Magnetic charge and Massive Photon*

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Maxwell's Equations for Massive Photons

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- In this talk we consider two changes to Maxwell's equations.
- First for a massive photon one has

$$\nabla \cdot \mathbf{E} + m^2 \phi = 4\pi \rho_{(e)} \quad ; \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{B} = 0 \quad ; \quad \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} + m^2 \mathbf{A} = 4\pi \mathbf{J}_{(e)}$$

- Above equations not gauge invariant, unlike standard Maxwell
- Above equations have a reduced (gauge) symmetry
- For ordinary Maxwell $\mathbf{E} = -\nabla \phi - \partial_t \mathbf{A} \quad \& \quad \mathbf{B} = \nabla \times \mathbf{A}$ invariant under

$$\phi' \rightarrow \phi - \partial_t \lambda \quad ; \quad \mathbf{A}' \rightarrow \mathbf{A} + \nabla \lambda$$

Electrostatic Interaction for Massive Photons

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- If the photon is massive one finds

$$\mathbf{E}_{Yukawa} = \frac{qe^{-mr}\hat{\mathbf{r}}}{r^2}(1+mr) \quad ; \quad \phi_{Yukawa} = \frac{qe^{-mr}}{r}$$

- The Yukawa field, Yukawa potential, and Yukawa Force

$$\mathbf{F}_{Yukawa} = \frac{q_1 q_2 e^{-mr}\hat{\mathbf{r}}}{r^2}(1+mr)$$

- Experimentally $m < 10^{-18} eV$ (see Particle Data Handbook).

Maxwell's Equations Electric & Magnetic Charges

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- Maxwell's equations for Magnetic and Electric Charges are

$$\nabla \cdot \mathbf{E} = 4\pi\rho_{(e)} \quad ; \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = -4\pi \mathbf{J}_{(m)}$$
$$\nabla \cdot \mathbf{B} = 4\pi\rho_{(m)} \quad ; \quad \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = 4\pi \mathbf{J}_{(e)}$$

- This makes Maxwell's equations symmetric under a dual rotation

$$\mathbf{E} = \mathbf{E}'\cos\theta + \mathbf{B}'\sin\theta \quad ; \quad \mathbf{B} = -\mathbf{E}'\sin\theta + \mathbf{B}'\cos\theta \ .$$
$$\rho_{(e)} = \rho'_{(e)}\cos\theta + \rho'_{(m)}\sin\theta \quad ; \quad \rho_{(m)} = -\rho'_{(e)}\sin\theta + \rho'_{(m)}\cos\theta \ .$$
$$\mathbf{J}_{(e)} = \mathbf{J}'_{(e)}\cos\theta + \mathbf{J}'_{(m)}\sin\theta \quad ; \quad \mathbf{J}_{(m)} = -\mathbf{J}'_{(e)}\sin\theta + \mathbf{J}'_{(m)}\cos\theta \ .$$

Magnetic Charges

- For electric charge q, $\phi = \frac{q}{r}$ gives $\mathbf{E} = -\nabla\phi = \frac{q\hat{\mathbf{r}}}{r^2}$
- For magnetic charge g one naively wants $\mathbf{B} = \nabla \times \mathbf{A} = \frac{g\hat{\mathbf{r}}}{r^2}$
- For electric charge $\nabla \cdot \mathbf{E} = q\left[\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2}\right)\right] = 4\pi q\delta^3(\mathbf{r}) = 4\pi\rho_{(e)}$
- For magnetic charge $\nabla \cdot \mathbf{B} = g\left[\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2}\right)\right] = 4\pi g\delta^3(\mathbf{r}) = 4\pi\rho_{(m)}$
- But canonically $\nabla \cdot \left(\nabla \times \mathbf{A}\right) = 0$
- Bullet point 4 contradicts bullet point 5. Problem?

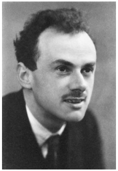
Dirac vector-potential for massless photon

- Bullet point 4 from above realized by the Dirac String*

$$\mathbf{A}_{\pm} = \frac{g}{r}\left(\frac{\pm 1 - \cos\theta}{\sin\theta}\right)\hat{\varphi} \implies \mathbf{B} = \nabla \times \mathbf{A}_{\pm} = \frac{g\hat{\mathbf{r}}}{r^2}$$

- The result is a Coulomb **B**-field.

- $\mathbf{A}_+$ singular at $(\theta = \pi)$; $\mathbf{A}_-$ singular at $(\theta = 0)$.



* *P.A.M. Dirac, Proc. Roy. Soc. A* **133**, *60-72 (1931)*

String vector-potential for massless photon

- Consistency with $\nabla \cdot \left(\nabla \times \mathbf{A}\right) = 0$ requires string contribution*

  $\mathbf{A}_{s\pm} = \pm\frac{2g\Theta(\rho)\Theta(\mp z)}{\rho}\hat{\varphi} \implies \mathbf{B}_{s\pm} = \frac{2g\delta(\rho)\Theta(\mp z)}{\rho}\hat{z}$

- $\mathbf{A}_{s\pm}$ is solenoid carrying flux $4\pi g$

- Adding $\mathbf{A}_{s\pm}$ gives $\mathbf{B}_{\pm} = \nabla \times (\mathbf{A}_{\pm} + \mathbf{A}_{s\pm})$ and $\nabla \cdot \mathbf{B}_{\pm} = 0$.

- Dirac condition makes this string "invisible".

* *R. Heras, Contemp. Phys.* **59**, *331-355 (2018).*

Doubly Modified Maxwell's Equations

- Doubly modified Maxwell → photon mass + magnetic charge

$$\nabla \cdot \mathbf{E} + m^2 \phi = 4\pi \rho_{(e)}$$
$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} + m^2 \mathbf{A} = 4\pi \mathbf{J}_{(e)}$$
$$\nabla \cdot \mathbf{B} = 4\pi \rho_{(m)}$$
$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = -4\pi \mathbf{J}_{(m)}$$

- Gauge symmetry spoiled; dual symmetry added

Magnetic charge plus photon mass I

- Allow a photon mass **and** a magnetic charge.
- Accomplished tentatively by multiplying **A** by the e^{-mr}
- So, $\mathbf{A}_{\pm} = g\frac{e^{-mr}}{r}\left(\frac{\pm 1 - \cos\theta}{\sin\theta}\right)\hat{\varphi}$*

  and $\mathbf{B}_{\pm} = \nabla \times \mathbf{A}_{\pm} = g\frac{e^{-mr}}{r^2}\left(\hat{\mathrm{r}} + mr\left[\frac{\pm 1 - \cos\theta}{\sin\theta}\right]\hat{\boldsymbol{\theta}}\right)$*

* *T. Evans and D. Singleton, Int. J. Mod. Phys. A* **33**, *1850064 (2018) &*

M. Dunia, T. Evans, and D. Singleton, Phys.Rev.D **103**, *128501 (2021)*

Magnetic charge plus photon mass II

- The above solution $\mathbf{A}_{\pm}$,$\mathbf{B}_{\pm}$ corrects an earlier wrong solution*
- Our solution assumes $\mathbf{A}=e^{-mr}\mathbf{A}_{\text{Dirac}}$; earlier work assumed $\mathbf{A}=\mathbf{A}_{Dirac}+\mathbf{A}_{mass}$
- String singularity is real due to its presence in the magnetic field

* *A. Y. Ignatiev and G. C. Joshi, Phys. Rev. D* **53**, *984 (1996)*

Magnetic charge plus photon mass III

- However, as in the massless case, $\mathbf{A}_\pm, \mathbf{B}_\pm$ modified so that $\nabla \cdot \mathbf{B}_\pm = 0$ & $\nabla \times \mathbf{B}_\pm + m^2 \mathbf{A}_\pm \neq 0$

- Introduce vector potential regularized by ϵ

$$\mathbf{A}_\pm = g \frac{\Theta(\rho - \epsilon) e^{-m\sqrt{\rho^2+z^2}}}{\rho} \left(\pm 1 - \frac{z}{\sqrt{\rho^2 + z^2}} \right) \hat{\varphi}$$

- Taking the curl and $\epsilon \to 0$ yields (with $\nabla \cdot \mathbf{B}_\pm = 0$)

$$\mathbf{B}_\pm = \frac{g e^{-mr}}{r^2} \left[\hat{\mathrm{r}} + mr \left(\frac{\pm 1 - \cos\theta}{\sin\theta} \right) \hat{\boldsymbol{\theta}} \right] \pm 4\pi g e^{-m|z|} \Theta(\mp z) \delta(x) \delta(y) \hat{\mathrm{z}}$$

Dirac Quantization Condition

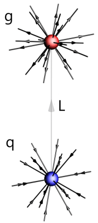
- Amazing feature of magnetic charge – Dirac Condition

$$qg = n\frac{\hbar}{2}$$

- Many Derivations (AB-Phase, Wu-Yang Fiber Bundle)
- Here we use the field angular momentum quantization method

Field Angular Momentum and Dirac Condition

- An at rest $q+g$ system carries an E&M field angular momentum
- For massless photon, $\mathbf{L}_{EM} = \frac{1}{4\pi}\int \mathrm{r} \times (\mathbf{E} \times \mathbf{B}_{\text{Dirac}}) d^3x = qg\hat{\mathrm{r}}_0$
- Quantizing $|\mathbf{L}_{EM}| = n\frac{\hbar}{2}$ yields the Dirac condition $qg = \frac{n\hbar}{2}$
- $\mathbf{L}_{EM}$ not dependent on r_0 [magnitude of $\mathbf{r}_0$]



Modified Field Angular Momentum

- With a photon mass, field angular momentum becomes

$$\mathbf{L}_{EM} = \frac{1}{4\pi}\int \mathbf{r} \times \left[\left(\mathbf{E} \times \mathbf{B}\right) + m^2\mathbf{A}\right] d^3x$$

- When $r_0 = 0$, this field angular momentum gives

$$\mathbf{L}_{EM} = qg\hat{\mathbf{z}}$$

- String along z-axis; same quantization condition as Dirac!
- Here $\mathbf{L}_{EM}$ comes from string, not the monopole itself.

Modified Dirac condition

- Now take q + string on z-axis and $r_0 \neq 0$
- Detailed calculation now gives E&M field angular momentum:
  - *String along* $\mp$*z-axis, electric charge at* $r = \mp r_0 \hat{\mathbf{z}}$*:*
    $\mathbf{L}_{EM} = \mathbf{L}_{EM}^{point} + \mathbf{L}_{EM}^{\mathbf{B}^\theta + \mathbf{A}} = \pm 2qge^{-\frac{mr_0}{\hbar}} \hat{\mathbf{z}}$
  - *String along* $\mp$*z-axis, electric charge at* $r = \pm r_0 \hat{\mathbf{z}}$*:*
    $\mathbf{L}_{EM} = \mathbf{L}_{EM}^{point} + \mathbf{L}_{EM}^{\mathbf{B}^\theta + \mathbf{A}} = 0$
- Quantizing this angular momentum, give:

$$2qg \exp[-\frac{mr_0}{\hbar}] = \frac{n\hbar}{2} \text{ for all } n = 1, 2, 3...$$

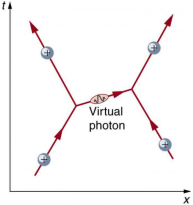
- Implies not only quantization of q and g but m and r_0.

Summary and Conclusions

- Combining magnetic charge plus photon mass leads to a Yukawa-Dirac String potential and magnetic field.
- Dirac string becomes a real object *i.e.* appears in **A** *and* **B**.
- Dirac quantization condition changes. Includes m and r_0.

Quantum Picture for Interactions (additional material)

- The Coulomb field described in QM via Feynman diagram.
- Exchanged force particle is massless.



- The Yukawa field described in QM via Feynman diagram.
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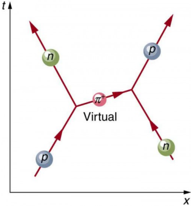

Field Angular Momentum and Dirac Condition (Additional Material)

- One can write the total angular momentum as the following

  $\mathbf{L}_{Tot} = \mathbf{L}_{Mech} + \mathbf{L}_{EM}$, where $\mathbf{L}_{Mech} = \mathbf{r} \times (\mathbf{p} - q\mathbf{A})$

- Yang* showed that the total angular momentum still satisfies

  $[\mathbf{L}_{Tot\ i}, \mathbf{L}_{Tot\ j}] = i\hbar\varepsilon_{ijk}\mathbf{L}_{Tot\ k}$

- Angular momentum commutator not satisfied by $\mathbf{L}_{Mech}$ alone

* *C.N. Yang, Annals New York Academy of Sciences* **294**, *86 - 97 (1977)*