Gravitational corrections to two-loop beta function in quantum electrodynamics

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Introduction

Although the Standard Model of particle physics is very successful in describing most of the fundamental interactions, it does not describe quantum gravity.

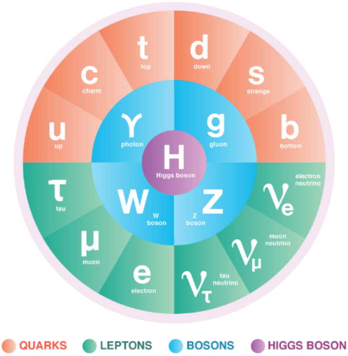

Figure 1: Particles of the SM

- A full theory of quantum gravity is nonrenormalizable;[1]
- However, we can treat gravity as an effective field theory.[2,3]

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \tag{1}$$

We also need to introduce higher derivatives terms in the action to take care of the divergences proportional to higher order in momentum coming from the gravitational interaction.
In this work, we will consider the one graviton exchange approximation (i.e. up to order κ^2) and we will use the Minimal Subtraction scheme. The results were obtained in an arbitrary gauge.

[1] G. 't Hooft and M. J. G. Veltman, Annales Poincare Phys. Theor. A **20**, 69 (1974).
[2] J. F. Donoghue, Phys. Rev. D **50**, 3874-3888 (1994).
[3] C. P. Burgess, Living. Rev. Rel. **7**, 5 (2004).

Renormalization group functions

- In 2005, Robinson and Wilczek suggested that gravity renders all gauge coupling constants asymptotically free;[4]
- The result was soon contested and showed to be gauge dependent;[5]
- The main goal of the present work is to compute the gravitational corrections to the two-loop beta function of the electrical charge of quantum electrodynamics.

[4] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **96**, 231601 (2006).
[5] A. R. Pietrykowski, Phys. Rev. Lett. **98**, 061801 (2007).

Einstein-QED model

The lagrangian we are working with is given by,

$$\mathcal{L} = \sqrt{-g}\Big\{\frac{2}{\kappa^2}R - \frac{1}{4}g^{\mu\nu}g^{\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + i\bar{\psi}(\nabla_\mu - ieA_\mu)\gamma^\mu\psi - m\bar{\psi}\psi\Big\} + \mathcal{L}_{HO} + \mathcal{L}_{GF} + \mathcal{L}_{CT}, \tag{2}$$

where the Dirac matrices are contracted with the vierbein ($\gamma^\mu \equiv \gamma^a e_a^\mu$), e is the electric charge and κ is the gravitational coupling ($\kappa^2 = 32\pi G = 32\pi/M_P^2$, with M_P being the Planck mass and G the Newtonian gravitational constant), $\mathcal{L}_{GF}$ is the gauge-fixing plus Faddeev-Popov ghost Lagrangian (for both graviton and photon), $\mathcal{L}_{CT}$ is the Lagrangian of counterterms and $\mathcal{L}_{HO}$ is the Lagrangian of higher derivative terms.

We expand the metric around the flat metric as,

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \text{ and } \sqrt{-g} = 1 + \frac{\kappa}{2} h + \cdots \,. \tag{3}$$

We also define the renormalized fields,

$$\psi_0 = Z_2^{1/2} \psi \text{ and } A_{0\mu} = Z_3^{1/2} A_\mu, \tag{4}$$

where,

$$Z = 1 + Z^{(1)} + Z^{(2)} + \cdots \tag{5}$$

and,

$$Z_2 Z_3^{1/2} e_0 = \mu^{2\epsilon} Z_1 e \tag{6}$$

Using the Ward-Takeshi identity,

$$e = \mu^{-2\epsilon} Z_3^{1/2} e_0. \tag{7}$$

Therefore, we need to compute only Z_3 to find the β function since,

$$\beta(e) = \mu \frac{de}{d\mu}. \tag{8}$$

To do so, we have used four Mathematica packages: FeynRules[6], FeynArts[7], FeynCalc[8] and TARCER[9].

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[6] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, Comput. Phys. Commun. **185**, 2250-2300 (2014) doi:10.1016/j.cpc.2014.04.012 [arXiv:1310.1921 [hep-ph]].

[7] Thomas Hahn, Comp. Phys. Comm. **140**, 418 (2001).

[8] R. Mertig and M. Bahm and A. Denne, Comp. Phys. Comm. **64**, 345 (1991).

[9] R. Mertig and R. Scharf, Comput. Phys. Commun. **111**, 265-273 (1998) doi:10.1016/S0010-4655(98)00042-3 [arXiv:hep-ph/9801383 [hep-ph]].

First, at one-loop order, we need to compute the following diagrams,

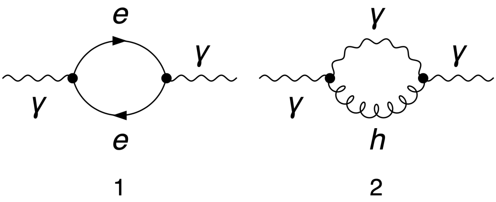

Figure 2: One-loop corrections to the photon self-energy. Continuous, wavy and wiggly lines represent the electron, photon and graviton propagators, respectively.

Using gauge invariance we can write the photon self-energy as,

$$\Pi^{\mu\nu}(p) = (p^2\eta^{\mu\nu} - p^\mu p^\nu)\Pi(p). \tag{9}$$

Then,

$$\Pi(p) = Z_3^{(1)} + \tilde{Z}_3^{(1)} p^2 + \frac{\left(8e^2 + \kappa^2 p^2\right)}{96\pi^2\epsilon} + \textit{finite}, \tag{10}$$

where $\tilde{Z}_3$ is the counterterm coming from the higher order term $F^{\mu\nu}\Box F_{\mu\nu}$ and it renormalizes the term proportional to p^2, while Z_3 renormalizes the Maxwell term,

$$Z_3^{(1)} = -\frac{e^2}{12\pi^2\epsilon}; \tag{11a}$$

$$\tilde{Z}_3^{(1)} = -\frac{\kappa^2}{96\pi^2\epsilon}. \tag{11b}$$

Therefore, at one-loop order, there is no gravitational corrections to the β function of the coupling constant.

At two-loops order, the diagrams are,

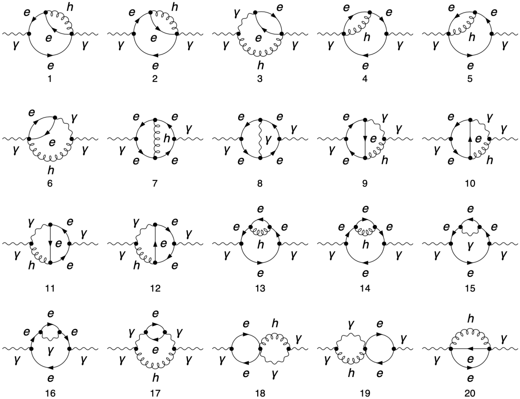

Figure 3: Two-loop corrections to the photon self-energy.

Proceding in the same manner as we did at one-loop, we have that the β function at two-loops order is given by

$$\beta(e) = \frac{e^2}{48\pi^2} + \frac{e^4}{128\pi^4} + \frac{e^2\kappa^2 m^2}{256\pi^4}. \tag{12}$$

From the above results, we can see that gravitational corrections do not render the coupling constant asymptotically free, on the contrary, it makes the electrical charge grow faster.

The Einstein-Scalar-QED model

We now consider the model described by

$$\mathcal{L} = \sqrt{-g}\Big\{\frac{2}{\kappa^2}R - \frac{1}{4}g^{\mu\nu}g^{\alpha\beta}F_{\mu\nu}F_{\alpha\beta} - g^{\mu\nu}(D_\mu\phi)^\dagger D_\nu\phi - m^2\phi^\dagger\phi - \frac{1}{4}\lambda(\phi^\dagger\phi)^2\Big\} + \mathcal{L}_{HO} + \mathcal{L}_{GF} + \mathcal{L}_{CT} \tag{13}$$

where $D_\mu = \partial_\mu - ieA_\mu$ is the covariant derivative, and we have an additional coupling constant, λ, for the self-interaction of the scalar field. Now, we expand the metric around the flat metric as done before.

The one-loop photon self-energy diagrams are,

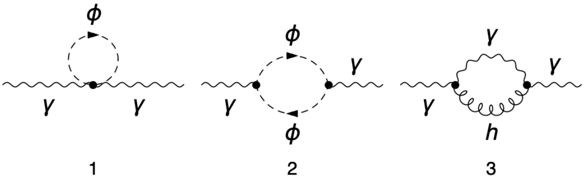

Figure 4: The one-loop photon polarization tensor in the Scalar QED coupled to gravity.

Computing then, we find,

$$\Pi(p) \quad = \quad Z_3^{(1)} + \tilde{Z}_3^{(1)} + \frac{e^2}{48\pi^2\varepsilon} + \frac{\kappa^2 p^2}{96\pi^2\varepsilon} + \text{finite}, \qquad (14)$$

where we have used gauge invariance to write $\Pi^{\mu\nu} = (p^2\eta^{\mu\nu} - p^\mu p^\nu)\Pi(p)$.

Therefore,

$$Z_3^{(1)} = -\frac{\mathrm{e}^2}{48\pi^2\varepsilon}, \tag{15}$$

$$\tilde{Z}_3^{(1)} = -\frac{\kappa^2}{96\pi^2\varepsilon}. \tag{16}$$

Now, at two-loops order we have

$$\begin{aligned} Z_3 &= 1 + Z_3^{(1)} + Z_3^{(2)} + \cdots \\ &= 1 - \frac{\mathrm{e}^2}{48\pi^2\varepsilon} - \frac{\mathrm{e}^4}{128\pi^4\varepsilon} - \frac{\mathrm{e}^2\kappa^2 m^2}{256\pi^4\varepsilon} + \cdots \end{aligned} \tag{17}$$

Then,

$$\beta(e) = \frac{e^2}{48\pi^2} + \frac{e^4}{128\pi^4} + \frac{e^2\kappa^2 m^2}{256\pi^4} \tag{18}$$

.

Conclusion

- In this work, we computed the two-loop corrections to the photon self-energy in the Einstein-QED and in the Einstein-Scalar-QED models;
- We used our results to study the behavior of the β function and notice that gravitational corrections do not induces asymptotically freedom;

Acknowledgement

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