat{T}_C^{\mu\nu} + \frac{1}{2}\partial_\lambda\left(\widehat{S}_C^{\lambda,\mu\nu} + \widehat{S}_C^{\mu,\nu\lambda} - \widehat{S}_C^{\nu,\lambda\mu}\right) \qquad \widehat{S}_B^{\lambda,\mu\nu} = 0$$

- gives exactly symmetric Hilbert $T^{\mu\nu}$ acting as the source of gravity in GR
- long-standing problem of physical significance of the spin tensor
- spin tensor is used by the community that studies the spin of proton
  X.S. Chen, X.F. Lu, W.M. Sun, F. Wang, T. Goldman, PRL 100 (2008) 232002;
  E. Leader, C. Lorce, Phys. Rep. 541 (2014) 163.

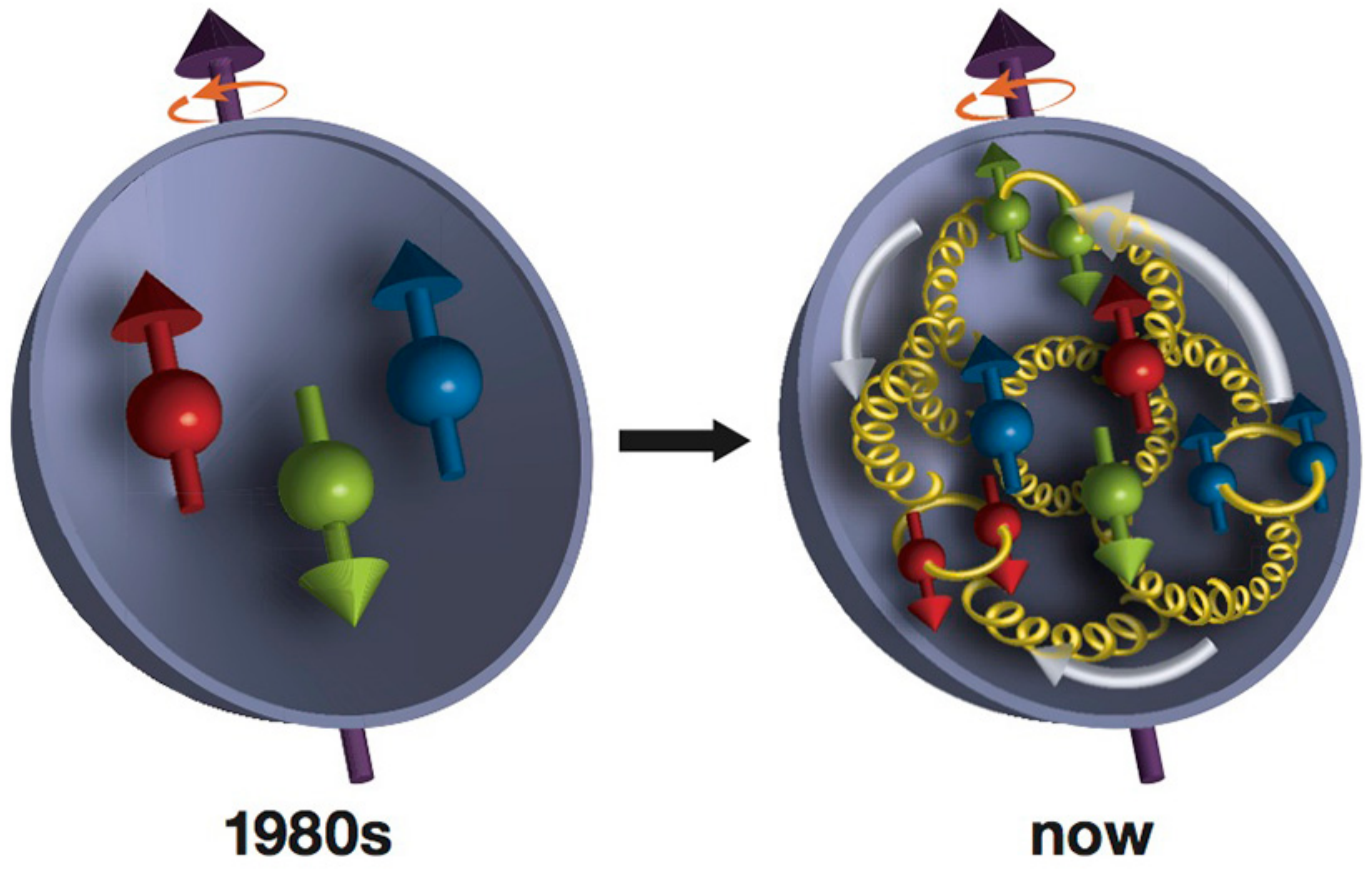


figure: Physics World

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If the energy-momentum tensor is symmetric the hydrodynamics with spin is given by

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$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

What are the constitutive relations which enter equations of motion?

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \omega, \xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta, \omega, \xi], \quad N^\mu = N^\mu[\beta, \omega, \xi]$$

Local equilibrium distributions

System without spin

$$f^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu\right]$$

De Groot, van Leeuwen, van Weert: Relativistic Kinetic Theory. Principles and Applications, 1980.
W. Florkowski, A. Kumar, R. R., PRC 98 (2018) 044906

System with spin

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi , Annals Phys. 338 (2013) 32
W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901
W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, PRD 97 (11) (2018) 116017

$$f^+_{rs}(x,p) = \frac{1}{2m}\bar{u}_r(p)X^+u_s(p)$$

$$f^-_{rs}(x,p) = -\frac{1}{2m}\bar{v}_s(p)X^-v_r(p)$$

$$X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu \pm \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$$

This is not thermal vorticity!

$$\hat{\Sigma}^{\mu\nu} = (i/4)\left[\gamma^\mu, \gamma^\nu\right]$$

$$\mathcal{W}^+_{\rm eq}(x,k) = \frac{1}{2}\sum_{r,s=1}^{2}\int dP\delta^{(4)}(k-p)u^r(p)\bar{u}^s(p)f^+_{rs}(x,p)$$

$$W^-_{\rm eq}(x,k) = -\frac{1}{2}\sum_{r,s=1}^{2}\int dP\delta^{(4)}(k+p)v^s(p)\bar{v}^r(p)f^-_{rs}(x,p)$$

$$\mathcal{W}_{\rm eq}(x,k) = \mathcal{W}^+_{\rm eq}(x,k) + \mathcal{W}^-_{\rm eq}(x,k)$$

$$T^{\beta\alpha}_{\rm eq}(x) = T^{\alpha\beta}_{\rm eq}(x)$$

Spin is conserved separately!

Classical approach to spin hydrodynamics

M.Mathisson

J. Weyssenhoff

In the classical treatments of particles with spin-1/2 one introduces internal angular momentum tensor of particles [M. Mathisson, APPB 6 (1937) 163–2900]

$$\boxed{s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta} p_\gamma s_\delta\,.}$$

$s^{\alpha\beta}$ is antisymmetric *i.e.* $s^{\alpha\beta} = -s^{\beta\alpha}$ and satisfies Frenkel (or Weyssenhoff) $p_\alpha s^{\alpha\beta} = 0$.

The spin four vector can be obtained by above equation,

$$\boxed{s^{\alpha} = \frac{1}{2m}\epsilon^{\alpha\beta\gamma\delta} p_\beta s_{\gamma\delta}}$$

In particle rest frame (PRF) where $p^\mu = (m, 0, 0, 0)$, $s^\alpha = (0, \mathbf{s}_*)$ with the length of spin vector given by $-s^2 = -s^\alpha s_\alpha = |\mathbf{s}_*|^2 = \mathfrak{s}^2 = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{3}{4}$.

Classical approach to spin hydrodynamics - perfect fluid

W. Florkowski, R. R., A. Kumar, Prog. Part. Nucl. Phys. 108 (2019) 103709 ;
J.-W. Chen, J.-y. Pang, S. Pu, Q. Wang, PRD 89 (9) (2014) 094003

$$\boxed{f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-p\cdot\beta(x) \pm \xi(x) + \frac{1}{2}\omega_{\alpha\beta}(x)s^{\alpha\beta}\right)}$$

$$\int dS \ldots = \frac{m}{\pi \mathfrak{s}} \int d^4 s\, \delta(s\cdot s + \mathfrak{s}^2)\, \delta(p\cdot s)\ldots$$

some 1D applications
+
3+1D implementation forthcoming

W.Florkowski, A. Kumar, RR, R. Singh, *Phys.Rev.C* 99 (2019) 4, 044910
R. Singh, G. Sophys, R.R, *Phys.Rev.D* 103 (2021) 7, 074024
R. Singh, M. Shokri, R.R., 2103.02592 (accepted to *Phys.Rev.D)*

$$N^{\mu}_{\rm eq} = \int dP \int dS\, p^{\mu} \left[f^{+}_{\rm eq}(x,p,s) - f^{-}_{\rm eq}(x,p,s)\right]$$

$$T^{\mu\nu}_{\rm eq} = \int dP \int dS\, p^{\mu}p^{\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right]$$

$$S^{\lambda\mu\nu}_{\rm eq} = \int dP \int dS\, p^{\lambda}\, s^{\mu\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right]$$

Explicit constitutive relations

$$N^{\alpha}_{\rm eq} = n u^{\alpha}$$

$$T^{\alpha\beta}_{\rm eq}(x) = \varepsilon u^{\alpha}u^{\beta} - P\Delta^{\alpha\beta}$$

$$S^{\lambda,\mu\nu}_{\rm eq} = S^{\lambda,\mu\nu}_{\rm GLW} = \mathcal{C}\left(n_0(T)u^{\lambda}\omega^{\mu\nu} + S^{\lambda,\mu\nu}_{\Delta\rm GLW}\right)$$

$$S^{\alpha,\beta\gamma}_{\Delta\rm GLW} = \mathcal{A}_0\, u^{\alpha}u^{\delta}u^{[\beta}\omega^{\gamma]}{}_{\delta} + \mathcal{B}_0\left(u^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_{\delta} + u^{\alpha}\Delta^{\delta[\beta}\omega^{\gamma]}{}_{\delta} + u^{\delta}\Delta^{\alpha[\beta}\omega^{\gamma]}{}_{\delta}\right)$$

For $|\omega_{\mu\nu}| < 1$ one obtains the formalism that agrees with that based on the quantum description of spin (in the GLW version).

Classical approach to spin hydrodynamics - dissipation

Use the relaxation time approximation for the collision terms in the classical kinetic equations

[S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, R. R. , *Phys.Lett.B* 814 (2021) 136096. *Phys.Rev.D* 103 (2021) 1, 014030

$$p^\mu \partial_\mu f_s^\pm(x,p,s) = C[f_s^\pm(x,p,s)], \qquad C[f_s^\pm(x,p,s)] = p\cdot u\, \frac{f_{s,\mathrm{eq}}^\pm(x,p,s) - f_s^\pm(x,p,s)}{\tau_{\mathrm{eq}}}.$$

Simple Chapman-Enskog expansion of the single particle distribution function around its equilibrium value in powers of space-time gradients

$$\boxed{\delta f_s^\pm = -\frac{\tau_{\mathrm{eq}}}{(u\cdot p)} e^{\pm\xi - p\cdot\beta}\left[\left(\pm p^\mu\partial_\mu\xi - p^\lambda p^\mu \partial_\mu \beta_\lambda\right)\left(1 + \frac{1}{2}s^{\alpha\beta}\omega_{\alpha\beta}\right) + \frac{1}{2}p^\mu s^{\alpha\beta}(\partial_\mu\omega_{\alpha\beta})\right]}$$

Dissipative corrections

$$\delta N^\mu = \int \mathrm{dP}\,\mathrm{dS}\; p^\mu(\delta f_s^+ - \delta f_s^-),$$

$$\delta T^{\mu\nu} = \int \mathrm{dP}\,\mathrm{dS}\; p^\mu p^\nu(\delta f_s^+ + \delta f_s^-),$$

$$\delta S^{\lambda,\mu\nu} = \int \mathrm{dP}\,\mathrm{dS}\; p^\lambda s^{\mu\nu}(\delta f_s^+ + \delta f_s^-).$$

$\Rightarrow$

$$\delta N^\mu = \nu^\mu = \tau_{\mathrm{eq}}\,\beta_n(\nabla^\mu\xi),$$

$$\delta T^{\mu\nu} = \pi^{\mu\nu} - \Delta^{\mu\nu}\Pi, \quad \pi^{\mu\nu} = 2\tau_{\mathrm{eq}}\beta_\pi\,\sigma^{\mu\nu}, \quad \Pi = -\tau_{\mathrm{eq}}\beta_\Pi\,\theta$$

$$\boxed{\delta S^{\lambda,\mu\nu} = \tau_{\mathrm{eq}}\left[B_\Pi^{\lambda,\mu\nu}\,\theta + B_n^{\kappa\lambda,\mu\nu}\,(\nabla_\kappa\xi) + B_\pi^{\alpha\kappa\lambda,\mu\nu}\,\sigma_{\alpha\kappa} + B_\Sigma^{\kappa\lambda\beta\alpha,\mu\nu}\,(\nabla_\kappa\omega_{\beta\alpha})\right]}$$

There are non-equilibrium corrections to spin tensor

Other developments towards hydrodynamics with spin

Lagrangian effective field theory approach

D. Montenegro, G. Torrieri, Phys.Rev. D94 (2016) no.6, 065042
D. Montenegro, L. Tinti, G. Torrieri, Phys. Rev. D 96(5) (2017) 056012; Phys. Rev. D 96(7) (2017) 076016
D. Montenegro, G. Torrieri, Phys. Rev. D 100, 056011 (2019)

Hydrodynamics with spin based on entropy-current analysis

K. Hattori, M. Hongo, X-G Huang, M. Matsuo, H. Taya, PLB 795 (2019) 100-106

Hydrodynamics of spin currents using presence of torsion

D. Gallegos, U. Gursoy, A. Yarom arXiv:2101.04759

Relativistic viscous hydrodynamics with spin using Navier-Stokes type gradient expansion analysis

D. She, A. Huang, D. Hou, J. Liao, arXiv:2105.04060

Relativistic viscous spin hydrodynamics from chiral kinetic theory

S. Shi, C. Gale, and S. Jeon, Phys. Rev. C 103, 044906 (2021)

Spin polarization generation from vorticity through nonlocal collisions

N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, arXiv:2005.01506, arXiv:2103.04896

Spin polarisation due to thermal shear

F. Becattini, M. Buzzegoli, and A. Palermo, arXiv:2103.10917
S. Y. F. Liu and Y. Yin, arXiv:2103.09200