Solving Adjoint QCD_2 with Asymptotic Basis Functions

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QCD_{2A} is a 2D theory of quarks in the adjoint representation coupled by non-dynamical gluon fields ("matrix quarks")

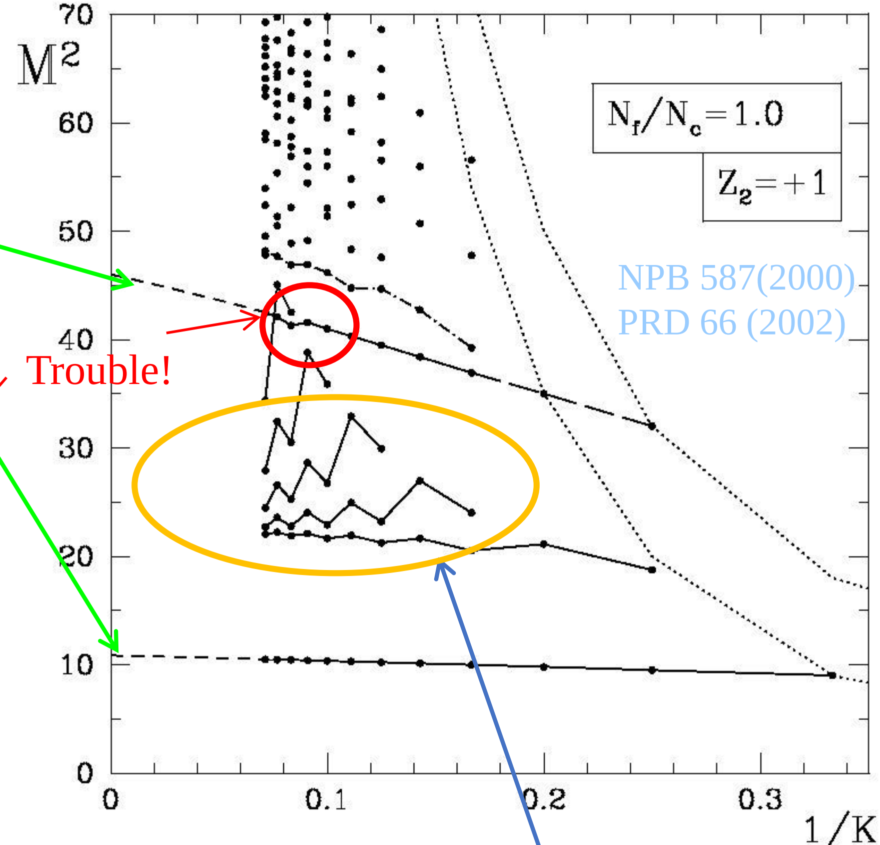
- The Problem: all known approaches are cluttered with multi-particle states (MPS)
- We want "the" bound-states, i.e. single-particle states (SPS)
- Get also tensor products of these SPS with relative momentum
- SPS interact with MPS!

(kink in trajectory)

- DLCQ calculation shown, but typical

(see Katz et al **JHEP 1405 (2014) 143**)

➔ Need to solve theory with new method → **eLCQ**



Group of approximate MPS

Algebraic Solution of the Asymptotic Theory

- Parton number violation is disallowed, the asymptotic theory splits into decoupled sectors of fixed parton number
- Wavefunctions are determined by 't Hooft-like integral equations

$$M^2\phi_r(x_1,\ldots,x_r) = -\sum_{i=1}^{r}(-1)^{(r+1)(i+1)}\int_{-\infty}^{\infty}\frac{\phi_r(y,x_i+x_{i+1}-y,x_{i+2},\ldots,x_{i+r-1})}{(x_i-y)^2}dy,$$

➔ Sinusoidal ansatz with correct number of excitation numbers:
n_i ; $i = 1 \ldots r\text{-}1$

$$|n_1,n_2,\ldots n_{r-1}\rangle \doteq \prod_j^{r-1} e^{i\pi n_j x_j} = \phi_r(x_1,x_2,\ldots,x_r)$$

- Use **symmetries** to construct asymptotic eigenfunctions from this ansatz

Exhaustively-symmetrized Light-Cone Quantization (eLCQ)

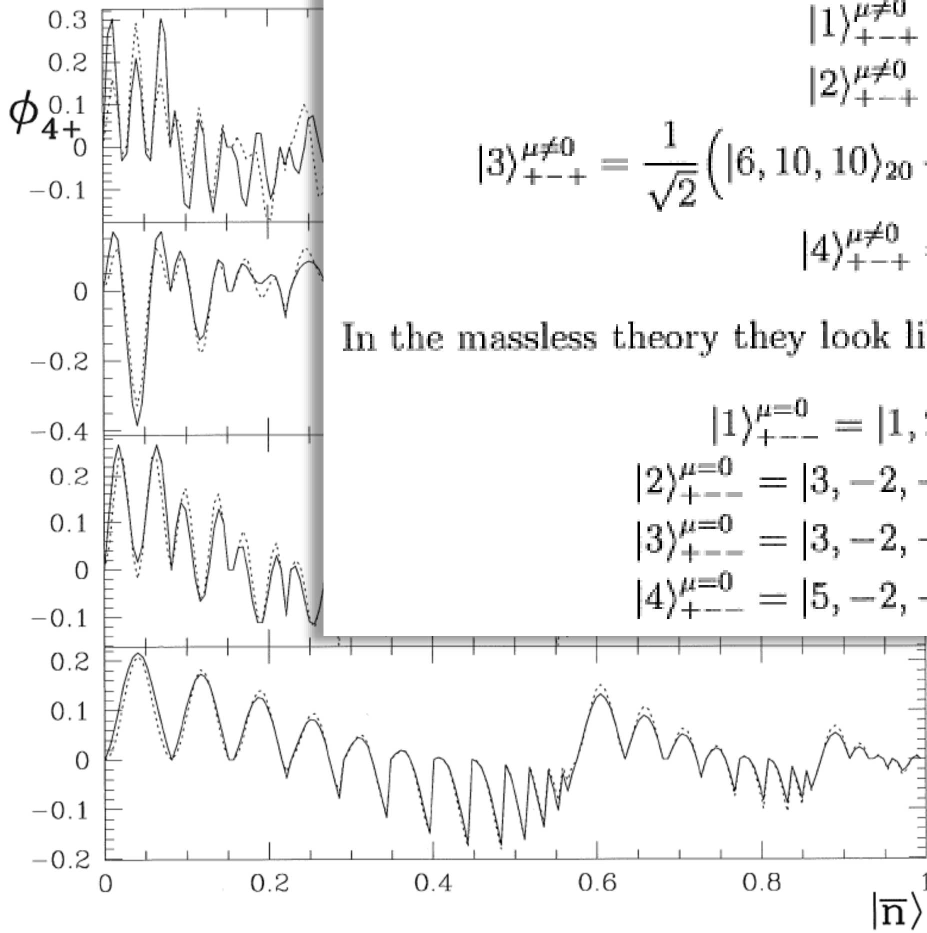
- Construct the eigenstates by completely symmetrizing states under all symmetries
- Here: $\mathcal{C}, \mathcal{T}, \mathcal{I}, \mathcal{S}$
  - Cyclicity $\mathcal{C}$ due to trace: $\phi_r(x_1, x_2, \ldots, x_r) = (-1)^{r+1}\phi_r(x_2, x_3 \ldots, x_r, x_1)$
  - Reorientation $\mathcal{T}$ due to symmetric color matrices
  - Inversion $\mathcal{I}$ due to inverse-square Coulomb force
  - Need low-dim inversion $\mathcal{S}$ to implement $\phi_n(0, x_2, \ldots, x_n) = 0,$

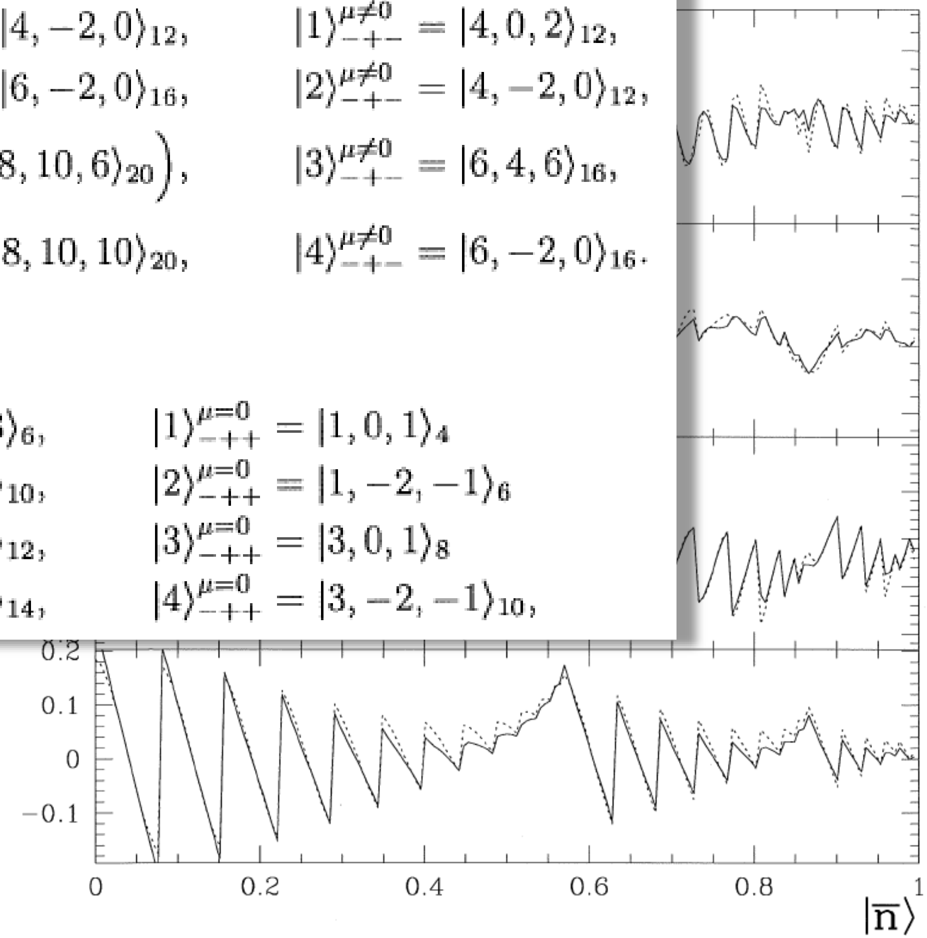
$$\mathcal{S} : (x_1, \ldots x_r) \to (x_1, 1 - x_2 - x_1, 1 - x_3, 1 - x_4, \ldots, 1 - x_r - x_1)$$

➔ $\mathit{eigenfunction\ of\ } 2r! \mathit{\ terms\ of\ the\ form\ } \mathcal{T}\mathcal{I}\mathcal{C} \cdots \mathcal{S} e^{i\pi \sum_j^{r-1} n_j x_j}$

Works! Massless Four-Parton Eigenfunctions

Numerical (solid) vs. Algebraic (dashed)





four states are in the massive theory ($\mu \neq 0$)

$$|1\rangle^{\mu\neq 0}_{+-+} = |4,-2,0\rangle_{12}, \qquad |1\rangle^{\mu\neq 0}_{-+-} = |4,0,2\rangle_{12},$$

$$|2\rangle^{\mu\neq 0}_{+-+} = |6,-2,0\rangle_{16}, \qquad |2\rangle^{\mu\neq 0}_{-+-} = |4,-2,0\rangle_{12},$$

$$|3\rangle^{\mu\neq 0}_{+-+} = \frac{1}{\sqrt{2}}\Big(|6,10,10\rangle_{20} + |8,10,6\rangle_{20}\Big), \qquad |3\rangle^{\mu\neq 0}_{-+-} = |6,4,6\rangle_{16},$$

$$|4\rangle^{\mu\neq 0}_{+-+} = |8,10,10\rangle_{20}, \qquad |4\rangle^{\mu\neq 0}_{-+-} = |6,-2,0\rangle_{16}.$$

In the massless theory they look like

$$|1\rangle^{\mu=0}_{+--} = |1,2,3\rangle_{6}, \qquad |1\rangle^{\mu=0}_{-++} = |1,0,1\rangle_{4}$$

$$|2\rangle^{\mu=0}_{+--} = |3,-2,-1\rangle_{10}, \qquad |2\rangle^{\mu=0}_{-++} = |1,-2,-1\rangle_{6}$$

$$|3\rangle^{\mu=0}_{+--} = |3,-2,-3\rangle_{12}, \qquad |3\rangle^{\mu=0}_{-++} = |3,0,1\rangle_{8}$$

$$|4\rangle^{\mu=0}_{+--} = |5,-2,-1\rangle_{14}, \qquad |4\rangle^{\mu=0}_{-++} = |3,-2,-1\rangle_{10},$$

T+ (even) under string reversal **(odd) T −**

Make Table of Asymptotic States → Extract Ground States in all Parton Sectors

r	T I S	$\text{Sector}^{\text{mass}}_{T_{state}}$	Excitation numbers of lowest states	Masses ($g^2N\pi$)
2	$-+$	$\|o\rangle^0_+$	$(1), (3), (5), (7)$	$2, 6, 10, 14$
	$--$	$\|e\rangle^\mu_+$	$(2), (4), (6), (8)$	$4, 8, 12, 16$
3	$++$	$\|ee\rangle^0_-$	$(0,0), (2,2), (4,2), (4,0), (6,2)$	$0, 4, 8, 8, 12$
	$--$	$\|ee\rangle^0_+$	$(4,2), (6,2), (8,2), (8,4)$	$8, 12, 16, 16$
	$+-$	$\|ee\rangle^\mu_-$	$(2,0), (4,0), (6,2), (6,0)$	$4, 8, 12, 12$
	$-+$	$\|ee\rangle^\mu_+$	$(6,2), (8,2), (10,4), (10,2)$	$12, 16, 20, 20$
4	$+--$	$\|oeo\rangle^0_+$	$(3,2,1), (5,4,3), (5,6,3), (7,6,3)$	$6, 10, 12, 14$
	$-++$	$\|oeo\rangle^0_-$	$(1,2,1), (3,2,1), (3,4,3), (3,6,3)$	$4, 6, 8, 10$
	$+-+$	$\|eee\rangle^\mu_+$	$(6,6,4), (8,8,6), (8,10,6), (10,10,6), (10,10,8)$	$12, 16, 20, 20, 20$
	$-+-$	$\|eee\rangle^\mu_-$	$(4,6,4), (6,6,4), (6,8,6), (6,10,6)$	$12, 12, 16, 16$
5	$+++$	$\|eeee\rangle^0_+$	$(0,0,0,0), (2,2,2,2), (2,4,4,4), (4,4,4,2), (4,4,4,4)$	$0, 4, 8, 8, 8$
	$---$	$\|eeee\rangle^0_-$	$(4,4,4,2), (4,6,6,6), (4,6,4,2), (6,6,4,2), (4,8,8,8)$	$8, 12, 12, 12, 16$
	$++-$	$\|eeee\rangle^\mu_+$	$(4,6,6,4), (6,8,8,6), (6,10,10,6), (8,10,10,6)$	$12, 16, 20, 20$
	$--+$	$\|eeee\rangle^\mu_-$	$(8,10,10,6), (8,12,10,6), (8,14,12,8)$	$20, 24, 28$
6	$-++$	$\|oeoeo\rangle^0_+$	$(1,2,3,2,1), (1,2,3,4,3), (5,4,3,2,1), (3,4,5,4,3)$	$6, 8, 10, 10$
	$+--$	$\|oeoeo\rangle^0_-$	$(1,2,3,4,3), (5,4,3,2,1), (3,6,5,4,3), (5,6,5,4,3)$	$8, 10, 12^{(2)}, 14^{(2)}, 16^{(4)}$
	$--+$	$\|eeeee\rangle^\mu_+$	$(6,10,12,10,6), (6,10,12,12,8), (8,12,14,12,8),$ $(8,14,14,12,8), (8,14,16,14,8), (8,14,18,14,8)$	$24, 24, 28, 28, 32, 36$
	$++-$	$\|eeeee\rangle^\mu_-$	$(8,12,12,10,6), (10,12,12,10,6), (8,12,14,14,10),$ $(8,14,14,12,8), (12,14,14,12,8)$	$24^{(2)}, 28^{(3)}, 32^{(5)}$
7	$---$	$\|eeeeee\rangle^0_+$	$(2,4,4,4,4,4), (4,6,6,6,4,2), (4,6,8,6,4,2), (4,6,8,8,8,4)$	$8, 12, 16^3$
	$+++$	$\|eeeeee\rangle^0_-$	$(0,0,0,0,0,0), (2,2,2,2,2,2), (2,4,4,4,4,2), (2,4,4,4,4,4)$	$0, 4, 8^3, 12^3$
	$-+-$	$\|eeeeee\rangle^\mu_+$	$(8,14,16,16,14,10), (8,14,18,18,16,10),$ $(8,14,18,18,16,12), (10,16,18,18,16,12)$	$32, 36^3$
	$+-+$	$\|eeeeee\rangle^\mu_-$	$(6,10,12,12,10,6), (8,14,16,16,14,8), (8,14,16,16,14,10),$ $(8,14,18,18,14,8)$	$24, 32^2, 36^5$
8	$+--$	$\|oeoeoeo\rangle^0_+$	$(3,4,5,4,3,2,1), (5,4,5,4,3,2,1), (3,6,5,4,3,2,1)$	$10, 12^2, 14$
	$-++$	$\|oeoeoeo\rangle^0_-$	$(1,2,3,4,3,2,1), (3,4,5,4,3,2,1), (3,4,5,6,5,4,3)$	$8, 10, 12^3$
	$+-+$	$\|eeeeeee\rangle^\mu_+$	$(10,16,20,20,18,14,8), (10,18,20,20,18,14,8)$	$40^3, 52$
	$-+-$	$\|eeeeeee\rangle^\mu_-$	$(8,14,18,20,18,14,8), (10,16,20,20,18,14,8)$	40^5

Extracting Physics from this Combinatorics Exercise: *Explaining Spectral Patterns*

- Lowest state is a three-parton, not two-parton state
  - Only odd-parton eLCQ asymptotic eigenstates can be massless, see Table
- Lowest bound states are isolated in mass, "pure" in parton number
  - Symmetry forces low-mass and high-mass states into same sector → mass gap → parton purity
  - Example: lowest fermionic T+ state is five-parton (asymptotically massless!) and lowest available three-parton excitation numbers (4,2) lead to $M^2=16\pi\approx 50$, lowest seven-parton state is also very massive, and nine-parton state gets mass from non-singular term. (see below)

- Consistent with known results
  - Earlier: String-inspired parametrizations of two-, four-, six-parton spectrum in half the T-sectors
  - Mostly correct, but eLCQ rules allow us to:
    - find additional states (complete basis!)
    - find states in missed sectors
    - purge states not compatible with full set of symmetries

Complete Basis! On to the Full Theory

- Have fully symmetrized states: |TIS; ***n*** > with quantum numbers under $\mathcal{T}$, $\mathcal{I}$, $\mathcal{S}$ and *r*-1 excitation numbers ***n***

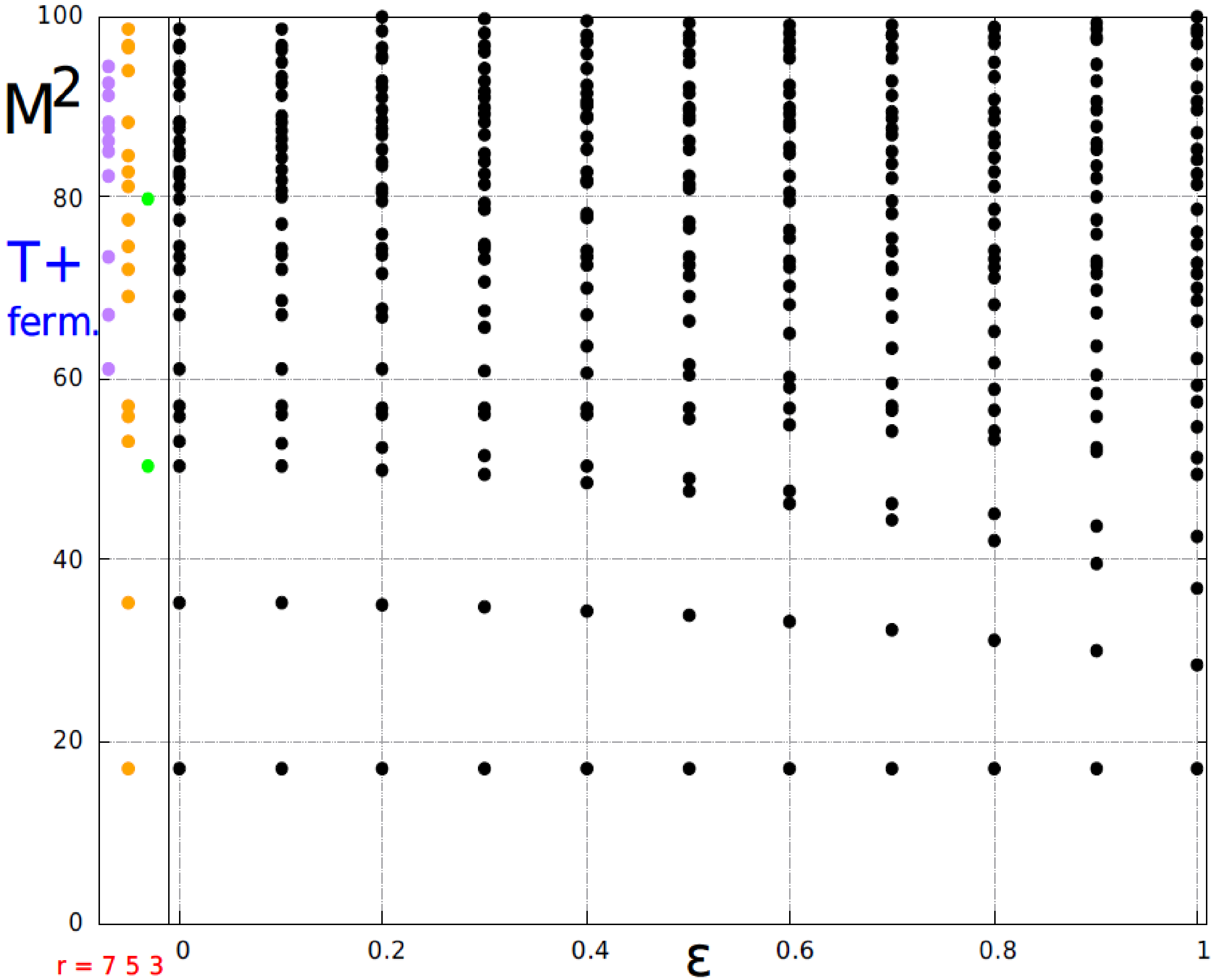
➔ Use in basis-function calculation

- Turn pair production back on
- Eventually integrate out parton momenta to diagonalize Hamiltonian matrix

$$H_{\mathbf{mn}} = <\mathrm{TIS};\, \boldsymbol{m} \,|\, \mathcal{H} \,|\, \mathrm{TIS};\, \boldsymbol{n} >$$

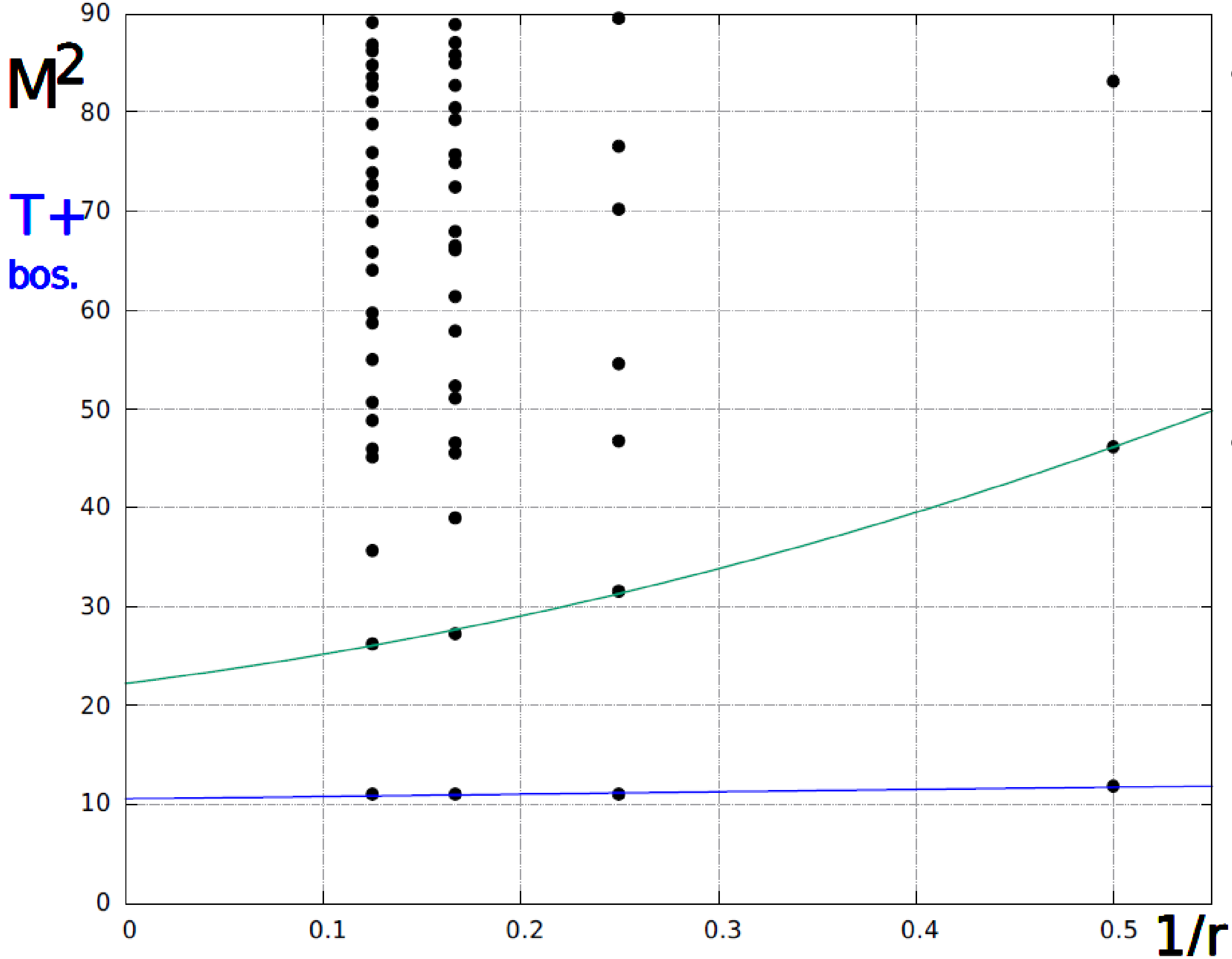
- For now: use discretized approach to avoid PV integrals

Mixing of Parton Sectors as Pair-Production is turned on



- Lowest states here are very pure 5-parton states (see above)
- Mixing changes bound state masses

Convergence with Parton Number



- Much slower than convergence with number of basis states or harmonic resolution K
- Trouble:
  - Limited to r < 9
  - State at $M^2 \approx 22$ likely a single-trace, multi-particle state

On to a continuous approach

- Replace sums over discretized momenta with integrals in $H_{mn} = \langle TIS; \boldsymbol{m} \,|\mathcal{H}|\, TIS; \boldsymbol{n} \rangle$
  - r=2,3 completed

- Advantages:
  - No numerical artifacts typical of extrapolations
  - Can split off most d.o.f.s from multidimensional integration; only 3 numerical integrations necessary – regardless of parton number r (5 for PNV term)
  - Might be able to separate and sum behavior as number of basis states and/or partons r becomes large

Conclusion/Outlook

- Full eLCQ spectrum of QCD_{2A} agrees with earlier work
- Unfortunately, degenerate multi-particle states still appear in all sectors
- Conclusion: appearance of MPS not due to method, but due to formulation of theory (fermions vs currents)
- Hope: continuum formulation may allow taking limits r, $N_\varphi \rightarrow \infty$, purging long (MPS) states
- Outlook: apply eLCQ to bosonized theory where (exact) MPS states are projected out a priori

Thanks for your attention!

- Questions?