
The photon PDF within the CT18 global analysis

Keping Xie

PITT PACC, University of Pittsburgh

DPF 2021, 14 July 2021

CTEQ-TEA Collaboration

Work with Tim J. Hobbs (SMU & IIT), Tie-Jiun Hou (Northeastern U., China),
Carl Schmidt (MSU), Mengshi Yan (PKU), and C.-P. Yuan (MSU)

[arXiv:2106.10299](https://arxiv.org/abs/2106.10299)

The precision requirements

The precision requirements

- The LHC becomes a precision machine.
- Theoretical cross sections have been achieved at NNLO in QCD, $\mathcal{O}(\alpha_s^2)$, for many processes.
- Due to $\alpha_e \sim \alpha_s^2$, we expect the QED corrections are the same level.
- The photon-initiated processes ($\gamma + \gamma, q, g \rightarrow X$) will have observable effects.

Many applications

The SM processes

- Drell-Yan: $\ell^+ \ell^-$
- $W^\pm H$
- $W^+ W^-$

BSM scenarios

- Heavy leptons: $L^+ L^-$
- Charged Higgs: $H^\pm, H^{\pm\pm}$

The existing photon PDFs

The first generation

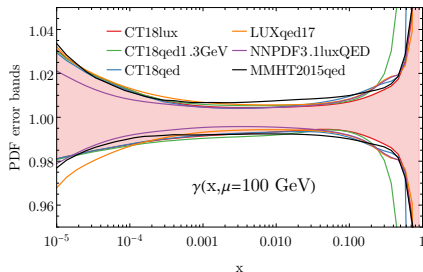
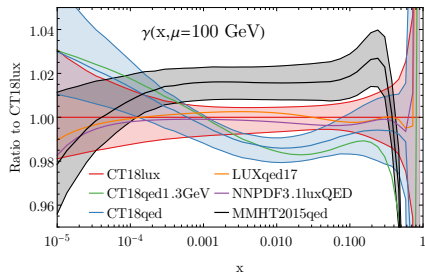
- MRST2004QED [0411040] models the photon PDF with an effective mass scale.
- NNPDF23QED [1308.0598] and NNPDF3.0QED [1410.8849] constrains photon PDF with the LHC Drell-Yan data, $q\bar{q}, \gamma\gamma \rightarrow \ell^+\ell^-$
- CT14qed_inc fits the inelastic ZEUS $ep \rightarrow e\gamma + X$ data [1509.02905], and include elastic component as well.

The second generation

- Recently, LUXqed directly takes the structure functions $F_{2,L}(x, Q^2)$ to constrain photon PDF uncertainty down to **percent level** [1607.04266,1708.01256]
- NNPDF3.1luxqed [1712.07053] initializes photon PDF with LUX formula at **$Q = 100$ GeV** (a high scale) and evolves DGLAP equation both upwardly and downwardly.
- MMHT2015qed [1907.02750] initializes photon at **1 GeV** (a low scale) and evolve DGLAP upwardly.
- Our work incorporates the LUX formalism with the CT18 [1912.10053] global analysis.

Two approaches: LUX vs DGLAP

- CT18lux: directly calculate the photon PDF with the **LUX** formalism
- CT18qed: initialize the inelastic photon PDF with the LUX formalism at low scales, and evolve the $\text{QED}_{\text{NLO}} \otimes \text{QCD}_{\text{NNLO}}$ **DGLAP** equations up to high scales, similar to MMHT2015qed.

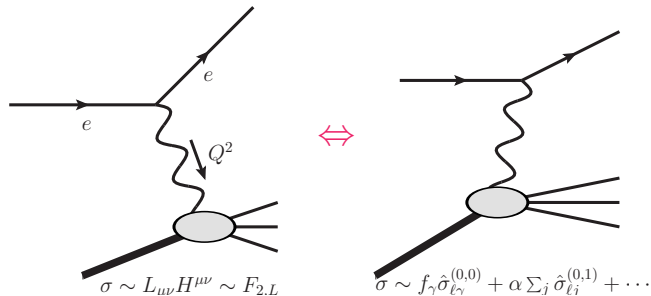


The take-home message:

- In the intermediate- x region, all photon PDFs give similar error bands.
- CT18lux photon PDF is **in between** LUXqed (also, NNPDF3.1luxQED) and MMHT2015qed, while CT18qed gives a **smaller** photon PDF.
- In the large- x region, the DGLAP approach (for both MMHT2015qed and CT18qed) gives a smaller photon than the LUX approach.

The LUX formalism [1607.04266,1708.01256]

- The DIS process: $ep \rightarrow e + X$



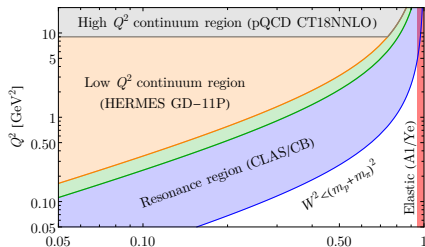
- Matching these two approaches leads to the LUX master formula:

$$x\gamma(x, \mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha_{\text{ph}}^2(-Q^2) \left[\left(zp\gamma_q(z) + \frac{2x^2 m_q^2}{Q^2} \right) \times \right. \right. \\ \left. \left. F_2(x/z, Q^2) - z^2 F_L(x/z, Q^2) \right] - \alpha^2(\mu^2) z^2 F_2(x/z, \mu^2) \right\}.$$

The square bracket term corresponds to the “**physical factorization**” scheme, while the second term is referred as the “ **$\overline{\text{MS}}$ -conversion**” term.

- The structure functions $F_{2,L}$ can be directly measured, or calculated through pQCD in the high-energy regime.

The breakup of (x, Q^2) plane



- In the resonance region $W^2 = m_p^2 + Q^2(1/x - 1) < W_{\text{Io}}^2$, the structure functions are taken from CLAS [0301204] or Christy-Bosted [0712.3731] fits.
- In the low- Q^2 continuum region $W^2 > W_{\text{hi}}^2$ GeV², the HERMES GD11-P [1103.5704] fits with ALLM [PLB1991] functional form.
- In the high- Q^2 region ($Q^2 > Q_{\text{PDF}}^2$), $F_{2,L}$ are determined through pQCD.
- The elastic form factors are taken from A1 [1307.6227] or Ye [1707.09063] fits of world data.

The difference between LUX and DGLAP

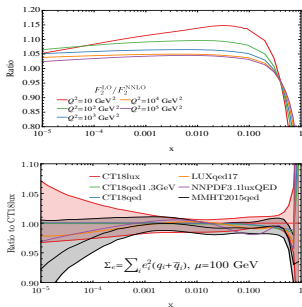
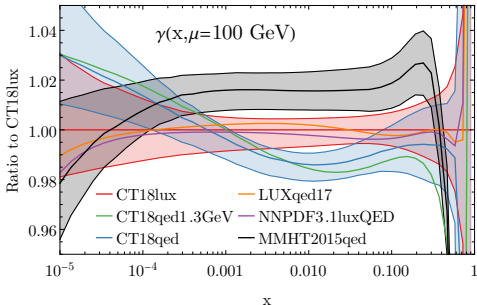
- The DGLAP only evolves the inelastic photon

$$\frac{dx\gamma^{\text{inel}}}{d\log\mu^2} = \frac{\alpha}{2\pi} \left(xP_{\gamma\gamma} \otimes x\gamma^{\text{inel}} + \sum_i e_i^2 xP_{\gamma q} \otimes xq_i \right)$$

- The first-order solution corresponds to the LO F_2 in LUX formalism

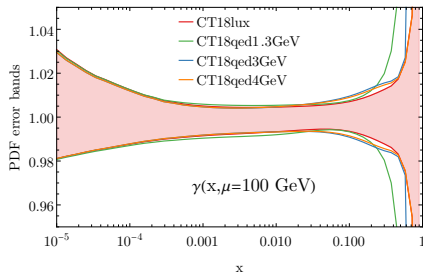
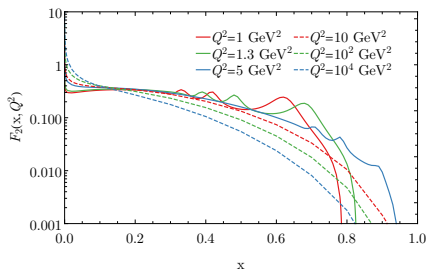
$$x\gamma^{\text{inel}}(x, \mu^2) \sim \int^{\mu^2} d\log Q^2 \frac{\alpha}{2\pi} \sum_i e_i^2 xP_{\gamma q} \otimes x f_{q_i} \rightarrow F_2^{\text{LO}} \text{ in LUX formula}$$

- It explains CT18qed gives larger photon at small x than CT18lux.
- MMHT2015qed gives smaller photon at small x , because the smaller charge-weighted singlet quark distributions.

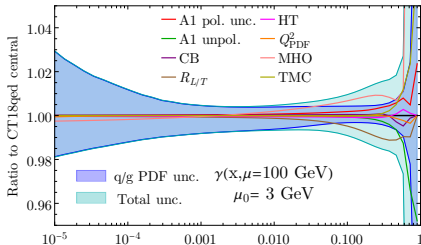
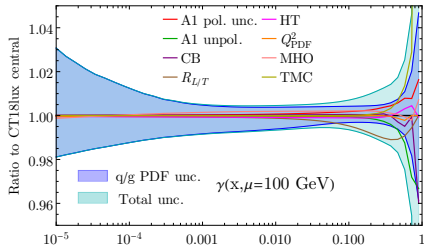


The large x behavior

- At large x , the LUX approach gives significantly larger PDF than the DGLAP one.
- It is resulted from the non-perturbative F_2 at low energy (resonance and low- Q^2 continuum regions).
- It induces a big uncertainty with the DGLAP low initialization scale approach, just because of scaling violation is not well behaved in the non-perturbative F_2 .
- It can be rescued with a slightly higher initialization scale above the pQCD matching scale $Q_{\text{PDF}} \sim 3 \text{ GeV}$.



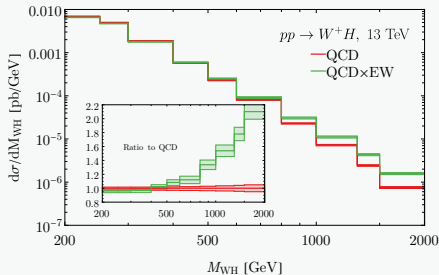
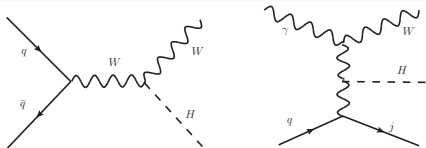
Other low- Q^2 uncertainties



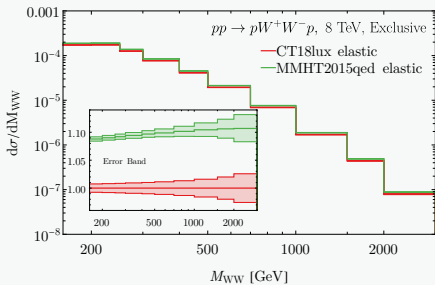
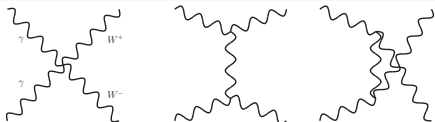
- A1 pol. unc.: the uncertainty of the A1 fit of the world^x polarized data
- A1 unpol.: Switching to A1 fit of the world unpolarized data
- CB: Changing resonance SF from CLAS to Christy-Bosted fit
- Variations of $R_{L/T} = \sigma_L/\sigma_T$ by 50% [1708.01256]
- HT: Adding higher-twist contribution to F_L [1708.01256] and F_2 [1602.03154].
- Q_{PDF}^2 : changing the matching scale $9 \rightarrow 5 \text{ GeV}^2$
- MHO: varying the scale to estimate the missing high-order uncertainty
- TMC: adding the target mass correction to the SFs.

The applications

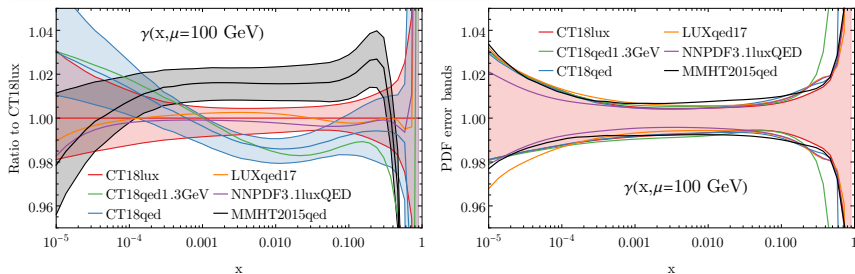
W^+H production



Exclusive $\gamma\gamma \rightarrow W^+W^-$



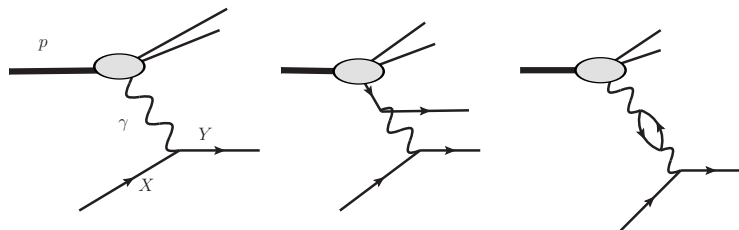
Summary and conclusions



- We have two photon PDF sets, CT18lux and CT18qed, based on the LUX and DGLAP approach, respectively.
- The overall uncertainties agree with the LUXqed(also NNPDF3.1luxQED) and MMHT2015qed.
- In the intermediate- x region, CT18lux is in between the LUXqed(also NNPDF3.1luxQED) and MMHT2015qed, while CT18qed is smaller.
- In the small- x region, the CT18qed is larger than CT18lux, due to the equivalent LO SF. The MMHT2015qed becomes smaller because of the smaller singlet PDFs Σ_e .
- In the large- x region, the DGLAP approach (MMHT2015qed and CT18qed) give smaller PDFs due to the non-perturbative SFs.
- The low- μ_0 DGLAP approach gives larger uncertainty at large x , due to non-perturbative SFs at low scales.

The cancellation in a higher order calculation

- Suppose we want to calculate a process $\gamma + X \rightarrow Y$.



- At one order higher, both photon and quark parton will participate.
- The PDFs are related with the DGLAP evolution, with divergence properly canceled.
- This can be also achieved in the LUX approach, with proper $\overline{\text{MS}}$ conversion terms order by order.

The scale variation of the $\overline{\text{MS}}$ conversion term

- In the default scale choice $\mu^2/(1-z)$, the $\overline{\text{MS}}$ -conversion term is

$$x\gamma^{\text{con}} \sim (-z^2)F_2(x/z, \mu^2),$$

which is negative

- When varying the scale as μ^2 , the conversion term should be change as well,

$$x\gamma^{\text{con}}([M]) = x\gamma^{\text{con}} + \frac{1}{2\pi\alpha} \int_x^1 \frac{dz}{z} \int_{M^2[z]}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2 z p_{\gamma q}(z) F_2(x/z, Q^2).$$

With $M^2[z] = \mu^2$, we have $\int_{\mu^2}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} = \log \frac{1}{1-z}$.

- The central MMHT2015qed corresponds to $M^2[z] = \mu^2$ choice at low scale $\mu_0 = 1 \text{ GeV}$.
- The DGLAP approach at low scale **DOES** give larger uncertainty due to the large non-perturbative contributions to structure functions.
- One method to avoid it is to start γ PDF at a higher scale in the pQCD region, i.e., $\mu_0^2 > Q_{\text{PDF}}^2$.

The DGLAP approach gives smaller PDFs at large x

- MMHT2015qed divides the integration into two regions:

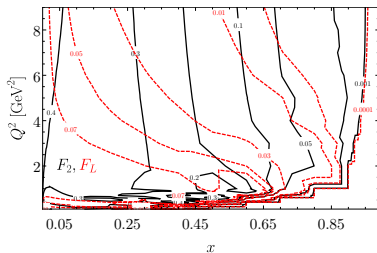
$$\left(\int_{\frac{x^2 m_p^2}{1-z}}^{Q_0^2} + \int_{Q_0^2}^{\frac{Q_0^2}{1-z}} \right) [\dots]$$

The second part is integrated semi-analytically:

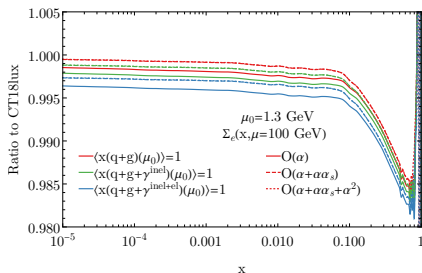
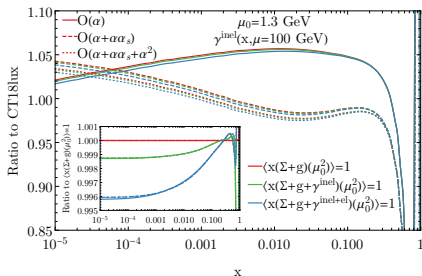
$$\int_{Q_0^2}^{\frac{Q_0^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2 \left(zp_{\gamma q} + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q_0^2) = \alpha^2(Q_0^2) \left(zp_{\gamma q} \log \frac{1}{1-z} + \frac{2x^2 m_p^2 z}{Q_0^2} \right) F_2\left(\frac{x}{z}, Q_0^2\right)$$

The F_L is dropped because $F_L \sim \mathcal{O}(\alpha_s) \ll F_2$.

- In contrast, we integrate over $F_2(x/z, Q^2)$ rather than $F_2(x/z, Q_0^2)$.
- It explains the MMHT2015qed gives smaller photon at large x than CT18qed.
- MMHT15 does not include the uncertainty induced by Q_0 variation.



The NLO QED evolution and momentum sum rules



- The NLO QED corrections to splitting functions

$$P_{ij} = \frac{\alpha}{2\pi} P_{ij}^{(0,1)} + \frac{\alpha}{2\pi} \frac{\alpha_s}{2\pi} P_{i,j}^{(1,1)} + \left(\frac{\alpha}{2\pi}\right)^2 P_{ij}^{(0,2)} + \dots$$

- The NLO QED correction is negative.
- The momentum sum rules: the impact is $\mathcal{O}(0.1\%)$, negligible compared with higher order QED evolution.

$$\langle x(\Sigma + g + \gamma^{\text{inel+el}}) \rangle = 1$$