

Phenomenology of T-odd jet function

Wai Kin Lai

South China Normal University
University of California, Los Angeles

in collaboration with Xiaohui Liu, Hongxi Xing, and Manman Wang

based on

X. Liu and H. Xing, [arXiv:2104.03328 \[hep-ph\]](https://arxiv.org/abs/2104.03328), and work in progress

July 14, 2021

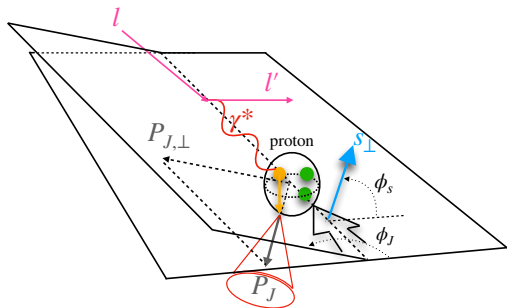
DPF 2021

MOTIVATION

- 3D structure of proton were studied typically using
 - semi-inclusive hadron production
[Mulders, Tangerman (1996), Brodsky, Hwang, Schmidt (2002), Bacchetta et al. (2007)]
 - jet production
[Kang, Metz, Qiu, Zhou (2011), Liu, Ringer, Vodelsang, Yuan (2019), Kang, Lee, Shao, Zhao (2021)]
- Jet used to be unable to distinguish quark flavor. This issue was solved by introducing jet charge. [Kang, Liu, Mantry, Shao (2020)]
- Jet was thought to be able to probe only T-even TMD PDFs.
- Aim: introduce T-odd jet function to probe T-odd TMD PDFs as well.

INCLUSIVE JET PRODUCTION IN DIS

Consider $l + p(P, S) \rightarrow l' + J(P_J) + X$



This is like SIDIS, but replace a hadron by a jet.

[Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2018)]

FACTORIZATION

- Focus on the region $\Lambda_{QCD} \sim |\mathbf{P}_{J\perp}| \ll Q$
- Factorization from SCET: $\sigma = H \otimes \Phi \otimes \mathcal{J}$
 H : hard function, Φ : TMD PDFs, \mathcal{J} : TMD jet functions (JFs)

[Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2018)]

$$\Phi^{ij}(x, p_T) = \int \frac{dy^- d^2 \mathbf{y}_T}{(2\pi)^3} e^{ip \cdot y} \langle P | \bar{\chi}_n^j(0) \chi_n^i(y) | P \rangle |_{y^+=0}$$

$$\mathcal{J}^{ij}(z, k_T) = \frac{1}{2z} \sum_X \int \frac{dy^- d^2 \mathbf{y}_T}{(2\pi)^3} e^{ik \cdot y} \langle 0 | \chi_{\bar{n}}^i(y) | JX \rangle \langle JX | \bar{\chi}_{\bar{n}}^j(0) | 0 \rangle |_{y^-=0}$$

- TMD PDFs and TMD JFs encoded in azimuthal asymmetries:

$$\begin{aligned} \frac{d\sigma}{dx dy dz d\psi d\phi_J dP_J^2} &= \frac{\alpha^2}{xyQ^2} \left\{ \left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1 - y) \cos(2\phi_J) F_{UU}^{\cos(2\phi_J)} \right. \\ &\quad \left. + S_{\parallel} (1 - y) \sin(2\phi_J) F_{UL}^{\sin(2\phi_J)} + S_{\parallel} \lambda_e y \left(1 - \frac{y}{2}\right) F_{LL} \right. \\ &\quad \left. + |S_{\perp}| \left[\left(1 - y + \frac{y^2}{2}\right) \sin(\phi_J - \phi_S) F_{UT,T}^{\sin(\phi_J - \phi_S)} + (1 - y) \sin(\phi_J + \phi_S) F_{UT}^{\sin(\phi_J + \phi_S)} \right. \right. \\ &\quad \left. \left. + (1 - y) \sin(3\phi_J - \phi_S) F_{UT}^{\sin(3\phi_J - \phi_S)} \right] + |S_{\perp}| \lambda_e y \left(1 - \frac{y}{2}\right) \cos(\phi_J - \phi_S) F_{LT}^{\cos(\phi_J - \phi_S)} \right\} \end{aligned}$$

F 's contain convolutions of TMD PDFs and JFs.

TMD PDFs AT LEADING TWIST

$$\Phi = \frac{1}{2} \left\{ f_1 \not{n} - f_{1T}^\perp \frac{\epsilon_{\alpha\beta} p_T^\alpha S_T^\beta}{M} \not{n} + \left(S_L g_{1L} - \frac{p_T \cdot S_T}{M} g_{1T} \right) \gamma_5 \not{n} \right. \\ \left. + h_{1T} \frac{[S_T, \not{n}] \gamma_5}{2} + \left(S_L h_{1L}^\perp - \frac{p_T \cdot S_T}{M} h_{1T}^\perp \right) \frac{[p_T, \not{n}] \gamma_5}{2M} + i h_1^\perp \frac{[p_T, \not{n}]}{2M} \right\}$$

quark \ hadron	unpolarized	chiral	transverse
U	f_1		h_1^\perp (Boer-Mulders)
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp (Sivers)	g_{1T}	h_{1T}, h_{1T}^\perp (transversity)

- 8 TMD PDFs at leading twist, functions of x and p_T^2
- T-even: $f_1, g_{1L}, g_{1T}, h_{1T}, h_{1L}^\perp, h_{1T}^\perp$
T-odd: f_{1T}^\perp, h_1^\perp
- 3 functions survive after p_T integration: f_1, g_{1L}, h_{1T}

T-ODD JET FUNCTION

- Traditionally, only jets with high p_T were of interest. Naive time reversal symmetry implies that jet functions are T-even.
- At low p_T , the jet is sensitive to nonperturbative physics. In particular, spontaneous chiral symmetry breaking breaks naive time reversal symmetry. This leads to a nonzero T-odd jet function (Collins effect). [Collins (2002)]

$$\mathcal{J}(z, k_T) = \mathcal{J}_1(z, k_T) \frac{\not{\eta}}{2} + i \mathcal{J}_T(z, k_T) \frac{k_T \not{\eta}}{2}$$

\mathcal{J}_1 : T-even, ordinary collinear part

\mathcal{J}_T :

- Analogue of Collins function, couples quark transverse spin with parton transverse momentum
- depends on jet algorithm, axis definition, jet size, etc.

ADVANTAGES OF T-ODD JET FUNCTION

- *Universality*

Like the T-even \mathcal{J}_1 , T-odd \mathcal{J}_T is process independent.

- *Flexibility*

Flexibility of choosing jet recombination schemes and hence the jet axis, allows us to adjust sensitivity to different nonperturbative contributions. This extra control could provide opportunity to “film” the QCD nonperturbative dynamics, if one manages to continuously change the axis from one to another.

- *High predictive power*

- Since a jet contains many hadrons, it is a more perturbative object compared to a hadron. This means that the jet function has more perturbatively calculable degrees of freedom than the fragmentation function. For instance, in the WTA scheme, the z -dependence in the jet function is completely determined.
- Similar to the study in [Becher, Bell (2014)], \mathcal{J}_T can be factorized into a product of a perturbative coefficient and a nonperturbative Sudakov factor. As a vacuum operator element, the nonperturbative Sudakov factor can be calculated on the lattice. This is unlike the TMD fragmentation function, which is an operator element of $|h + X\rangle$.

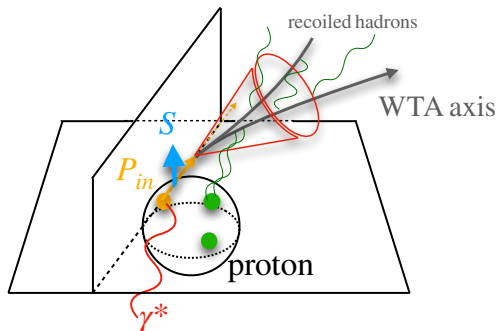
WTA SCHEME

In the WTA scheme, there is an extra simplification

$$\mathcal{J}(z, k_T, R) = \delta(1 - z)\tilde{J}(k_T) + \mathcal{O}\left(\frac{k_T^2}{p_J^2 R^2}\right)$$

[Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2018)]

$$\tilde{J}(k_T) = J(k_T)\frac{\not{k}_T}{2} + iJ_T(k_T)\frac{k_T\not{k}_T}{2}$$



Collins asymmetry:

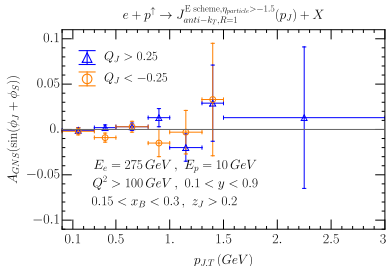
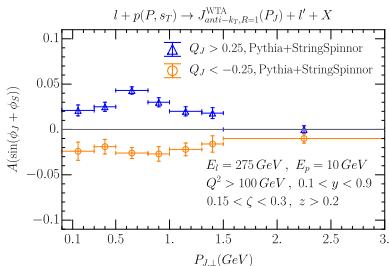
$$A(\zeta, y, \phi_s, \phi_J, P_{J\perp}) = 1 + \epsilon |S_\perp| \sin(\phi_J + \phi_s) \frac{F_{UT}}{F_{UU}}$$

- $F_{UT} \sim h_1 \otimes J_T$, probes transversity
- We simulate using Pythia 8.2+StringSpinner [Kerbizi, Loenblad (2021)], with jet charge [Kang, Liu, Mantry, Shao (2020)] measured to avoid cancellation between different quark flavors, with EIC kinematics.
- Use the spherically-invariant jet algorithm [Cacciari, Salam, Soyez (2012)]

$$d_{ij} = \min(E_i^{2p}, E_j^{2p}) \frac{1 - \cos \theta_{ij}}{1 - \cos R}, \quad d_{iB} = E_i^{2p}$$

(Conventional k_T -algorithms using p_T instead of E not good for low p_T jets)

- Use two different jet axis schemes: WTA and E-scheme.
- Result shows significant dependence on jet axis definitions.

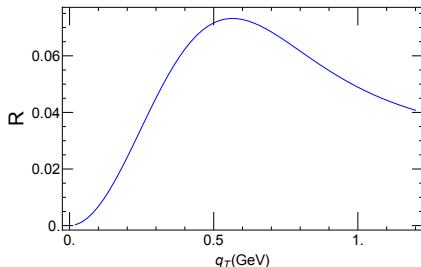
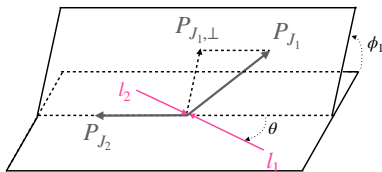


e^+e^- ANNIHILATION

We demonstrate prediction on azimuthal asymmetry in e^+e^- annihilation at $\sqrt{s} = \sqrt{110}$ GeV, with WTA scheme and parametrized nonperturbative sudakov for J_T , and $Q_J > 0.25$ and $Q_J < -0.25$ for the two jets respectively.

$$R^{J_1 J_2} = 1 + \cos(2\phi_1) \frac{\sin^2 \theta}{1 + \cos^2 \theta} \frac{F_T(q_T)}{F_U(q_T)}$$

$$R = 2 \int d \cos \theta \frac{d\phi_1}{\pi} \cos(2\phi_1) R^{J_1 J_2}$$

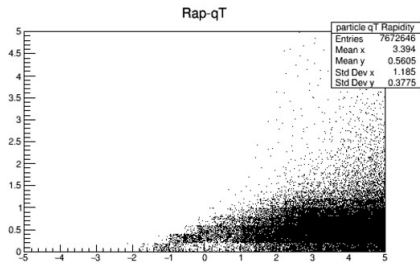


SUMMARY AND OUTLOOK

- We introduce the T-odd jet function, which is nonzero for low p_T jets.
- With T-odd jet function, we can probe T-odd TMD PDFs.
- T-odd jet function has the advantages of universality, flexibility, and high predictive power.
- This opens a new direction in applications and understanding of jet physics.

Thank you.

Backup slides



AZIMUTHAL ASYMMETRY AT LEADING TWIST

Couplings of Φ and \mathcal{J} encoded in angular distribution:

$$\begin{aligned} \frac{d\sigma}{dx dy dz d\psi d\phi_J dP_J^2} = & \frac{\alpha^2}{xyQ^2} \left\{ \left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1 - y) \cos(2\phi_J) F_{UU}^{\cos(2\phi_J)} \right. \\ & \left. + S_{\parallel} (1 - y) \sin(2\phi_J) F_{UL}^{\sin(2\phi_J)} + S_{\parallel} \lambda_e y \left(1 - \frac{y}{2}\right) F_{LL} \right. \\ & \left. + |\mathbf{S}_{\perp}| \left[\left(1 - y + \frac{y^2}{2}\right) \sin(\phi_J - \phi_S) F_{UT,T}^{\sin(\phi_J - \phi_S)} + (1 - y) \sin(\phi_J + \phi_S) F_{UT}^{\sin(\phi_J + \phi_S)} \right. \right. \\ & \left. \left. + (1 - y) \sin(3\phi_J - \phi_S) F_{UT}^{\sin(3\phi_J - \phi_S)} \right] + |\mathbf{S}_{\perp}| \lambda_e y \left(1 - \frac{y}{2}\right) \cos(\phi_J - \phi_S) F_{LT}^{\cos(\phi_J - \phi_S)} \right\} \end{aligned}$$

- $F_{UU,T}, F_{LL}, F_{UT,T}^{\sin(\phi_J - \phi_S)}, F_{LT}^{\cos(\phi_J - \phi_S)}$: contain T-even parts of Φ and \mathcal{J}
- $F_{UU}^{\cos(2\phi_J)}, F_{UL}^{\sin(2\phi_J)}, F_{UT}^{\sin(\phi_J + \phi_S)}, F_{UT}^{\sin(3\phi_J - \phi_S)}$: contain T-odd parts of Φ and \mathcal{J}

The F 's are convolutions of TMD PDFs and jet functions:

$$\mathcal{C}[w f J] \equiv x \sum_a e_q^2 \int d^2 \mathbf{p}_T \int d^2 \mathbf{k}_T \delta^{(2)}(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T) w(\mathbf{p}_T, \mathbf{k}_T) f(x, p_T^2) J(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}[f_1 \mathcal{J}_1], \quad F_{LL} = \mathcal{C}[g_{1L} \mathcal{J}_1]$$

$$F_{UT,T}^{\sin(\phi_J - \phi_S)} = \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} f_{1\perp} \mathcal{J}_1 \right], \quad F_{UT,T}^{\cos(\phi_J - \phi_S)} = \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} g_{1T} \mathcal{J}_1 \right],$$

$$F_{UU}^{\cos(2\phi_J)} = \mathcal{C} \left[-\frac{(2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T)}{M} h_1^\perp \mathcal{J}_T \right]$$

$$F_{UL}^{\sin(2\phi_J)} = \mathcal{C} \left[-\frac{(2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T)}{M} h_{1L}^\perp \mathcal{J}_T \right]$$

$$F_{UT}^{\sin(\phi_J + \phi_S)} = \mathcal{C} \left[-\hat{\mathbf{h}} \cdot \mathbf{k}_T h_1 \mathcal{J}_T \right]$$

$$F_{UT}^{\sin(3\phi_J - \phi_S)} = \mathcal{C} \left[\frac{2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)(\mathbf{p}_T \cdot \mathbf{k}_T) + \mathbf{p}_T^2 (\hat{\mathbf{h}} \cdot \mathbf{k}_T) - 4(\hat{\mathbf{h}} \cdot \mathbf{p}_T)^2 (\hat{\mathbf{h}} \cdot \mathbf{k}_T)}{2M^2} h_{1T}^\perp \mathcal{J}_T \right]$$

where $\hat{\mathbf{h}} \equiv \mathbf{P}_{J\perp} / |\mathbf{P}_{J\perp}|$ and $h_1 \equiv h_{1T} + \frac{\mathbf{p}_T^2}{2M^2} h_{1T}^\perp$